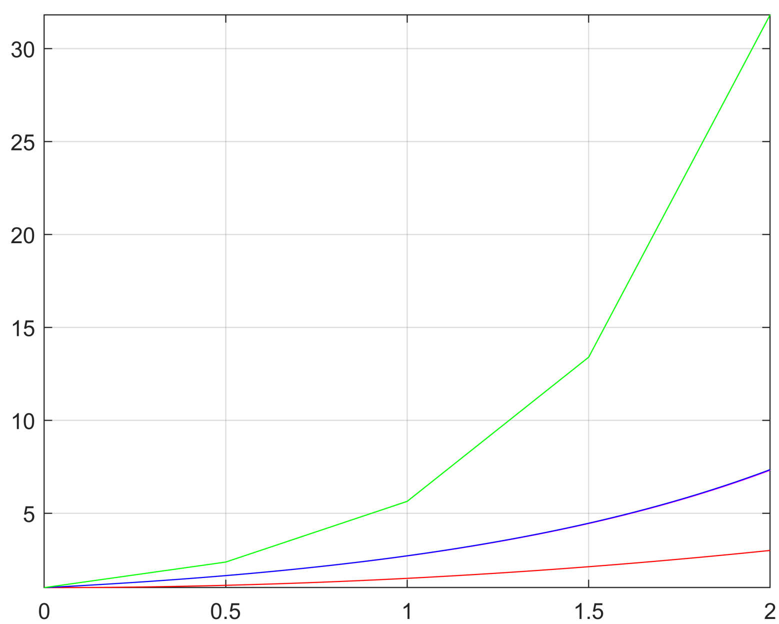
# Ecuaciones lineales de primer orden

# Problemas de valor inicial

Use separation of variables to find solutions of the IVP given by y(0) = 1 and the following differential equations:

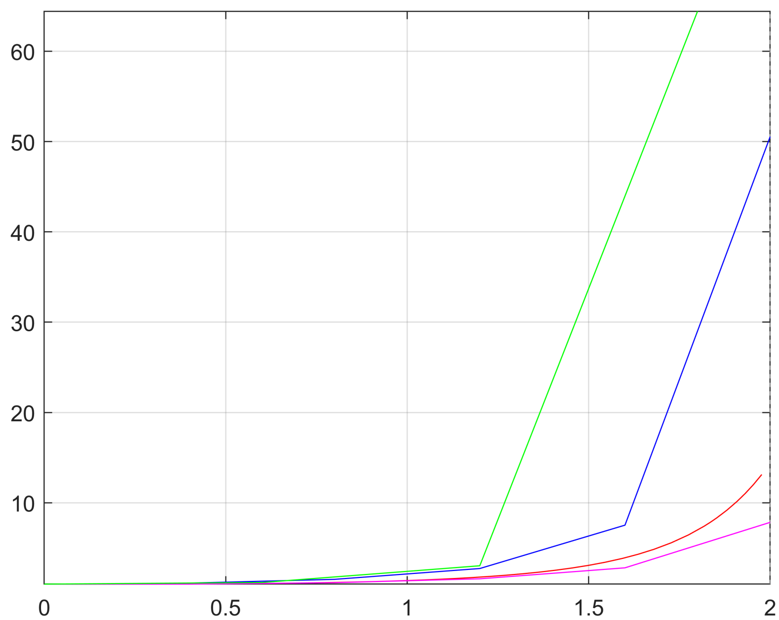
(a) $\frac{\mathrm{d}}{\mathrm{d}t}y = t$ (b) $\frac{\mathrm{d}}{\mathrm{d}t}y = t^2 y$ (c) $\frac{\mathrm{d}}{\mathrm{d}t}y = 2(t+1)y$

```
% t en [0,2]

%a)
y=@(t) 1+t.^2/2;
fplot(y,[0 2], 'r')
hold on
grid on
f= @(y,t) t;
[t,y] = odeEuler(f,1,0,2,0.01);
plot(t,y,'magenta')
[t,y] = odeMidpoint(f,1,0,2,0.01);
plot(t,y,'blue')
[t,y] = odeTrapecio(f,1,0,2,0.5);
plot(t,y,'green')
hold off
```

