Algoritmos Numéricos por Computadora

COM - 14105

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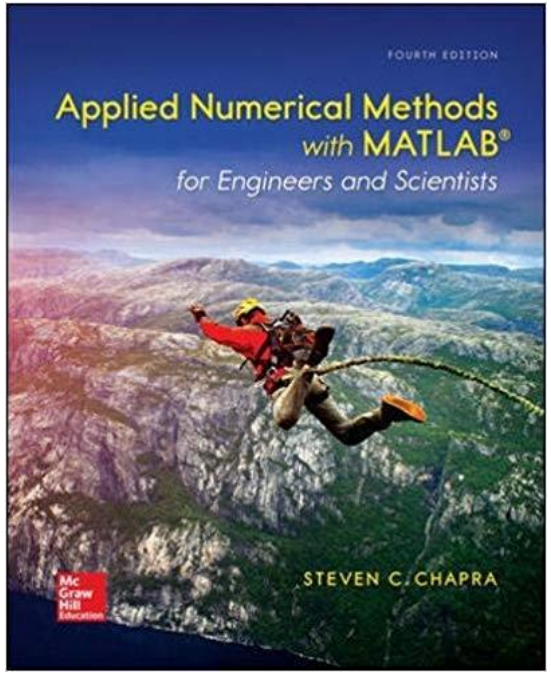
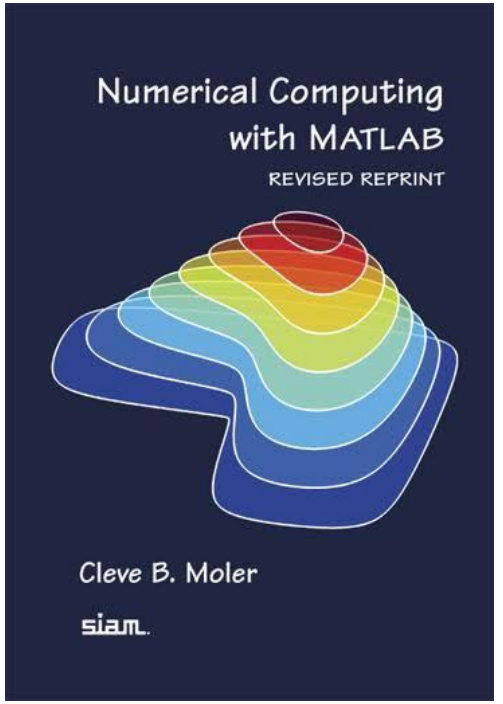
4. Ecuaciones diferenciales parciales (lineales de segundo orden)*

Bibliografía

Steven Chapra, *Applied Numerical Methods with MATLAB for Engineers and Scientists*, McGraw- Hill, Fourth edition, 2018.





Cleve Moler, *Numerical Computing with MATLAB*, SIAM, 2008.

“Simplicity is a great virtue but it requires hard work to achieve it and education to appreciate it. And to make matters worse: complexity sells better.”

Edsger Wybe Dijkstra

“Actually, a person does not really understand something until he can teach it to a computer.”

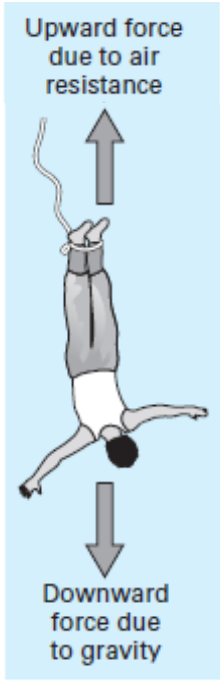
Donald Knuth

Teclado IBM PC/Compatible IBM PC/Microsoft Windows (distribución latinoamericana).

