Assignment 3 - Written Exercises

CMPSCI 603: Robotics

Sanuj Bhatia

March 6, 2018

Chapter 5

6. A locomotive with a mass of 2×10^4 kg is moving along a straight track at 40 m/s at 43 deg$N$. Calculate the magnitude and direction of the transverse horizontal force on the track.

   Solution: Consider a stationary intertial frame $\boldsymbol{A}$ and a rotating frame $\boldsymbol{B}$ attached to the earth.

$$v_A = {}_AR_B(t)v_B$$
$$F_A = \frac{d(mv_A)}{dt} = \frac{d({}_AR_B(t)mv_B)}{dt}$$
$$F_A = m\dot{v_A} = {}_AR_B[\dot{v_B} + (\omega \times v_B)]$$

so that an observer that travels with frame B:

$$F_B = {}_BR_AF_A - (\omega \times mv_B)$$

   The $-(\omega \times m_B)$ term is the coriolis force. The minus sign is explained in the derivation itself, and we include a 2 factor in the final form:

$$F = -2m\omega \times v$$
$$F = -2 \times 2 \times 10^4 \times \frac{2\pi}{24 \times 60 \times 60} \times 40 \times sin(43^{\circ})$$
$$F = 79.62\ N \text{ in the right direction, using the right hand rule}$$

7. Write the dynamic equation of motion using Newton/Euler equations, and Lagrangian dynamics:
   Solution: Handwritten solution shown on the next page -

Outward Iteration

$$^1\omega_1 = {}_1R_0\,{}^0\omega_0 + \dot{\theta}_1 \hat{Z}_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$^1\dot{\omega}_1 = {}_1R_0\,{}^0\dot{\omega}_0 + \left({}_1R_0\,{}^0\omega_0 \times \dot{\theta}_1 \hat{Z}_1\right) + \ddot{\theta}_1 \hat{Z}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix}$$

$$^1\dot{v}_1 = {}_1R_0\left[{}^0\dot{v}_0 + \left({}^0\dot{\omega}_0 \times {}^0P_1\right) + \left({}^0\omega_0 \times {}^0\omega_0 \times {}^0P_1\right)\right] = {}_1R_0\,{}^0\dot{v}_0$$

$$= \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} gc_1 \\ -gs_1 \\ 0 \end{bmatrix}$$

$$^1\dot{v}_{cm} = \left({}^1\dot{\omega}_1 \times {}^1P_{cm}\right) + \left({}^1\omega_1 \times {}^1\omega_1 \times {}^1P_{cm}\right) + {}^1\dot{v}_1$$

$$= \begin{bmatrix} 0 \\ l_1\ddot{\theta}_1 \\ 0 \end{bmatrix} + \begin{bmatrix} -l_1\dot{\theta}_1^2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} +gc_1 \\ -gs_2 \\ 0 \end{bmatrix} = \begin{bmatrix} gc_1 - l_1\dot{\theta}_1^2 \\ -gs_1 + l_1\ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$$^1F_1 = m_1\,{}^1\dot{v}_{cm} = m_1 \begin{bmatrix} gc_1 - l_1\dot{\theta}_1^2 \\ -gs_1 + l_1\ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$$^1N_1 = I_1\,{}^1\dot{\omega}_1 + \left({}^1\omega_1 \times I_1\,{}^1\omega_1\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Inward Iteration: External forces zero

$$^1f_1 = {}^1F_1 + {}_1R_2\,{}^2f_2$$

$$= \begin{bmatrix} m_1gc_1 - m_1l_1\dot{\theta}_1^2 \\ -m_1gs_1 + m_1l_1\ddot{\theta}_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Figure 1: Newton/Euler Inward/Outward Iterations (continued)

$$ {}^1\eta_1 = {}^1N_1 + \overset{0}{\cancel{{}^1R_2\,{}^2\eta_2}} + ({}^1P_{cm} \times {}^1F_1) + \overset{0}{\cancel{({}^1P_2 \times {}^1R_2\,{}^2f_2)}} $$

$$ = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{vmatrix} i & j & k \\ l_1 & 0 & 0 \\ ({}^1F_1)_x & ({}^1F_1)_y & 0 \end{vmatrix} $$

$$ = \begin{bmatrix} 0 \\ 0 \\ l_1\,({}^1F_1)_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tau_1 \end{bmatrix} $$

where τ_1 comes out to be

$$ \tau_1 = m_1 l_1^2 \ddot{\theta}_1 - m_1 g l_1 \sin\theta_1 $$

$$ \boxed{\begin{array}{l} \tau_1 = M\ddot{\theta}_1 + G \\ M = m_1 l_1^2 \quad \text{and} \quad G = -mgl\sin\theta \end{array}} $$

Dynamic Equation of motion

b) Lagrangian Dynamics.

