Modes of internal symmetry - taking a closer look at Messiaen's modes of limited transposition

Stefaan.Himpe@gmail.com

06/March/2014 - last updated: Friday 14th March, 2014 at 22:28

1 Introduction

Before we start, make sure to get the latest version of this document from https://github.com/shimpe/mints . If you are unsure what to do on that page, click "Download ZIP". It will download an archive containing all the files required to recreate this document. In the *out* folder is a document.pdf file which corresponds to the article itself. In the *output* folder are .pdf and .midi files of the modes listed in sections 8, 9, 10, 11, 12, 13 and 14. Feel free to send your questions and remarks with respect to this document to stefaan.himpe@gmail.com. Feel free to report errors, or corrections, additions, to https://github.com/shimpe/mints/issues

After reading Olivier Messiaen's book "The technique of my Musical Language" I started wondering how exactly his "modes of limited transposition" create the sound they create. Major questions for me became:

- What is so special about the property of *limited transposition* that makes the music derived from it work so well?

- Is music derived from a random subset of a chromatic scale on some sense "crippled" compared to music written based on Messiaen's modes? Can we find some kind of intuitive explanation why Messiaen's modes work well, arguably better than other non-diatonic modes? Can we use this insight to propose other modes that perhaps do not have limited transposition, but still might result in an interesting, fresh, sound?

I set out to do a series of experiments and came up with some insights that I intend to explain here. I apologize in advance if what I'm about to describe is already well-known and obvious to more informed readers. Please accept that I'm writing this down only to further my own understanding of the matter. I do not claim to have found something new.

2 Symmetry? What symmetry?

Messiaen himself explains how his modes can be thought of as consisting of *symmetrical* groups of notes. The exact nature of this symmetry was not entirely clear to me. In fact, at first sight it seemed more like repeating patterns than symmetry, and this is what I wanted to clarify.

As a starting point I set out to systematically enumerate all modes derived from a chromatic scale that have intervals symmetrically distributed around the f# (the middle note of the chromatic scale starting on c). Note that during the experiments I don't directly take into account any property of *limited transposition* but I will find back many of Messiaen's modes by considering only symmetry arguments anyway.

2.1 Systematic enumeration

The first thing to explain is how the modes with intervals symmetrically distributed around f# can be systematically enumerated.

I started from a chromatic scale. On that scale I added symbols under each note. Note how the symbols that occur left of f# return later on to the right of f#. This is important to keep symmetry of the intervals around f#, as will hopefully become clearer in the next step. For now, remember that the symbols under the complete chromatic scale form a *palindrome*, i.e. if you read them left-to-right you get exactly the same sequence of symbols as when you read them right-to-left.

In what follows we will now construct modes by assigning values 0 or 1 to each of the symbols p,q,r,s,t,u,v. A value of 0 indicates that the note to which the symbol is attached is not to be selected from the chromatic scale while constructing a mode. A value of 1 indicates that the notes to which the symbol is attached are to be selected from the chromatic scale.

E.g. if we set p=1, q=1, r=1, s=1, t=1, u=1, v=1 we retain all the notes from the chromatic scale and end up with the chromatic scale itself. We can say that the chromatic scale is characterized by a "key" (p,q,r,s,t,u,v) = (1,1,1,1,1,1,1). If we set p=1, q=0, r=1, s=0, t=0, u=0, v=1, we end up with a mode that contains the notes c, d, f#, a#, c'. This mode is characterized by a key (p,q,r,s,t,u,v) = (1,0,1,0,0,0,1).

This method for constructing modes has some properties:

- Any combination of values 0,1 assigned to all of p, q, r, s, t, u, v results in a mode derived from the chromatic scale with intervals distributed symmetrically around f#.

- A different combination of values 0,1 assigned to p, q, r, s, t, u, v results in a different mode. In other words, there are no two different keys

(p,q,r,s,t,u,v) that result in the same set of notes selected from the chromatic scale.

- If we look at all possible ways that we can assign 0, 1 to p, q, r, s, t, u, v, we construct all possible subsets of the chromatic scale with intervals distributed symmetrically around f#. In other words, we don't skip any modes symmetrical around f# by using this method.

- All in all, this means that any (p,q,r,s,t,u,v) key uniquely defines one mode with intervals symmetrically distributed around f#.

- There are $2^7 = 128$ different ways to assign the values 0, 1 to the variables p, q, r, s, t, u, v. This means that there are 128 unique modes with symmetrical distribution of intervals around f#. One of those modes is mode 0, which has no notes at all. We don't consider it further.

2.2 Intermezzo: binary numbers

In computer science a key like (0,1,1,0,0,1,0) can be interpreted as a binary number. Each number 1 or 0 is called a "bit". For every binary number, there's an equivalent decimal number and vice versa. In the systematic enumeration of modes in the appendices of this explanation, I use decimal equivalents of binary numbers, because they take much less typesetting space. If you want to make sense of the explanations that follow, it's useful to understand how binary numbers relate to decimal numbers, as explained now:

2.2.1 Conversion from binary number to decimal number

In order to convert from a binary number to a decimal number, one writes powers of two underneath the binary number and then multiplies and adds the results. As an example, consider conversion of number (0,1,1,0,0,1,0) to decimal:

0	1	1	0	0	1	0	← write binary digits	
64	32	16	8	4	2	1	← write powers of 2	
$*\downarrow$	$*\downarrow$	$*\downarrow$	$*\downarrow$	$*\downarrow$	$*\downarrow$	$*\downarrow$	$\downarrow$ multiply one by one	
0	32	16	0	0	2	0	→ sum to get:	$32 + 16 + 2 = 50$

Note that if I add extra 0's to the left of a binary number, its value doesn't change. The same is true for a decimal number: if you write 6 or you write 06 you really have the same number.

2.2.2 Conversion from a decimal number to a binary number

In order to convert a decimal number back to a binary number one keeps on dividing the number by 2, and notes down the rest after division. As an example, consider converting 50 back to binary representation:

- We start from 50

- divide 50 by 2 to get 25, with rest after division=0

- divide 25 by 2 to get 12, with rest after division=1

- divide 12 by 2 to get 06, with rest after division=0

- divide 06 by 2 to get 03, with rest after division=0

- divide 03 by 2 to get 01, with rest after division=1

- divide 01 by 2 to get 00, with rest after division=1

If you now look at the rests after division from bottom to top, you get (1,1,0,0,1,0). Remember from section 2.2.1 that one can add zeros to the left of any number without changing its value. Since we prefer to work with binary numbers (keys) of length 7 (i.e. the number of symbols p,q,...,v) we turn the binary number into key (0, 1, 1, 0, 0, 1, 0).

2.2.3 Tip about binary/decimal conversion

Practically all operating systems nowadays have some built-in calculator application that knows how to convert between binary and decimal should you ever need to do so.

3 What's the point? How do these binary numbers help us analyze music?

Using this enumeration method, I listed all the 127 non-empty modes that are symmetrical around f# as can be seen in section 8. I've sorted the modes by length. Mode 0 with zero notes was left out. Each mode is annotated with a few numbers, e.g. 74:(2)7-71. The number 74 means it's the 74th mode in the list of modes sorted by length. The number (2) means that we're looking at modes wity binary symmetry (the mode is divided in two symmetrical parts). The number 7 means that the mode consists of 7 notes, distributed symmetrically around f#. The number 71 is the decimal number that corresponds to the binary key (p,q,r,s,t,u,v) = (1,0,0,0,1,1,1) that uniquely defines the mode:

3.1 The symmetry of Messiaen's modes around f#

A first thing that struck me as interesting is that all of Messiaen's modes can be found back in the list of 127 modes enumerated in section 8 (albeit not always in first transposition, which is caused by our restriction to only look at modes symmetrical around f#, and not to look at modes symmetrical around – say – g.):

- Messiaen's mode 1 corresponds to our mode (2)85 (p,q,r,s,t,u,v) = (1,0,1,0,1,0,1)

- Messiaen's mode 2 corresponds to our mode (2)54 (p,q,r,s,t,u,v) = (0,1,1,0,1,1,0)

- Messiaen's mode 3 corresponds to our mode (2)93 (p,q,r,s,t,u,v) = (1,0,1,1,1,0,1)

- Messiaen's mode 4 corresponds to our mode (2)103 (p,q,r,s,t,u,v) = (1,1,0,0,1,1,1)

- Messiaen's mode 5 corresponds to our mode (2)99 (p,q,r,s,t,u,v) = (1,1,0,0,0,1,1)

- Messiaen's mode 6 corresponds to our mode (2)107 (p,q,r,s,t,u,v) = (1,1,0,1,0,1,1)

- Messiaen's mode 7 corresponds to our mode (2)119 (p,q,r,s,t,u,v) = (1,1,1,0,1,1,1)

3.2 More symmetry in Messiaen's modes

Now take a close look at the binary keys for Messiaen's modes. *In all but one of his modes, the binary keys are themselves palindromes*, indicating that not only are the modes symmetrical around f#, but they also have extra internal symmetry around d# (left-hand side of f#) and around a (right-hand side of f#). Having a palindromic key thus implies that there's a second level of symmetrical interval distribution inside the upper and lower halves of the modes.

It would not be correct to say that only modes with palindromic binary keys sound good. Look e.g. at mode of binary internal symmetry number (2)90 (1,0,1,1,0,1,0), which is better known as "c dorian". The binary key is not

palindromic but it does show other extra symmetries in its structure (observe how its binary key becomes a palindrome if you leave out the last 0). Similarly, Messiaen's fourth mode of limited transposition also becomes a palindrome if you leave out the last 1 in its binary key, indicating that it has significant internal symmetries beyond the always present symmetry around f#.

I'd like to speculate here that additional internal symmetries play an important role in making modes (and their "modes") sound good. The human brain is optimized for pattern matching. It is sensitive to symmetries and the listener probably subconsciously picks up the patterns and hears the symmetries present in the intervals that make up the mode. (If that were true, we might even nominate the "Dorian" mode as the most natural of modes based on a diatonic scale.) Modes with extra internal symmetries look like good candidates for harmony and melody experiments.

3.3 Are there any modes with similar internal symmetries that are not Messiaen modes?

3.3.1 Perfect internal symmetry

In section 9 all modes are listed that have palindromic binary keys. Note that almost all of Messiaen's modes appear here in one form or another, and that some modes appear that are not part of Messiaen's musical language. This is because we used *internal symmetry* instead of *limited transposition* as criterion. From these results, it's quite clear that there's a close connection between the two criteria. It's also interesting that a number of (shorter) modes appear which, to the best of my knowledge, were not used directly by Messiaen, but which may be interesting for further harmonic and melodic experiments.

In what follows, remember that the notation "mode (2)x-y" means "a mode with binary symmetry, x notes and key y, where y should be converted to binary to see which notes are present in the mode. Compare the descriptions given here to the modes as listed in section 9.

- Mode (2)02-008 (0,0,0,1,0,0,0) is a ditonic mode. Two notes may be a bit limited for a composition. Interesting though that it is a tritone, which is one of the basic building blocks for Messiaen's musical language.

- Mode (2)04-020 (0,0,1,0,1,0,0) is a tetratonic mode.

- Mode (2)06-028 (0,0,1,1,1,0,0) is a hexatonic mode.

- Mode (2)04-034 (0,1,0,0,0,1,0) is a tetratonic mode. Interesting about this mode is that there's even more symmetry present in the lower and upper half of the binary key. This is a third level of symmetry in the mode.

- Mode (2)06-042 (0,1,0,1,0,1,0) is a hexatonic mode. Interesting about this mode is that there's even more symmetry present in the lower and upper half of the binary key. This is a third level of symmetry in the mode.

- Mode (2)08-054 (0,1,1,0,1,1,0) is an octotonic mode. It's also known as Messiaen's second mode of limited transposition.

- Mode (2)10-062 (0,1,1,1,1,1,0) is a decatonic mode. This mode is not listed by Messiaen. However, if we extend this mode with a c# at the right, we get a "mode" of Messiaen's seventh mode of limited transposition built on note d.

- Mode (2)03-065 (1,0,0,0,0,0,1) is a tritonic mode. It consists of 2 tritones.

- Mode (2)05-073 (1,0,0,1,0,0,1) is a pentatonic mode.

- Mode (2)07-085 (1,0,1,0,1,0,1) is a heptatonic mode. It's also known as Messiaen's first mode of limited transposition, or as the whole-tone scale. Interesting about this mode is that there's even more symmetry present in the lower and upper half of the binary key. This is a third level of symmetry in the mode.