Introducción

Modelado de sistemas dinámicos

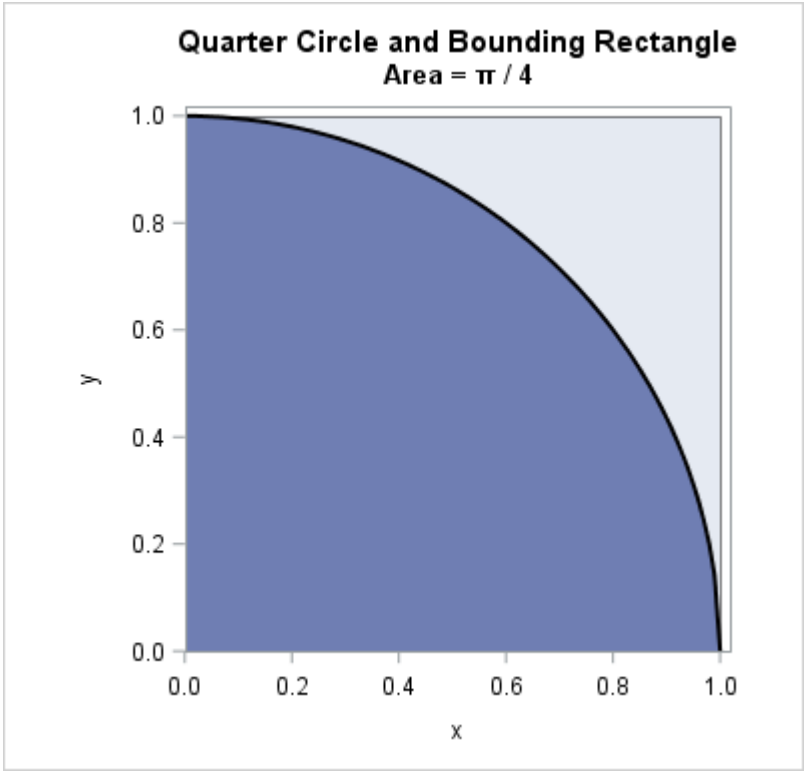
1. Fuerzas que actúan en caída libre



Introducción

Modelado de sistemas dinámicos

2. Montecarlo



https://blogs.sas.com/content/iml/2016/03/14/monte-carlo-estimates-of-pi.html

Introducción

Modelado de sistemas dinámicos

3. Sistema solar

Introducción

Truncamiento y redondeo

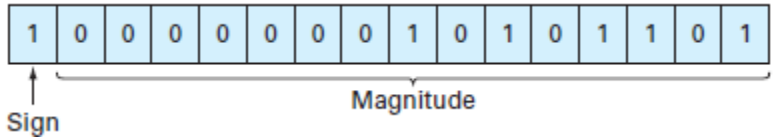
Double-precision floating-point numbers

ANSI/IEEE Standard 754

Introducción

Redondeo

¿-173 en binario?



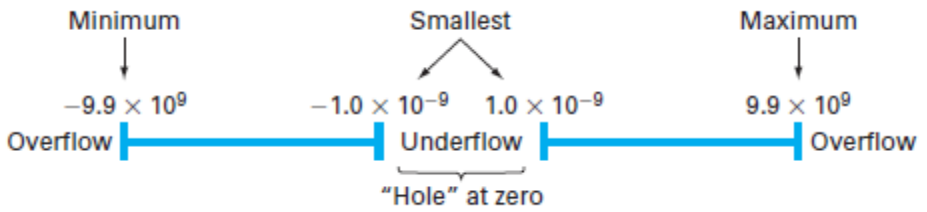
¿Números decimales?

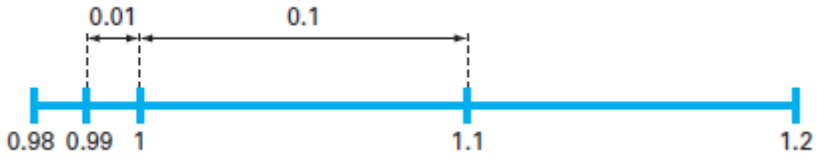
- Punto flotante
- Normalización

Ejemplo

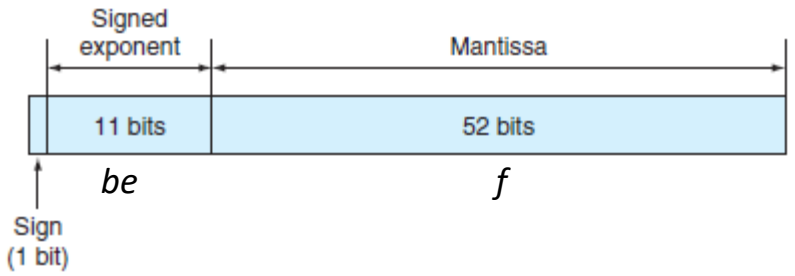
Base-10 floating-point computer with 5-digit word size

$s_1d_1d_2$ X 10^{s0d0}





IEEE double-precision format



1	2	3	. . .	12	13	. . .	64

$$x = \pm(1 + f) \cdot 2^e$$

$$0 \leq f < 1$$

$$f = (\text{integer} < 2^{52})/2^{52}$$

$$-1022 \leq e \leq 1023$$

$e = be - 1023$

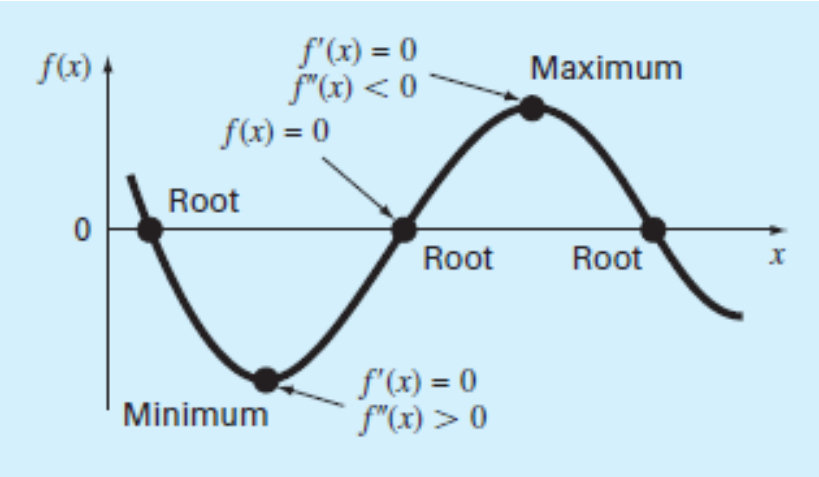
Finite f implies finite *precision*.

Finite e implies finite *range*

Floating point numbers have discrete spacing,
a maximum and a minimum.

