

Control of Double-Inverted Pendulum

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Objectives

- ▷ System Identification
- ▷ System Design
- ▷ Synthesis of Controllers
 - Integral
 - Integral + Oscillator
 - Repetitive + Feedback

Dynamics of system

$$x_1 = \tilde{\theta}, x_2 = \tilde{\omega}, u = \tilde{i}, w = \tilde{T_d}$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_u \end{bmatrix} u \begin{bmatrix} 0 \\ K_w \end{bmatrix} w$$

$$\omega_n^2 = \frac{mgl_c \cos \theta_e}{J_{pend}}, 2\zeta\omega_n = \frac{K_m^2}{J_{pend}R}, K_u = \frac{K_m V_s}{J_{pend}R}, K_w = \frac{-1}{J_{pend}}$$

Uncontrolled, Observed

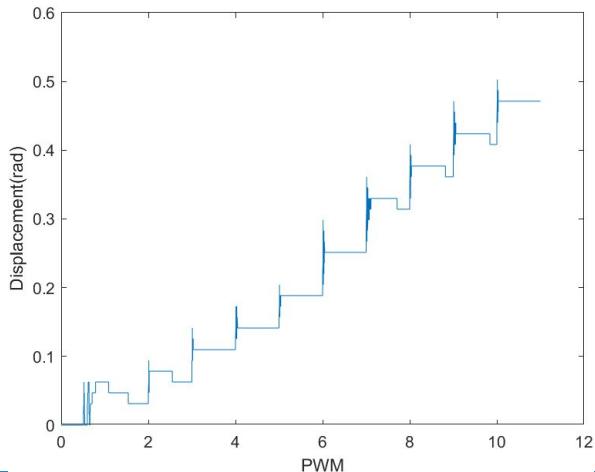
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A - k_c B_d C & k_c B_d C \\ k_c B_d C & A - k_c B_d C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u - \begin{bmatrix} B_d \\ 0 \end{bmatrix} T_w$$

Controlled, Unobserved

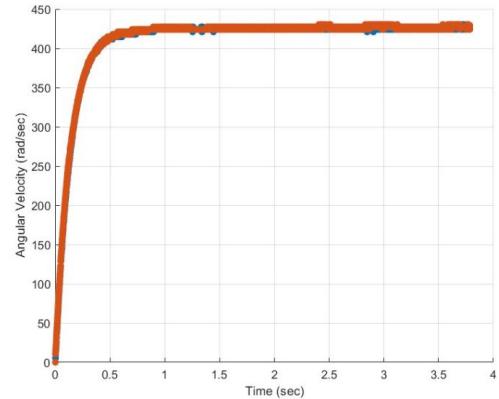
$$y = [C \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

System Identification

- Damping b: Negligible, assumed to be zero.
- Inductance L: Negligible, assumed to be zero.
- Resistance R: Measured with multimeter, $8.985\ \Omega$
- Supply Voltage V_s : Measured with multimeter, $10.72V$
- Torque constant k_t and Inertia J: first order data fitting
- Spring constant k_c : $0.095\ Nm/rad$

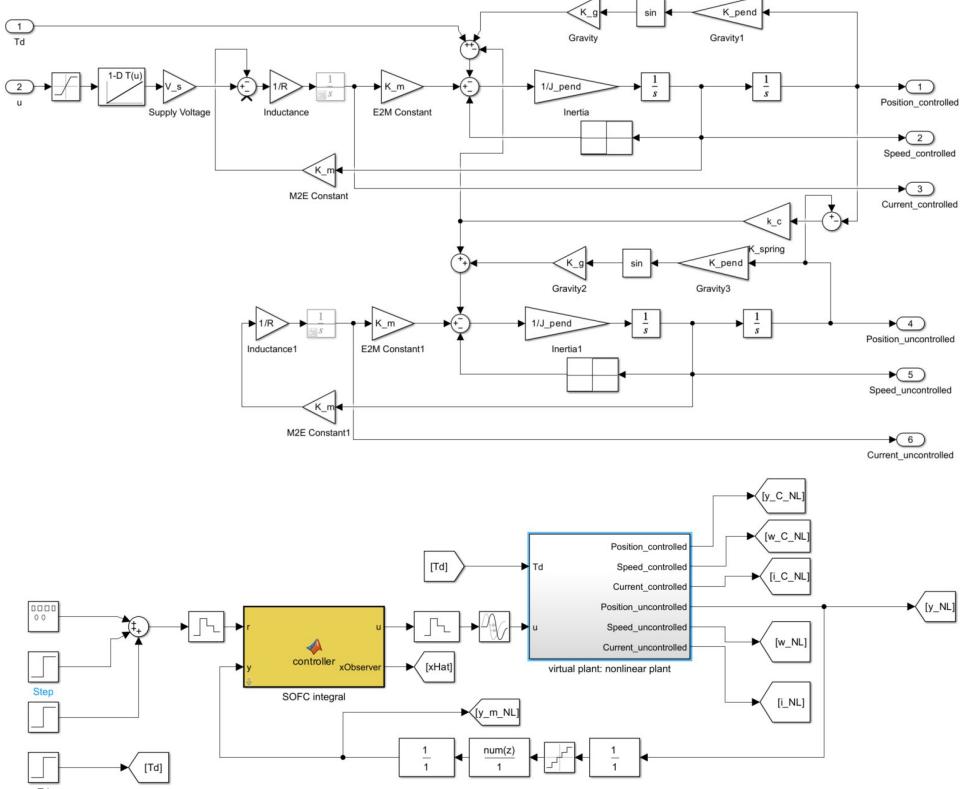


PWM percent	Displacement(rad)
0	0
10	0.031
20	0.0628
30	0.110
40	0.141
50	0.188
60	0.251
70	0.314
80	0.361
90	0.408
100	0.471

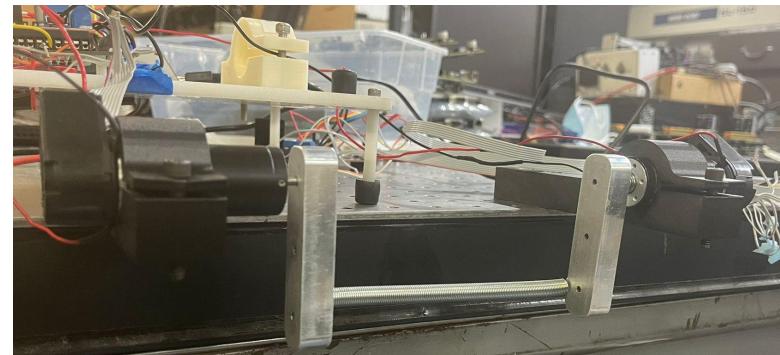


$$\frac{\frac{K_m V_s}{bR+K_m^2}}{\frac{J_{rotor}Rs}{bR+K_m^2} + 1} = \frac{\kappa}{\tau s + 1}$$

Simulation Simulink



Hardware



Integral Controller

$$AA = \begin{bmatrix} A_d - B_d * K_{sf} & B_d * K_{sf} \\ 0 & A_d - L_{pred} * C_d \end{bmatrix} BB = \begin{bmatrix} B_d * N & B_d & B_w \\ 0 & B_d & B_w \end{bmatrix}$$

For Oscillator, small non zero values of order 10^{-6} are used.

$$CC = \begin{bmatrix} C_d & 0 \\ -K_{sf} & -K_{sf} \\ -C_d & 0 \end{bmatrix} DD = \begin{bmatrix} 0 & 0 & 0 \\ N & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Q_pred= [15, 0, 0, 0;
0, 5000, 0, 0;
0, 0, 0, 0;
0, 0, 0, 0],

R_pred = [1]

Q_cont = [0, 0, 0, 0, 0;
0, 0, 0, 0, 0;
0, 0, 1000, 0, 0;
0, 0, 0, 0, 0;
0, 0, 0, 0, 0.1];
R_cont = [1]

Integral

```
L_pred = [0.9962, 5.1994, 0.0045,  
1.7757]
```

```
K_aug = [0.881, 0.0393, 0.9005,  
0.0369, 0.0098];
```

```
Poles = [0.9961 ± 0.0572i, 0.9788,  
0.9884 ± 0.0117i, 0.0098, 0.9894,  
0.9968 ± 0.039i]
```