- Mode (2)09-093 (1,0,1,1,1,0,1) is a nonatonic mode. This is Messiaen's third mode of limited transposition. Interesting about this mode is that there's even more symmetry present in the lower and upper half of the binary key. This is a third level of symmetry in the mode.

- Mode (2)07-099 (1,1,0,0,0,1,1) is a heptatonic mode. This is Messiaen's fifth mode of limited transposition.

- Mode (2)09-107 (1,1,0,1,0,1,1) is a nonatonic mode. This is a "mode" of Messiaen's sixth mode of limited transposition built on note c#.

- Mode (2)11-119 (1,1,1,0,1,1,1) is an undecatonic mode. This is a "mode" of Messiaen's seventh mode of limited transposition built on note b. Interesting about this mode is that there's even more symmetry present in the lower and upper half of the binary key. This is a third level of symmetry in the mode.

- Mode (2)13-127 (1,1,1,1,1,1,1) is the chromatic scale itself. This mode also has a third and even fourth level of symmetry.

3.3.2 Partial internal symmetry

Now follows a list of modes that are partially palindromic as follows: the binary keys of the modes listed here become palindromic if you leave out either the first or last bit. They are listed in musical form in section 10. These form another subset of modes (slightly less "perfect" than the modes in the previous section). Probably some of these modes are better known under other names, but my knowledge of existing scales is not large enough to recognize all of them.

By reducing the constraints on symmetry also other interesting modes can be selected (e.g. modes that are a palindromic if you leave out 2 outer bits), but listing those is left as an exercise to the interested reader.

- (2)001 (0,0,0,0,0,0,1) is a mode consisting of a single note f#. This is a bit limited to compose with :)

- (2)012 (0,0,0,1,1,0,0) tetratonic

- (2)018 (0,0,1,0,0,1,0) tetratonic

- (2)024 (0,0,1,1,0,0,0) tetratonic

- (2)025 (0,0,1,1,0,0,1) tetratonic. Sounds quite exotic (arabic?).

- (2)030 (0,0,1,1,1,1,0) octatonic

- (2)033 (0,1,0,0,0,0,1) tritonic

- (2)036 (0,1,0,0,1,0,0) tetratonic

- (2)037 (0,1,0,0,1,0,1) pentatonic

- (2)045 (0,1,0,1,1,0,1) heptatonic. This is c# aeolian mode (natural minor diatonic scale).

- (2)051 (0,1,1,0,0,1,1) heptatonic. Sounds quite exotic (arabic?)

- (2)060 (0,1,1,1,1,0,0) octatonic

- (2)061 (0,1,1,1,1,0,1) nonatonic

- (2)063 (0,1,1,1,1,1,1) undecatonic. Like a chromatic scale but without note c.

- (2)064 (1,0,0,0,0,0,0) ditonic. Consists of only notes c.

- (2)066 (1,0,0,0,0,1,0) tetratonic. Consists of the notes of a sus4 chord built on c.

- (2)067 (1,0,0,0,0,1,1) pentatonic.

- (2)076 (1,0,0,1,1,0,0) hexatonic.

- (2)082 (1,0,1,0,0,1,0) hexatonic.

- (2)090 (1,0,1,1,0,1,0) octatonic. This is really just the dorian mode of c.

- (2)091 (1,0,1,1,0,1,1) nonatonic. Like the dorian mode of c but with f# added.

- (2)094 (1,0,1,1,1,1,0) decatonic.

- (2)097 (1,1,0,0,0,0,1) pentatonic.

- (2)102 (1,1,0,0,1,1,0) octatonic. Sounds quite exotic (arabic?).

- (2)103 (1,1,0,0,1,1,1) is the fourth mode of limited transposition of Messiaen.

- (2)109 (1,1,0,1,1,0,1) nonatonic. Left halve sounds darker than right halve.

- (2)115 (1,1,1,0,0,1,1) nonatonic.

- (2)126 (1,1,1,1,1,1,0) dodecatonic. Like a chromatic scale, but with f# left out.

- (2)127 (1,1,1,1,1,1,1) is the chromatic scale. This is the only key that also appears in the fully palindromic modes.

4 Ternary symmetries

So far we've only considered symmetries that divide the chromatic scale in a symmetrical left half and a right half. But a chromatic scale consists of 12 half tones, and therefore it can also be divided in groups of 3. The way of working remains more or less the same. First we propose an enumeration scheme. Note that to find ternary symmetries, we leave out the repeated tonic at the end of the mode. We then need a 4-bit binary key. Note how the mode now is divided in three parts $p, q, r, s \rightarrow s, r, q, p \rightarrow p, q, r, s$. This unfolded key (p,q,r,s,s,r,q,p,p,q,r,s) is not a palindrome anymore. This is a fundamentally different form of symmetry, namely between d#,e and between g,g#.

p=1 q=1 r=1 s=1 s=1 r=1 q=1 p=1 p=1 q=1 r=1 s=1

p=1 q=0 r=0 s=1 s=1 r=0 q=0 p=1 p=1 q=0 r=0 s=1

We can now reuse the knowledge we gathered before: given that we use 4-bit binary keys, there must be $2^4 = 16$ such modes (of course mode 0 has no notes, so we don't consider it further). As before we can translate the binary keys to decimal numbers, but the decimal denote a different mode than the same decimal numbers we used while examining binary symmetries. The difference lies in the structure of the unfolded key. In the binary symmetry case we had a key (p,q,r,s,t,u,v) that after unfolding becomes (p,q,r,s,t,u,v,u,t,s,r,q,p). Now in the ternary case we have a key (p,q,r,s) that after unfolding becomes (p,q,r,s,s,r,q,p,p,q,r,s). To distinguish the decimal numbers for modes with binary symmetry from the decimal numbers for modes with ternary symmetry, we preceded the former ones with (2) and we precede the latter ones with (3).

Sections 15, 16, 17, 18, 19 list all 16 modes with ternary symmetry.

5 Deriving harmonies from modes of internal symmetry

Another question I was struggling with was why Messiaen chose to build chords based on the interval of a fourth. During my investigation I think I saw a possible explanation as follows:

- Sections 11, 12, 13, 14 systematically list the chords built on the modes of binary internal symmetry by taking notes from consecutive mode (scale) degrees, every third mode degree, every fourth mode degree, every fifth mode degree respectively. Sections 16, 17, 18 and 19 do the same for modes of ternary internal symmetry.

- As is clearly visible, the fewer notes are in a mode that is symmetrical around f#, the more widely spaced those notes are (on average).

- In wider spaced modes (many 0's in the binary key), it makes sense to build chords from mode degrees that are close enough to each other.

- In more dense modes (many 1's in the binary key), it makes sense to build chords from mode degrees that are spaced further apart. If we use notes too close together, every chord sounds very dissonant and there's not much room for creating harmonic contrasts.

- Since Messiaen's modes are relatively dense compared to - say - a diatonic scale (Messiaen's modes all have 8 or more notes, whereas a diatonic scale has 7 notes), my guess is that the harmonies as built using every third note sound a bit too harsh, and Messiaen therefore decided to to use fourths.

It would seem that when deriving harmonies from a set of notes we want to avoid uneven spreading of intervals over the mode, to avoid ending up with a series of very closely spaced chords, followed by a series of much more widely spaced chords. In other words: a mode that consists of a left part with many half tones, and a right part with many whole tones will tend to result in much more dissonant chords derived from that left part, and much more open sounding chords derived from the right part. Can the binary keys can help us see in which modes the intervals are better spread out than in other modes?

5.1   From binary key to intervals in symmetric modes

First we can convert the binary keys back to intervals between successive notes in the mode. Doing so is simple once we think back of what the binary keys really mean: each 1 or 0 is an inclusion respectively exclusion of a particular note from the chromatic scale built on c. We can find back the intervals in the mode as follows:

- Write down the binary key (p,q,r,s,t,u,v) and unfold it to from the complete mode: (p,q,r,s,t,u,v,u,t,s,r,q,p). Or in the case of a ternary symmetrical mode, take the key (p,q,r,s) and unfold it to (p,q,r,s,s,r,q,p,p,q,r,s).

- Now each time count the number of 0's between successive 1's. Each count is one less than the number of half steps in the interval.

An abstract description like the above begs for an example. Let's take mode 83:
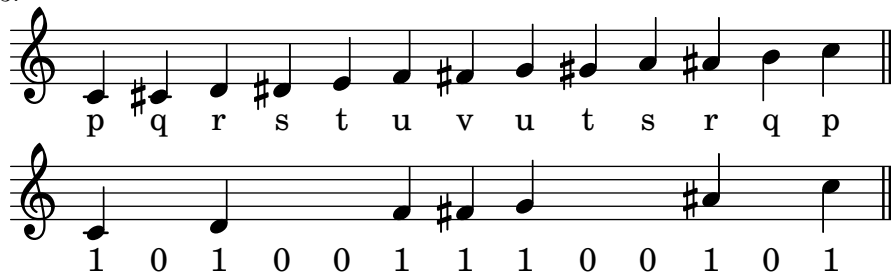
Number (2)83 in binary is (1,0,1,0,0,1,1). After unfolding we get (1,0,1,0,0,1,1,1,0,0,1,0,1) (see music example). Now we count zeros between successive 1's. We find 1 zero, 2 zeros, 0 zeros, 0 zeros, 2 zeros, 1 zero:

If we count zero 0's, then the interval between successive notes is 1/2 tone. If we count one 0, the interval is 1 tone. In general, if z is the number of zeros

count 0's between
successive 1's

1 0 1 0 0 1 1 1 0 0 1 0 1

1 zero

2 zeros

0 zeros

0 zeros

2 zeros   1 zero
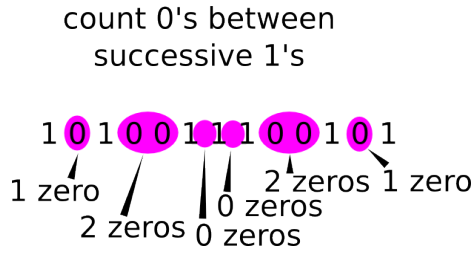
Figure 1: Turning binary keys back into intervals

we counted, the number of half tones in the interval is $h = (z + 1)$. And then the number of tones in the interval is $t = h/2 = (z + 1)/2$. In the case of mode (2)83, we apply this formula as follows:

- we counted 0's between successive 1's: (1,2,0,0,2,1).

- we apply the formula: ((1+1)/2, (2+1)/2, (0+1)/2, (0+1)/2, (2+1)/2, (1+1)/2) = (1, 1.5, 0.5, 0.5, 1.5, 1).

- so we have a mode with intervals 1 tone, 1.5 tones, 0.5 tones, 0.5 tones, 1.5 tones, 1 tone

- we double check with the musical example and see indeed that $c \to d = 1$ tone, $d \to f = 1.5$ tones, $f \to f\# = 0.5$ tones, $f\# \to g = 0.5$ tones, $g \to a\# = 1.5$ tones, $a\# \to c = 1$ tone

5.2 Ideal interval size. Avoiding clusters and gaps

Intuitively, when one wants to derive harmonies by stacking notes from every n-th scale degree, one could decide to avoid too small intervals (clusters) and too large intervals (gaps), e.g. to avoid a concentration of too dissonant chords on one side of the scale. For this reason it's useful to think about what intervals one reasonably can expect to occur in a mode.

Suppose you create a mode of only 2 different notes. These 2 notes have to span an octave, i.e. 12 half tones. To have maximal spreading of the notes over the mode one should have intervals between the notes of 6 half tones ($c \to f\#$). The interval ($f\# \to c$) closes the construction by repeating the first note c, so this second "c" it is not a "third note" in our mode. More general, for n notes spanning an octave (12 half tones), the ideal interval has $12/n$ half tones.

Suppose this assumption makes sense, then in an octotonic scale (8 distinct notes) the ideal interval between notes is $12/8 = 1.5$ half tones. There's one problem with this: 1.5 half tones leads to microtonal music. If we don't want to go there, we need to do the next best thing: round to multiples of 1. We can't round all 1.5 half tone intervals to 2 half tone intervals because then we end up with way too many half tones to fit in an octave. Similarly we can't round all 1.5 intervals down to 1 because then we end up with too few half tones to span the octave. We can however approximate this 1.5 by alternating between rounding 1.5 up to 2 and rounding 1.5 down to 1 to get the following configuration of half tone intervals: (1,2,1,2,1,2,1,2) which nicely adds up to

$1 + 2 + 1 + 2 + 1 + 2 + 1 + 2 = 12$ half tones over 8 notes. But now look closely at what we really constructed...

