EE 357 – Communication Systems
Laboratory Session

FM Demodulation

A frequency modulated passband signal can be written as

$$x(t) = A_c \cos\left(2\pi f_c t + 2\pi f_\Delta \int_0^t m(\tau)d\tau\right)$$

where A_c and f_c are the carrier amplitude and carrier frequency, $m(\tau)$ is the baseband input signal and f_Δ is the frequency deviation in Hz.

In order to down convert the passband signal into the baseband we perform

$$y_i(t) = x(t)\cos(2\pi f_c t)$$

$$y_q(t) = x(t)\sin(2\pi f_c t)$$

Next, a low pass filter can be employed to extract the down-converted term and filter out the up-converted term in $y_i(t)$ and $y_q(t)$. Let the outputs of the low pass filters be written as $\tilde{y}_i(t)$ and $\tilde{y}_q(t)$.

We now form the complex FM baseband signal as:

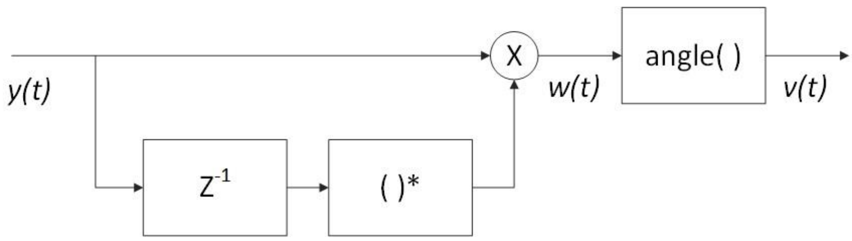
$$y(t) = \tilde{y}_i(t) + j\tilde{y}_q(t)$$

or

$$y(t) = \frac{A_c}{2} e^{j2\pi f_\Delta \int_0^t m(\tau)d\tau} = \frac{A_c}{2} e^{j\phi(t)}$$

with $\phi(t) = 2\pi f_\Delta \int_0^t m(\tau)d\tau$ which shows that the baseband input signal is a scaled version of the time derivate of $\phi(t)$.

A baseband delay FM demodulator is used to recover the message signal from $y(t)$.



A delayed and conjugated copy of the received signal is subtracted from the signal itself,

$$w(t) = \frac{A_c^2}{4} e^{j\phi(t)} e^{-j\phi(t-T)} = \frac{A_c^2}{4} e^{j(\phi(t)-\phi(t-T))}$$

where T is the sample period. In discrete time, $w_n = w(nT)$ and

$$w_n = \frac{A_c^2}{4} e^{j(\phi_n - \phi_{n-1})}$$

$$v_n = \phi_n - \phi_{n-1}$$

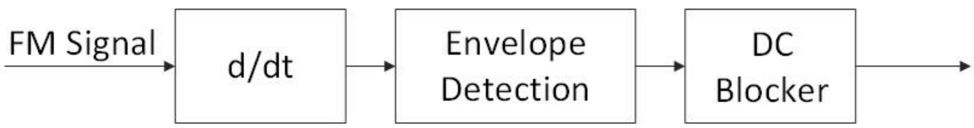
The signal v_n is the approximate derivative of ϕ_n so that $v_n \approx x_n$.

Demodulation using Differentiation

The differentiated FM signal can be written as

$$\frac{d\,x(t)}{dt} = -A_c(2\pi f_c + 2\pi f_\Delta m(t)) \sin\left(2\pi f_c t + 2\pi f_\Delta \int_0^t m(\tau) d\tau\right)$$

The envelope of the differentiated FM signal is linearly related to the input message signal. Hence, $x(t)$ can be recovered by an envelope detection of $\frac{d\,x(t)}{dt}$.

