Assignment 2 - Written Exercises

CMPSCI 603: Robotics

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Chapter 3

2. **Harmonic Oscillators**

   (a) **Critical Damping**

   i. Find the damping coefficient, B, for critical damping in the second order system $\ddot{x} + B\dot{x} + 16x = 0$

   Solution: The given system has the characteristic equation

   $$s^2 + Bs + 16 = 0$$

   with characteristic roots

   $$\frac{-B \pm \sqrt{B^2 - 64}}{2}$$

   Critical damping occurs when the discriminant is zero. Therefore,

   $$\begin{aligned} B^2 &= 64 \\ B &= 8 \ (B \geq 0) \end{aligned}$$

   ii. Show that when $\zeta = 1$ and the roots of the characteristic equation $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ are $s_1, s_2 = -\omega_n$, that terms in the time domain solution like $Ae^{-\omega_n t}$ and $Ate^{-\omega_n t}$ both satisfy the original differential equation $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$.

   Solution: The roots of the given characteristic equation are

   $$\frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n}}{2}$$

   For $\zeta = 1$ and $s_1, s_2 = -\omega_n$, the discriminant is zero. First, we show that the term $Ae^{-\omega_n t}$ satisfies the original differential equation:

   $$\begin{aligned} \frac{d^2(Ae^{-\omega_n t})}{dt} + 2\omega_n \frac{d(Ae^{-\omega_n t})}{dt} + \omega_n^2 Ae^{-\omega_n t} &= 0 \\ -A\omega_n \frac{d(e^{-\omega_n t})}{dt} - 2A\omega_n^2 e^{-\omega_n t} + A\omega_n^2 e^{-\omega_n t} &= 0 \\ A\omega_n^2 e^{-\omega_n t} - 2A\omega_n^2 e^{-\omega_n t} + A\omega_n^2 e^{-\omega_n t} &= 0 \\ 0 &= 0 \end{aligned}$$

Hence, the solution term $Ae^{-\omega_n t}$ satisfies the original differential equation. We test the term $Ate^{-\omega_n t}$ next:

$$\begin{aligned}
\frac{d^2(Ate^{-\omega_n t})}{dt} + 2\omega_n \frac{d(Ate^{-\omega_n t})}{dt} + \omega_n^2 Ate^{-\omega_n t} &= 0 \\
\frac{Ad(e^{-\omega_n t} - \omega_n te^{-\omega_n t})}{dt} + 2A\omega_n[e^{-\omega_n t} - \omega_n te^{-\omega_n t}] + A\omega_n^2 te^{-\omega_n t} &= 0 \\
-A\omega_n e^{-\omega_n t} - A\omega_n[e^{-\omega_n t} - \omega_n te^{-\omega_n t}] + 2A\omega_n[e^{-\omega_n t} - \omega_n te^{-\omega_n t}] + A\omega_n^2 te^{-\omega_n t} &= 0 \\
-A\omega_n e^{-\omega_n t} + A\omega_n[e^{-\omega_n t} - \omega_n te^{-\omega_n t}] + A\omega_n^2 te^{-\omega_n t} &= 0 \\
-A\omega_n^2 te^{-\omega_n t} + A\omega_n^2 te^{-\omega_n t} &= 0 \\
0 &= 0
\end{aligned}$$

Hence, the solution term $Ate^{-\omega_n t}$ also satisfies the original differential equation.

(b) **Natural Frequency**

Design a second order control law, $m\ddot{x} + B\dot{x} + Kx$ with natural frequency, $\omega_n = 50\ rad/sec$:

i. For $m = 1$, find K and B for critical damping.

Solution: For the given second order law, we have the characteristic equation (with $m = 1$):

$$s^2 + Bs + K = 0$$
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

where $\zeta = \frac{B}{2\sqrt{K}}$ and $\omega_n = \sqrt{K} = 50$. Therefore, $K = 2500$, and for critical damping, $\zeta = 1$:

$$\begin{aligned}
\frac{B}{2\sqrt{K}} &= 1 \\
B &= 2\sqrt{K} \\
B &= 100
\end{aligned}$$

ii. Comment on why it might be useful to design the natural frequency of a controlled system.

Solution: The natural frequency is the frequency at which a system would oscillate if there was no damping effect. If the system is excited at or near this frequency, large amplitude oscillations can occur, leading to resonance. It is thus important to model the natural frequency to keep it separate from the excitation frequency.

3. **Characteristic Equation**

Consider the second order characteristic equation

$$3s^2 + 24s + 21 = 0$$

(a) What is the natural frequency of the system?

