Bounded Rationality as Subjective Menus: Contraction Consistency and Intertemporal Choice

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Abstract

How would a boundedly rational agent react to a larger menu? I model bounded rationality as choice from an unobservable, subjective consideration subset. Consideration sets satisfy Sen's (1969) property α: larger objective choice sets can generate smaller consideration sets. In a single-period model, the "weak weak axiom of revealed preference" is represented by such choice; contraction consistency underlies two recent representations of this axiom. The main contribution is a representation of intertemporal choice: sets are as valuable as their subjective subsets. Applications suggest that bounded rationality could explain some phenomena often ascribed to behavioral preferences, including some apparent impatience.

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1 Introduction

How would a boundedly rational agent react to a larger choice set? Evidence suggests that people choose intransitively, especially out of large sets, and elect to restrict their options.

I model bounded rationality as choice out of subjective subsets: agents do not consider their entire objective choice set as it is observed by a decision theorist, but only an unobserved subset of their options.[1] In particular, in this model, consideration subsets are formed according to Sen's (1969) property α, also known as contraction consistency, which captures a key feature of bounded rationality: larger objective choice sets can generate smaller subjective consideration sets.

A single period representation identifies property α as the feature that unifies two recent representations by Manzini and Mariotti (2009) and Cherepanov et al. (2008) of the so-called the Weak Weak Axiom of Revealed Preference with very different interpretations. The main contribution of this essay is a model of intertemporal choice that provides necessary and sufficient conditions for boundedly rational selection of future choice sets that are valued only according to their subjective consideration sets. The essay concludes with applications that suggest that bounded rationality, rather than present-biased or loss aversive preferences, could explain some instances of apparent impatience or the endowment effect.

1.1 Bounded rationality

Conlisk (1996) reviews evidence for many varieties of bounded rationality. While the term has several uses — and sometimes merely refers to non-standard, "behavioral" preferences — I use it here to mean failure to optimize preferences (behavioral or otherwise) over final options. Experiments provide evidence both of inconsistent choice out of choice sets, and of

[1] Roberts and Lattin (1997) review evidence from the marketing literature that consumers only consider a subset of their options. Eliaz and Spiegler (2007) model consideration subsets formed by the active competition and marketing of firms. Eliaz et al. (2009) provide representation theorems for the choice of observable consideration sets. In this essay, subjective consideration sets are inferred from the agent's behavior.

preference for smaller choice sets when participants may choose their menus.

A central property of optimizing choices is the independence of choice from irrelevant alternatives. For example, if Mexican food is chosen out of Mexican, Italian, or Indian food, then Mexican is chosen even when Indian is unavailable. Experimental findings of "menu effects" question whether this applies to many agents' choices. Manzini and Mariotti (2009) find that half of participants in a paid lab experiment violate Condorcet consistency, which requires that if an option is chosen out of two-item choice sets with every other option in a larger menu, it is again chosen out of the larger menu. In a well-known experiment, Iyengar and Lepper (2000) found that grocery shoppers were more likely to purchase a jam out of a choice set of six than out of a superset of 24. Similarly, Iyengar and Kamenica (2007) show that out of larger sets of gambles or 401(k) plans people tend to choose simpler options.

Other evidence suggests that, given the option, agents sometimes prefer smaller choice sets, even when this may eliminate good options.[2] Salgado (2006) invites laboratory experiment participants to choose a lottery out of a large set. They are given the option to commit to choosing out of a subset randomly selected by a computer. Across experimental treatments, from one-third to one-half of participants chose to choose from a random subset, even though this runs a large risk of discarding valuable options.

1.2 Selection of future choice sets

This essay adds to a growing set of representation theorems for preferences over future choice sets. Kreps (1979) models agents who prefer larger sets to permit flexibility; Ergin (2003) elaborates on this model to represent choice by agents with consideration costs. Gul and Pesendorfer (2001) model agents who might choose to restrict their options in order to forgo the costs of resisting temptation. Sarver (2008) considers choice sets of lotteries and similarly argues that agents may prefer smaller menus in order to avoid regret. Section 3.2 compares the axioms of these models with the intertemporal representation in section 3. Importantly,

[2] Although, for arguable counter-examples, see Shin and Ariely (2004) and Brown et al. (2003).

and unlike temptation preferences, bounded rationality with consideration sets permits two choice sets to both be strictly preferred to their union.

