

# Forecasting old-age dependency

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24 February 2022

# Outline

- 1 Demographic change and old-age dependency
- 2 Forecasting population age structure
- 3 Sustainable pension age schemes

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# Ageing population



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- 2 high fertility rates after World War II
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- 2 high fertility rates after World War II
- 3 large decline in fertility rates over past thirty years.

**Consequently:**

- older people leaving workforce, fewer workers replacing them
- increasing old-age dependency ratio (OADR)

# Population data

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$$\text{OADR}_t = \frac{\sum_{x \geq \lfloor a_t \rfloor} P_t(x) - (a_t - \lfloor a_t \rfloor)P_t(\lfloor a_t \rfloor)}{\sum_{x=15}^{\lfloor a_t \rfloor - 1} P_t(x) + (a_t - \lfloor a_t \rfloor)P_t(\lfloor a_t \rfloor)}$$

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in year  $t$

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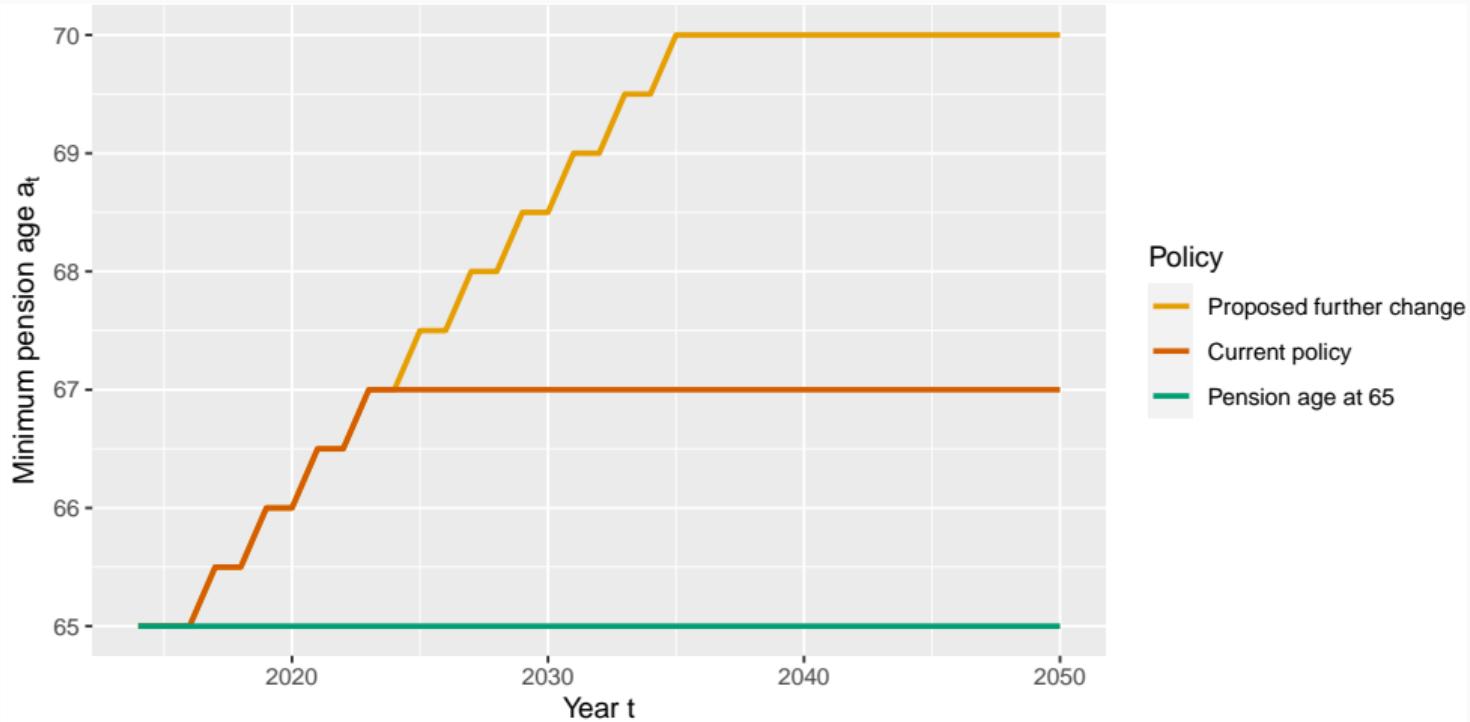
$P_t(x)$  = population age  $x$   
in year  $t$

$a_t$  = pension age  
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Assumptions:

- everyone retires at pension age
- pension age can only be adjusted at beginning of year
- birthdays are uniformly distributed over year

# The age pension in Australia



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# Available data

$B_t(x)$  = Births in calendar year  $t$  to females age  $x$

$D_t(x)$  = Deaths in calendar year  $t$  of persons age  $x$

$P_t(x)$  = Population age  $x$  at 1 January year  $t$

$E_t(x)$  = Population age  $x$  at 30 June year  $t$

- Ages:  $x = 0, 1, 2, \dots, 99, 100+$
- Year:  $t = 1, 2, \dots, T$
- $m_t^F(x) = D_t^F(x)/E_t^F(x)$  = female central death rates, year  $t$
- $m_t^M(x) = D_t^M(x)/E_t^M(x)$  = male central death rates, year  $t$
- $f_t(x) = B_t(x)/E_t^F(x)$  = fertility rates, year  $t$

# Mortality rates

# Fertility rates

## Migration rates

$$G_t(x) = P_{t+1}(x + 1) - P_t(x) + D_t(x)$$

# Functional linear model

$$y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

$$s_t(x) = \mu(x) + \sum_{j=1}^J \beta_{t,j}\phi_j(x) + e_t(x)$$

- $y_t(x)$  = observed data, year  $t$ , age  $x$
- $s_t(x)$  = smoothed version of  $y_t(x)$
- $\varepsilon_{t,x} \sim \text{iid } N(0,1)$
- $\mu(x)$  = mean  $s_t(x)$  across years.
- $\phi_j(x)$  and  $\beta_{t,j}$  are principal components & scores of  $s_t(x) - \mu(x)$ .
- $e_t(x)$  serially uncorrelated with mean zero