Solution:

$$3s^2 + 24s + 21 = 0$$
$$s^2 + 8s + 7 = 0 \tag{1}$$

Comparing (1) with the standard form:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{2}$$
$$w_n = \sqrt{7} = 2.64\ rad/sec$$

(b) What is the damping ratio?
Solution: Comparing (1) and (2):

$$2\zeta\omega_n = 8$$
$$\zeta = \frac{4}{\sqrt{7}} = 1.51$$

(c) What is the bandwidth of the system?
Solution: From equation 3.28 in the book, the magnitude of the equation with a forcing function $s = i\omega$

$$\frac{1}{[(1-(\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2]^{1/2}}$$

Putting $\omega = \omega_b$ at which the gain falls to $1/\sqrt{2}$, we have:

$$\frac{1}{[(1-(\omega_b/\omega_n)^2)^2 + (2\zeta(\omega_b/\omega_n))^2]^{1/2}} = \frac{1}{\sqrt{2}}$$
$$(1-(\omega_b/\omega_n)^2)^2 + (2\zeta(\omega_b/\omega_n))^2 = 2$$
$$(\omega_b/\omega_n)^4 + (4\zeta^2-2)(\omega_b/\omega_n)^2 - 1 = 0$$

Let $(\omega_b/\omega_n)^2 = \gamma$, then we have a quadratic equation in γ, which upon solving using the quadratic formula (and taking the positive root) yields:

$$\sqrt{\gamma} = \frac{\omega_b}{\omega_n} = \sqrt{1-2\zeta^2+\sqrt{(1-2\zeta^2)^2+1}}$$

Putting $\zeta = 4/\sqrt{7}$ and $\omega_n = \sqrt{7}$, we get

$$\omega_b = 0.98\ rad/sec$$

4. **Time Domain Solutions**
Suppose that the plant described by the following characteristic equations are released from state $x(0) = 1,\ \dot{x}(0) = 0,\ x(\infty) = 0$. Derive the time domain response $x(t)$ of the system.

(a) $\ddot{x} + 5\dot{x} + 6x$
Solution: Writing this equation in the standard form after taking Laplace transform

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

with $\zeta = \frac{5}{2\sqrt{6}} = 1.02 \approx 1$ and $\omega_n = \sqrt{6}$. Assuming this to be a critically damped system, there is one solution of the equation, $s = -\omega_n$ and $x(t)$ takes the form

$$x(t) = A_0 + (A_1 + A_2 t)e^{-\omega_n t}$$
$$\text{given,}\ \ x(0) = 1 = A_0 + A_1$$
$$\dot{x}(0) = 0 = -\omega_n A_1 + A_2$$
$$x(\infty) = 0 = A_0$$

Solving these equations, we get $A_0 = 0,\ A_1 = 1,\ A_2 = \omega_n$ and the time domain response:

$$x(t) = (1+\sqrt{6}t)e^{-\sqrt{6}t}$$

(b) $\ddot{x} + 2\dot{x} + 10x$
Solution: Writing this equation in the standard form after taking Laplace transform

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

with $\zeta = \frac{1}{\sqrt{10}} < 1$ and $\omega_n = \sqrt{10}$. Since this is an under-damped system, there are two solutionx of the equation,

$$\begin{aligned} s_1, s_2 &= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \\ s_1 &= -1 + j3,\ s_2 = -1 - j3 \end{aligned}$$

and $x(t)$ takes the form

$$\begin{aligned} x(t) &= A_0 + A_1 e^{s_1 t} + A_2 e^{s_2 t} \\ \text{given,}\ \ x(0) &= 1 = A_0 + A_1 + A_2 \\ \dot{x}(0) &= 0 = s_1 A_1 + s_2 A_2 \\ x(\infty) &= 0 = A_0 \end{aligned}$$

Solving these equations, we get $A_0 = 0,\ A_1 = 0.5 - j/6,\ A_2 = 0.5 + j/6$, and the time domain response:

$$(0.5 - j/6)e^{(-1+j3)t} + (0.5 + j/6)e^{(-1-j3)t}$$

9. **Stability - CLTF -** For the simple closed-loop system illustrated:

(a) Write the closed loop transfer function.
Solution:

$$\frac{G}{1+GH} = \frac{Bs^2}{1 + Bs^2 \times \frac{1}{s}} = \frac{Bs^2}{1 + Bs}$$

(b) Determine if the system is stable.
Solution: The denominator of the transfer function is the characteristic equation, and its roots are the poles of the transfer function. We have one pole

$$s = -\frac{1}{B}$$

which is a real pole in the left half of the s plane, because the damping coefficient B is positive. Since this will generate a decaying component in the homogeneous response, the system is stable.