Section 2 offers the single-period representation, and section 3 the representation of intertemporal choice. Section 4 considers some applications of these models, and section 5 concludes.

2 Single-period choice: Weak WARP and property α

An agent faces a standard choice problem. She is offered a choice set out of which she chooses one option. Options x are members of a finite grand choice set X; choice sets A are from the set of non-empty subsets of X, $\mathcal{A}$.

The conditions when the agent's behavior reveal a utility function are well-known. A boundedly rational agent, however, may not satisfy them. In particular, she may consider only a subset of her objectively available options, and ignore the rest. I will write the subjective choice set considered when offered A as $\hat{A}$. Standard rationality is the special case where $\hat{A} = A$ for all A. Therefore, $\wedge : \mathcal{A} \to \mathcal{A}$ is a function that maps a set to its consideration set, with $\hat{A} \subseteq A$.

In this situation, consideration sets are unobservable to decision theorists; we only know the agent's choice function $c : \mathcal{A} \to X$, which reports the element chosen out of each (objective) choice set. Thus $c(A) \in A$ and $c(A)$ is single-valued for all A. With no restrictions on $\wedge$, which governs the formation of subjective consideration sets, what type of choice functions are possible? That is, for which choice functions c could it be that

$$c(A) = \arg\max_{x \in \hat{A}} u(x) \tag{1}$$

for some utility $u : X \to \mathbb{R}$ and some $\wedge$ function such that $\hat{A} \subseteq A$?

The simple answer is that, with no further restrictions on $\wedge$, any choice function is possible. To see this, selecting any choice function, for each A let $\hat{A} = \{c(A)\}$ as a singleton, and the desired choice function is ensured. Clearly more structure is required for $\wedge$ to be a useful summary of bounded rationality.

2.1 Sen's property α and consideration sets

The evidence discussed in section 1.1 suggests that larger choice sets can overwhelm decision makers, causing them to ignore potentially valuable options. I model this possibility with consideration sets governed by Sen's (1969) property α.[3]

Property α. $A \subseteq B \Rightarrow A \cap \hat{B} \subseteq \hat{A}$.

Thus, anything that is in the consideration set of a (larger) set must also be in the consideration set of all of its subsets. If we imagine an objective choice set A becoming larger, as elements are added to A:

- options that were otherwise thought about can drop out of the consideration set,
- newly available members of the objective choice set may or may not be added to the consideration set, and
- options that were previously ignored (that is, not in $\hat{A}$) cannot rejoin the consideration set.

Section 4.1 will consider choice procedures that would generate subjective consideration sets that would satisfy property α.

2.2 Weak WARP

Manzini and Mariotti (2007) characterize cyclical choice as the result of a "rational short-list" decision process. In their representation, agents also choose out of an unobservable

[3] Sen explains property α as requiring that "if the world champion in some game is a Pakistani, then he must also be the champion in Pakistan" (384). Property α combines with property β ("if some Pakistani is a world champion, then *all* champions of Pakistan must be champions of the world.") to identify a rational choice function.

consideration subset. This subset is a rational choice set because it is the set of maximal elements according to some binary relation, though not necessarily a preference. They show that this representation tractably captures a range of heuristics and intransitive behaviors.

One of two conditions on behavior that they identify for rational shortlist choice is a property they call "Weak WARP."[4] This further weakening of the weak axiom of revealed preference requires that

Weak WARP. $\{x, y\} \subseteq A \subseteq B$ *and* $c(\{x, y\}) = c(B) = x \neq y$ *together imply that* $c(A) \neq y$.

For intuition, if x is chosen out of $\{x, y\}$, it is in some sense better than y; so, if y is chosen out of A, x must be somehow unavailable from A. Then, Weak WARP rules out that x again becomes available in the larger set B, a restriction reminiscent of the formation of subjective consideration sets under property α.

Two recent papers weaken Manzini and Mariotti's (2007) representation by requiring only Weak WARP. With this one axiom, they construct representations with very different interpretations. Manzini and Mariotti (2009) propose that agents "categorize then choose": the agent organizes options into subjective categories and eliminates any options in a category that is dominated by another category before making a final selection from the options that remain. Cherepanov et al. (2008) model an agent with one preference and various "rationales;" the agent chooses the most preferred option among those that are best according to some rationale.

