Reference

Thursday, May 2, 2024 11:08 PM

1. https://www.rareskills.io/post/encrypted-polynomial-evaluation
2. https://www.rareskills.io/post/elliptic-curve-qap
3. https://www.rareskills.io/post/groth16

Notes

Thursday, June 20, 2024 10:32 AM

Trusted Setup (powers of tau)

Recall:
-> Problem statement
-> R1CS (L, R, O matrices)
-> Lagrange Interpolation (U, V, W array of polys)
-> Evaluate QAP at a random point (guaranteed by Schwartz-Zippel)

$\uparrow$

Need trusted setup

But why ?

Suppose random point tau is publicly known, QAP formula becomes:

Q: how to generate a fake proof if tau is leaked

$$((U\cdot a)(\tau))\,((V\cdot a)(\tau)) = (W(\tau)\cdot a) + h(\tau)\,t(\tau)$$

Prover can come up with fake numbers that still satisfy the equality, therefore forge fake proof and cheat verifier.

Trusted setup computes and publishes "encryptions" of tau, called powers of tau:

$$(\tau^0 G_1, \tau^1 G_1, \tau^2 G_1, \tau^3 G_1, \cdots, \tau^6 G_1)$$
$$(\tau^0 G_2, \tau^1 G_2, \tau^2 G_2, \tau^3 G_2, \cdots, \tau^6 G_2)$$

Prover computes evaluation of polys in QAP according to powers of tau. For example, U * a can be computed as:

$[\ ]_1$ means G_1 point

(either G_1 or G_2)

$\uparrow \quad \uparrow$

$[\ ]_1 \quad [\ ]_2$

$$[A]_1 = [(\underbrace{U\cdot a})(\tau)]_1 = [(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)(\tau)]_1$$

inner product, still poly

$$= [C_n\tau^n + C_{n-1}\tau^{n-1} + \cdots + C_1\tau + C_0]_1$$

EC points addition $= [C_n\tau^n]_1 + [C_{n-1}\tau^{n-1}]_1 + \cdots + [C_1\tau]_1 + [C_0]_1$

EC point scalar multiplication $= C_n\underline{(\tau^n G_1)} + C_{n-1}\underline{(\tau^{n-1}G_1)} + \cdots + C_1\underline{(\tau G_1)} + C_0(G_1)$

$\uparrow \quad \uparrow \quad \uparrow$

Come from trusted setup

$$= (\cdots, \cdots)$$

Compute $[B]_2 = [(V\cdot a)(\tau)]_2 = ((\cdots,\cdots), (\cdots,\cdots))$

$$[C']_1 = [(W \cdot a)(\tau)]_1 = (\cdots, \cdots)$$

$$\underline{[HT]_1 = [(h(x)t(x))(\tau)]_1} = (\cdots, \cdots)$$

not that easy, will discuss later

$$[C]_1 = [C']_1 + [HT]_1 = (\cdots, \cdots)$$

Verifier verifies if:

pretty bad

at this moment

$$\text{pairing}(\underset{G_1 \text{ point}}{[A]_1}, \underset{G_2 \text{ point}}{[B]_2}) = \text{pairing}(\underset{G_1 \text{ point}}{[C]_1}, G_2)$$

Issue: evaluating h(x)t(x)

When we evaluate h(tau)t(tau), we are "multiplying" two G1 points, which will introduce a pairing -> unwanted

(But why not compute h(x) * t(x) first and evaluate ht(tau)?
I think h(x) * t(x) computation is unwanted too because it is expensive)

Solution: embed t(tau) in another powers of tau

$$(\tau^0 t(\tau)G_1, \tau^1 t(\tau)G_1, \tau^2 t(\tau)G_1, \cdots, \tau^5 t(\tau)G_1) \rightarrow \text{3rd powers of tau generated by trusted setup}$$

