

Seeing is Believing

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A sum identity

Theorem

Let $n \in \mathbb{N}$. Then

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$



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Proof.

Induct on n .

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k^2 + k) + (2k + 2)}{2} = \frac{(k+1)(k+2)}{2}.$$



The question

Why should you expect the formula to look like this?



Consternation

- ▶ Bothered me a lot in high school
 - ▶ Teachers: “because the induction works”
 - ▶ Eventually gave up on a deeper perspective
- ▶ College
 - ▶ Didn’t question the inductive proof
- ▶ Winter break



The challenge

'I think that I just don't work well in abstraction. I care about the tangible, and things that I can see.' My Neighbor



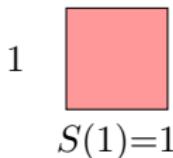
Desired explanation

- ▶ Brief
- ▶ Accessible
- ▶ Visualizable



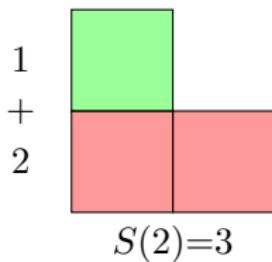
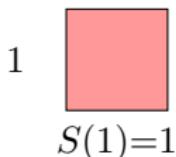
Drawing a picture

Partial sums

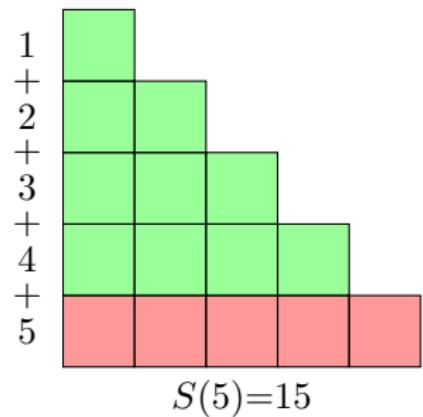
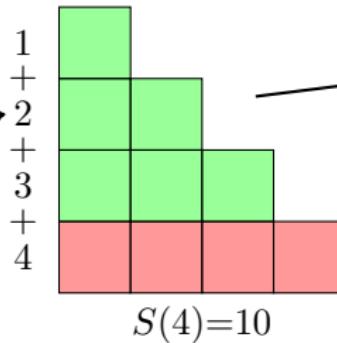
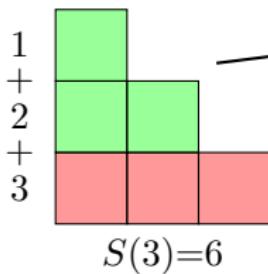


Drawing a picture

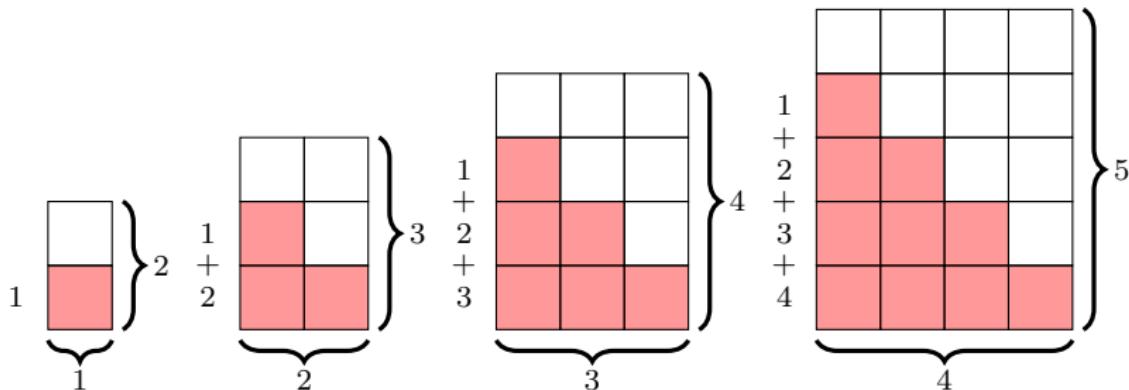
Partial sums



Continuing



Calculating the shaded area



Challenge

*'Ok, that's pretty cool. But can you do it with Calculus? Because then I'd be **really** impressed. I didn't understand **any** of Calculus, and I mean it.'*