We find Messiaen's second mode of limited transpostion! Even though I didn't present a rigorous mathematical proof here, I hope it is clear that Messiaen's second mode of limited transposition is a best possible approximation of evenly spread intervals in an octotonic scale (best possible if we don't allow microtonal intervals).[1]

When we select modes for our own compositions, we may want to pay some attention to avoiding clusters and gaps in the intervals that lie between the notes in the modes we select. An interval G can be named a "gap" if it has more half tones than the ideal interval rounded up, and an interval C can be named a "cluster" if it has less half tones than the ideal interval rounded down. In all this, the ideal interval is itself a function of the number of notes in the mode. Summarizing:

- The ideal interval I(n) in a mode is $I(n) = 12/n$ where n is the number of distinct notes in the mode.

- An interval G is a gap if $B > \lceil I(n) \rceil$. (The symbols $\lceil$ and $\rceil$ should be read as "round up")

- An interval C is a cluster if $S < \lfloor I(n) \rfloor$. (The symbols $\lfloor$ and $\rfloor$ should be read as "round down")

6 Conclusions

To summarize:

- First we systematically listed all modes with internal symmetry around f# derived from a chromatic scale. We found that there are 127 non-empty such modes.

- Then, we saw how all of Messiaen's modes are part of this list of 127 modes, meaning that they at least have some symmetry in interval distribution.

- After that, we noticed how all but one of the Messiaen modes contain extra symmetries in the lower half and upper half of the mode (palindromic keys). We listed all modes that have both symmetry around f# and the extra symmetries in the lower half and upper half of the mode, and discovered some potentially interesting modes not directly used by Messiaen (with fewer notes). Even though we didn't care about the property of *limited transposition*, by just considering symmetry arguments we arrived at a very similar set of modes.

- We continued by looking for other forms of symmetry and looked in detail at ternary symmetries yielding different set of modes.

- While doing so, we also formulated a possible explanation for why Messiaen may have chosen to build his chords using fourths, and we speculated about how internal symmetry and even spreading of intervals may contribute to make Messiaen's modes work better than other, randomly chosen, modes.

---

[1]This makes me wonder how octotonic scales sound if we do allow microtonal intervals.

- In the reference sections 20 - 161 we list all the modes examined in this document and create a detailed passport, which can be used while exploring its musical possibilities.

7 Some resources

While investigating I made extensive use of Jackson Hardaker's Messiaen mode visualizer: http://messiaen.jacksonhardaker.com/

The complete code required to reproduce the experiments and the text in this document (together with midi files for the listed modes and chords in the following sections) can be found online at https://github.com/shimpe/mints. To recreate this document you need at the very least the following free software:

- python 2.x

- lilypond

- LaTeX

- a .pdf viewer (I used okular, but any viewer should do).

The provided build script is written in bash. For windows or other systems, you may need to translate to an appropriate format.

Feel free to send your questions and remarks with respect to this document to stefaan.himpe@gmail.com. Feel free to report errors, or corrections, additions, to https://github.com/shimpe/mints/issues

8   All modes with binary internal symmetry around f#, ordered by length

9 All modes of binary internal symmetry with palindromic keys

10 All modes of binary internal symmetry with partially palindromic keys

11 All chords built by stacking every second note from the modes of internal binary symmetry

18

57:(2)6-82   58:(2)6-84   59:(2)6-88

60:(2)6-98   61:(2)6-100   62:(2)6-104

63:(2)6-112   64:(2)7-15   65:(2)7-23

66:(2)7-27   67:(2)7-29

68:(2)7-39   69:(2)7-43

70:(2)7-45   71:(2)7-51

72:(2)7-53   73:(2)7-57

74:(2)7-71   75:(2)7-75   76:(2)7-77

77:(2)7-83   78:(2)7-85   79:(2)7-89

80:(2)7-99   81:(2)7-101   82:(2)7-105

83:(2)7-113   84:(2)8-30

85:(2)8-46   86:(2)8-54

87:(2)8-58   88:(2)8-60

89:(2)8-78   90:(2)8-86

91:(2)8-90

92:(2)8-92

93:(2)8-102

94:(2)8-106

95:(2)8-108

96:(2)8-114

97:(2)8-116

98:(2)8-120

99:(2)9-31

100:(2)9-47

101:(2)9-55

102:(2)9-59

103:(2)9-61

104:(2)9-79

105:(2)9-87

106:(2)9-91

107:(2)9-93

108:(2)9-103

109:(2)9-107

110:(2)9-109

111:(2)9-115

112:(2)9-117

113:(2)9-121

114:(2)10-62

115:(2)10-94

116:(2)10-110

117:(2)10-118

118:(2)10-122

119:(2)10-124

120:(2)11-63

121:(2)11-95

122:(2)11-111

123:(2)11-119

124:(2)11-123

125:(2)11-125

126:(2)12-126

127:(2)13-127

12 All chords built by stacking every third note from the modes of internal binary symmetry

66:(2)7-27

67:(2)7-29

68:(2)7-39

69:(2)7-43

70:(2)7-45

71:(2)7-51

72:(2)7-53

73:(2)7-57

74:(2)7-71

75:(2)7-75

76:(2)7-77

77:(2)7-83

78:(2)7-85

79:(2)7-89

80:(2)7-99

81:(2)7-101

82:(2)7-105

83:(2)7-113

84:(2)8-30

85:(2)8-46

86:(2)8-54

87:(2)8-58

88:(2)8-60

89:(2)8-78

90:(2)8-86

91:(2)8-90

92:(2)8-92

93:(2)8-102

94:(2)8-106

95:(2)8-108

96:(2)8-114

97:(2)8-116

98:(2)8-120

99:(2)9-31

100:(2)9-47

101:(2)9-55

102:(2)9-59

103:(2)9-61

104:(2)9-79

105:(2)9-87
106:(2)9-91
107:(2)9-93
108:(2)9-103
109:(2)9-107
110:(2)9-109
111:(2)9-115
112:(2)9-117
113:(2)9-121
114:(2)10-62
115:(2)10-94
116:(2)10-110
117:(2)10-118
118:(2)10-122
119:(2)10-124
120:(2)11-63
121:(2)11-95
122:(2)11-111
123:(2)11-119
124:(2)11-123
125:(2)11-125
126:(2)12-126
127:(2)13-127

13 All chords built by stacking every fourth note from the modes of internal binary symmetry

64:(2)7-15 65:(2)7-23 66:(2)7-27

67:(2)7-29 68:(2)7-39 69:(2)7-43

70:(2)7-45 71:(2)7-51 72:(2)7-53

73:(2)7-57 74:(2)7-71 75:(2)7-75 76:(2)7-77

77:(2)7-83 78:(2)7-85 79:(2)7-89 80:(2)7-99

81:(2)7-101 82:(2)7-105 83:(2)7-113 84:(2)8-30

85:(2)8-46 86:(2)8-54 87:(2)8-58

88:(2)8-60 89:(2)8-78 90:(2)8-86

91:(2)8-90 92:(2)8-92 93:(2)8-102

94:(2)8-106 95:(2)8-108 96:(2)8-114

97:(2)8-116 98:(2)8-120 99:(2)9-31

100:(2)9-47 101:(2)9-55

102:(2)9-59 103:(2)9-61

104:(2)9-79 105:(2)9-87 106:(2)9-91

107:(2)9-93 108:(2)9-103 109:(2)9-107

110:(2)9-109 111:(2)9-115 112:(2)9-117

113:(2)9-121 114:(2)10-62

115:(2)10-94 116:(2)10-110 117:(2)10-118

118:(2)10-122 119:(2)10-124

120:(2)11-63 121:(2)11-95

122:(2)11-111 123:(2)11-119

124:(2)11-123 125:(2)11-125

126:(2)12-126 127:(2)13-127

14 All chords built by stacking every fifth note from the modes of internal binary symmetry

55:(2)6-74   56:(2)6-76   57:(2)6-82   58:(2)6-84

59:(2)6-88   60:(2)6-98   61:(2)6-100   62:(2)6-104   63:(2)6-112

64:(2)7-15   65:(2)7-23   66:(2)7-27   67:(2)7-29

68:(2)7-39   69:(2)7-43   70:(2)7-45   71:(2)7-51

72:(2)7-53   73:(2)7-57   74:(2)7-71   75:(2)7-75

76:(2)7-77   77:(2)7-83   78:(2)7-85   79:(2)7-89

80:(2)7-99   81:(2)7-101   82:(2)7-105   83:(2)7-113

84:(2)8-30   85:(2)8-46   86:(2)8-54

87:(2)8-58   88:(2)8-60   89:(2)8-78

90:(2)8-86   91:(2)8-90   92:(2)8-92

93:(2)8-102   94:(2)8-106   95:(2)8-108

96:(2)8-114   97:(2)8-116   98:(2)8-120

99:(2)9-31   100:(2)9-47   101:(2)9-55

102:(2)9-59   103:(2)9-61   104:(2)9-79

105:(2)9-87   106:(2)9-91   107:(2)9-93

108:(2)9-103   109:(2)9-107   110:(2)9-109

111:(2)9-115   112:(2)9-117   113:(2)9-121

114:(2)10-62   115:(2)10-94   116:(2)10-110

117:(2)10-118   118:(2)10-122   119:(2)10-124

120:(2)11-63   121:(2)11-95

122:(2)11-111   123:(2)11-119

124:(2)11-123   125:(2)11-125

126:(2)12-126   127:(2)13-127

15 All modes with ternary internal symmetry ordered by length

16 All chords built by stacking every second note from the modes of ternary internal symmetry

17 All chords built by stacking every third note from the modes of ternary internal symmetry

18 All chords built by stacking every fourth note from the modes of ternary internal symmetry

19 All chords built by stacking every fifth note from the modes of ternary internal symmetry

20   Characterizing mode (2)1

Mode (2)1
(binary key: 0000001)
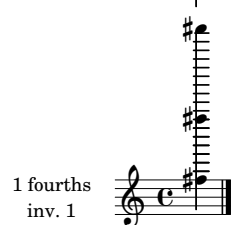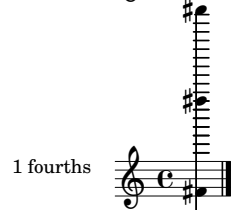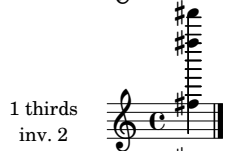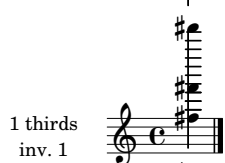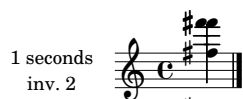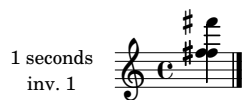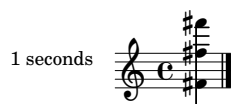
1 fourths
inv. 2

1 fifths

1 fifths
inv. 1

1 fifths
inv. 2

1 sus4

1 sus4
inv. 1

1 sus4
inv. 2

1 sus2

1 sus2
inv. 1

1 sus2
inv. 2
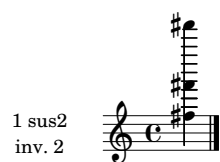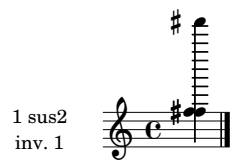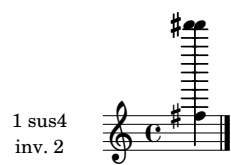
21 Characterizing mode (2)2

Mode (2)2
(binary key: 0000010)

2 plain

2 seconds

2 seconds
inv. 1

2 seconds
inv. 2

2 thirds

2 thirds
inv. 1

2 thirds
inv. 2

2 fourths

2 fourths
inv. 1

2 fourths
inv. 2

2 fifths

2 fifths
inv. 1
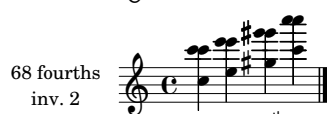
2

2 fifths
inv. 2

2 sus4

2 sus4
inv. 1

2 sus4
inv. 2

2 sus2

2 sus2
inv. 1
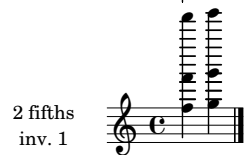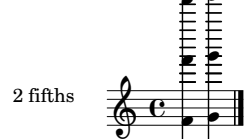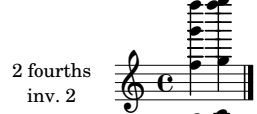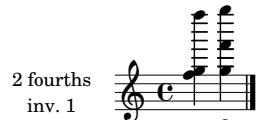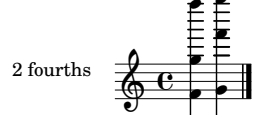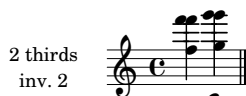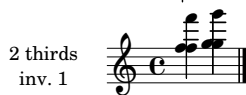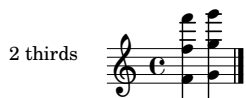
2 sus2
inv. 2
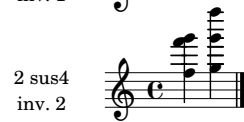
22   Characterizing mode (2)3

