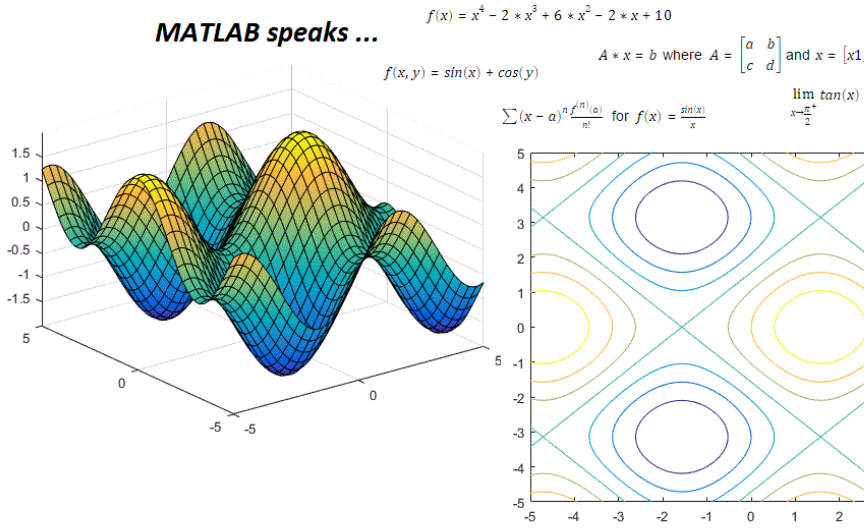
Computational Mathematics in the Symbolic Toolbox



This example provides an overview of the Symbolic Math Toolbox which offers a complete set of tools for compu

- Equations, expressions and functions
- Substitution and solving
- Simplification and manipulation
- Calculus (Differentiation, Integration, Limits, Series)
- Differential Equations
- Linear Algebra
- Graphics

For more details see Getting Started with Symbolic Math Toolbox. For more details on documenting and sharin

Variables, expressions, functions and equations

The Symbolic Math Toolbox supports mathematical expressions, functions and equations. It supports a wide br logarithmic, exponential, and special functions.

Express numbers in exact rational form instead of decimal approximations.

```
sym(pi/6) + sym(pi/4)
```

```
ans =

 5π
 ──
 12
```

```
pi/6 + pi/4
```

```
ans = 1.3090
```

Create symbolic variables, vectors and matrices using sym. Create symbolic expressions and perform mathema

```
syms x y
log(x) + exp(y)
```

ans = $e^{y} + \log(x)$

Create and evaluate functions, find the value of $f(x) = x^4 - 2 * x^3 + 6 * x^2 - 2 * x + 10$ at $x = -5$.

```
syms f(x)
f(x) = x^4-2*x^3+6*x^2-2*x+10
```

f(x) = $x^4 - 2x^3 + 6x^2 - 2x + 10$

```
f(-5)
```

ans = 1045

Create equations using the == operator; find the intersection between lines $y1$ and $y2$ using solve.

```
syms y1 y2
y1 = x+3; y2 = 3*x;
solve(y1 == y2)
```

ans =

$$\frac{3}{2}$$

Make assumptions on symbolic variables. There are 4 solutions to $x^4 = 1$, two real and two complex. Assuming solution.

```
syms x
solve(x^4 == 1)
```

ans =

$$\begin{pmatrix} -1 \\ 1 \\ -i \\ i \end{pmatrix}$$

```
assume(x,'real')
assumeAlso( x > 0)
assumptions(x)
```

ans = $(0 < x \quad x \in \mathbb{R})$

```
solve(x^4 == 1)
```

ans = 1

```
assume(x,'clear')
```

Substitution and Solving

The Symbolic Math Toolbox supports evaluation of mathematical functions by substituting for any part of an exp values, other symbolic variables or expressions, vectors, or matrices. The Symbolic Math Toolbox supports the using solve. It supports solving multi-variate equations, solving inequalities and solving with assumptions. Soluti with high precision variable precision arithmetic.