Then the computation of h(tau)t(tau) by prover becomes:

say $t(x) = x-1$ $\quad \tau = 5$

$h(x) = 2x+1$

Q: what if I evaluate h(tau)t(tau) directly without doing the extra powers of tau

$$h(\tau)t(\tau) = 11 \cdot 4 = 44$$

$$(h(x) \cdot t(\tau))(\tau) = (2x+1) \cdot 4$$

$$= 8x+4$$

$$= 40+4 = 44$$

$$[h(\tau)t(\tau)]_1 = [ht(\tau)]_1$$

$$= [(h(x) \cdot t(\tau))(\tau)]_1$$

$$= [((h_0 + h_1x + h_2x^2 + \cdots + h_nx^n) \cdot t(\tau))(\tau)]_1$$

$$= [h_0t(\tau) + h_1t(\tau)x + h_2t(\tau)x^2 + \cdots + h_nt(\tau)x^n)(\tau)]_1$$

$$= [h_0t(\tau) + h_1\tau t(\tau) + h_2\tau^2 t(\tau) + \cdots + h_n\tau^n t(\tau)]_1$$

EC addition

$$= [h_0t(\tau)]_1 + [h_1\tau t(\tau)]_1 + [h_2\tau^2 t(\tau)]_1 + \cdots + [h_n\tau^n t(\tau)]_1$$

EC scalar multiplication

$$= h_0\underline{(t(\tau)G_1)} + h_1\underline{(\tau t(\tau)G_1)} + h_2\underline{(\tau^2 t(\tau)G_1)} + \cdots + h_n\underline{(\tau^n t(\tau)G_1)}$$

Come from trusted setup

Implementation of powers of tau

```python
# polynomial degree is 6

# Powers of tau for A
def generate_powers_of_tau_G1(tau):
    return [multiply(G1, int(tau ** i)) for i in range(t.degree)] # up to tau**6
```

```python
# Powers of tau for B
def generate_powers_of_tau_G2(tau):
    return [multiply(G2, int(tau ** i)) for i in range(t.degree)] # up to tau**6
```

```python
# Powers of tau for h(tau)t(tau)
def generate_powers_of_tau_HT(tau):
    before_delta_inverse = [multiply(G1, int(tau ** i * t_evaluated_at_tau)) for i in range(t.degree - 1)] # up to tau**5
    return [multiply(entry, int(delta_inverse)) for entry in before_delta_inverse]
```

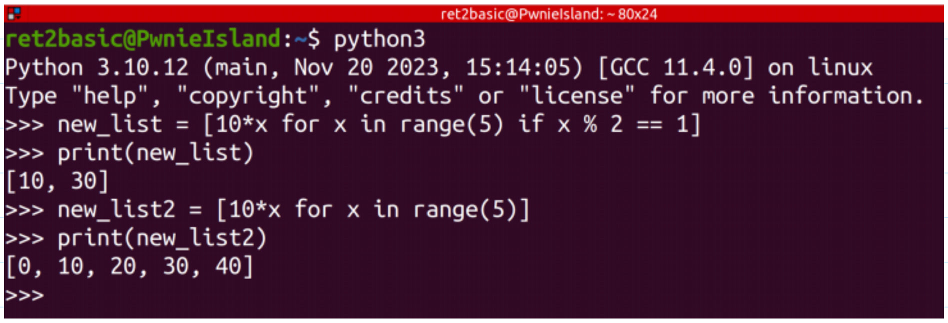
Q: why the 3rd powers of tau has 1 less term?

Python list comprehension
https://realpython.com/list-comprehension-python/

Syntax:
new_list = [expression for member in iterable <u>if conditional</u>]