Integral+Oscillator

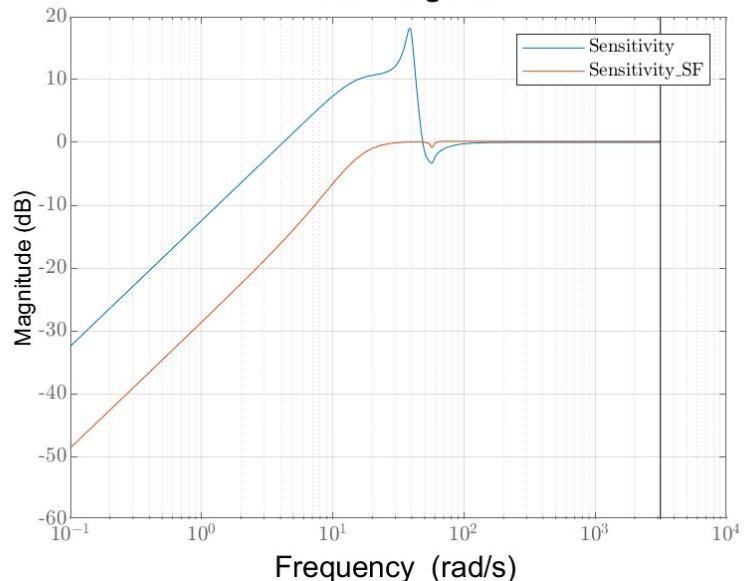
```
L_pred = [0.9962, 5.1994, 0.0045,  
1.7757]
```

```
K_aug = [1.0251, 0.0433, 1.0249, 0.0396,  
0.0087, -0.0053, 0.0053];
```

```
Poles = [0.9962 ± 0.0572i, 0.9794,  
0.9891 ± 0.011i, 0.9975 ± 0.0059i,  
0.0098, 0.9894, 0.9968 ± 0.039i]
```