Introducción

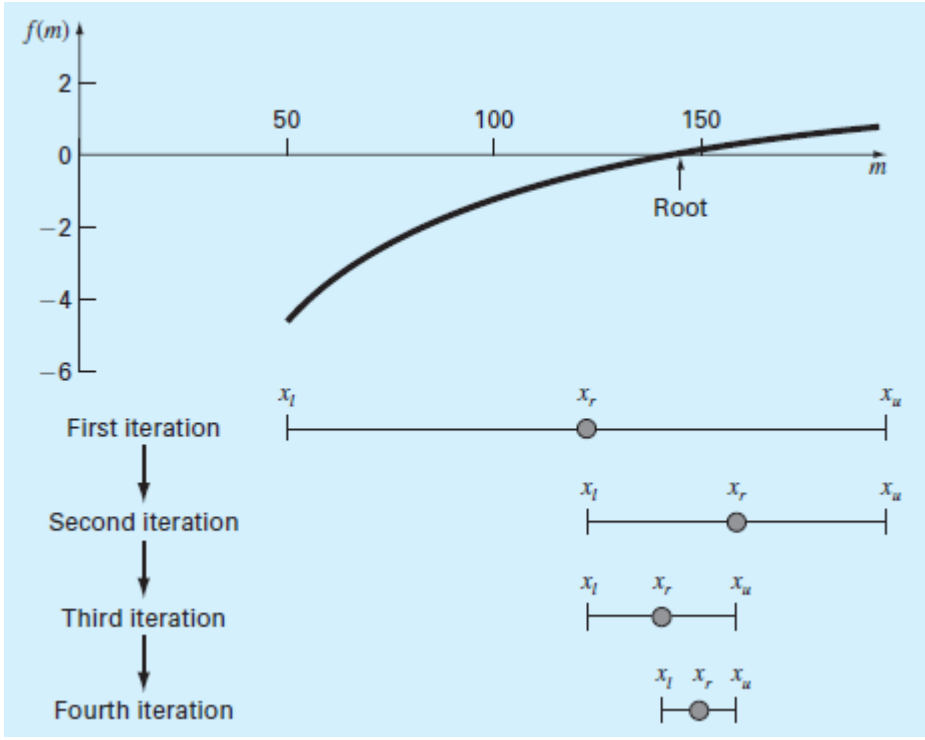
Raíces y optimización



Raíces:

- Métodos cerrados: bisección, interpolación lineal
- Métodos abiertos: Newton-Raphson, secante