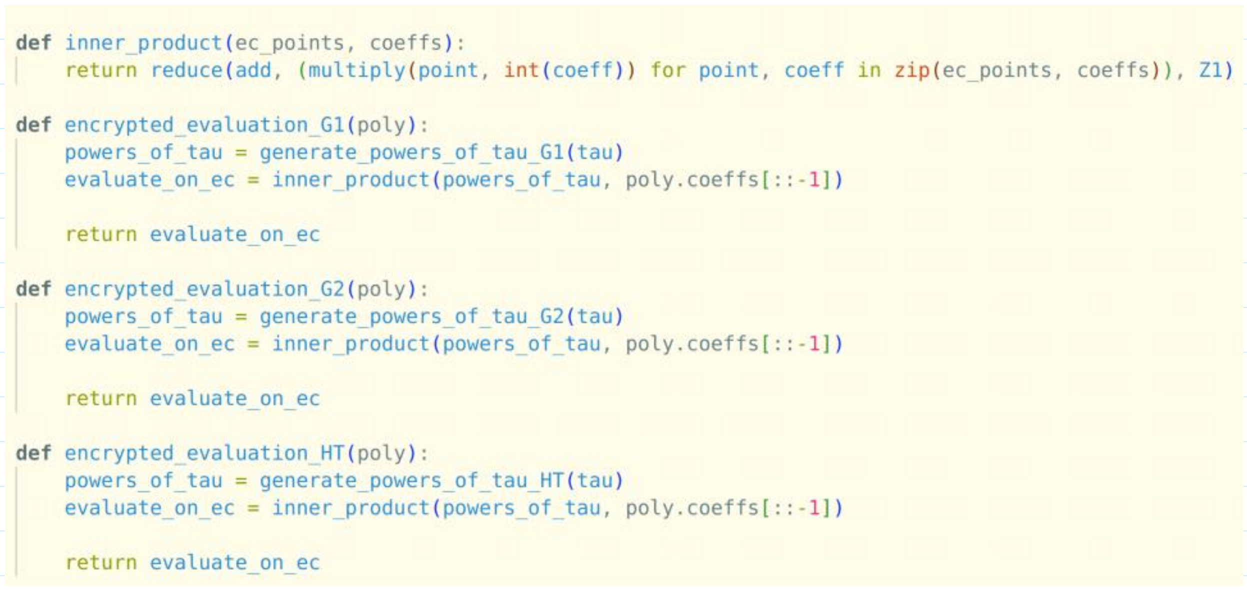
optional

Example:



```
ret2basic@PwnieIsland:~$ python3
Python 3.10.12 (main, Nov 20 2023, 15:14:05) [GCC 11.4.0] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> new_list = [10*x for x in range(5) if x % 2 == 1]
>>> print(new_list)
[10, 30]
>>> new_list2 = [10*x for x in range(5)]
>>> print(new_list2)
[0, 10, 20, 30, 40]
>>>
```

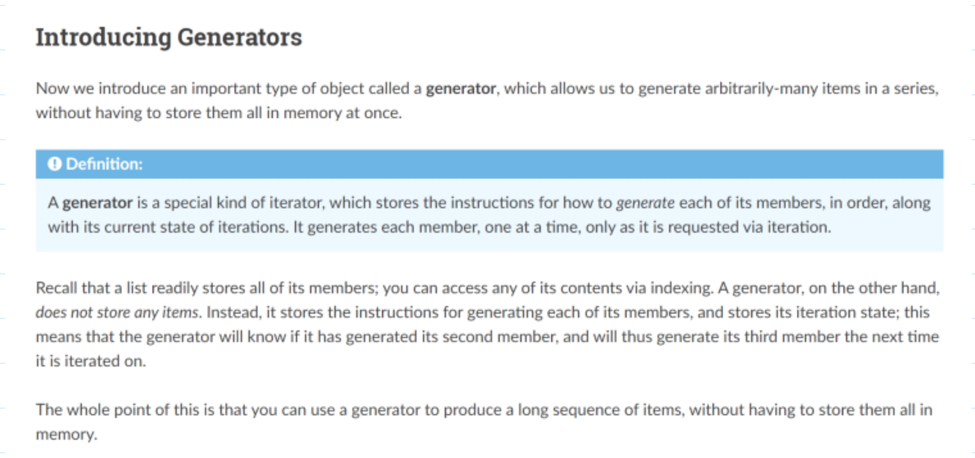
Implementation for encrypted poly evaluation:



```python
def inner_product(ec_points, coeffs):
    return reduce(add, (multiply(point, int(coeff)) for point, coeff in zip(ec_points, coeffs)), Z1)

def encrypted_evaluation_G1(poly):
    powers_of_tau = generate_powers_of_tau_G1(tau)
    evaluate_on_ec = inner_product(powers_of_tau, poly.coeffs[::-1])

    return evaluate_on_ec

def encrypted_evaluation_G2(poly):
    powers_of_tau = generate_powers_of_tau_G2(tau)
    evaluate_on_ec = inner_product(powers_of_tau, poly.coeffs[::-1])

    return evaluate_on_ec

def encrypted_evaluation_HT(poly):
    powers_of_tau = generate_powers_of_tau_HT(tau)
    evaluate_on_ec = inner_product(powers_of_tau, poly.coeffs[::-1])

    return evaluate_on_ec
```

