Question 1 Part 1

The G, C, f, and b matrices are given below:

$$G = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 1 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ 0 & 0 & -\frac{1}{R_2} & \frac{1}{R_2} & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} C_1 & -C_1 & 0 & 0 & 0 \\ -C_1 & C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -L_1 \end{bmatrix}$$

$$f = \begin{bmatrix} 0 \\ e^{v_2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_S \\ E_S \\ 0 \end{bmatrix}$$

Question 1 Part 2

The G, C, f, and b matrices are given below:

$$G = \begin{bmatrix} \frac{1}{R_2}+\mu & 0 & 0 & \mu & 0 & -\frac{1}{R_2} & 0 & 0 \\ 0 & \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1} & \frac{1}{R_1} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{R_2} & 0 & 0 & 0 & 0 & \frac{1}{R_2} & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -\sigma & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} C_1+C_2 & 0 & -C_1 & -C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -C_1 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 \\ -C_2 & 0 & 0 & C_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$f = \begin{bmatrix} -I_o\left(e^{(v_5-v_1)/V_T}-1\right) \\ 0 \\ 0 \\ 0 \\ I_o\left(e^{(v_5-v_1)/V_T}-1\right) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} -I_S \\ I_S \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Question 3 Part 1

The OCAML code for computing the Γ and Γ inverse matrices is shown below. It uses the GNU Scientific library for all matrix operations though this is largely hidden by the stylized operators. As you can see, the Γ matrix is computed directly from the Γ inverse matrix through a simple transpose and then scaling operation.

```
(* Compute the gamma matrix *)
let gamma' n =
  let m = ones n n in
    for row = 0 to (n-1) do
      for col = 1 to (n/2) do
        let value = ((float (row+1)) *. (float col) *. 2. *. pi) /. (float n) in
          (m => (row, col*2-1)) (cos value);
          if (col*2) < n then (m => (row,col*2)) (sin value);
      done
    done;
    m

(* Compute the gamma matrix based on the gamma' matrix *)
let gamma n =
  let m = gamma' n in
  let f1 = 1. /. (float n) in
  let f2 = 2. /. (float n) in
    M.transpose_in_place m;
    for col = 0 to (n-1) do
      (m => (0,col)) ((m $@ (0,col)) *. f1)
    done;
    for row = 1 to (n-1) do
      for col = 0 to (n-1) do
        (m => (row,col)) ((m $@ (row,col)) *. f2)
      done
    done;
    m
```

The matrices satisy the relation given by Eq 2.10 in the notes. This is demonstrated for the two cases of H=5 and H=9 below:

H=5

$$\left(\Gamma^{-1}\right)^T = \begin{pmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 2.5 & 0 & 0 & 0 \\ 0 & 0 & 2.5 & 0 & 0 \\ 0 & 0 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 0 & 2.5 \end{pmatrix}$$

$$\Gamma^{-1} = \begin{pmatrix} 1 & 0.309017 & 0.951057 & -0.809017 & 0.587785 \\ 1 & -0.809017 & 0.587785 & 0.309017 & -0.951057 \\ 1 & -0.809017 & -0.587785 & 0.309017 & 0.951057 \\ 1 & 0.309017 & -0.951057 & -0.809017 & -0.587785 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.123607 & -0.323607 & -0.323607 & 0.123607 & 0.4 \\ 0.380423 & 0.235114 & -0.235114 & -0.380423 & 0 \\ -0.323607 & 0.123607 & 0.123607 & -0.323607 & 0.4 \\ 0.235114 & -0.380423 & 0.380423 & -0.235114 & 0 \end{pmatrix}$$

$$\Gamma^{-1}\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

H=9

$$\left(\Gamma^{-1}\right)^T = \begin{pmatrix} 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.5 \end{pmatrix}$$

$$\Gamma^{-1} = \begin{pmatrix} 1 & 0.766044 & 0.642788 & 0.173648 & 0.984808 & -0.5 & 0.866025 & -0.939693 & 0.34202 \\ 1 & 0.173648 & 0.984808 & -0.939693 & 0.34202 & -0.5 & -0.866025 & 0.766044 & -0.642788 \\ 1 & -0.5 & 0.866025 & -0.5 & -0.866025 & 1 & 0 & -0.5 & 0.866025 \\ 1 & -0.939693 & 0.34202 & 0.766044 & -0.642788 & -0.5 & 0.866025 & 0.173648 & -0.984808 \\ 1 & -0.939693 & -0.34202 & 0.766044 & 0.642788 & -0.5 & -0.866025 & 0.173648 & 0.984808 \\ 1 & -0.5 & -0.866025 & -0.5 & 0.866025 & 1 & 0 & -0.5 & -0.866025 \\ 1 & 0.173648 & -0.984808 & -0.939693 & -0.34202 & -0.5 & 0.866025 & 0.766044 & 0.642788 \\ 1 & 0.766044 & -0.642788 & 0.173648 & -0.984808 & -0.5 & -0.866025 & -0.939693 & -0.34202 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$\Gamma =$

$$\begin{pmatrix} 0.111111 & 0.111111 & 0.111111 & 0.111111 & 0.111111 & 0.111111 & 0.111111 & 0.111111 & 0.111111 \\ 0.170232 & 0.0385885 & -0.111111 & -0.208821 & -0.208821 & -0.111111 & 0.0385885 & 0.170232 & 0.222222 \\ 0.142842 & 0.218846 & 0.19245 & 0.0760045 & -0.0760045 & -0.19245 & -0.218846 & -0.142842 & 0 \\ 0.0385885 & -0.208821 & -0.111111 & 0.170232 & 0.170232 & -0.111111 & -0.208821 & 0.0385885 & 0.222222 \\ 0.218846 & 0.0760045 & -0.19245 & -0.142842 & 0.142842 & 0.19245 & -0.0760045 & -0.218846 & 0 \\ -0.111111 & -0.111111 & 0.222222 & -0.111111 & -0.111111 & 0.222222 & -0.111111 & -0.111111 & 0.222222 \\ 0.19245 & -0.19245 & 0 & 0.19245 & -0.19245 & 0 & 0.19245 & -0.19245 & 0 \\ -0.208821 & 0.170232 & -0.111111 & 0.0385885 & 0.0385885 & -0.111111 & 0.170232 & -0.208821 & 0.222222 \\ 0.0760045 & -0.142842 & 0.19245 & -0.218846 & 0.218846 & -0.19245 & 0.142842 & -0.0760045 & 0 \end{pmatrix}$$

$$\Gamma^{-1}\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$