Bisección



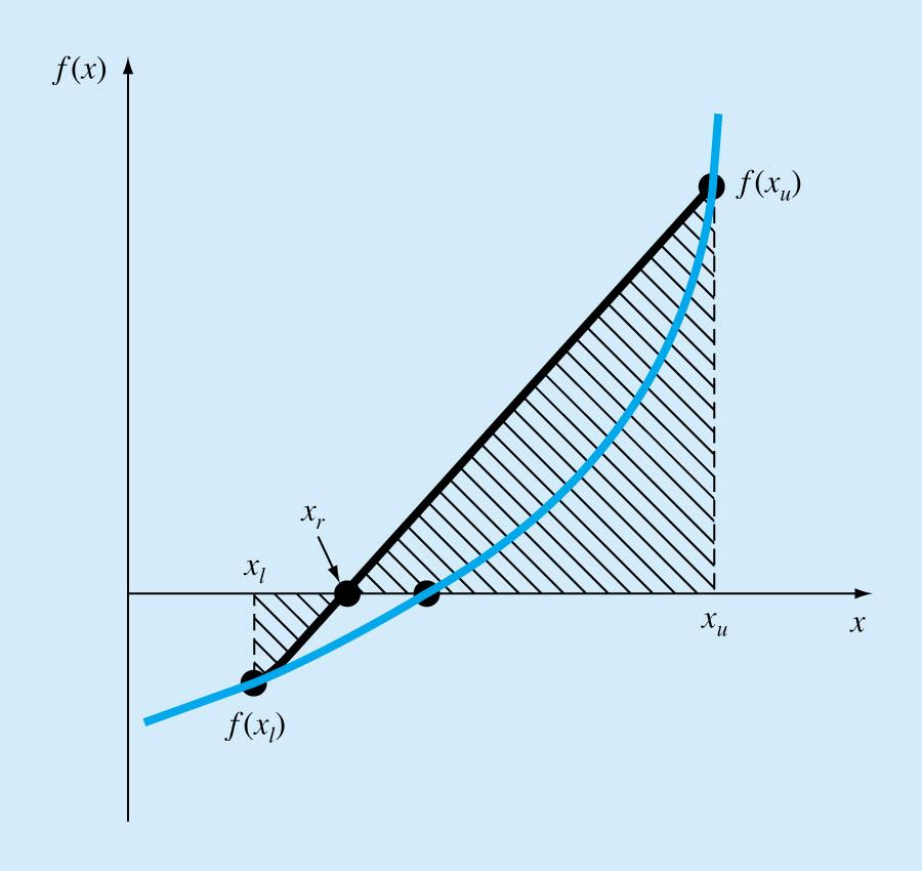
Convergencia lineal

Rapidez de convergencia

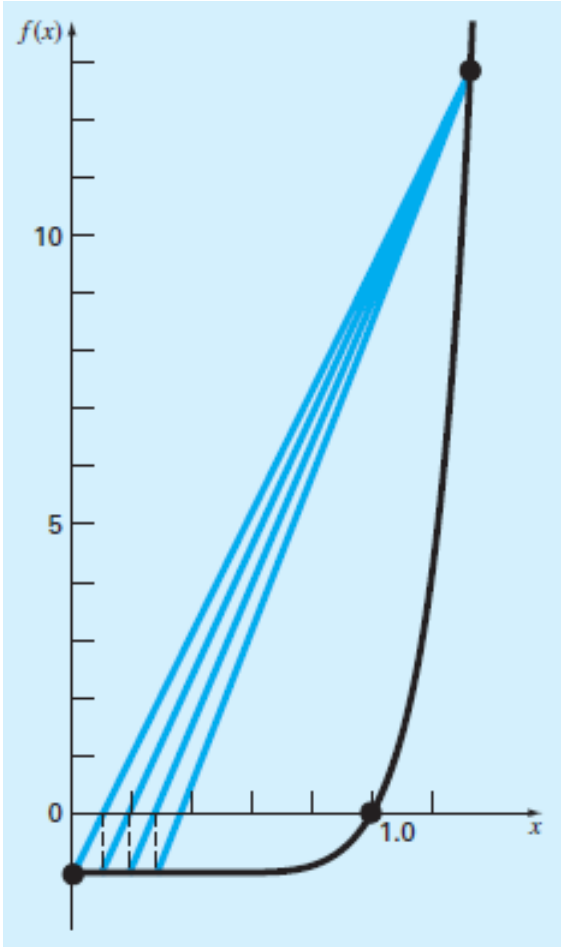
La sucesión p_n converge a p con orden α y una constante de error asintótica λ si

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^\alpha} = \lambda$$

Interpolación lineal (falsa posición)

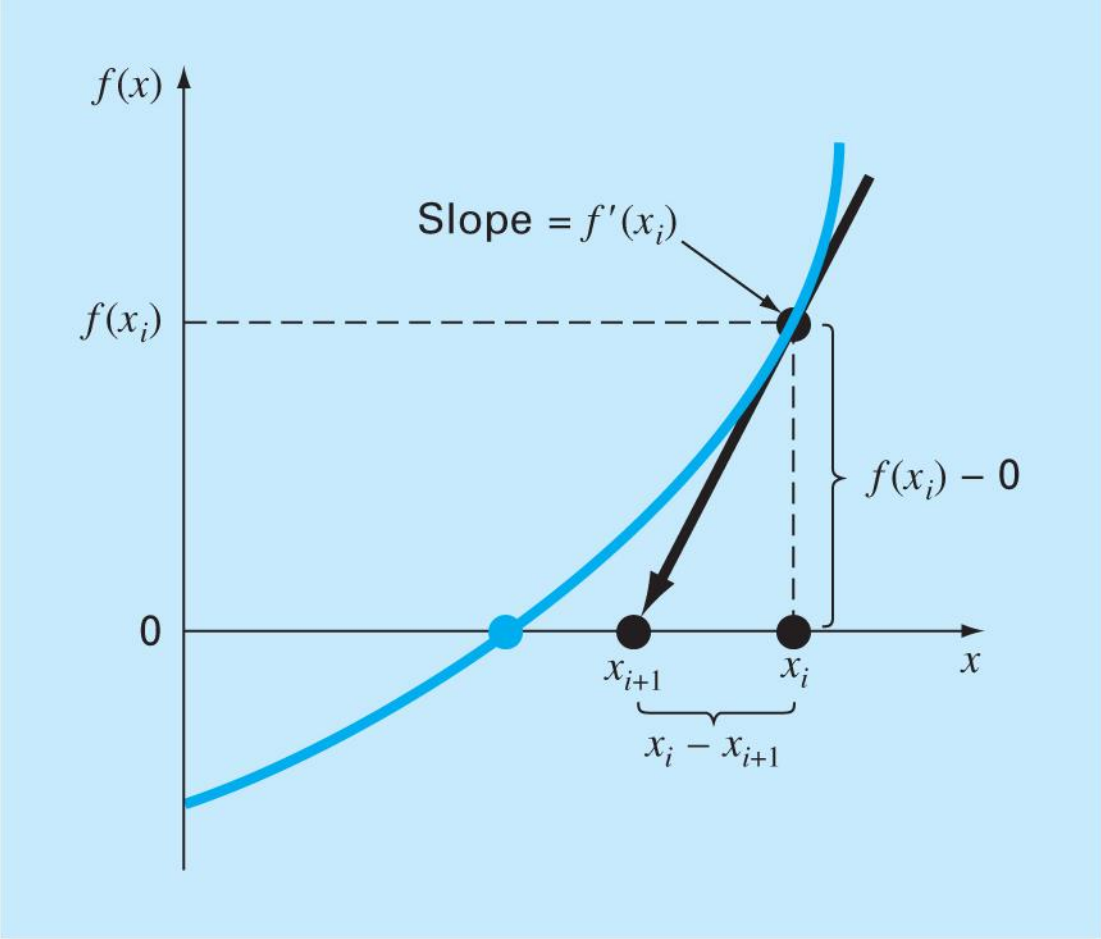


Interpolación lineal (falsa posición)



