Assignment 4 - Written Exercises

CMPSCI 603: Robotics

Sanuj Bhatia

April 4, 2018

Chapter 11

5.1 (a) Discuss the qualification for the range of motion potential presented in Equation 11.7 as a navigation function. Write the expression for the control Jacobian J_c.

Solution:

1. Analyticity

The taylor series for $\cos(q)$ about q_0

$$\cos(q) = \cos(q_0)\frac{(q-q_0)^0}{0!} - \sin(q_0)\frac{(q-q_0)^1}{1!} - \cos(q_0)\frac{(q-q_0)^2}{2!} + \ldots$$

for q close to q_0, at $q_0 = 0$

$$\cos(0) = 1 = \cos(0)\,1 - 0 - 0 + \ldots$$

Hence, the equation is analytic.

2. Polar

$$\frac{\partial \Phi_{rom}}{\partial \theta_1} = \sin(\Psi(\theta_1)), \quad \frac{\partial \Phi_{rom}}{\partial \theta_2} = \sin(\Psi(\theta_2)), \quad \frac{\partial \Phi_{rom}}{\partial \theta_3} = \sin(\Psi(\theta_3))$$

All three gradients are zero at $\Psi(\theta_i) = 0$, and therefore terminate at a unique minimum. The function is therefore polar.

3. Morse

$$\frac{\partial^2 \phi_{rom}}{\partial \theta_1^2} = \cos(\Psi(\theta_1)), \frac{\partial^2 \phi_{rom}}{\partial \theta_2^2} = \cos(\Psi(\theta_2)), \frac{\partial^2 \phi_{rom}}{\partial \theta_3^2} = \cos(\Psi(\theta_3))$$

The slope is zero at $\Psi(\theta_i) = 0$, the curvature is not simultaneously zero at that point. There are no other minima in the range $(-\pi/2, \pi/2)$, and the function is morse.

4. Admissibility

Since the slope $(\sin(\Psi(\theta_i))$ is bounded between $(-1, 1)$ in the range $(-\pi/2, \pi/2)$, the function is admissible.

Control Jacobian J_c

$$J_c = \begin{bmatrix} \sin(\Psi(\theta_1)) & \sin(\Psi(\theta_2)) & \sin(\Psi(\theta_3)) \end{bmatrix}$$

5.4 **Convexity**

1. Given function of one variable, determine the convexity of the function if there exist two points on the function, which, when joined by a line, cut the function at two other points as well.
   Answer: The function is not convex, since the line cuts two points on the function and this tells us that it is not always curving upwards.
2. Is the function x^3 convex? On what interval?
   Answer: The second derivative is 6x, and hence it is convex on the interval $[0, \infty]$.
3. Does a function need to be monotonically increasing to be convex? Conversely, is a monotonically increasing function always convex? What about the convexity of $\sqrt{x}$ (monotonically increasing) and x^2 (not monotonically increasing)?
   Answer: The second derivative of $\sqrt{x}$ is $-\frac{1}{4x^{\frac{3}{2}}} < 0$. Hence, it is not convex. $f''(x^2) = 2 > 0$ and it is convex. Hence, monoticity does not guarantee convexity, nor is a convex function necessarily monotonic.

Chapter 10

Indicate true or false with a brief explanation:

1. Neonate is the name given to the early human being from whom modern humans have evolved.
   False. Neonate is a newborn during the first 28 days of birth.

2. The pharyngeal reflex closes the vocal folds and the airway.
   False. The laryngeal reflex closes the airway, the pharyngeal reflex is responsible for the swallowing of food.

3. Sneezing due to bright light, called a photic sneeze reflex, is an uncommon reflex in humans.
   True. It is present in about 1 in 4 people.

4. The Moro reflex is present at birth and continues throughout a human's life.
   False. It is inhibited at 16 weeks and is tranformed into the Strauss reflex in adults.

5. The weight ratio of the human brain to the whole body remains relatively stable from birth onward.
   False. It increases from 25% at birth to 90% by 6 years of age.

Assignment 4 - Programming Project

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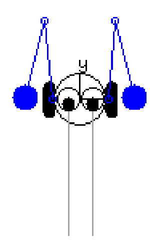
3.4 Project #4 - SearchTrack()

(1) A description of control distribution.
Solution: *Search:* The heading is distributed to the eyes and the base with a factor of 0.6 and 0.4 respectively. 0.6 of the angle is added to the current angle of the eyes, and if the angle exceeds 67.5 degrees in magnitude, the eyes are set to be 67.5 and the difference is added onto the base. This is done so that a 45 degree viewing angle is retained in the eyes at all times, with the edge of vision being perpendicular to the direction Roger is facing ($67.5 + 45/2 = 90$). This angle was chosen to avoid Roger's vision being obstructed by his hands, which are tucked at his side be default, but are just behind the line of vision perpendicular to the base. The 0.6 and 0.4 distribution was done to let the eyes move more than the base since they require less power and move faster. A more unequal distribution resulted in the eyes being at 67.5 degrees for most of the samples, and 0.6 was chosen for aesthetic reasons. *Track:* Roger's eyes are set to gaze at the centre of the ball, and he always attempts to rotate his body to center his eyes at the ball, so that the angles of his eyes are equal and opposite. To prevent excessive rotation, however, the base only get a set-point scaled by 0.9 towards the direction of ball. This makes it rotate a little slower and satisfies the function of keeping the ball in vision even when moving at relatively high speeds. The default position for Roger is shown in the screenshot below.