Mode (2)3
(binary key: 0000011)

3 sus4
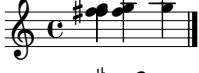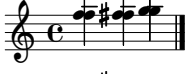
3 sus4
inv. 1

3 sus4
inv. 2

3 sus2

3 sus2
inv. 1

3 sus2
inv. 2

23 Characterizing mode (2)4

Mode (2)4
(binary key: 0000100)

4 fifths
inv. 2

4 sus4

4 sus4
inv. 1

4 sus4
inv. 2

4 sus2

4 sus2
inv. 1

4 sus2
inv. 2

24   Characterizing mode (2)5

Mode (2)5
(binary key: 0000101)

5 sus4

5 sus4
inv. 1

5 sus4
inv. 2

5 sus2

5 sus2
inv. 1

5 sus2
inv. 2

25   Characterizing mode (2)6

Mode (2)6
(binary key: 0000110)

6 plain

6 seconds
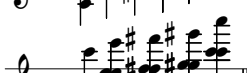
6 seconds
inv. 1

6 seconds
inv. 2

6 thirds

6 thirds
inv. 1

6 thirds
inv. 2

6 fourths

6 fourths
inv. 1

6 fourths
inv. 2

6 fifths

6 fifths
inv. 1

6 fifths
inv. 2

6 sus4

6 sus4
inv. 1

6 sus4
inv. 2

6 sus2

6 sus2
inv. 1

6 sus2
inv. 2

26 Characterizing mode (2)7

Mode (2)7
(binary key: 0000111)

7 plain

7 seconds

7 seconds
inv. 1

7 seconds
inv. 2

7 thirds

7 thirds
inv. 1

7 thirds
inv. 2

7 fourths

7 fourths
inv. 1

7 fourths
inv. 2

7 fifths

7 fifths
inv. 1

7 fifths
inv. 2

7 sus4

7 sus4
inv. 1

7 sus4
inv. 2

7 sus2

7 sus2
inv. 1

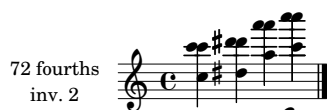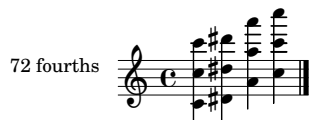7 sus2
inv. 2

27 Characterizing mode (2)8

Mode (2)8
(binary key: 0001000)

8 plain

8 seconds

8 seconds
inv. 1

8 seconds
inv. 2

8 thirds

8 thirds
inv. 1

8 thirds
inv. 2

8 fourths

8 fourths
inv. 1

8 fourths
inv. 2

8 fifths

8 fifths
inv. 1

8 fifths
inv. 2

8 sus4

8 sus4
inv. 1

8 sus4
inv. 2

8 sus2

8 sus2
inv. 1
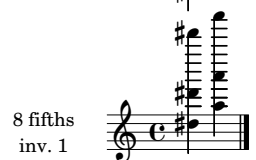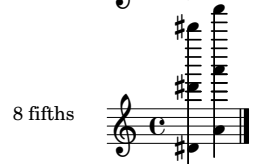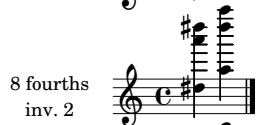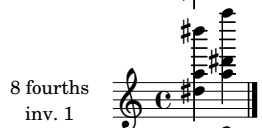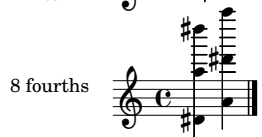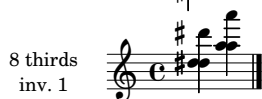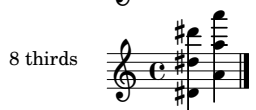
8 sus2
inv. 2
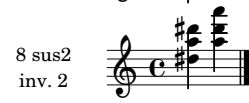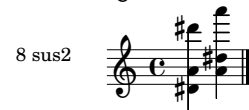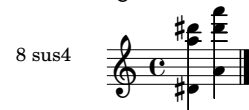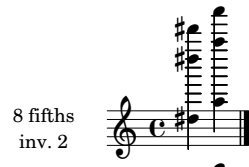
28 Characterizing mode (2)9

Mode (2)9
(binary key: 0001001)

9 plain

9 seconds

9 seconds
inv. 1

9 seconds
inv. 2
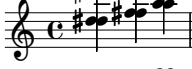
9 thirds

9 thirds
inv. 1

9 thirds
inv. 2

9 fourths

9 fourths
inv. 1

9 fourths
inv. 2

9 fifths

9 fifths
inv. 1

9 fifths
inv. 2

9 sus4

9 sus4
inv. 1

9 sus4
inv. 2

9 sus2

9 sus2
inv. 1
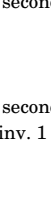
9 sus2
inv. 2
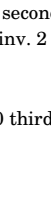
29 Characterizing mode (2)10

Mode (2)10
(binary key: 0001010)
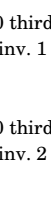
10 plain

10 seconds
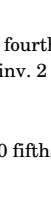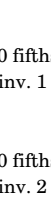
10 seconds
inv. 1

10 seconds
inv. 2

10 thirds

10 thirds
inv. 1

10 thirds
inv. 2

10 fourths

10 fourths
inv. 1

10 fourths
inv. 2

10 fifths

10 fifths
inv. 1

10 fifths
inv. 2

10 sus4

10 sus4
inv. 1

10 sus4
inv. 2

10 sus2

10 sus2
inv. 1

10 sus2
inv. 2

Mode (2)11
(binary key: 0001011)

11 plain

11 seconds

11 seconds
inv. 1

11 seconds
inv. 2

11 thirds

11 thirds
inv. 1

11 thirds
inv. 2

11 fourths

11 fourths
inv. 1

11 fourths
inv. 2

11 fifths

11 fifths
inv. 1

11 fifths
inv. 2

11 sus4

11 sus4
inv. 1

11 sus4
inv. 2

11 sus2

11 sus2
inv. 1

11 sus2
inv. 2

31   Characterizing mode (2)12

Mode (2)12
(binary key: 0001100)

12 plain

12 seconds

12 seconds
inv. 1

12 seconds
inv. 2

12 thirds

12 thirds
inv. 1

12 thirds
inv. 2

12 fourths

12 fourths
inv. 1

12 fourths
inv. 2

12 fifths

12 fifths
inv. 1
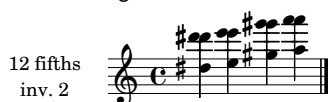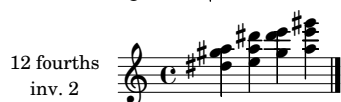
12 fifths
inv. 2

12 sus4

12 sus4
inv. 1

12 sus4
inv. 2

12 sus2

12 sus2
inv. 1

12 sus2
inv. 2

32 Characterizing mode (2)13

Mode (2)13
(binary key: 0001101)

13 plain

13 seconds

13 seconds
inv. 1

13 seconds
inv. 2
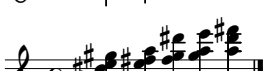
13 thirds

13 thirds
inv. 1

13 thirds
inv. 2
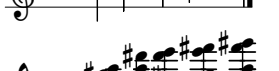
13 fourths

13 fourths
inv. 1

13 fourths
inv. 2

13 fifths

13 fifths
inv. 1

13 fifths
inv. 2

13 sus4

13 sus4
inv. 1

13 sus4
inv. 2

13 sus2

13 sus2
inv. 1

13 sus2
inv. 2

Mode (2)14
(binary key: 0001110)

14 sus4

14 sus4
inv. 1
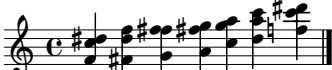
14 sus4
inv. 2

14 sus2

14 sus2
inv. 1

14 sus2
inv. 2

Mode (2)15
(binary key: 0001111)

15 sus4

15 sus4
inv. 1

15 sus4
inv. 2

15 sus2

15 sus2
inv. 1

15 sus2
inv. 2

35 Characterizing mode (2)16

Mode (2)16
(binary key: 0010000)

16 plain

16 seconds

16 seconds
inv. 1

16 seconds
inv. 2

16 thirds

16 thirds
inv. 1

16 thirds
inv. 2

16 fourths

16 fourths
inv. 1

16 fourths
inv. 2

16 fifths

16 fifths
inv. 1

16 fifths
inv. 2

16 sus4

16 sus4
inv. 1

16 sus4
inv. 2

16 sus2

16 sus2
inv. 1

16 sus2
inv. 2

36 Characterizing mode (2)17

Mode (2)17
(binary key: 0010001)

17 plain

17 seconds

17 seconds
inv. 1
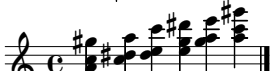
17 seconds
inv. 2

17 thirds
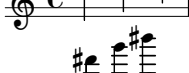
17 thirds
inv. 1

17 thirds
inv. 2

17 fourths

17 fourths
inv. 1

17 fourths
inv. 2

17 fifths

17 fifths
inv. 1

17 fifths
inv. 2

17 sus4

17 sus4
inv. 1

17 sus4
inv. 2

17 sus2

17 sus2
inv. 1

17 sus2
inv. 2

37 Characterizing mode (2)18

Mode (2)18
(binary key: 0010010)

18 sus4

18 sus4
inv. 1

18 sus4
inv. 2

18 sus2

18 sus2
inv. 1

18 sus2
inv. 2

Mode (2)19
(binary key: 0010011)

19 sus4

19 sus4
inv. 1

19 sus4
inv. 2

19 sus2

19 sus2
inv. 1

19 sus2
inv. 2

Mode (2)20
(binary key: 0010100)

20 sus4

20 sus4
inv. 1

20 sus4
inv. 2

20 sus2

20 sus2
inv. 1

20 sus2
inv. 2

Mode (2)21
(binary key: 0010101)

21 sus4

21 sus4
inv. 1

21 sus4
inv. 2

21 sus2

21 sus2
inv. 1

21 sus2
inv. 2

41 Characterizing mode (2)22

Mode (2)22
(binary key: 0010110)

2

22 sus4
inv. 1

22 sus4
inv. 2

22 sus2

22 sus2
inv. 1

22 sus2
inv. 2

Mode (2)23
(binary key: 0010111)

23 plain

23 seconds

23 seconds
inv. 1
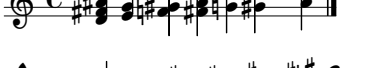
23 seconds
inv. 2

23 thirds

23 thirds
inv. 1
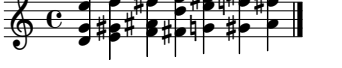
23 thirds
inv. 2

23 fourths

23 fourths
inv. 1

23 fourths
inv. 2

23 fifths

23 fifths
inv. 1

23 fifths
inv. 2

23 sus4

23 sus4 inv. 1

23 sus4 inv. 2

23 sus2

23 sus2 inv. 1

23 sus2 inv. 2

43   Characterizing mode (2)24

Mode (2)24
(binary key: 0011000)

24 sus4

24 sus4
inv. 1

24 sus4
inv. 2

24 sus2

24 sus2
inv. 1

24 sus2
inv. 2

Mode (2)25
(binary key: 0011001)

25 sus4

25 sus4
inv. 1

25 sus4
inv. 2

25 sus2

25 sus2
inv. 1

25 sus2
inv. 2

Mode (2)26
(binary key: 0011010)

26 sus4

26 sus4
inv. 1

26 sus4
inv. 2

26 sus2

26 sus2
inv. 1

26 sus2
inv. 2

Mode (2)27
(binary key: 0011011)
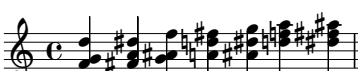
27 sus4

27 sus4
inv. 1

27 sus4
inv. 2

27 sus2

27 sus2
inv. 1

27 sus2
inv. 2

Mode (2)28
(binary key: 0011100)