$f(x) = x^{10} - 1$

Newton-Raphson



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Newton-Raphson (raíz cuadrada de M)

$$f(x) = x^2 - M$$

$$\begin{aligned} x_{n+1} &= x_n - \frac{x_n^2 - M}{2x_n} \\ &= \frac{1}{2}\left(x_n + \frac{M}{x_n}\right) \end{aligned}$$

Newton-Raphson (convergencia cuadrática)

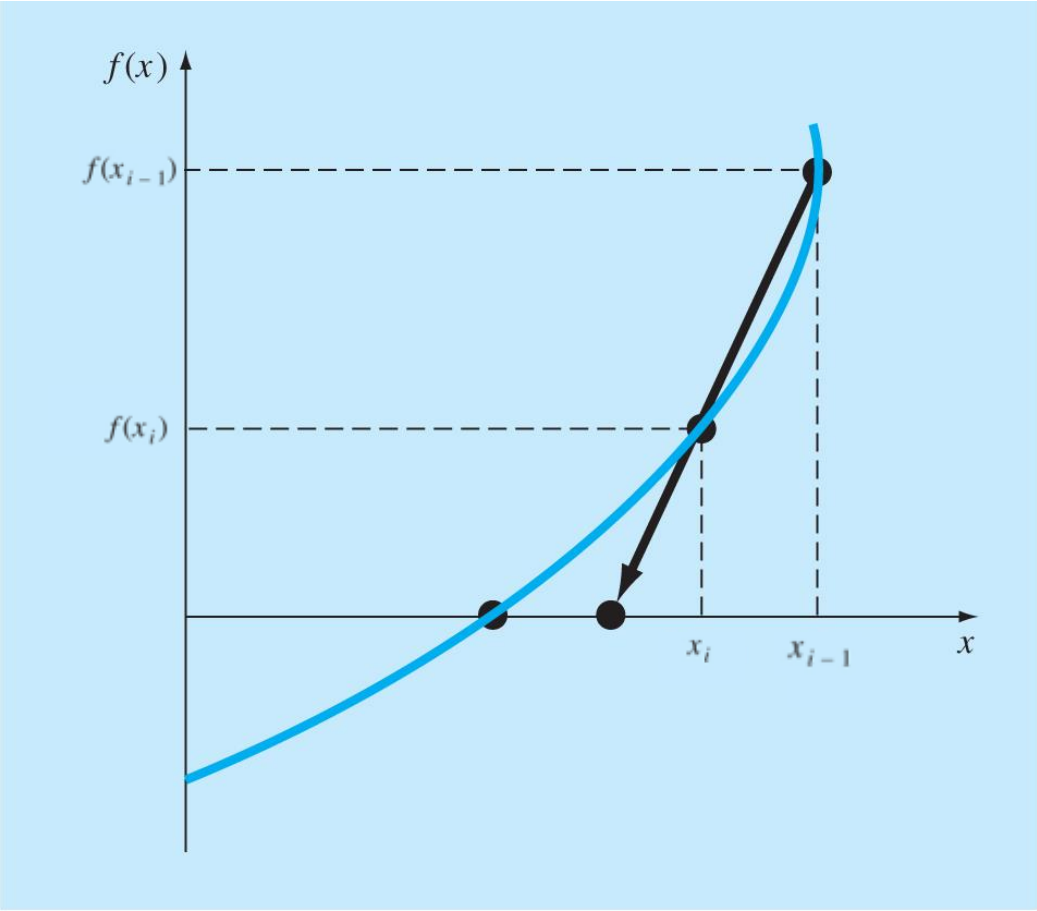
$$f(x_r) = f(x_{n+1}) + f'(x_{n+1})(x_r - x_{n+1}) + \frac{f''(\xi)}{2}(x_r - x_{n+1})^2 = 0$$

$$e_{n+1} = -\frac{1}{2}\frac{f''(\xi)}{f'(x_n)}e_n^2$$

$$e_{n+1} = O(e_n^2)$$

Secante

La derivada se aproxima con una diferencia finita hacia atrás:



$$s_n = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{s_n}$$

Problema

6.22 You are designing a spherical tank (Fig. P6.22) to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

$$V = \pi h^2 \frac{[3R - h]}{3}$$

where V = volume [m^3], h = depth of water in tank [m], and R = the tank radius [m].

