T.C. Trakya University
Faculty of Engineering
Department of Electrical and Electronics Engineering
Assist. Prof. Işık İlber Sırmatel

EEM/EEE314 Automatic Control Systems

Exam-style questions with solutions

Part 1: Electrical systems

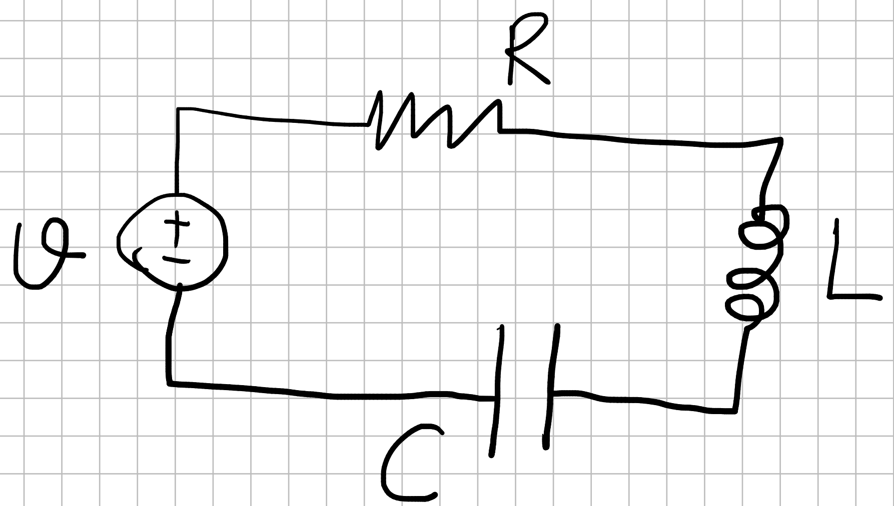
Abbreviations:

KVL: Kirchhoff's voltage law

KCL: Kirchhoff's voltage law

LHS: left-hand side (of the equation)

Question 1: Consider the schematic of an electrical circuit depicted below, consisting of a resistor, an inductor, and a capacitor (with parameters R, L, and C, respectively). The charge accumulated on the capacitor is denoted as q(t). An external voltage is applied to the circuit, denoted as v(t). Find the differential equation model of the system relating v(t) and q(t).



Solution:

Denoting the voltage drops across the circuit elements as V_R, V_L, and V_C, respectively, and invoking KVL, we write:

$$v = V_R + V_L + V_C$$

Denoting the current flowing through the circuit as $i(t)$, and using linear models of the circuit elements, we write:

$$V_R = R \cdot i \qquad V_L = L \cdot \dot{i}$$

$$V_C = \frac{1}{C} q$$

Note that, for the capacitor, the following are true:

1. $q = \int i \, dt$ (accumulation of charge)

(or, equivalently: $i = \dot{q}$)

2. voltage drop across the capacitor:

$$V_C = \frac{1}{C} q = \frac{1}{C} \int i \, dt$$

(or, equivalently: $i = C \cdot \dot{V}_C$)

The question asks us to relate v and q, thus we need to rewrite V_R and V_L in terms of q (using $i = \dot{q}$):

