Assignment 1 - Programming Project

CMPSCI 603: Robotics

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3.1.1 Oculomotor Control

(1) Describe a procedure for searching for critically damped gains, report gains you end up with and describe any issues you encounter.

Solution: We start with the mass and length parameters of the eye, obtained from *roger.h*. The moment of inertia, I, is easy to calculate as ml^2. Beginning with a small, arbitrary K value, say 0.1, and setting ζ to be 1 in the equation $\zeta = \frac{B}{2\sqrt{KI}}$ (for a theoretically critically damped solution), a starting value for B was obtained. This value moves the eyes very slowly, and, being the lightest and vital part of the robot, it is important and easiest for them to fast. Therefore, increasing the K (and calculating B) is the next step.
The eyes, however, appear to oscillate at $K = 15.0$ and a critically damped $B = 0.069$. This can be explained as follows: the equations are suitable for an ideal, isolated motor unit with one degree of freedom. Roger, however, is an 8 degree of freedom, coupled, dynamical system. Perturbations in one motor trigger base movement (rotation and translation), change in moment of inertia due to arms, and torque generation in other motor units, leading to effects that cannot be fully expressed in a mathematical equation. This explanation is valid for many of the irregularities and deviations from the theoretical response.
The damping had to be increased slowly until a satisfactory movement of eyes is observed, with the theta error (printing on the console) does not change sign or remains very close to zero, and the eyes move fast enough to cover the full range of motion in less than 30 milliseconds. The final gains for the eye motor are:

$$K = 15\ Nm$$
$$B = 0.079\ kg \cdot m^2/s$$

(2) Hold K fixed and change B to generate plots for under-, over-, and critically-damped responses for the eye.

Solution: Holding $K = 15$ and varying B, we obtain the following curves. At $B = 0.079$, I obtain a critically damped response, with a satisfactory, fast movement of the eyes. Hence, the final set point. The higher values of B can be seen to oscillate, being under-damped. The values of B lower than 0.079 are over-damped, and approach the set point much slower.