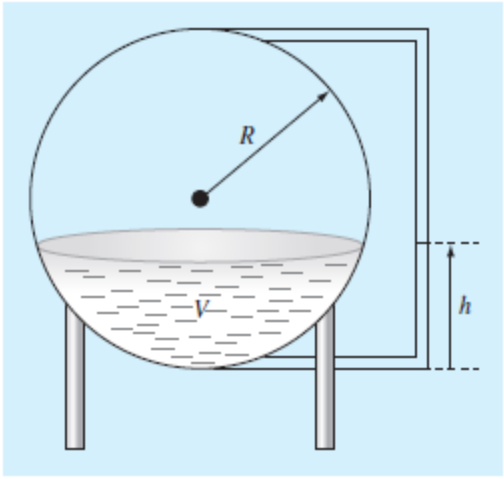
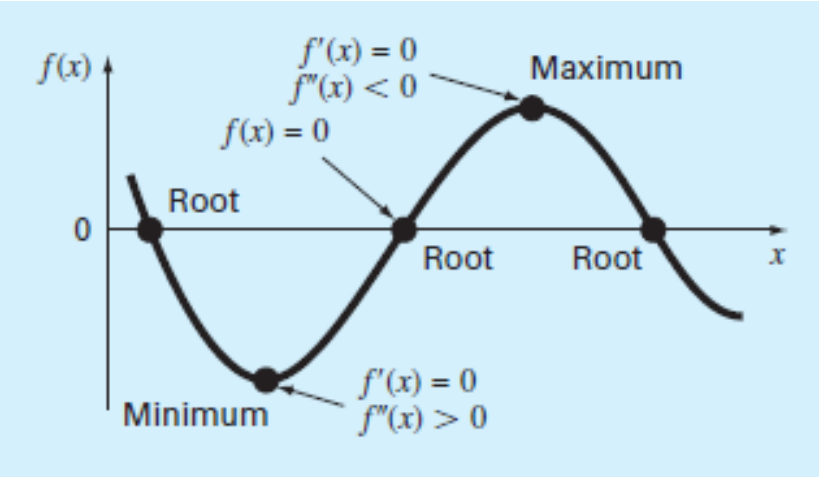


FIGURE P6.22

If $R = 3$ m, what depth must the tank be filled to so that it holds 30 m^3? Use three iterations of the most efficient numerical method possible to determine your answer. Determine the approximate relative error after each iteration. Also, provide justification for your choice of method. Extra information: **(a)** For bracketing methods, initial guesses of 0 and R will bracket a single root for this example. **(b)** For open methods, an initial guess of R will always converge.

Introducción

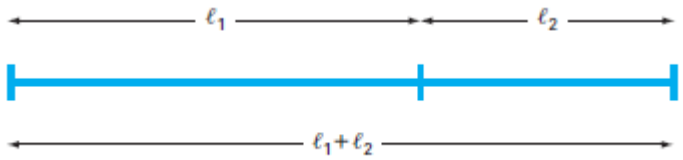
Raíces y optimización



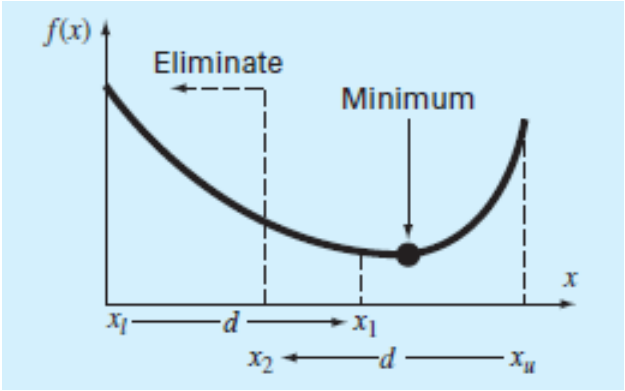
Optimización:

- Proporción áurea (golden ratio)
- Interpolación parabólica

Proporción áurea

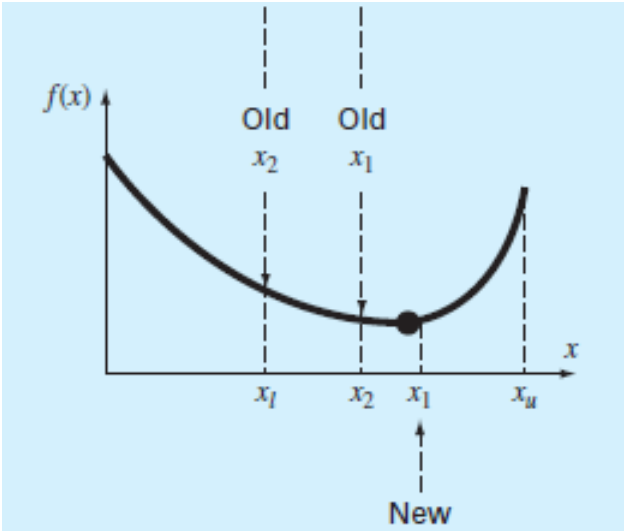


$$\frac{l_1+l_2}{l_1} = \frac{l_1}{l_2} = \varphi$$



$$d = (\emptyset - 1)(x_u - x_l)$$
$$d = g\,(x_u - x_l)$$

$$x_1 = x_l + d$$
$$x_2 = x_u - d$$

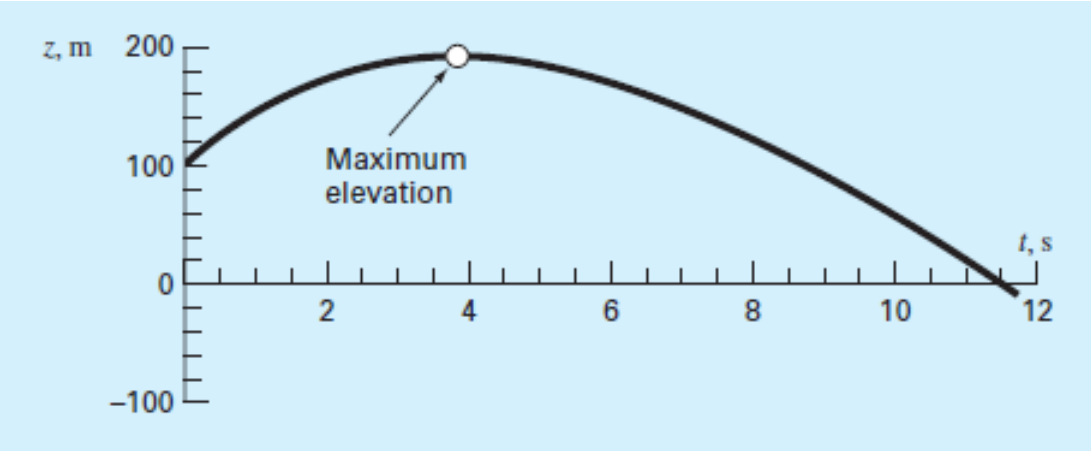


$$d_{i+1} = g\ di$$

$$x_l < x_2 < x_1 < x_u$$

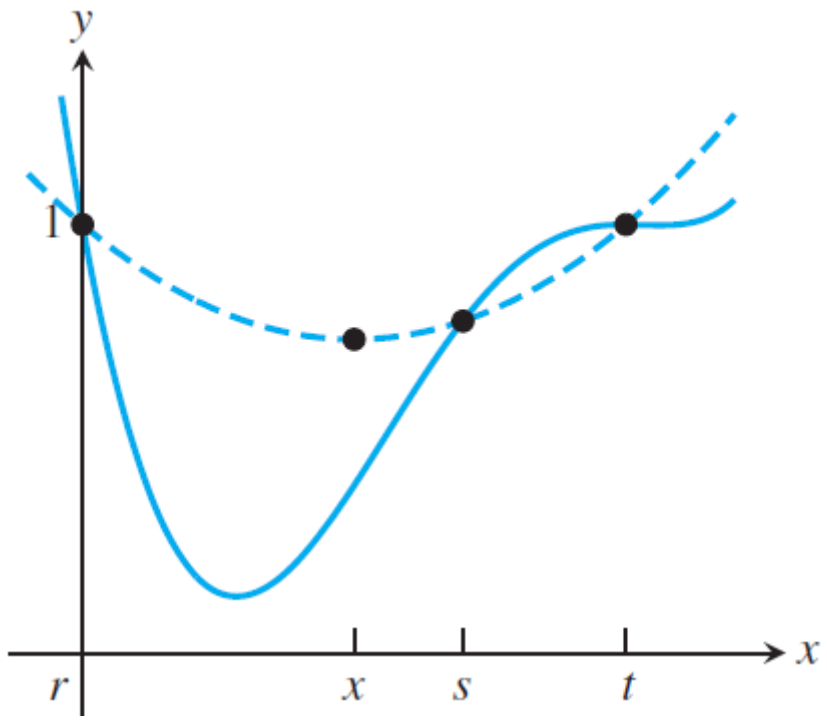
Problema

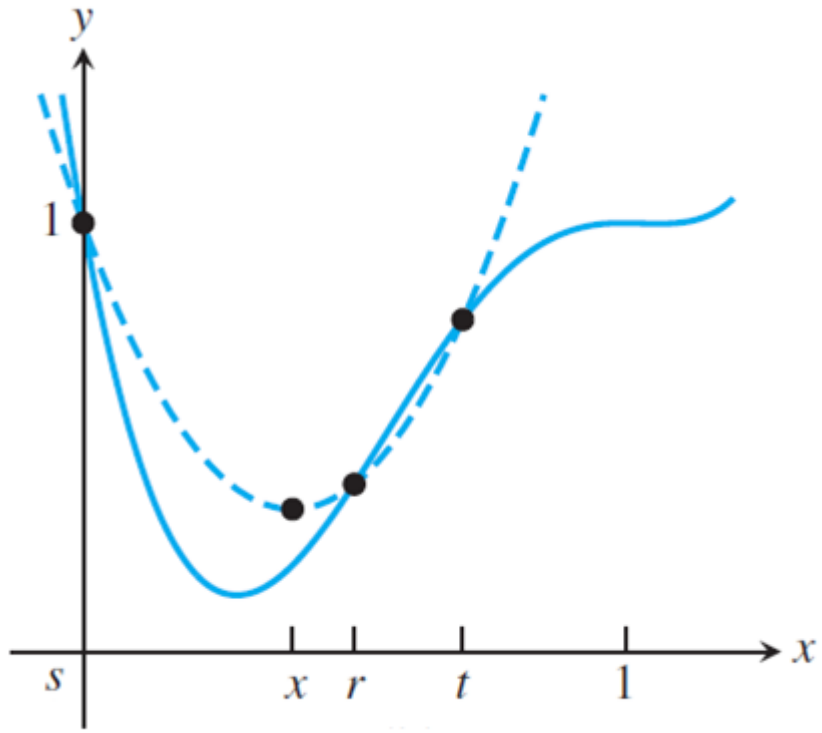
Elevación de un objeto proyectado inicialmente hacia arriba con una velocidad inicial (resistencia lineal)



$$z = z_0 + \frac{m}{c}\left(v_0 + \frac{mg}{c}\right)\left(1 - e^{-(c/m)t}\right) - \frac{mg}{c}t$$

Interpolación parabólica (sucesiva)