Make substitutions with your symbolic variables; substitute $x = xo - 1$ into $x^2 + 1$

```
syms x xo
subs(x^2+1,x,xo-1)
```

ans = $\left(\mathrm{xo} - 1\right)^2 + 1$

Substitute multiple values, evaluate $cos(a) + sin(b) - e^{2C}$ by substituting $a = \frac{\pi}{2}, b = \frac{\pi}{4}, c = -1$.

```
syms a b c
subs(cos(a) + sin(b) - exp(2*c), [a b c], [pi/2 pi/4 -1])
```

ans =

$$\frac{\sqrt{2}}{2} - \mathrm{e}^{-2}$$

Create and solve equations; Find the zeros of $9x^2 - 1 = 0$.

```
solve(9*x^2 - 1 == 0)
```

ans =

$$\begin{pmatrix} -\frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

Or solve the general quadratic equation $ax^2 + bx + c = 0$ and use subs to evaluate that solution for $a = 9, b = 0$

```
eqn = a*x^2 + b*x + c == 0;
sol = solve(eqn)
```

sol =

$$\begin{pmatrix} -\frac{b + \sqrt{b^2 - 4ac}}{2a} \\ -\frac{b - \sqrt{b^2 - 4ac}}{2a} \end{pmatrix}$$

```
subs(sol,[a b c],[9 0 -1])
```

ans =

$$\begin{pmatrix} -\frac{1}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

Solve equations numerically using variable precision arithmetic when high accuracy or speed is required.

```
syms f(x)
f(x) = 6*x^7-2*x^6+3*x^3-8;
sol = vpasolve(f)
```

sol =

$$\begin{pmatrix} 1.024024075905370294144831656333 7 \\ -0.880806200517621496392056722983 26 + 0.504340588401275843763318065924 05\,i \\ -0.880806200517621496392056722983 26 - 0.504340588401275843763318065924 05\,i \\ -0.229747952261181639630985706107 24 + 0.967746155767440310739990106951 71\,i \\ -0.229747952261181639630985706107 24 - 0.967746155767440310739990106951 71\,i \\ 0.765208781492784655617293267590 3 + 0.831873314310497132183672393171 21\,i \\ 0.765208781492784655617293267590 3 - 0.831873314310497132183672393171 21\,i \end{pmatrix}$$

Simplification and Manipulation

The Symbolic Math Toolbox supports the simplification and manipulation of mathematical functions. Most mathe different, but mathematically equivalent forms and the Symbolic Math Toolbox supports a number of operations combining terms, rewriting or rearranging expressions and simplification based on assumptions.

Perform polynomial multiplication and simplify the results, show that $(x-1)*(x+1)*(x^2+x+1)*(x^2+1)*($

```
simplify((x - 1)*(x + 1)*(x^2 + x + 1)*(x^2 + 1)*(x^2 - x + 1)*(x^4 - x^2 + 1))
```

ans = $x^{12} - 1$

Apply trigonometric identities to simplifications, for example $sin^2(x) = \frac{1 - cos(2*x)}{2}$

```
combine(2*sin(x)*cos(x) + (1- cos(2*x))/2 + cos(x)^2,'sincos')
```

ans = $\sin(2x) + 1$

Factor or expand multivariate polynomials

```
factor(y^6-x^6)
```

ans = $\begin{pmatrix} -1 & x-y & x+y & x^2+xy+y^2 & x^2-xy+y^2 \end{pmatrix}$

```
f(x) = (x^3 + 7);
expand(f(y-1))
```

ans = $y^3 - 3y^2 + 3y + 6$

Find the functional composition $f(g(x))$

```
f(x) = sqrt(log(x));
g(x) = sqrt(1-x);
h = compose(g,f,x)
```

$$h = \sqrt{1 - \sqrt{\log(x)}}$$

Calculus (Differentiation, Integration, Limits, Series, etc)

The Symbolic Math Toolbox has a full set of calculus tools for applied mathematics. It can perform multivariate s
generate, manipulate and perform calculations with series.

Find the derivative of $\frac{d}{dx}\left(sin(x)\right)$.

```
diff(sin(x))
```

$$ans = \cos(x)$$

Find the derivative of $\frac{d}{dx}\left(x^2 + sin(2x^4) + 1\right)$ using the chain rule.

```
diff(x^2+sin(2*x^4)+1,x)
```

$$ans = 2x + 8x^3\cos(2x^4)$$

Find the indefinite integral $\int f(x)\,dx$ for $f(x) = e^{\frac{-x^2}{2}}$.

```
int(exp(-x^2/2),x)
```

ans =

$$\frac{\sqrt{2}\ \sqrt{\pi}\ \mathrm{erf}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Find the definite integral $\int_a^b f(x)\,dx$ for $f(x) = x\log(1 + x)$ from *0* to *1*.

```
int(x*log(1+x),0,1)
```

ans =

$$\frac{1}{4}$$

Show that $\frac{sin(x)}{x} = 1$ at $x = 0$ by computing the taylor series expansion $\sum (x-a)^n \frac{f^{(n)}(a)}{n!}$ for $f(x) = \frac{sin(x)}{x}$ arc

```
syms x
```

```
T = taylor(sin(x)/x)
```

```
T =

   x^4/120 - x^2/6 + 1
```

```
subs(T,x,0)
```

```
ans = 1
```

