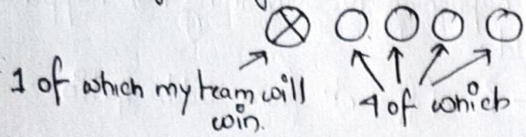
Odd Ratios and log (Odd) Ratios

Odds → For example, we might say that the odds in favour of my team (⊗) winning the game are 1 to 4 in below set. Visually we have 5 games total

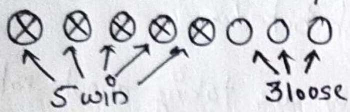


So the odds are 1. $= \frac{\text{Number of my win}}{\text{No of my team loss}}$

$= \frac{1}{4}$

odds (my team will win) = 0.25

Another example (second) → You may say thats odds in favour of my team winning the game 5 to 3



odds $= \frac{5}{3} = 1.7$ So, odds are 1.7 in favour of my team winning the game.

NOTE → Odds are not probability. Odds are ratio of something happening divide by something not happening. Probability is ratio of something happening divide by everything that could happen.

$$\text{Odds} = \frac{\text{something happening (winning)}}{\text{something not happening (not winning)}}$$

$$\text{Probability} = \frac{\text{something happening (winning)}}{\text{everything that could happen (winning \& loosing)}}$$

In second example, odds (win) $= \frac{5}{3} = 1.7$ probability (win) $= \frac{5}{5+3} = \frac{5}{8} = 0.625$

How to derive odds from probability?

→ In second example, odds (winning) $= \frac{5}{3}$ probability (win) $= \frac{5}{8}$.

probability (loose) $= 3/8$ or $1 - p(\text{win})$ $= 1 - \frac{5}{8} = 3/8$.

$$\text{odds (win)} = \frac{P(\text{win})}{P(\text{loose})} = \frac{5/8}{3/8} = 5/3$$

or $\text{odds (win)} = \frac{\text{Probability(win)}}{\text{Probability (loose)}} = \frac{\text{Probability (win)}}{1 - \text{Probability (win)}}$ ← Derived from this

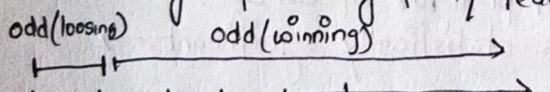
In many problems, formula of odd is ~~decied~~ derived as $\boxed{\text{odds} = \frac{P}{(1-P)}}$

Log (Odds) → Suppose my team is bad odd $= 1/4 = 0.25$, more worse $1/8 = 0.125$, more more worse $1/16 = 0.06$

Full worse $= 0/\text{anything} = 0$. So range will be from 0 to 1. (Denominator > Numerator)

Suppose my team is good, $5/3 = 1.7$ or improve $9/3 = 3$, more $27/3 = 9$, it will go to ∞.

So range of winning if my team strong is 1 to ∞. (Numerator > denominator)



Asymmetry makes it difficult to compare the odds for or against my team winning.

For example if odds are against 1 to 6, then odds $= 1/6 = 0.17$ but if odds are in favour 6 to 1 then odds $= 6/1 = 6$.

Magnitude of odds (negative only 1 and positive 1 to ∞) are different.

So taking logs of this odd solve the problem by making everything symmetrical.

so if odds are against 1 to 6 $= \log(\text{odds}) = \log(1/6) = \log(0.17) = -1.79$

if odds are in favour 6 to 1 $= \log(\text{odds}) = \log(6/1) = \log(6) = 1.79$

$\log(\text{odds}) = \log\left(\frac{P}{1-P}\right)$, this is known as the log of the ratio of the probabilities is called the logit function and form the basics for logistic regression.

Summary → Odds are the ratio of something happening to something not happening. And log(odds) is log of the odds.

NOTE – Even if Odds formula is ratio but it is different from odds Ratio.

So what's big deal?

→ If we take random numbers, that all those adds up to 100 (for example) and use them to calculate odds it will not be normally distributed. So, if we log (odd) it will become normally distributed.

Odds Ratio → when people say about "odds' ratio", they are taking about ratio of odds

$$\text{Odd Ratio} = \frac{\text{Odd}_1}{\text{Odd}_2} = \frac{\frac{XX}{OOOO}}{\frac{OOO}{X}} = \frac{2/4}{3/1} = 0.17$$

So, when we calculate the odds of something,

→ if the denominator is larger than the numerator, odds ratio goes from 0 to 1.

→ if the numerator is larger than the denominator, odds ratio goes from 1 to ∞ (infinity)

Taking logs(odd ratio) making things symmetrical.

Example of odds Ratio.

		Has Cancer Yes	Has Cancer NO
Has the mutated gene	Yes	23	117
	NO	6	210

Total people → 356.
Total cancer → 29 (23+6) No cancer → 327 (117+210)
Mutated gene → 140 (23+117) No mutated gene → 216 (6+210)

So we can use odd ratio to determine if there is relationship between mutated gene and cancer.

Given a person has mutated gene, odds they have cancer $= \frac{23}{117}$

Given a person does not have mutated gene, odds they have cancer $= \frac{6}{210}$.

$$\text{odds ratio} = \frac{23/117}{6/210} = \frac{0.2}{0.03} = 6.88$$

so the odds ratio tell us that the odds are 6.88 times greater that someone with mutated gene will also have cancer.

$\log(\text{odd ratio}) = \log(6.88) = 1.93$

Larger value means that the mutated gene is a good predictor of cancer. Smaller value means that the mutated gene is not a good predictor of cancer.

3 ways to determine odds ratio or log (odds ratio) is statistically significant →

1. Fisher's Exact Test
2. Chi-Square Test (to calculat p-value)
3. The Wald Test (to calculate confidence interval and p value)

→ The odd's ratio (and logs(odds ratio)) tells us if there is strong or weak relationship between two things, like whether or not having a mutated gene increased the odds of having a cancer.