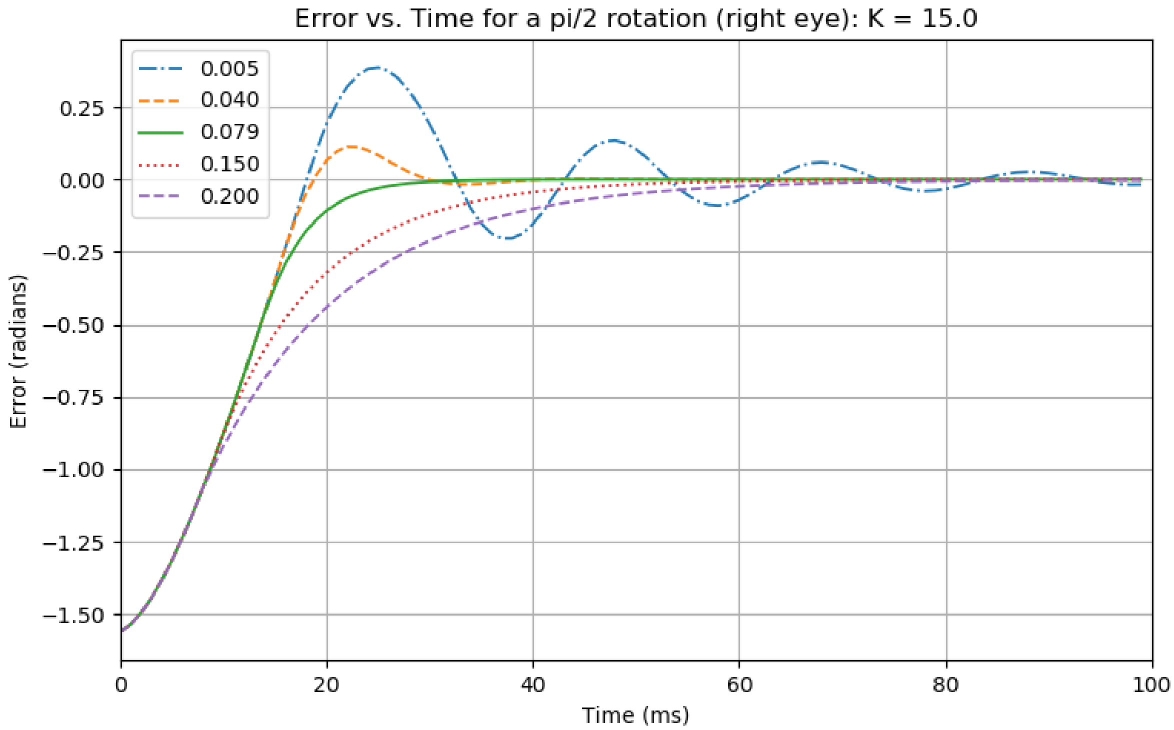


Figure 1: Eye - under-, over-, and critically damped experimental responses

(3) Construct an experiment for comparing the critically damped response of your implementation to the theoretical response. Determine the initial and final posture for the experimental movement. On the same plot, compare the error $(\theta_{ref} - \theta_{act})$ vs. time from your implementation to the analytical result.

Solution: With the conditions of the experiment as follows:

$$\theta(0) = -\pi/2$$
$$\theta(\infty) = 0$$
$$\dot{\theta(0)} = 0$$

we get the following time-domain solution:

$$\theta(t) = (-1.56 - 675.48t)e^{-433t}$$

In the plot below, the orange colored, critically damped theoretical curve approaches the set point at the 15 ms mark, with the experimental curve approaching around 27 ms. The difference of 12 milliseconds between these curves can be reduced by decreasing the B value at the cost of a slight oscillation in the eyes, which has currently been avoided for the purposes of this assignment. The experimental curve cannot reach the zero set point earlier while maintaining its shape because of not being an isolated unit, as explained in (1) earlier.