# Functional linear model

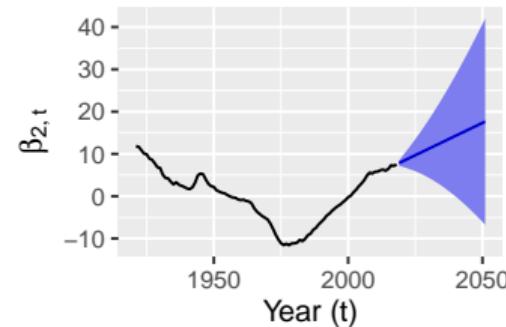
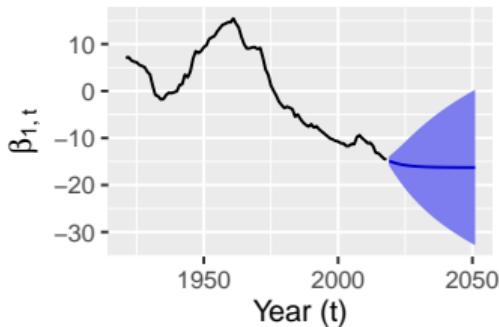
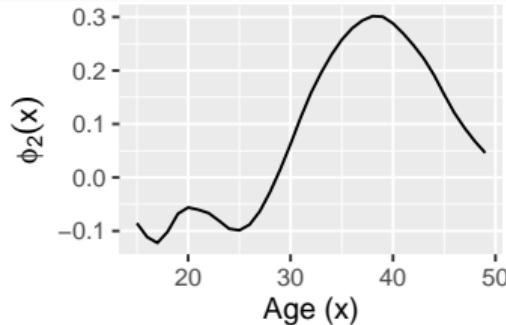
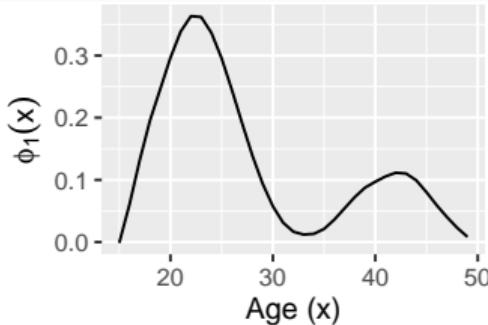
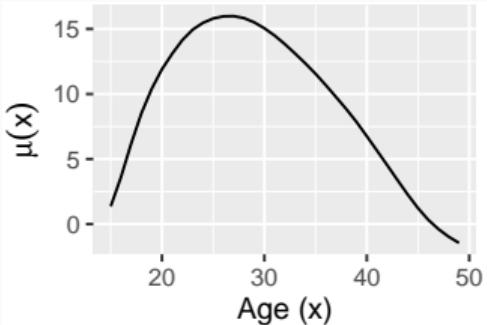
$$y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

$$s_t(x) = \mu(x) + \sum_{j=1}^J \beta_{t,j}\phi_j(x) + e_t(x)$$

- $\{\beta_{1,j}, \dots, \beta_{T,j}\}$  is automatically modelled with ARIMA or ARFIMA process
- $\hat{\beta}_{T+h|T,j}$  is an  $h$ -step forecast from the AR(F)IMA model

$$\hat{y}_{T+h|T}(x) = \hat{\mu}(x) + \sum_{j=1}^J \phi_j(x)\hat{\beta}_{T+h|T,j}$$