Both of these representations suggest plausible choice procedures in some situations. Their convergence on Weak WARP alone indicates that this is a key property of boundedly rational choice. Both representations restrict choice to a subjective subset: members of undominated categories, and optimizers of rationales. Yet, their different interpretations

[4] The other condition that with Weak WARP is necessary and sufficient for rational shortlist choice is Expansion: "An alternative chosen from each of two sets is also chosen from their union" (1828). Manzini and Mariotti (2009) find widespread experimental consistency with Weak WARP.

invite the question of what exactly is critical to a representation of boundedly rational choice following Weak WARP. The answer is that Weak WARP characterizes choice out of consideration subsets formed according to property α:

Theorem 1. *For a choice function c, Weak WARP is necessary and sufficient for there to exist a complete and asymmetric binary relation $\rhd$ on X and function $\wedge$ mapping $\mathcal{A}$ to $\mathcal{A}$ such that $\hat{A} \subseteq A$ for all A, $\wedge$ satisfies property α, and*

$$c(A) = x \in \hat{A} \text{ such that } x \rhd y \text{ for all } y \in \hat{A}, y \neq x.$$

Proof: See appendix A.1.

While property α does not itself specify any mental or deliberative processes, it defines the key feature of any boundedly rational process that is compatible with Weak WARP.

3 Intertemporal choice

How might such a boundedly rational agent select future choice sets? A standard decision maker — optimizing a simple utility function out of all available options — values a menu exactly as much as she values the best option it contains. However, an agent who only selects among options in a consideration subset values the objective choice set only as much as the best option in the smaller, subjective set. Options that she ignores do not increase the value of the set.

I model an agent who, like in the single period model, ignores options in A outside of an unobservable (to the decision theorist) subset $\hat{A}$, where A is again mapped into $\hat{A}$ by a function respecting property α. The agent can undervalue sets or the opportunity to expand a set, relative to utility over final options, and can even strictly prefer each of two sets to their union. This is because out of a larger objective choice set, ever less is considered in the subjective choice set.

Specifically, I offer axioms characterizing a binary relation $\succeq$ defined on $\mathcal{A}$ such that $\succeq$ is represented by

$$U(A) = \max_{x \in \hat{A}} u(x), \tag{2}$$

where $u : X \to \mathbb{R}$ is ordinally unique, $\hat{A} \subseteq A$ for all A, and $\wedge$ satisfies property α.

3.1 Axiomatization

Axiom 1. $\succeq$ *is a preference relation.*

Axiom 2. *For all A, there exists $x \in A$ such that $A \sim \{x\}$.*

Axiom 3. *If $A \succ B$ and $A \succ C$ then $A \succ B \cup C$.*

Axiom 4. *If $A \cup B \succ A$ then there exists $x \in B$ such that $\{x\} \sim A \cup B$.*

Each of these axioms would be true of standard implicit preferences over choice sets, where each menu is exactly as good as its best element. Therefore, standard preferences are a special case of this representation.

Axioms 3 and 4 best illustrate the intuition of this model: bigger sets can be worse for a boundedly rational agent. According to axiom 3, a union of two sets cannot be preferred to any set that is preferred to both of the constituent sets; as a lemma will show, combining sets can make the agent worse off, but not better off. According to axiom 4, in the event that adding options to a set does make the agent better off, the larger set must be as good as one of the new options: none of the original options will be thought about that was not already, and some may no longer be considered.

Theorem 2. *Axioms 1 through 4 are necessary and sufficient for $\succ$ to be represented by $U : \mathcal{A} \to \mathbb{R}$ defined in equation 2.*

Proof: Necessity is straightforward and will be omitted. The proof of sufficiency makes use of the following lemma, which claims that, given these axioms, each set has an essential element, with the property that no subset containing the essential element is worse than the original set, again suggesting that smaller sets can be better.

Lemma 1. *Axioms 1 through 4 are sufficient for each $A \in \mathcal{A}$ to contain an* essential element *x_A such that $\{x_A\} \sim A$ and $x_A \in B \subseteq A$ implies $B \succeq A$.*

The proof of the lemma is given in appendix A.2. To prove sufficiency, notice that because $\succ$ on the set of singletons is a preference relation, it defines U restricted to singletons ordinally uniquely. Then, simply let $u(x) = U(\{x\})$.

