moving_average_models

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1 Moving Average (MA) Models

Moving average models are expressed as depended of lagged white noise (a_t) values.

$$X_t = \mu + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - ... - \theta_q a_{t-q}$$

The operator form (zero-mean) is

$$X_t = \left(1 - \theta_1 B - ... - \theta_q B^q\right) a_t$$

Which provides the characteristic

$$1 - \theta_1 z - ... - \theta_q z^q = 0$$

1.1 Properties

- An MA(q) is a finite GLP that is always stationary
- $E[X_t] = \mu$
- $\rho_0 = 1$
- $\rho_k = 0$ for $k > q$

1.1.1 MA(1) Autocorrelations

The autocorrelations for MA(1) are as follows

$$\rho_1 = \frac{-\theta_1}{1 + \theta_1^2}$$

$$\rho_k = 0|_{k>1}$$

Note: The max absolute value of $\rho_1 = 0.5$ for MA(1).

1.1.2 MA(2) Autocorrelations

The autocorrelations for MA(2) are as follows

$$\rho_1 = \frac{-\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2}$$

$$\rho_1 = \frac{-\theta_2}{1 + \theta_1^2 + \theta_2^2}$$

$$\rho_k = 0|_{k>2}$$

1.2 Invertability

Due to model multiplicity in MA models, MA models are restricted to have all roots outside the unit circle. This is to make models identifible i.e. a singlar mapping between parameters and models.

More details

1.3 References

- [1] W. Woodward and B. Salder, "Moving Average (MA(q)) and ARMA(p,q) Models", SMU, 2019