Python generator comprehension

https://www.pythonlikeyoumeanit.com/Module2_EssentialsOfPython/Generators_and_Comprehensions.html



Introducing Generators

Now we introduce an important type of object called a **generator**, which allows us to generate arbitrarily-many items in a series, without having to store them all in memory at once.

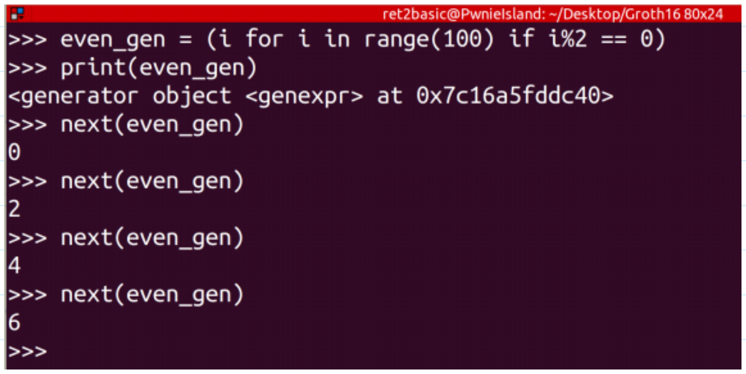
Definition:

A **generator** is a special kind of iterator, which stores the instructions for how to *generate* each of its members, in order, along with its current state of iterations. It generates each member, one at a time, only as it is requested via iteration.

Recall that a list readily stores all of its members; you can access any of its contents via indexing. A generator, on the other hand, *does not store any items*. Instead, it stores the instructions for generating each of its members, and stores its iteration state; this means that the generator will know if it has generated its second member, and will thus generate its third member the next time it is iterated on.

The whole point of this is that you can use a generator to produce a long sequence of items, without having to store them all in memory.

Example:

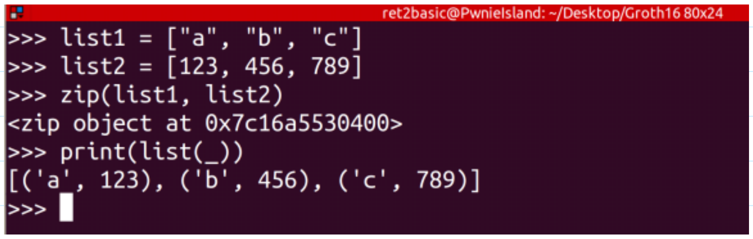


ret2basic@PwnieIsland: ~/Desktop/Groth16 80x24

```
>>> even_gen = (i for i in range(100) if i%2 == 0)
>>> print(even_gen)
<generator object <genexpr> at 0x7c16a5fddc40>
>>> next(even_gen)
0
>>> next(even_gen)
2
>>> next(even_gen)
4
>>> next(even_gen)
6
>>>
```

Python zip() -> return the cartesian product of two iterators

Example:


ret2basic@PwnieIsland: ~/Desktop/Groth16 80x24

```
>>> list1 = ["a", "b", "c"]
>>> list2 = [123, 456, 789]
>>> zip(list1, list2)
<zip object at 0x7c16a5530400>
>>> print(list(_))
[('a', 123), ('b', 456), ('c', 789)]
>>> 
```

py_ecc.bn128.Z1 -> point at infinity over FQ