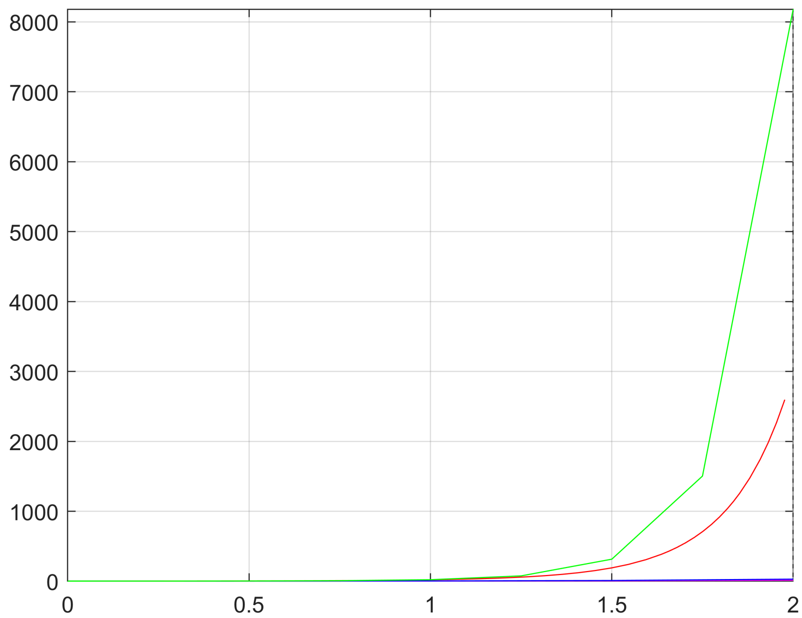



```
%b)

y=@(t) exp(t.^3/3);
fplot(y,[0 2], 'r')
hold on
```

```
grid on
f= @(y,t) (t.^2)*y;
[t,y] = odeEuler(f,1,0,2,0.4);
plot(t,y,'m')
[t,y] = odeMidpoint(f,1,0,2,0.4);
plot(t,y,'b')
[t,y] = odeTrapecio(f,1,0,2,0.6);
plot(t,y,'g')
hold off
```




```
%c)

y=@(t) exp(t.^2+2*t);
fplot(y,[0 2], 'r')
hold on
grid on
f= @(y,t) 2*(t+1)*y;
[t,y] = odeEuler(f,1,0,2,0.5);
plot(t,y,'m')
[t,y] = odeMidpoint(f,1,0,2,0.5);
plot(t,y,'b')
[t,y] = odeTrapecio(f,1,0,2,0.25);
plot(t,y,'g')
hold off
```

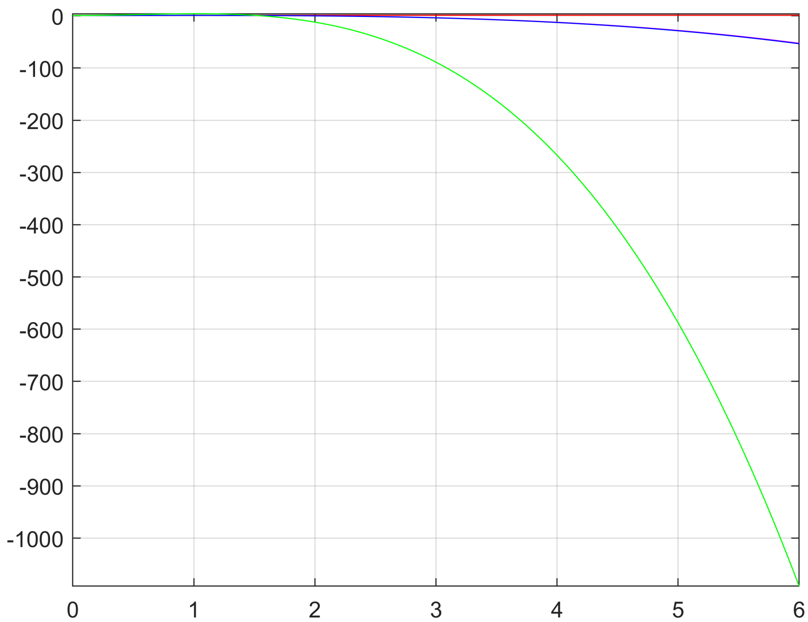



$y' = y(1-y) \qquad y(0) = 1/2$

```
% t en [0,6]
y=@(t) exp(t)/(1+exp(t));
fplot(y,[0 6] , 'r')
```

```
Warning: Function behaves unexpectedly on array inputs. To improve performance, properly vectorize your
 function to return an output with the same size and shape as the input arguments.
```

