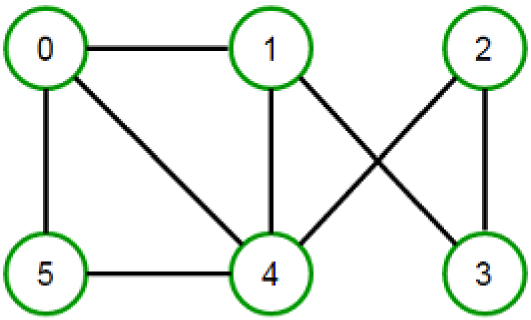
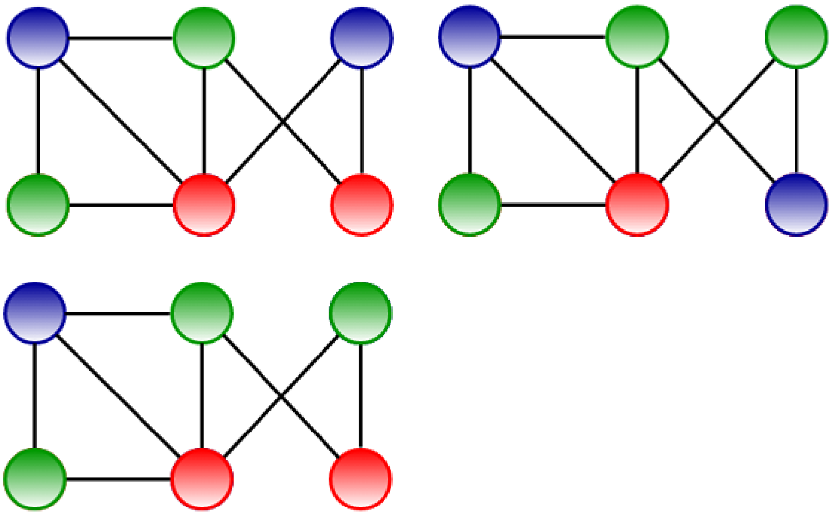
Graph Coloring Problem

Graph coloring (also called vertex coloring) is a way of coloring a graph's vertices such that no two adjacent vertices share the same color. This post will discuss a greedy algorithm for graph coloring and minimize the total number of colors used.

For example, consider the following graph:



We can color it in many ways by using the minimum of 3 colors.



Please note that we can't color the above graph using two colors.

Before discussing the greedy algorithm to color graphs, let's talk about basic graph coloring terminology.

K–colorable graph:

A coloring using at most `k` colors is called a (proper) *k*–coloring, and a graph that can be assigned a (proper) *k*–coloring is *k*–colorable.

K–chromatic graph:

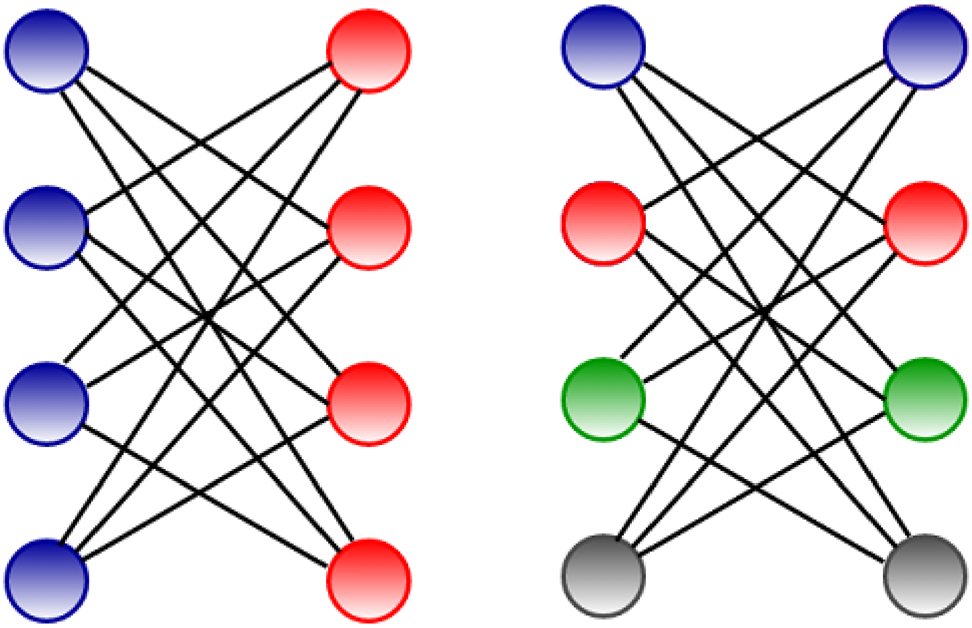
The smallest number of colors needed to color a graph `G` is called its chromatic number, and a graph that is *k*–chromatic if its chromatic number is exactly `k`.

