Basic naming conventions

Let's start with the naming conventions of basic notions.

A *function* f, from the set E to F, denoted $f : E \to F$ maps objects $x \in E$ to objects $y \in F$, as $y = f(x)$.
Its *domain* $\mathcal{D}_f = E$ is the set of objects onto which it is defined.
Objects of its domain $\mathcal{D}_f$ are mapped to objects of its *codomain* $\mathcal{D}^c_f = F$.
We say that f is *taking values* in its codomain.
The *image per* f of the subset $U \subset E$, denoted $f(U)$, is $\{y \in F, \exists x \in E, y = f(x)\}$.
The *image of* f is the image of its domain. We denote $\mathcal{I}_f$.
The *fiber* of the object $y \in \mathcal{I}_f$ is the object $x \in E$ such that $y = f(x)$.
The *inverse image per* f of the subset $V \subset F$, denoted $f^{-1}(V)$ is $\{x \in E, \exists y \in F, y = f(x)\}$.

A finite dimensional vector space E is defined as $\mathbb{R}^n$, and is equipped with pointwise addition and scalar multiplication.

Signals and datasets

A *signal* s is a function taking values in a finite dimensional vector space. A *dataset* is a finite set of signals. A dataset is said to be *static* if its signals are defined on the same domain, it is said to be *non-static* otherwise.

For examples, images are signals defined on a set of pixels. Typically, an image s in RGB representation is a mapping from pixels p to a 3d vector space, as $s : p \mapsto (r, g, b)$. Image datasets used in practice are usually static. Non-static would mean that there are images of different sizes and/or different scales; in which case, they are usually rescaled and/or padded with zeros.

A *graph* $G = (V, E)$ is defined as a set of vertices V, and a set of edges $E \subseteq \binom{V}{2}$. A *graph signal* is a signal defined on the vertices of a graph.

Regular and irregular domains

In this subsection, we are going to redefine the notion of *regularity* of a function's domain relatively to the context of deep learning and convolutional representations.

Definition 1 *Regular Domain*

Definition 2 *Irregular Domain*

Invariance

Definition 3

In order to be observed, invariances must be defined relatively to an observation. Let's give a formal definition to support our discussion.

Definition 4 *A function* f :