Using lemma 1, for all A define the consideration subset $\hat{A} = A \cap \{x_B | A \subseteq B, B \in \mathcal{A}\}$; $\hat{A}$ is the set of all essential elements from sets containing A. Clearly, because $A \subseteq A$, $x_A \in \hat{A}$.

To show that this satisfies property α — using essentially the same proof as for the single-period theorem — take any $A \subseteq B$ with a $y \in \hat{B} \cap A$. Then, because $y \in \hat{B}$, by the construction of $\wedge$ it must be that y is x_C for some C such that $B \subseteq C$. But A is, in turn, contained in B, so x_C — that is, y — must be in $\hat{A}$.

To show that x_A is maximal in $\hat{A}$, select any y in $\hat{A}$. Again, by the construction of $\wedge$, $y = x_B$ for some $B \supseteq A$ and $y \in A$. According to the lemma, then, $A \succeq B$. Then, because $\succeq$ is a preference relation, $\{x_A\} \sim A \succeq B \sim \{x_B\} = \{y\}$.

To finish the proof, let $U(A) = \max_{x \in \tilde{A}} u(x)$ for all A. Then $U(A) = u(x_A)$; $A \sim \{x_A\}$, so $U(A)$ represents $\succeq$.

3.2 Relationship with other models of menu choice

The standard economic model is the special case of this representation where $\hat{A} = A$ for all A. How does this relate to other models of preference over choice sets? Unlike other representations, this model allows $A \succ A \cup B$ and $B \succ A \cup B$.

Models of a preference for flexibility such as Kreps (1979) or Ergin's (2003) extension focusing on costly contemplation permit $A \cup B \succ A$ and $A \cup B \succ B$. Uncertainty leads the agent to prefer having more options. Such preferences are ruled out by this model's lemma.

If $A \cup B \succ A$, then $x_{A \cup B}$ must be in B, so $B \succeq A \cup B$.

Gul and Pesendorfer's (2001) model of temptation preferences hinges upon "set betweenness," an axiom holding that if $A \succeq B$ then $A \succeq A \cup B \succeq B$. Set betweenness is unrelated to axioms 1 through 4; either this model or set betweenness can be satisfied without the other being true.

To see that this essay's model does not imply set betweenness, consider the following example, where larger numbers represent more preferred options:

	A	B	$A \cup B$
set	$\{5,6\}$	$\{1,2\}$	$\{1,2,5,6\}$
$\widehat{\text{set}}$	$\{6\}$	$\{1,2\}$	$\{1\}$

This model satisfies the representation — notice that 1 is included in $\hat{B}$ because it is included in the consideration set $A \cup B$, in accordance with property α. However, $A \succ B \succ A \cup B$, in violation of set betweenness.

To see the reverse, let $X = \{1,2,3\}$ and define $\succeq$ such that

$$\begin{gathered} \{3\} \sim \{1,3\} \succ \\ \{2\} \sim \{2,3\} \sim \{1,2,3\} \succ \\ \{1\} \sim \{1,2\}. \end{gathered}$$

Set betweenness is satisfied. However, lemma 1 fails. Because $\{1,2,3\} \sim \{2\}$, it is clear that 2 is the essential element of $\{1,2,3\}$. Yet, $2 \in \{1,2\}$ and $\{1,2,3\} \succ \{1,2\}$, a contradiction.

The key axiom of Sarver's (2008) model of regret is that $\{x\} \succeq \{y\}$ implies that if $x \in A$ then $A \succeq A \cup \{y\}$. The proof is omitted, but this neither implies nor is implied by axioms 1 through 4.

4 Applications

4.1 Property α and procedural rationality

This representation can be applied to many choice procedures by selecting a special interpretation of consideration sets. Indeed, this is exactly what Manzini and Mariotti's (2009) categorize then choose model and Cherepanov et al.'s (2008) rationalization model do.[5] While I believe property α is a useful characterization of boundedly rational consideration sets even without an ultimately unobservable procedural interpretation, procedural interpretations may helpfully motivate applications of the concept of subjective consideration sets to applied problems.