28 plain

28 seconds

28 seconds
inv. 1

28 seconds
inv. 2

28 thirds

28 thirds
inv. 1

28 thirds
inv. 2

28 fourths

28 fourths
inv. 1

28 fourths
inv. 2

28 fifths

28 fifths
inv. 1

28 fifths
inv. 2

28 sus4

28 sus4
inv. 1

28 sus4
inv. 2

28 sus2

28 sus2
inv. 1

28 sus2
inv. 2

Mode (2)29
(binary key: 0011101)

29 sus4

29 sus4
inv. 1

29 sus4
inv. 2

29 sus2

29 sus2
inv. 1

29 sus2
inv. 2

49   Characterizing mode (2)30

Mode (2)30
(binary key: 0011110)

30 plain

30 seconds

30 seconds
inv. 1

30 seconds
inv. 2

30 thirds
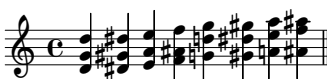
30 thirds
inv. 1

30 thirds
inv. 2

30 fourths

30 fourths
inv. 1

30 fourths
inv. 2

30 fifths

30 fifths
inv. 1

30 fifths
inv. 2

95

30 sus4

30 sus4
inv. 1

30 sus4
inv. 2

30 sus2

30 sus2
inv. 1

30 sus2
inv. 2

Mode (2)31
(binary key: 0011111)

31 plain

31 seconds

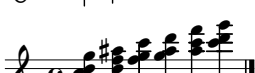31 seconds
inv. 1

31 seconds
inv. 2

31 thirds

31 thirds
inv. 1

31 thirds
inv. 2

31 fourths

31 fourths
inv. 1

31 fourths
inv. 2

31 fifths

31 fifths
inv. 1

31 fifths
inv. 2

31 sus4

31 sus4
inv. 1

31 sus4
inv. 2

31 sus2

31 sus2
inv. 1

31 sus2
inv. 2

Mode (2)32
(binary key: 0100000)

32 plain

32 seconds

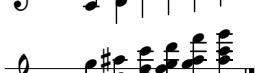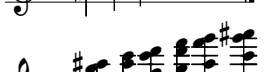32 seconds
inv. 1

32 seconds
inv. 2

32 thirds

32 thirds
inv. 1

32 thirds
inv. 2

32 fourths

32 fourths
inv. 1

32 fourths
inv. 2

32 fifths

32 fifths
inv. 1

32 fifths
inv. 2

32 sus4

32 sus4
inv. 1

32 sus4
inv. 2

32 sus2

32 sus2
inv. 1

32 sus2
inv. 2
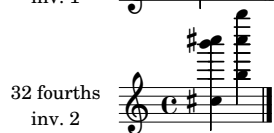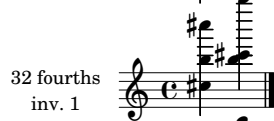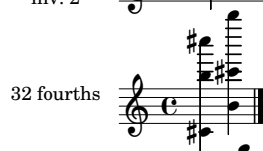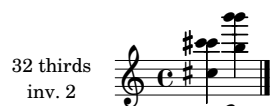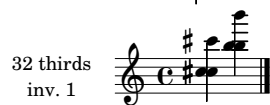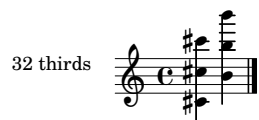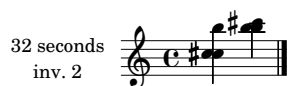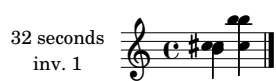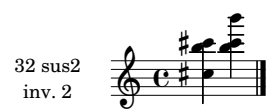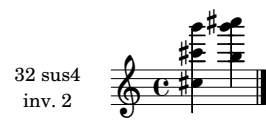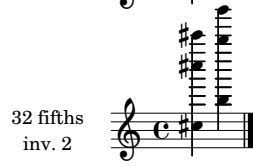
52 Characterizing mode (2)33

Mode (2)33
(binary key: 0100001)

33 plain

33 seconds
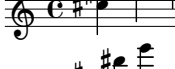
33 seconds
inv. 1

33 seconds
inv. 2

33 thirds

33 thirds
inv. 1

33 thirds
inv. 2

33 fourths

33 fourths
inv. 1

33 fourths
inv. 2

33 fifths

33 fifths
inv. 1

33 fifths
inv. 2

33 sus4

33 sus4
inv. 1

33 sus4
inv. 2

33 sus2

33 sus2
inv. 1

33 sus2
inv. 2

53   Characterizing mode (2)34

Mode (2)34
(binary key: 0100010)

34 plain

34 seconds

34 seconds inv. 1

34 seconds inv. 2

34 thirds

34 thirds inv. 1

34 thirds inv. 2

34 fourths

34 fourths inv. 1

34 fourths inv. 2

34 fifths

34 fifths inv. 1
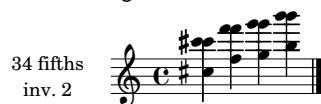
34 fifths inv. 2

34 sus4

34 sus4
inv. 1

34 sus4
inv. 2

34 sus2

34 sus2
inv. 1

34 sus2
inv. 2

Mode (2)35
(binary key: 0100011)

35 plain

35 seconds

35 seconds
inv. 1

35 seconds
inv. 2

35 thirds

35 thirds
inv. 1

35 thirds
inv. 2

35 fourths

35 fourths
inv. 1

35 fourths
inv. 2

35 fifths

35 fifths
inv. 1

35 fifths
inv. 2

35 sus4

35 sus4
inv. 1

35 sus4
inv. 2

35 sus2

35 sus2
inv. 1

35 sus2
inv. 2

Mode (2)36
(binary key: 0100100)
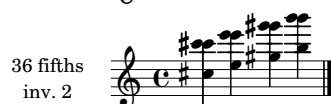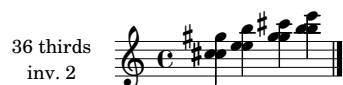
36 sus4

36 sus4
inv. 1

36 sus4
inv. 2

36 sus2

36 sus2
inv. 1

36 sus2
inv. 2

Mode (2)37
(binary key: 0100101)

37 sus4

37 sus4
inv. 1

37 sus4
inv. 2

37 sus2

37 sus2
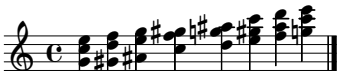inv. 1

37 sus2
inv. 2

Mode (2)38
(binary key: 0100110)

38 plain

38 seconds

38 seconds
inv. 1
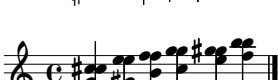
38 seconds
inv. 2

38 thirds

38 thirds
inv. 1

38 thirds
inv. 2

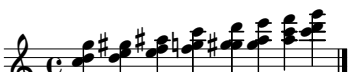38 fourths

38 fourths
inv. 1

38 fourths
inv. 2

38 fifths

38 fifths
inv. 1

38 fifths
inv. 2

38 sus4

38 sus4
inv. 1

38 sus4
inv. 2

38 sus2

38 sus2
inv. 1

38 sus2
inv. 2

Mode (2)39
(binary key: 0100111)

39 plain

39 seconds

39 seconds
inv. 1

39 seconds
inv. 2

39 thirds

39 thirds
inv. 1

39 thirds
inv. 2

39 fourths

39 fourths
inv. 1

39 fourths
inv. 2

39 fifths

39 fifths
inv. 1

39 fifths
inv. 2

39 sus4

39 sus4
inv. 1

39 sus4
inv. 2

39 sus2

39 sus2
inv. 1

39 sus2
inv. 2

Mode (2)40
(binary key: 0101000)

40 sus4

40 sus4
inv. 1

40 sus4
inv. 2

40 sus2

40 sus2
inv. 1

40 sus2
inv. 2

Mode (2)41
(binary key: 0101001)
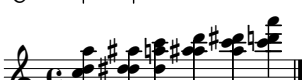
41 plain

41 seconds

41 seconds
inv. 1

41 seconds
inv. 2

41 thirds

41 thirds
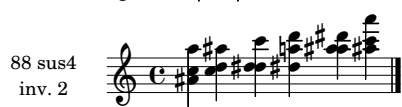inv. 1

41 thirds
inv. 2

41 fourths

41 fourths
inv. 1

41 fourths
inv. 2

41 fifths

41 fifths
inv. 1

41 fifths
inv. 2

41 sus4

41 sus4
inv. 1

41 sus4
inv. 2

41 sus2

41 sus2
inv. 1

41 sus2
inv. 2

Mode (2)42
(binary key: 0101010)

42 plain

42 seconds

42 seconds
inv. 1

42 seconds
inv. 2

42 thirds

42 thirds
inv. 1

42 thirds
inv. 2

42 fourths

42 fourths
inv. 1

42 fourths
inv. 2

42 fifths

42 fifths
inv. 1
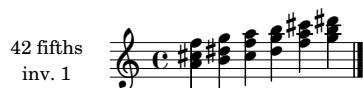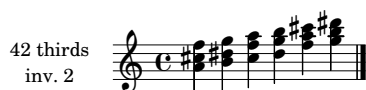
42 fifths
inv. 2

42 sus4

42 sus4
inv. 1

42 sus4
inv. 2

42 sus2

42 sus2
inv. 1

42 sus2
inv. 2

Mode (2)43
(binary key: 0101011)

43 sus4

43 sus4
inv. 1

43 sus4
inv. 2

43 sus2

43 sus2
inv. 1

43 sus2
inv. 2

63 Characterizing mode (2)44

Mode (2)44
(binary key: 0101100)

44 plain

44 seconds

44 seconds
inv. 1

44 seconds
inv. 2

44 thirds

44 thirds
inv. 1

44 thirds
inv. 2

44 fourths

44 fourths
inv. 1

44 fourths
inv. 2

44 fifths

44 fifths
inv. 1

44 fifths
inv. 2

123

44 sus4

44 sus4
inv. 1

44 sus4
inv. 2

44 sus2

44 sus2
inv. 1

44 sus2
inv. 2

Mode (2)45
(binary key: 0101101)

45 sus4

45 sus4
inv. 1

45 sus4
inv. 2

45 sus2

45 sus2
inv. 1

45 sus2
inv. 2

Mode (2)46
(binary key: 0101110)

46 sus4

46 sus4
inv. 1

46 sus4
inv. 2

46 sus2

46 sus2
inv. 1

46 sus2
inv. 2

Mode (2)47
(binary key: 0101111)

47 plain

47 seconds

47 seconds
inv. 1

47 seconds
inv. 2

47 thirds

47 thirds
inv. 1

47 thirds
inv. 2

47 fourths

47 fourths
inv. 1

47 fourths
inv. 2

47 fifths

47 fifths
inv. 1

47 fifths
inv. 2

47 sus4
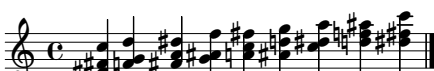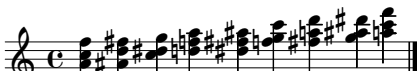
47 sus4
inv. 1

47 sus4
inv. 2

47 sus2

47 sus2
inv. 1

47 sus2
inv. 2

Mode (2)48
(binary key: 0110000)

48 plain

48 seconds

48 seconds
inv. 1

48 seconds
inv. 2

48 thirds

48 thirds
inv. 1

48 thirds
inv. 2

48 fourths

48 fourths
inv. 1

48 fourths
inv. 2

48 fifths

48 fifths
inv. 1
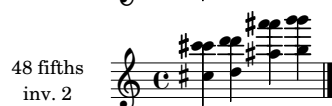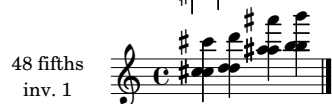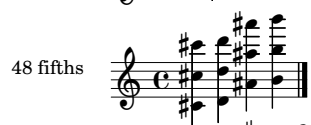
48 fifths
inv. 2

48 sus4

48 sus4
inv. 1

48 sus4
inv. 2

48 sus2

48 sus2
inv. 1

48 sus2
inv. 2

Mode (2)49
(binary key: 0110001)

49 sus4

49 sus4
inv. 1

49 sus4
inv. 2

49 sus2

49 sus2
inv. 1

49 sus2
inv. 2

Mode (2)50
(binary key: 0110010)

50 plain

50 seconds
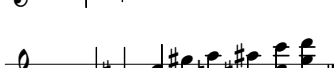
50 seconds
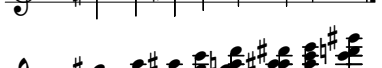inv. 1

