THE UNIVERSITY OF ADELAIDE
AUSTRALIA

Primary Examination, Semester 1, 2022

Algorithm and Data Structure Analysis
COMPSCI 2201, 7201

Writing Time: 120 mins

Questions	Time	Marks
Answer all 7 questions	120 mins	120 marks
		120 Total

Instructions

- Begin each answer on a new page
- Examination papers must not be removed from the examination room
- Calculators allowed
- This is a open book exam

Materials

- 1 blue book
- Lecture notes (handwritten or printed)
- Textbooks (all books permitted)
- 1 dictionary for translation purposes only

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Right or Wrong?

Question 1

(a) Indicate whether each of the following statements is true or false. There is one mark for each correct answer and zero marks for each incorrect answer. In the following statements, n is a positive integer.

	Statement
1	$\log(n^{10}) \in \Omega(\log(n))$
2	$2.1 \cdot n^{2.1} + 1500n^2 \in \Theta(n^{2.1})$
3	$3n^2 + 1 \in O(n)$
4	For multiplying two n digit integers, Karatsuba Multiplication's complexity is $O(n)$
5	It is known that there are NP-complete problems that are not in P
6	All connected undirected graphs must contain cycles
7	The height of a BST tree with n nodes is always in $O(\log n)$
8	The height of an AVL tree with n nodes is always in $O(\log n)$
9	The Dijkstra single-source shortest path algorithm can only work for graphs where the edge costs are integers
10	Hash-tables can maintain $O(1)$ access times in the worst case.

[10 marks]

[Total for Question 1: 10 marks]

Proofs and Sequences

Question 2

(a) Briefly describe the recursive multiplication algorithm for multiplying two n digit integers.

[4 marks]

(b) A simplified version of the Master Theorem is as follows:
a, b, c and d are constants. For $n = b^k$ for some integer k, consider the recurrence

$$r(n) = \begin{cases} a & \text{if} \quad n = 1 \\ cn + d \cdot r(n/b) & \text{if} \quad n > 1 \end{cases}$$

We have

$$r(n) = \begin{cases} \Theta(n) & \text{if} \quad d < b \\ \Theta(n \log n) & \text{if} \quad d = b \\ \Theta(n^{\log_b d}) & \text{if} \quad d > b \end{cases}$$

Use the above to derive the complexity of recursive multiplication for multiplying two n digit integers. For this question, you only need to consider $n = 2^k$ for some integer k.

[4 marks]

(c) Let T be a function that satisfies
1) $T(n) = n^2$ for all positive integer n that is a multiple of 10.
2) $T(n)$ is increasing in n.
Prove that $T(n) \in O(n^2)$.

[8 marks]

[Total for Question 2: 16 marks]

Hashing and Skiplists

Question 3

(a) Assume you have a hash table of size $m = 11$ and hash keys are lowercase alphabetic strings. Given is the following hash function

$$h(k) = code(lastLetter(k)) \bmod 11,$$

where $lastLetter$ extracts the last letter from its input and $code$ returns an integer representing the position of the letter in the alphabet. So, for example $h(\text{anna})$ returns 1, $h(\text{job})$ returns 2 and $h(\text{noon})$ returns 3. Draw the hashtable described above after the insertion of the elements:

`"plate" "pass" "band" "print" "case" "class" "zoo"`

when collisions are resolved through chaining.

[7 marks]

(b) Explain why, when resolving hash-table collisions via linear probing, one cannot remove an entry from the hash table by resetting the slot to NIL.

[3 marks]

(c) Assume that the size of a hash table is given by $m = 2^r$ for some integer $r > 1$. We map a key k into one of the m slots using the hash function $h(k) = k \; mod \; m$. Give one reason why this might be a poorly chosen hash function.

[4 marks]

(d) Given the following sequence of coin tosses, where H stands for head and T stands for tails:
T H T T H H T T H T H T H H T H H T T H H T H T T H T T T H T H T.
Build a skip list containing the following values: 1 40 11 85 86 5. Show the full state of the skip list after each insertion. **Hint:** explicitly state the meaning you attach to heads and tails at the start of your answer.

[5 marks]

[Total for Question 3: 19 marks]

Trees

Question 4

(a) Briefly describe the process for deleting a value from a binary search tree.

[6 marks]

(b) Is the following statement correct?

Let v be the largest node of a binary search tree T. The parent of v must be the second largest node.

Please prove the above statement or provide a counter example.

[5 marks]

(c) Draw a sequence of diagrams showing the insertion of the values:

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[9,1,2,3,8,7,4,6,5]
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into an empty AVL tree, in the order shown above.

You must:

- Show the resulting tree immediately after each insertion step (that is *before* any balancing has taken place).
- Show the resulting tree after balancing operation(s).