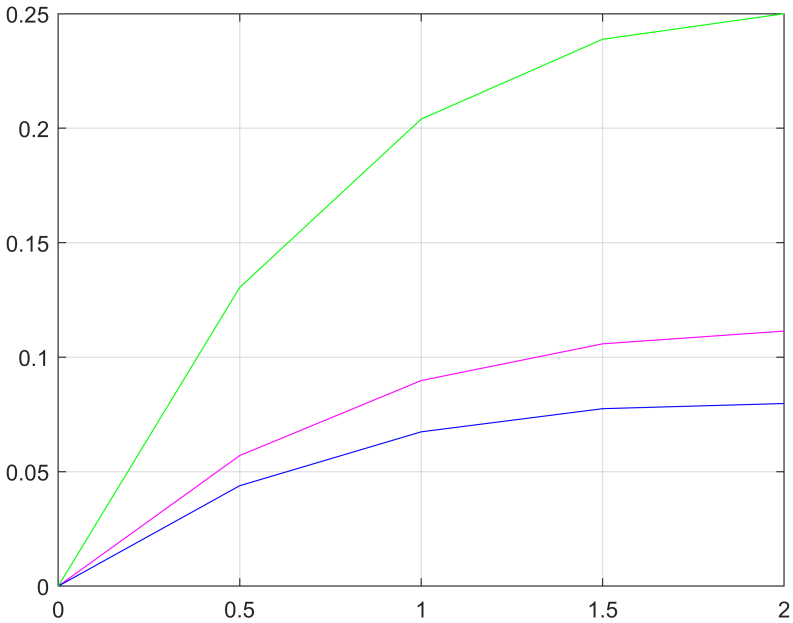
```
hold on
grid on
f= @(y,t) y*(1-y);
[t,y] = odeEuler(f,0.5,0,6,0.05);
plot(t,y,'m')
[t,y] = odeMidpoint(f,0.5,0,6,0.05);
plot(t,y,'b')
[t,y] = odeTrapecio(f,0.5,0,6,0.05);
plot(t,y,'g')
hold off
```




$y' = k(a_0 - y)^2(b_0 - y/2) \qquad y(0) = 0$

$k = 0.00713;\ a_0 = 4;\ b_0 = 1$

```
% t en [0,400]
k=0.00713;
a0=4;
b0=1;


f= @(y,t) k*((a0-y).^2)*(b0-y/2);
[t,y] = odeEuler(f,0,0,2,0.5);
plot(t,y,'m')
hold on
grid on
[t,y] = odeMidpoint(f,0,0,2,0.5);
plot(t,y,'b')
[t,y] = odeTrapecio(f,0,0,2,0.5);
plot(t,y,'g')
hold off
```




Resuelve el siguiente problema de valor inicial en el intervalo de $t = 0$ a $t = 2$

$y' = yt^2 - 1.1y \qquad y(0) = 1$

```
% Solucion analitica
syms y(t)
eqn = diff(y,t) == y*t^2 - 1.1*y;
yG = dsolve(eqn)
```