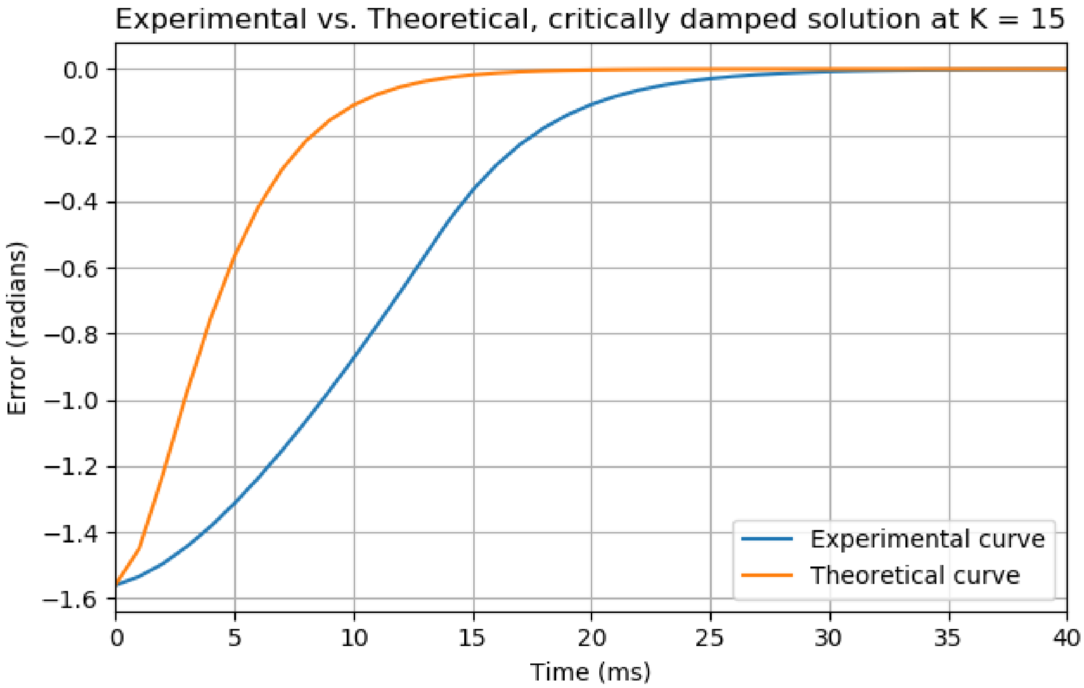


Figure 2: Eye - under-, over-, and critically damped experimental responses for the eye

3.1.2 Arm Control

The arms, although having two separate degrees of freedom (shoulder and elbow joints), are modelled with the same K and B values for both. We begin the same way as for the eyes, by first calculating the moment of inertia, setting a low K value, and using the theoretical, critically damped B. One consideration in setting the gains for the arms is that the robot should have a controlled movement for a natural range of motion, i.e. the base should not substantially rotate due to the force exerted in the shoulder and elbow joints. Because of having more mass with longer links, and consequently having a larger moment of inertia, their movement introduces more significant perturbations than the eyes.
To mitigate this effect, I have encoded a start flag in the project to set the shoulder and elbow joint angles at the beginning of the simulation to a position close to the base, so as to reduce the moment of inertia component due to the arm span. All experiments in this assignment have been performed with that default position. The gains used for the arms are

$$K = 80\ Nm$$
$$B = 11.2\ kg \cdot m^2/s$$

The plot below shows the theta errors of joints *q1* and *q2* for a simultaneous movement from $q1 = q2 = 0$ to $q1 = q2 = \pi/2$. The elbow joint starts off with a steeper slope than the shoulder joint, even though they have the same gain values. The slight difference in the curve shape might be because of the following: when the shoulder and elbow joints begin actuating, the moment of inertia is greater from the shoulder joint up until the end effector, than it is from the elbow joint onward. This is because of the increased length of links and mass the shoulder joint is trying to move. It, therefore, starts with a slower rate of descent, but speeds up as the elbow joint rotates closer to the base, decreasing the moment of inertia. They eventually both reach the set point, effectively stabilizing at around 650ms.