50 seconds
inv. 2

50 thirds

50 thirds
inv. 1

50 thirds
inv. 2

50 fourths

50 fourths
inv. 1

50 fourths
inv. 2

50 fifths

50 fifths
inv. 1

50 fifths
inv. 2

50 sus4

50 sus4
inv. 1

50 sus4
inv. 2

50 sus2

50 sus2
inv. 1

50 sus2
inv. 2

Mode (2)51
(binary key: 0110011)
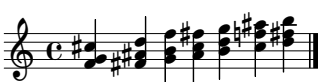
51 plain

51 seconds

51 seconds
inv. 1

51 seconds
inv. 2

51 thirds

51 thirds
inv. 1

51 thirds
inv. 2

51 fourths

51 fourths
inv. 1

51 fourths
inv. 2

51 fifths

51 fifths
inv. 1

51 fifths
inv. 2

51 sus4

51 sus4
inv. 1

51 sus4
inv. 2

51 sus2

51 sus2
inv. 1

51 sus2
inv. 2

71 Characterizing mode (2)52

Mode (2)52
(binary key: 0110100)

52 sus4

52 sus4
inv. 1

52 sus4
inv. 2

52 sus2

52 sus2
inv. 1

52 sus2
inv. 2

Mode (2)53
(binary key: 0110101)

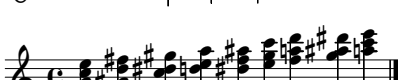53 plain

53 seconds

53 seconds
inv. 1

53 seconds
inv. 2

53 thirds

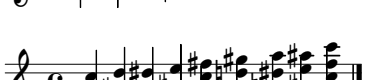53 thirds
inv. 1
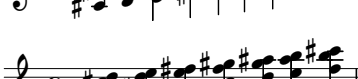
53 thirds
inv. 2

53 fourths

53 fourths
inv. 1

53 fourths
inv. 2

53 fifths

53 fifths
inv. 1

53 fifths
inv. 2

53 sus4

53 sus4
inv. 1

53 sus4
inv. 2

53 sus2

53 sus2
inv. 1

53 sus2
inv. 2

Mode (2)54
(binary key: 0110110)

54 plain

54 seconds

54 seconds
inv. 1

54 seconds
inv. 2

54 thirds

54 thirds
inv. 1

54 thirds
inv. 2

54 fourths

54 fourths
inv. 1

54 fourths
inv. 2

54 fifths

54 fifths
inv. 1

54 fifths
inv. 2

54 sus4

54 sus4
inv. 1

54 sus4
inv. 2

54 sus2

54 sus2
inv. 1

54 sus2
inv. 2

Mode (2)55
(binary key: 0110111)

55 sus4

55 sus4
inv. 1

55 sus4
inv. 2

55 sus2

55 sus2
inv. 1

55 sus2
inv. 2

75 Characterizing mode (2)56

Mode (2)56
(binary key: 0111000)
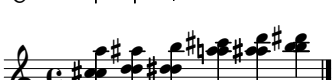
56 sus4

56 sus4
inv. 1

56 sus4
inv. 2

56 sus2

56 sus2
inv. 1

56 sus2
inv. 2

Mode (2)57
(binary key: 0111001)
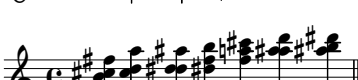
57 sus4

57 sus4
inv. 1

57 sus4
inv. 2

57 sus2

57 sus2
inv. 1

57 sus2
inv. 2

Mode (2)58
(binary key: 0111010)

58 sus4

58 sus4
inv. 1

58 sus4
inv. 2

58 sus2

58 sus2
inv. 1

58 sus2
inv. 2

Mode (2)59
(binary key: 0111011)
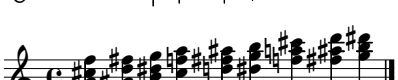
59 sus4

59 sus4
inv. 1

59 sus4
inv. 2

59 sus2

59 sus2
inv. 1

59 sus2
inv. 2

Mode (2)60
(binary key: 0111100)

60 sus4

60 sus4
inv. 1

60 sus4
inv. 2

60 sus2

60 sus2
inv. 1

60 sus2
inv. 2

Mode (2)61
(binary key: 0111101)

61 plain

61 seconds

61 seconds inv. 1

61 seconds inv. 2

61 thirds

61 thirds inv. 1

61 thirds inv. 2

61 fourths

61 fourths inv. 1

61 fourths inv. 2

61 fifths

61 fifths inv. 1

61 fifths inv. 2

61 sus4

61 sus4
inv. 1

61 sus4
inv. 2

61 sus2

61 sus2
inv. 1

61 sus2
inv. 2

81 Characterizing mode (2)62

Mode (2)62

(binary key: 0111110)

62 sus4

62 sus4
inv. 1

62 sus4
inv. 2

62 sus2

62 sus2
inv. 1

62 sus2
inv. 2

Mode (2)63
(binary key: 0111111)

63 sus4

63 sus4
inv. 1

63 sus4
inv. 2

63 sus2

63 sus2
inv. 1

63 sus2
inv. 2

Mode (2)64
(binary key: 1000000)

64 plain

64 seconds

64 seconds
inv. 1

64 seconds
inv. 2

64 thirds

64 thirds
inv. 1

64 thirds
inv. 2
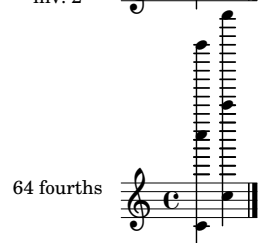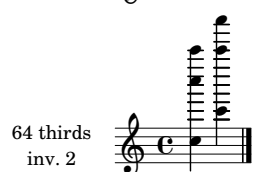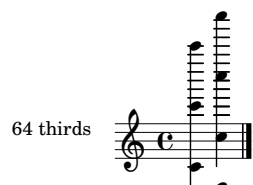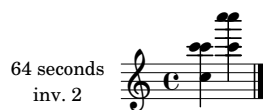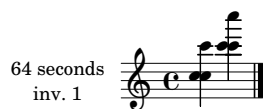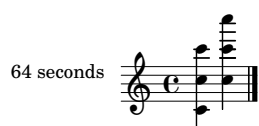
64 fourths

2
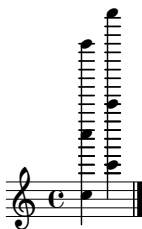
64 fourths
inv. 1

64 fourths
inv. 2
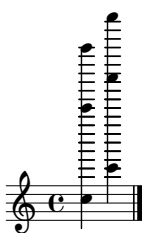
64 fifths
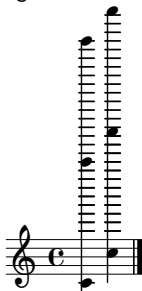
64 fifths
inv. 1

64 fifths
inv. 2

64 sus4

64 sus4
inv. 1

64 sus4
inv. 2

64 sus2

64 sus2
inv. 1

64 sus2
inv. 2
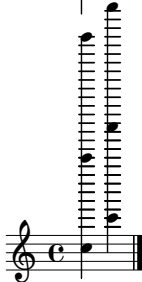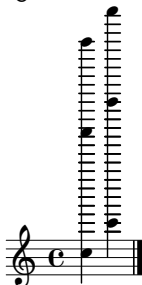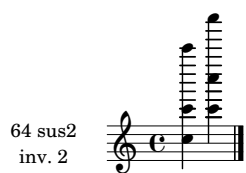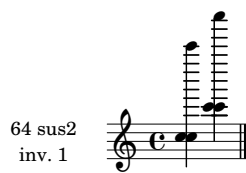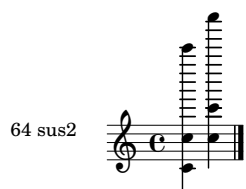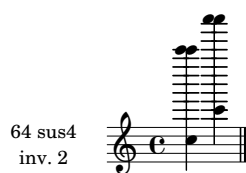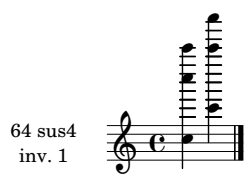
Mode (2)65
(binary key: 1000001)

65 plain

65 seconds

65 seconds
inv. 1

65 seconds
inv. 2

65 thirds

65 thirds
inv. 1

65 thirds
inv. 2

65 fourths

65 fourths
inv. 1

65 fourths
inv. 2

65 fifths

65 fifths
inv. 1

65 fifths
inv. 2

65 sus4

65 sus4
inv. 1

65 sus4
inv. 2

65 sus2

65 sus2
inv. 1

65 sus2
inv. 2
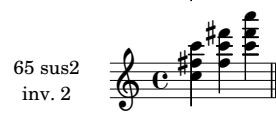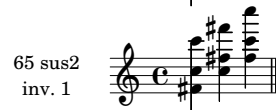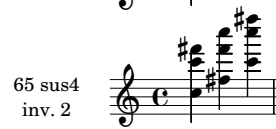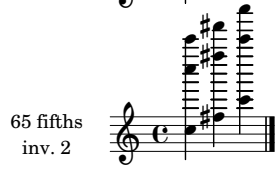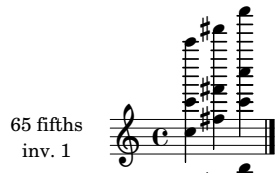
Mode (2)66
(binary key: 1000010)

66 plain

66 seconds

66 seconds inv. 1

66 seconds inv. 2

66 thirds

66 thirds inv. 1

66 thirds inv. 2

66 fourths

66 fourths inv. 1

66 fourths inv. 2

66 fifths

66 fifths inv. 1

66 fifths inv. 2

66 sus4

66 sus4
inv. 1

66 sus4
inv. 2

66 sus2

66 sus2
inv. 1

66 sus2
inv. 2
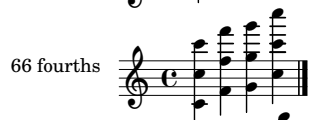
Mode (2)67
(binary key: 1000011)

67 plain

67 seconds

67 seconds
inv. 1

67 seconds
inv. 2

67 thirds

67 thirds
inv. 1

67 thirds
inv. 2

67 fourths

67 fourths
inv. 1

67 fourths
inv. 2

67 fifths

67 fifths
inv. 1

67 fifths
inv. 2

67 sus4

67 sus4
inv. 1

67 sus4
inv. 2

67 sus2

67 sus2
inv. 1

67 sus2
inv. 2

87 Characterizing mode (2)68

Mode (2)68
(binary key: 1000100)

68 plain

68 seconds

68 seconds
inv. 1

68 seconds
inv. 2

68 thirds

68 thirds
inv. 1

68 thirds
inv. 2

68 fourths

68 fourths
inv. 1

68 fourths
inv. 2

68 fifths

68 fifths
inv. 1

172

2

68 fifths
inv. 2

68 sus4

68 sus4
inv. 1

68 sus4
inv. 2

68 sus2

68 sus2
inv. 1

68 sus2
inv. 2

Mode (2)69
(binary key: 1000101)

69 plain

69 seconds

69 seconds inv. 1

69 seconds inv. 2

69 thirds

69 thirds inv. 1

69 thirds inv. 2

69 fourths

69 fourths inv. 1

69 fourths inv. 2

69 fifths

69 fifths inv. 1

69 fifths inv. 2

69 sus4

69 sus4
inv. 1

69 sus4
inv. 2

69 sus2

69 sus2
inv. 1

69 sus2
inv. 2

Mode (2)70
(binary key: 1000110)

70 plain

70 seconds

70 seconds
inv. 1

70 seconds
inv. 2

70 thirds

70 thirds
inv. 1

70 thirds
inv. 2

70 fourths

70 fourths
inv. 1

70 fourths
inv. 2

70 fifths

70 fifths
inv. 1

70 fifths
inv. 2

70 sus4

70 sus4
inv. 1

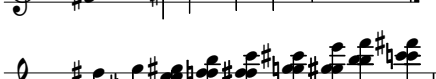70 sus4
inv. 2

70 sus2

70 sus2
inv. 1

70 sus2
inv. 2

Mode (2)71
(binary key: 1000111)

71 plain

71 seconds

71 seconds
inv. 1

71 seconds
inv. 2

71 thirds

71 thirds
inv. 1

71 thirds
inv. 2

71 fourths

71 fourths
inv. 1

71 fourths
inv. 2

71 fifths

71 fifths
inv. 1

71 fifths
inv. 2

71 sus4

71 sus4
inv. 1

71 sus4
inv. 2

71 sus2

71 sus2
inv. 1

71 sus2
inv. 2

Mode (2)72
(binary key: 1001000)

