Math 80, Spring 2019

Prof: Pippenger

Name: **Your Name Here!**

HW #: **1**

Day: **Mon.** **Tue.** **Wed.** **Thu.** **Fri.**

Date: **3/27/2018**

No.	Points	Acknowledgments
1		
2		
3		
4		
Total		

This Assignment is (check one):

☒ On Time ☐ Late, without deduction ☐ Late, with deduction

For Homework assignments, you may work with others, but you must write up the solution you submit yourself, and it must reflect your own understanding of the solution. You may use any resources (books, papers, the Internet) in working on homework assignments, but in accordance with scholarly conventions, you should acknowledge any sources you benefit from.

For homework assignments, in solving a particular problem or part of a problem, you may use results stated in any previous problem or parts of problems (in the same or earlier assignments), even if you have not solved those previous problems.

Problem 1.

Suppose $y_1(x)$ and $y_2(x)$ are two solutions of

$$y''(x) + P(x)y'(x) + Q(x)y(x) = 0.$$

Show that if the Wronskian

$$W(x) = \det \begin{pmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{pmatrix}$$

does not vanish for $x = 0$, then it does not vanish for any value of x. (Hint: Show that $W'(x) + P(x)W(x) = 0$).

Solution:

Problem 2.

For the wave equation

$$\frac{\partial^2}{\partial r^2}G(r,\theta,t) + \frac{1}{r}\frac{\partial}{\partial r}G(r,\theta,t) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}G(r,\theta,t) - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}G(r,\theta,t) = 0$$

in polar coordinates, look for a solution $G(r,\theta,t) = R(r)\Theta(\theta)T(t)$ that is not necessarily radially symmetric. Find the solutions for $T(t)$ and $\Theta(\theta)$, and express $R(r)$ in terms of the solutions of the equation

$$y''(s) + \frac{1}{s}y'(s) + \left(1 - \frac{n^2}{s^2}\right)y(x) = 0$$

which is known as "Bessel's equation of order n." Explain why n must be an integer.

Solution:

Problem 3.

Find two linearly independent power-series solutions of

$$y''(x) + xy'(x) + y(x) = 0$$

Solution:

Problem 4.

Find two linearly independent power-series solutions of

$$(1 - x^2)y''(x) - xy'(x) + p^2y(x) = 0$$

where p is a constant. Show that if p is a non-negative integer, there is a solution that is a polynomial of degree p.

Solution: