Horowitz & Hill, The Art of Electronics

JH

The NIC therefore converts a capacitor to a "backward" inductor:

$$Z_C = 1/j\omega C \rightarrow Z_{\text{in}} = j/\omega C$$

i.e., it is inductive in the sense of generating a current that lags the applied voltage, but its impedance has the wrong frequency dependence (it goes down, instead of up, with increasing frequency). The gyrator, on the other hand, converts a capacitor to a true inductor:

$$Z_C = 1/j\omega C \rightarrow Z_{\text{in}} = j\omega CR^2$$

i.e., an inductor with inductance $L = CR^2$.

The existence of the gyrator makes it intuitively reasonable that inductorless filters can be built to mimic any filter using inductors: Simply replace each inductor by a gyrated capacitor. The use of gyrators in just that manner is perfectly OK, and in fact the telephone filter illustrated previously was built that way. In addition to simple gyrator substitution into preexisting *RLC* designs, it is possible to synthesize many other filter configurations. The field of inductorless filter design is extremely active, with new designs appearing in the journals every month.

Sallen-and-Key filter

Figure 5.6 shows an example of a simple and even partly intuitive filter. It is known as a Sallen-and-Key filter, after its inventors. The unity-gain amplifier can be an op-amp connected as a follower, or just an emitter follower. This particular filter is a 2-pole high-pass filter. Note that it would be simply two cascaded *RC* high-pass filters except for the fact that the bottom of the first resistor is bootstrapped by the output. It is easy to see that at very low frequencies it falls off just like a cascaded *RC*, since the output is essentially zero. As the output rises at increasing frequency, however, the bootstrap action tends to reduce the attenuation, giving a sharper knee. Of course, such hand-waving cannot substitute for honest analysis, which luckily has already been done for a prodigious variety of nice filters. We will come back to active filter circuits in Section 5.06.

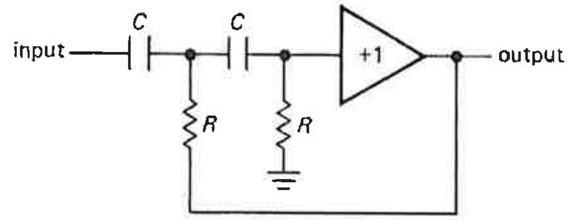

Figure 5.6

5.04 Key filter performance criteria

There are some standard terms that keep appearing when we talk about filters and try to specify their performance. It is worth getting it all straight at the beginning.

Frequency domain

The most obvious characteristic of a filter is its gain versus frequency, typified by the sort of low-pass characteristic shown in Figure 5.7.

The *passband* is the region of frequencies that are relatively unattenuated by the filter. Most often the passband is considered to extend to the −3dB point, but with certain filters (most notably the "equiripple" types) the end of the passband may be defined somewhat differently. Within the passband the response may show variations or *ripples*, defining a *ripple band*, as shown. The *cutoff frequency*, f_c, is the end of the passband. The response of the filter then drops off through a *transition region* (also colorfully known as the *skirt* of the filter's response) to a *stopband*, the region of significant attenuation. The stopband may be defined by some minimum attenuation, e.g., 40dB.

Along with the gain response, the other parameters of importance in the frequency

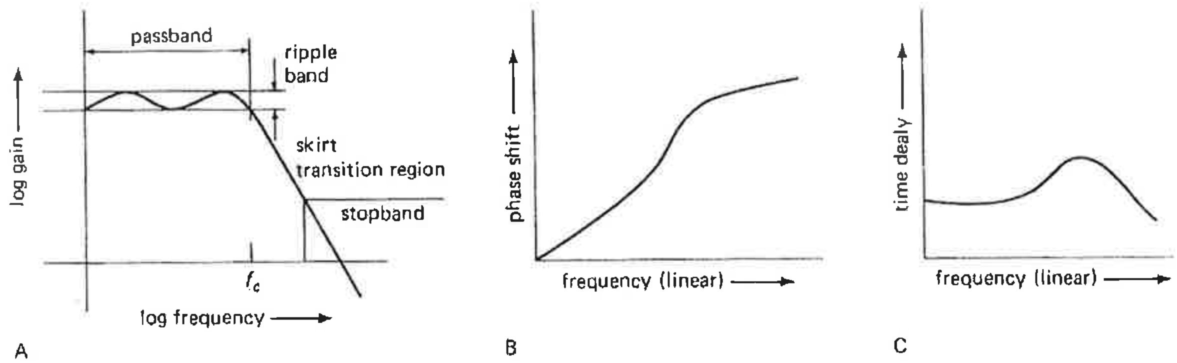

Figure 5.7. Filter characteristics versus frequency.

domain is the *phase shift* of the output signal relative to the input signal. In other words, we are interested in the *complex* response of the filter, which usually goes by the name of **H**(s), where s = $j\omega$, where **H**, s, and ω all are complex. Phase is important because a signal entirely within the passband of a filter will emerge with its waveform distorted if the time delay of different frequencies in going through the filter is not constant. Constant time delay corresponds to a phase shift increasing linearly with frequency; hence the term *linear-phase filter* applied to a filter ideal in this respect. Figure 5.8 shows a typical graph of phase shift and amplitude for a low-pass filter that is definitely not a linear-phase filter. Graphs of phase shift versus frequency are best plotted on a linear-frequency axis.

Time domain

As with any ac circuit, filters can be described in terms of their *time-domain* properties: rise time, overshoot, ringing, and settling time. This is of particular importance where steps or pulses may be used. Figure 5.9 shows a typical low-pass-filter step response. Here, *rise time* is the time required to reach 90% of the final value, whereas *settling time* is the time required to get within some specified amount of the final value and stay there. *Overshoot* and *ringing* are self-explanatory terms for some undesirable properties of filters.

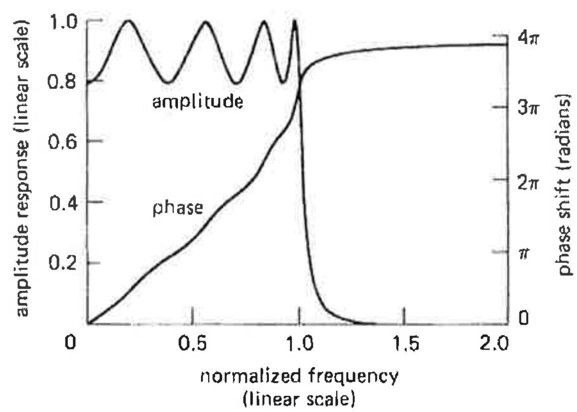

Figure 5.8. Phase and amplitude response for an 8-pole Chebyshev low-pass filter (2dB passband ripple).

5.05 Filter types

Suppose you want a low-pass filter with flat passband and sharp transition to the stopband. The ultimate rate of falloff, well into the stopband, will always be 6ndB/octave, where n is the number of "poles." You need one capacitor (or inductor) for each pole, so the required ultimate rate of falloff of filter response determines, roughly, the complexity of the filter.

Now, assume that you have decided to use a 6-pole low-pass filter. You are guaranteed an ultimate rolloff of 36dB/octave at high frequencies. It turns out

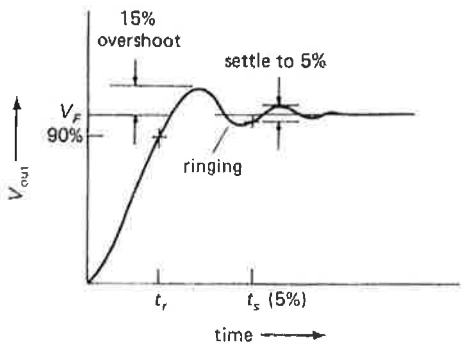

Figure 5.9

that the filter design can now be optimized for maximum flatness of passband response, at the expense of a slow transition from passband to stopband. Alternatively, by allowing some ripple in the passband characteristic, the transition from passband to stopband can be steepened considerably. A third criterion that may be important is the ability of the filter to pass signals within the passband without distortion of their waveforms caused by phase shifts. You may also care about rise time, overshoot, and settling time.

There are filter designs available to optimize each of these characteristics, or combinations of them. In fact, rational filter selection will not be carried out as just described; rather, it normally begins with a set of requirements on passband flatness, attenuation at some frequency outside the passband, and whatever else matters. You will then choose the best design for the job, using the number of poles necessary to meet the requirements. In the next few sections we will introduce the three popular favorites, the Butterworth filter (maximally flat passband), the Chebyshev filter (steepest transition from passband to stopband), and the Bessel filter (maximally flat time delay). Each of these filter responses can be produced with a variety of different filter circuits, some of which we will discuss later. They are all available in low-pass, high-pass, and bandpass versions.

Butterworth and Chebyshev filters

The Butterworth filter produces the flattest passband response, at the expense of steepness in the transition region from passband to stopband. As you will see later, it also has poor phase characteristics. The amplitude response is given by

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{[1 + (f/f_c)^{2n}]^{\frac{1}{2}}}$$

where n is the order of the filter (number of poles). Increasing the number of poles flattens the passband response and steepens the stopband falloff, as shown in Figure 5.10.

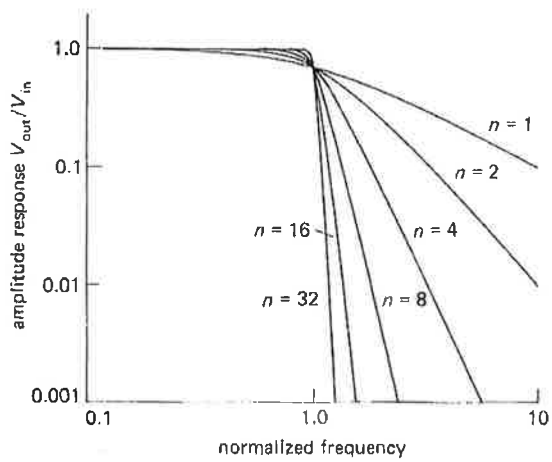

Figure 5.10. Normalized low-pass Butterworth-filter response curves. Note the improved attenuation characteristics for the higher-order filters.

The Butterworth filter trades off everything else for maximum flatness of response. It starts out extremely flat at zero frequency and bends over near the cutoff frequency f_c (f_c is usually the -3dB point).

In most applications, all that really matters is that the wiggles in the passband response be kept less than some amount, say 1dB. The Chebyshev filter responds to this reality by allowing some ripples throughout the passband, with greatly improved sharpness of the knee. A Chebyshev filter is specified in terms of its number of poles and passband ripple. By allowing greater passband ripple, you get a sharper knee. The amplitude is given by

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{[1+\epsilon^2 C_n^2(f/f_c)]^{\frac{1}{2}}}$$

where C_n is the Chebyshev polynomial of the first kind of degree n, and ϵ is a constant that sets the passband ripple. Like the Butterworth, the Chebyshev has phase characteristics that are less than ideal.

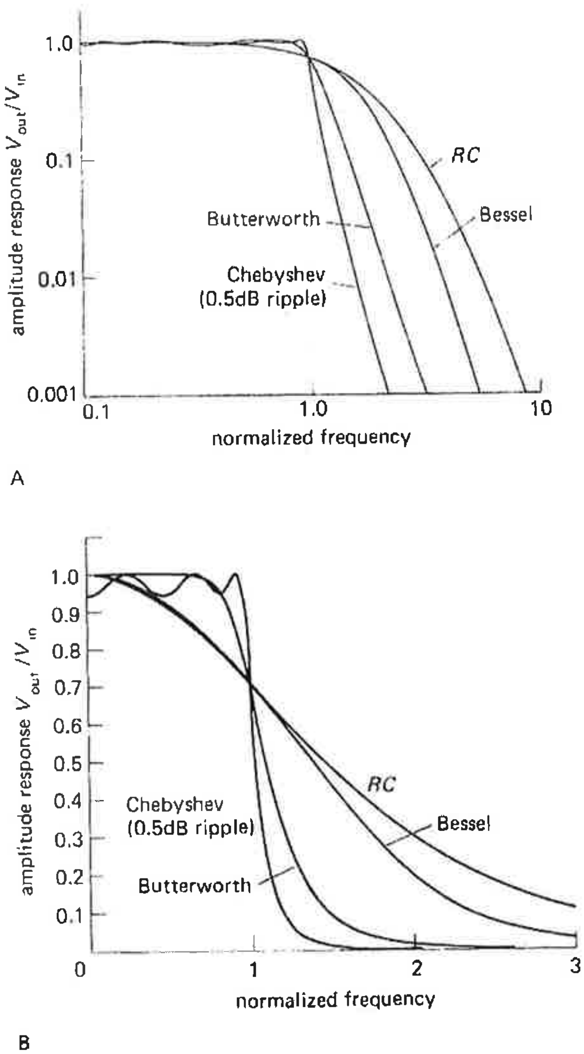

Figure 5.11. Comparison of some common 6-pole low-pass filters. The same filters are plotted on both linear and logarithmic scales.

Figure 5.11 presents graphs comparing the responses of Chebyshev and Butterworth 6-pole low-pass filters. As you can see, they're both tremendous improvements over a 6-pole RC filter.

Actually, the Butterworth, with its maximally flat passband, is not as attractive as it might appear, since you are always accepting some variation in passband response anyway (with the Butterworth it is a gradual rolloff near f_c, whereas with the Chebyshev it is a set of ripples spread throughout the passband). Furthermore, active filters constructed with components of finite tolerance will deviate from the predicted response, which means that a real Butterworth filter will exhibit some passband ripple anyway. The graph in Figure 5.12 illustrates the effects of worst-case variations in resistor and capacitor values on filter response.

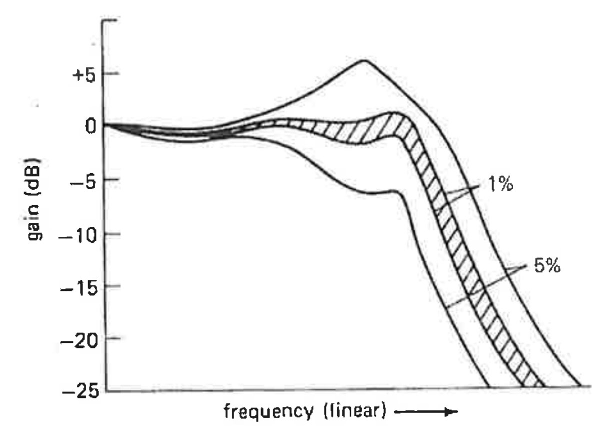

Figure 5.12. The effect of component tolerance on active filter performance.

Viewed in this light, the Chebyshev is a very rational filter design. It is sometimes called an equiripple filter: It manages to improve the situation in the transition region by spreading equal-size ripples throughout the passband, the number of ripples increasing with the order of the filter. Even with rather small ripples (as little as 0.1dB) the Chebyshev filter offers considerably improved sharpness of the knee

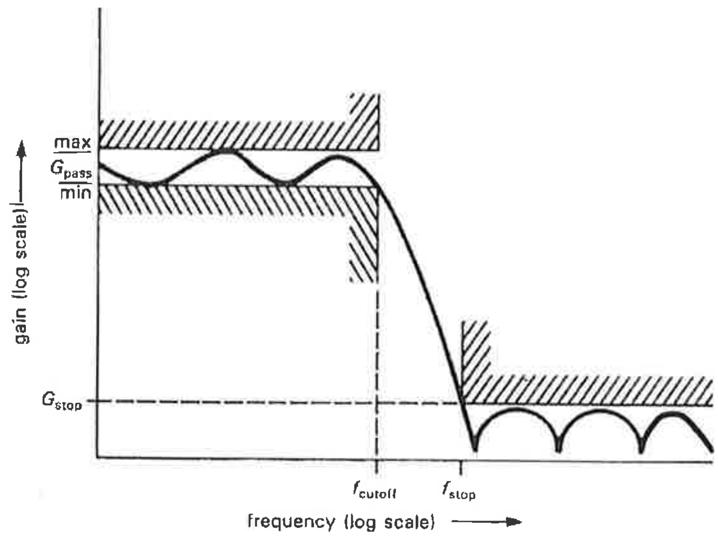

Figure 5.13. Specifying filter frequency response parameters.

as compared with the Butterworth. To make the improvement quantitative, suppose that you need a filter with flatness to 0.1dB within the passband and 20dB attenuation at a frequency 25% beyond the top of the passband. By actual calculation, that will require a 19-pole Butterworth, but only an 8-pole Chebyshev.

The idea of accepting some passband ripple in exchange for improved steepness in the transition region, as in the equiripple Chebyshev filter, is carried to its logical limit in the so-called elliptic (or Cauer) filter by trading ripple in both passband and stopband for an even steeper transition region than that of the Chebyshev filter. With computer-aided design techniques, the design of elliptic filters is as straightforward as for the classic Butterworth and Chebyshev filters.

Figure 5.13 shows how you specify filter frequency response graphically. In this case (a low-pass filter) you indicate the allowable range of filter gain (i.e., the ripple) in the passband, the minimum frequency at which the response leaves the passband, the maximum frequency at which the response enters the stopband, and the minimum attenuation in the stopband.

Bessel filter

As we hinted earlier, the amplitude response of a filter does not tell the whole story. A filter characterized by a flat amplitude response may have large phase shifts. The result is that a signal in the passband will suffer distortion of its waveform. In situations where the shape of the waveform is paramount, a linear-phase filter (or constant-time-delay filter) is desirable. A filter whose phase shift varies linearly with frequency is equivalent to a constant time delay for signals within the passband, i.e., the waveform is not distorted. The Bessel filter (also called the Thomson filter) had maximally flat time delay within its passband, in analogy with the Butterworth, which has maximally flat amplitude response. To see the kind of improvement in time-domain performance you get with the Bessel filter, look at Figure 5.14 for a comparison of time delay versus normalized frequency for 6-pole Bessel and Butterworth low-pass filters. The poor time-delay performance of the Butterworth gives rise to effects such as overshoot when driven with pulse signals. On the other hand, the price you pay for the Bessel's constancy of time delay is an amplitude response with even less steepness than that of the Butterworth in the transition region between passband and stopband.

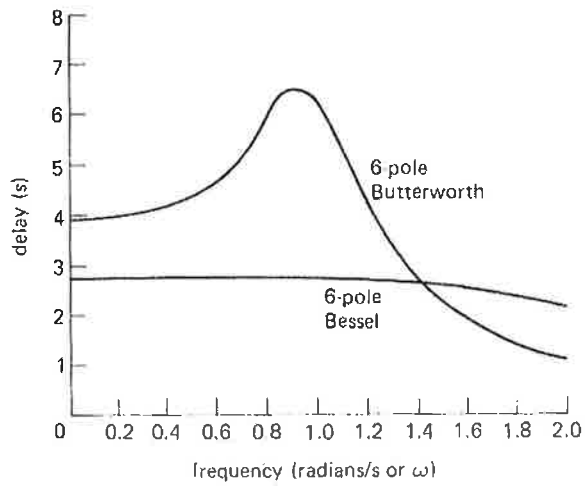

Figure 5.14. Comparison of time delays for 6-pole Bessel and Butterworth low-pass filters. The excellent time-domain performance of the Bessel filter minimizes waveform distortion.

There are numerous filter designs that attempt to improve on the Bessel's good time-domain performance by compromising some of the constancy of time delay for improved rise time and amplitude-versus-frequency characteristics. The Gaussian filter has phase characteristics nearly as good as those of the Bessel, with improved step response. In another class there are interesting filters that allow uniform ripples in the passband time delay (in analogy with the Chebyshev's ripples in its amplitude response) and yield approximately constant time delays even for signals well into the stopband. Another approach to the problem of getting filters with uniform time delays is to use all-pass filters, also known as delay equalizers. These have constant amplitude response with frequency, with a phase shift that can be tailored to individual requirements. Thus, they can be used to improve the time-delay constancy of any filter, including Butterworth and Chebyshev types.

Filter comparison

In spite of the preceding comments about the Bessel filter's transient response, it still has vastly superior properties in the time domain, as compared with the Butterworth and Chebyshev. The Chebyshev, with its highly desirable amplitude-versus-frequency characteristics, actually has the poorest time-domain performance of the three. The Butterworth is in between in both frequency and time-domain properties. Table 5.1 and Figure 5.15 give more information about time-domain performance for these three kinds of filters to complement the frequency-domain graphs presented earlier. They make it clear that the Bessel is a very desirable filter where performance in the time domain is important.

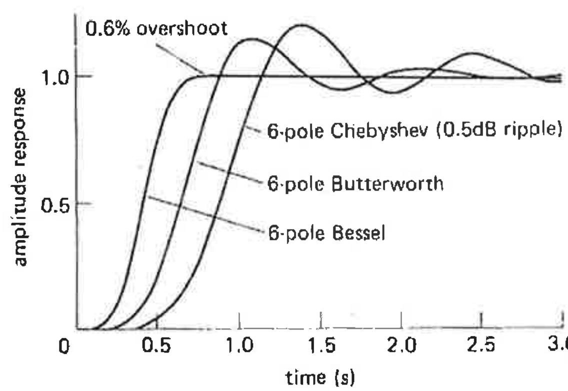

Figure 5.15. Step-response comparison for 6-pole low-pass filters normalized for 3dB attenuation at 1Hz.

ACTIVE FILTER CIRCUITS

A lot of ingenuity has been used in inventing clever active circuits, each of which can be used to generate response functions such as the Butterworth, Chebyshev, etc. You might wonder why the world needs more than one active filter circuit. The reason is that various circuit realizations excel in one or another desirable property, so there is no all-around best circuit.

Some of the features to look for in active filters are (a) small numbers of parts, both

TABLE 5.1. TIME-DOMAIN PERFORMANCE COMPARISON FOR LOW-PASS FILTERS[a]

Type	f_{3dB} (Hz)	Poles	Step rise time (0 to 90%) (s)	Over-shoot (%)	Settling time to 1% (s)	Settling time to 0.1% (s)	Stopband attenuation f = 2f_c (dB)	Stopband attenuation f = 10f_c (dB)
Bessel	1.0	2	0.4	0.4	0.6	1.1	10	36
(-3.0dB at	1.0	4	0.5	0.8	0.7	1.2	13	66
f_c = 1.0Hz)	1.0	6	0.6	0.6	0.7	1.2	14	92
	1.0	8	0.7	0.3	0.8	1.2	14	114
Butterworth	1.0	2	0.4	4	0.8	1.7	12	40
(-3.0dB at	1.0	4	0.6	11	1.0	2.8	24	80
f_c = 1.0Hz)	1.0	6	0.9	14	1.3	3.9	36	120
	1.0	8	1.1	16	1.6	5.1	48	160
Chebyshev	1.39	2	0.4	11	1.1	1.6	8	37
0.5dB ripple	1.09	4	0.7	18	3.0	5.4	31	89
(-0.5dB at	1.04	6	1.1	21	5.9	10.4	54	141
f_c = 1.0Hz)	1.02	8	1.4	23	8.4	16.4	76	193
Chebyshev	1.07	2	0.4	21	1.6	2.7	15	44
2.0dB ripple	1.02	4	0.7	28	4.8	8.4	37	96
(-2.0dB at	1.01	6	1.1	32	8.2	16.3	60	148
f_c = 1.0Hz)	1.01	8	1.4	34	11.6	24.8	83	200

(a) a design procedure for these filters is presented in Section 5.07.

active and passive, (b) ease of adjustability, (c) small spread of parts values, especially the capacitor values, (d) undemanding use of the op-amp, especially requirements on slew rate, bandwidth, and output impedance, (e) the ability to make high-Q filters, and (f) sensitivity of filter characteristics to component values and op-amp gain (in particular, the gain-bandwidth product, f_T). In many ways the last feature is one of the most important. A filter that requires parts of high precision is difficult to adjust, and it will drift as the components age; in addition, there is the nuisance that it requires components of good initial accuracy. The VCVS circuit probably owes most of its popularity to its simplicity and its low parts count, but it suffers from high sensitivity to component variations. By comparison, recent interest in more complicated filter realizations is motivated by the benefits of insensitivity of filter properties to small component variability.

In this section we will present several circuits for low-pass, high-pass, and bandpass active filters. We will begin with the popular VCVS, or controlled-source type, then show the state-variable designs available as integrated circuits from several manufacturers, and finally mention the twin-T sharp rejection filter and some interesting new directions in switched-capacitor realizations.

5.06 VCVS circuits

The voltage-controlled voltage-source (VCVS) filter, also known simply as a controlled-source filter, is a variation of the Sallen-and-Key circuit shown earlier. It replaces the unity-gain follower with a noninverting amplifier of gain greater than 1. Figure 5.16 shows the circuits for low-pass, high-pass, and bandpass realizations. The resistors at the outputs of the op-amps create a noninverting voltage amplifier

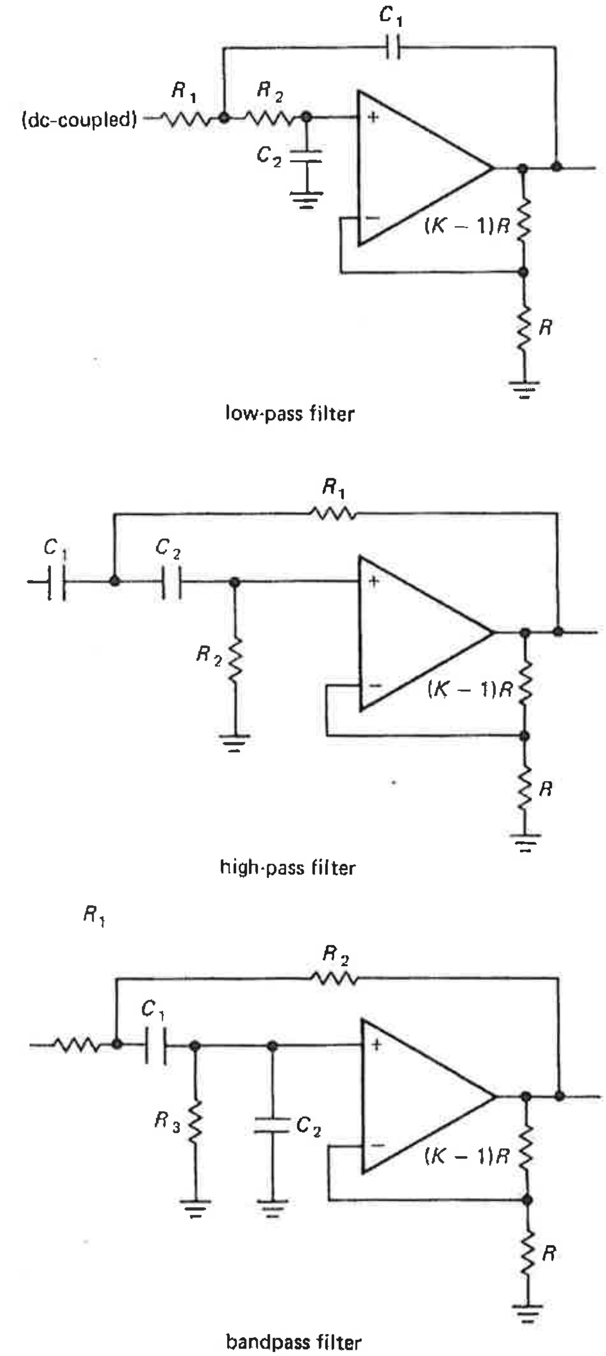

Figure 5.16. VCVS active filter circuits.

of voltage gain K, with the remaining Rs and Cs contributing the frequency response properties for the filter. These are 2-pole filters, and they can be Butterworth, Bessel, etc., by suitable choice of component values, as we will show later. Any number of VCVS 2-pole sections may be cascaded to generate higher-order filters. When that is done, the individual filter sections are, in general, not identical. In fact, each section represents a quadratic polynomial factor of the nth-order polynomial describing the overall filter.

There are design equations and tables in most standard filter handbooks for all the standard filter responses, usually including separate tables for each of a number of ripple amplitudes for Chebyshev filters. In the next section we will present an easy-to-use design table for VCVS filters of Butterworth, Bessel, and Chebyshev responses (0.5dB and 2dB passband ripple for Chebyshev filters) for use as low-pass or high-pass filters. Bandpass and band-reject filters can be easily made from combinations of these.

5.07 VCVS filter design using our simplified table

To use Table 5.2, begin by deciding which filter response you need. As we mentioned earlier, the Butterworth may be attractive if maximum flatness of passband is desired, the Chebyshev gives the fastest roll-off from passband to stopband (at the

TABLE 5.2. VCVS LOW-PASS FILTERS

Poles	Butter-worth	Bessel		Chebyshev (0.5dB)		Chebyshev (2.0dB)	
	K	f_n	K	f_n	K	f_n	K
2	1.586	1.272	1.268	1.231	1.842	0.907	2.114
4	1.152	1.432	1.084	0.597	1.582	0.471	1.924
	2.235	1.606	1.759	1.031	2.660	0.964	2.782
6	1.068	1.607	1.040	0.396	1.537	0.316	1.891
	1.586	1.692	1.364	0.768	2.448	0.730	2.648
	2.483	1.908	2.023	1.011	2.846	0.983	2.904
8	1.038	1.781	1.024	0.297	1.522	0.238	1.879
	1.337	1.835	1.213	0.599	2.379	0.572	2.605
	1.889	1.956	1.593	0.861	2.711	0.842	2.821
	2.610	2.192	2.184	1.006	2.913	0.990	2.946

expense of some ripple in the passband), and the Bessel provides the best phase characteristics, i.e., constant signal delay in the passband, with correspondingly good step response. The frequency responses for all types are shown in the accompanying graphs (Fig. 5.17).

To construct an n-pole filter (n is an even number), you will need to cascade $n/2$ VCVS sections. Only even-order filters are shown, since an odd-order filter requires as many op-amps as the next higher-order filter. Within each section, $R_1 = R_2 = R$, and $C_1 = C_2 = C$. As is usual in op-amp circuits, R will typically be chosen in the range 10k to 100k. (It is best to avoid small resistor values, because the rising open-loop output impedance of the op-amp at high frequencies adds to the resistor values and upsets calculations.) Then all you need to do is set the gain, K, of each stage according to the table entries. For an n-pole filter there are $n/2$ entries, one for each section.

Butterworth low-pass filters

If the filter is a Butterworth, all sections have the same values of R and C, given simply by $RC = 1/2\pi f_c$, where f_c is the desired -3dB frequency of the entire filter. To make a 6-pole low-pass Butterworth filter, for example, you cascade three of the low-pass sections shown previously, with gains of 1.07, 1.59, and 2.48 (preferably in that order, to avoid dynamic range problems), and with identical Rs and Cs to set the 3dB point. The telescope drive circuit in Section 8.31 shows such an example, with $f_c = 88.4$Hz ($R = 180$k, $C = 0.01\mu$F).

Bessel and Chebyshev low-pass filters

To make a Bessel or Chebyshev filter with the VCVS, the situation is only slightly more complicated. Again we cascade several 2-pole VCVS filters, with prescribed gains for each section. Within each section we again use $R_1 = R_2 = R$, and $C_1 = C_2 = C$. However, unlike the situation with the Butterworth, the RC products for the different sections are different and must be scaled by the normalizing factor f_n (given for each section in Table 5.2) according to $RC = 1/2\pi f_n f_c$. Here f_c is again the -3dB point for the Bessel filter, whereas for the Chebyshev filter it defines the end of the passband, i.e., it is the frequency at which the amplitude response falls out of the ripple band on its way into the stopband. For example, the response of a Chebyshev low-pass filter with 0.5dB ripple and $f_c = 100$Hz will be flat within +0dB to -0.5dB from dc to 100Hz, with 0.5dB attenuation at 100Hz and a rapid falloff for frequencies greater than 100Hz. Values are given for Chebyshev filters with 0.5dB and 2.0dB passband ripple; the latter have a somewhat steeper transition into the stopband (Fig. 5.17).

High-pass filters

To make a high-pass filter, use the high-pass configuration shown previously, i.e., with the Rs and Cs interchanged. For Butterworth filters, everything else remains unchanged (use the same values for R, C, and K). For the Bessel and Chebyshev filters, the K values remain the same, but the normalizing factors f_n must be inverted, i.e., for each section the new f_n equals $1/(f_n$ listed in Table 5.2).

A bandpass filter can be made by cascading overlapping low-pass and high-pass filters. A band-reject filter can be made by summing the outputs of nonoverlapping low-pass and high-pass filters. However, such cascaded filters won't work well for high-Q filters (extremely sharp bandpass filters) because there is great sensitivity to the component values in the individual (uncoupled) filter sections. In such cases a high-Q single-stage bandpass circuit (e.g., the VCVS bandpass circuit

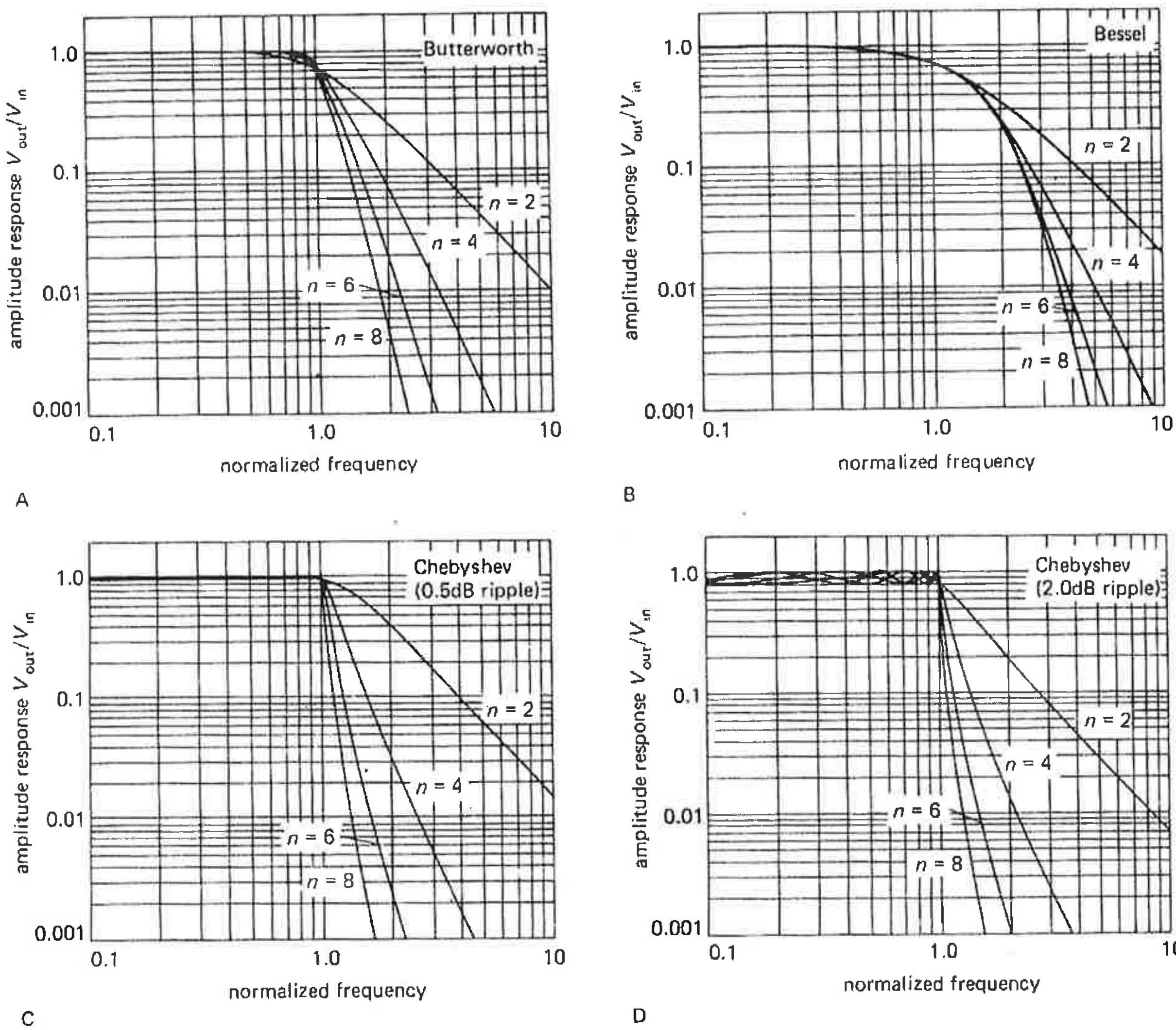

Figure 5.17. Normalized frequency response graphs for the 2-, 4-, 6-, and 8-pole filters in Table 5.2. The Butterworth and Bessel filters are normalized to 3dB attenuation at unit frequency, whereas the Chebyshev filters are normalized to 0.5dB and 2dB attenuations.

illustrated previously, or the state-variable and biquad filters in the next section) should be used instead. Even a single-stage 2-pole filter can produce a response with an extremely sharp peak. Information on such filter design is available in the standard references.

VCVS filters minimize the number of components needed (2 poles/op-amp) and offer the additional advantages of noninverting gain, low output impedance, small spread of component values, easy adjustability of gain, and the ability to operate at high gain or high Q. They suffer from high sensitivity to component values and amplifier gain, and they don't lend themselves well to applications where a tunable filter of stable characteristics is needed.

EXERCISE 5.3

Design a 6-pole Chebyshev low-pass VCVS filter with a 0.5dB passband ripple and 100Hz cutoff frequency f_C. What is the attenuation at $1.5f_C$?

5.08 State-variable filters

The 2-pole filter shown in Figure 5.18 is far more complex than the VCVS circuits,