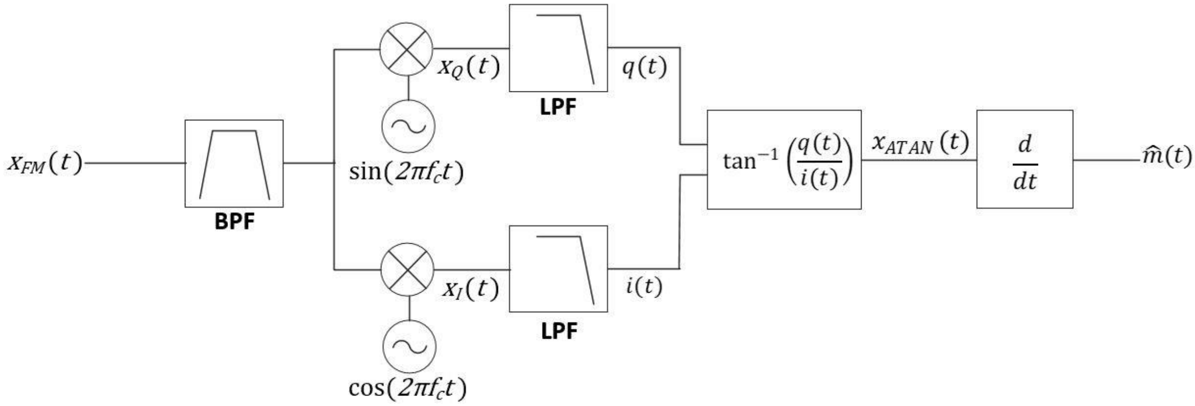


Exercise:

1 Write a Matlab program to implement the baseband delay FM demodulator. Assume a 1 kHz sinusoidal message signal and a suitable carrier signal. Plot the message signal, FM signal and the demodulated message signal.

2 Now consider the FM demodulation using differentiation method. Write a Matlab program and demodulate (a) 1 kHz sinusoidal message signal and (b) a 1 kHz rectangular pulse train with a pulse width of $1/2$ millisecond as the message signal. Assume a suitable carrier signal.

FM Arctangent Demodulator



$$i(t) = \frac{A_c}{2}\cos\left(2\pi f_\Delta \int_0^t m(\tau)d\tau\right)$$

$$q(t) = \frac{A_c}{2}\sin\left(2\pi f_\Delta \int_0^t m(\tau)d\tau\right)$$

Hence $x_{ATAN}(t)$ can be written as

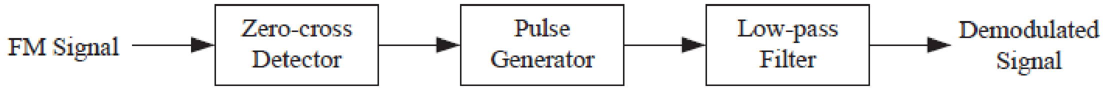
$$\begin{aligned} x_{ATAN}(t) &= \tan^{-1}\left(\frac{i(t)}{q(t)}\right) \\ &= \tan^{-1}\left(\tan\left(2\pi f_\Delta \int_0^t m(\tau)d\tau\right)\right) \\ &= 2\pi f_\Delta \int_0^t m(\tau)d\tau \end{aligned}$$

Therefore, differentiating $\mathrm{x_{ATAN}(t)}$ we can recover the message signal, $\mathrm{m(t)}$. Note that the above demodulator requires implementing the ***four-quadrant inverse tangent*** operation (in Matlab use **atan2** function)

Exercise:

Write a Matlab program to implement the FM arctangent demodulator. Assume a 1 kHz sinusoidal message signal and a suitable carrier signal. Plot the message signal, FM signal and the demodulated message signal. Use another message and repeat the steps as above.

FM Zero-Crossing Demodulation



The *zero-cross detector* is used to find the positive zero-crossing points. Specifically, when the amplitude of the input signal changes from negative to positive values, the zero-cross detector generates an impulse. Next, the *pulse generator* converts the impulses into a pulse train (width τ and amplitude A).

If the instantaneous frequency of the FM signal is

$$f_i = f_c + \Delta f . m(t)$$

And

$$T = \frac{1}{f_i}$$

The output of the lowpass filter can be written as

$$A\frac{\tau}{T} = A\tau(f_c + \Delta f . m(t)) = A\tau f_c + A\tau\Delta f . m(t)$$

Hence the dc value is $A\tau f_c$ and $A\tau\Delta f m(t)$ is the demodulated message signal.

Exercise

Assume that a 25 Hz sine wave is used as the message signal and the carrier signal is a 300 Hz sinusoidal signal. Set the maximum frequency deviation Δf to 20 Hz.

Plot (a) message signal (b) FM signal (c) zero-crossing points (d) the pulse train and (d) the demodulated message signal in the time interval $[0 \text{ to } 0.5]$ seconds.