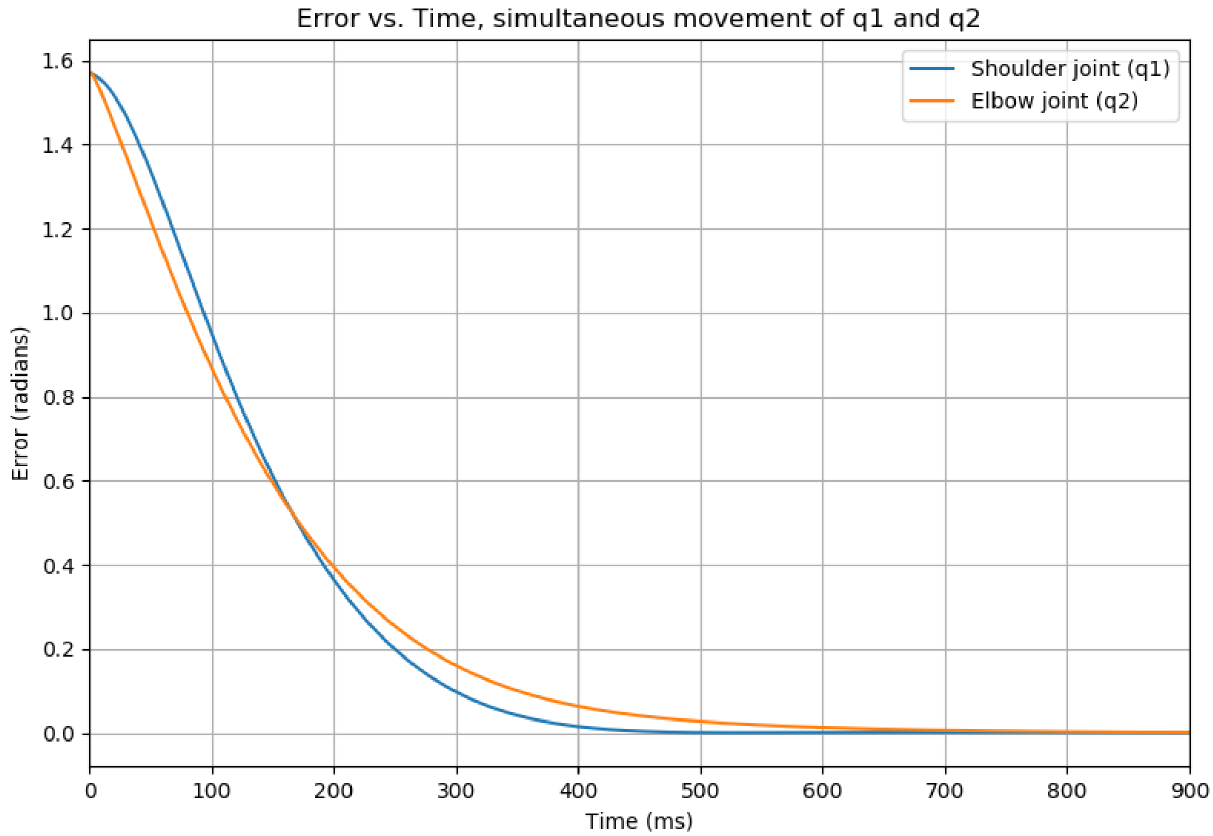


Figure 3: Simultaneous movement of elbow and shoulder joints, error vs. time

The gains were tuned to be close to the experimental critically damped response, with a lower value of B making the arms reach the set point faster but oscillate slightly. Apart from a bump in the beginning, this closely depicts the theoretical second-order response with critical damping, which would have a similar shape but converge faster to the set point.

3.1.3 Base Controller

(1) Describe your criteria for selecting control gains and your final choices.

Solution: In the calculation of gains for the base controller, we have to keep in mind that the arms and eyes are attached to the base, and so the weight and moment of inertia associated with it are the highest of all components we have worked on so far. This makes the base oscillate much more, and the value of B is very far from the theoretical value for both the rotational and translational part of the controller.
The K value was set much larger for the rotation controller because it needs to dominate the translation to minimize errors in the $\hat{y_B}$ direction when reaching the set point. Since the robot cannot travel laterally, it needs to align itself in the correct angle before it starts accelerating towards the set point. At a K value when the robot moves to a set point in a satisfactory amount of time and is aesthetically satisfying, the damping factor was searched for by increasing it until

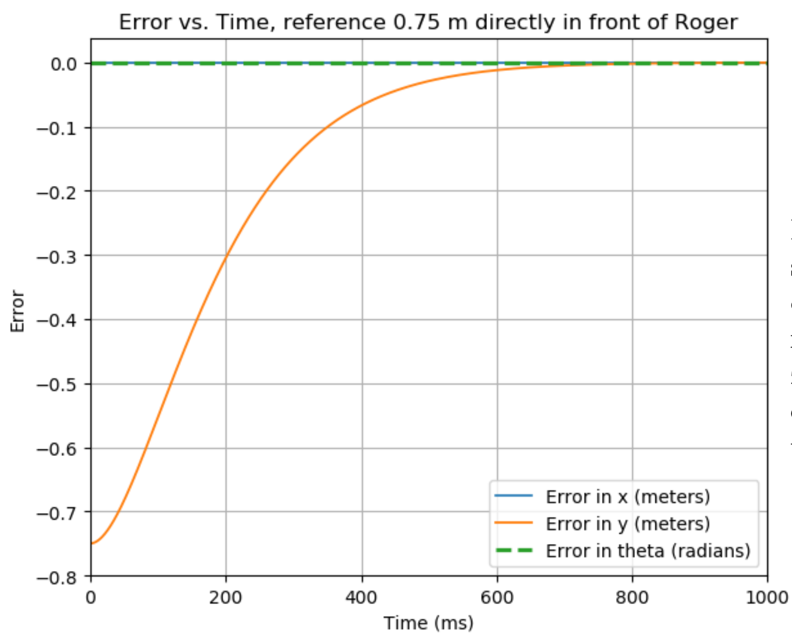
it does not oscillate. The final gain values are:

$$\begin{aligned} K_{trans} &= 185 \\ B_{trans} &= 36 \quad \text{and,} \\ K_{rot} &= 580 \\ B_{rot} &= 36.9 \end{aligned}$$

As explained above, B_{trans} is bigger than expected (comparable to B_{rot}, even though K_{rot} is much higher) because it tends to oscillate around the set point for lower values of the damping coefficients. The gains set for the base controller are able to quickly bring it back to stability if it is disturbed due to sweeping motions of the arms.

(2) Plot x, y, and θ errors in the base position as a function of time for a reference:

(a) 0.75 meters directly in front of the robot (in the $\hat{x_B}$ direction)



For a linear translation straight ahead, Roger reduces the error in y, with the x and θ errors remaining at zero. Please note that these errors are reported in the world frame.

Figure 4

(b) 0.75 meters to the left of the robot (in the $\hat{y_B}$ direction)
In this experiment, Roger rotates and translates simultaneously, and ends up introducing a slight error in y (around 0.04m) as it nears the θ set point. This, as discussed before, is caused (apart from the nonholonomic constraint) due to the arms changing their angles as the base rotates and exerting their own correcting torques at the shoulder and elbow joints.

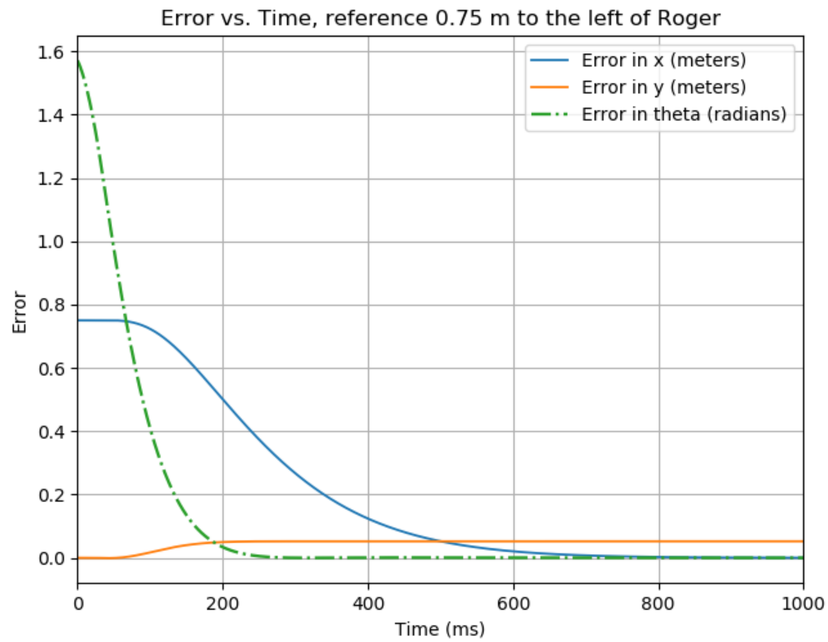


Figure 5

(c) a pure $\pi/2$ rotation around the $\hat{z_B}$ axis

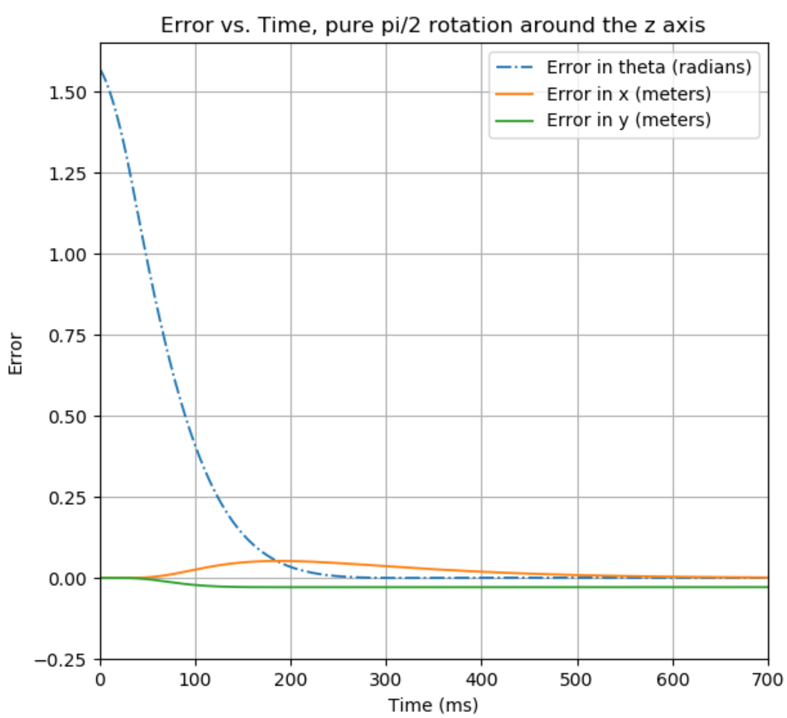


Figure 6

A pure rotation around the z-axis introduces an error in x and y, the first of which is stabilized slower than the error in θ (due to K_{rot} being much higher than K_{trans}). A small error in y remains (around 0.02 m) due to reasons discussed earlier.

Assignment 1 - Written Exercises

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1.6.1 **Knowledge -** Are the following most appropriately considered tacit, implicit, or explicit sources of knowledge? Please support your answer.