A sufficient but not necessary condition for $\wedge$ to satisfy property α is that there is some binary relation R such that $\hat{A}$ is the set of best elements in A according to R, that is $\hat{A} = \{x \in A | \forall y \in A, \quad xRy\}$. In this case, xRy represents the relationship "x is thought about even when y is available," a potentially appealing interpretation when some options are attractive, worrisome, or attention-getting, and may keep the agent from considering better possibilities. In the single period model, this is what Manzini and Mariotti's (2007) theory of sequentially rationalizable choice proposes.

However, other procedures for forming $\hat{A}$ are possible that cannot be matched to a binary relation.[6] For example, consider assigning an attractiveness rating $r(x)$ to each x, where options with higher r are more attractive. A boundedly rational decision maker could think only about the most attractive options, with a higher standard for larger sets, so $\hat{A} = \{x \in A : |A| \leq r(x)\}$.

[5] Clearly an element which is not dominated by the category of any other element or which is best according to an available rationale will continue to be in a set with fewer other options.

[6] While this illustrative procedure is compatible with property α and do not require a binary relation, axioms 1 through 4 are not sufficient for them.

4.2 Apparent impatience as bounded rationality

If $u(x) > u(y)$ — that is, if x is valued more than y as an ultimate outcome — then an agent with standard preferences would strictly prefer $\{x, y\}$ to $\{y\}$, and would indeed be willing to pay to expand her future choice set. Many issues in intertemporal decision-making — how much to save and spend, whether to continue an education, whether to invest in a project — involve deciding now whether or not to expand our future options. Sometimes, as in the case of saving, investing, or working, having more options tomorrow comes at a cost today.

However, agents who behave according to theorem 2 can[7] undervalue opportunities to expand their future choices: to a boundedly rational agent following property α, bigger choice sets do not always appear better, even when they contain better options. As section 5 will discuss, this could be either because the agent, when selecting a future menu, sophisticatedly anticipates choosing boundedly rationally (as in theorem 1), or because she has particular trouble predicting and imagining the future, and does not consider all of the future options that an action now would make available in her choice set. As a result, she may not choose the larger future choice set, especially if there is a cost to doing so. While some observers might attribute such behavior to impatient preferences, the actual cause would be failure to maximize.

Consider an young man in a rural village who, without education, will have the options of subsistence farming s and agricultural labor ℓ. If he goes to school, he could also choose a wage job in a town w. He prefers the wage job: $w \succ \ell \succ s$. Will he go to school? That is, does this situation imply $\{s, \ell, w\} \succ \{s, \ell\}$? Not necessarily. For example, the prospect of subsistence agriculture may be so distressing that it distracts him from considering wage labor. If so, he may choose not to invest in education — not because of impatient preferences, but because of bounded rationality.

[7]This is not a necessary implication of the model.

4.3 The endowment effect

Thaler (1980) identified the endowment effect, a now well-known behavioral anomaly in which people apparently value objects more when they own them, so willingness to accept exceeds willingness to pay. While this effect is often interpreted as evidence of loss aversive preferences, in some cases it could be explained by boundedly rational menu choice.[8]

An offer of money is ultimately an offer of the choice set of options one can choose with the money. Consider an agent with a sum of money generating the choice set $A = \{\text{mug}, \ldots\}$; it includes a mug as well as other options. Theorems 1 and 2 are independent representations of preferences over different domains: an agent could be boundedly rational when predicting the implications for current choice on future options, but unboundedly rational when choosing out of a choice set. Such an agent could choose $c(A) = \text{mug}$, selecting the mug, only to be unwilling to trade the mug back for A. Because A is a larger choice set, $\{\text{mug}\} \succ A$ is possible if the agent does not consider using A to repurchase the mug.

5 Discussion

People choose inconsistently out of choice sets and sometimes prefer smaller future choice sets. Both of these phenomena can be explained by bounded rationality as choice out of a subjective consideration subset: only some options are thought about. Moreover, as choice sets grow larger, previously considered options can become ignored. Selecting out of consideration sets formed according to Sen's principle α is identical to choosing according to Weak WARP; this representation highlights what unifies earlier procedural interpretations. Additionally, in intertemporal choice where an agent only values a set according to such a consideration subset, agents can strictly prefer smaller sets — including preferring both A and B to $A \cup B$.

Theorems 1 and 2 are two independent results characterizing preference over two different

[8] I thank Abhijit Banerjee for calling my attention to this application.