Kinetic energy : $T = \frac{1}{2} m (l\dot{\theta})^2 \qquad (v = l\dot{\theta})$

Potential Energy $V = -mgl\cos\theta$ ($l\cos\theta$ is height of mass m from the ground)

<u>Lagragian</u>

$L = T - V$, where L is the Lagrangian

(1) $$ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \Upsilon = \tau \quad \text{(External forces} = \tau) $$

$$ \frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta}, \quad \frac{\partial L}{\partial \theta} = mgl\sin\theta, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta} $$

From (1) $\boxed{ml^2\ddot{\theta} - mgl\sin\theta = \tau}$

Figure 2: (b) Deriving the dynamic equation of motion using Lagrangian formulation

Assignment 3 - Programming Project

CMPSCI 603: Robotics

Sanuj Bhatia

March 6, 2018

3.2 Project #3.1.4 - Feedforward Inertial Compensation

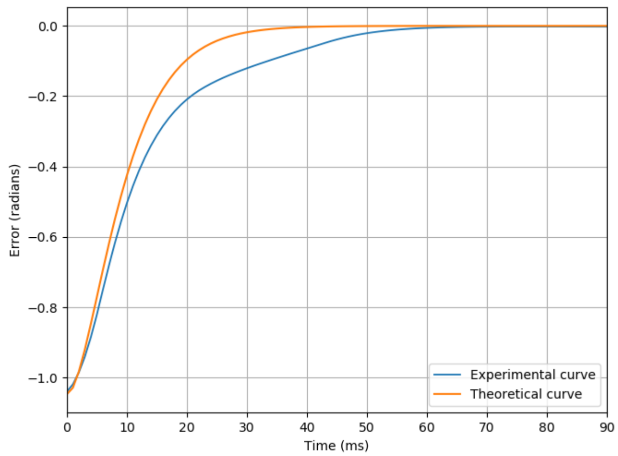
(1) A short problem statement/motivation.
Solution: With this assignment, we hope to *linearize* and *decouple* Roger's 8 degrees of freedom and use energy from the motors to compensate the inertial effects propagating through the system. We then conduct an experiment of simultaneously providing unit step inputs to Roger's arms, eyes, and the base, and compare the results to the theoretical response. It is expected that they will be closer to the theoretical response than those in earlier experiments, since our desired acceleration is now the actual acceleration that the simulator produces.

(2) Choice of the final gains in light of the feedforward compensator for each type of DOF (eyes, shoulder, elbow)
Solution: The gains being used earlier for the coupled system were thoroughly inadequate and had to be increased by up to 2000 times for the eyes, and between 2-10 times for the others. The mathematical reason for this could be that we are increasing the moment of inertia of all them to 1, which was earlier the lowest for the eyes (0.0008). Because of this, we have had to increase our K value proportionally. Further, the B value is fixed for any K in this model, since we can simple take the critically damped form of the two, with $\zeta = 1$ and $I = 1$, giving us $B = 2\sqrt{K}$. Shown below are the values for all DOFs for Roger:

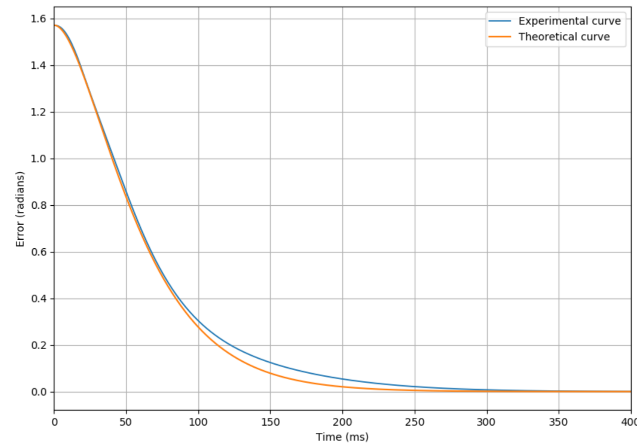
$$\begin{aligned} \text{Eyes: } & K = 40,000 \quad B = 400 \\ \text{Arms: } & K = 1,000 \quad B = 63.24 \\ \text{Base, Rotation: } & K = 1,000 \quad B = 63.24 \\ \text{Base, Translation: } & K = 1,000 \quad B = 63.24 \end{aligned}$$