yG =

$$C_1\, e^{\frac{t\,(10\,t^2-33)}{30}}$$

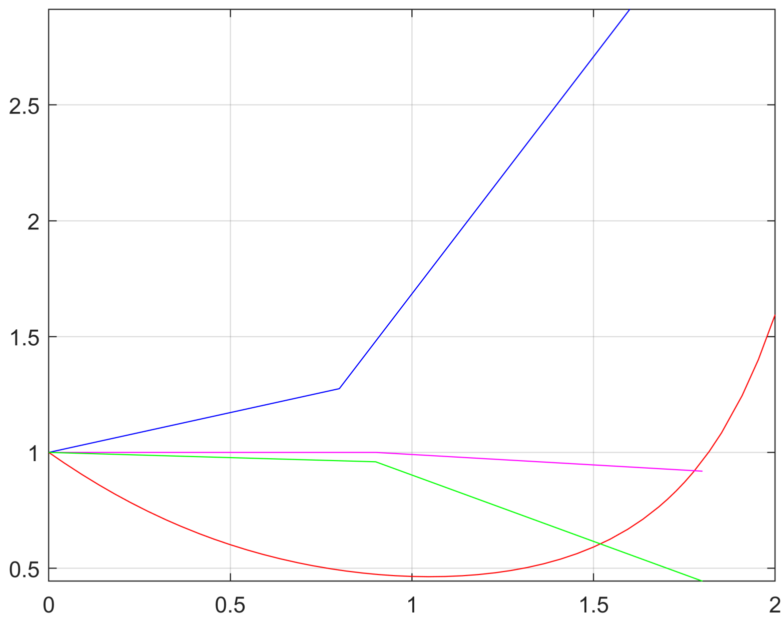
```
cond = y(0) == 1;
yP = dsolve(eqn,cond)
```

yP =

$$e^{\frac{t\,(10\,t^2-33)}{30}}$$

```
fplot(yP, [0 2],'r');
hold on
grid on

f= @(y,t) y*t.^2 -1.1*y;
[t,y] = odeEuler(f,1,0,2,0.9);
plot(t,y,'m')
[t,y] = odeMidpoint(f,1,0,2,0.8);
plot(t,y,'b')
[t,y] = odeTrapecio(f,1,0,2,0.9);
```

```
plot(t,y,'g')

hold off;
```




```

```

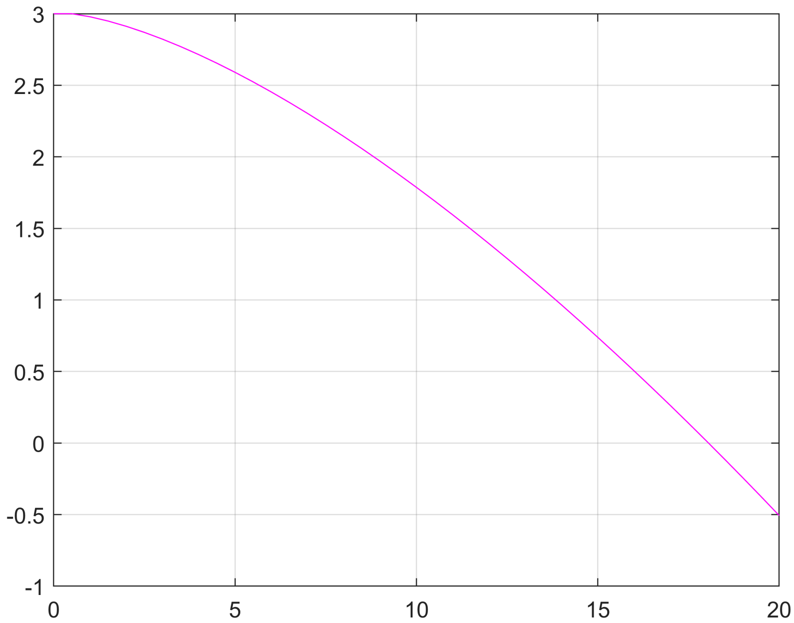
Si se drena el agua desde un tanque cilindrico vertical abriendo una válvula en la base, el líquido fluirá rápido cuando el tanque esté lleno y despacio cuando se drene. Como resultado, la tasa a la que el nivel del agua disminuye es:

$y' = -k\sqrt{y}$ (*y* en metros y *t* en minutos)

donde *k* es una constante que depende de la forma del agujero y del área de la sección transversal del tanque y del agujero.