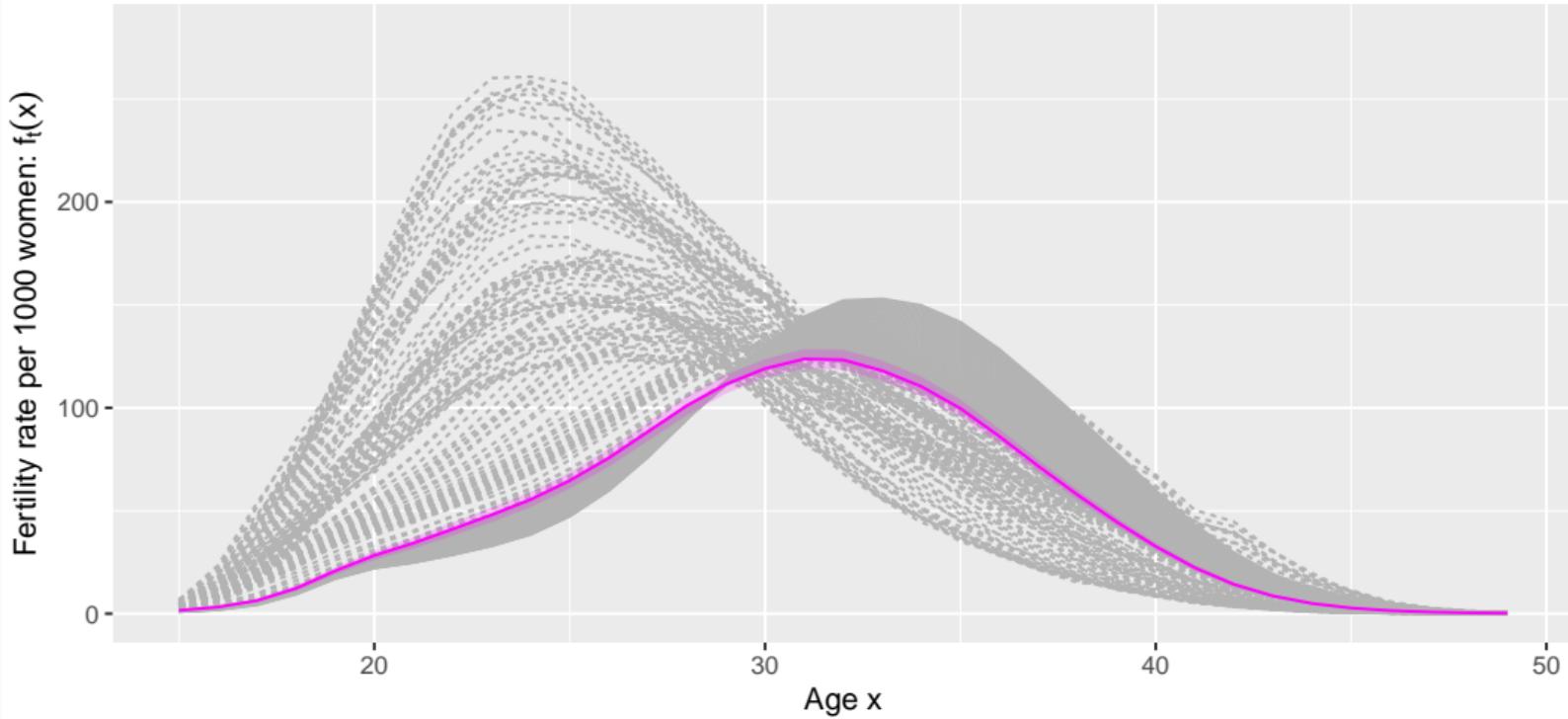
# Fertility forecasts



# Fertility forecasts

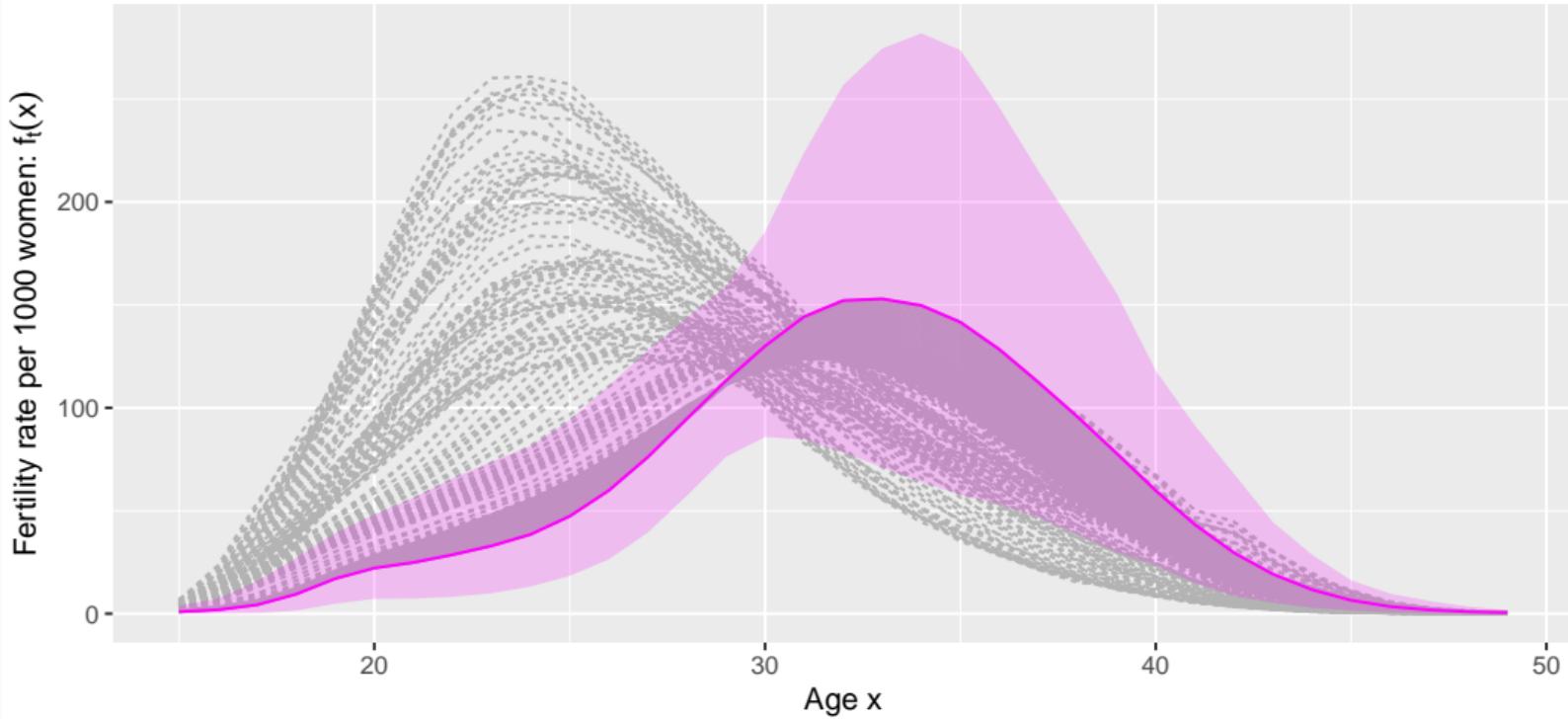
# Fertility forecasts

Year: 2019



# Fertility forecasts

Year: 2051



# Population forecasts

- Fit similar models to fertility rates, mortality rates, migration numbers
- Constrained so sex differences do not diverge
- Assume mortality, fertility and migration are independent processes
- Simulate births, deaths and migrants for future years
- Assume births and deaths are Poisson, migrant distribution is bootstrapped
- Compute populations by age and sex for future years