My Neighbor



Starting simple

Theorem

Let $n \in \mathbb{N}$. Then

$$\frac{dx^n}{dx} = nx^{n-1}.$$

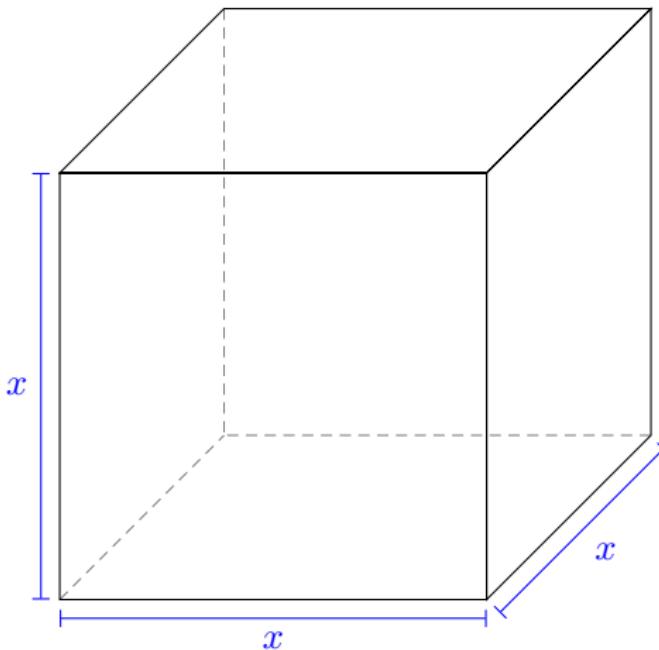
Proof.

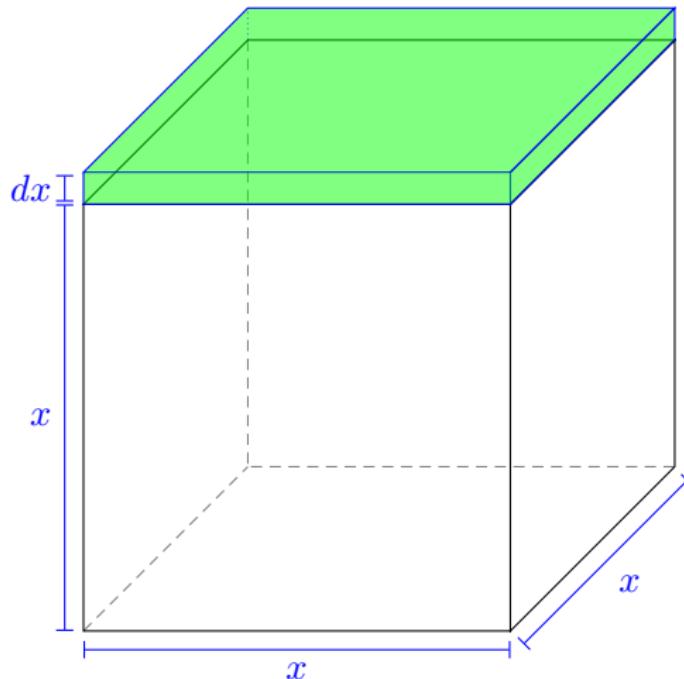
Take the difference quotient and apply the identity

$$(x^n - y^n) = (x - y)(x^{n-1} + x^{n-2}y + \cdots + y^{n-1})$$

□



Special case: $n = 3$ 

Figure: Extending one side by dx 

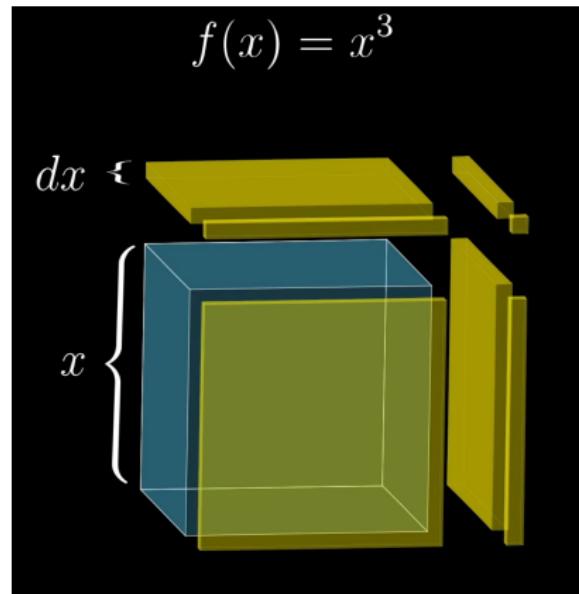


Figure: 3blue1brown's visualization ([3Bl17])



A final quote



References



3Blue1Brown.

Derivative formulas through geometry | Essence of calculus,
chapter 3, Apr 2017.

[Online; accessed 17. Feb. 2019].