72 plain

72 seconds

72 seconds
inv. 1

72 seconds
inv. 2

72 thirds

72 thirds
inv. 1

72 thirds
inv. 2

72 fourths

72 fourths
inv. 1

72 fourths
inv. 2

72 fifths

72 fifths
inv. 1

72 fifths
inv. 2

72 sus4

72 sus4
inv. 1

72 sus4
inv. 2

72 sus2

72 sus2
inv. 1

72 sus2
inv. 2

Mode (2)73
(binary key: 1001001)

73 plain

73 seconds

73 seconds
inv. 1

73 seconds
inv. 2

73 thirds

73 thirds
inv. 1

73 thirds
inv. 2

73 fourths

73 fourths
inv. 1

73 fourths
inv. 2

73 fifths

73 fifths
inv. 1

73 fifths
inv. 2

2

73 sus4

73 sus4
inv. 1

73 sus4
inv. 2

73 sus2

73 sus2
inv. 1

73 sus2
inv. 2

Mode (2)74
(binary key: 1001010)

74 sus4

74 sus4
inv. 1

74 sus4
inv. 2

74 sus2

74 sus2
inv. 1

74 sus2
inv. 2

Mode (2)75
(binary key: 1001011)

75 plain

75 seconds

75 seconds
inv. 1

75 seconds
inv. 2

75 thirds

75 thirds
inv. 1

75 thirds
inv. 2

75 fourths

75 fourths
inv. 1

75 fourths
inv. 2

75 fifths

75 fifths
inv. 1

75 fifths
inv. 2

75 sus4

75 sus4
inv. 1

75 sus4
inv. 2

75 sus2
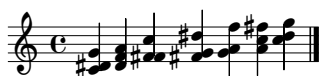
75 sus2
inv. 1

75 sus2
inv. 2

Mode (2)76
(binary key: 1001100)

76 sus4

76 sus4
inv. 1

76 sus4
inv. 2

76 sus2

76 sus2
inv. 1

76 sus2
inv. 2

Mode (2)77
(binary key: 1001101)

2

77 sus4

77 sus4
inv. 1

77 sus4
inv. 2

77 sus2

77 sus2
inv. 1

77 sus2
inv. 2

Mode (2)78
(binary key: 1001110)

78 plain

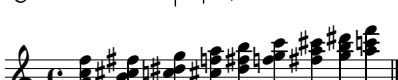78 seconds

78 seconds
inv. 1

78 seconds
inv. 2

78 thirds

78 thirds
inv. 1

78 thirds
inv. 2
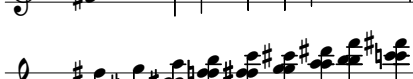
78 fourths

78 fourths
inv. 1

78 fourths
inv. 2

78 fifths

78 fifths
inv. 1

78 fifths
inv. 2

78 sus4

78 sus4
inv. 1

78 sus4
inv. 2

78 sus2

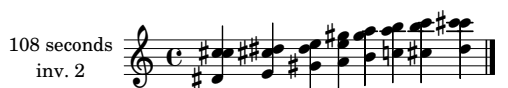78 sus2
inv. 1

78 sus2
inv. 2

Mode (2)79
(binary key: 1001111)

79 sus4

79 sus4
inv. 1

79 sus4
inv. 2

79 sus2

79 sus2
inv. 1

79 sus2
inv. 2

Mode (2)80
(binary key: 1010000)

80 plain

80 seconds

80 seconds
inv. 1

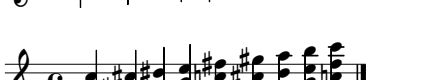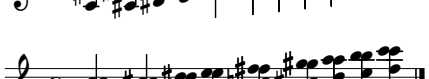80 seconds
inv. 2

80 thirds

80 thirds
inv. 1

80 thirds
inv. 2

80 fourths

80 fourths
inv. 1

80 fourths
inv. 2

80 fifths

80 fifths
inv. 1

80 fifths
inv. 2

80 sus4

80 sus4
inv. 1

80 sus4
inv. 2

80 sus2

80 sus2
inv. 1

80 sus2
inv. 2
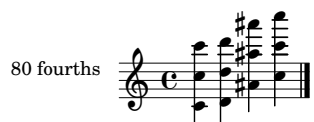
Mode (2)81
(binary key: 1010001)

81 plain

81 seconds

81 seconds inv. 1

81 seconds inv. 2

81 thirds

81 thirds inv. 1

81 thirds inv. 2

81 fourths

81 fourths inv. 1

81 fourths inv. 2

81 fifths

81 fifths inv. 1

81 fifths inv. 2

81 sus4
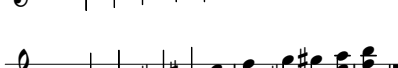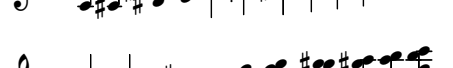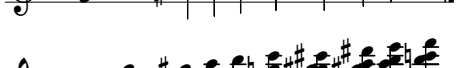
81 sus4
inv. 1

81 sus4
inv. 2

81 sus2

81 sus2
inv. 1

81 sus2
inv. 2

Mode (2)82
(binary key: 1010010)

82 plain

82 seconds

82 seconds
inv. 1

82 seconds
inv. 2

82 thirds

82 thirds
inv. 1

82 thirds
inv. 2

82 fourths

82 fourths
inv. 1

82 fourths
inv. 2

82 fifths

82 fifths
inv. 1

82 fifths
inv. 2

82 sus4

82 sus4
inv. 1

82 sus4
inv. 2

82 sus2

82 sus2
inv. 1

82 sus2
inv. 2

Mode (2)83
(binary key: 1010011)

83 plain

83 seconds

83 seconds
inv. 1

83 seconds
inv. 2

83 thirds

83 thirds
inv. 1

83 thirds
inv. 2

83 fourths

83 fourths
inv. 1

83 fourths
inv. 2

83 fifths

83 fifths
inv. 1

83 fifths
inv. 2

83 sus4

Mode (2)84
(binary key: 1010100)

84 plain

84 seconds

84 seconds
inv. 1

84 seconds
inv. 2

84 thirds

84 thirds
inv. 1

84 thirds
inv. 2

84 fourths

84 fourths
inv. 1

84 fourths
inv. 2

84 fifths

84 fifths
inv. 1

84 fifths
inv. 2

84 sus4

84 sus4
inv. 1

84 sus4
inv. 2

84 sus2

84 sus2
inv. 1

84 sus2
inv. 2

Mode (2)85
(binary key: 1010101)

85 sus4

85 sus4
inv. 1

85 sus4
inv. 2

85 sus2

85 sus2
inv. 1

85 sus2
inv. 2

Mode (2)86
(binary key: 1010110)

86 sus4

86 sus4
inv. 1

86 sus4
inv. 2

86 sus2

86 sus2
inv. 1

86 sus2
inv. 2

Mode (2)87
(binary key: 1010111)

87 sus4

87 sus4
inv. 1

87 sus4
inv. 2

87 sus2

87 sus2
inv. 1

87 sus2
inv. 2

Mode (2)88
(binary key: 1011000)

88 sus4

88 sus4
inv. 1

88 sus4
inv. 2

88 sus2

88 sus2
inv. 1

88 sus2
inv. 2

Mode (2)89
(binary key: 1011001)

89 sus4

89 sus4
inv. 1

89 sus4
inv. 2

89 sus2

89 sus2
inv. 1

89 sus2
inv. 2

Mode (2)90
(binary key: 1011010)

2

90 sus4

90 sus4
inv. 1

90 sus4
inv. 2

90 sus2

90 sus2
inv. 1

90 sus2
inv. 2

Mode (2)91
(binary key: 1011011)

91 plain

91 seconds

91 seconds
inv. 1

91 seconds
inv. 2

91 thirds

91 thirds
inv. 1

91 thirds
inv. 2

91 fourths

91 fourths
inv. 1

91 fourths
inv. 2

91 fifths

91 fifths
inv. 1

91 fifths
inv. 2

Mode (2)92
(binary key: 1011100)

92 sus4

92 sus4
inv. 1

92 sus4
inv. 2

92 sus2

92 sus2
inv. 1

92 sus2
inv. 2

Mode (2)93
(binary key: 1011101)

93 sus4

93 sus4
inv. 1

93 sus4
inv. 2

93 sus2

93 sus2
inv. 1

93 sus2
inv. 2

113   Characterizing mode (2)94

Mode (2)94
(binary key: 1011110)

94 sus4

94 sus4
inv. 1

94 sus4
inv. 2

94 sus2

94 sus2
inv. 1

94 sus2
inv. 2

Mode (2)95
(binary key: 1011111)

95 sus4

95 sus4
inv. 1

95 sus4
inv. 2

95 sus2

95 sus2
inv. 1

95 sus2
inv. 2

Mode (2)96
(binary key: 1100000)

96 fifths
inv. 2

96 sus4

96 sus4
inv. 1

96 sus4
inv. 2

96 sus2

96 sus2
inv. 1

96 sus2
inv. 2
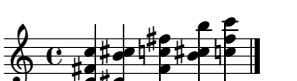
Mode (2)97
(binary key: 1100001)

97 plain

97 seconds

97 seconds inv. 1
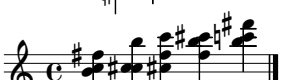
97 seconds inv. 2

97 thirds

97 thirds inv. 1

97 thirds inv. 2

97 fourths

97 fourths inv. 1

97 fourths inv. 2

97 fifths

97 fifths inv. 1

97 fifths inv. 2

97 sus4

97 sus4
inv. 1
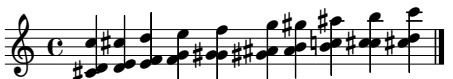
97 sus4
inv. 2

97 sus2

97 sus2
inv. 1

97 sus2
inv. 2

Mode (2)98
(binary key: 1100010)

98 plain

98 seconds

98 seconds
inv. 1

98 seconds
inv. 2

98 thirds

98 thirds
inv. 1

98 thirds
inv. 2

98 fourths

98 fourths
inv. 1

98 fourths
inv. 2

98 fifths

98 fifths
inv. 1

98 fifths
inv. 2

98 sus4

98 sus4
inv. 1

98 sus4
inv. 2

98 sus2

98 sus2
inv. 1

98 sus2
inv. 2

Mode (2)99
(binary key: 1100011)

99 plain

99 seconds

99 seconds inv. 1

99 seconds inv. 2

99 thirds

99 thirds inv. 1
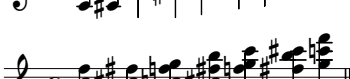
99 thirds inv. 2

99 fourths

99 fourths inv. 1

99 fourths inv. 2

99 fifths

99 fifths inv. 1

99 fifths inv. 2

99 sus4

99 sus4
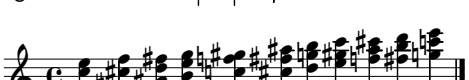inv. 1

99 sus4
inv. 2

99 sus2

99 sus2
inv. 1

99 sus2
inv. 2

Mode (2)100
(binary key: 1100100)

2

100 sus4

100 sus4
inv. 1

100 sus4
inv. 2

100 sus2

100 sus2
inv. 1

100 sus2
inv. 2

Mode (2)101
(binary key: 1100101)

101 sus4

101 sus4
inv. 1

101 sus4
inv. 2

101 sus2

101 sus2
inv. 1

101 sus2
inv. 2

Mode (2)102
(binary key: 1100110)

102 sus4

102 sus4
inv. 1

102 sus4
inv. 2

102 sus2

102 sus2
inv. 1

102 sus2
inv. 2

Mode (2)103
(binary key: 1100111)

103 plain

103 seconds

103 seconds
inv. 1

103 seconds
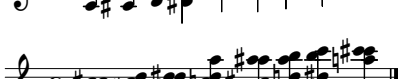inv. 2

103 thirds

103 thirds
inv. 1

103 thirds
inv. 2

103 fourths

103 fourths
inv. 1

103 fourths
inv. 2

103 fifths

103 fifths
inv. 1

103 fifths
inv. 2

103 sus4

103 sus4
inv. 1

103 sus4
inv. 2

103 sus2

103 sus2
inv. 1

103 sus2
inv. 2

Mode (2)104
(binary key: 1101000)