## For each year of each simulated population

- Draw from forecast distribution of fertility rates:  $f_t(x)$
- Generate number of births from Poisson population:  $B_t$
- Draw from forecast distribution of mortality rates:  $m_t^F(x)$ ,  $m_t^M(x)$
- Generate number of deaths by age and sex:

$$D_t(x) \sim \text{Poisson}(P_t(x)m_t(x))$$

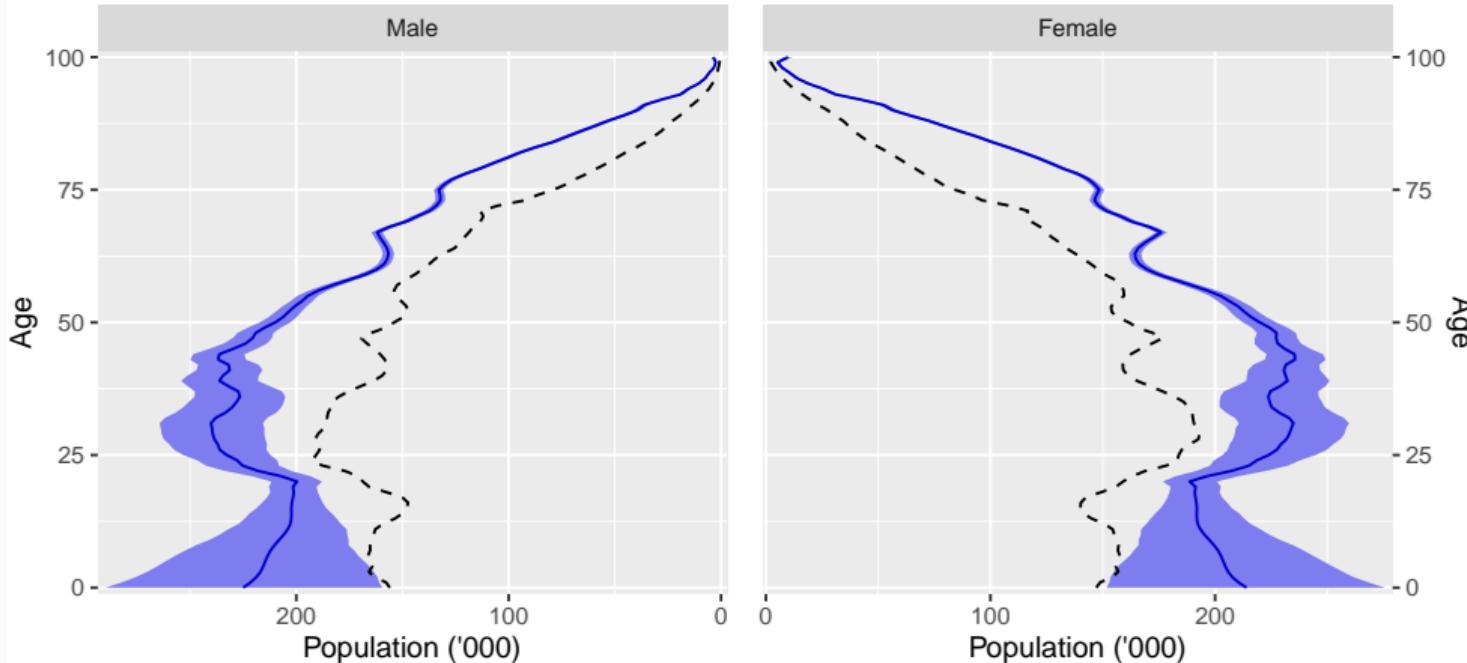
- Draw from forecast distribution of migration:  $G_t^F(x)$ ,  $G_t^M(x)$
- Compute population for next year from

$$P_{t+1}(x) = P_t(x) + B_t 1(x = 0) - D_t(x) + G_t(x)$$

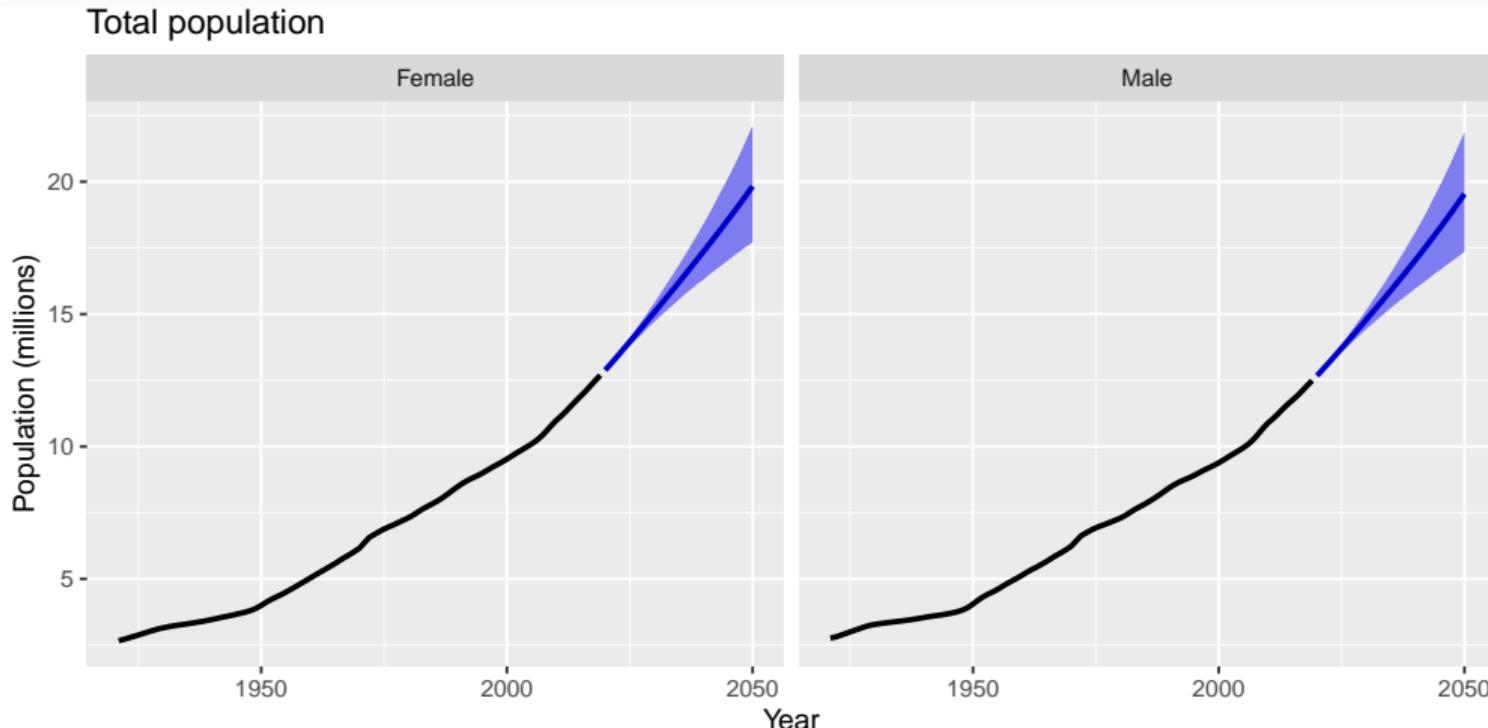
Repeat for all future years. Repeat for multiple simulated populations.<sup>20</sup>