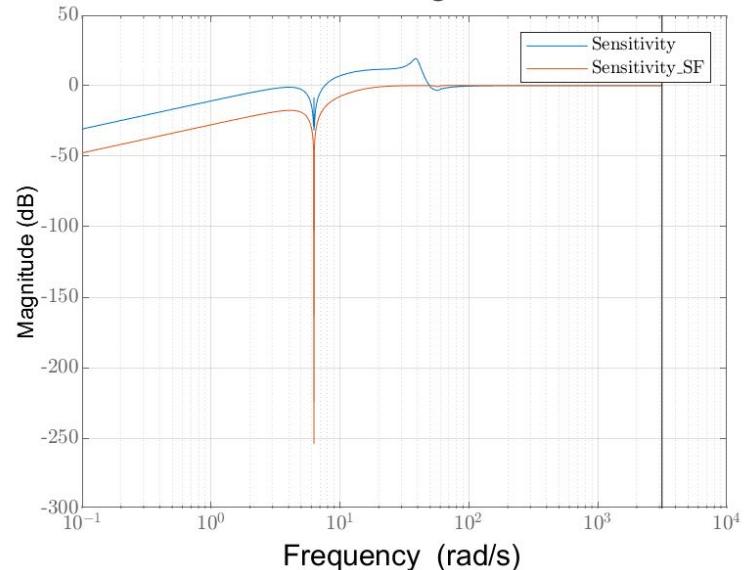
Sensitivity

Bode Diagram



Integral

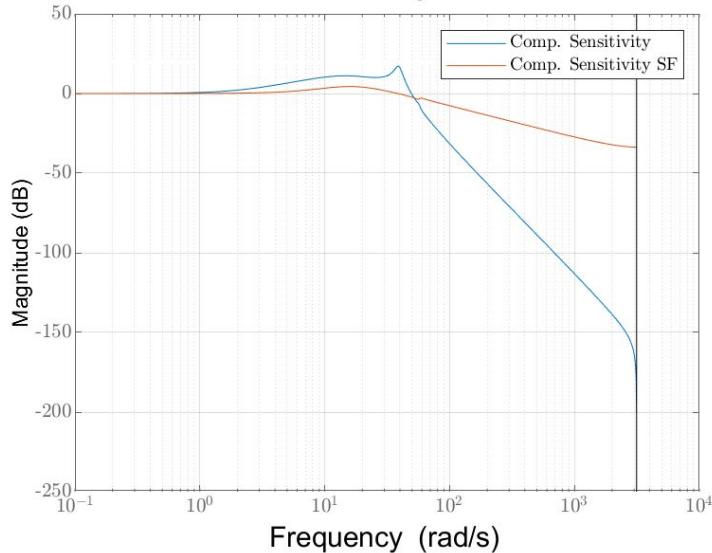
Bode Diagram



Integral + Oscillator

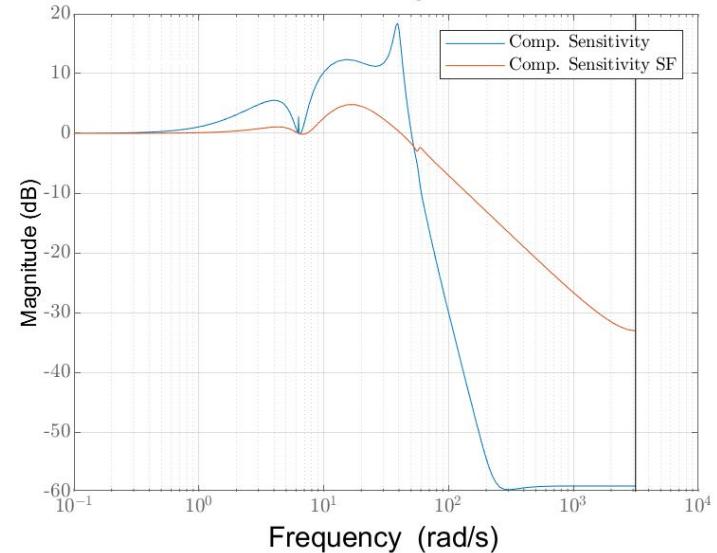
Complementary Sensitivity

Bode Diagram



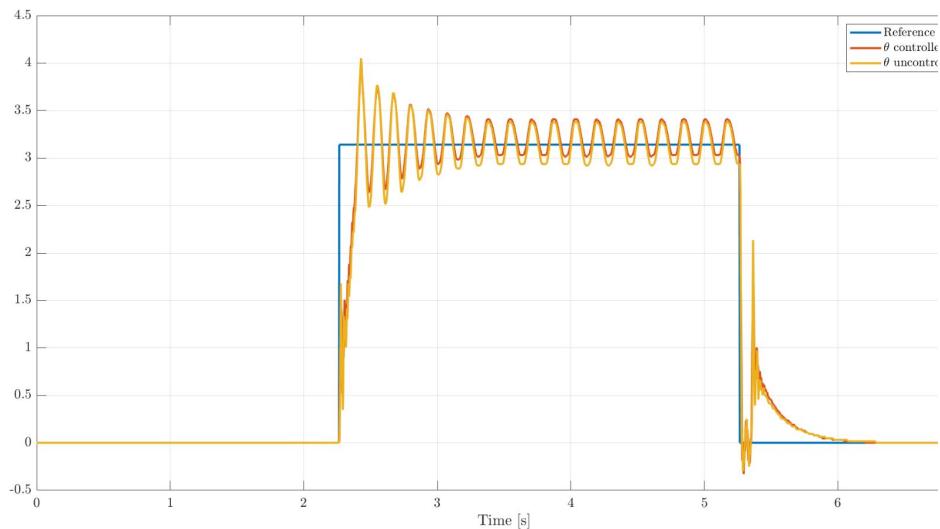
Integral

Bode Diagram

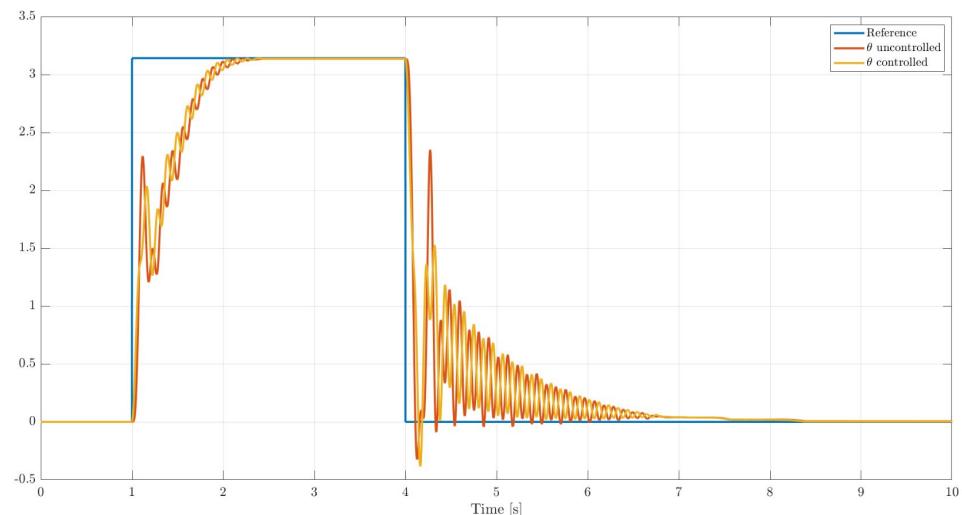


Integral + Oscillator

Swinging - up and down

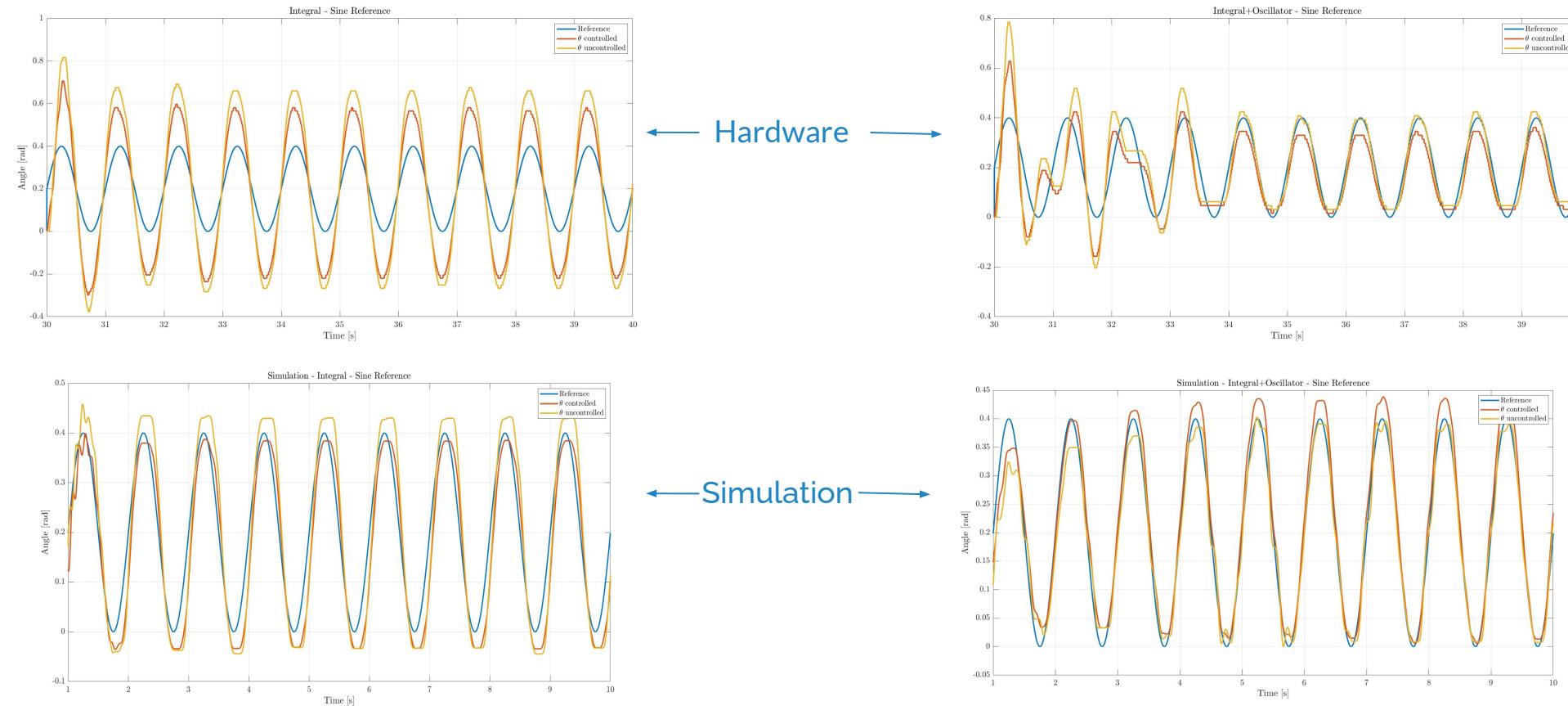


Hardware

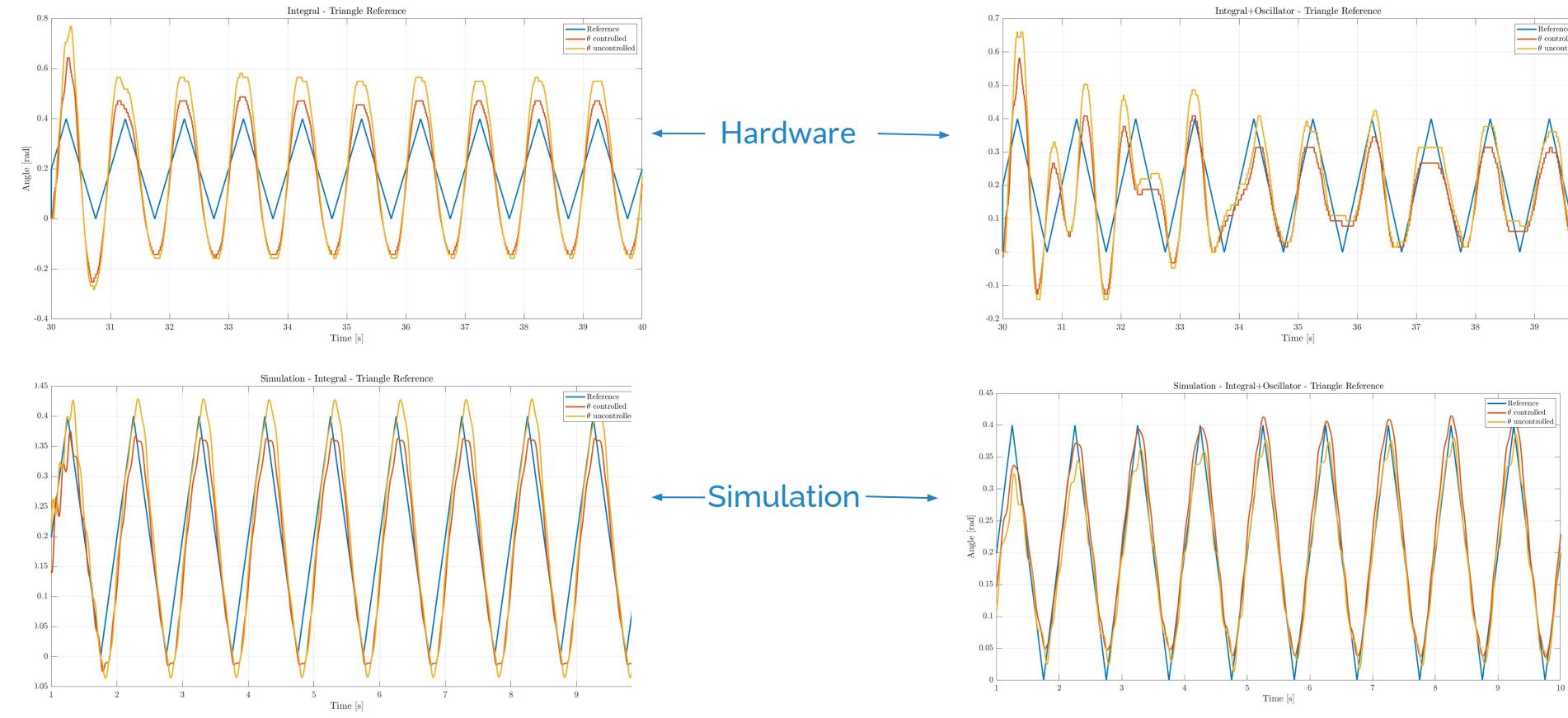


Simulation

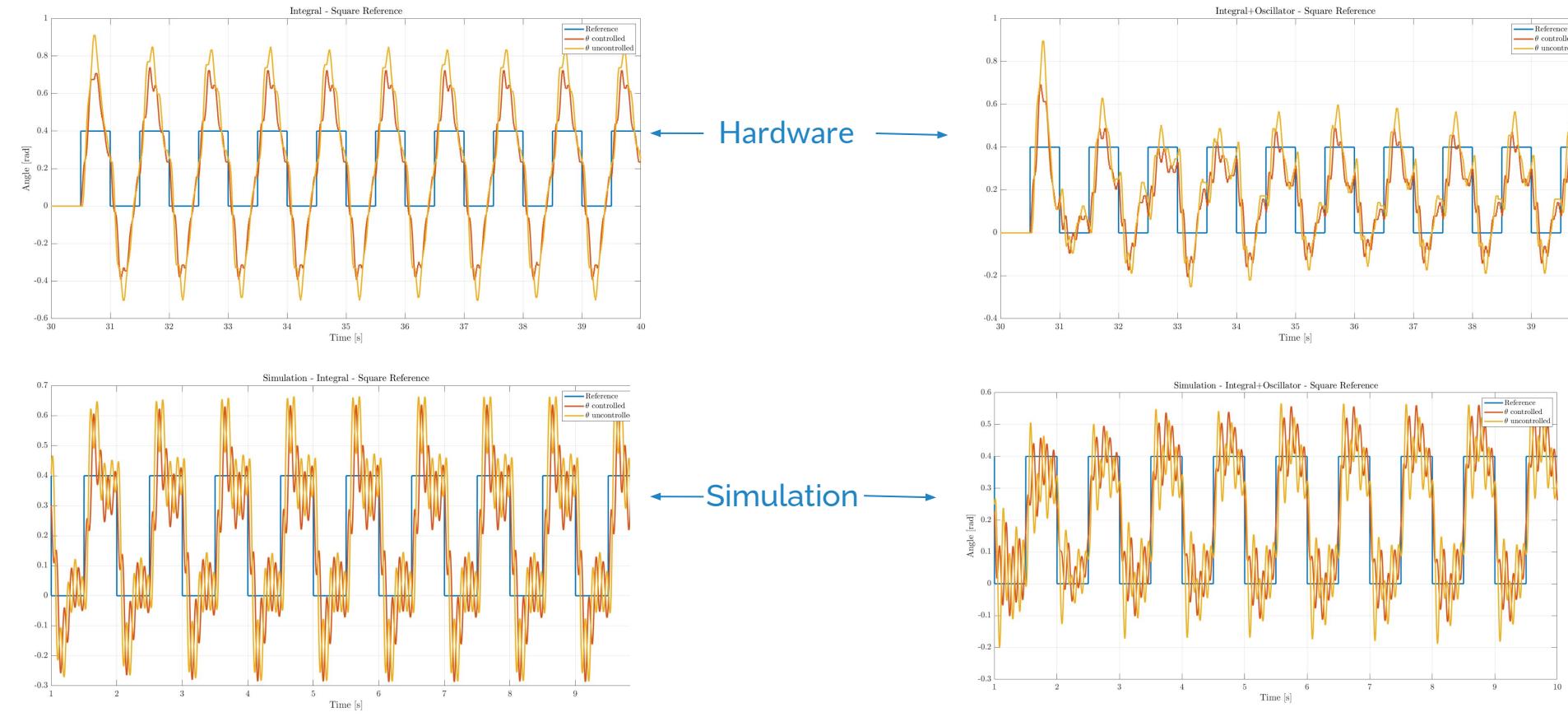