(3) Plots that compare the experimental and theoretical responses for simultaneous unit step inputs to base translate, base rotate, a shoulder, an elbow, and an eye.
Solution: Simultaneous unit step inputs were given to Roger from his default initial position. Base rotate was left out because of the discussion involving non-holonomic constraint and there not being a force to account for inertial effects orthogonal to the wheels' current heading. Therefore, the eyes, the left elbow, right shoulder, and the base all were given setpoints using the Enter Params function and the resulting errors were plotted against the computed theoretical error responses for each of them. The table below details the boundary conditions and w_n for each of the critically damped responses, with w_n simply being $\sqrt{k}$ or $\frac{B}{2}$, and Figure 1 shows the plots.

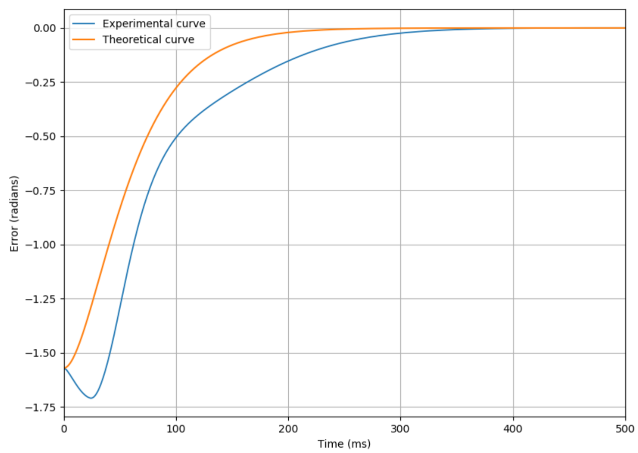
Component	$\theta(0)$	$\dot{\theta}(0)$	$\theta(\infty)$
Eyes	$-\pi/3$	0	0
Left Shoulder	$\pi/2$	0	0
Right Elbow	$-\pi/2$	0	0
Base Translate	$y(0) = -0.5$	$\dot{y}(0) = 0$	$y(\infty) = 0$



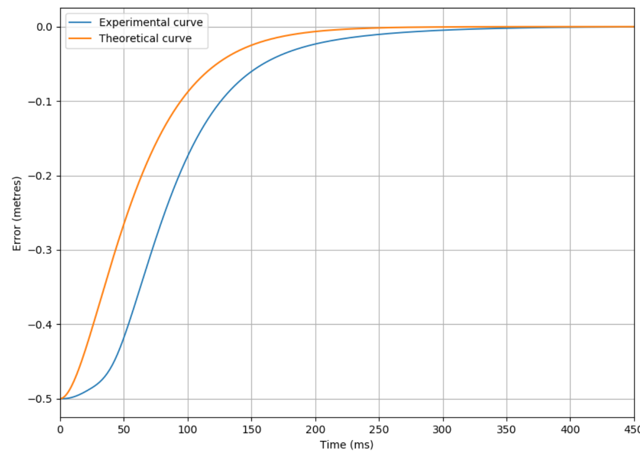
(a) Error in the right eye



(b) Error in the left shoulder



(c) Error in the right elbow



(d) Error in y in the base's position

Figure 1: Simultaneous errors due to movement in the eyes, arms, and the base of Roger

(4) A short discussion of your results.
Solution: Figure 1 tells us that the experimental results are very close to the theoretical response. The left shoulder is almost ideal, and does not get affected by the other parts of Roger as its own inertia dominates its motion. As it moves from $\pi/2$ to a zero position, it makes the base compensate for its motion, which can explain why the experimental curve for the base is seen to be a little delayed in catching up to the theoretical curve. This sudden torque in the base propagates to the right arm, perturbing the elbow in the wrong direction before it turns back to begin reducing the error. As the base and the shoulder slow down, the elbow compensates a little in the backward direction, and we can see the curve turning into a straight line around 120ms in the elbow because of this. The eyes, though very close to ideal, also have a straight line curve at around 20ms, possibly due to the tendency of the base to rotate in response to the shoulder's movement. Overall, a lot of the inconsistencies in movement from the expected behaviour can also be explained due to the fact that there is a Δt time gap between the input dynamics being processed and the torque acting in the motors - we are in discontinuous time with a 1ms step size. Also, there are non-linearities in Roger that this model does not take care of, as well the constraints in the rotation of the base that was discussed earlier. This model is a good exercise in understanding the theory behind how Roger works, but we probably would have a more stable, predictable, and fine-tuned system with the earlier, coupled 8-DOF dynamical Roger.