CircularHole.nb

$$In[2] := ClearAll[ux, \phi, x, y];$$

$$In[5] := \phi = ((x^2 - y^2) / (x^2 + y^2) * (x^2 + y^2 + R^4 / (x^2 + y^2) - 2R^2) + x^2 + y^2 - R^2 * Log[x^2 + y^2]) / 4$$

$$\textit{Out[5]} = \frac{1}{4} \left( x^2 + y^2 + \frac{(x^2 - y^2) \left( -2 R^2 + x^2 + y^2 + \frac{R^4}{x^2 + y^2} \right)}{x^2 + y^2} - R^2 \text{Log}[x^2 + y^2] \right)$$

In[7]:= 
$$\sigma yy = Simplify[D[\phi, x, x]]$$

$$Out[7] = \frac{2 (x^2 + y^2)^{\frac{4}{3}} + 3 R^4 (x^4 - 6 x^2 y^2 + y^4) + R^2 (x^6 + 13 x^4 y^2 + 7 x^2 y^4 - 5 y^6)}{2 (x^2 + y^2)^{\frac{4}{3}}}$$