(2) State-action table for the SearchTrack function.
Solution:

Case	Track	Search	Action
0	0	0	*search* set points
1	0	1	no action
2	0	2	*search* set points
3	1	0	*track* set points
4	1	1	*track* set points
5	1	2	*track* set points
6	2	0	no action
7	2	1	no action
8	2	2	no action

(3) Plot showing the base control error vs time.
Solution: The ball was put behind Roger initially and the base control error was plotted versus time.

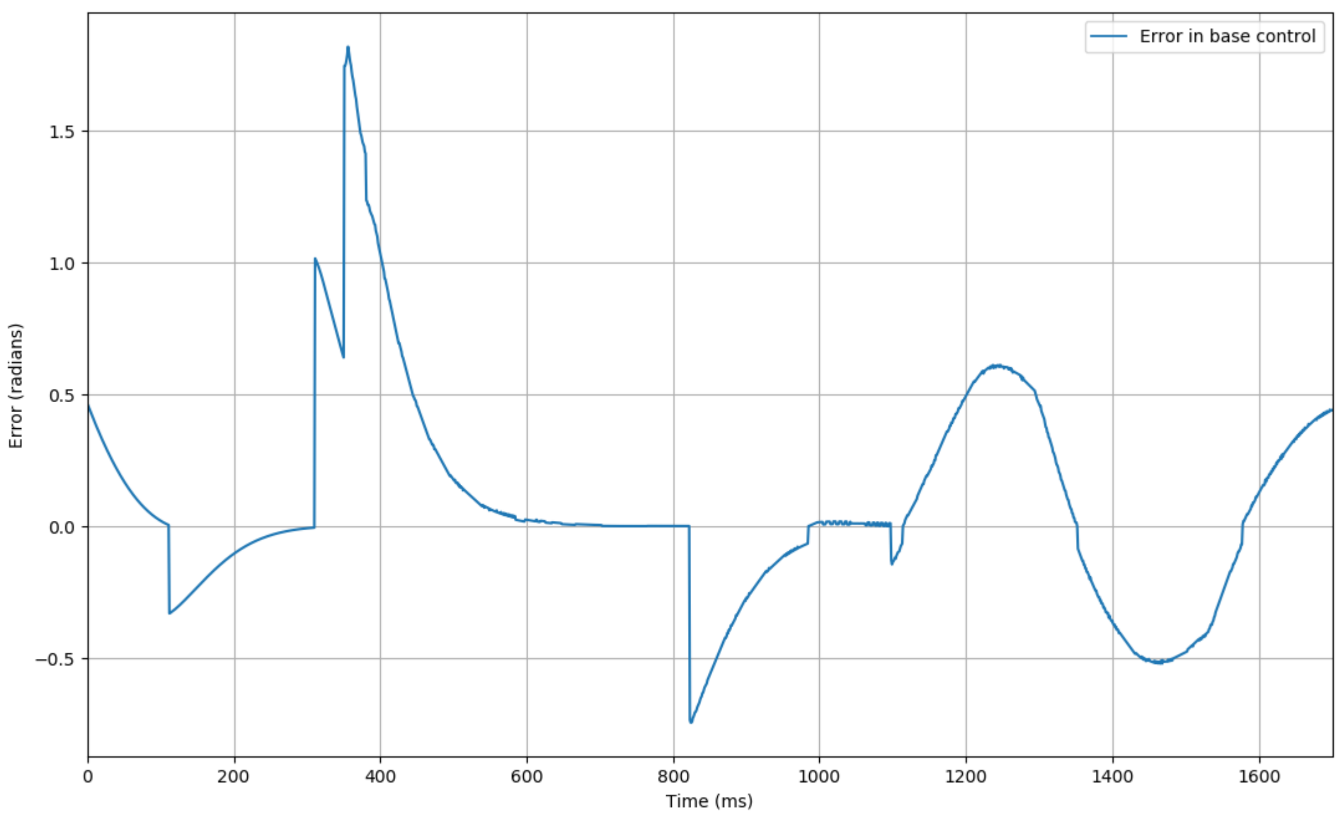


Figure 1: Error in the base angle for a complete search track episode

(4) A short discussion of your results.
Solution: Figure 1 is a complete search track episode. The first search sample begins at 0, and can be seen to converge at around 110 ms when the base error is zero. Immediately, a second sample heading is generated, and this converges at 300 ms. The third sample is set and begins its transient state when the ball comes into vision at 350 ms, and the base set point is now directed by the track function, which centers the base to directly gaze at the ball. This converges with a line that is not perfectly smooth because of the simultaneous movement of the eyes back towards the centre of the ball as the base re-centers itself. At 820 ms, the ball is set near the wall until convergence, and is then bounced off of it at 1100 ms. A wave-like plot follows, with the crest and trough marking the ball bouncing again, and a zero error at the point nearest to Roger when

he looks directly at the ball. It is observed that unless the ball moves really fast, Roger is able to keep it in its vision at all times. If it is lost, searching takes at most 3-4 episodes before the ball is found again. Depending on the use case, the restriction that Roger be looking directly at the ball with the angle of his eyes equal and opposite can be made less strict - say in a range of -30 to 30 degrees for the eyes, the base will remain stationary.

