Machine Learning Homework 4
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1.

(1)

$$X = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}, \quad Y = \begin{bmatrix} 6 \\ 10 \\ 16 \end{bmatrix}$$

(a) $\hat{Y_i} = \theta_0 + \theta_1 x_i$

(b) initial $(\theta_0, \theta_1) = (0, 0)$

θ_1, θ_2 →

Error $= \sum \varepsilon_i^2 = \sum (Y_i - \theta_0 - \theta_1 x_i)^2$

$= 392 + 3\theta_0^2 + 46\theta_1^2 + 20\theta_0\theta_1 - 64\theta_0$
$\quad - 264\theta_1$

Iter 1

fix $\theta_0 = 0$

∴ Error$(\theta_1) = 392 + 46\theta_1^2 - 264\theta_1$

$\dfrac{\partial \text{Error}}{\partial \theta_1} = 0$

$\Rightarrow \theta_1 = \dfrac{51}{92} \approx 0.56$

fix $\theta_1 = 0$

then similarly,

$\theta_0 = 10.6$

∴ $(\theta_0, \theta_1) = \text{out}, (10.6, 0.56)$ **Ans**

Iter 2

fix $\theta_0 = 10.6 \Rightarrow \theta_1 = 0.95$

fix $\theta_1 = 0.56 \Rightarrow \theta_0 = 8.83$

$(\theta_0, \theta_1) = (0.95, 8.83)$ **Ans**

Iter 3 similarly,

$(\theta_0, \theta_1) = \approx (7.1, 1.2)$ **Ans**

(b) initially , $(\theta_0, \theta_1) = (0, 0)$.

$\quad\quad\quad \alpha = 0.01$.

It. 1

$\quad\quad (\theta_0, \theta_1) = (0, 0)$

$\quad\quad \theta_0 = \theta_0 - \cdot \alpha \left(\dfrac{\partial}{\partial \theta_0} \; Error \right)$

$\quad\quad\quad = 0.12$

$\quad\quad \theta_1 = \theta_1 - \alpha \; \dfrac{\partial}{\partial \theta_1} \; Error.$

$\quad\quad\quad = \theta - 0.01 \times$

$\quad\quad\quad = 0.12$

$\quad\quad (\theta_0, \theta_1) = (0.12, 0.12)$

It. 2

$\quad\quad$ Simmilarly,

$\quad\quad (\theta_0, \theta_1) = (0.34, 0.72)$.

It. 3

$\quad\quad (\theta_0, \theta_1) = (0.62, 1.8)$

(c) Normal Equation

$\quad\quad \delta^2 = \lambda = 1.$

$\quad\quad \theta = \left(X^T X + \lambda I' \right)^{-1} X^T Y$

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 6 \end{bmatrix}, \quad T = \begin{bmatrix} 6 \\ 10 \\ 16 \end{bmatrix}$$

$$\therefore \quad X^T X + \lambda I' = \begin{bmatrix} 4 & 10 \\ 10 & 47 \end{bmatrix}$$

$$\left[X^T X + \lambda I' \right]^{-1} = \frac{1}{32} \left[\quad \right.$$

$$= \frac{1}{88} \begin{bmatrix} 47 & -10 \\ -10 & 4 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \\ 16 \end{bmatrix}$$

$$= \begin{bmatrix} 32 \\ 132 \end{bmatrix}$$

$$\therefore \quad \theta = \frac{1}{88} \begin{bmatrix} 47 & -10 \\ -10 & 4 \end{bmatrix} \begin{bmatrix} 32 \\ 132 \end{bmatrix}$$

$$\begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 2.09 \\ 2.363 \end{bmatrix} \quad Ans$$

2. Link- https://gist.github.com/iMshivji/8f1b06f0fb8b363acc9915ea5e6773f0
3. Link https://gist.github.com/iMshivji/b7264893e27d9c9326f6f1ce750b680d
4. Link https://gist.github.com/iMshivji/9ed392bdc20eaee0d4cfcc2ba03ce759
5. https://gist.github.com/iMshivji/22d42858d837283d88a3d793cd354dfa