Reference

Wednesday, June 5, 2024 10:29 PM

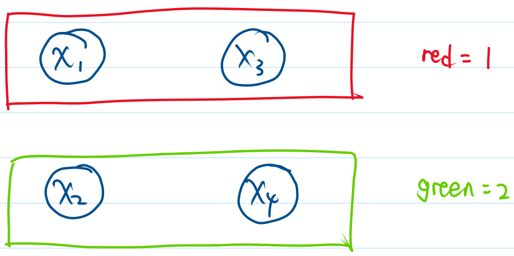
1. https://www.rareskills.io/zk-book
2. https://www.rareskills.io/post/circom-tutorial
3. https://docs.circom.io/circom-language/basic-operators/

Notes

Friday, April 26, 2024 1:21 PM

Problem statement

Given vertices (x_1, x_2, x_3, x_4). prove the graph is bipartite.



The solution to this problem is a "witness vector":

$$a = [1, \underbrace{x_1, x_2, x_3, x_4}_{\text{how to "2-color" this graph (private input)}}]$$

$\uparrow$ Constant term (public input)

- ➢ Verifier has access to public inputs
- ➢ Prover has access to both public and private inputs
- ➢ Intermediate signals can exist but we don't need them for this specific problem.

Arithmetic circuit

1. Each vertex should have color 1 or 2

$$(x_1 - 1)(x_1 - 2) = 0 \rightarrow x_1^2 - 3x_1 + 2 = 0$$
$$(x_2 - 1)(x_2 - 2) = 0 \rightarrow x_2^2 - 3x_2 + 2 = 0$$
$$(x_3 - 1)(x_3 - 2) = 0 \rightarrow x_3^2 - 3x_3 + 2 = 0$$
$$(x_4 - 1)(x_4 - 2) = 0 \rightarrow x_4^2 - 3x_4 + 2 = 0$$

2. Vertices from different "groups" should have different colors:

$$x_1 x_2 - 2 = 0 \quad ①$$
$$x_1 x_4 - 2 = 0$$
$$x_2 x_3 - 2 = 0 \quad ②$$

($x_1 x_3 - 2 = 0$ is implied by ① and ② implicitly)

R1CS

Turn arithmetic circuits into system of quadratic equations (constraints).

1. Each vertex should have color 1 or 2

$x_1 x_1 = 3x_1 - 2$

$x_2 x_2 = 3x_2 - 2$

$x_3 x_3 = 3x_3 - 2$

$x_4 x_4 = 3x_4 - 2$

2. Vertices from different "groups" should have different colors:

$x_1 x_2 = 2$

$x_1 x_4 = 2$

$x_2 x_3 = 2$

Turn system of equations into matrix form

Recall that witness vector $a = [1, x_1, x_2, x_3, x_4]$

We turn systems of equation into $L \cdot a \circ R \cdot a = O \cdot a$

$\uparrow$ Hadamard product

where L, R, O are 3 matrices with same dimension:

- # rows == # Constraints
- # columns == dimension of witness vector

Begin transformation:

<u>Constraint 1:</u> $x_1 x_1 = 3x_1 - 2$

$$\Rightarrow \underset{L}{(x_1)} \cdot \underset{R}{(x_1)} = \underset{O}{\underbrace{3x_1 - 2}}$$

$$\underset{L}{\begin{array}{c} \begin{matrix} 1 & x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ & & \text{don't know yet} & & \end{bmatrix} \end{array}} \cdot \underset{a}{\begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}} \circ \underset{R}{\begin{array}{c} \begin{matrix} 1 & x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ & & \text{don't know yet} & & \end{bmatrix} \end{array}} \cdot \underset{a}{\begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}} = \underset{O}{\begin{array}{c} \begin{matrix} 1 & x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} -2 & 3 & 0 & 0 & 0 \\ & & \text{don't know yet} & & \end{bmatrix} \end{array}} \cdot \underset{a}{\begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}$$

Check if this transformation is done correctly:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \circ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \end{bmatrix} \circ \begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} -2+3x_1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \cdot x_1 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2 \end{bmatrix}$$