Brooks' theorem:

Brooks' theorem states that a connected graph can be colored with only `x` colors, where `x` is the maximum degree of any vertex in the graph except for complete graphs and graphs containing an odd length cycle, which requires `x+1` colors.

Greedy coloring *considers the vertices of the graph in sequence and assigns each vertex its first* *available color*, i.e., vertices are considered in a specific order `v1`, `v2`, … `vn`, and `vi` and assigned the smallest available color which is not used by any of `vi`'s neighbors.

Greedy coloring doesn't always use the minimum number of colors possible to color a graph. For a graph of maximum degree `x`, greedy coloring will use at most `x+1` color. Greedy coloring can be arbitrarily bad; for example, the following crown graph (a complete bipartite graph), having `n` vertices, can be 2–colored (refer left image), but greedy coloring resulted in `n/2` colors (refer right image).



The algorithm can be implemented as follows in C++, Java, and Python:

C++

```
#include <iostream>
#include <vector>
#include <unordered_map>
#include <set>
using namespace std;

// Data structure to store a graph edge
struct Edge {
    int src, dest;
};

class Graph
{
public:
    // a vector of vectors to represent an adjacency list
    vector<vector<int>> adjList;

    // Constructor
```

```
    Graph(vector<Edge> const &edges, int N)
    {
        // resize the vector to hold `N` elements of type `vector<int
        adjList.resize(N);

        // add edges to the undirected graph
        for (Edge edge: edges)
        {
            int src = edge.src;
            int dest = edge.dest;

            adjList[src].push_back(dest);
            adjList[dest].push_back(src);
        }
    }
};

// Add more colors for graphs with many more vertices
string color[] =
{
    "", "BLUE", "GREEN", "RED", "YELLOW", "ORANGE", "PINK",
    "BLACK", "BROWN", "WHITE", "PURPLE", "VOILET"
};

// Function to assign colors to vertices of a graph
void colorGraph(Graph const &graph, int N)
{
    // keep track of the color assigned to each vertex
    unordered_map<int, int> result;

    // assign a color to vertex one by one
    for (int u = 0; u < N; u++)
    {
        // set to store the color of adjacent vertices of `u`
        set<int> assigned;

        // check colors of adjacent vertices of `u` and store them ir
        for (int i: graph.adjList[u])
        {
            if (result[i]) {
                assigned.insert(result[i]);
            }
        }

        // check for the first free color
        int color = 1;
        for (auto &c: assigned )
        {
            if (color != c) {
                break;
            }
            color++;
        }

        // assign vertex `u` the first available color
        result[u] = color;
    }

    for (int v = 0; v < N; v++)
    {
        cout << "The color assigned to vertex " << v << " is "
```

```
                << color[result[v]] << endl;
    }
}

// Greedy coloring of a graph
int main()
{
    // vector of graph edges as per the above diagram
    vector<Edge> edges = {
        {0, 1}, {0, 4}, {0, 5}, {4, 5}, {1, 4}, {1, 3}, {2, 3}, {2, 4
    };

    // total number of nodes in the graph
    int N = 6;

    // build a graph from the given edges
    Graph graph(edges, N);

    // color graph using the greedy algorithm
    colorGraph(graph, N);

    return 0;
}
```

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Java

```
import java.util.*;

// A class to store a graph edge
class Edge
{
    int source, dest;

    public Edge(int source, int dest)
    {
        this.source = source;
        this.dest = dest;
    }
}

// A class to represent a graph object
class Graph
{
    // A list of lists to represent an adjacency list
    List<List<Integer>> adjList = null;

    // Constructor
    Graph(List<Edge> edges, int N)
    {
        adjList = new ArrayList<>();
        for (int i = 0; i < N; i++) {
            adjList.add(new ArrayList<>());
        }

        // add edges to the undirected graph
```