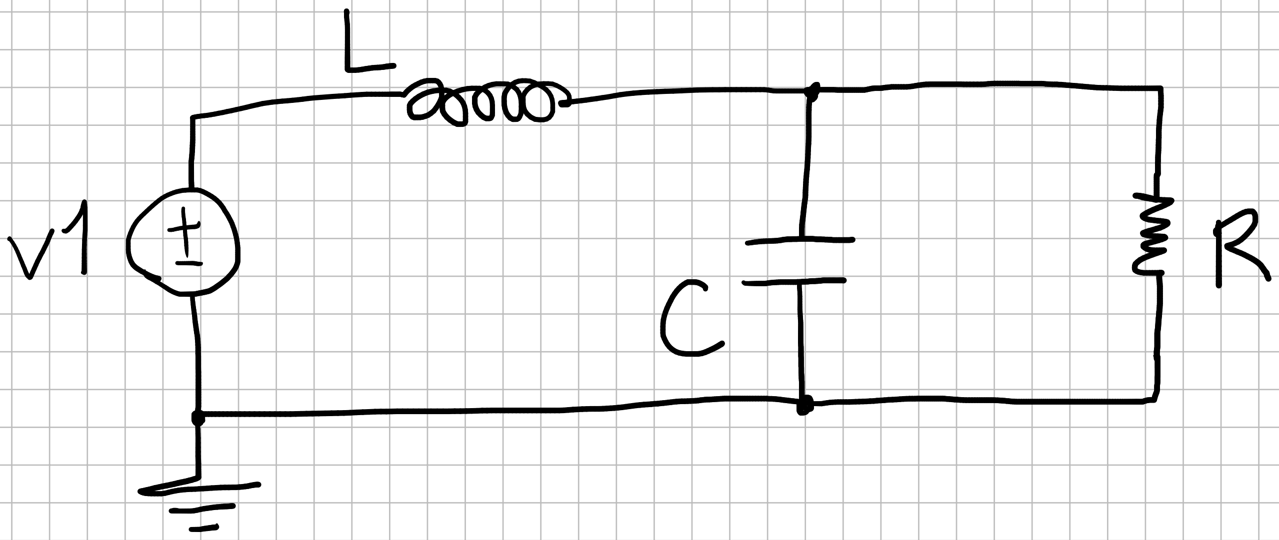
$$V_R = R \cdot i = R \cdot \dot{q}$$

$$V_L = L \cdot \dot{i} = L \cdot \ddot{q}$$

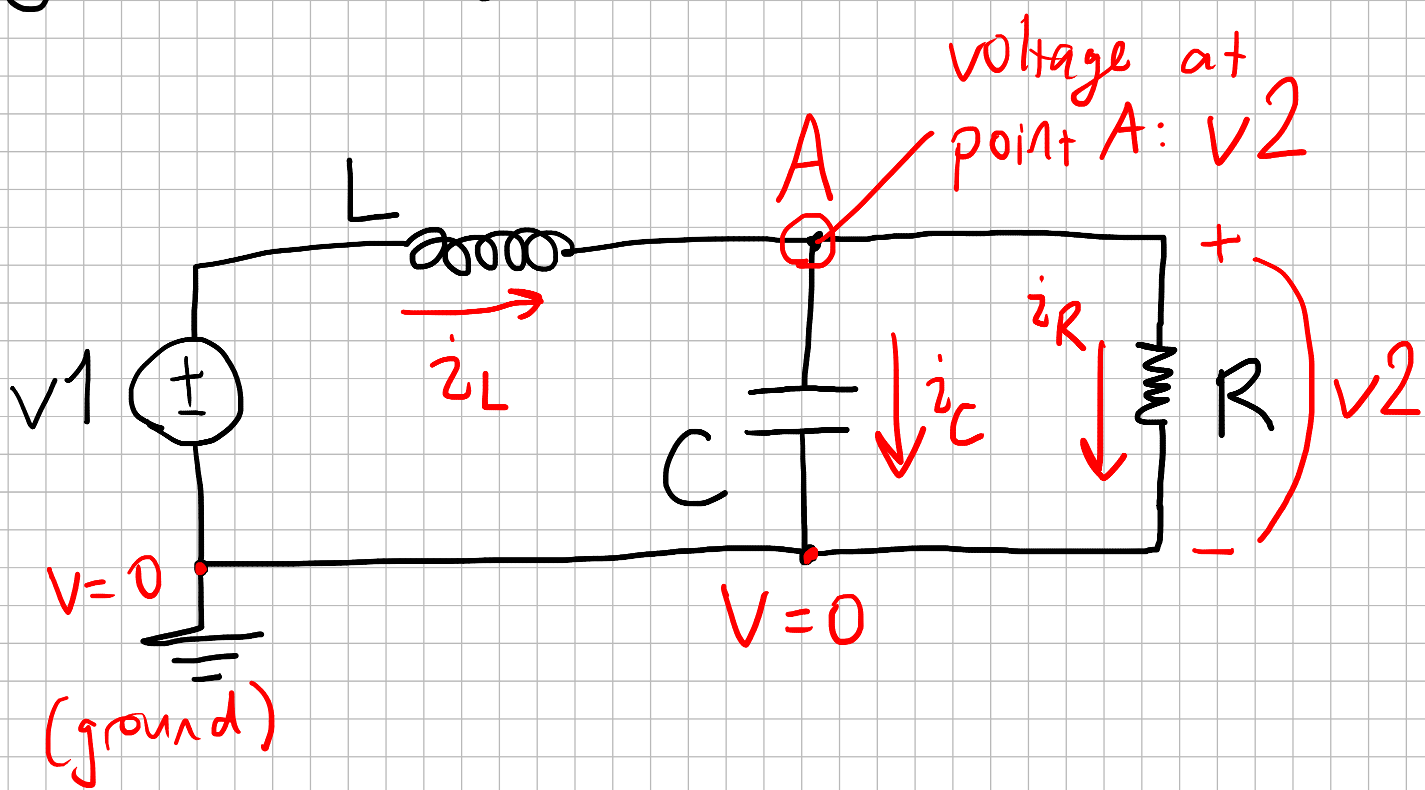
Rewriting KVL with these, we find:

$$\boxed{v = R\dot{q} + L \cdot \ddot{q} + \frac{1}{C} q}$$

Question 2: Consider the schematic of an electrical circuit depicted below, consisting of a resistor, an inductor, and a capacitor (with parameters R, L, and C, respectively). The voltage drop across the resistor is denoted as v2(t). An external voltage is applied to the circuit, denoted as v1(t). Find the differential equation model of the system relating v1(t) and v2(t).



Solution: Define the following symbols for the circuit:



Invoking KCL at point A:

$$i_L = i_c + i_R$$

Invoking KVL for left loop:

$$v1 = V_L + v2$$

The question asks us to relate v1 and v2, thus we need to rewrite V_L and V_c in terms of v2.

Writing the linear model of the inductor: $V_L = L \cdot \dot{i}_L$

Rewriting KVL with this:

$$v1 = L \cdot \dot{i}_L + v2$$

We still need to rewrite $\dot{i}_L$ in terms of v2.

We found KCL for point A as follows: $i_L = i_C + i_R$

Differentiating both sides, we get : $\dot{i}_L = \dot{i}_C + \dot{i}_R$

From linear models of the capacitor and resistor, we can write:

$$i_C = C \cdot \dot{v}2 \qquad i_R = \frac{1}{R} v2$$

Differentiating both:

$$\dot{i}_C = C \cdot \ddot{v}2 \qquad \dot{i}_R = \frac{1}{R} \cdot \dot{v}2$$

Rewriting $\dot{i}_L = \dot{i}_C + \dot{i}_R$

with these, we have:

$$\dot{i}_L = C \cdot \ddot{v}2 + \frac{1}{R} \cdot \dot{v}2$$

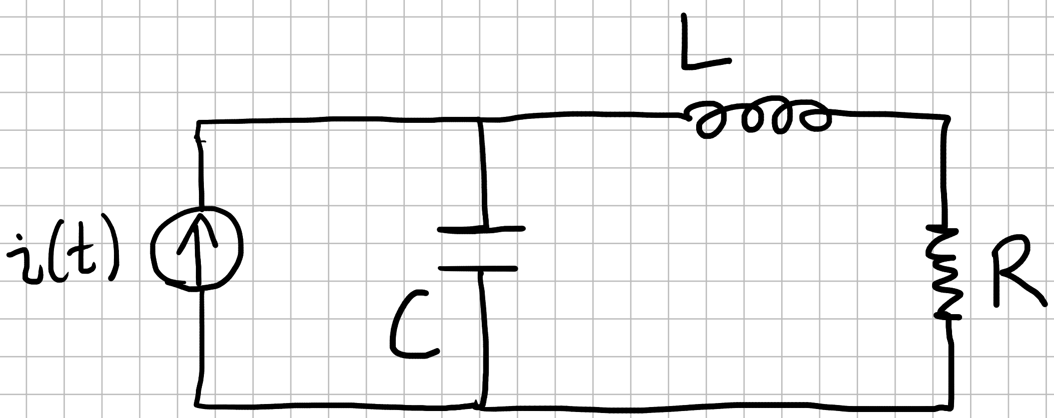
Substituting this in the KVL:

$$v1 = L \cdot \dot{i}_L + v2$$

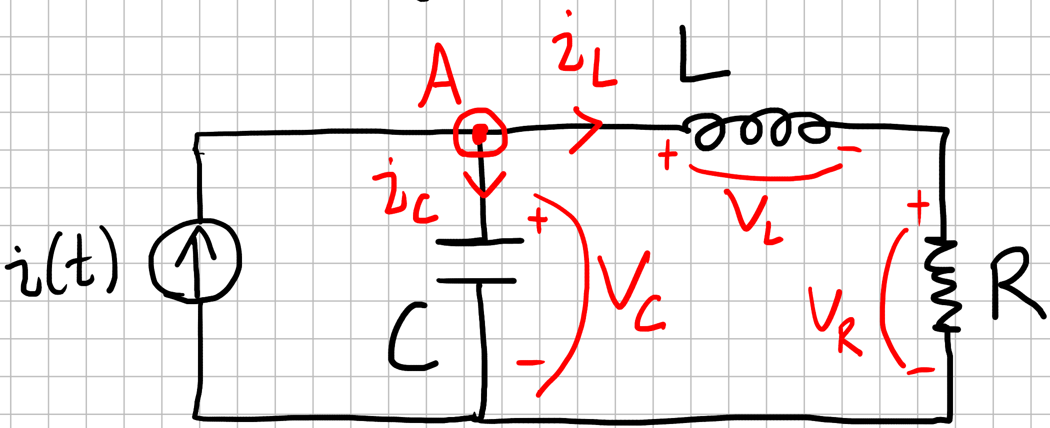
$$v1 = L \cdot \left(C \cdot \ddot{v}2 + \frac{1}{R} \dot{v}2 \right) + v2$$

$$\boxed{v1 = L \cdot C \cdot \ddot{v}2 + \frac{L}{R} \cdot \dot{v}2 + v2}$$

Question 3: Consider the schematic of an electrical circuit depicted below, consisting of a resistor, an inductor, and a capacitor (with parameters R, L, and C, respectively). The current flowing through the inductor is denoted as iL(t). An external current is supplied to the circuit, denoted as i(t). Find the differential equation model of the system relating i(t) and iL(t).



Solution: Define the following symbols for the circuit:



Invoking KCL at point A:

$$i = i_C + i_L$$

Invoking KVL for right loop:

$$V_C = V_L + V_R$$

From linear models of the inductor and the resistor, we can write:

$$V_L = L \cdot \dot{i}_L \qquad V_R = R \cdot i_L$$

Rewriting KVL with these:

$$V_C = L \cdot \dot{i}_L + R \cdot i_L$$

Differentiating both sides:

$$\dot{V}_C = L \cdot \ddot{i}_L + R \cdot \dot{i}_L$$

Multiplying both sides by C:

$$C \cdot \dot{V}_C = L \cdot C \cdot \ddot{i}_L + R \cdot C \cdot \dot{i}_L$$

From linear model of the capacitor: $i_C = C \cdot \dot{V}_C$

This equation thus becomes:

$$i_C = L \cdot C \cdot \ddot{i}_L + R \cdot C \cdot \dot{i}_L$$

From KCL at point A we found:

$$i = i_c + i_L \text{ , thus: } i_C = i - i_L$$

Substituting i_C, this equation becomes:

$$i - i_L = L \cdot C \cdot \ddot{i}_L + R \cdot C \cdot \dot{i}_L$$

rearranging to have i_L terms on LHS:

$$\boxed{LC\,\ddot{i}_L + RC\,\dot{i}_L + i_L = i}$$