(a) "apples are red"
Explicit: A simple sentence relaying the color of a fruit is information that can be understood directly.

(b) an expert golf swing
Implicit: It is a skill that cannot be communicated completely.

(c) the distance between your eyes
Tacit: It is encoded in the physical configuration of the person.

(d) your hair/skin color
Tacit: These are natural characteristics of a person.

(e) the difference between keys and loose change in your pocket
Explicit: The shape and structural difference between the two can be perfectly explained.

(f) shivering/perspiring
Tacit: These are functions or adaptations that are built-in in animals.

(g) the ratio of the circumference of a circle to its diameter
Explicit: A simple mathematical formulation.

1.6.2 **Invent your own homework**
Question: Consider an aggressor Braitenberg vehicle that can be flipped to reverse its acceleration using a switch. In this flipped state, it will go in the reverse direction, away from a light source (it does not have to turn away, but go in reverse gear). The aggressor and the flipped vehicle have different, but constant accelerations as they face a light source. Such an aggressor vehicle starts a distance $d = 5\ m$ away from a light source, directly facing it, with an initial velocity $u_0 = 0$. It accelerates at a constant rate of $a_1 = 2m/s^2$ while facing it. At what distance S from the starting point should the behaviour of the vehicle be flipped, given it decelerates at a constant rate of $a_2 = 1m/s^2$, if the aim of the experiment is to just touch the light source without hitting it? (**Note:** This questions aims to be a simple refresher on classical mechanics, since the course is heavily based on physics and mathematics)