Integral vs. Integral+Oscillator



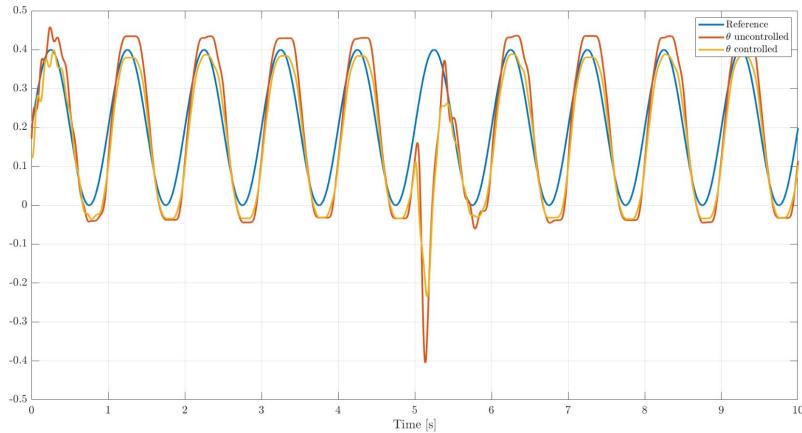
Integral vs. Integral+Oscillator



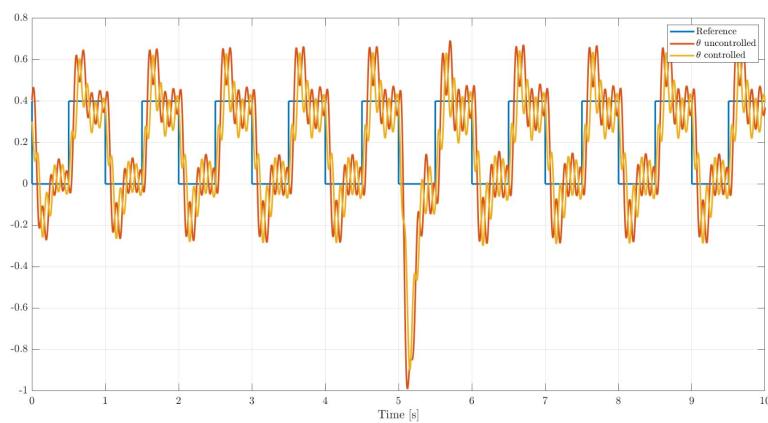
Integral vs. Integral+Oscillator



Disturbance Rejection - Integral

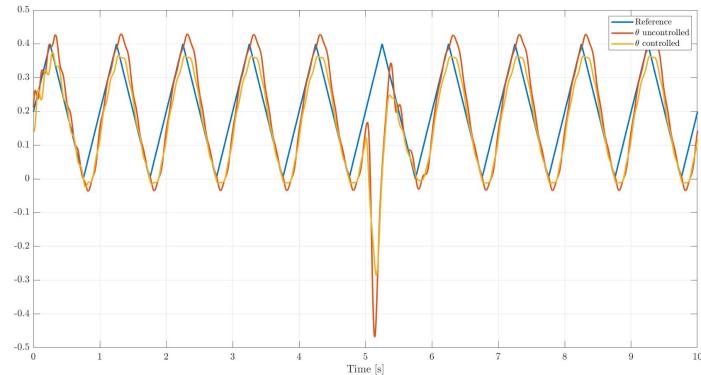


Sine

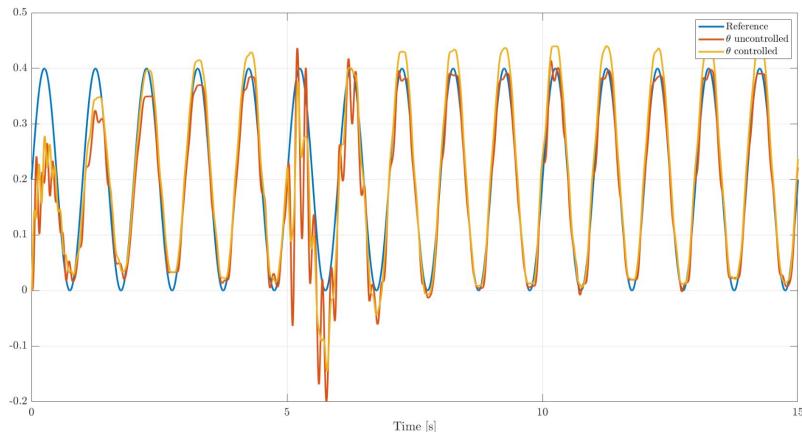


Square

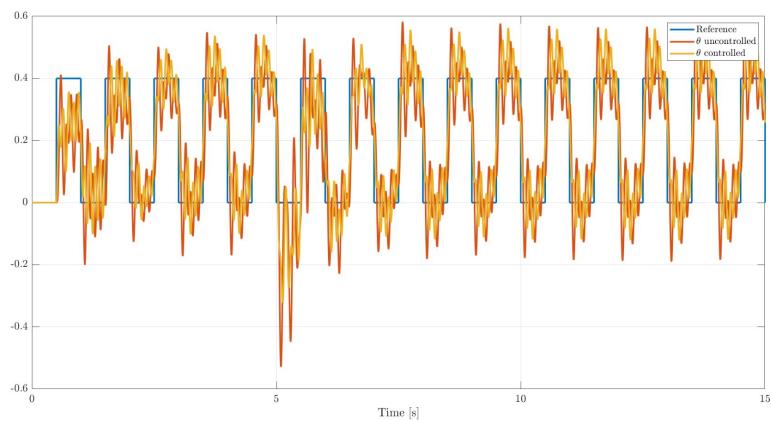
Triangle



Disturbance Rejection - Integral + Oscillator

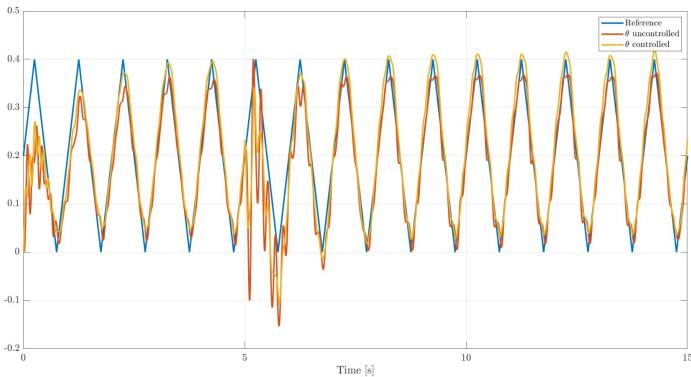


Sine

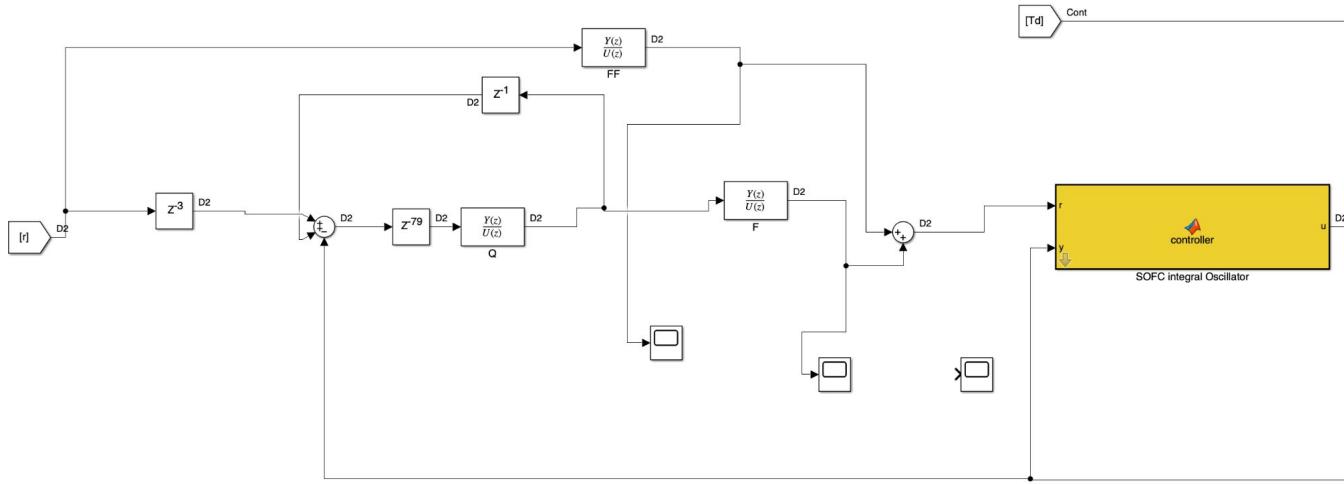


