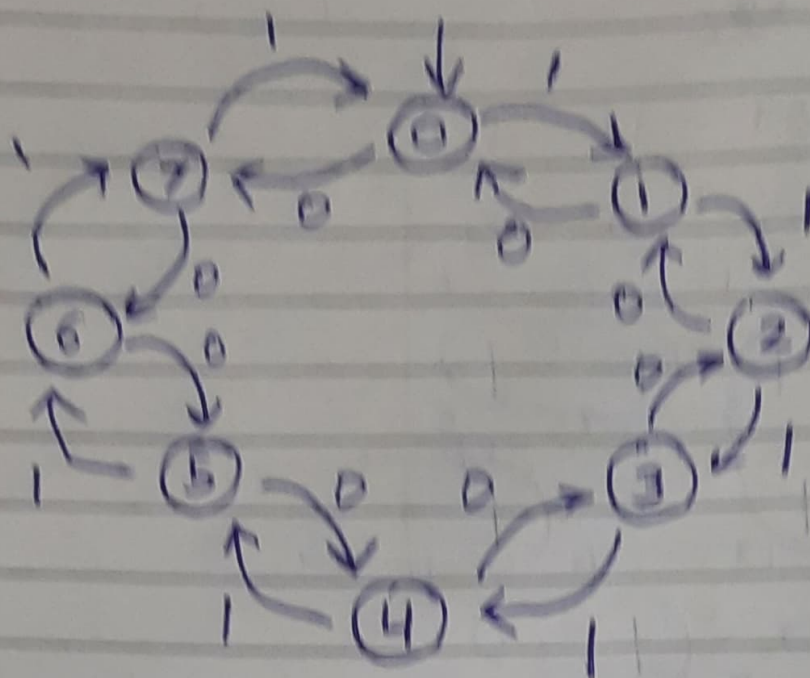


## Question 2

FSM

State encoding



|   |   |     |
|---|---|-----|
| 0 | : | 000 |
| 1 | : | 001 |
| 2 | : | 010 |
| 3 | : | 011 |
| 4 | : | 100 |
| 5 | : | 101 |
| 6 | : | 110 |
| 7 | : | 111 |

As  $S = \lambda$ , output is specified by the next state, thus no need to mention alongside input

$$M = \{I, O, S, S_0, S, \lambda\}$$

$$I = \{0, 1\} \text{ ('up' is the input)}$$

$$O = \{000, 001, \dots, 111\} \text{ (count values)}$$

$$S = \{0, 1, \dots, 7\} \text{ (states)}$$

8 states, thus  $\log_2 8 = 3$  storage elements

$$S_0 = 0$$

~~$S = \lambda$~~

$$\lambda_i(\text{up}, s_2, s_1, s_0) = s_i \text{ (for } i = 0, 1, 2)$$

(STT)

| up | $s_2$ | $s_1$ | $s_0$ | $t_2$ | $t_1$ | $t_0$ | m  |
|----|-------|-------|-------|-------|-------|-------|----|
| 0  | 0     | 0     | 0     | 0     | 0     | 0     | 0  |
| 0  | 0     | 0     | 1     | 0     | 0     | 0     | 1  |
| 0  | 0     | 1     | 0     | 0     | 0     | 1     | 2  |
| 0  | 0     | 1     | 1     | 0     | 1     | 0     | 3  |
| 0  | 1     | 0     | 0     | 0     | 1     | 1     | 4  |
| 0  | 1     | 0     | 1     | 1     | 0     | 0     | 5  |
| 0  | 1     | 1     | 0     | 1     | 0     | 1     | 6  |
| 0  | 1     | 1     | 1     | 1     | 1     | 0     | 7  |
| 1  | 0     | 0     | 0     | 0     | 0     | 1     | 8  |
| 1  | 0     | 0     | 1     | 0     | 1     | 0     | 9  |
| 1  | 0     | 1     | 0     | 0     | 1     | 1     | 10 |
| 1  | 0     | 1     | 1     | 1     | 0     | 0     | 11 |
| 1  | 1     | 0     | 0     | 1     | 0     | 1     | 12 |
| 1  | 1     | 0     | 1     | 1     | 1     | 0     | 13 |
| 1  | 1     | 1     | 0     | 1     | 1     | 1     | 14 |
| 1  | 1     | 1     | 1     | 0     | 0     | 0     | 15 |



| $s_1 s_0$ |    | $t_0$ |    |    |    |
|-----------|----|-------|----|----|----|
| $up\ s_2$ |    | 00    | 01 | 11 | 10 |
|           | 00 | 1     | 0  | 0  | 1  |
|           | 01 | 1     | 0  | 0  | 1  |
|           | 11 | 1     | 0  | 0  | 1  |
|           | 10 | 1     | 0  | 0  | 1  |

$$\delta_0(up, s_2, s_1, s_0) = \boxed{\overline{s_0}}$$

$S_1 S_0$   
 $t_1$   
 $U P S_2$

|    | 00              | 01              | 11              | 10              |
|----|-----------------|-----------------|-----------------|-----------------|
| 00 | 1 <sub>0</sub>  | 0 <sub>1</sub>  | 1 <sub>3</sub>  | 0 <sub>2</sub>  |
| 01 | 1 <sub>4</sub>  | 0 <sub>5</sub>  | 1 <sub>7</sub>  | 0 <sub>6</sub>  |
| 11 | 0 <sub>12</sub> | 1 <sub>13</sub> | 0 <sub>15</sub> | 1 <sub>14</sub> |
| 10 | 0 <sub>8</sub>  | 1 <sub>9</sub>  | 0 <sub>11</sub> | 1 <sub>10</sub> |

let up be denoted by  $U$

$$\begin{aligned}
 S_1(U, S_2, S_1, S_0) &= \overline{U} \cdot \overline{S_1} \cdot \overline{S_0} \\
 &\quad + \overline{U} \cdot S_1 \cdot S_0 \\
 &\quad + U \cdot \overline{S_1} \cdot S_0 \\
 &\quad + U \cdot S_1 \cdot \overline{S_0}
 \end{aligned}$$

$$= \overline{U} (\overline{S_1} \overline{S_0} + S_1 S_0) + U (\overline{S_1} S_0 + S_1 \overline{S_0})$$

$$= \overline{U} (S_1 \oplus S_0) + U (S_1 \oplus S_0)$$

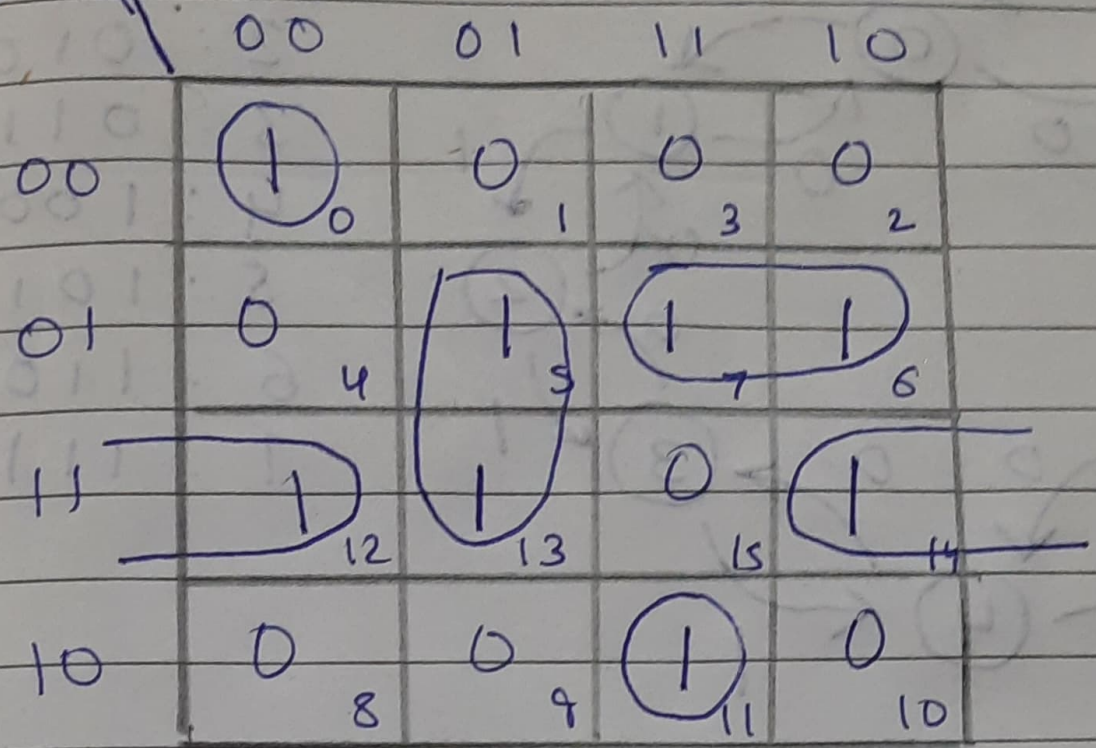


S<sub>2</sub>

U S<sub>2</sub>

S<sub>1</sub> S<sub>0</sub>

t<sub>2</sub>



$$\begin{aligned} f_2(U, S_2, S_1, S_0) = & \bar{U} \bar{S}_2 \bar{S}_1 \bar{S}_0 \\ & + \\ & U \bar{S}_2 S_1 S_0 \\ & + \\ & S_2 \bar{S}_1 S_0 \\ & + \\ & S_1 \bar{U} S_2 \\ & + \\ & \bar{S}_0 U S_2 \end{aligned}$$