# Population forecasts

Forecast population for 2038 compared to 2019



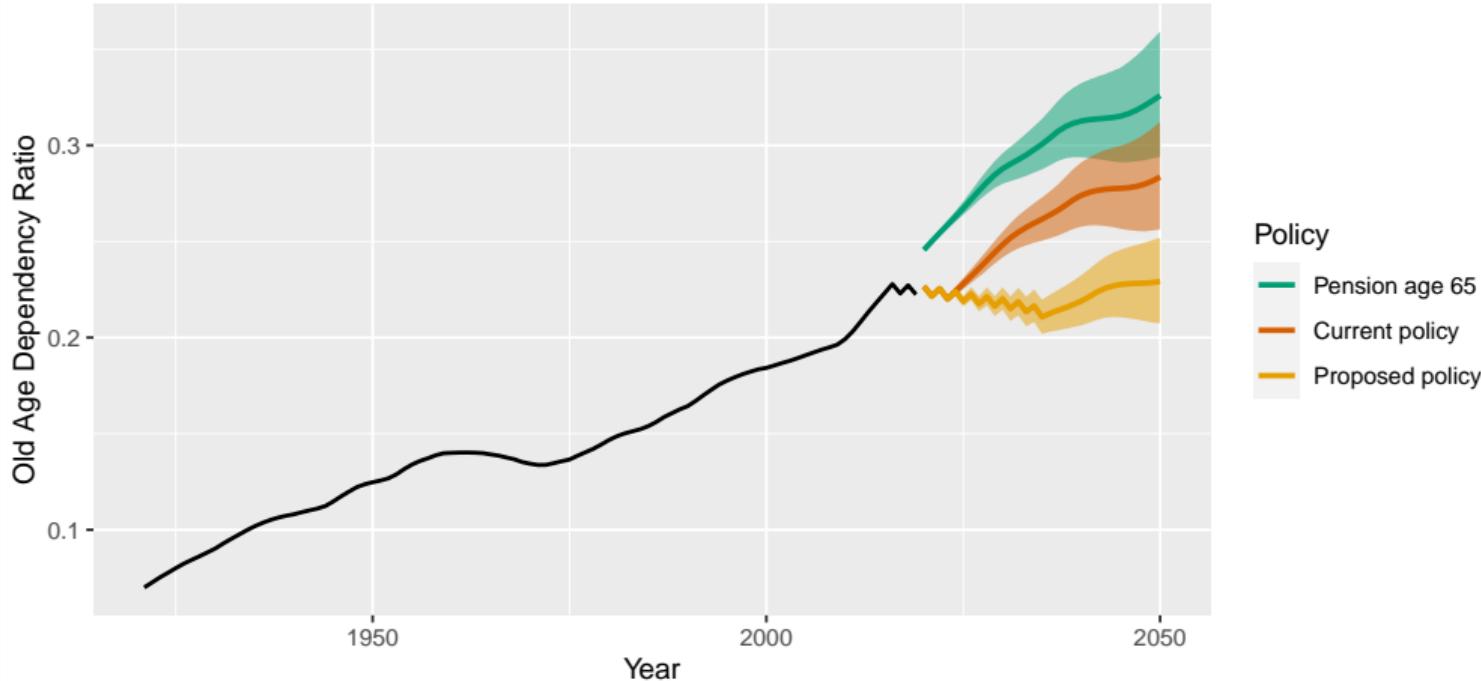
# Population forecasts



Fifty-year forecasts with 80% prediction intervals.

# OADR forecasts

OADR forecasts under different pension schemes



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# Sustainable pension age schemes

$a_t$  = pension age in year  $t$

$O_t$  = OADR in year  $t$

$O^*$  = target OADR

**Find  $a_{T+1}, \dots, a_{T+H}$  where**

- $a_t$  is minimum pension age such that  $O_t \leq O^*$ .
- $a_{t-1} \leq a_t < a_{t-1} + 1$  to prevent (a) years where no-one is able to retire; and (b) years where retired people become ineligible for the pension.
- $a_t$  must be in increments of one month.