Table 1: Two independent representations

		$t = 1$ (theorem 2)	
		standard A	$\hat{A}$ and α
$t = 2$ (theorem 1)	standard A	standard	boundedly rational about the future
	$\hat{A}$ and α	naïve	sophisticated

domains: final options and choice sets. As table 1 demonstrates, either could hold with or without the other. If the agent is boundedly rational when making her final selection, but not when choosing future choice sets, she is naïve about her bounded rationality; if she is ultimately boundedly rational and takes it into consideration when choosing menus, she behaves sophisticatedly. Finally, it is possible that an agent experiences special bounded rationality when planning for the future: predicting and imagining may be different tasks than selecting from immediate options. If so, an agent could ultimately maximize a utility function over final outcomes, but choose future menus boundedly rationally.

Much of behavioral economics has modified the types of preferences that agents are modeled to optimize. As the example applications in section 4 illustrate, boundedly rational choice among choice sets could give rise to behavior that appears impatient or loss aversive. If so, some behaviors might be better explained not by optimizing special preferences over final options, but by ultimately not optimizing at all.

A Proofs

A.1 Proof of theorem 1

Sufficiency:

First, define $x \rhd y$ if $x = c(\{x, y\})$. Because c is a function and is defined for all A, $\rhd$ is complete and asymmetric. Next let $\hat{A} = A \cap \{y | y = c(B), A \subseteq B\}$. Then $c(A) \in \hat{A}$ because $A \subseteq A$ and, by construction, $\hat{A} \subseteq A$.

To show that this satisfies property α select $x \in A \subseteq B$ such that $x \in \hat{B}$. That $x \in \hat{B}$

implies that $x = c(C)$ for some $C \supseteq B$. Then $A \subseteq C$, so by the construction of $\hat{A}$ it is clear that $x \in \hat{A}$ and property α is satisfied.

For optimization, assume for indirect proof that there exists y in $\hat{A}$ such that $y \rhd c(A)$ but $y \neq c(A)$. Then, by the construction of $\rhd$, $y = c(\{c(A), y\})$ and by the construction of $\hat{A}$, $y = c(B)$ for some B such that $A \subseteq B$. But $\{c(A), y\} \subseteq A \subseteq B$, which violates Weak WARP. Thus, by contradiction, no such y exists. Because $\rhd$ is complete, and because for no y in $\hat{A}$ other than $c(A)$ is $y \rhd c(A)$, we can conclude that for all such y that $c(A) \rhd y$.

Necessity:

Because $c(A) \in \hat{A}$ and $\hat{A} \subseteq A$, c is a choice function.

Assume for indirect proof that $\{x, y\} \subseteq A \subseteq B$, $x = c(\{x, y\}) = c(B) \neq y$ and $y = c(A)$. Then, because $x \in \hat{B}$ and $x \in A$, by property α $x \in \hat{A}$. Also, $y \in c(A)$ requires $y \rhd x$ and $y \in \hat{A}$. But then, by property α, $y \in \widehat{\{x, y\}}$, so $c(\{x, y\}) = y$, a contradiction.

A.2 Proof of lemma 1

Fix any $A \in \mathcal{A}$. If $B \succeq A$ for all $B \subseteq A$ then let any $x \in A$ such that $A \sim \{x\}$ (which, by axiom 2, exists) be x_A and the proof is done.

Otherwise, let $\{B_i\}$ be the set of all subsets of A such that $A \succ B_i$. Because X is finite, so is $\{B_i\}$. Therefore, by repeated application of axiom 3, $A \succ \bigcup_i B_i$.

Assume for indirect proof that $A \setminus \bigcup_i B_i = \emptyset$. Then $A = \bigcup_i B_i$, which would contradict $A \succ \bigcup_i B_i$, so $A \setminus \bigcup_i B_i \neq \emptyset$. Summarizing this and rewriting A, we have that

$$A = \left(A \setminus \bigcup_i B_i \right) \cup \left(\bigcup_i B_i \right) \succ \bigcup_i B_i.$$

By axiom 4, there exists $x \in A \setminus \bigcup_i B_i$ such that $A \sim \{x\}$. Re-label x as x_A. Take B such that $x_A \in B \subseteq A$. Because $x_A \in B$, we can conclude $B \notin \{B_i\}$. Therefore $B \succeq A$.

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