Square

Triangle



Repetitive + FeedForward Control



Repetitive

$$\left[\frac{z + 2 + \frac{1}{z}}{4} \right]^{120}$$

QFilter

$$\left[\frac{z + 2 + \frac{1}{z}}{4} \right]^{15}$$

Additional FeedForward
Lowpass Filter

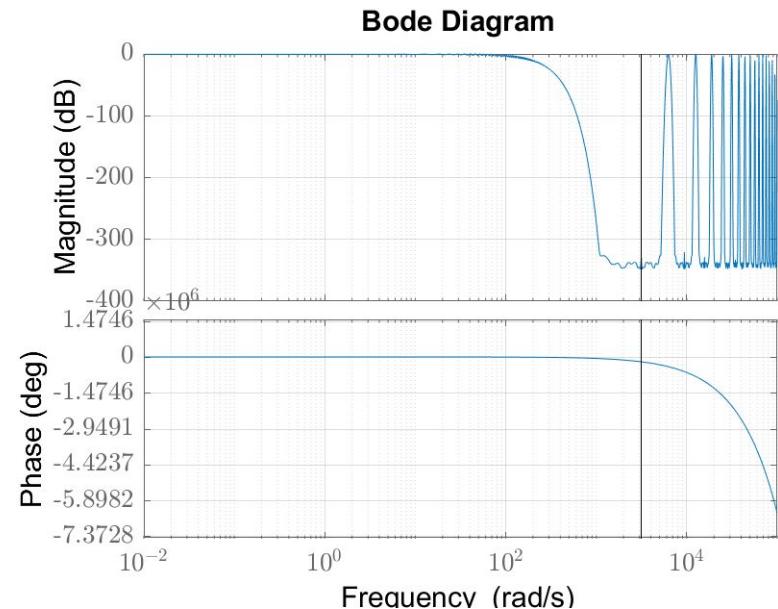
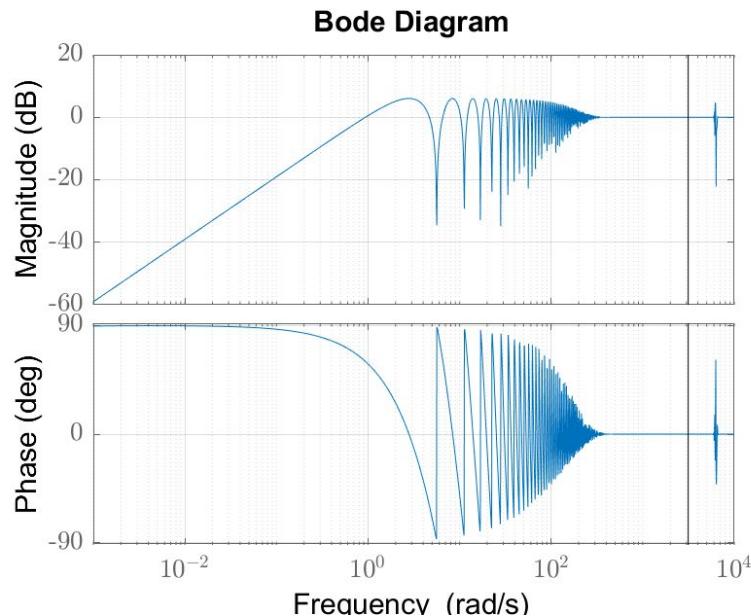
StablePlantTF
$$\frac{7.333e - 07z^6 + 5.858e - 06z^5 - 1.386e - 05z^4 - 2.404e - 08z^3 + 1.382e - 05z^2 - 5.797e - 06z - 7.239e - 07}{z^7 - 6.658z^6 + 18.97z^5 - 30.01z^4 + 28.44z^3 - 16.15z^2 + 5.088z - 0.686}$$