104 plain

104 seconds

104 seconds
inv. 1

104 seconds
inv. 2

104 thirds
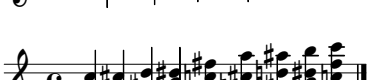
104 thirds
inv. 1

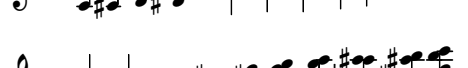104 thirds
inv. 2

104 fourths

104 fourths
inv. 1

104 fourths
inv. 2

104 fifths

104 fifths
inv. 1

104 fifths
inv. 2

104 sus4

104 sus4
inv. 1

104 sus4
inv. 2

104 sus2

104 sus2
inv. 1

104 sus2
inv. 2

Mode (2)105
(binary key: 1101001)

105 sus4

105 sus4
inv. 1

105 sus4
inv. 2

105 sus2

105 sus2
inv. 1

105 sus2
inv. 2

Mode (2)106
(binary key: 1101010)
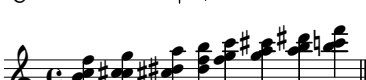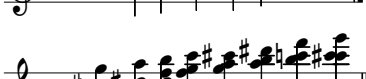
106 sus4

106 sus4
inv. 1

106 sus4
inv. 2

106 sus2

106 sus2
inv. 1

106 sus2
inv. 2

Mode (2)107
(binary key: 1101011)

107 sus4

107 sus4
inv. 1

107 sus4
inv. 2

107 sus2

107 sus2
inv. 1

107 sus2
inv. 2

Mode (2)108
(binary key: 1101100)

108 sus4

108 sus4
inv. 1

108 sus4
inv. 2

108 sus2

108 sus2
inv. 1

108 sus2
inv. 2

Mode (2)109
(binary key: 1101101)

109 plain

109 seconds

109 seconds inv. 1

109 seconds inv. 2

109 thirds

109 thirds inv. 1

109 thirds inv. 2

109 fourths

109 fourths inv. 1

109 fourths inv. 2

109 fifths

109 fifths inv. 1

109 fifths inv. 2

109 sus4

109 sus4
inv. 1

109 sus4
inv. 2

109 sus2

109 sus2
inv. 1

109 sus2
inv. 2

Mode (2)110
(binary key: 1101110)

110 sus4

110 sus4
inv. 1

110 sus4
inv. 2

110 sus2

110 sus2
inv. 1

110 sus2
inv. 2

Mode (2)111
(binary key: 1101111)

111 sus4

111 sus4
inv. 1

111 sus4
inv. 2

111 sus2

111 sus2
inv. 1

111 sus2
inv. 2

131   Characterizing mode (2)112

Mode (2)112
(binary key: 1110000)

112 sus4

112 sus4
inv. 1

112 sus4
inv. 2

112 sus2

112 sus2
inv. 1

112 sus2
inv. 2

Mode (2)113
(binary key: 1110001)

113 plain
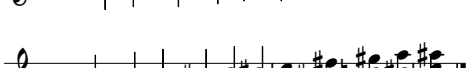
113 seconds

113 seconds
inv. 1

113 seconds
inv. 2

113 thirds

113 thirds
inv. 1

113 thirds
inv. 2

113 fourths

113 fourths
inv. 1

113 fourths
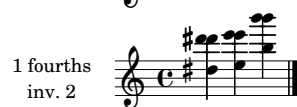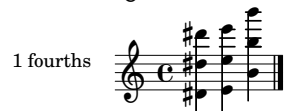inv. 2

113 fifths

113 fifths
inv. 1

113 fifths
inv. 2

113 sus4

113 sus4
inv. 1

113 sus4
inv. 2

113 sus2

113 sus2
inv. 1

113 sus2
inv. 2

133 Characterizing mode (2)114

Mode (2)114
(binary key: 1110010)

114 sus4

114 sus4
inv. 1

114 sus4
inv. 2

114 sus2

114 sus2
inv. 1

114 sus2
inv. 2

Mode (2)115
(binary key: 1110011)

115 sus4

115 sus4
inv. 1

115 sus4
inv. 2

115 sus2

115 sus2
inv. 1
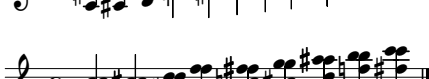
115 sus2
inv. 2

135 Characterizing mode (2)116

Mode (2)116
(binary key: 1110100)

268

116 sus4

116 sus4
inv. 1

116 sus4
inv. 2

116 sus2

116 sus2
inv. 1

116 sus2
inv. 2

Mode (2)117
(binary key: 1110101)

117 plain

117 seconds

117 seconds inv. 1

117 seconds inv. 2

117 thirds

117 thirds inv. 1

117 thirds inv. 2

117 fourths

117 fourths inv. 1

117 fourths inv. 2

117 fifths

117 fifths inv. 1

117 fifths inv. 2

117 sus4

117 sus4
inv. 1

117 sus4
inv. 2

117 sus2

117 sus2
inv. 1

117 sus2
inv. 2

137   Characterizing mode (2)118

Mode (2)118
(binary key: 1110110)

118 sus4

118 sus4
inv. 1

118 sus4
inv. 2

118 sus2

118 sus2
inv. 1

118 sus2
inv. 2

Mode (2)119
(binary key: 1110111)

119 plain

119 seconds

119 seconds
inv. 1

119 seconds
inv. 2

119 thirds

119 thirds
inv. 1

119 thirds
inv. 2

119 fourths

119 fourths
inv. 1

119 fourths
inv. 2

119 fifths

119 fifths
inv. 1

119 fifths
inv. 2

119 sus4

119 sus4
inv. 1

119 sus4
inv. 2

119 sus2

119 sus2
inv. 1

119 sus2
inv. 2

Mode (2)120
(binary key: 1111000)

120 sus4

120 sus4
inv. 1

120 sus4
inv. 2

120 sus2

120 sus2
inv. 1

120 sus2
inv. 2

Mode (2)121
(binary key: 1111001)

121 plain

121 seconds

121 seconds
inv. 1

121 seconds
inv. 2

121 thirds

121 thirds
inv. 1

121 thirds
inv. 2

121 fourths

121 fourths
inv. 1

121 fourths
inv. 2

121 fifths

121 fifths
inv. 1

121 fifths
inv. 2

121 sus4

121 sus4
inv. 1

121 sus4
inv. 2

121 sus2

121 sus2
inv. 1

121 sus2
inv. 2

141 Characterizing mode (2)122

Mode (2)122
(binary key: 1111010)

122 sus4

122 sus4
inv. 1

122 sus4
inv. 2

122 sus2

122 sus2
inv. 1

122 sus2
inv. 2

Mode (2)123
(binary key: 1111011)

123 sus4

123 sus4
inv. 1

123 sus4
inv. 2

123 sus2

123 sus2
inv. 1

123 sus2
inv. 2

143 Characterizing mode (2)124

Mode (2)124
(binary key: 1111100)

124 plain

124 seconds

124 seconds
inv. 1

124 seconds
inv. 2

124 thirds

124 thirds
inv. 1

124 thirds
inv. 2

124 fourths

124 fourths
inv. 1

124 fourths
inv. 2

124 fifths

124 fifths
inv. 1

124 fifths
inv. 2

124 sus4

124 sus4
inv. 1

124 sus4
inv. 2

124 sus2

124 sus2
inv. 1

124 sus2
inv. 2

Mode (2)125
(binary key: 1111101)

125 sus4

125 sus4
inv. 1

125 sus4
inv. 2

125 sus2

125 sus2
inv. 1

125 sus2
inv. 2

145 Characterizing mode (2)126

Mode (2)126
(binary key: 1111110)

126 plain

126 seconds

126 seconds
inv. 1

126 seconds
inv. 2

126 thirds

126 thirds
inv. 1

126 thirds
inv. 2

126 fourths

126 fourths
inv. 1

126 fourths
inv. 2

126 fifths

126 fifths
inv. 1

126 fifths
inv. 2

126 sus4

126 sus4
inv. 1

126 sus4
inv. 2

126 sus2

126 sus2
inv. 1

126 sus2
inv. 2

Mode (2)127
(binary key: 1111111)

127 sus4

127 sus4
inv. 1

127 sus4
inv. 2

127 sus2

127 sus2
inv. 1

127 sus2
inv. 2

Mode (3)1
(binary key: 0001)

1 plain

1 seconds

1 seconds
inv. 1

1 seconds
inv. 2

1 thirds

1 thirds
inv. 1

1 thirds
inv. 2

1 fourths

1 fourths
inv. 1

1 fourths
inv. 2

1 fifths

1 fifths
inv. 1

1 fifths
inv. 2

2

1 sus4

1 sus4
inv. 1

1 sus4
inv. 2

1 sus2

1 sus2
inv. 1

1 sus2
inv. 2

Mode (3)2
(binary key: 0010)

2 plain

2 seconds

2 seconds
inv. 1

2 seconds
inv. 2

2 thirds

2 thirds
inv. 1

2 thirds
inv. 2

2 fourths

2 fourths
inv. 1

2 fourths
inv. 2

2 fifths

2 fifths
inv. 1

2 fifths
inv. 2

2 sus4

2 sus4
inv. 1

2 sus4
inv. 2

2 sus2

2 sus2
inv. 1

2 sus2
inv. 2

Mode (3)3
(binary key: 0011)

3 sus4

3 sus4
inv. 1

3 sus4
inv. 2

3 sus2

3 sus2
inv. 1

3 sus2
inv. 2

Mode (3)4
(binary key: 0100)

4 sus4

4 sus4
inv. 1

4 sus4
inv. 2

4 sus2

4 sus2
inv. 1

4 sus2
inv. 2

Mode (3)5
(binary key: 0101)

5 plain

5 seconds

5 seconds
inv. 1

5 seconds
inv. 2

5 thirds

5 thirds
inv. 1

5 thirds
inv. 2

5 fourths

5 fourths
inv. 1

5 fourths
inv. 2

5 fifths

5 fifths
inv. 1

5 fifths
inv. 2

5 sus4

5 sus4
inv. 1

5 sus4
inv. 2

5 sus2

5 sus2
inv. 1

5 sus2
inv. 2

Mode (3)6
(binary key: 0110)

6 sus4

6 sus4
inv. 1

6 sus4
inv. 2

6 sus2

6 sus2
inv. 1

6 sus2
inv. 2

153   Characterizing mode (3)7

Mode (3)7
(binary key: 0111)

7 sus4

7 sus4
inv. 1

7 sus4
inv. 2

7 sus2

7 sus2
inv. 1

7 sus2
inv. 2

Mode (3)8
(binary key: 1000)

8 sus4

8 sus4
inv. 1

8 sus4
inv. 2

8 sus2

8 sus2
inv. 1

8 sus2
inv. 2

Mode (3)9
(binary key: 1001)

9 sus4

9 sus4
inv. 1

9 sus4
inv. 2

9 sus2

9 sus2
inv. 1

9 sus2
inv. 2

Mode (3)10
(binary key: 1010)

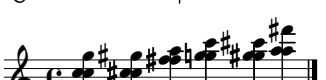10 plain

10 seconds

10 seconds
inv. 1
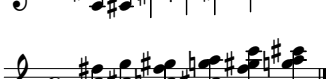
10 seconds
inv. 2

10 thirds

10 thirds
inv. 1

10 thirds
inv. 2

10 fourths

10 fourths
inv. 1

10 fourths
inv. 2

10 fifths

10 fifths
inv. 1

10 fifths
inv. 2

10 sus4

10 sus4
inv. 1

10 sus4
inv. 2

10 sus2

10 sus2
inv. 1

10 sus2
inv. 2

Mode (3)11
(binary key: 1011)

11 sus4

11 sus4
inv. 1

11 sus4
inv. 2

11 sus2

11 sus2
inv. 1

11 sus2
inv. 2

Mode (3)12
(binary key: 1100)

12 sus4

12 sus4
inv. 1

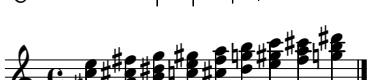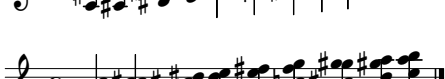12 sus4
inv. 2

12 sus2

12 sus2
inv. 1

12 sus2
inv. 2

Mode (3)13
(binary key: 1101)

13 sus4

13 sus4
inv. 1

13 sus4
inv. 2

13 sus2

13 sus2
inv. 1

13 sus2
inv. 2

Mode (3)14
(binary key: 1110)

14 sus4

14 sus4
inv. 1

14 sus4
inv. 2

14 sus2

14 sus2
inv. 1

14 sus2
inv. 2

161 Characterizing mode (3)15

Mode (3)15
(binary key: 1111)

15 sus4

15 sus4
inv. 1

15 sus4
inv. 2

15 sus2

15 sus2
inv. 1

15 sus2
inv. 2