Chapter 4

1. **Inverting the Homogeneous Transform**
Given the general expression for the homogeneous transform and its inverse, prove that ${}_B\mathbf{T}_A$ is the inverse of ${}_A\mathbf{T}_B$, that is, that ${}_B\mathbf{T}_A\ {}_A\mathbf{T}_B = \mathbf{I}_{4\times4}\ (=\ {}_B\mathbf{T}_B)$.
Solution: Given,

$$ {}_A\mathbf{T}_B = \begin{bmatrix} \bar{x} & \bar{y} & \bar{z} & | & p \\ - & - & - & | & - \\ 0 & 0 & 0 & | & 1 \end{bmatrix}_{4\times4} \qquad {}_B\mathbf{T}_A = {}_A\mathbf{T}_B^{-1} = \begin{bmatrix} \bar{x}^T & | & -p\cdot\bar{x} \\ \bar{y}^T & | & -p\cdot\bar{y} \\ \bar{z}^T & | & -p\cdot\bar{z} \\ -- & | & --- \\ 0 & | & 1 \end{bmatrix}_{4\times4}$$

We need to prove that ${}_B\mathbf{T}_A\ {}_A\mathbf{T}_B = I_{4\times4}\ (=\ {}_B\mathbf{T}_B)$

$$
\begin{aligned}
{}_B\mathbf{T}_A\ {}_A\mathbf{T}_B &= \begin{bmatrix} \bar{x}^T & -p\cdot\bar{x} \\ \bar{y}^T & -p\cdot\bar{y} \\ \bar{z}^T & -p\cdot\bar{z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x} & \bar{y} & \bar{z} & p \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \bar{x}\bar{x}^T & \bar{x}\bar{y}^T & \bar{x}\bar{z}^T & 0 \\ \bar{y}\bar{x}^T & \bar{y}\bar{y}^T & \bar{y}\bar{z}^T & 0 \\ \bar{z}\bar{x}^T & \bar{z}\bar{y}^T & \bar{z}\bar{z}^T & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{I}_{4\times4}
\end{aligned}
$$

where $\bar{x}\bar{x}^T = 1$ since the rows are columns are of unit length (and similarly for $\bar{y}$ and $\bar{z}$), and $\bar{x}\bar{y}^T = 0$ since the columns and rows are orthogonal (similarly for any other combination of two different rows or columns).Hence proved.

2. **Properties of the Rotation Matrix**
   Given the following homogeneous transform

$$
\begin{bmatrix} ? & 0 & -1 & 0 \\ ? & 0 & 0 & 1 \\ ? & -1 & 0 & 2 \\ ? & 0 & 0 & 1 \end{bmatrix}
$$

   find the elements designated by the question marks.
   Solution: For a homogeneous transform, we know that

   i. The last row should be [0 0 0 1]
   ii. The rows and columns of the rotation matrix are orthogonal and of unit length.
   iii. $det(R) = 1$ for right-handed conventions (taking this assumption here)

   Using the above, we can say that

   1. The ***?*** in the last row is 0, using ***i.***
   2. The ***?*** in the first row should satisfy $?^2 + 0^2 + (-1)^2 = 1$ using ***ii.*** It's correct value is therefore 0.
   3. Similarly, using ***ii.***, the missing value in the third row is 0 as well.
   4. The missing value in the second row can either be +1 or -1, using ***ii.*** Using ***iii.*** we can find the correct sign for the value:

$$
\mathbf{R} = \begin{bmatrix} 0 & 0 & -1 \\ X & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}
$$
$$
det(R) = 0(0-0) + 0(0-0) - 1(-X-0) = X
$$

Therefore, we can conclude that X = 1, making the final homogeneous transform:

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. **Solving Spatial Relationships**
   Using the transforms given:

   (a) Solve for ${}_{cam}\mathbf{T}_{fing}$
   Solution:

$${}_{W}\mathbf{T}_{cam} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_{cam}\mathbf{T}_{W} = {}_{W}\mathbf{T}_{cam}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   using the formula for the inverse of a homogeneous transformation matrix. And we have

$${}_{W}\mathbf{T}_{fing} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Combining the transforms, } {}_{cam}\mathbf{T}_{fing} = {}_{cam}\mathbf{T}_{W}\ {}_{W}\mathbf{T}_{fing}$$

$${}_{cam}\mathbf{T}_{fing} = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_{cam}\mathbf{T}_{fing} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (b) Compute the position of the hand's fingertip in the camera's coordinate frame.
   Solution: Since the position of the fingertip in the finger's coordinate frame is at it's origin, we can define the homogeneous position vector as

$$\mathbf{r}_{fing} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and we get

$$\mathbf{r}_{cam} = {}_{cam}\mathbf{T}_{fing}\ \mathbf{r}_{fing}$$

$$\mathbf{r}_{cam} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{r}_{cam} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

Therefore, the coordinates of the fingertip in the camera's coordinate frame are $(1, 0, 3)$

4. **Solving Spatial Relationships**

(a) Use the given transforms for ${}_W\mathbf{T}_A$ and ${}_W\mathbf{T}_B$ to calculate the transform from A to B, ${}_A\mathbf{T}_B$.
Solution:

$${}_W\mathbf{T}_A = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_A\mathbf{T}_W = {}_W\mathbf{T}_A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

using the formula for the inverse of a homogeneous transformation matrix. And we have

$${}_W\mathbf{T}_B = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combining the transforms, ${}_A\mathbf{T}_B = {}_A\mathbf{T}_W\ {}_W\mathbf{T}_B$

$${}_A\mathbf{T}_B = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

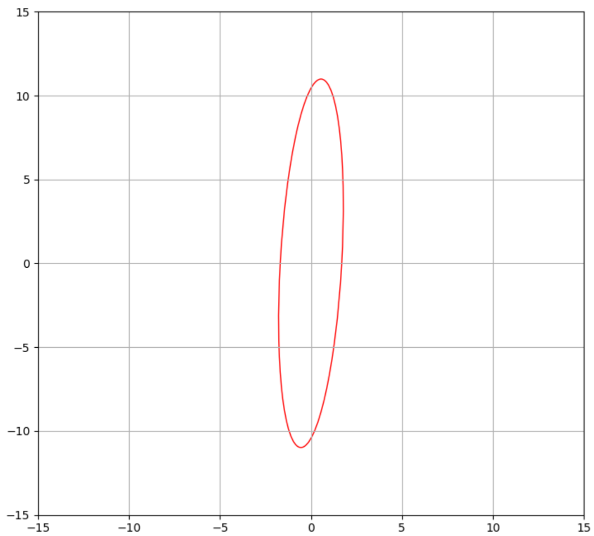
$${}_A\mathbf{T}_B = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Given point P with $\mathbf{r}_B = [-1,\ 1.5,\ 0,\ 1]$, find $\mathbf{r}_A$.
Solution: From the previous result,

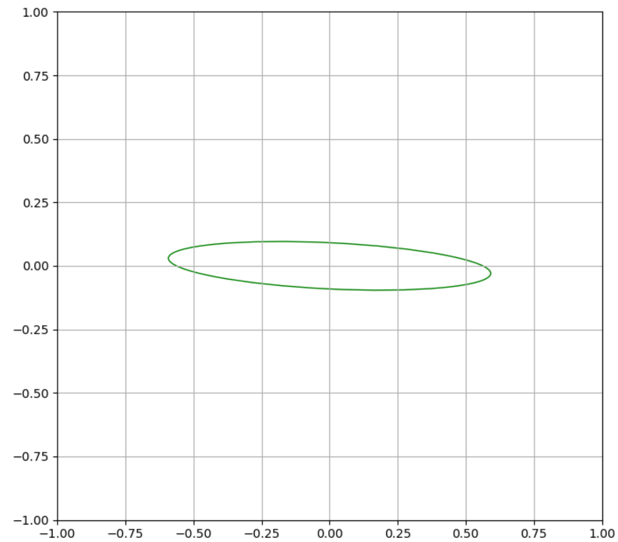
$$\mathbf{r}_A = {}_A\mathbf{T}_B\mathbf{r}_B$$

$$\mathbf{r}_A = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1.5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1.5 \\ 0 \\ 1 \end{bmatrix}$$

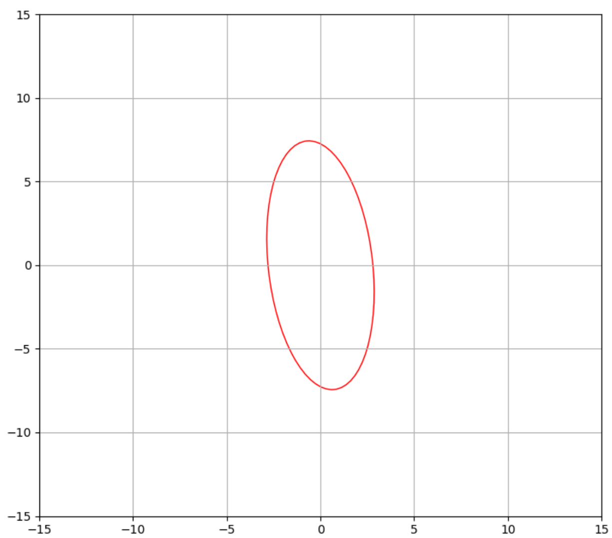
8. **Force and Velocity Ellipsoids**
For this exercise, I used the planar 2R manipulator configuration as shown in the slides to approximate $\theta_1, \theta_2, l_1, l_2$ for both Lucy and the chimp. Given below the plots are the values I used to derive them, and the eigenvalues and eigenvectors obtained for force and velocity ellipsoids. These were solved in code (Python) using angles and link lengths as parameters, and the ellipsoids (with $z = 0,\ k = 1$) plotted in matplotlib. Lucy has been built to have approximately straight legs, with link lengths a little longer than the chimp, who has a bent posture due to the chosen angles.



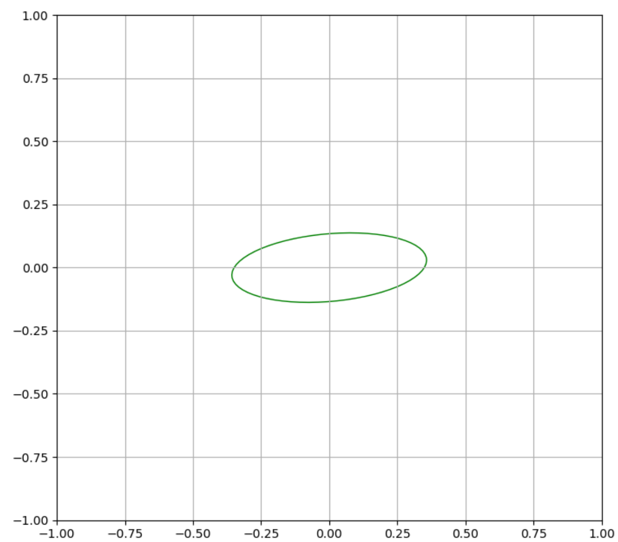
(a) Lucy's Force Ellipsoid



(b) Lucy's Velocity Ellipsoid



(c) Chimp's Force Ellipsoid



(d) Chimp's Velocity Ellipsoid

Figure 1: Force and Velocity Ellipsoids for Lucy - a, b - and the chimp - c, d.

Lucy:

$$\theta_1 = 85^\circ,\ \theta_2 = 25^\circ,\ l_1 = 0.33\ m,\ l_2 = 0.28\ m$$
$$\text{Force ellipsoid: } \lambda_1 = 0.35,\ \lambda_2 = 0.008,\ \hat{e}_1 = [0.99\ -0.05],\ \hat{e}_2 = [0.05\ 0.99]$$
$$\text{Velocity ellipsoid: } \lambda_1 = 2.84,\ \lambda_2 = 121.03,\ \hat{e}_1 = [0.99\ -0.05],\ \hat{e}_2 = [0.05\ 0.99]$$

Chimp:

$$\theta_1 = 40^\circ,\ \theta_2 = 85^\circ,\ l_1 = 0.25\ m,\ l_2 = 0.24\ m$$
$$\text{Force ellipsoid: } \lambda_1 = 0.12,\ \lambda_2 = 0.017,\ \hat{e}_1 = [0.99\ 0.09],\ \hat{e}_2 = [-0.09\ 0.99]$$
$$\text{Velocity ellipsoid: } \lambda_1 = 7.77,\ \lambda_2 = 55.77,\ \hat{e}_1 = [0.99\ 0.09],\ \hat{e}_2 = [-0.09\ 0.99]$$

As we can see from the Lucy's velocity ellipsoid, Fig 1(b), Lucy is able to transform small joint angle velocities into large Cartesian velocities, and is identified by the smaller value λ_1 (its reciprocal is bigger), which contributes to the longer axis of the ellipse in the $\hat{e}_1$ direction - an axis which is parallel to the ground. This means that Lucy's lower body is better evolved for walking. Comparing with the chimp's velocity ellipsoid, we can see that the axes are more closer in length, and the kinematics of the ape are not efficient at transforming velocities parallel to the ground, but are sufficient in both directions.

The force ellipsoid for Lucy tells us that she efficiently converts small joint torques into large endpoint forces into the ground - this enables her to keep her legs straight, while balancing the weight of here body, as she walks. This supports the hypothesis that Lucy is better evolved for endurance walking. For the chimp, the force ellipsoid is more evenly distributed, and can transform forces well in both directions, which suggests that they might instead be good at climbing trees with dexterity.

The results can be validated by observing that the eigenvalues for the force and velocity ellipsoids are reciprocals, and the eigenvectors are identical, meaning the ellipsoid is similarly rotated, but the major and minor axes vary in length.

Assignment 2 - Programming Project
CMPSCI 603: Robotics

Sanuj Bhatia

February 22, 2018

3.2 Project #2 - Arm Kinematics

(1) Describe your criteria for selecting inverse kinematic solutions.
Solution: After the coordinate transformations are done and we have the reference of the clicked point in the frame of the shoulder, we check if the point is in range of the arm. If not, FALSE is returned, and no further action is taken. If it is in range, I check if the limb is the left one or the right one, and select the negative and positive values of θ_1 and θ_2, respectively. I do this to make the movement natural and *human-like*, in which we extend our arms with our elbow joint going outward, so as to:

1. Not block our vision, although this is not really a serious consideration in Roger's case - as the links and joints in his arms (except the end effector) are *invisible*, so to speak - but is a valid reason in practical use cases.
2. Not collide with its own elbow and arms, and give it space to work right in front of his base and eyes. This type of interaction, again, is simply ignored in the simulator, but is easy to imagine happening to a real robot or a human.
3. In order to bring to itself some object (after grasping it, say), Roger would be in a better position to begin with if his elbow is extended away from his base, since he would otherwise have to either switch to the correct configuration (wasting time) or move in an awkward way with his elbow going through his base (unrealistic).

This was the most natural choice of kinematic solutions for both the limbs. As for when the set-point is behind Roger, either of the choices would be situationally beneficial, and has not been handled as a special for the current project.

(2) Plot the Cartesian error vs. time for the endpoint frame of the arm in an experimental demonstration of your choice.
Solution: The default position of Roger's arms for my projects has been set to a pose with the end effector near the base wheels, elbows extending backward ($\hat{x}_B = 0$). From this position, the setpoint was given to the left arm as directly in front of Roger at a distance of 0.85m in $\hat{x}_B$, in a wide punching motion. The graph of error in x and y between the end effector and the setpoint, from the robot's shoulder frame of reference is plotted in Figure 1. The angle setpoints, with a default value of 3 rad and -2.8 rad for θ_1 and θ_2 respectively, move to a final value of around 0.5 rad and -1 rad.

(3) Discuss your results.
Solution: Figure 1 starts in time from the moment when the GUI is clicked. From the initial error in y, we see an increase first before it decays to zero. This increase in y error is seen as the elbow joint opens up and moves further away from the base, before the shoulder joint's rotation moves it closer to the setpoint. There is no way to control the end effector's movement directly for now, since we have not defined a path for it to the given setpoint, only the joint angles are currently being set to reach the goal. The error in x constantly decreases, as expected from such a configuration.

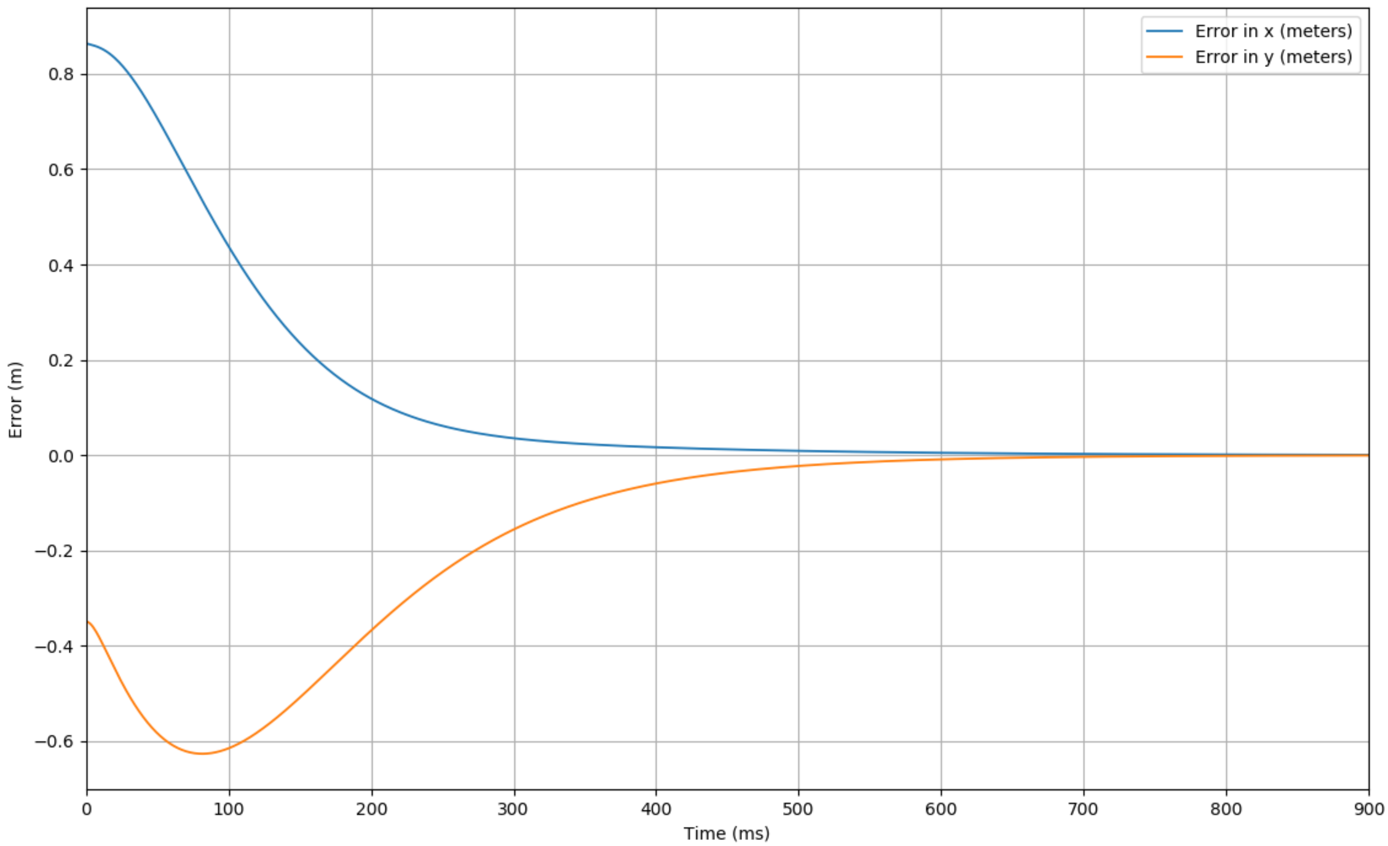
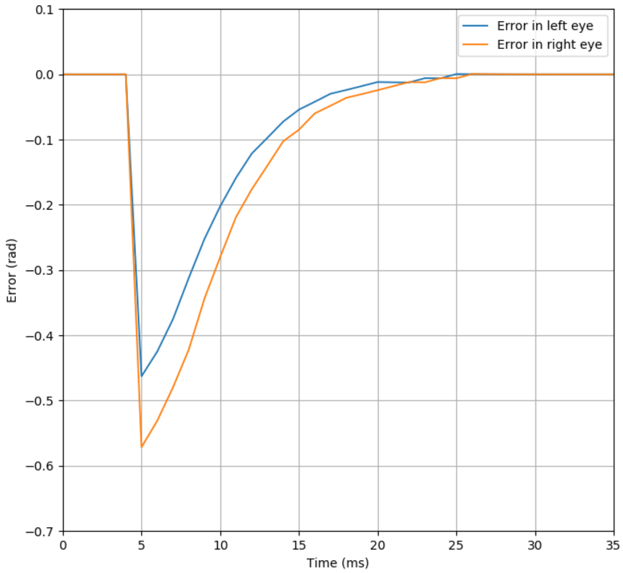