$$L_{pred} = [0.2023, 17.8975, 0.0162, 1.6858]$$

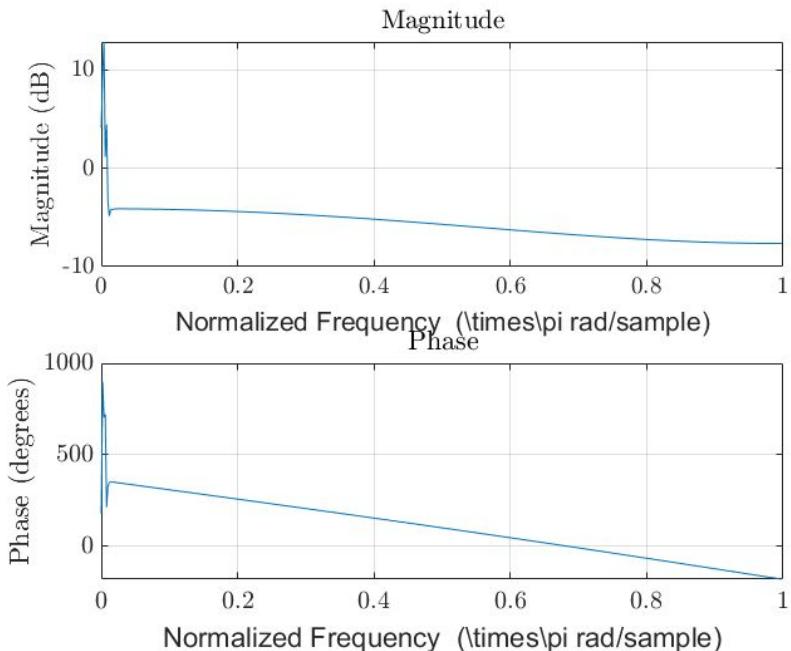
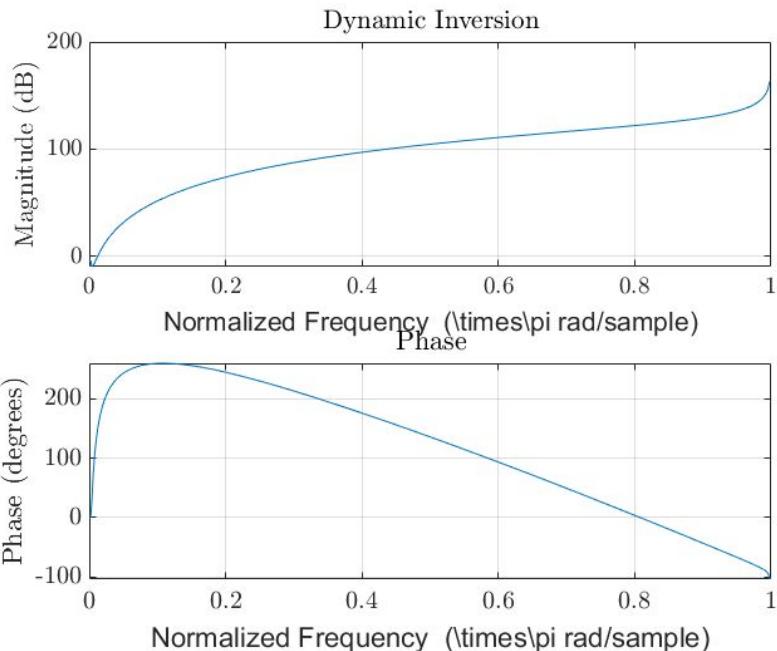
$$K_{aug} = [-9.7870, 0.225, 21.9725, 0.2909, 0.0301, -0.0438, 0.0440];$$

$$Poles = [0.993 + 0.006i, 0.993 - 0.006i, 0.991, 0.991, 0.765, 0.961, 0.961]$$

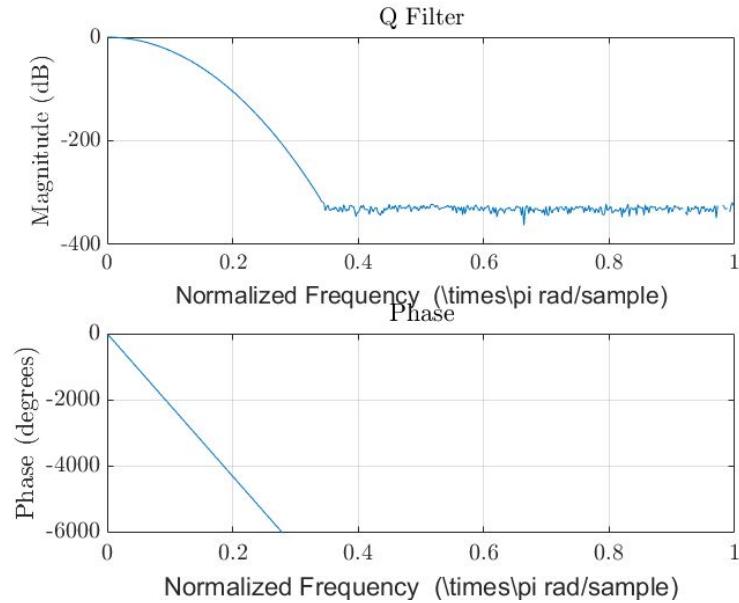
Sensitivity & Complementary Sensitivity



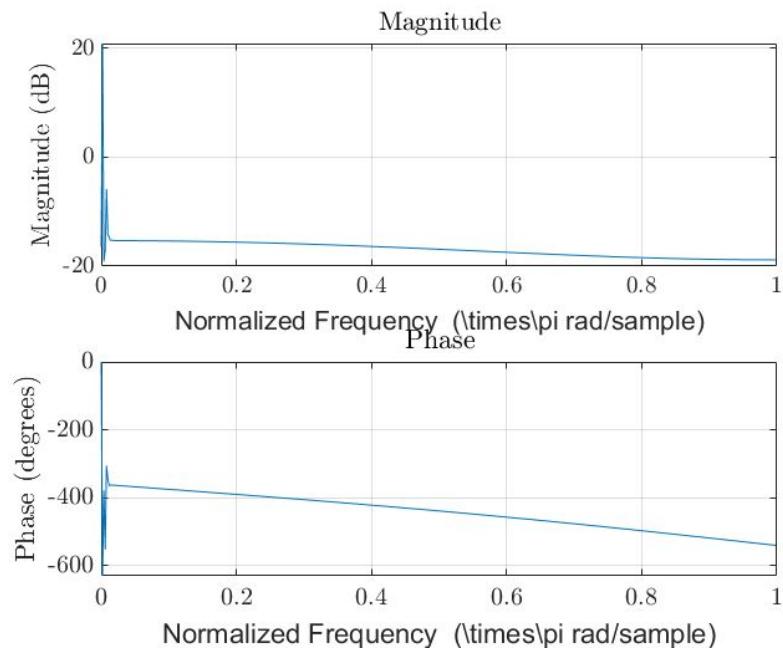
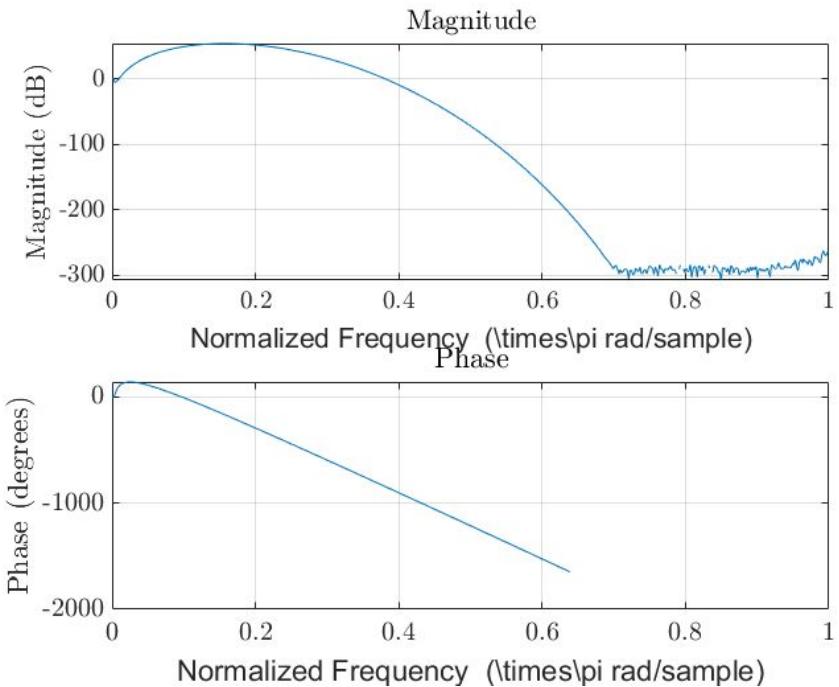
Dynamic Inversion