Answer: The velocity of the aggressor at distance S when it is flipped, in say time t_1 is

$$v_1 = u_0 + a_1 t_1$$
$$v_1 = 0 + 2 \times t_1$$
$$\boxed{v_1 = 2t_1}$$

The final velocity at a distance of 5 m is 0, and it takes t_2 seconds to reach the light source after it is flipped

$$0 = 2t_1 - 1 \times t_2$$
$$\boxed{2t_1 = t_2}$$

At a distance S when it is flipped

$$S = u_0 t_1 + \frac{1}{2} a_1 t_1^2$$
$$S = 0 + \frac{1}{2} \times 2 \times t_1^2$$
$$\boxed{S = t_1^2}$$

The remaining distance *(d - S)*

$$d - S = v_1 t_2 + \frac{1}{2} a_2 t_2^2$$
$$5 - S = 2t_1 t_2 - \frac{1}{2} \times 1 \times t_2^2$$
$$5 - S = 2t_1(2t_1) - 0.5(2t_1)^2$$
$$5 - S = 4t_1^2 - 2t_1^2$$
$$5 = S + 2t_1^2 = S + 2S$$
$$\boxed{S = \frac{5}{3} m}$$

2.3.2 (a) What is the steady state angular velocity of the **load** for 10V in?

Solution: At steady state, input voltage is equal to the back emf generated. Therefore,

$$V = V_b = K_b \dot{\theta_M}$$
$$10 = 0.02 \times \dot{\theta_M}$$
$$\dot{\theta_M} = 500$$
$$\dot{\theta_L} = \eta \dot{\theta_M}$$
$$\boxed{\dot{\theta_L} = 5\ rad/sec}$$

(b) When the rotor is locked (i.e., $\dot{\theta} = 0$) and 10V is applied, how much torque is generated on the **load**?

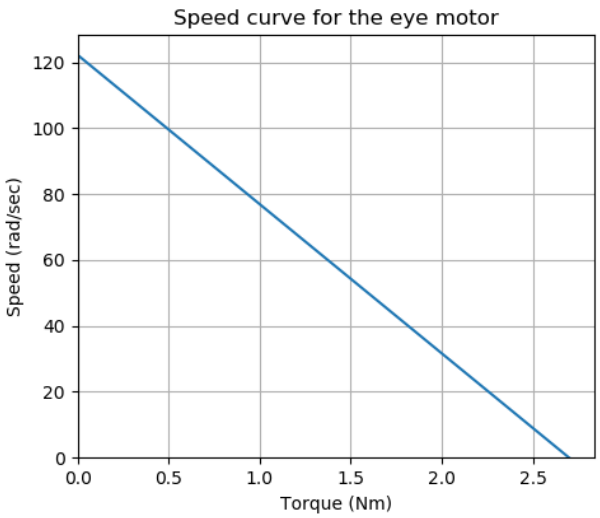
Solution:

$$V = IR$$
$$10 = I \times 10$$
$$I = 1A$$
$$\tau = K_t I$$
$$\tau_M = 0.02 \times 1 = 0.02$$
$$\tau_L = \frac{1}{\eta} \tau_M$$
$$\boxed{\tau_L = 2\ Nm}$$

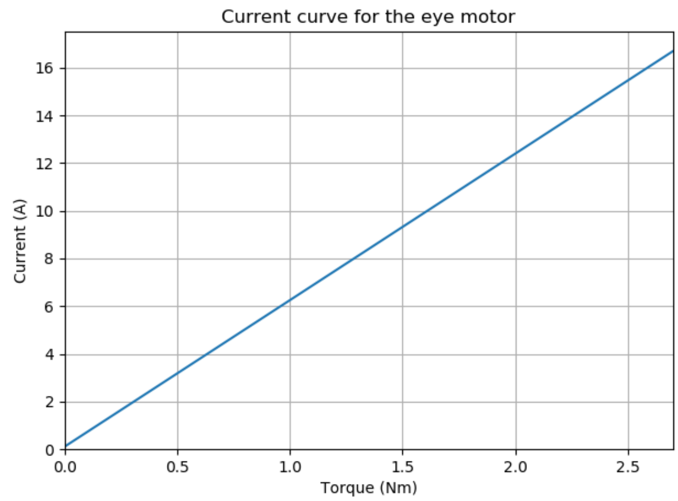
(c) If $J_M = 0.005\ kg \cdot m^2$ and $J_L = 1.0\ kg \cdot m^2$, which is most significant in the output criteria?
Solution: The net inertia of the compound load, $J_{net} = J_M + \eta^2 J_L$. Therefore, J_L is attenuate by 0.0001, making J_M have more impact in the output criteria.

