

```
ClearAll[ux, v, x, y];
```

```
uy[v_, x_, y_] = (4 * v - 2) * y / (x^2 + y^2) + (x^2 * y - y^3) / (x^2 + y^2)^2
```

$$\frac{x^2 y - y^3}{(x^2 + y^2)^2} + \frac{y(-2 + 4v)}{x^2 + y^2}$$

```
ux[v_, x_, y_] = (x^3 - x * y^2) / (x^2 + y^2)^2
```

$$\frac{x^3 - x y^2}{(x^2 + y^2)^2}$$

```
Needs["VectorFieldPlots`"]
```

```
Get::noopen : Cannot open VectorFieldPlots`. More...
```

```
Needs::nocont : Context VectorFieldPlots` was not created when Needs was evaluated. More...
```

```
$Failed
```

```
VectorFieldPlot[{ux[0.3, x, y], uy[0.3, x, y]}, {x, 1, 3}, {y, 0, 1}]
```

```
VectorFieldPlot[{ $\frac{x^3 - x y^2}{(x^2 + y^2)^2}$ ,  $-\frac{0.8 y}{x^2 + y^2} + \frac{x^2 y - y^3}{(x^2 + y^2)^2}$ }, {x, 1, 3}, {y, 0, 1}]
```

```
εxx = D[ux[v, x, y], {x, 1}]
```

$$\frac{3 x^2 - y^2}{(x^2 + y^2)^2} - \frac{4 x (x^3 - x y^2)}{(x^2 + y^2)^3}$$

```
εyy = D[uy[v, x, y], {y, 1}]
```

$$\frac{x^2 - 3 y^2}{(x^2 + y^2)^2} - \frac{4 y (x^2 y - y^3)}{(x^2 + y^2)^3} - \frac{2 y^2 (-2 + 4 v)}{(x^2 + y^2)^2} + \frac{-2 + 4 v}{x^2 + y^2}$$

```
εxy = (D[uy[v, x, y], {x, 1}] + D[ux[v, x, y], {y, 1}]) / 2
```

$$\frac{1}{2} \left(-\frac{4 y (x^3 - x y^2)}{(x^2 + y^2)^3} - \frac{4 x (x^2 y - y^3)}{(x^2 + y^2)^3} - \frac{2 x y (-2 + 4 v)}{(x^2 + y^2)^2} \right)$$

```
σyy = v / (1 - 2 * v) * (εxx + εyy) + εyy
```

$$\frac{x^2 - 3 y^2}{(x^2 + y^2)^2} - \frac{4 y (x^2 y - y^3)}{(x^2 + y^2)^3} - \frac{2 y^2 (-2 + 4 v)}{(x^2 + y^2)^2} + \frac{-2 + 4 v}{x^2 + y^2} + \frac{v \left(\frac{x^2 - 3 y^2}{(x^2 + y^2)^2} + \frac{3 x^2 - y^2}{(x^2 + y^2)^2} - \frac{4 x (x^3 - x y^2)}{(x^2 + y^2)^3} - \frac{4 y (x^2 y - y^3)}{(x^2 + y^2)^3} - \frac{2 y^2 (-2 + 4 v)}{(x^2 + y^2)^2} + \frac{-2 + 4 v}{x^2 + y^2} \right)}{1 - 2 v}$$

```
y = 0;
```

```
Simplify[σyy]
```

$$\frac{-1 + 2 v}{x^2}$$

```
ClearAll[ux, uy, v, x, y];
```

```

phi = -2 * pi * R^2 * Log[Sqrt[x^2 + y^2]]
-2 pi R^2 Log[ $\sqrt{x^2 + y^2}$ ]

psi = pi * R^2 * (x^2 + y^2) * (1 - Log[Sqrt[x^2 + y^2]])
pi R^2 (x^2 + y^2) (1 - Log[ $\sqrt{x^2 + y^2}$ ])

Simplify[D[psi, {y, 2}] + D[psi, {x, 2}] - 2 * phi]
0

ux = Simplify[D[psi, x, x, y] / 4 / pi / (1 - v) - D[phi, y] / 2 / pi]

$$\frac{R^2 y (x^2 (-3 + 2 v) + y^2 (-1 + 2 v))}{2 (x^2 + y^2)^2 (-1 + v)}$$


uy = Simplify[D[psi, y, x, y] / 4 / pi / (1 - v) - D[phi, x] / 2 / pi]

$$\frac{R^2 x (y^2 (-3 + 2 v) + x^2 (-1 + 2 v))}{2 (x^2 + y^2)^2 (-1 + v)}$$


exx = Simplify[D[ux, x]]

$$-\frac{R^2 x y (x^2 (-3 + 2 v) + y^2 (1 + 2 v))}{(x^2 + y^2)^3 (-1 + v)}$$


eyy = Simplify[D[uy, y]]

$$-\frac{R^2 x y (y^2 (-3 + 2 v) + x^2 (1 + 2 v))}{(x^2 + y^2)^3 (-1 + v)}$$


exy = Simplify[(D[uy, x] + D[ux, y]) / 2]

$$-\frac{R^2 (x^4 - 6 x^2 y^2 + y^4)}{2 (x^2 + y^2)^3 (-1 + v)}$$


syy = Simplify[2 * v / (1 - 2 * v) * (exx + eyy) + 2 * exy]

$$-\frac{2 R^2 x y (x^2 - 3 y^2)}{(x^2 + y^2)^3 (-1 + v)}$$


y = 0;

Simplify[syy]

$$\frac{R^2 (1 - 2 v)}{4 x^2 (-1 + v)}$$


```