¿En cuántos minutos se vaciará un tanque que inicialmente tiene 3 metros de agua?

```
%
k=0.06;
f= @(y,t) -k*sqrt(y);
[t,y] = odeEuler(f,3,0,20,0.5);
%Aproximadamente en el segundo 18
plot(t,y,'m')
grid on
```




Para simular la dinámica de una poblacion se utiliza el modelo logistico:

$$p' = k_{\mathrm{gm}}\left(1 - \frac{p}{p_{\max}}\right)p$$

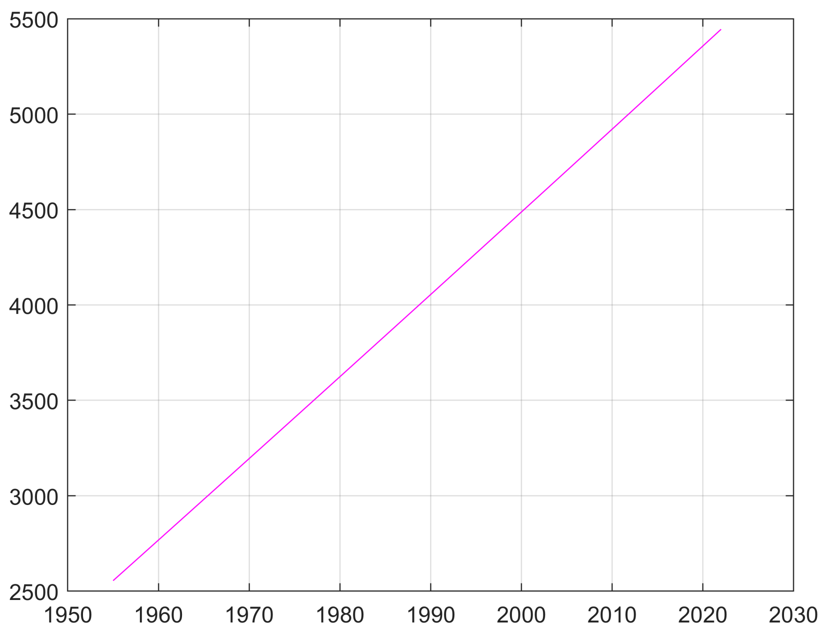
donde *p* es la población, $k_{\mathrm{gm}}$ es la tasa máxima de crecimiento en condiciones ilimitadas y $p_{\max}$ es la capacidad de carga. Simula la población mundial entre 1950 y 2022, sabiendo que en 1950 había 2555 millones de personas. Para la simulación utiliza los siguientes valores de parametros: $k_{\mathrm{gm}} = 0.026/\text{año}$ y $p_{\max} = 12000$ millones de personas-

¿Cuántos millones de personas habrá en 2022?

```
%
k=0.026;
pM=12000;
f= @(y,t) k*(1-y/pM)*y;
[t,y] = odeEuler(f,2555,1955,2022,0.5);
%Aproximadamente en el segundo 18
poblacion2022=max(y)
```

```
poblacion2022 = 5.4446e+03
```

```
plot(t,y,'m')
grid on
```




More examples

```
% y(0) = 1; 0 <=t<= 10;
% f         Exact solution
% 0         1
% t         1+t^2/2
% y         exp(t)
% -y        exp(-t)
% 2*y-y^2   2/(1+exp(-2*t))
```

```
function [t,y] = odeEuler(f,y0,t0,tf,h)
    t = t0:h:tf; %crea el vector de las ts que inicia en t0, aumenta de h en h y termina en tf
    n = length(t);
    m = length(y0);
    y = zeros(m,n);
    y(1) = y0; % valor inicial de y
    for i=1:1:n-1
        phi = f(t(i),y(i));
        y(i+1) = y(i) + phi*h;
    end
end

function [t,y] = odeMidpoint(f,y0,t0,tf,h)
    t = t0:h:tf; %crea el vector de las ts que inicia en t0, aumenta de h en h y termina en tf
    n = length(t);
    m = length(y0);
    y = zeros(m,n);
    y(1) = y0; % valor inicial de y
    aux=h/2;
```

```
    for i=1:1:n-1
        phi = f(t(i)+aux,y(i)+aux);
        y(i+1) = y(i) + phi*h;
    end
end

function [t,y] = odeTrapecio(f,y0,t0,tf,h)
    t = t0:h:tf;
    n = length(t);
    y = zeros(1,n);
    y(1) = y0; % valor inicial de y

    for i=1:1:n-1
        phi = f(t(i),y(i));
        predictor = y(i) + phi*h;
        y(i+1) = y(i) + phi+((f(t(i+1),predictor))/2)*h;
    end

end
```