

Liquidity, Default Risk, and the Information Sensitivity of Sovereign Debt

Stelios Fourakis*

October 30, 2025

[CLICK HERE FOR MOST RECENT VERSION](#)

Abstract

In this paper, I document that, during the height of the Eurozone Debt Crisis in Spain, 1.) Spanish government bonds became substantially less liquid and less traded in secondary markets, 2.) the first appearance of this phenomenon came years after the initial jump in interest rate spreads in late 2008, and 3.) it persisted throughout the period of peak interest rate spreads and only subsided after the worst of the crisis had passed. I show that a model of sovereign default in which some traders may have private information about the country's future economic growth reproduces the delayed reaction of bid-ask spreads, their peak during the height of the crisis, and their relationship with trading volumes. The model-implied relationship between current bid-ask spreads and future GDP generates a GDP forecast that significantly outperforms a standard, benchmark forecast. Finally, the model-implied losses to investors from lower liquidity rose to 0.26% of Spanish GDP during the peak of the crisis.

*Department of Economics, Johns Hopkins University. 617 Wyman Park Building, 3100 Wyman Park Drive, Baltimore, MD 21211. Email: sfourakis@jhu.edu

I am deeply indebted to Manuel Amador and Tim Kehoe for their advice throughout this project. I am also very thankful for the comments of Mark Aguiar, Marco Bassetto, Dean Corbae, Olivier Jeanne, and Francesco Bianchi. Finally, I am grateful for the comments of seminar participants at the University of Wisconsin and Princeton University.

1 Introduction

The Eurozone Debt Crises of 2008-2014 were an unprecedented event for the modern, developed world. Greece was forced to default. Ireland, Portugal, and Cyprus had to be rescued with bailouts. And Spain and Italy required unprecedented support to avoid disaster. Since the end of the Second World War, both policymakers and academics had thought sovereign debt crises and default to be almost exclusively problems of Emerging Market Economies and Developing Countries. While the cases of Portugal and especially Greece bear some resemblance to debt crises in Emerging Market Economies, which have been studied at length¹, the cases of Spain, Italy, and Ireland had no such obvious parallel. Spain and Ireland had relatively low levels of government debt prior to the crisis, and Italy had been steadily reducing its debts over the decade prior to the crisis. Furthermore, secondary market events played a significantly larger role in the evolution and narrative of the crisis overall.

As the periphery countries came under stress, the markets for their outstanding debts began to break down. This is not necessarily a natural outcome. Certainly, equilibrium prices should fall as default risk rises, but that does not imply that it should become harder to find someone willing to trade at those prices. However, the liquidity of each country's debt declined sharply during the peak of their crisis,² and those debts become significantly harder to trade. This paper focuses on documenting and rationalizing secondary market patterns in the experience of Spain, as well as measuring the cost of secondary market disruptions during the crisis. I focus on the case of Spain because of the availability of granular, high frequency data on trading activity in secondary markets for its bonds, which I describe below.

I study the interactions between government borrowing and default decisions, interest rate spreads, trading volumes, and the liquidity of government debt securities. First, I provide novel data on the relationship between realized trading volumes of Spanish bonds and their liquidity, as measured by bid-ask spreads in secondary markets. When bid-ask spreads widen

¹Both ran large current account deficits and issued debt at a rapid pace in order to fund a consumption binge during the relatively good economic times of 2001 – 2008. When circumstances soured in late 2008 and early 2009, before deteriorating much further over the next few years, both were caught with huge stocks of debt that were unsustainable as their economies contracted rapidly.

²See e.g. [Chaumont \(2024\)](#) for evidence from Greece or [Pelizzon et al. \(2016\)](#) for evidence from Italy.

beyond about 10 basis points, trading volumes drop dramatically. Second, I show that there is a highly nonlinear relationship between bid-ask spreads and both interest rate spreads and implied default risk. Specifically, bid-ask spreads barely reacted at all to the initial rise in interest rates that began in 2008. Once interest rates rose higher and passed a certain threshold in 2011, bid-ask spreads jumped from their prior levels and began to comove closely with interest rate spreads.

I then build a quantitative model of sovereign default incorporating frictional secondary markets. I use the qualitative nature of the patterns in the data to discipline my choice of friction. Bid-ask spreads are traditionally (see e.g. [Huang and Stoll \(1997\)](#)) decomposed into 1) the presence of private information, 2) transaction costs and operating costs charged by intermediaries, and 3) inventory costs and risk. Combining the first, which can lead to adverse selection, with costly information acquisition yields the most plausible explanation for the key features of the data from Spain.

Asymmetric information among traders can of course generate bid-ask spreads that are correlated with realized trading volumes. Furthermore, if private information is acquired at cost, then the proportion of market participants who decide to acquire information will vary with the value of that information. During periods with low credit risk, the ex ante expected value of any information for pricing debt may be very low. After all, low credit risk implicitly limits the dispersion of possible payoffs to a bondholder. On the other hand, as dispersion in those payoffs rises, information can become more valuable. For small fluctuations in credit risk, this may have no effect on the decision to acquire information, so it is possible for default risk and interest rate spreads to rise some without immediately triggering problems in secondary markets. Once they rise enough, however, liquidity changes sharply as some agents pay to acquire information and other agents react to that choice. In other contexts, [Benmelech and Bergman \(2018\)](#) and [Feldhutter and Poulsen \(2018\)](#) also argue that partially discontinuous, asymmetric patterns in the data on corporate bonds can be explained by adverse selection.

Neither of the other classic explanations can produce this partially discontinuous relationship. Transaction costs and operating costs should not generally vary significantly over time, or

covary with features of the business cycle or measures of default risk. Models of inventory risk, on the other hand, would predict a continuous relationship between future expected risks and liquidity, rather than the partially discontinuous one observed in the data.

In my model, the only friction in secondary markets is induced by some investors acquiring, at cost, private information about the country's future economic prospects. I show this mechanism can produce the patterns observed in the data. The key assumptions of my setting are 1) random differences in fundamental valuations of bonds between buyers and sellers, 2) a secondary market with anonymous (with respect to information acquisition and fundamental valuation) trading, and 3) the ability of some agents to acquire, at cost, private information that is useful for pricing the bond. The reason for assumption 3 is outlined above. Assumption 1 rules out indeterminacy when no agent acquires information, as well as the applicability of standard no-trade theorems should any agent acquires information. Finally, assumptions 1 and 2 together guarantee that both sides cannot consistently invert each other's actions to infer their information. When there is a possible surplus as well as the possibility that one party has more information than the other, the less informed party cannot be sure whether the transaction is clearing because there actually is a real surplus or because the other party is exploiting its informational advantage.

I calibrate the model to match the experience of Spain before and during the Eurozone Debt Crises. In order to validate my assumptions about the nature of secondary market frictions, I exploit a relationship the model predicts between bid-ask spreads in secondary markets and future realizations of the country's GDP. In particular, I filter realized bid-ask spreads through the lens of the model to produce one step ahead forecasts of GDP and show that this forecast does significantly better than a standard benchmark. I then examine the model's predictions of how crises should evolve and show they match the experience of Spain quite well. I also develop measures of liquidity based on whether gains from trade between buyers and sellers in secondary markets are actually achieved, and show that periods with high bid-ask spreads (and high rates of information acquisition) tend to be characterized by much lower realized gains from trade. Finally, I use the model to measure the magnitude of the losses to investors during the peak of the crisis that can be attributed to falls in realized gains

from trade. I find that the cost of these secondary market disruptions was about 0.26% of Spain’s GDP from January 2011 to June 2012. The scale of these losses is large and suggests there are substantial gains (to both the borrowing country and its foreign investors) from interventions aimed at reducing private information acquisition. While I focus on the market for government debt in Spain, the framework I use could in theory could also be applied to other asset markets.

2 Literature Review

This paper connects to the broader literature on the information sensitivity of debt, including [Gorton and Ordoñez \(2014\)](#), [Gorton and Ordoñez \(2020\)](#), [Dang et al. \(2015\)](#), and [Dang et al. \(2020\)](#), who all consider environments with interconnected choices of borrowing and information acquisition.³ A common theme in these papers is that in normal times, when average risk levels are lower, most borrowing choices do not trigger information acquisition. On the other hand, in periods of higher risk, the information insensitive set of borrowing values shrinks, more borrowers make choices which end up triggering information acquisition, and equilibrium prices must “compensate” creditors for the cost of acquiring that information. This paper will feature similar dynamics, albeit in a relatively simple secondary market. Furthermore, I apply quantitative discipline to the magnitude of information sensitivity in this market and use a calibrated model to measure the costs of breakdowns in trading that can arise during periods of higher risk.

The papers in the sovereign default literature most closely related to this one are [Passadore and Xu \(2022\)](#) and [Chaumont \(2024\)](#). Both incorporate frictional secondary markets into otherwise standard model of sovereign defaults in order to produce endogenous bid-ask spreads which covary with interest rates and default risk. [Passadore and Xu \(2022\)](#) study the case of Argentina’s 2001 default. Their model includes investors of two types (high and low, with high types randomly transitioning to low types). Low type investors have a larger exogenous holding cost for the bond, which leads to them having less bargaining power when

³This paper of course also connects to the work studying trading patterns when some traders have private information, including [Kyle \(1985\)](#) [Glosten and Milgrom \(1985\)](#).

their bondholdings are in default, so rises in default risk increase bid-ask spreads. Crucially, however, the probability of trading is assumed to be constant, which implies that trading volumes are constant and independent of bid-ask spreads.

[Chaumont \(2024\)](#) studies the case of Greece's 2012 default. The basic mechanism which produces the differences in valuations that lead to gains from trade is identical to that of [Passadore and Xu \(2022\)](#) (high and low type investors who vary by their preference for owning the government's bond). However, he also explicitly models the primary and secondary markets separately and distinguishes between primary dealers and normal investors. Investors and dealers engage in a directed search problem to choose which market to trade in. Markets are distinguished by the transaction fees for trading in them. These transaction fees, net of participation costs, are dealer profits, and bid-ask spreads are driven by the equilibrium distribution of transaction fees. As default risk rises, low type investors begin to panic and move to markets with higher transaction costs in order to quickly sell their bonds. Dealers and high type investors adjust so that markets clear, but the equilibrium outcome yields higher average transaction fees as well as higher trading volumes. In this setting, bid-ask spreads and trading volumes therefore rise together when default risk rises.

In the data for Spain, I observe that trading volumes fall as bid-ask spreads and default risk rise, which these mechanisms either miss or contradict. Furthermore, these mechanisms predict a continuous relationship between default risk and bid-ask spreads, rather than the partially discontinuous one observed in the data. I do not directly model intermediaries, but instead incorporate a simple trading protocol that allows me to easily characterize behavior when one party has an informational advantage over the other. This allows me to capture the above two key features of my data that these mechanisms would miss. My model also makes measuring investor welfare cleaner by avoiding variation due to changes in incidence of exogenous holding costs.

In the context of countries where inflation risk (rather than actual default risk) is the primary worry for investors, [Bassetto and Miller \(2025\)](#) studies a model in which deficits, debt levels, and inflation can be either persistently disconnected or tightly linked. Bondholders can acquire information about future government surpluses for a cost. When the value of this

information is low, agents do not acquire information, and bond prices do not respond to realized deficits or debt levels. However, once debt levels are high enough and surpluses variable enough, agents become worried enough about repayment to acquire the information, causing a reconnect between deficits, debt, and inflation. As in [Bianchi and Ilut \(2017\)](#), debt suddenly becomes informationally sensitive and prices respond to changes in fundamentals. My model also features a reconnect between key outcomes, interest rate spreads and bid-ask spreads, that is driven by endogenous information acquisition decisions. Whereas [Bassetto and Miller \(2025\)](#) effectively focus on primary market implications, I focus on the effects of this reconnect on liquidity and trading volumes in secondary markets.

A final pair of related papers are [Cole et al. \(2025\)](#) and [Cole et al. \(2022\)](#). Both consider the effects of information asymmetry among investors on outcomes in the primary market. The model of [Cole et al. \(2025\)](#) is more stylized and designed to illustrate a mechanism by which asymmetric information about default risk can impact interest rates paid by the government when auctioning debt. Motivated by data on Mexican government debt auctions, [Cole et al. \(2022\)](#) is a more quantitative paper. The authors show that bids from larger bidders are significantly more likely to be accepted, and that this observation is consistent only with the presence of asymmetric information among investors. After calibrating the model to match the specific patterns they observe in the data, they find that information asymmetries can have economically significant effects on the yields paid by the government when it auctions new debt. While my model also incorporates information asymmetries among investors, I focus on the effects of these asymmetries in secondary markets because secondary market frictions were a key source of concerns during the Eurozone Debt Crises. In primary markets, my model features symmetric information among all participants.

3 Data

The data on bond pricing and trading volumes used in this paper comes from two primary sources. First, until mid-2018, the Bank of Spain maintained public, detailed, complete records of trading in the secondary market for Spanish government debt (as well as pur-

chases in the primary market).⁴ For each individual security (distinguished by its ISIN) and type of transaction on each trading day, these records include trading volumes, the average transacted price, and the range of transacted prices. In constructing the data for trading volumes, I use only transactions where the security permanently changes hands (rather than including repos as well). In order to constructed bond-month specific weights, I use another component of these records, which is produced monthly and contains the total value of each security held by the public at the end of the month.