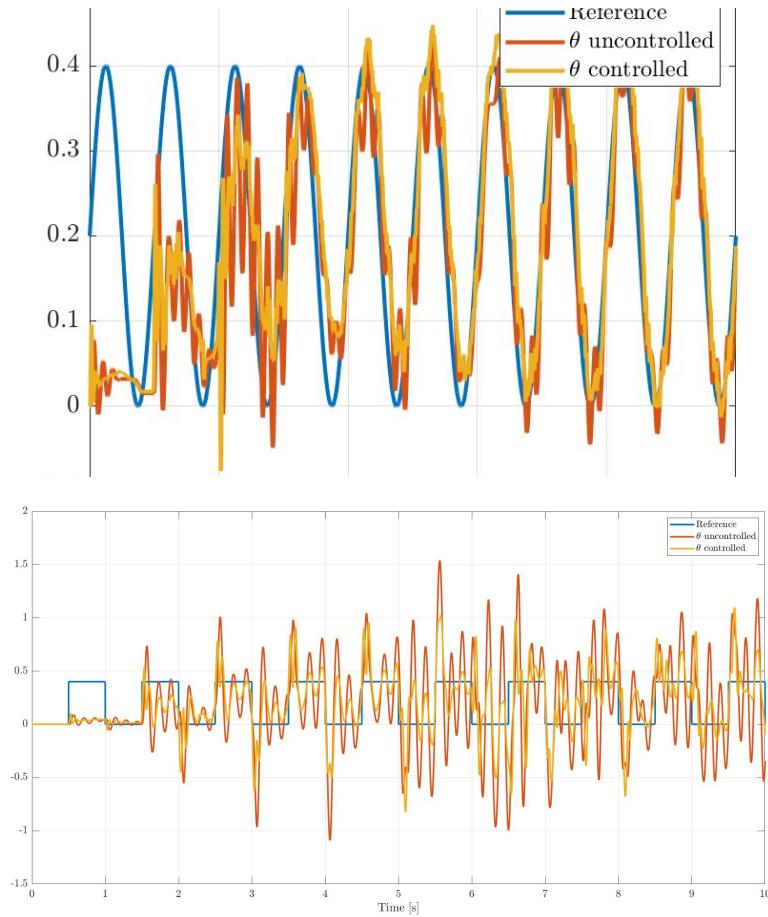
Disturbance Model - Q Filter



Feedforward Filter



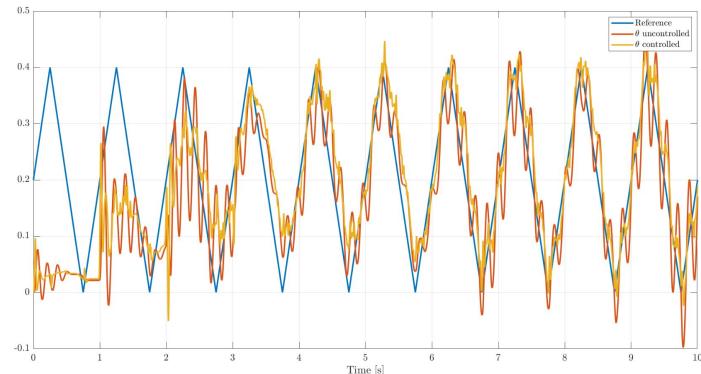
Repetitive + FeedForward



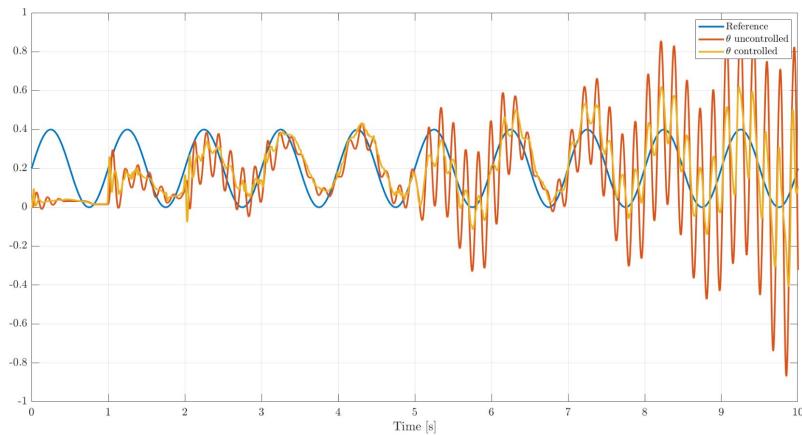
← Sine

Triangle →

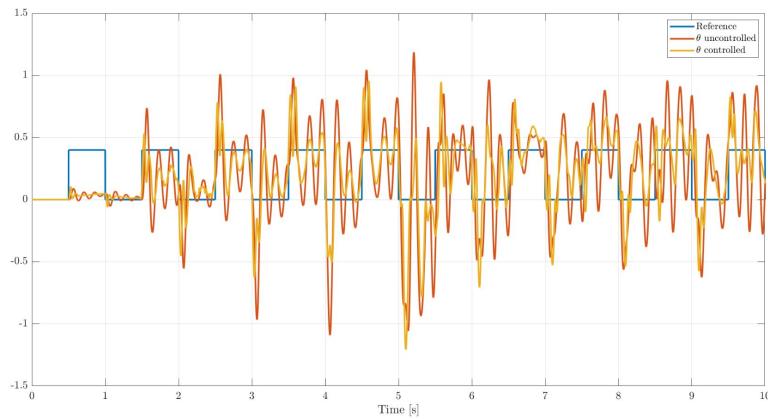
← Square



Disturbance Rejection - Repetitive + FeedForward

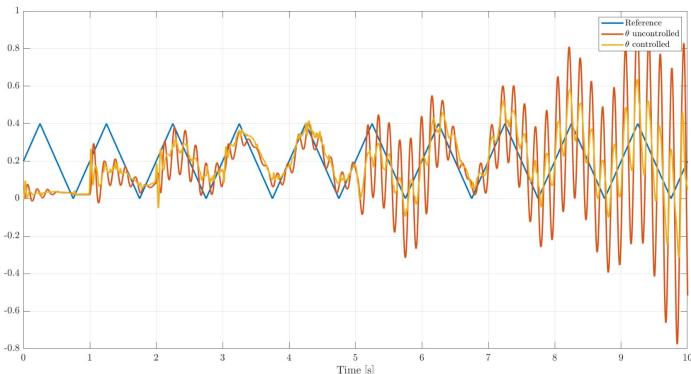


Sine



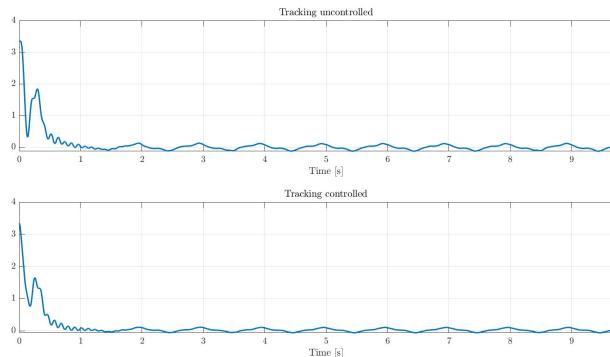
Square

Triangle

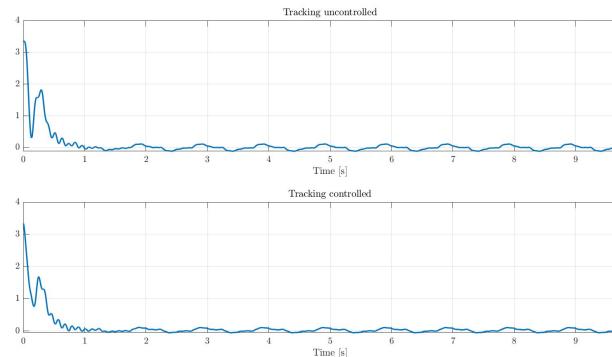


Tracking error - steady state - Integral

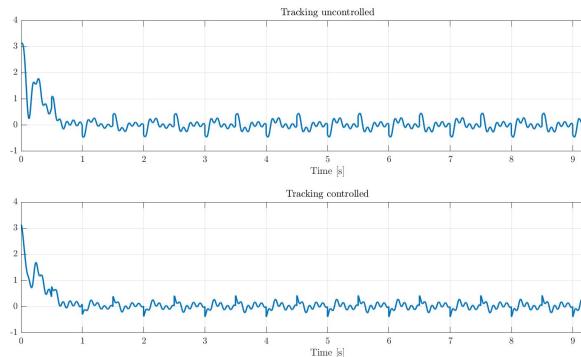
	Controlled		Uncontrolled	
	Mean	Standard Deviation	Mean	Standard Deviation
Sine	0.0226	0.0467	-6.477e-4	0.0668
Triangle	0.0238	0.0471	-1.6052e-5	0.0639
Square	0.0230	0.1335	-2.39e-4	0.1847