Repeat this process for each constraint in the system of equation, or equivalently, for each row in the matrices. In the end this is what we get:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot a \circ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \cdot a = \begin{bmatrix} -2 & 3 & 0 & 0 & 0 \\ -2 & 0 & 3 & 0 & 0 \\ -2 & 0 & 0 & 3 & 0 \\ -2 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot a$$

Implementing R1CS in Python

groth16.py ×

groth16.py > ...

```python
import numpy as np
import galois
from functools import reduce
from py_ecc.bn128 import G1, G2, multiply, add, curve_order, Z1, pairing, neg, final_exponentiate, FQ12

# curve_order = 1151
GF = galois.GF(curve_order) # we work with bn128/bn254 curve

```

galois library handles modular arithmetic over scalar field:

```
>>> import galois
>>> curve_order = 17
>>> GF = galois.GF(curve_order)
>>> operand1 = GF(6)
>>> operand2 = GF(14)
>>> operand1 + operand2
GF(3, order=17)
>>> int(_)
3
>>> operand1 * operand2
GF(16, order=17)
>>> int(_)
16
>>>
```

_ = return value from previous computation

operand1/operand2 is equivalent to operand1 * pow(operand2, -1, curve_order).

```
>>> operand1 - operand2
GF(9, order=17)
>>> int(_)
9
>>> operand1 / operand2
GF(15, order=17)
>>> int(_)
15
>>>
```

galois can also handle matrices and polynomials:

```
ret2basic@PwnieIsland: ~ 80x24
>>> import numpy as np
>>> np.array([[1,0,0],[0,1,0],[0,0,1]])
array([[1, 0, 0],
       [0, 1, 0],
       [0, 0, 1]])
>>> GF(_)
GF([[1, 0, 0],
    [0, 1, 0],
    [0, 0, 1]], order=17)
>>> A = _
>>> B = GF(np.array([[0,2,0],[4,0,0],[0,0,8]]))
>>> A + B
GF([[1, 2, 0],
    [4, 1, 0],
    [0, 0, 9]], order=17)
>>> A * B
GF([[0, 0, 0],
    [0, 0, 0],
    [0, 0, 8]], order=17)
>>>
```

```
ret2basic@PwnieIsland: ~ 80x24
>>> poly1 = galois.Poly([1, 2, 3, 4], field=GF)
>>> poly1
Poly(x^3 + 2x^2 + 3x + 4, GF(17))
>>> poly2 = galois.Poly([5, 2, 3], field=GF)
>>> poly2
Poly(5x^2 + 2x + 3, GF(17))
>>> poly1 + poly2
Poly(x^3 + 7x^2 + 5x + 7, GF(17))
>>> poly1 * poly2
Poly(5x^5 + 12x^4 + 5x^3 + 15x^2 + 12, GF(17))
>>>
```

Back to groth16:

```python
# R1CS matrices

L = np.array([
    [0, 1, 0, 0, 0],
    [0, 0, 1, 0, 0],
    [0, 0, 0, 1, 0],
    [0, 0, 0, 0, 1],
    [0, 1, 0, 0, 0],
    [0, 1, 0, 0, 0],
    [0, 0, 1, 0, 0],
])

R = np.array([
    [0, 1, 0, 0, 0],
    [0, 0, 1, 0, 0],
    [0, 0, 0, 1, 0],
    [0, 0, 0, 0, 1],
    [0, 0, 1, 0, 0],
    [0, 0, 0, 0, 1],
    [0, 0, 0, 1, 0],
])
```

```python
O = np.array([
    [curve_order-2, 3, 0, 0, 0],
    [curve_order-2, 0, 3, 0, 0],
    [curve_order-2, 0, 0, 3, 0],
    [curve_order-2, 0, 0, 0, 3],
    [2, 0, 0, 0, 0],
    [2, 0, 0, 0, 0],
    [2, 0, 0, 0, 0],
])

L_galois = GF(L)
R_galois = GF(R)
O_galois = GF(O)
```

Negative numbers such as -2 must be converted to curve_order - 2.