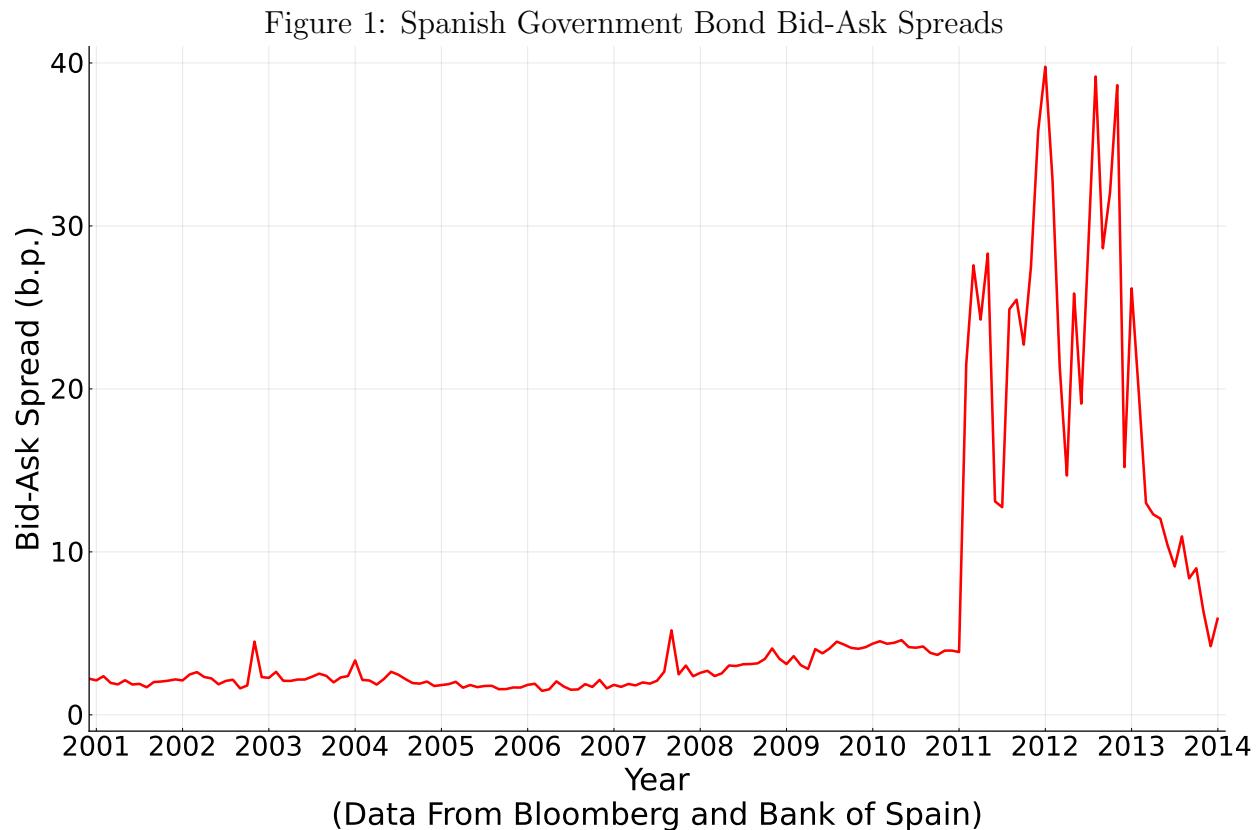
The second source I use in this section is Bloomberg data on bond-specific (ISIN) daily bid yields and ask yields. To construct daily average yields for the entire stock of Spanish government debt, I apply the weights from the Bank of Spain data to these yields. Differencing the bid yields and the ask yields produces the bid-ask spreads that I use throughout this paper. This definition of bid-ask spreads is tailored to the case of bonds and designed to be comparable in meaning to interest rate spreads. It differs from the more standard price based definition $\frac{p_{ask} - p_{bid}}{p_{mid}}$ where $p_{mid} = \frac{p_{ask} + p_{bid}}{2}$ (which can easily be applied to a broader range of securities, including equities). The results below are robust to using a price-based definition instead of my yield-based one.

The bid-ask spread is the most commonly used proxy for the liquidity of an asset. It is the difference between exchange-posted prices for an immediate sale and for an immediate purchase. The bid price is the highest price attached to an active, unfilled buy order submitted to the exchange, and is therefore, to an investor not holding the asset, the price for an immediate sale. The ask price is the lowest price attached to an active, unfilled sell order submitted to the exchange, and is therefore, to an investor holding the asset, the price for an immediate purchase. The difference between the two must be positive, and is known as the bid-ask spread. When bid-ask spreads are narrow, perhaps accounted for fully by technological costs, almost all trades for which a real surplus exists occur. For any pair of distributions of seller valuations and potential buyer valuations, if observed bid-ask spreads widen, then, all else equal, it must be the case that relatively more trades for which a real surplus exists

⁴The archive of these data can be found in [Bank of Spain \(2018\)](#). Since mid-2018, these data have been recorded by Bolsas y Mercados Españoles Renta Fija, S.A., but the detailed data for periods after the handover are no longer publicly available.

do not occur. At least one potential participant in each such trade is choosing not to trade by bidding below their true valuation or asking above it. Markets and circumstances which lead to more and larger potential surpluses from trade not being realized are often termed “illiquid,” which is one reason why bid-ask spreads are a good proxy for liquidity.

Throughout 2001 – 2016, Spanish government debt constituted one of the ten biggest single markets in the world. During that time, the average outstanding face value of its debt securities averaged just over 500 billion Euros. The average turnover in secondary markets as a percent of units outstanding over the same period was 0.8% daily (or 15.4% monthly). Even during the worst stages of the crisis in 2011 – 2013, the 1st and 5th percentiles for turnover were, respectively, 0.06% and 0.14% daily (or 4.5% and 5.6% monthly). Just the daily numbers represent total exchanges of 300 – 700 million euros worth of face value. Observed turnover rates imply substantial activity, even during the peak of the crisis.



That said, during the height of the Spanish debt crisis, the liquidity of Spanish government bonds changed sharply. Figure 1 plots the monthly weighted average bid-ask spread, the

difference between interest rates demanded by buyers of Spanish government bonds and those demanded by sellers. The beginning of the period of when bid-ask spreads rose significantly away from their pre-crisis average is January 2011. To confirm that the period from then until June 2012 (the last month before Mario Draghi made his “Whatever It Takes” speech) is associated with markedly subdued activity in secondary markets, I plot in Figure 2 daily weighted average bid-ask spreads against the average daily trading volumes in secondary markets (measured as total units exchanged divided by total units outstanding).

Figure 2: Spanish Government Bond Trading Volumes and Bid-Ask Spreads (Daily)

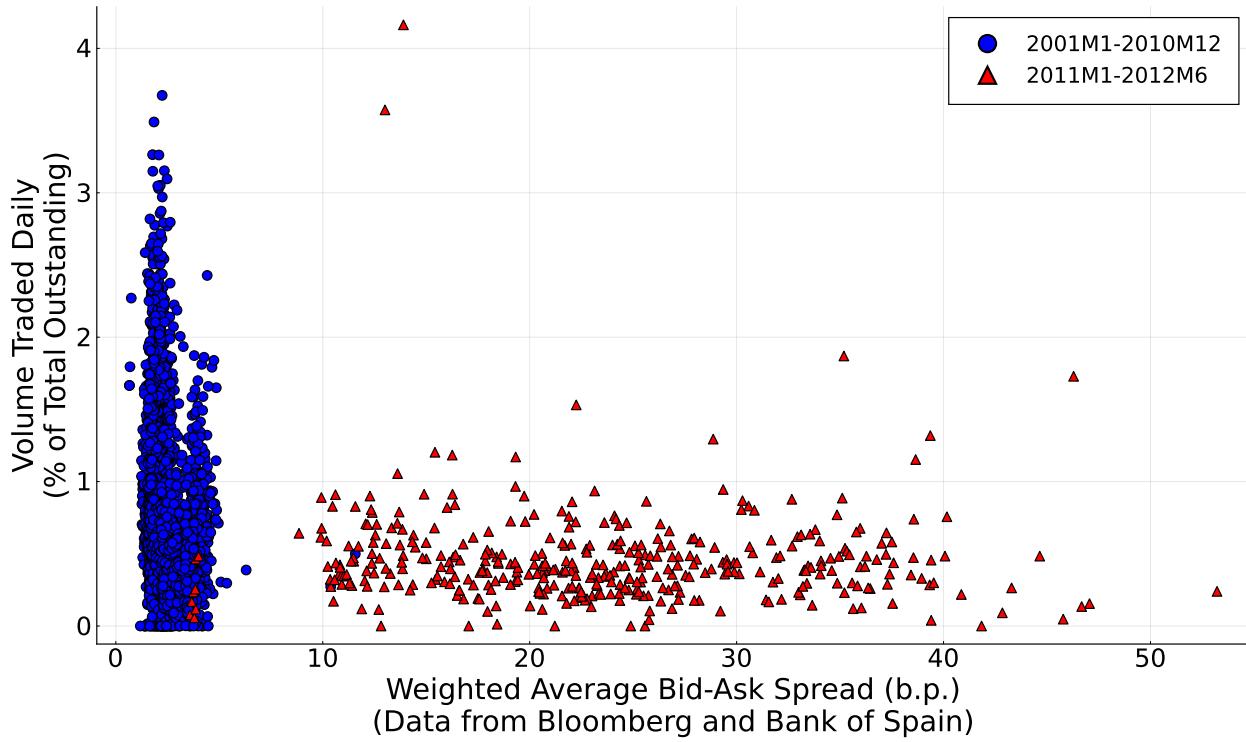
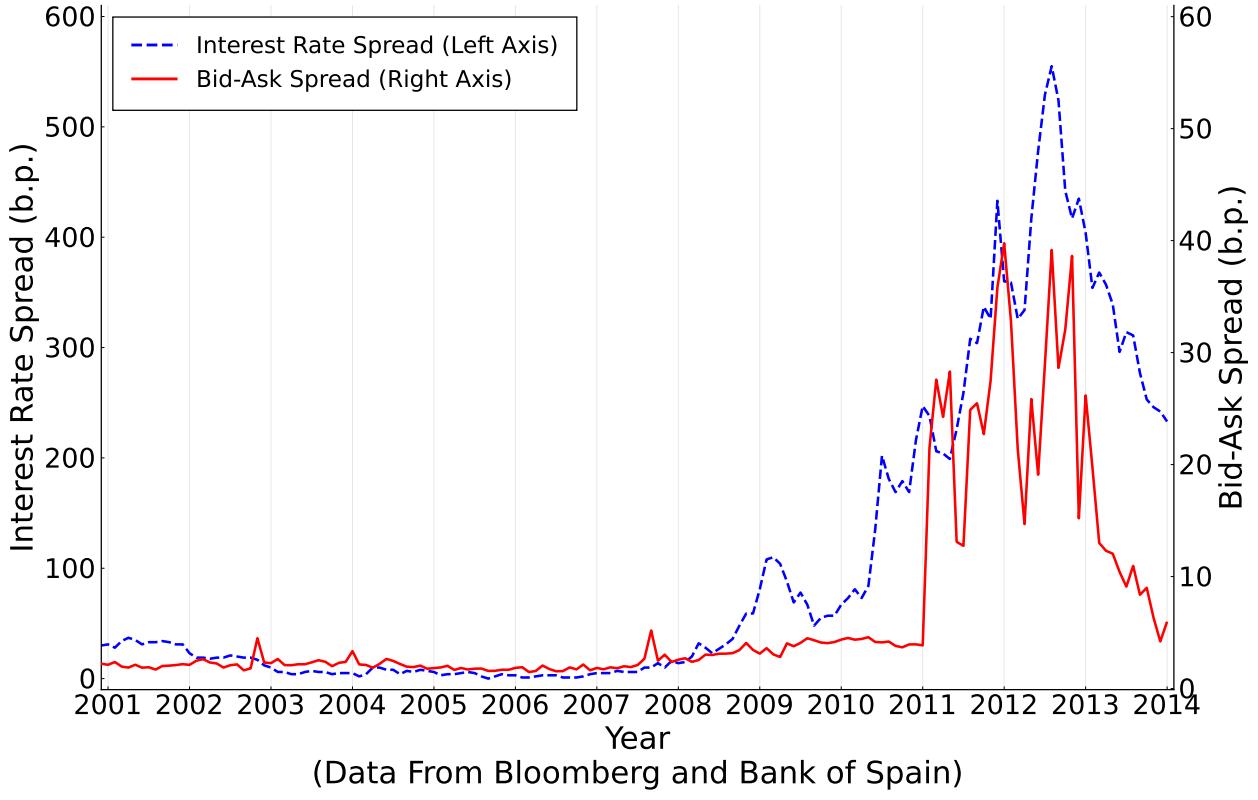


Figure 2 shows that when bid-ask spreads are elevated, the range of possible outcomes for volumes is almost always far narrower than when bid-ask spreads are low.⁵ Furthermore, while 20-50 basis points might be small relative to average interest rate spreads in EMEs, these bid-ask spreads occurred in an advanced economy whose interest rate spreads averaged just 0.35% over the prior 10 years and just 0.72% over the full period 2001M1-2012M6.