Show that $tan(x)$ is discontinuous at $x = \frac{\pi}{2}$ by showing that the left and right limits are not equal. $\lim_{x \to \frac{\pi}{2}^+} tan(x) \neq$

```
limit(tan(x),x,pi/2,'left')
```

```
ans = ∞
```

```
limit(tan(x),x,pi/2,'right')
```

```
ans = -∞
```

```
limit(tan(x),x,pi/2)
```

```
ans = NaN
```

Differential Equations

The Symbolic Math Toolbox can analytically solve systems of ordinary differential equations using `dsolve`.

Solve the first order ODE $\frac{dy}{dx} = -ay$

```
syms a b y(x)
dsolve(diff(y) == -a*y)
```

```
ans = C_4 e^(-a x)
```

Solve the same ODE with the intial condition $y(0) = b$

```
dsolve(diff(y)== -a*y,y(0)==b)
```

```
ans = b e^(-a x)
```

Solve the system of coupled first order ODEs $\frac{dx}{dt} = y$ and $\frac{dy}{dt} = -x$

```
syms x(t) y(t)
z = dsolve(diff(x) == y, diff(y) == -x);
disp([z.x;z.y])
```

$$\begin{pmatrix} C_7\cos(t) + C_6\sin(t) \\ C_6\cos(t) - C_7\sin(t) \end{pmatrix}$$

Linear Algebra

The Symbolic Math Toolbox can work with symbolic vectors and matricies. It can compute eigenvalues and eige

Perform the matrix vector multiplication $A * x = b$ where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $x = [x1, x2]$

```
syms a b c d
syms x1 x2
x = [x1; x2];
A = [a b ; c d];
b = A*x
```

```
b =
```

$$\begin{pmatrix} a\,x_1 + b\,x_2 \\ c\,x_1 + d\,x_2 \end{pmatrix}$$

Find the determinant of A

```
det(A)
```

```
ans = a d - b c
```

Find the eigenvectors of A

```
lambda = eig(A)
```

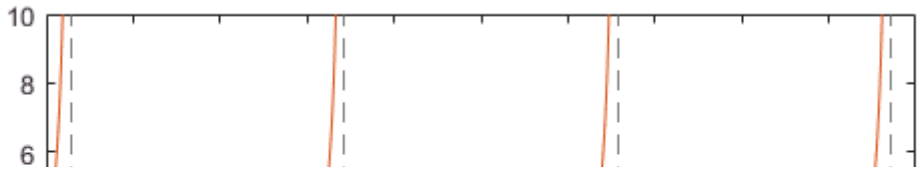
```
lambda =
```

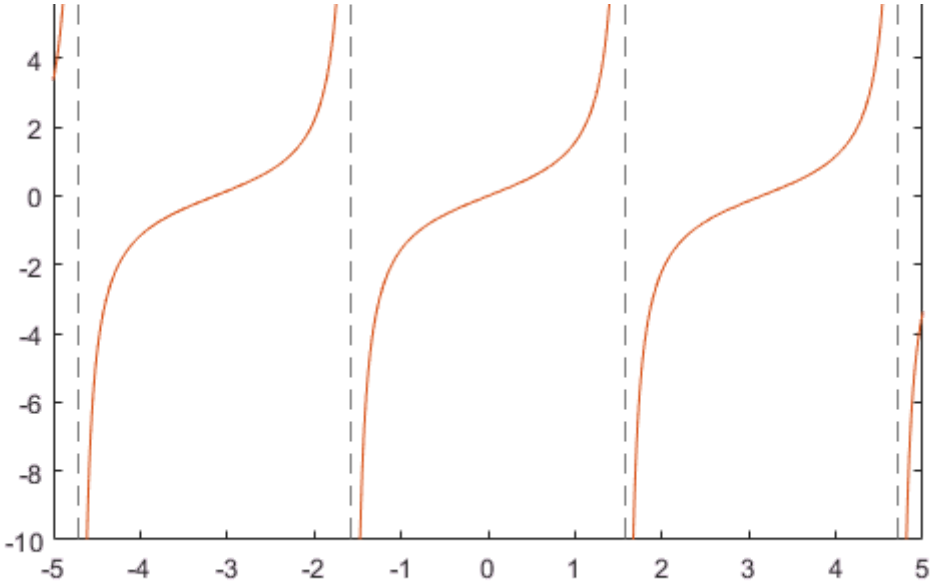
$$\begin{pmatrix} \frac{a}{2} + \frac{d}{2} - \frac{\sqrt{a^2 - 2\,a\,d + d^2 + 4\,b\,c}}{2} \\ \frac{a}{2} + \frac{d}{2} + \frac{\sqrt{a^2 - 2\,a\,d + d^2 + 4\,b\,c}}{2} \end{pmatrix}$$