# Sustainable pension age schemes

Starting with  $h = 1$ :

- 1 Set  $a_{T+h} = a_{T+h-1}$ .
- 2 Increment  $a_{T+h}$  by one-month intervals until either  $\hat{O}_{T+h|T} \leq O^*$  or  $a_{T+h} - a_{T+h-1} = 11$  months, where  $\hat{O}_{T+h|T}$  denotes the mean of the simulated  $O_{T+h|T}$  values.

Repeat for  $h = 2, \dots, H$ .

# Sustainable pension age schemes

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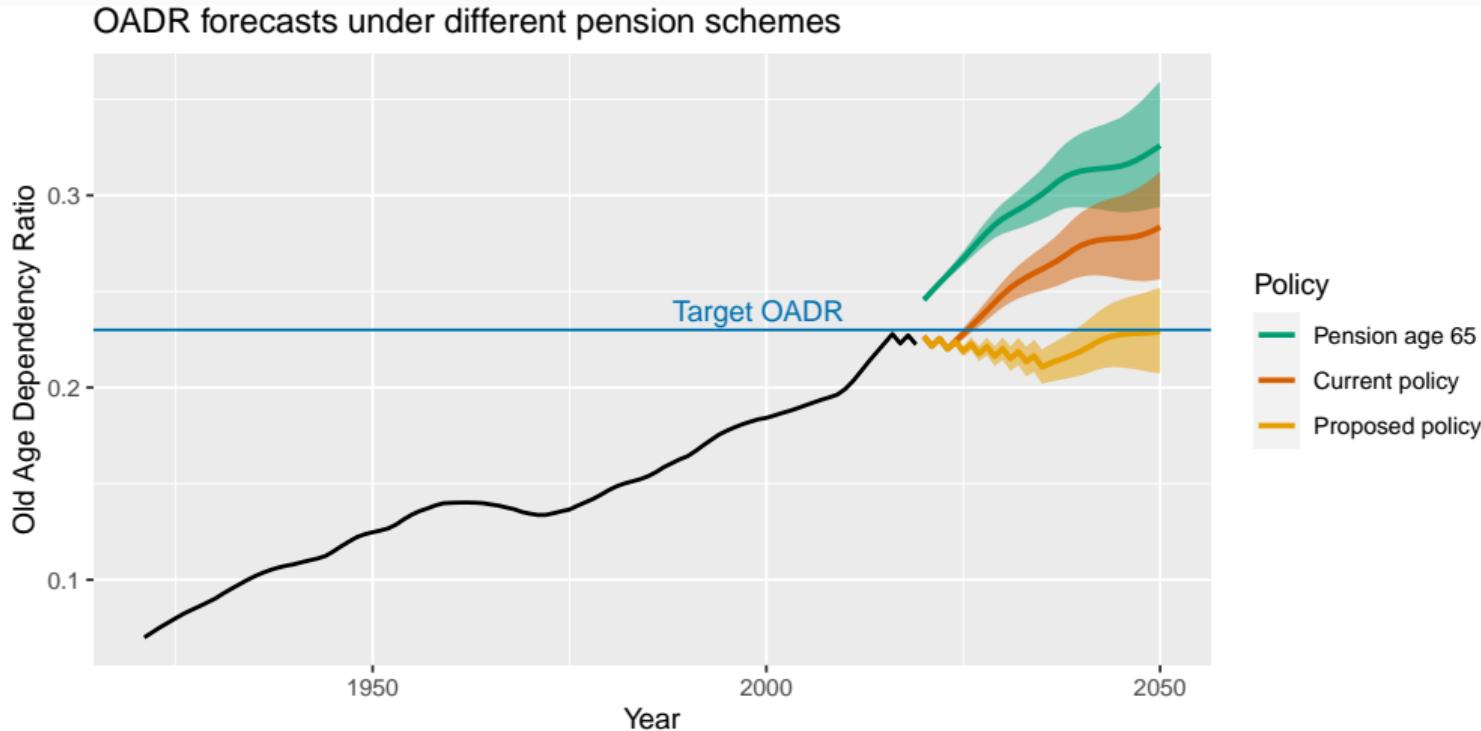
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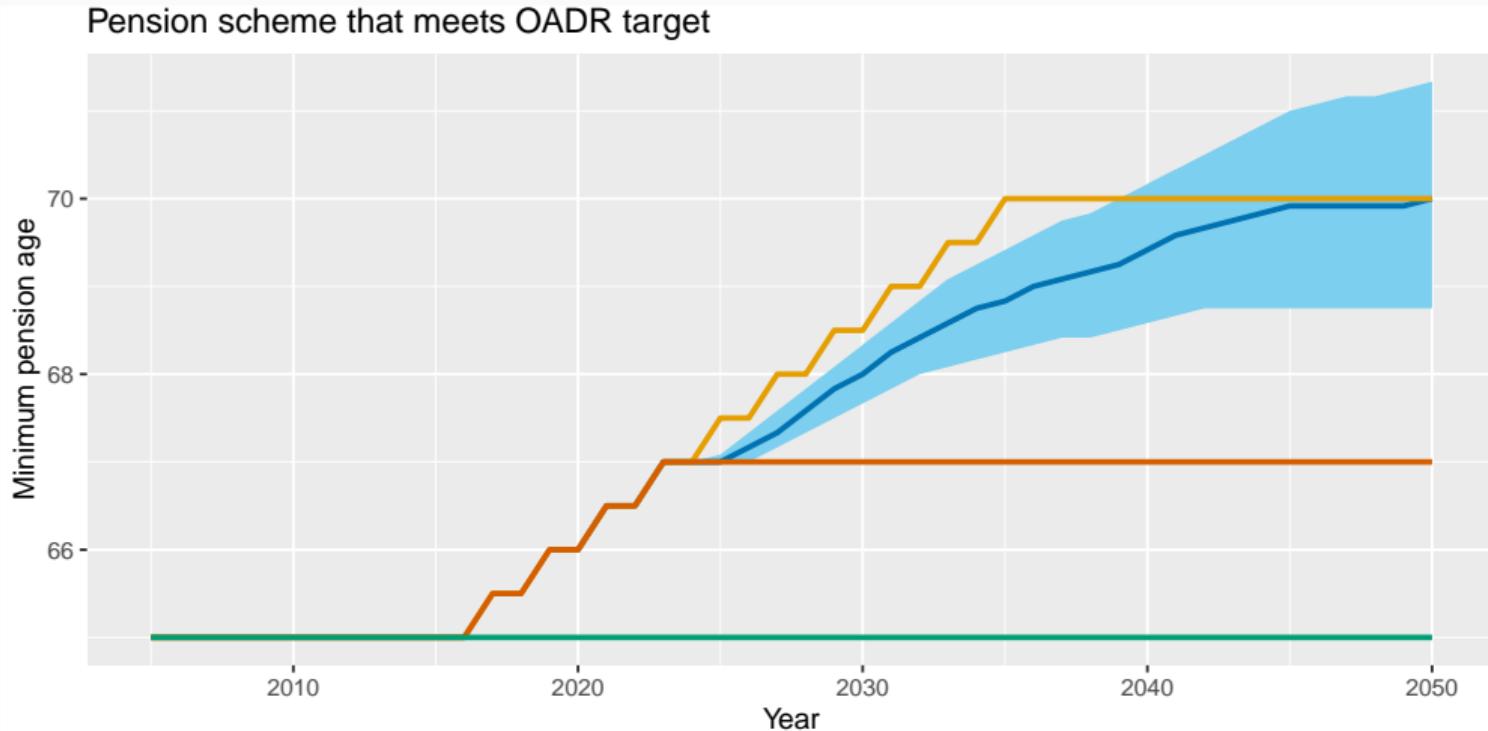
## Confidence intervals