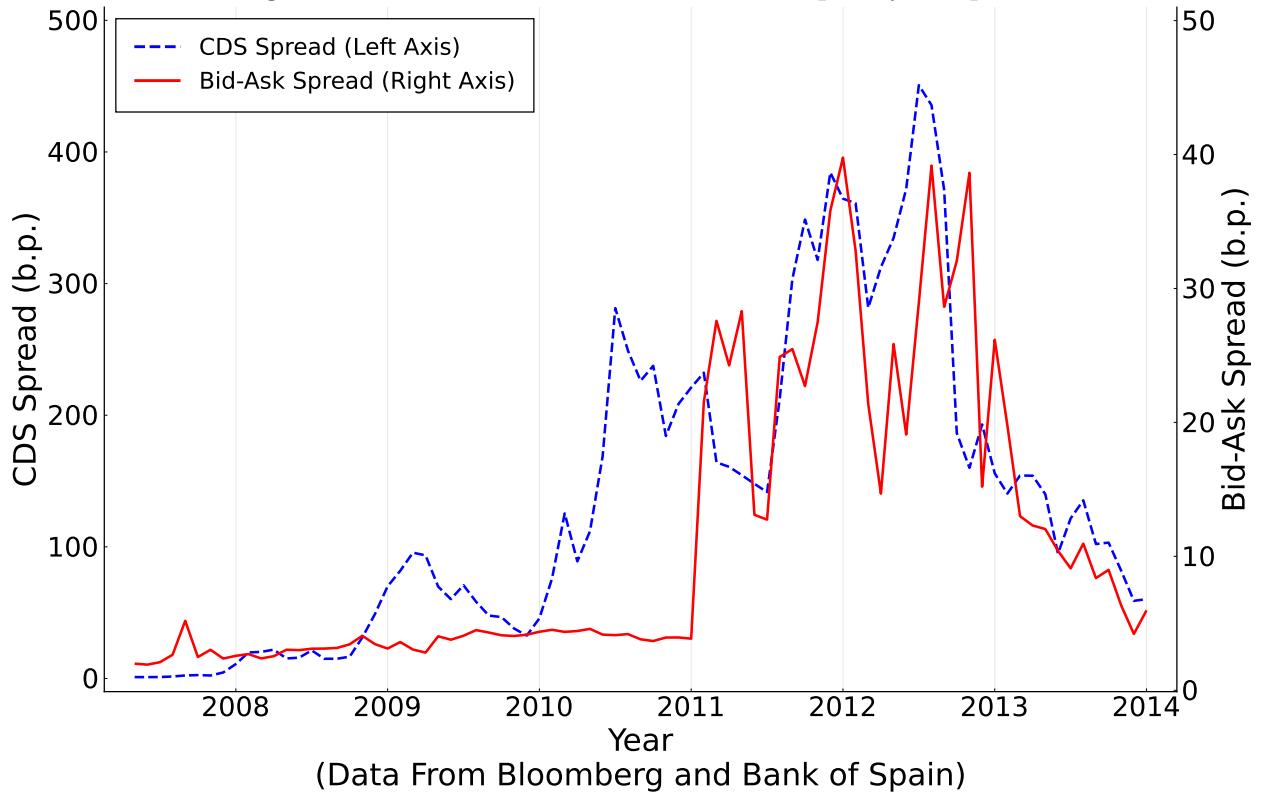
⁵The two outlier observations (high volumes with elevated bid ask spreads) correspond to Friday, March 9, 2012, the day Greece completed its restructuring and announced total participation in the exchange was 82.5%, and Monday, March 12, 2012, the first day those restructured bonds traded. I interpret this burst of trading as the result of uncertainty resolution regarding restructuring outcomes in Europe.

Figure 3: Spanish Government Bond Spreads



In any case, these data do much more than simply confirm that bid-ask spreads are a useful proxy for liquidity. The time dimension is also an important part of the story. The narrow time period in which we observe high bid ask-spreads and low trading volumes coincides almost exactly with the chapter of the crisis in which interest rate spreads on Spanish government debt were highest. To illustrate this, Figure 3 plots the spread between benchmark Spanish government bonds and their German counterparts. In the three years from 2008 to 2011, interest rate spreads rose from essentially zero to about 2%, with barely any reaction at all from bid-ask spreads. As they continued to rise during 2011, bid-ask spreads suddenly began responding. They jumped significantly away from their prior low levels and covaried moderately with interest rate spreads until the end of the crisis. In order to tease apart the effects of expected future liquidity risks from those of default risk itself, I turn to the probability of default over various horizons implied by the price of credit default swaps. Figure 4 plots the monthly average 1 year CDS spread for contracts on Spanish government debt from 2007 to 2014. As before, the period of unusually high bid-ask

Figure 4: Short Term Default Risk and Liquidity in Spain



spreads also corresponds to the period during which CDS spreads peak. Again, however, the CDS spreads began to rise much earlier, approximately in line with the interest rate spread series.

Finally, the spike in bid-ask spreads near the beginning of 2011 does not coincide with any change in the country's credit ratings. Between September 2010 and March 2011, Spain had ratings just 1-2 levels below AAA for all three agencies. S&P rated Spain AA from April 2010, until October 2011, when they downgraded it to AA-. Moody's rated Spain AA1 from October 2010 until March 2011, when they downgraded it to AA2. Fitch rated Spain AA+ from May 2010 until October 2011, when they downgraded it to AA-. The only rating event in the six month window from September 2010 to March 2011 is Moody's adding a negative outlook on December 15, 2010, and there is no discernible change in behavior on or around that date. Bid-ask spreads were 3.36 b.p. on December 14, 3.33 b.p. on December 15, and 3.46 b.p. on December 16. From December 1 to December 14, they averaged 3.57 b.p., and from December 16 to December 31, they averaged 3.49 b.p..

4 Model

To rationalize these data, I build on the setting of Chatterjee and Eyigungor (2012). Time is discrete and infinite. There is a small open economy with a representative consumer and a benevolent government who have identical recursive preferences over consumption and continuation values given by

$$V = U(c, V'). \quad (1)$$

$U : \mathbb{R}_{++}^2 \rightarrow \mathbb{R}_{++}$ is a continuously differentiable function that is strictly increasing, homogeneous of degree 1, and satisfies $\lim_{c \downarrow 0} U_1(c, V') = +\infty$ and $\lim_{V' \downarrow 0} U_2(c, V') = +\infty$. The exogenous stochastic state of the world s is a Markov Process and governs the country's GDP $y(s)$.

The government may borrow on international markets using a defaultable long term bond. Following Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), I use a probabilistic characterization of maturity. Each bond matures with constant probability λ each period. With complementary probability, it instead pays a coupon κ . There is a continuum $[0, \bar{B}]$ of risk-neutral, competitive, international lenders, each of whom can hold a unit of the bond. \bar{B} is assumed to be large enough that it is never binding in equilibrium. When the country defaults, it enters financial autarky and suffers an output penalty $\phi(s)$. It exits autarky at constant rate θ .

When in good standing, the government's problem at the beginning of the period is

$$W(s, b) = \max_{d \in \{0, 1\}} (1 - d)W^R(s, b) + dW^D(s). \quad (2)$$

Conditional on repayment, it solves

$$W^R(s, b) = \max_{c, b'} U(c, \bar{W}(s, b')) \quad (3)$$

such that

$$c + (\lambda + (1 - \lambda)\kappa)b = y(s) + q(s, b')(b' - (1 - \lambda)b), \quad (4)$$

where $\mu(\cdot)$ is a certainty equivalent operator and the value of continuing in good standing, $\bar{W}(s, b')$, is

$$\bar{W}(s, b') = \mu(W(s', b')|s). \quad (5)$$

Conditional on default, the government's value is

$$W^D(s) = U(y(s) - \phi(s), \bar{W}^D(s)), \quad (6)$$

where $\phi(s)$ is the cost of default, and the value of exiting the period while in default, $\bar{W}^D(s')$, is given by

$$\bar{W}^D(s) = \mu(W(s', 0), W^D(s')|s). \quad (7)$$

So far, this setting is simply a generalization of the one presented in [Chatterjee and Eyigunor \(2012\)](#) to allow for non-expected utility preferences. It will differ only with respect to the functional equation that the bond price $q(s, b')$ must satisfy. I now turn to describing the market structure in order to derive that equation.

Once the government has made its borrowing decision and the auction has been completed, there is a set $[0, b']$ of bondholders. Below, I refer to this set of lenders as “current investors,” which I distinguish from the “new investors,” with whom they may trade. Each current investor has a random discount factor $\hat{\delta} \sim F(\cdot)$. These discount factors are i.i.d. across agents and time, with $supp(F) = [\underline{\delta}, \bar{\delta}]$. They are not known at the beginning of the period. Once the auction is completed, a signal \hat{s}' about the future exogenous state of the world s' is realized. Current investors are experts on the country’s economy and may pay a cost $f(\pi)$ to attempt to learn the value of this signal. With probability π , they observe the signal, and with complementary probability, they learn nothing. After they have made that decision and observed the signal, their random discount factors are realized. The secondary market then opens and trading begins.

I now turn to describing the trading protocol in this secondary market. Each current investor is matched with a single new investor. All new investors have a fixed, known discount factor δ and cannot observe the signal \hat{s}' (but do observe the full public state of the world

$(s, b, d, b'))$.⁶ New investors do not observe the discount factor of their match or whether their match observed the signal \hat{s}' . After being matched, current investors submit an ask price and new investors simultaneously submit a bid price. If the bid price exceeds the ask price, the transaction clears and the current investor is replaced by the new investor who bought their bondholdings. For tractability, I assume that transactions clear at the bid price, so this is a “seller’s market.”⁷

Since at the time of the initial auction, prospective investors are risk neutral, competitive, and identical, the price of the bond on the primary market must be exactly the expected value of going to the secondary market with a bond:

$$q(s, b') = \max_{\pi \in [0,1]} (1 - \pi)q_U(s, b') + \pi q_I(s, b') - f(\pi) \quad (8)$$

where $q_U(\cdot), q_I(\cdot)$ denote the value of being uninformed or informed, respectively. $\pi^*(s, b') \in (0, 1)$ therefore implies

$$q_I(s, b') - f'(\pi) = q_U(s, b'), \quad (9)$$

whenever interior π is optimal. Let $\pi_S(s, b')$ be the equilibrium proportion of current investors who access the signal \hat{s}' .

5 Secondary Market Equilibrium

In order to derive the value of being informed or uninformed, as well as the equilibrium proportion acquiring information, I will introduce some new notation. Let v denote the undiscounted unit value of the asset to an uninformed agent.

$$v(s, b') = \mathbb{E}[(1 - d(s', b'))(\lambda + (1 - \lambda)(\kappa + q(s', b''(s', b'))))|s] \quad (10)$$

⁶I assume these differences between current investors and new investors simply for tractability. The basic mechanisms at work in the model are robust to relaxing them.

⁷In the corporate finance literature on the relationship between credit risk and liquidity, assumptions leading to similar results are common (see e.g. [Zhiguo and Milbradt \(2014\)](#)).

Let $\hat{v} \sim G(\cdot)$ denote the random variable which is the undiscounted unit value of the asset to an informed agent (and of course $\mathbb{E}[\hat{v}] = v$),

$$\hat{v}(s, \hat{s}', b') = \mathbb{E}[(1 - d(s', b'))(\lambda + (1 - \lambda)(\kappa + q(s', b''(s', b'))))|s, \hat{s}']. \quad (11)$$

Since transactions clear at the bid price, truth telling is a dominant strategy for sellers, so

$$p_{S,U}^*(\hat{\delta}) = \hat{\delta}v \quad p_{S,I}^*(\hat{\delta}, \hat{v}) = \hat{\delta}\hat{v}. \quad (12)$$

Knowing this, the problem solved by buyers is

$$\max_{p_B} (1 - \pi_S)(\delta v - p_B)F\left(\frac{p_B}{v}\right) + \pi_S \left(-Pr(\hat{v} = 0)p_B + \int_V (\delta\hat{v} - p_B)F\left(\frac{p_B}{\hat{v}}\right)dG(\hat{v}) \right). \quad (13)$$

The first term is the payoff to the buyer from meeting an uninformed seller, which occurs with probability $(1 - \pi_S)$. Both the probability the offer is accepted $\pi_{Accept}^{Uninformed}(p_B, v) = F\left(\frac{p_B}{v}\right)$ as well as the buyer's payoff $(\delta v - p_B)$ are independent of next period's state. The second term is the payoff to the buyer from meeting an informed seller, which occurs with probability π_S . Here, that independence ceases to hold because the seller can condition their decision on their private knowledge \hat{v} about the state of the world in the next period. The buyer's payoffs, $(\delta\hat{v} - p_B)$, are therefore negatively correlated with the probability of meeting an agent with a discount factor low enough to accept the offer p_B , $\pi_{Accept}^{Informed}(p_B, \hat{v}) = F\left(\frac{p_B}{\hat{v}}\right)$. Adverse selection (by the seller) arises: as \hat{v} falls, $(\delta\hat{v} - p_B)$ falls while $\pi_{Accept}^{Informed}(p_B, \hat{v})$ rises.

Therefore, if there is any dispersion in \hat{v} , the value to the buyer at any given bid is decreasing in the proportion of sellers which acquire information. This results in equilibrium average bid prices for any fixed v that decrease as the dispersion of the \hat{v} values rises. Given any bidding strategy for buyers, dispersion in \hat{v} raises the value of acquiring information from the perspective of sellers, so more sellers acquire that information. Buyers react to this by submitting lower bid prices. However, the equilibrium average ask price is always v multiplied by the average discount factor of sellers. Since when default risk is high, information about the value of s' helps relatively more in discerning future values, average bid prices drop while

average ask prices remain the same, and the average bid-ask spread rises while the average volume of transactions falls. As the average bid price falls, however, the ex ante value to both uninformed as well as informed sellers of bond falls. This in turn affects demand when the government sells new debt on the primary market at the beginning of the period, where prospective lenders demand relatively higher interest rates when purchasing the bond. This is the channel by which liquidity in the model flows from secondary markets back to the price schedule faced by the government at the primary market.

In order to understand the way all of these forces interact, I walk through the determination of an equilibrium at an example point in the calibrated model. In the calibrated model, $\hat{\delta}$ has a uniform distribution and the cost of acquiring information is linear $f(\pi) = f * \pi$, so the seller's problem involves comparing q^U and $q^I - f$. I begin with a plot of these ex ante values for a range of bid prices in Figure 5.

Figure 5: Seller Value Functions

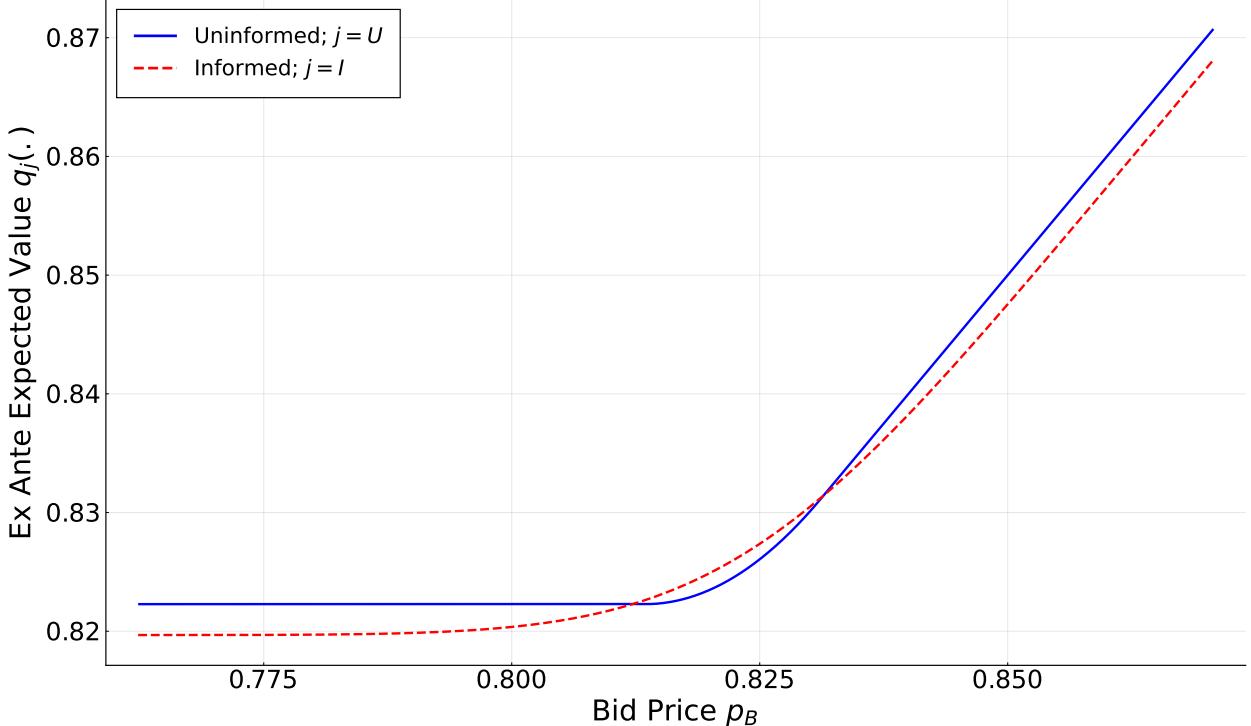
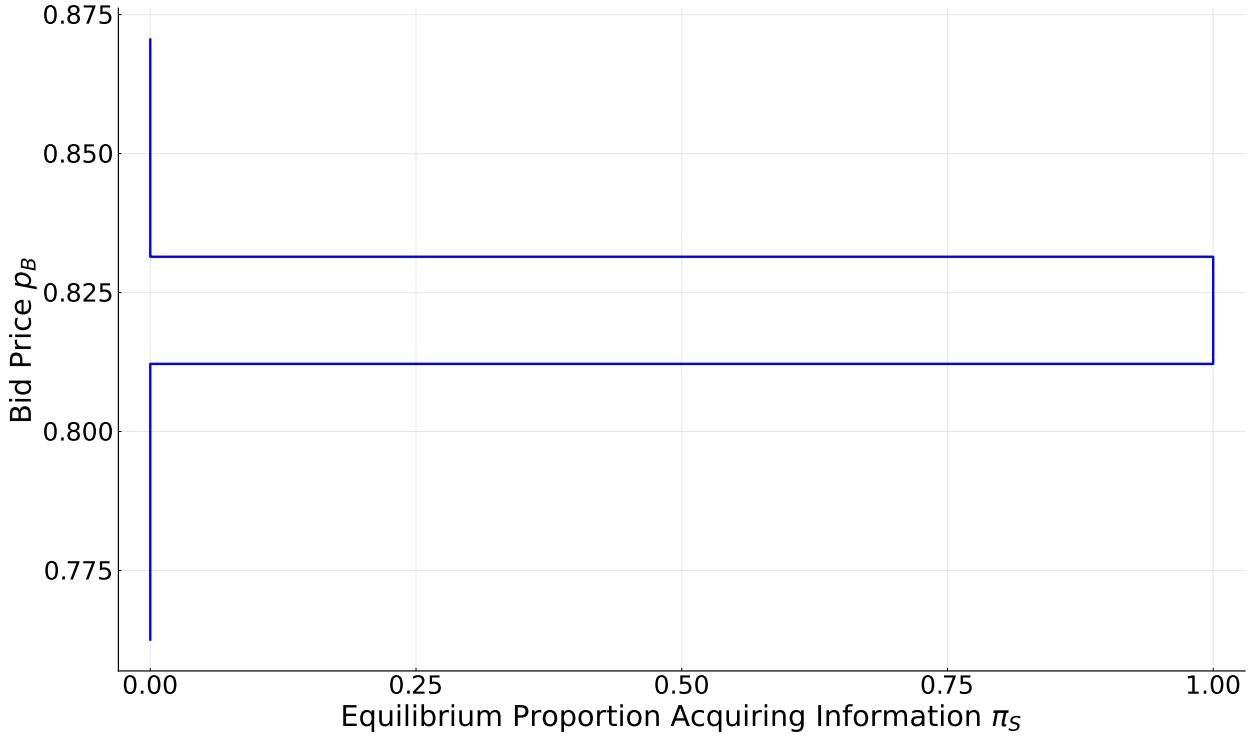


Figure 5 illustrates how information is not always worth the cost associated with acquiring it. In particular, when buyers bid very high values, sellers accept their offers almost regardless of

the signal \hat{s}' , so knowing that signal does not allow them to make any extra profit. Similarly, when buyers bid very low values, sellers reject their offers similarly frequently, so observing \hat{s}' again does not allow them to extract anything extra from buyers. For intermediate values, however, having access to \hat{s}' is very useful and allows the seller to adjust their accept/reject decision quite often. In these cases, the sellers can use their information advantage to extract enough extra value from the buyers to more than cover the cost of acquiring the information in the first place. Using this logic, I construct the set of π_S values consistent with seller optimization and plot them in Figure 6.

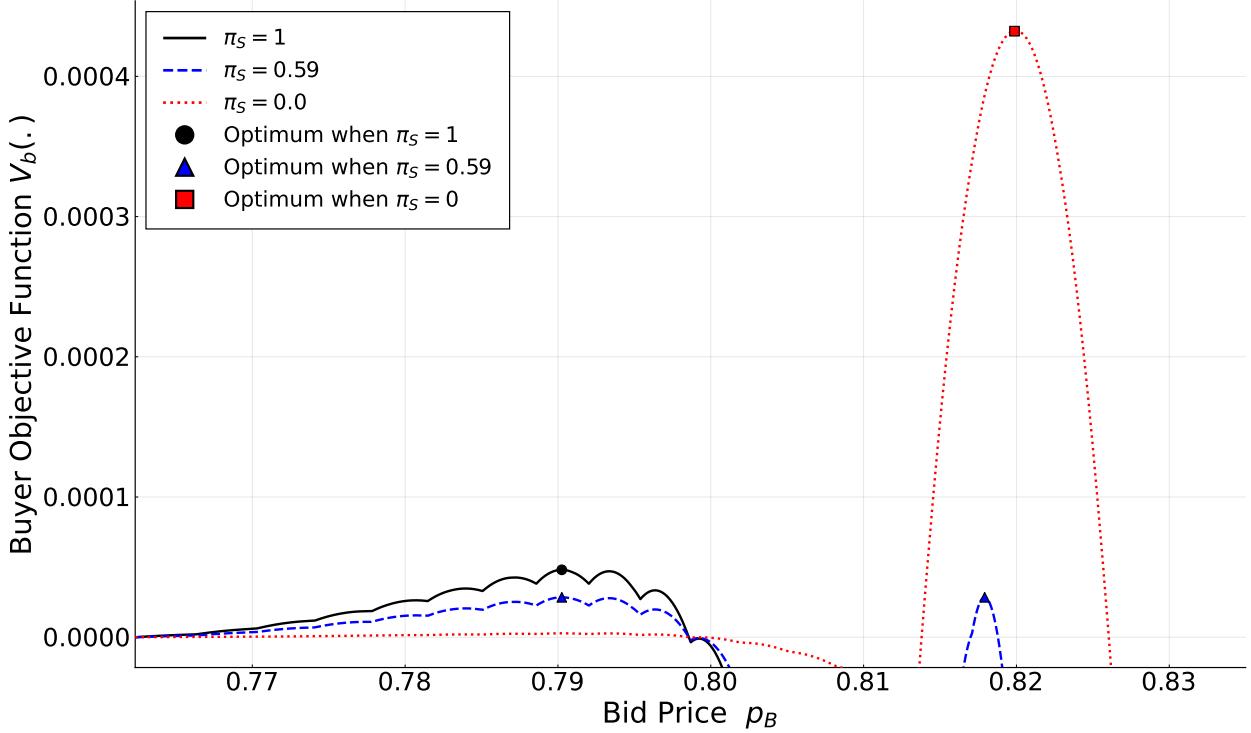
Figure 6: Seller Equilibrium Behavior



The levels at which this best response becomes flat correspond exactly to the points at which the sellers' values crossed in Figure 5.

I now turn to the buyer's problem. Figure 7 plots the value to the buyer of various bids at a few specific values of π_S . When no sellers acquire information, buyers are confident that they will be trading with uninformed sellers and submit relatively high bids (the red square). When all sellers acquire information, buyers are certain that they will be at a huge

Figure 7: Buyer Objective Functions



informational disadvantage (the black circle). In order to protect themselves, they submit relatively low bids, and trade only occurs following relatively poor signals. Some trade does still occur, and buyers do gain some of the surplus, in expectation. At the intermediate value of π_S , there are two bid prices that yield the exact same value to the buyer (the blue triangles). The lower of the two is exactly the same price buyers choose when 100% of sellers acquire information. This is because, by bidding so low, these buyers know they will never end up trading with uninformed sellers, so the only outcomes which concern them are meetings with informed sellers. When π_S takes this intermediate value, there is another bid price that allows buyers to trade with both uninformed sellers and informed sellers. This higher price exposes these buyers to much heavier information rent extraction when they meet informed sellers, but the opportunity for large gains from trade with uninformed traders just makes up for that. This logic generates the buyer's best response function plotted in Figure 8.

Figure 8: Buyer Best Response

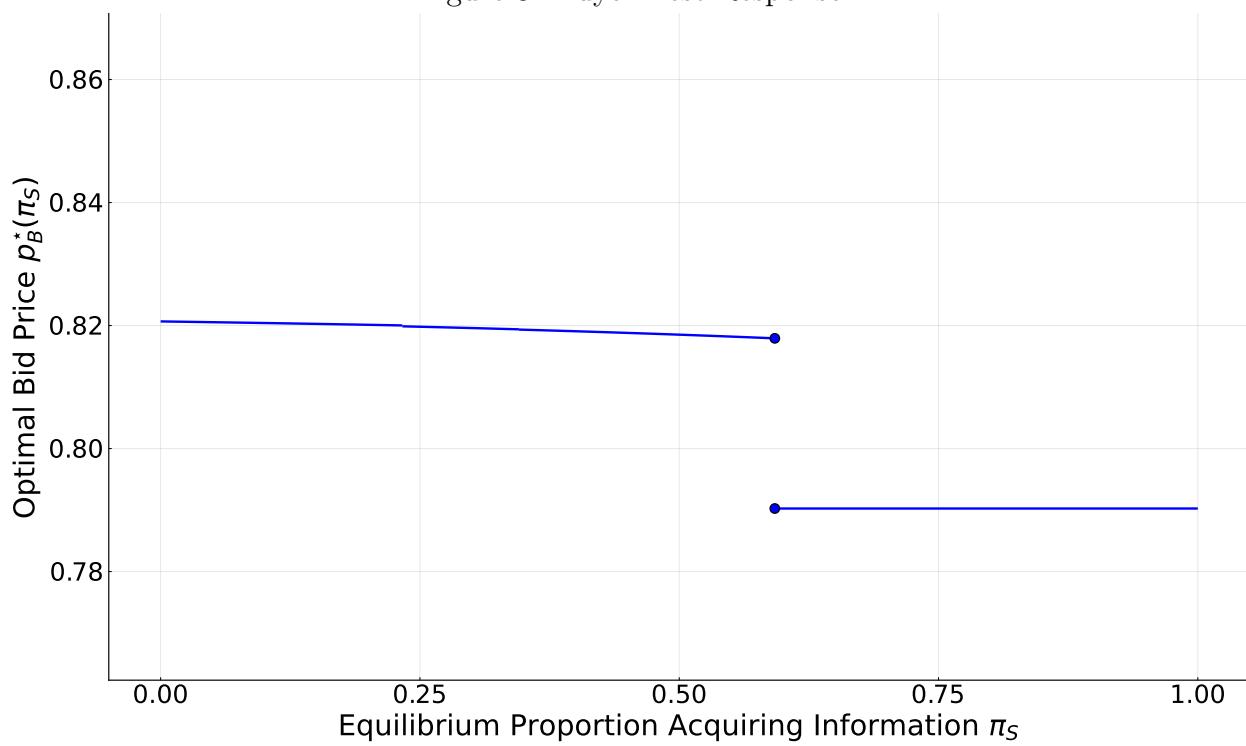
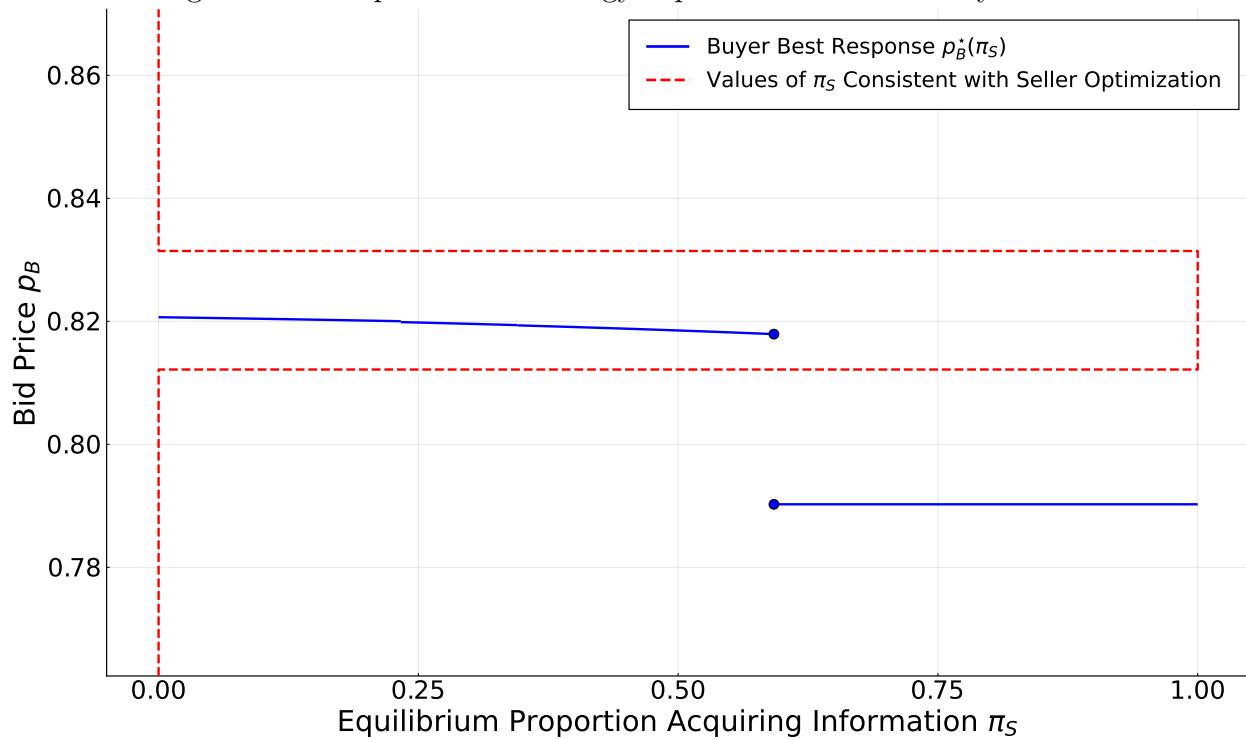


Figure 9: Example Mixed Strategy Equilibrium in Secondary Market



If we overlay the two best response plots on top of one another, we obtain Figure 9. In this case, there is a unique equilibrium, and it involves sellers choosing an intermediate value of π^* and some buyers bidding a relatively high price while the remainder bid a relatively low price. The plot of seller value function in Figure 5 shows that there must be some mixing probability for buyers at these two bid prices that makes sellers indifferent among all possible choices of π , including the one that makes buyers indifferent between these two bid prices. The unique equilibrium in this case is for both buyers and sellers to mix with those probabilities. There are four distinct types of equilibria in the secondary market:

1. No seller acquires information, buyers play identical pure strategies.
2. All sellers acquire information, buyers play identical pure strategies.
3. Only some sellers acquire information, buyers play identical pure strategies.
4. Only some sellers acquire information, buyers mix between two pure strategies.

The equilibrium obtained in the above example was of type 4. In practice, all four types occur with nontrivial frequency in the calibrated model.

6 Calibration

The model is calibrated using data on the economy and government borrowing activities of Spain. GDP is assumed to be the sum of a persistent process y_t and an i.i.d. process m_t .⁸ The persistent component of the income process y is parametrized as an $AR(1)$ process in logs with normally distributed innovations,

$$\ln(y_t) = \rho \ln(y_{t-1}) + \eta_t \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2). \quad (14)$$

The m process is parametrized as a symmetrically truncated normal random variable $m \sim \mathcal{T}\mathcal{N}(0, \sigma_m^2, -\bar{m}, \bar{m})$. The parameters $\rho, \sigma_\eta, \sigma_m$ are estimated using OECD data on Spanish Real GDP from 1986Q1 (the first quarter after Spain's accession to the European Union

⁸The role of m_t is primarily to ensure convergence of the computational algorithm used to solve the model. See [Chatterjee and Eyigunor \(2012\)](#) for a more detailed explanation.

on January 1 1986) to 2012Q2 using standard state space methods, after the removal of a quadratic time trend. \bar{m} is set to be $2\sigma_m$, a wide enough range to ensure convergence for a broad set of parameter values. Since liquidity is inherently a very short run issue, I estimate all parameters at monthly values and set the time step in the computational model to one month. Table 1 contains the resulting estimates for the income process parameters.

Table 1: Estimated Income Process Parameters (Monthly Values)

Parameter	Value	SE
ρ	0.9918	0.007
σ_η	0.0049	0.0005
σ_m	0.0015	0.0004
\bar{m}	0.0031	—

The maturity and coupon parameters for the bond are estimated using data on the full portfolio of Spanish bonds and notes outstanding in each month from 2001M1 to 2012M6. The coupon parameter is set to be the weighted average coupon rate on outstanding debts during the sample period. The maturity parameter is set to match the mean of the weighted average maturity of Spain's portfolio of outstanding debt obligations during the sample period. At each date t , that portfolio promises payment $P_{t,\tau}$ for some set of future dates $\tau > t$. The weighted average maturity of that portfolio is therefore

$$\bar{M}_t = \frac{1}{\sum_{\tau \in \mathcal{T}} P_{t,\tau}} \sum_{\tau \in \mathcal{T}} (\tau - t) P_{t,\tau}, \quad (15)$$

where \mathcal{T} is the set of future dates with nonzero payments. The utility function is assumed to be Epstein Zin. Aggregation over time follows

$$U(c, V') = \begin{cases} ((1 - \beta)c^{1-\psi} + \beta V'^{1-\psi})^{\frac{1}{1-\psi}} & \psi \neq 1 \\ c^{1-\beta} V'^\beta & \psi = 1 \end{cases}, \quad (16)$$

while aggregation across states follows

$$\mu(V'(s')|s) = \begin{cases} \left(\int_{\mathcal{S}} V'(s')^{1-\gamma} dF(s'|s) \right)^{\frac{1}{1-\gamma}} & \gamma \neq 1 \\ \exp \left(\int_{\mathcal{S}} \ln(V'(s')) dF(s'|s) \right) & \gamma = 1 \end{cases}. \quad (17)$$

The reentry parameter is set to the monthly equivalent of the quarterly 0.0385 value used by [Chatterjee and Eyigungor \(2012\)](#). The distribution of $\hat{\delta}$ is assumed to be uniform on the interval $[\underline{\delta}, \bar{\delta}]$. The values of $\bar{\delta}$, $\underline{\delta}$, and δ are chosen to match certain features of the secondary market prior to 2010. Jointly, the chosen values imply that 1) the implied annualized risk free interest rate faced by the government when $\pi_S = 0$ is 4%, 2) the average bid-ask spread in secondary markets when $\pi_S = 0$ is 2.5 b.p., and 3) the percent of outstanding debt traded every month is 37%. Table 2 contains the non-income process parameters set outside the model.

Table 2: Non-Income Parameters (Monthly Values)

Parameter	Value	Notes
θ	0.0130	Chatterjee and Eyigungor (2012)
$\underline{\delta}$	0.990	Fix implied $r_f = 0.33\%$ when $\pi_S = 0$
δ	0.999	Fix B-A Spread = 2.5 b.p. when $\pi_S = 0$
$\bar{\delta}$	1.001	Fix volumes=37% when $\pi_S = 0$
λ	0.0122	Weighted Average Maturity of Debt
κ	0.0041	Average Coupon of Debt

The default cost function is set, following [Chatterjee and Eyigungor \(2012\)](#), to be

$$\phi(y) = \max\{d_0y + d_1y^2, 0\}. \quad (18)$$

The signal process \hat{y}' is assumed to be $\ln(\hat{y}') = \ln(y') + \epsilon$ with $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$, and the cost function associated with trying to observe the signal is set to be $f(\pi) = f * \pi$. There are seven parameters that are internally calibrated. These are 1) the government's inverse elasticity of intertemporal substitution (inverse EIS) ψ , 2) the government's coefficient of relative risk aversion γ , 3) the government's discount factor β , 4) the linear default cost d_0 , 5) the quadratic default cost d_1 , the conditional variance of the signal σ_ϵ^2 , the cost of observing the signal f .

These parameters are set to minimize the distance between seven empirical moments and their model counterparts. Three of them are standard moments in the sovereign default literature (see [Chatterjee and Eyigunor \(2012\)](#) or [Bocola and Dovis \(2019\)](#), for example): the mean spread, the volatility of spreads, and the mean of external debt to GDP. As is standard, since there is no recovery after default in the model, I adjust by the estimated face value haircut rate that would have been applied in the event of a default. I use the face value haircut applied in Greece's 2012 restructuring, as detailed in [Zettelmeyer et al. \(2013\)](#), as the analogue for what would have happened had Spain defaulted.

To calculate the first and second moments of Spanish interest rate spreads, I use data on monthly interest rates on Spanish and German government bonds from the IMF's International Financial Statistics. The spread is calculated as the value of the Spanish interest rate minus the value of the German one. Since the model's debt is held by foreign investors, I produce an analogous quantity using several data series. Spain reports the nominal value of general government total debt held by external investors in the Quarterly Public Sector Debt Statistics. However, this statistic contains liabilities that are not debt securities, in addition to the debt securities that correspond to the debt in the model (and make up the vast majority of Spain's borrowing from external private investors).⁹ To strip these additional debts out, I turn to Spain's Financial Accounts. Here, nonmarketable debts (essentially everything except debt securities) are recorded at nominal value, rather than market value.¹⁰ The Financial Accounts include a breakdown of Spain's liabilities to the Rest of the World into debt securities and other categories. These allow me to calculate the nominal value of nonmarketable debt held by foreigners as $LIAB_{TOT,MV}^{FA,RoW} - LIAB_{DS,MV}^{FA,RoW}$, which I then subtract from the QPSD entry for total nominal value of debt held by foreign investors to obtain the nominal value of debt securities held by foreign investors.¹¹ Over the full sample period, from 2001Q1 to 2012Q2, ownership of Spanish debt securities splits almost exactly

⁹The relevant QPSD series IDs are DP.DOD.DECX.CR.GG, DP.DOD.DECN.CR.GG, DP.DOD.DLDS.CR.GG.

¹⁰This can be confirmed by comparing values of $LIAB_{TOT,MV}^{FA} - LIAB_{DS,MV}^{FA}$ and $LIAB_{TOT,NV}^{QPSD} - LIAB_{DS,NV}^{QPSD}$ produced using the two data sources.

¹¹This approach differs from that taken in certain other papers studying the Eurozone Debt Crises. Many of those, following [Bocola et al. \(2019\)](#), have simply assumed private financial markets are complete, which makes total public debt the relevant state variable.

50/50 between foreign and domestic investors. Dividing the resulting external debt series by GDP yields the measure of indebtedness used to calculate mean debt to GDP.

I augment these three standard moments with a pair characterizing cyclical patterns in debt stocks and flows, the correlation between log GDP and the debt to GDP ratio and the correlation between log GDP and the trade balance. These serve primarily to identify the inverse EIS ψ and the government relative risk aversion coefficient γ . The final two moments, which are chosen primarily to identify the accuracy of information and the cost of acquiring it, are the long run average bid-ask spread and the correlation of bid-ask spreads and interest rates. The monthly bid-ask spreads are calculated as described in the data section. The security-specific bid and ask price data were acquired from Bloomberg, and the weights used in aggregating across bond issues are based on security-specific data on quantity outstanding produced by the Bank of Spain.

The model is solved in Julia by value function iteration with discrete state space methods. I use a grid with 201 points for GDP, evenly distributed in log space across six ergodic standard deviations of $\ln(y_t)$, centered at its mean, 0, and a grid with 1201 points for debt, equally spaced between 0 and 4. The full set of internally calibrated parameters is detailed in Table 3.

Parameter	Value	Notes
ψ	11.73	Govt Inverse EIS
γ	4.83	Govt Relative Risk Aversion
β	0.992	Govt Discount Factor
d_0	-0.110	Linear Default Cost
d_1	0.142	Quadratic Default Cost
f	0.000125	Cost of Information (Linear)
σ_ϵ	0.037	SD of Noise in \hat{y}

In order to produce debt to GDP that is countercyclical as well as the countercyclical trade balance typical of developed economies, the inverse EIS and risk aversion coefficient must be set quite far from the standard value of 2 used in the literature. In other work focused on developed economies, such as [Bocola and Dovis \(2019\)](#) and [Bocola et al. \(2019\)](#), non-homotheticities in preferences (specifically, a subsistence level of consumption) have been

used to try to replicate these features. In this paper, I show that adjusting the inverse EIS (and possibly the CRRA coefficient) away from 2 can be used to obtain the same result, while preserving the homotheticity of utility. The full set of targeted moments is detailed in Table 4.

Table 4: Targeted Moments (Annualized Values)

Moment	Period	Data	Model
$\mathbb{E}[B'/Y]$	Jan 1 2001 - June 30 2012	11.9%	13.5%
$\rho(B'/Y, \ln(Y))$	Jan 1 2001 - June 30 2012	-0.76	-0.49
$\rho(NX/Y, \ln(Y))$	Jan 1 2001 - June 30 2012	-0.78	-0.10
$\mathbb{E}[r - r^f]$	Jan 1 2001 - June 30 2012	0.72%	0.83%
$\sigma(r - r^f)$	Jan 1 2001 - June 30 2012	1.13%	1.05%
$\mathbb{E}[BA]$	Jan 1 2001 - June 30 2012	5.5 b.p.	5.4 b.p.
$\rho(BA, r - r^f)$	Jan 1 2001 - June 30 2012	0.84	0.80

7 Results

In this section, I first validate some key predictions the model makes about the relationships between output, bid-ask spreads, interest rate spreads, and debt levels. In particular, a key mechanism of the model is the relationship between current bid-ask spreads and expected future output. I incorporate this relationship into an exercise that uses the particle filter to estimate the distribution of future values of the persistent component of GDP. The forecasts produced by this filter are substantially better than a standard, benchmark forecast (produced using the Kalman Filter) that does not use information about secondary market liquidity. After that, I show that the model’s predictions about how the average debt crisis evolves are qualitatively similar to what occurred in Spain. Finally, I use the model to measure the costs of illiquidity.

7.1 Validation

I now turn to validating the model’s main mechanism, the connection between current realizations of bid-ask spreads and future realizations of output. To do this, I employ the particle filter, following [Bocola \(2016\)](#) and [Bocola and Dovis \(2019\)](#). Specifically, I assume that the

evolution underlying states S_t and observable outcomes X_t can be described as

$$X_t = f(S_t) + \zeta_t \quad S_t = g(S_{t-1}, \nu_t).$$

Since output in the data is measured at a quarterly frequency, I allow for three subperiods in each period for the filter. The underlying state S_t is then defined as

$$S_t = \{y_{t,1}, y_{t,2}, y_{t,3}, y_{t+1,1}, m_{t,1}, m_{t,2}, m_{t,3}, B_{t,1}, B_{t,2}, B_{t,3}, B_{t+1,1}\},$$

and the observable outcomes X_t are

$$X_t = \left\{ \sum_{\ell=1}^3 y_{t,\ell} + m_{t,\ell}, BA_{t,1}, BA_{t,2}, BA_{t,3}, d_{t,1}, d_{t,2}, d_{t,3} \right\}.$$

In both cases, $z_{t,\ell}$ denotes observation ℓ in quarter t . The functions $f(\cdot)$ and $g(\cdot)$ are implicitly defined by the model's parametrization and solution. The vector ϵ_t collects structural shocks, and the vector η_t collects uncorrelated, normally distributed measurement errors in the observable outcomes. I set the measurement error for output $\sum_{\ell=1}^3 y_{t,\ell} + m_{t,\ell}$ to 2% of its sample variance and the measurement error for the bid-ask spread to 5% of its sample variance.¹² I use $N = 10,000,000$ total particles.

The process of applying this filter over a single quarter works as follows. In each quarter, I begin each quarter t with a prior over $\{y_{t,1}, B_{t,1}\}$ (these are elements of the posterior over S_{t-1} in the previous quarter). Using $g(\cdot)$, I then sample paths for the remaining exogenous elements of S_t , $\{y_{t,2}, y_{t,3}, y_{t+1,1}, m_{t,1}, m_{t,2}, m_{t,3}\}$. Using these shocks and the starting value of debt $B_{t,1}$, I use the model's solution to obtain $B_{t,2}, B_{t,3}, B_{t+1,1}$, as well as $\{BA_{t,1}, BA_{t,2}, BA_{t,3}, d_{t,1}, d_{t,2}, d_{t,3}\}$. I then use these outcomes to calculate the likelihood of each path. Here, I place a large enough weight on the simulated default decision matching the data to ensure that paths leading to default have ≈ 0 likelihood. Using these likelihood values for the current sample of S_t , I produce a posterior distribution over S_t . As mentioned

¹²The bid-ask spreads series is substantially more volatile than output, which I interpret as containing somewhat more measurement error (especially because the series is produced using end of day values for yields). That said, the filter tends to fit the bid-ask-spreads extremely well, regardless of whether I use matching scales for the measurement errors or assume relatively more noise in the bid-ask spreads.

above, the key pieces of this which carry over to the next period in a meaningful way are $y_{t+1,1}$ and $B_{t+1,1}$. This posterior distribution is used to resample paths at the beginning of the filtering process in the next quarter. I plot the means of the model-implied series for quarterly output and monthly bid-ask spreads in Figure 10 and the means of the model-implied sequences for debt to GDP and interest rate spreads in Figure 11.

Figure 10: Particle Filter Fit (In Sample)

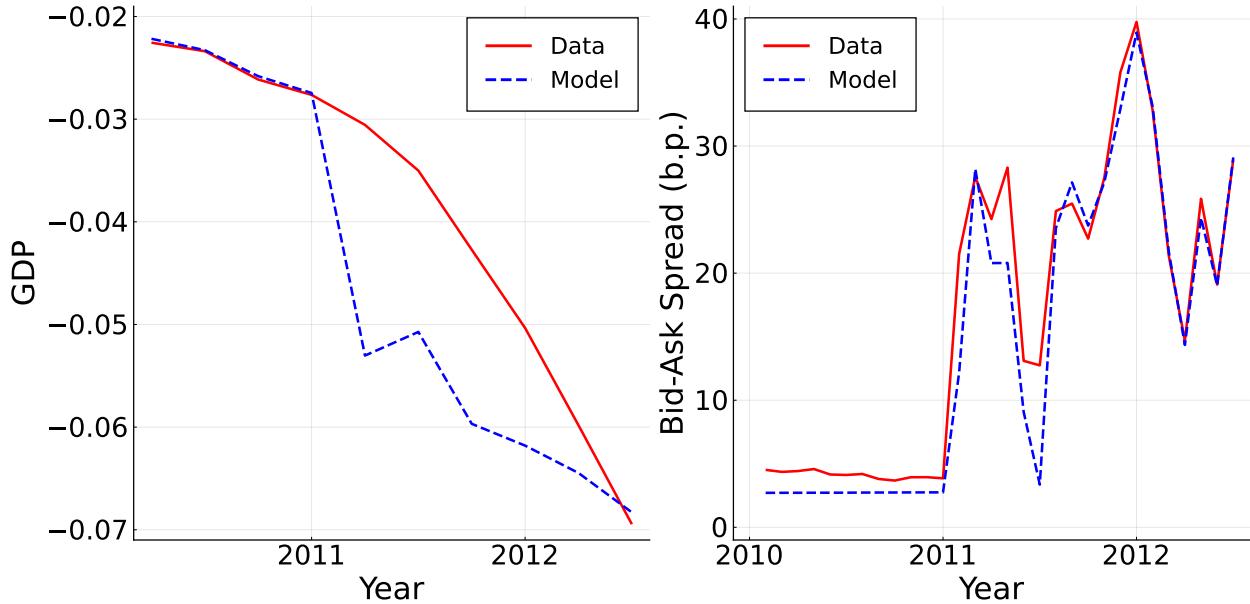
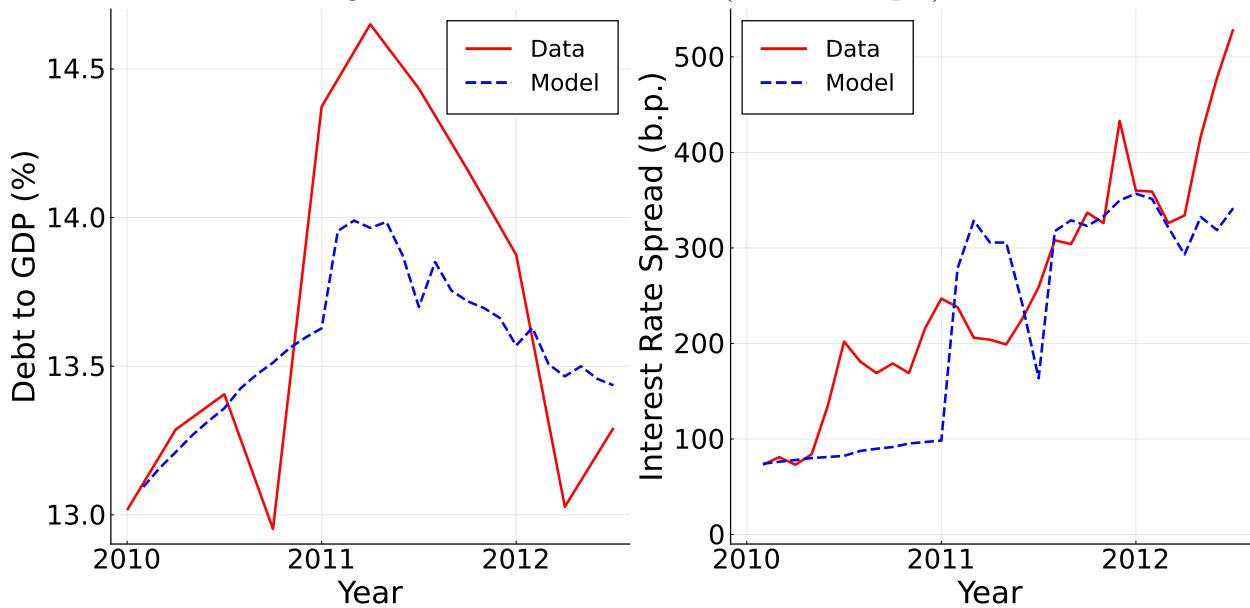
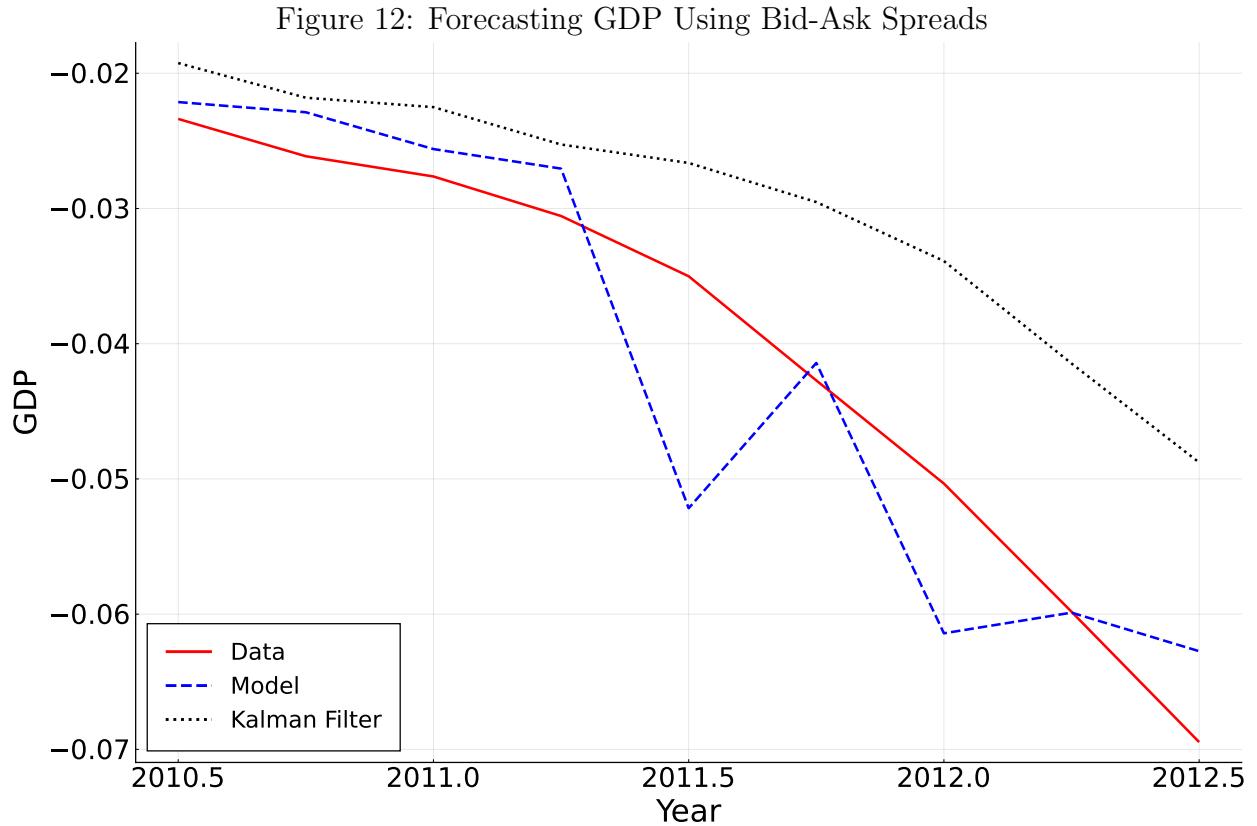


Figure 11: Particle Filter Fit (Out of Sample)



The model-filtered series for output indicates a somewhat steeper drop in output at the beginning of 2010 than is observed in the measured data, but otherwise the implied sequences of measurement errors are quite small. These paths are key features of the Eurozone Debt Crisis in Spain that were not directly targeted during the filtering process. Overall, they match well the general timing and dynamics of the crisis (although they due miss the scale of the rise in debt to GDP in 2011 and the 100 b.p. jumps in spreads in mid 2010 and mid 2012).

Finally, I turn to evaluating the predictions of the model for future output based on current bid-ask spreads. I compare these predictions to those made using the Kalman Filter (and incorporating all information up until the end of quarter t to predict total output in quarter $t + 1$). I begin this comparison at the end of 2009, a full year prior to the rise in bid-ask spreads and the run up in spreads that peaks in the middle of 2012. In Figure 12, I plot



the forecasts produced by the particle filter and the Kalman Filter as well as the actual, realized level of output. In this plot, we see that realized bid-ask spreads, filtered through

the lens of the model, provide substantial additional information on the state of the economy. To evaluate this improvement rigorously, I use the test of Diebold and Mariano (1995), as modified by Harvey et al. (1997), which is designed to evaluate whether two forecasts have equal accuracy. Under this test, the model's improvement in forecast accuracy over the up-to-date Kalman Filter is significant at the 5% level ($p = 0.038$).

7.2 Crisis Dynamics

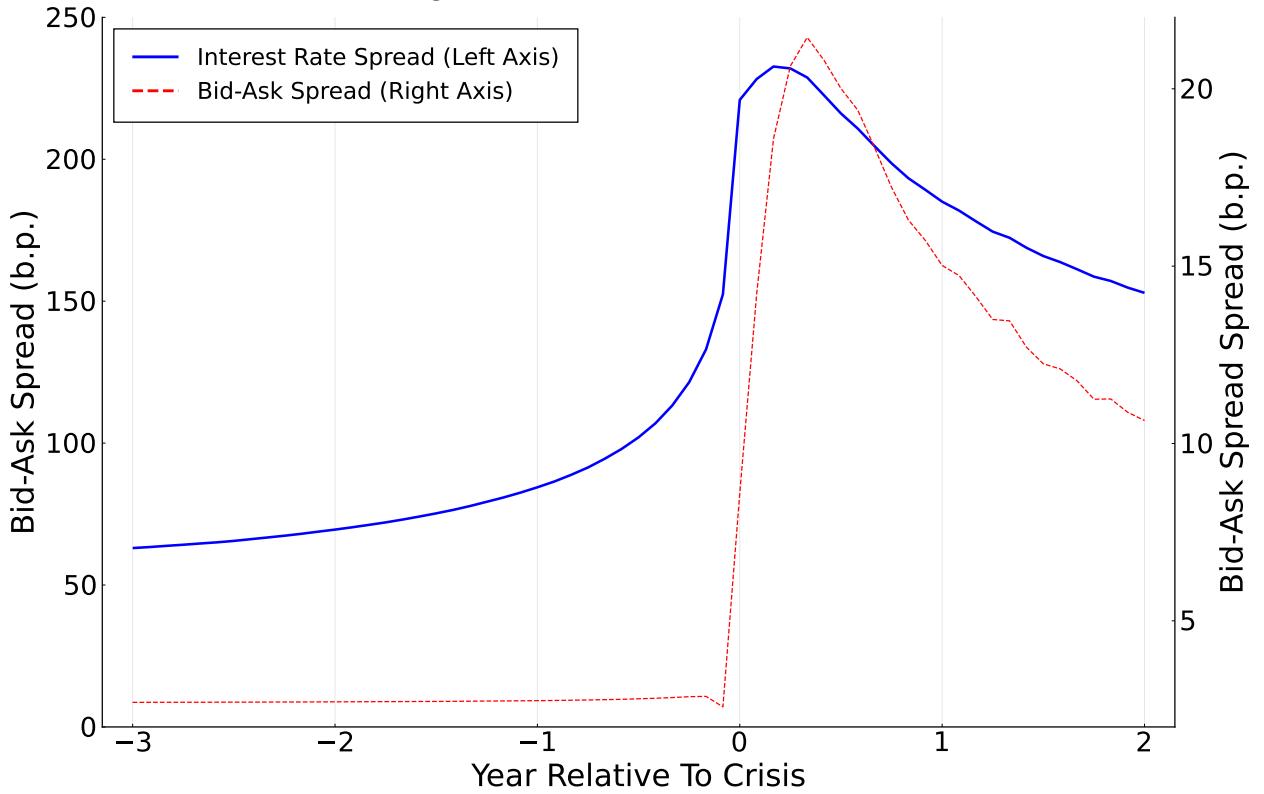
Having validated the model-implied relationship between current bid-ask spreads and future output, I turn to describing what we learn from the model about the dynamics of debt crises and secondary market turbulence. Following Bocola et al. (2019), I define a crisis in simulated data to be a period in which 1) no crisis or default has occurred recently (within the last three years), and 2) the interest rate spread rises more than one standard deviation above its mean. The spread level implied by this rule is about 2%, which is similar to spread on Spanish government debt first achieved in January, 2011. Figure 13 plots the average paths of interest rate spreads and bid-ask spreads over the three years prior to a crisis and the two years after (conditional on not defaulting).

In Figure 13, we see that interest rates rise quite sharply in the lead up to a crisis and its onset. Thereafter, that rise tapers off, and interest rates begin to fall steadily back towards their long term level. Until the actual onset of the crisis, bid-ask spreads do not react essentially at all, consistent with the patterns in the data. Recall that, in the data, bid-ask spreads were essentially unchanged from their pre-crisis levels until spreads rose above 2% at the beginning of 2011. In the simulation, once the crisis begins and interest rate spreads become significantly elevated, bid-ask spreads also rise away from zero and stay elevated throughout the next two years. Again, this is qualitatively exactly consistent with the patterns observed in the Spanish data.

7.3 Welfare Costs of Illiquidity

Next, I discuss the losses to investors associated with these elevated bid-ask spreads, measured as a change in liquidity. To do so, I develop several measures based the gains from

Figure 13: Prices Around a Crisis



trade. One direct cost of illiquidity is a reduction in the gains from trade that can be realized in a market. Based on this observation, I measure liquidity as how close the post-trading allocation of debt is to its efficient allocation. This requires some new notation, which I introduce below.

Call the pre-trading measure of asset owners be B and the pre-trading measure of potential buyers be A . Each individual agent can hold a single unit of the asset. Suppose that there is a CDF F of valuations for current owners of an asset and a CDF G of valuations for potential buyers. Based on these fundamentals, we can derive two key quantities are the maximum possible surplus from trade S^{max} and the minimum number of trades required to achieve that surplus N^{eff} .

If the maximum surplus from trade were to be achieved, then the allocation would be Pareto Efficient, and no further trades would be Pareto Improving. This implies that this allocation must be characterized by a valuation threshold v^* , such that all agents (both potential buyers and potential sellers) with valuation strictly higher than v^* end up holding a bond, while no

agent with valuation strictly less than v^* holds the bond. Since the outstanding amount of bonds is B and each agent holds one bond, v^* is the smallest number satisfying:

$$\lim_{v \uparrow v^*} B(1 - F(v)) + A(1 - G(v)) \leq B$$

Here we use a limit to acknowledge that there may be rationing among agents with valuation exactly v^* if the distribution for either group has a point mass at that value. This v^* is the minimum valuation for any asset holder if the maximum surplus from trade is to be achieved.

Then define N^{eff} as:

$$N^{eff} = \begin{cases} BF(v^*) & \lim_{x \uparrow v^*} F(x) = F(v^*) \\ A(1 - G(v^*)) & \lim_{x \uparrow v^*} F(x) < F(v^*) \end{cases}$$

When the CDF of valuations of initial owners is continuous at v^* , every asset held by initial owners with valuations less than v^* must be traded to buyers. When the CDF of initial owners is not continuous at v^* , then in addition to the assets of initial owners with valuations less than v^* , some of the assets of agents with valuation exactly v^* must be transferred, and the total number of transfers must result in all buyers with valuation strictly greater than v^* holding an asset. Next define S^{max} as the difference between the total value of the asset (to its holders) under the initial allocation and the total value under the efficient allocation:

$$\begin{aligned} S^{max} = & \lim_{x \downarrow v^*} \int_x^{+\infty} v(AdG(v) + BdF(v)) \\ & + v^* \max \left\{ BF(v^*) - A(1 - G(v^*)), 0 \right\} \\ & - \int_{-\infty}^{\infty} BvdF(v) \end{aligned}$$

The first term corresponds to holders of the with valuation strictly greater than v^* . The second term only matters when there is a point mass for either side at v^* , some of assets must be reallocated to or from that point mass in order to ensure that every agent with valuation strictly greater than v^* holds an asset. The final term refers to the initial distribution of

assets.

Suppose that $S^{max}(F, B, G, A) > 0$ and $N^{eff}(F, B, G, A) > 0$. Let F', G' denote the valuation distributions of current asset holders and current non-asset holders after a trading protocol X has been applied. Define raw liquidity under that protocol as

$$L_{raw} = \frac{N^{eff}(F, B, G, A) - N^{eff}(F', B, G', A)}{N^{eff}(F, B, G, A)},$$

and weighted liquidity as

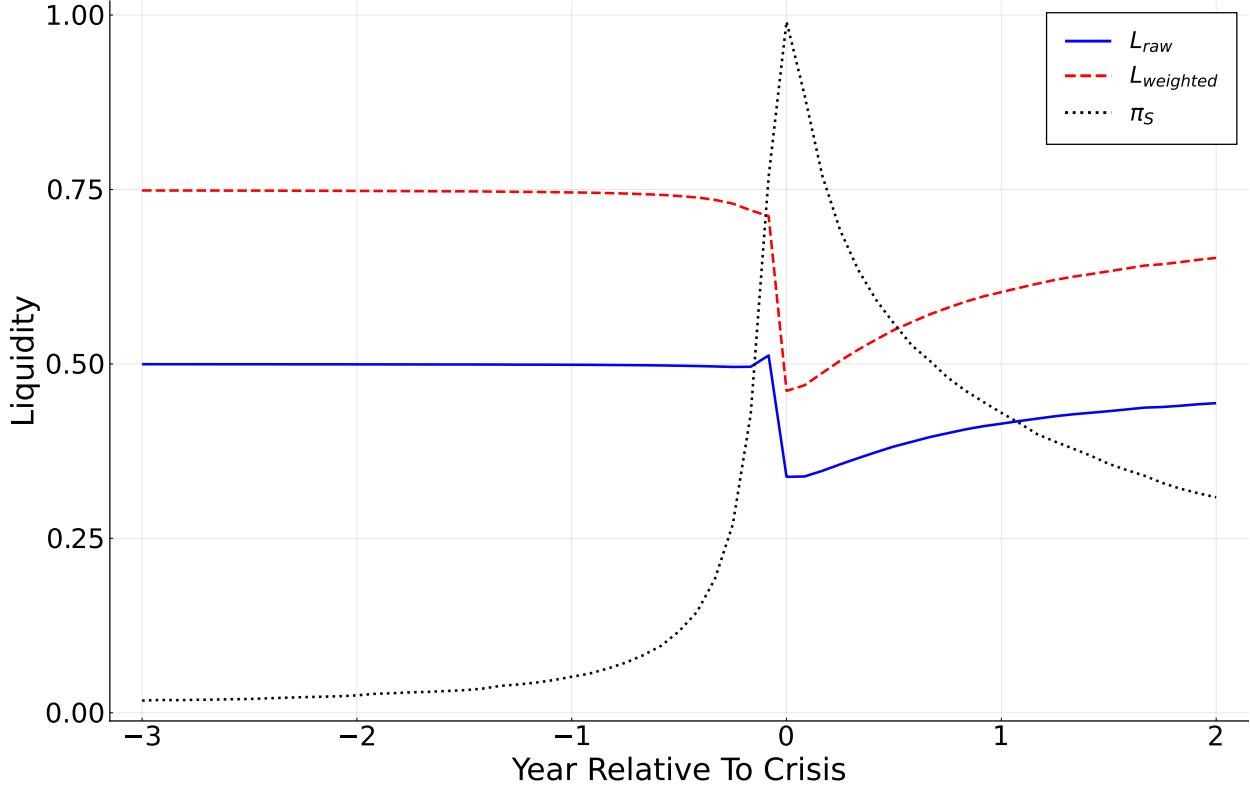
$$L_{weighted} = \frac{S^{max}(F, B, G, A) - S^{max}(F', B, G', A)}{S^{max}(F, B, G, A)}.$$

Both $1 - L_{raw}$ and $1 - L_{weighted}$ measure the distance between the post-trade distribution and the efficient distribution. $1 - L_{raw}$ measures this distance in terms of the number of trades required to move from the post-trade distribution to the efficient distribution. On the other hand, $1 - L_{weight}$ measures the potential surplus associated with making those remaining trades. Each is then normalized appropriately. High values of L_{raw} ($L_{weighted}$) imply that the trading protocol X , in circumstance F, B, G, A , results in most trades for which there is a surplus occurring (most of the possible surpluses from trade being attained).