Figure 1: Error in x and y for Roger's arm for a setpoint directly in front of his eyes.

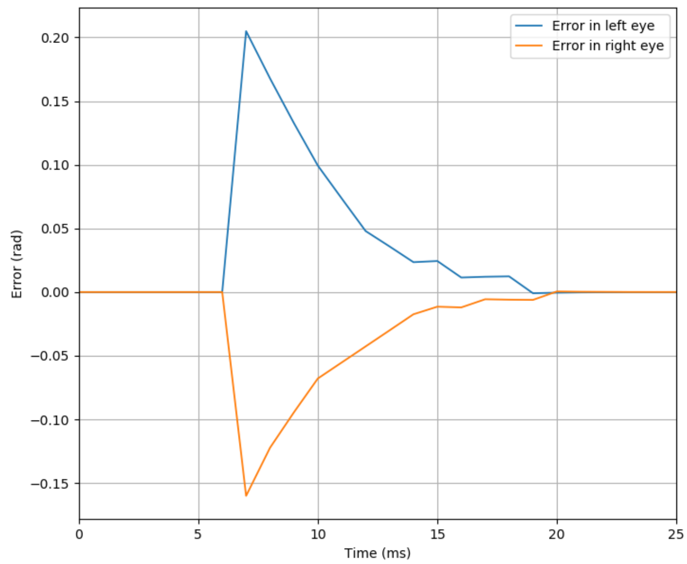
3.4 **Project #3 - Vision**

1. Describe the *compute_average_location_of_red_pixels()* algorithm.
   Solution: In this algorithm, I begin processing the input image from the leftmost pixel and look for a red pixel to begin with. Once found, I set a *start* and *end* variable to that index and keep checking for red pixels, incrementing the value of *end*, until either the end of array is reached or a non-red pixel is encountered. The average location is then computed as a float value of $(start + end)/2$ and returned. To compute u_L and u_R, a value of 63.5 is deducted from the average value.
2. Plot the oculomotor error for both eyes in response to three different placements of the red ball within the field of view as a target.
   Solution: The plots of the errors in both eyes are shown in Figure 2. *Experiment 1* is of the ball appearing in the field of vision of the robot from a parallel gaze, on the left, at the edge of the field of view. *Experiment 2* places the ball directly in front of Roger, between his eyes. *Experiment 3* bounces the ball off the wall and plots the error as Roger's eyes track the ball in motion.
3. A short discussion.
   Roger has been given a default parallel gaze directly in front of him if there is no red object in his vision. If the ball goes out of vision of both the eyes, Roger will reset his gaze to the default position. Using the u_L and u_R and the focal length, we can find the difference in Roger's current eye angles and the setpoint, and then set the correct goals. As we can see in Figure 2(a), the robot is really quick in reaching the setpoint as it appears as a spike. The error in the left eye is slightly less to begin with, since the ball is placed to the left of Roger. In Figure 2(b), the errors are of different signs as the ball is places between his eyes, and converge together to the centre of