Prover computes witness:

```python
# In reality this witness is prover's secret, only prover knows it
x1 = GF(1)
x2 = GF(2)
x3 = GF(1)
x4 = GF(2)
# a is the witness
a = GF(np.array([1, x1, x2, x3, x4]))

assert all(np.equal(np.matmul(L_galois, a) * np.matmul(R_galois, a), np.matmul(O_galois, a))), "not equal"
```

Separate public inputs and private inputs (for future use):

```python
# witness = [1, x1, x2, x3, x4]
# Only the first entry [1] is public input
# [x1, x2, x3, x4] are private inputs that only the prover knows
l = 0
public_inputs = a[:l+1]
private_inputs = a[l+1:]
```

Do it in circom

A simple piece of circom I wrote to represent the bipartite graph problem above: https://zkrepl.dev/?gist=810ac7fb657dc07bd933096cb36b7d5f

main.circom × + Add File

```
pragma circom 2.1.6;

// bipartite graph problem arithmetization

template Bipartite(n) {
    // coloring for 4 vertices x1, x2, x3, x4
    // in[0] -> x1
    // in[1] -> x2
    // in[2] -> x3
    // in[3] -> x4
    signal input in[n];

    // Condition 1: color is either 1 or 2 for each vertex
    (in[0] - 1) * (in[0] - 2) === 0;
    (in[1] - 1) * (in[1] - 2) === 0;
    (in[2] - 1) * (in[2] - 2) === 0;
    (in[3] - 1) * (in[3] - 2) === 0;

    // Condition 2: vertices from different "groups" have different colors
    in[0] * in[1] === 2;
    in[0] * in[3] === 2;
    in[1] * in[2] === 2;

}
component main = Bipartite(4);

/* INPUT = {"in": [1, 2, 1, 2]} */
```

```
template instances: 1
non-linear constraints: 7
linear constraints: 0
public inputs: 0
public outputs: 0
private inputs: 4
private outputs: 0
wires: 5
labels: 5
Written successfully: ./main.r1cs
Written successfully: ./main.sym
Written successfully: ./main_js/main.wasm
Everything went okay, circom safe
Compiled in 0.76s

ARTIFACTS:
Finished in 0.87s
• main.wasm (35.02KB)
• main.js (9.18KB)
• main.wtns (0.24KB)
• main.r1cs (1.14KB)
• main.sym (0.07KB)

SAVE:
Saved to Github
<iframe src="https://zkrepl.dev/?gist=810ac7fb657dc07bd933096cb36b7d5f" height="400" width="1000" style="border:1px solid #ddd"></iframe>

KEYS + SOLIDITY + HTML:
Groth16    PLONK    Verify
```

Note: circom operates in scalar field (curve_order) as well:

https://docs.circom.io/circom-language/basic-operators/

Field Elements

A field element is a value in the domain of Z/pZ, where p is the prime number set by default to

`p = 21888242871839275222246405745257275088548364400416034343698204186575808495617.`

As such, field elements are operated in arithmetic modulo p.

The circom language is parametric to this number, and it can be changed without affecting the rest of the language (using `GLOBAL_FIELD_P`).

```
ret2basic@PwnieIsland: ~/Desktop/zero-knowledge-puzzles 79x22
>>> from py_ecc.bn128 import field_modulus, curve_order
>>> field_modulus
21888242871839275222246405745257275088696311157297823662689037894645226208583
>>> curve_order
21888242871839275222246405745257275088548364400416034343698204186575808495617
>>>
```

Circom behind the scene

First we check if it actually compiles:

```
ret2basic@PwnieIsland: ~/Desktop/bipartite 80x24
ret2basic@PwnieIsland:~/Desktop/bipartite$ ll
total 12
drwxrwxr-x  2 ret2basic ret2basic 4096 Jun  5 22:47 ./
drwxr-xr-x 30 ret2basic ret2basic 4096 Jun  5 22:46 ../
-rw-rw-r--  1 ret2basic ret2basic  664 Jun  5 22:47 bipartite.circom
ret2basic@PwnieIsland:~/Desktop/bipartite$ circom bipartite.circom
template instances: 1
Everything went okay
ret2basic@PwnieIsland:~/Desktop/bipartite$
```

Generate R1CS file:

```
ret2basic@PwnieIsland: ~/Desktop/bipartite 80x24
ret2basic@PwnieIsland:~/Desktop/bipartite$ circom bipartite.circom --r1cs --sym
template instances: 1
non-linear constraints: 7
linear constraints: 0
public inputs: 0
private inputs: 4
public outputs: 0
wires: 5
labels: 5
Written successfully: ./bipartite.r1cs
Written successfully: ./bipartite.sym
Everything went okay
ret2basic@PwnieIsland:~/Desktop/bipartite$
```

Here we generate the .sym file so that we can provide symbolic input.json later, such as {"in": [1, 2, 1, 2]}.