Graphics

The Symbolic Math Toolbox supports analytical plotting in 2D and 3D.

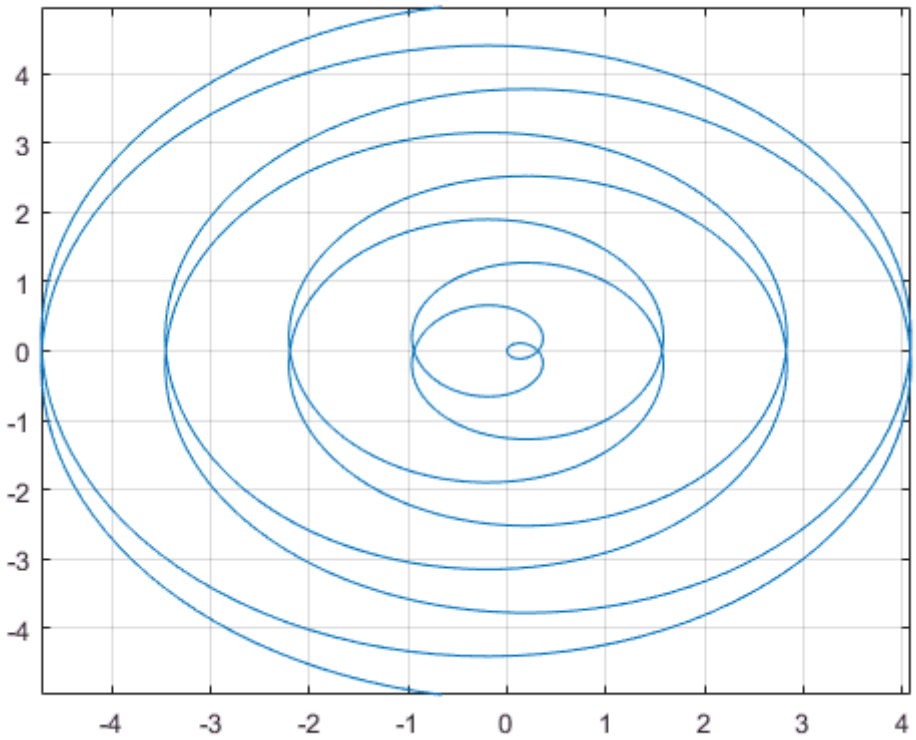
```
fplot(tan(x))
```





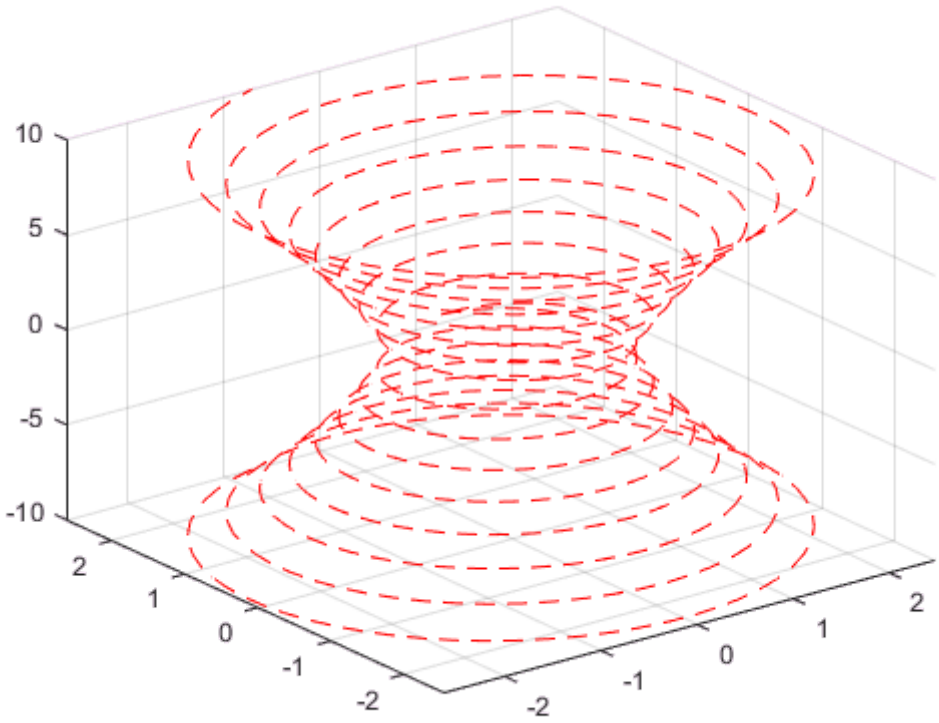
Plot the parametric curve $x(t) = t * sin(5t)$ and $y(t) = t * cos(5t)$.

```
syms t
x = t*sin(5*t);
y = t*cos(5*t);
fplot(x, y)
grid on
```



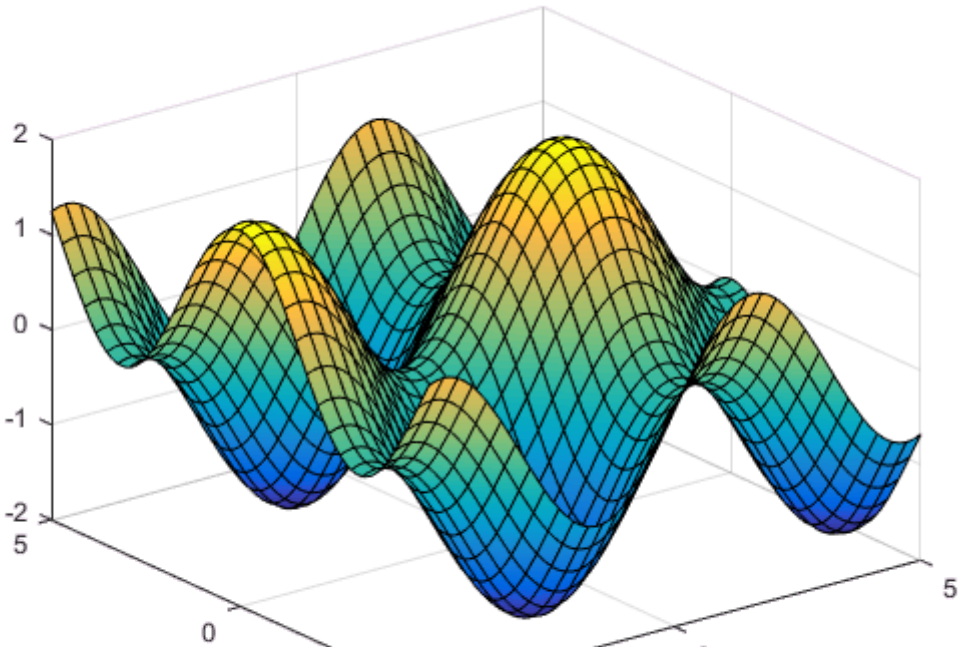
Plot the 3D parametric curve $x(t) = e^{\frac{|t|}{10}} sin(5|t|)$, $y(t) = e^{\frac{|t|}{10}} cos(5|t|)$ and $z(t) = t$ from [-10,10] with a dashed

```
syms t
xt = exp(abs(t)/10).*sin(5*abs(t));
yt = exp(abs(t)/10).*cos(5*abs(t));
zt = t;
h = fplot3(xt,yt,zt, [-10,10],'--r');
```



Plot the 3D surface $f(x, y) = sin(x) + cos(y)$.

```
syms x y
fsurf(sin(x) + cos(y))
```



Plot the 2D contours of the same surface.

```
fcontour(sin(x) + cos(y))
```