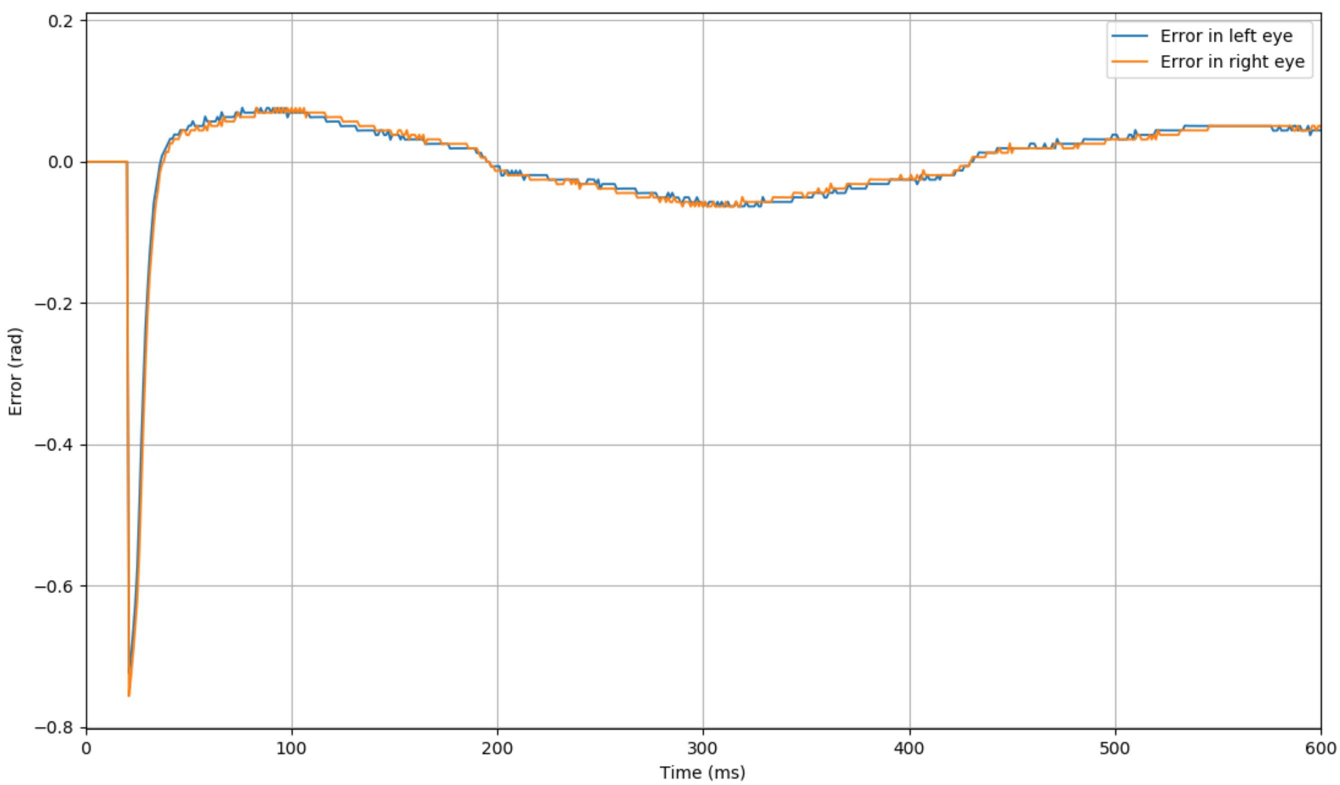
the ball. We can observe some discontinuities as we approach a smaller x axis, since the erros are being sampled every millisecond. Figure 2(c) is interesting to observe, and we can see how Roger is never able to look directly at the centre of the ball while it moves unobstructed (only a small error), but as soon as it hits the wall, the error passes zero and continues to be non-zero, but in the opposite direction.



(a) Experiment 1



(b) Experiment 2



(c) Experiment 3

Figure 2: Errors in Roger's oculomotor control as he tracks the red ball