3.5 Project #5 - Stereo Triangulation

1. A description of the theory and your implementation.
   Solution: A triangulation feature is implemented for localizing visual features by estimating the position and covariance of the observation of the ball. The position of the ball is estimated using the centre of the ball observed on the image plane, using the formulas

$$p_x = 2d\frac{cos(\gamma_R)cos(\gamma_L)}{sin(\gamma_R - \gamma_L)} \qquad p_y = d + 2d\frac{cos(\gamma_R)sin(\gamma_L)}{sin(\gamma_R - \gamma_L)}$$

   These x and y coordinates are converted to the world frame as

$$pos_W = \ _WT_B \ pos_B$$

   The localizability jacobian in the base is given by

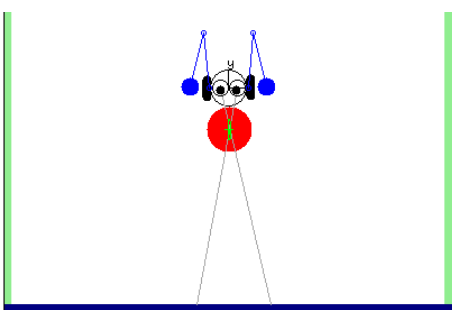
$$\frac{2d}{sin^2(\gamma_R - \gamma_L)}\begin{bmatrix} cos^2(\gamma_R) & -cos^2(\gamma_L) \\ sin(\gamma_R)cos(\gamma_R) & -sin(\gamma_L)cos(\gamma_L) \end{bmatrix}$$

   which can be rotated to world coordinates as

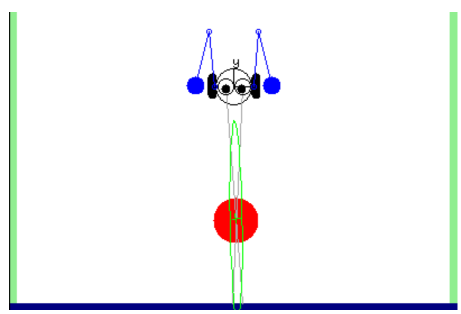
$$J_W = \ _WR_BJ_B$$

   The covariance matrix is proportional to $J_W J_W^T$. This position and covariance scaled by a factor of 0.005 is set into the Observation typedef data structure. It is then sent to the draw_observation function to plot the ellipsoid on the screen using the visualise button.

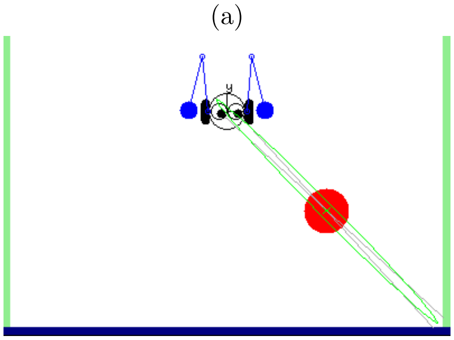
2. The screengrabs of Roger's localizability ellipsoid in various positions have been shown below.



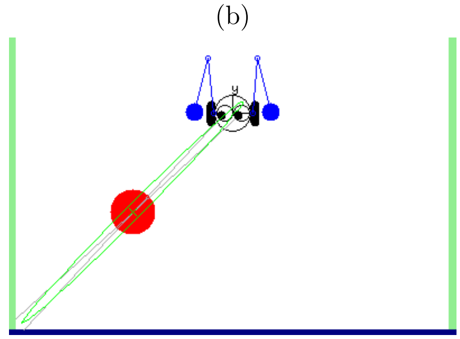
(a)



(b)



(c)



(d)

3. The localizability ellipsoid has principal arcs of length $k\sqrt{\lambda_1}$ and $k\sqrt{\lambda_2}$, where λ_1 and λ_2 are the eigenvalues of JJ_T. The crosshairs in the ellipsoids drawn above are are the mean, and is seen to be at the center of the ball for most cases. The error covariance increases radially as the ball goes farther away from the eyes. The lateral error is relatively small, and the ellipsoid as a whole is the smallest right in front of Roger where the stereo acuity is maximized. As the ball moves away from Roger, the uncertainty in its position with respect to the depth increases. This happens due to pixelization, since Roger sees the same image for more than one positions of the ball - the pixels change by 1 for a particular minimum movement of the ball, until which distance the image roger sees is the same.