The measures defined above have many uses. In other contexts, these two values could be used to measure the effectiveness of different trading protocols. In this paper, however, I use them to measure how easily the asset moves from those who would like to sell the asset to those who would like to buy it under various circumstances. Here, I interpret them along these lines, as measures of how liquidity varies with the state of the world. Figure 14 plots the evolution of these liquidity measures, as well as the equilibrium level of information acquisition, over the course of a crisis in the model.

For reference, when there is no information acquisition, $L_{raw} = 0.5$ and $L_{weighted} \approx 0.75$. There are several important features of this picture. First, consider the period before the onset of the crisis. Over this time period, bid-ask spreads are essentially constant and equal to their level when $\pi_S = 0$. That is not because there is no information acquisition before the crisis starts. Indeed, in Figure 14, we can see that there is indeed some information acquisi-

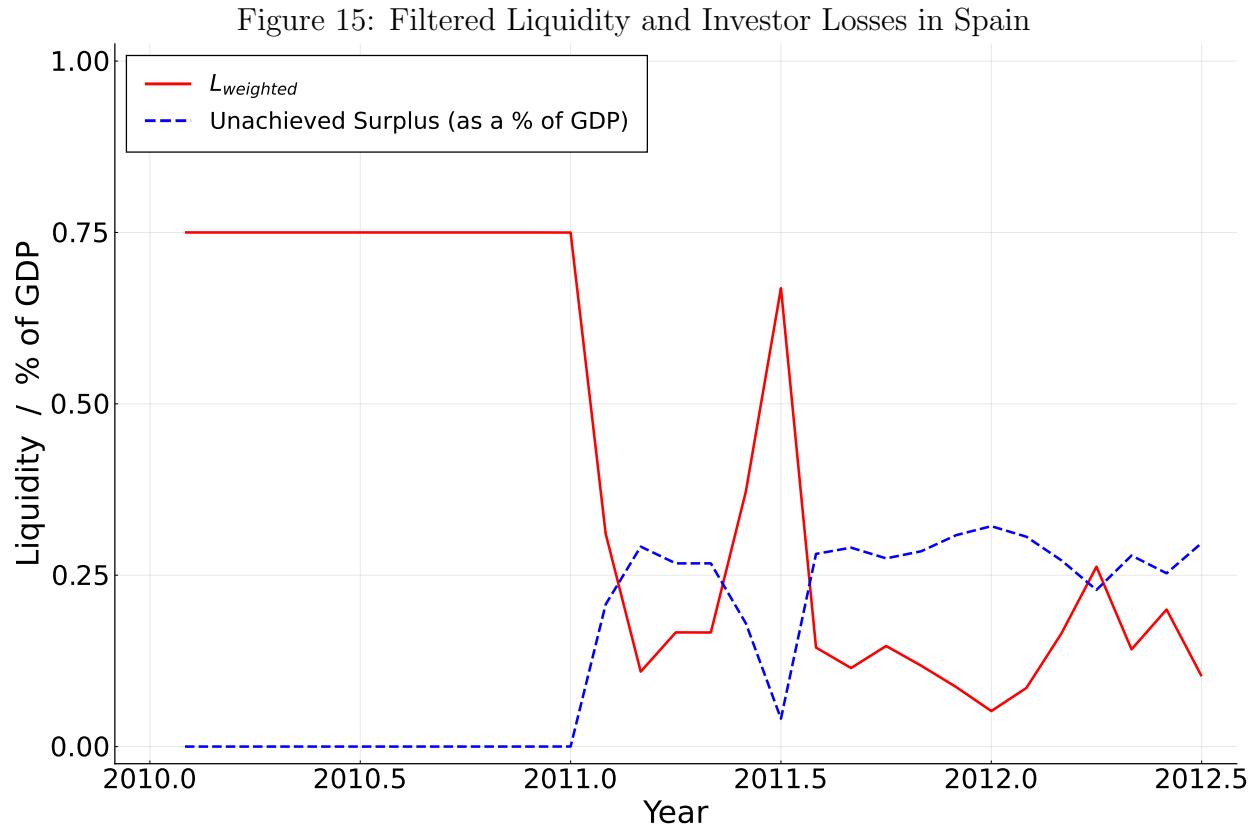
Figure 14: Liquidity Around a Crisis



tion over the year prior to this period. It is just that, along these paths, that information is not particularly useful and does not result in informed sellers submitting ask prices that are significantly different from those submitted by uninformed sellers. As buyers react to this information acquisition and submit relatively lower bid prices, the realized gains from trade fall, as we can see in the slight decline in $L_{weighted}$ over the sixth months leading up to a crisis. The slight rise in L_{raw} during the period prior to default is due to the signal \hat{s}' in that period delivering very bad news. Since many sellers are informed at this point, many more efficient trades actually occur, although those trades do not result in large enough increase in the surplus realized to induce a similar rise in $L_{weighted}$.

After the crisis begins, information acquisition peaks and there are significant drops in both measures of liquidity. Both the number of efficient trades made in equilibrium and the surplus produced by those trades fall about a third. Furthermore, those drops in liquidity are relatively persistent, and efficiency of trade suffers for years, even as information acquisition falls away from its peak.

Finally, I can use these liquidity measures to analyze the costs of secondary market turbulence in Spain during the peak of the crisis. In order to do so, I use the distributions over exogenous and endogenous states implied by the particle filter in the validation exercise. Under these distributions I calculate a filtered series for $L_{weighted}$ as well as a measure of the scale of losses to investors. In particular, I subtract the realized gains from trade from the gains from trade had π_S been equal to 0. The resulting difference measures the total loss to market participants that can be attributed to the trading frictions that result from endogenous information acquisition during times of turbulence. Figure 15 plots both series.



As we can see in figure 15, the measured scale of losses during the high stress period was substantial, averaging 0.26% of GDP from January 2011 to June 2012. Furthermore, this number does not include the costs paid by investors in order to acquire information or the welfare costs to the borrower (in higher interest rates) due to expected future secondary market frictions. These costs are large and are borne by foreign investors. In this environment, it is therefore possible that a bailout program run by the benevolent government of

country (or countries) where the foreign investors live could be Pareto improving. Avoiding defaults not only benefits the borrower, but also benefits the investors who live in the foreign country. To the extent that actual bailouts are rare enough and/or small enough that in the long run their costs are less than the restored trading surpluses, such a program could be optimal from the perspective of the foreign government. Other work (including [Bocola and Dovis \(2019\)](#)) has found that the ECB interventions after June 2012 may have raised bailout expectations. Based on my results, it is entirely possible that such a policy change is actually optimal from the perspective of the funding countries, rather than simply a transfer to the borrowers.

8 Conclusion

In this paper, I have documented novel facts about the relationships between bid-ask spreads, realized trading volumes, and interest rate spreads in Spain. I have shown that realized trading volumes are almost always low when bid-ask spreads are elevated, and that the relationship between bid-ask spreads and interest rate spreads differs by the level of interest rate spreads. Specifically, bid-ask spreads are not affected by variation in interest rate spreads when default risk is low. However, they react quite sharply once default risk rises to significant levels.

In order to rationalize these relationships, I built a model of sovereign default with frictional secondary markets where bid-ask spreads are produced by the presence of traders with private information. This private information pertains to a future realization of output (which is valuable to traders because the government’s default and borrowing decisions depend on the realization of output). After characterizing the nature of equilibria in secondary markets, I calibrate the model to the experience of Spain before and during the Eurozone Debt Crises. I validate the model’s mechanism by showing that incorporating information about bid-ask spreads into forecasts of future output by using the model-implied relationship between realized secondary market behavior and actual future output improves the accuracy of those forecasts. I then use the model to illustrate the dynamics of crises and measure how liquidity changes over the course of the average crisis. Finally, I measure the costs of secondary market

turbulence during the peak of the debt crisis in Spain, and find that these market dislocations cost foreign investors over 0.25% of Spanish GDP.

While the trading protocol used in this paper was sufficient to replicate the patterns predicted by the presence of asymmetric information, it does not directly incorporate a notion of financial intermediaries, which are known to have played a significant role in secondary markets for government debt in Europe. The role of their activities and interactions are exactly the focus of [Chaumont \(2024\)](#). The implications of combining the two mechanisms are unknown, and present a promising avenue for future research.

References

- Bank of Spain**, “Institutional Repository: Series históricas de deuda pública y del mercado de dinero (1988-2018),” 2018.
- Bassetto, Marco and David S. Miller**, “A Monetary-Fiscal Theory of Sudden Inflations,” *The Quarterly Journal of Economics*, 7 2025, 140 (3), 1959–2000.
- Benmelech, Efraim and Nittai Bergman**, “Debt, Information, and Illiquidity,” 9 2018.
- Bianchi, Francesco and Cosmin Ilut**, “Monetary/Fiscal policy mix and agents’ beliefs,” *Review of Economic Dynamics*, 10 2017, 26, 113–139.
- Bocola, Luigi**, “The pass-through of sovereign risk,” *Journal of Political Economy*, 8 2016, 124 (4), 879–881.
- and Alessandro Dovis, “Self-Fulfilling Debt Crises: A Quantitative Analysis,” *American Economic Review*, 2019, 109 (12), 4343–77.
- , Gideon Bornstein, and Alessandro Dovis, “Quantitative sovereign default models and the European debt crisis,” *Journal of International Economics*, 2019.
- Chatterjee, Satyajit and Burcu Eyigungor**, “Maturity, indebtedness, and default risk,” *American Economic Review*, 10 2012, 102 (6), 2674–2699.
- Chaumont, Gaston**, “Sovereign Debt, Default Risk, and the Liquidity of Government Bonds,” 2024.
- Cole, Harold, Daniel Neuhann, and Guillermo Ordoñez**, “Asymmetric Information and Sovereign Debt: Theory Meets Mexican Data,” *Journal of Political Economy*, 8 2022, 130 (8), 2055–2109.
- Cole, Harold L, Daniel Neuhann, and Guillermo Ordoñez**, “Information Spillovers and Sovereign Debt: Theory Meets the Eurozone Crisis,” *The Review of Economic Studies*, 1 2025, 92 (1), 197–237.

Dang, Tri Vi, Gary Gorton, and Bengt Holmström, “Ignorance, Debt and Financial Crises,” 2015.

—, —, and Bengt Holmstrom, “The Information View of Financial Crises,” *Annual Review of Financial Economics*, 11 2020, 12 (Volume 12, 2020), 39–65.

Diebold, Francis X and Roberto S Mariano, “Comparing Predictive Accuracy,” *Journal of Business & Economic Statistics*, 1995, 13 (3), 253–263.

Feldhutter, Peter and Thomas Kjaer Poulsen, “What Determines Bid-Ask Spreads in Over-the-Counter Markets?,” 11 2018.

Glosten, Lawrence R. and Paul R. Milgrom, “Bid, ask and transaction prices in a specialist market with heterogeneously informed traders,” *Journal of Financial Economics*, 3 1985, 14 (1), 71–100.

Gorton, Gary and Guillermo Ordoñez, “Collateral crises,” *American Economic Review*, 2 2014, 104 (2), 343–378.

— and —, “Good Booms, Bad Booms,” *Journal of the European Economic Association*, 4 2020, 18 (2), 618–665.

Harvey, David, Stephen Leybourne, and Paul Newbold, “Testing the equality of prediction mean squared errors,” *International Journal of Forecasting*, 6 1997, 13 (2), 281–291.

Hatchondo, Juan Carlos and Leonardo Martinez, “Long-duration bonds and sovereign defaults,” *Journal of International Economics*, 2009.

Huang, Roger D and Hans R Stoll, “The Components of the Bid-Ask Spread: A General Approach,” *The Review of Financial Studies*, 1997, 10 (4), 995–1034.

Kyle, Albert S, “Continuous Auctions and Insider Trading,” *Econometrica*, 1985, 53 (6), 1315–1335.

Passadore, Juan and Yu Xu, “Illiquidity in sovereign debt markets,” *Journal of International Economics*, 7 2022, 137, 103618.

Pelizzon, Loriana, Marti G. Subrahmanyam, Davide Tomio, and Jun Uno,
“Sovereign credit risk, liquidity, and European Central Bank intervention: Deus ex machina?,” *Journal of Financial Economics*, 10 2016, 122 (1), 86–115.

Zettelmeyer, Jeromin, Christoph Trebesch, and Mitu Gulati, “The Greek debt restructuring: An autopsy,” *Economic Policy*, 7 2013, 28 (75), 513–563.

Zhiguo, He and Konstantin Milbradt, “Endogenous Liquidity and Defaultable Bonds,” *Econometrica*, 7 2014, 82 (4), 1443–1508.