Check out R1CS file:

```
ret2basic@PwnieIsland: ~/Desktop/bipartite 80x24
ret2basic@PwnieIsland:~/Desktop/bipartite$ ll
total 20
drwxrwxr-x  2 ret2basic ret2basic 4096 Jun  5 22:48 ./
drwxr-xr-x 30 ret2basic ret2basic 4096 Jun  5 22:46 ../
-rw-rw-r--  1 ret2basic ret2basic  664 Jun  5 22:47 bipartite.circom
-rw-rw-r--  1 ret2basic ret2basic 1136 Jun  5 22:48 bipartite.r1cs
-rw-rw-r--  1 ret2basic ret2basic   68 Jun  5 22:48 bipartite.sym
ret2basic@PwnieIsland:~/Desktop/bipartite$ snarkjs r1cs print bipartite.r1cs
[INFO]  snarkJS: [ 218882428718392752222464057452572750885483644004160343436982041865758084956161 +main.in[0] ] * [ 218882428718392752222464057452572750885483644004160343436982041865758084956151 +main.in[0] ] - [  ] = 0
[INFO]  snarkJS: [ 218882428718392752222464057452572750885483644004160343436982041865758084956161 +main.in[1] ] * [ 218882428718392752222464057452572750885483644004160343436982041865758084956151 +main.in[1] ] - [  ] = 0
[INFO]  snarkJS: [ 218882428718392752222464057452572750885483644004160343436982041865758084956161 +main.in[2] ] * [ 218882428718392752222464057452572750885483644004160343436982041865758084956151 +main.in[2] ] - [  ] = 0
[INFO]  snarkJS: [ 218882428718392752222464057452572750885483644004160343436982041865758084956161 +main.in[3] ] * [ 218882428718392752222464057452572750885483644004160343436982041865758084956151 +main.in[3] ] - [  ] = 0
[INFO]  snarkJS: [ main.in[0] ] * [ main.in[1] ] - [ 21 ] = 0
[INFO]  snarkJS: [ main.in[0] ] * [ main.in[3] ] - [ 21 ] = 0
[INFO]  snarkJS: [ main.in[1] ] * [ main.in[2] ] - [ 21 ] = 0
ret2basic@PwnieIsland:~/Desktop/bipartite$
```

Could be circom bug. Circom is printing 1 at the end of each huge number.

Generate wasm file as preparation for computing witness:

```
ret2basic@PwnieIsland: ~/Desktop/bipartite 80x24
ret2basic@PwnieIsland:~/Desktop/bipartite$ circom bipartite.circom --r1cs --sym --wasm
template instances: 1
non-linear constraints: 7
linear constraints: 0
public inputs: 0
private inputs: 4
public outputs: 0
wires: 5
labels: 5
Written successfully: ./bipartite.r1cs
Written successfully: ./bipartite.sym
Written successfully: ./bipartite_js/bipartite.wasm
Everything went okay
ret2basic@PwnieIsland:~/Desktop/bipartite$
```

In ./bipartite_js directory, create input.json:

```
ret2basic@PwnieIsland: ~/Desktop/bipartite/bipartite_js 80x24
  GNU nano 6.2                    input.json *
{"in": [1, 2, 1, 2]}
```

Generate witness and check out:

```
ret2basic@PwnieIsland: ~/Desktop/bipartite/bipartite_js 80x24
ret2basic@PwnieIsland:~/Desktop/bipartite/bipartite_js$ node generate_witness.js
 bipartite.wasm input.json witness.wtns
ret2basic@PwnieIsland:~/Desktop/bipartite/bipartite_js$ snarkjs wtns export json
 witness.wtns
ret2basic@PwnieIsland:~/Desktop/bipartite/bipartite_js$ cat witness.json
[
 "1",
 "1",
 "2",
 "1",
 "2"
]ret2basic@PwnieIsland:~/Desktop/bipartite/bipartite_js$ 
```