arma_models

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1 Autoregressive Moving Average (ARMA) Models

ARMA models are an extention of AR and MA by combination. ARMA is described as follows

$$X_t - \phi_1 X_{t-1} - ... - \phi_p X_{t-p} = \beta + a_t - \theta_1 a_{t-1} - ... - \theta_q a_{t-a}$$

where
$ \beta = (1 - \phi_1 - \ldots - \phi_p) \mu and (a_t) $iswhitenoise$
and is constained with

- ϕ_i and θ_j are real valued constants
- $\phi_i \neq 0$ and $\theta_j \neq 0$
- $\phi(z)$ and $\theta(z)$ have no common factors

The operator for is

$$\left(1 - \phi_1 B - ... - \phi_p B^p\right) X_t = \left(1 - \theta_1 B - ... - \theta_q B^q\right)$$

And can generally be expressed as

$$\phi(B) X_t = \theta(B)$$

An ARMA model is **valid** if and only if model is stationary and invertable i.e.

1. All the roots of $\phi(z)$ are outside the unit circle
2. All the roots of $\theta(z)$ are outside the unit circle

1.1 Notation

Since ARMA is a combination of two models (AR and MA), the separate indexes are used to indicate the order of each model. Typically, p refers to the AR order and q refers to the MA order i.e. ARMA(p,q). Additionally, the order of the models are indexed with i and j in formulas, respectively.

- An MA(q) model is an ARMA(0, q) model
- An AR(p) model is an ARMA(p, 0) model

1.2 References

- [1] W. Woodward and B. Salder, "Moving Average (MA(q)) and ARMA(p,q) Models", SMU, 2019