filtering

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1 Filtering

Concept: A filter takes in a signal and applies a transfomation that affects the frequency structure.

1.1 Types

In general, there are four basic types of filters

- Low pass - filters out high frequency
- High pass - filters out low frequency
- Band pass - filters out frequencies outside the band
- Band stop - filters out frequencies inside the band

1.2 Examples

Two simple examples of filters are the 1st order difference and the moving average.

1.2.1 1st Order Difference

The first order difference is defined as

$$X_t = Z_t - Z_{t-1}$$

where Z_t is an input time series.
The first order difference filter is a **high pass** filter.

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1.2.2 Moving Average (5-point)

The 5-point moving average is defined as

$$X_t = \frac{Z_{t+2} + Z_{t+1} + Z_t + Z_{t-1} + Z_{t-2}}{5}$$

where Z_t is an input time series.

Moving average filters are **low pass** filters. The cut off frequency is set by the size of teh average.

1.3 More Advanced Filters

A well known more advanced filter is the butterworth filter, which is known for haing a **maximally flat magnitude** frequency response.

1.4 Expression as a Tranformation

Filters can be expressed as transformations using backshift notation where the transformation is typically denoted as $H(B)$.

For example, the 3-point moving average filter can be expressed as

$$X_t = \frac{Z_{t+1} + Z_t + Z_{t-1}}{3}$$

Substituting for the backshift, we have

$$X_t = Z_t \frac{B^{-1} + B^0 + B^{-1}}{3}$$

Then the backshift terms can be represented as a transformation $H(B)$

$$H(B) = \frac{B^{-1} + B^0 + B^{-1}}{3}$$

And the 3-point moving average filter is expressed as

$$X_t = Z_t H(B)$$

2 General Linear Processes

An general, a general linear process (GLP) is a linear filter that is an infinite linear combination of white noise. A general linear process is represented by the transfer function $H(B)$.

2.1 Relation to AR, MA, and ARMA models

Any ARMA(p, q) model can be written as GLP.

$$X_t = \frac{\theta(B)}{\phi(B)} a_t$$

The polynomial in the equation above can be thought of as a linear filter. When white noise is fed through this filter, the resulting autocorrelations may be non-zero.

The division of the polynomial in the equation above will yield the ψ values of the GLP representation of the ARMA process. These ψ values are used to determine the forecast intervals.

2.2 References

- [1] W. Woodward and B. Salder, "Autoregressive Models and Filtering", SMU, 2019