preliminaries

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1 Preliminaries

1.1 Expected Values

$E[X]$ is the mean of X.
For a discrete random variable X,

$$E[X] = \sum^{N}(x_i)P(x_i)$$

For a continuous random variable X,

$$E[X] = \int_a^b xf(x)\,dx$$

where $f(x)$ is the probablity distribution of the random variable X.

1.1.1 Properties of Expection Values

Some useful properties of expectation values.

- $E[a] = a$
- $E[aX] = aE[X] = a\mu$
- $E[aX + b] = aE[X] + b = a\mu$ + b

Where a, b are constants.

1.2 Variance

Definition of variance of a distribution

$$Var(X) = E[(X-\mu)^2] = \int_{-\infty}^{\infty}(X-\mu)^2 f(x)\,dx = \sigma^2$$

Estimation of variance

$$\hat{Var}(X) = \sum^{N} p_i (x_i - \bar{x})^2$$

where p_i represents the probability mass function of X.

1.3 Autocorrelation

1.3.1 Independence

An event is independent if the probability of its occurence does not depend on events in the past.

$$P(x_{t+1}|x_t) = P(x_t)$$

Thm: If two events are independent, thier correlation is 0.
Corollary: If the correlation between two variables is non-zero, they are not independent.

1.3.2 Serial Dependence

Serial dependence can be accessed with autocorrelation. Autocorrelcation is denoted as ρ_k, where k represents the number of lags. Autocorrelation is expressed as

$$\rho_k = \frac{E[(X_t - \mu)(X_{t+k} - \mu)]}{\sigma_x^2}$$

Let γ_k be defined as

$$\gamma_k = E[(X_t - \mu)(X_{t+k} - \mu)]$$

And noting that

$$\sigma_x^2 = E[(X_t - \mu)(X_{t+k} - \mu)] \mid_{k=0}$$

Then

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

Note: γ_k is known as autocovariance.

1.4 References

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[1] W. Woodward and B. Salder, “Stationary”, SMU, 2019