- Range of pension age schemes for which  $O^*$  is contained within 80% prediction intervals of simulated  $O_{T+H|T}$  values.
- Same algorithm but  $\hat{O}_{T+H|T}$  replaced by 10% and 90% quantiles.

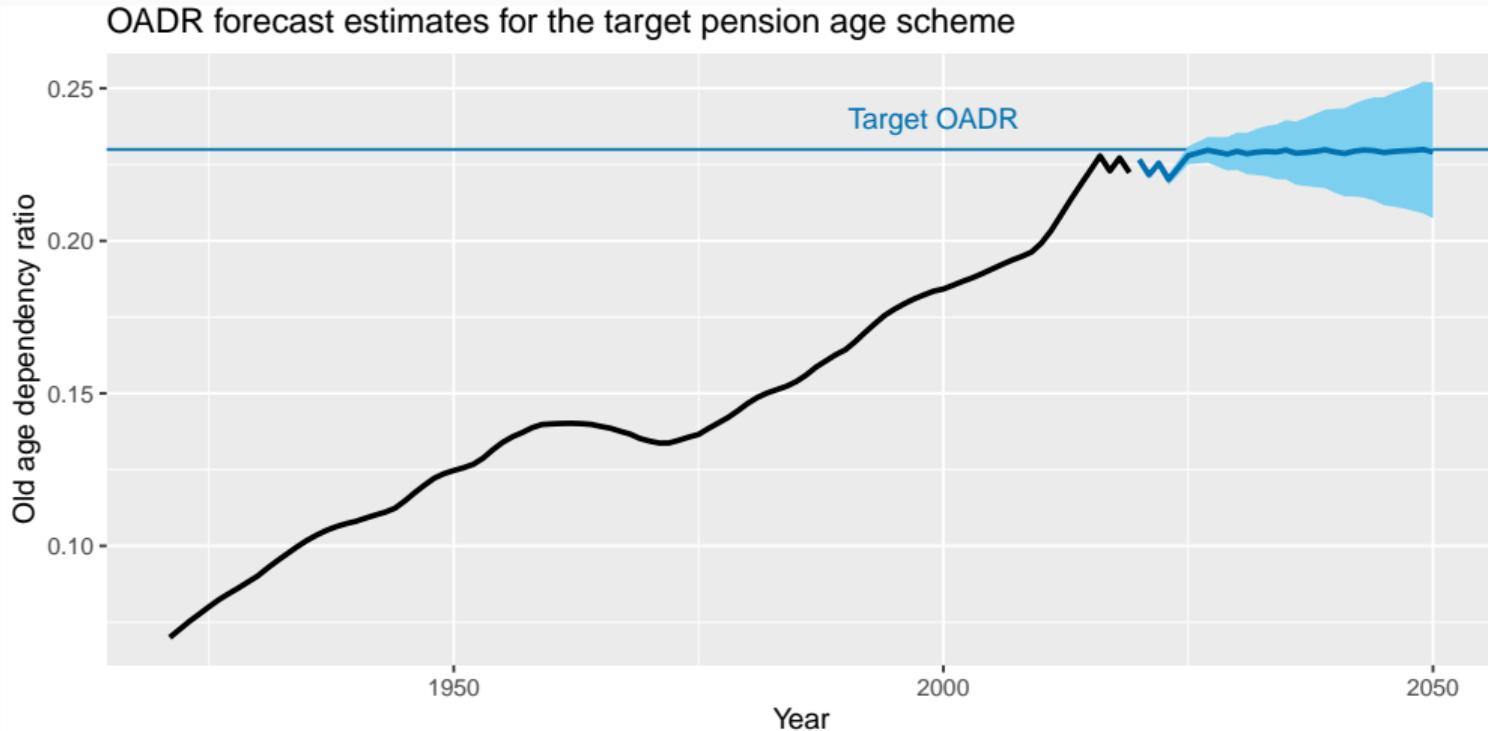
# Target OADR



# Target pension age scheme

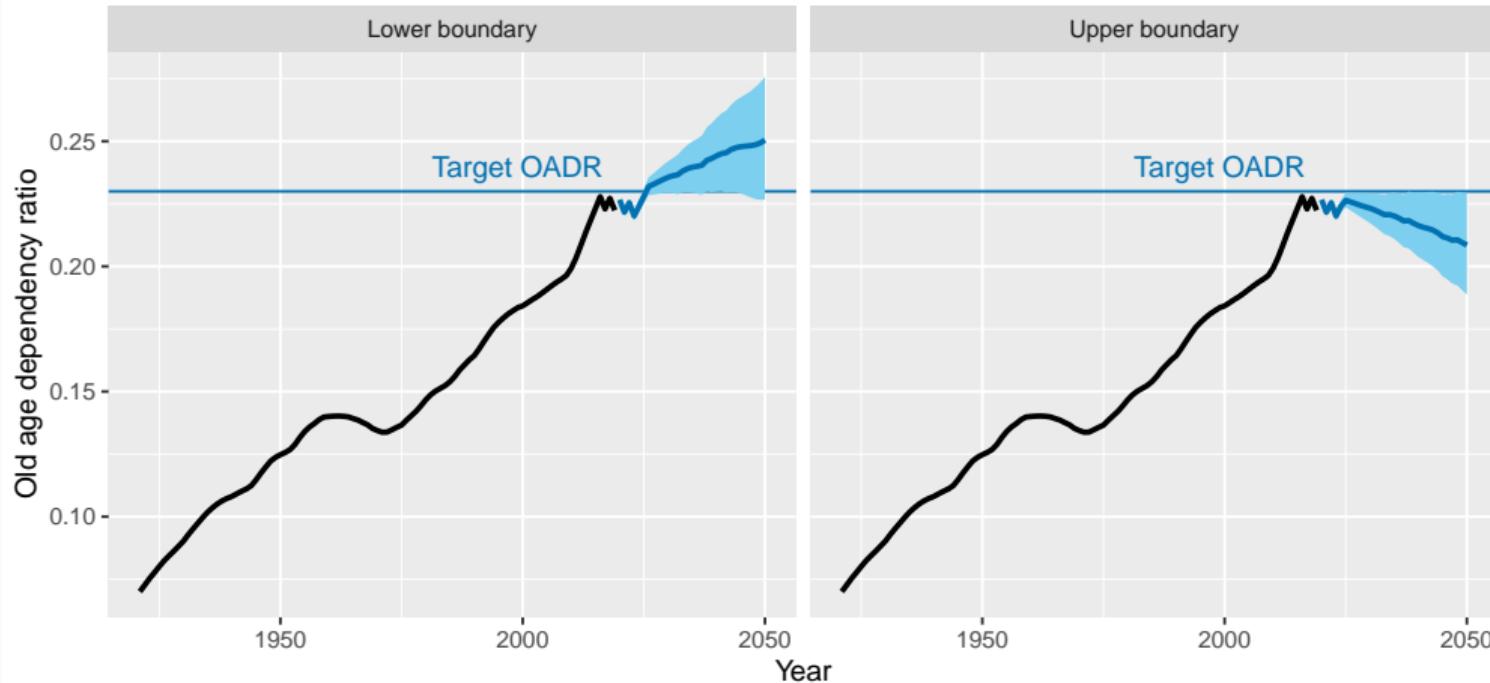


# Target pension age scheme



# Target pension age scheme

OADR forecasts for boundary pension age schemes



OADR when pension age set at prediction interval boundaries.

# Target pension age scheme

This analysis does not allow for:

- superannuation and other sources of income.
- changing GDP per capita
- COVID-19

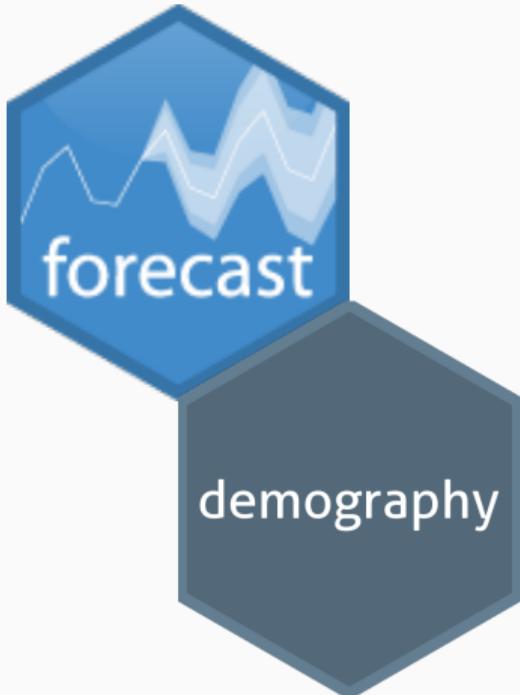


# Relevant papers

- Rob J Hyndman & Shahid Ullah (2007) Robust forecasting of mortality and fertility rates: a functional data approach. *Computational Statistics & Data Analysis* **51**, 4942–4956.
- Rob J Hyndman & Heather Booth (2008) Stochastic population forecasts using functional data models for mortality, fertility and migration. *International Journal of Forecasting* **24**(3), 323–342.
- Rob J Hyndman, Heather Booth & Farah Yasmeen (2013) Coherent mortality forecasting: the product-ratio method with functional time series models. *Demography* **50**(1), 261–283.
- Rob J Hyndman, Yijun Zeng & Han Lin Shang (2021) Forecasting the old-age dependency ratio to determine a sustainable pension age. *Australian & New Zealand Journal of Statistics*, **63**(2), 241–256.

# More information

[robjhyndman.com/seminars/oadr/](http://robjhyndman.com/seminars/oadr/)



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