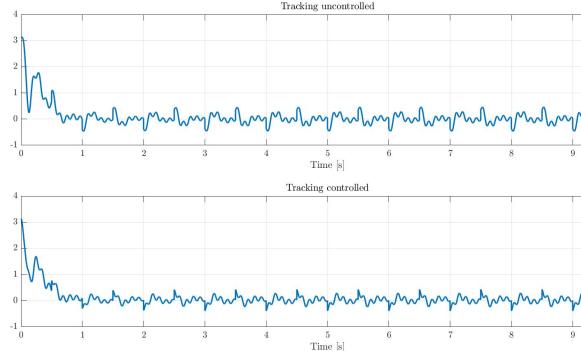
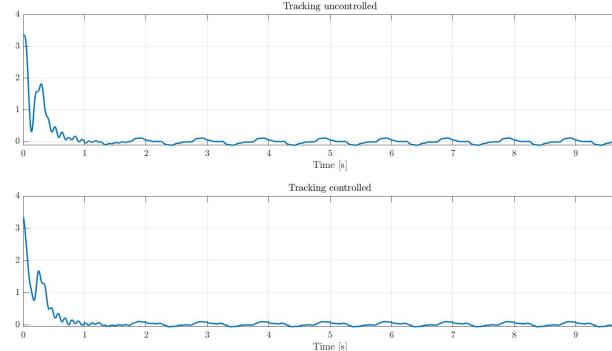
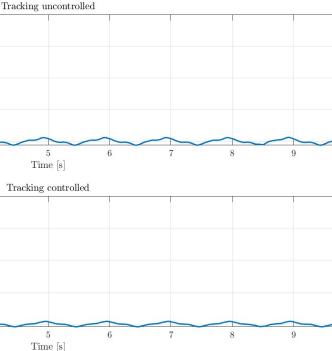
Sine



Triangle

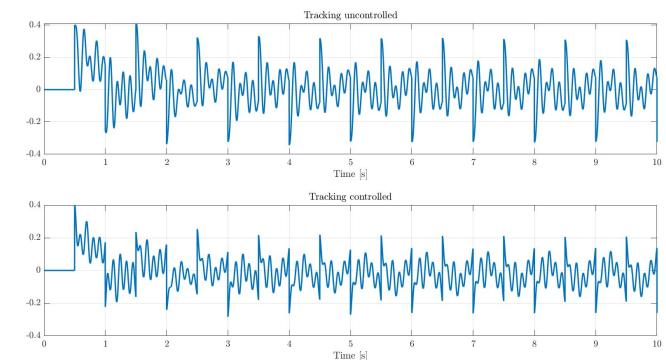
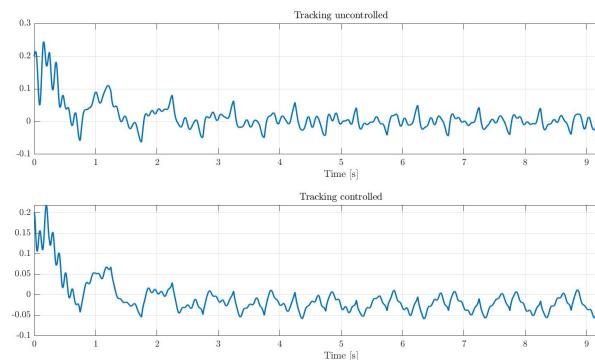
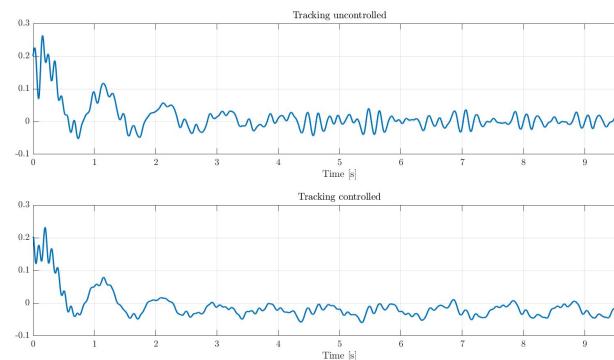


Square



Tracking error - Steady State - Integral + Oscillator

	Controlled		Uncontrolled	
	Mean	Standard Deviation	Mean	Standard Deviation
Sine	-0.0227	0.0160	0.0012	0.0122
Triangle	-0.0241	0.0175	-2.39e-4	0.0163
Square	-0.0239	0.0783	-3.98e-4	0.1121



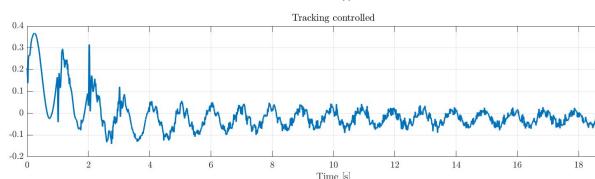
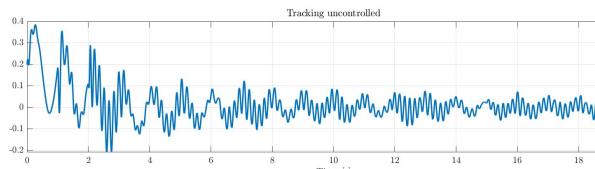
Sine

Triangle

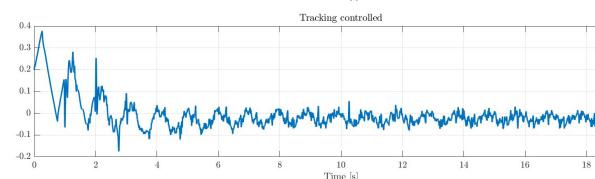
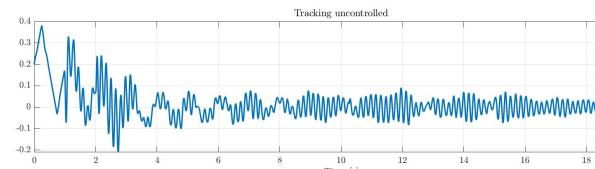
Square

Tracking error - Steady State - Repetitive

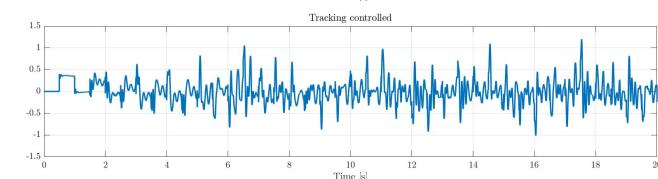
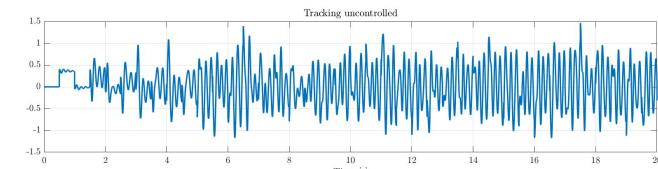
	Controlled		Uncontrolled	
	Mean	Standard Deviation	Mean	Standard Deviation
Sine	-0.0234	0.0276	3.11e-4	0.0334
Triangle	-0.0232	0.018	-8.61e-4	0.04255
Square	-0.0245	0.2836	-0.0017	0.5110



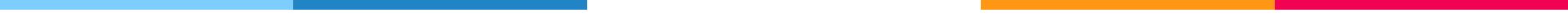
Sine



Triangle



Square



Acknowledgements:

- ▷ Prof. Tsao, for guiding us throughout the course
- ▷ Group D and E for collaborating with hardware