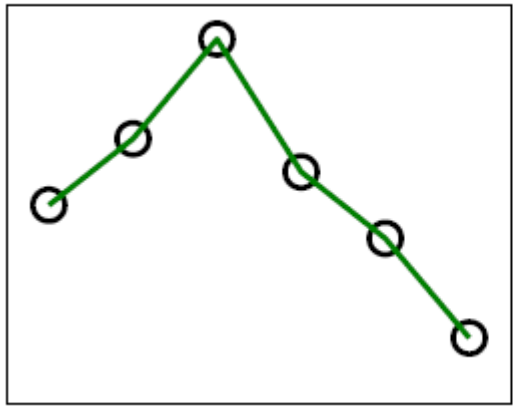




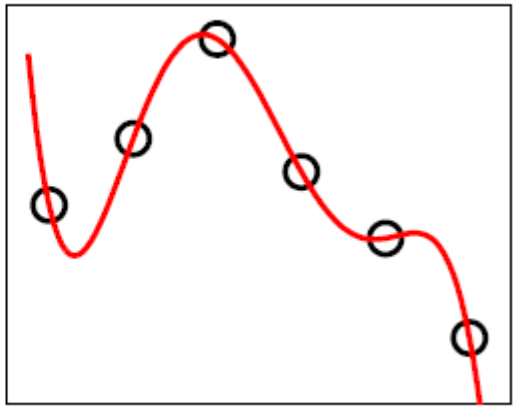
$$x = \frac{r+s}{2} - \frac{(f(s) - f(r))(t-r)(t-s)}{2[(s-r)(f(t) - f(s)) - (f(s) - f(r))(t-s)]}$$

Interpolación

Piecewise linear interpolation



Full degree polynomial interpolation



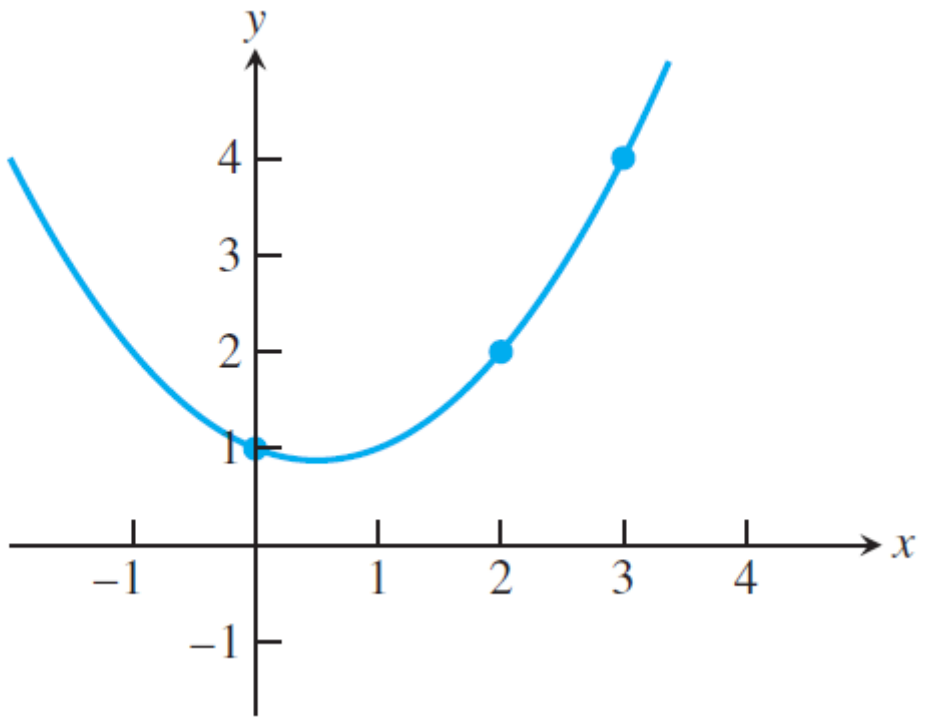
Lagrange

$$P_n(x_k) = y_k, \;\; k = 1, \ldots, n$$

$$P_n(x) = \sum_k \left(\prod_{j \neq k} \frac{x - x_j}{x_k - x_j} \right) y_k$$

$$P_2(x) = y_1 \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} + y_2 \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} + y_3 \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}$$

Interpolación



Interpolación

Newton

Diferencias divididas

$$P_2(x) = f[x_1] + f[x_1\ \ x_2](x - x_1) + f[x_1\ \ x_2\ \ x_3](x - x_1)(x - x_2)$$

$$\begin{array}{c|ccc} x_1 & f[x_1] & & \\ & & f[x_1\ \ x_2] & \\ x_2 & f[x_2] & & f[x_1\ \ x_2\ \ x_3] \\ & & f[x_2\ \ x_3] & \\ x_3 & f[x_3] & & \end{array}$$

$$f[x_k] = f(x_k)$$

$$f[x_k\ \ x_{k+1}] = \frac{f[x_{k+1}] - f[x_k]}{x_{k+1} - x_k}$$

$$f[x_k\ \ x_{k+1}\ \ x_{k+2}] = \frac{f[x_{k+1}\ \ x_{k+2}] - f[x_k\ \ x_{k+1}]}{x_{k+2} - x_k}$$