2.3.3 Torque-Speed curves

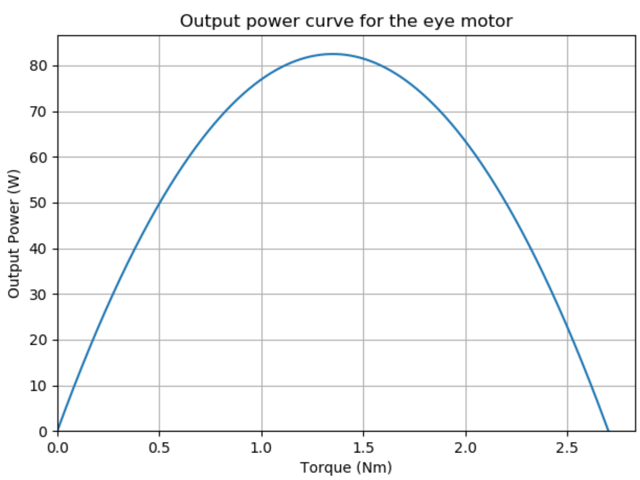
Using the formulas for torque, speed, current, power, and efficiency:



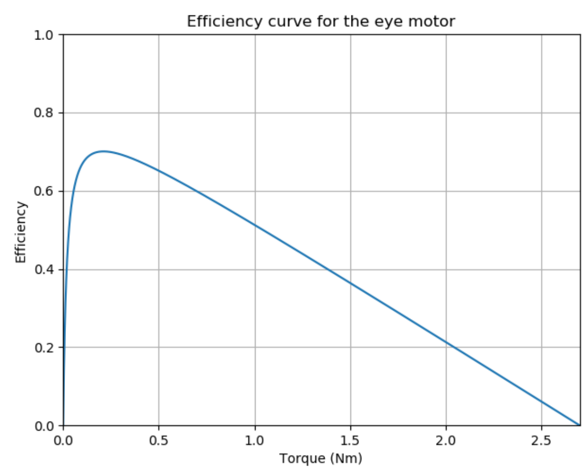
(a) Speed Curve



(b) Current Curve



(c) Power Curve



(d) Efficiency Curve

2.3.4 **Invent your own homework**

Question: Given the slope of the speed-torque curve for a DC motor as -k and the stall torque τ_s, find the load torque for which the power output is maximum. What does this expression depend upon? What does the expression for the maximum power depend upon? Compare with the characteristic power curve of a DC motor.

Answer: The output power relation

$$P_{out} = -k\tau_{load}^2 + \omega_0\tau_{load}$$
$$P_{out} = -\frac{\omega_0}{\tau_s}\tau_{load}^2 + \omega_0\tau_{load}$$

Differentiating with respect to τ_{load} and setting it to zero

$$0 = -2\frac{\omega_0}{\tau_s}\tau_{load} + \omega_0$$
$$2\frac{\omega_0}{\tau_s}\tau_{load} = \omega_0$$
$$\tau_{load} = \frac{\tau_s}{2}$$

Therefore, the power output is maximum at half the stall torque, and only depends on the stall torque. The expression that indicates the maximum power output depends upon the no-load speed in addition to the stall torque. The power curve indicates an inverted parabola with a peak at 50% of the stall torque.