[10 marks]

(d) What are the maximum numbers of rotations needed for

1) inserting a new value into an AVL tree with n nodes

2) deleting a value from an AVL tree with n nodes

[6 marks]

[Total for Question 4: 27 marks]

Graph Representations and Traversals

Question 5

(a) For each of the following problems state whether you think the best graph representation of the problem is an adjacency list or an an adjacency matrix. Include an explanation of your choice in your answer.

i. You need to trace the ancestors of living members of the extended British royal family back 600 years.

[2 marks]

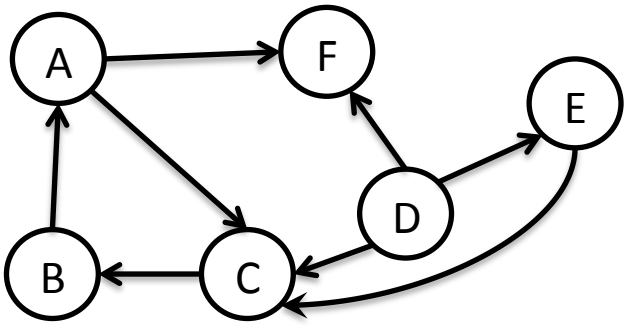
ii. You need to tabulate the minimum road distances between all cities with more than $50,000$ people in Japan

[3 marks]

iii. For an engineering project you are given a large list of tasks; the estimated time required for each task; and a set of dependencies between tasks. You need to calculate an estimated time of completion for the project.

[2 marks]

(b) Use the strongly connected components algorithm described in lectures to find the strongly connected components of following graph. Start your traversal at node A. In your answer show your working including any graph artefacts and other information you use while you apply the algorithm.



[7 marks]

(c) What is the running time of depth-first search, as a function of the number of vertices $|V|$ and the number of edges $|E|$, if the input graph is represented by an adjacency matrix instead of an adjacency list? Does it depend on the number of edges?

[4 marks]

[Total for Question 5: 18 marks]

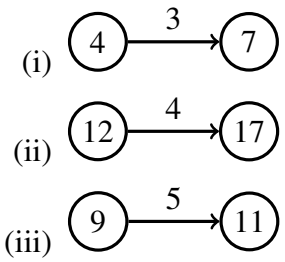
Shortest Path Algorithms

Question 6

(a) Consider a weighted directed graph $G = (V, E, w)$ and let X be a shortest $s - t$ path for $s, t \in V$. If we double the weight of every edge in the graph, setting $w'(e) = 2 \cdot w(e)$ for each $e \in E$, then will X be still a shortest $s - t$ path in G? Explain your answer.

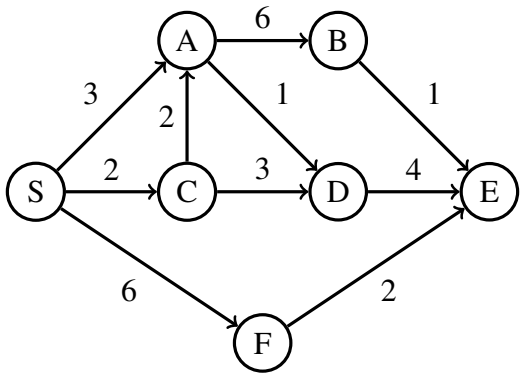
[3 marks]

(b) Draw the result of relaxing the following edges. Note: in the diagram the number in each node represents the current known minimum distance to the source.



[3 marks]

(c) Run Dijkstra's algorithm on the following directed graph, starting at vertex S. What is the order in which vertices get removed from the priority queue? What is the resulting shortest-path tree?



[7 marks]

(d) Briefly explain, with the help of an example, why negative weight cycles in graphs prevent the calculation of finite path lengths to nodes on the far side of such cycles.

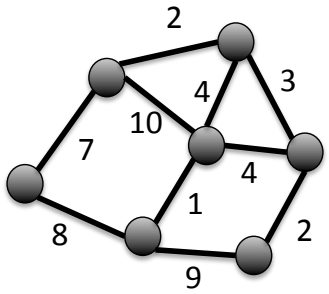
[3 marks]

[Total for Question 6: 16 marks]

Minimum Spanning Trees and P vs NP

Question 7

(a) Draw two different minimum spanning trees for the graph below.



In your answer show the final trees including the weights on the links of the trees.

[6 marks]

(b) Give an example problem that is NP-Complete.

[2 marks]

(c) Give an example problem that is in P.

[2 marks]

(d) Is the minimum spanning tree problem in P? Please explain your answer.

[4 marks]

[Total for Question 7: 14 marks]

End of exam