

backshift_operator

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1 The BackShift Operator

The backshift operator B is used to rewrite autoregressive models as factorable polynomials. The backshift operator shifts a time series back one time step.

$$BX_t = X_{t-1}$$

When the backshift operator shifts more or less than one time step, this is indicated with a exponent. B^n represents the n th backshift. Thus, more generally

$$B^k X_t = X_{t-k}$$

For a single step back in time B^1 , the exponent is typically omitted.

1.1 Usage Examples

The following are real application of using backshift notation.

1.1.1 The AR(1) Model

The zero-mean AR(1) is expressed as

$$X_t - \phi X_{t-1} = a_t$$

This can be rewritten in backshift notation as

$$(1 - \phi B)X_t = a_t$$

1.1.2 The 1st Order Difference Filter

The first order difference filter is expressed as

$$X_t = Z_t - Z_{t-1}$$

This can be rewritten in backshift notation as

$$X_t = Z_t(1 - B)$$

1.1.3 The Moving Average Filter

The 3 point moving average filter can be expressed as

$$X_t = \frac{Z_{t+1} + Z_t + Z_{t-1}}{3}$$

This can be rewritten in backshift notation as

$$X_t = Z_t \frac{B^{-1} + B^0 + B^1}{3}$$

1.2 References

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- [1] W. Woodward and B. Salder, “Autoregressive Models and Filtering”, SMU, 2019