Test Gauss Part

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1 Chosen values

$$\begin{aligned}
n_v &= 3 \\
n_h &= 2 \\
\text{batch size} = n_b &= 2 \\
i_{\text{death}} &= 0 \\
i_{\text{birth}} &= 0 \\
i_{\text{predator}} &= 1 \\
i_{\text{prey}} &= 0 \\
\mu &= \begin{pmatrix} 10 & 8 & 4 & 20 & 3 \end{pmatrix}
\end{aligned} \tag{1}$$

Cholesky decomposition of precision matrix $B = LL^\intercal$:

$$L = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 5 & 8 & 0 & 0 & 0 \\ -3 & 4 & 7 & 0 & 0 \\ -1 & -3 & -4 & 9 & 0 \\ -3 & 8 & 10 & 9 & 20 \end{pmatrix} \tag{2}$$

2 FourierLatentGaussLayer

$$\begin{aligned}
\text{Freqs} = \boldsymbol{f} &= \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \\
\text{No freqs} = L &= 3 \\
\text{Offset} = c &= 0 \\
\text{Cos coeffs} = \boldsymbol{a} &= \begin{pmatrix} 1 & 2 & 4 \end{pmatrix} \\
\text{Sin coeffs} = \boldsymbol{b} &= \begin{pmatrix} 1 & 5 & 4 \end{pmatrix} \\
\text{Timepoint (zero indexed)} = t &= 2
\end{aligned} \tag{3}$$

Fourier equation:

$$s(\boldsymbol{a}, \boldsymbol{b}, t) = c + \frac{\sum_{l=1}^{L} \left(a_l \cos(f_l t) + b_l \sin(f_l t) \right)}{\max\left(\sum_{l=1}^{L} (|a_l| + |b_l|), 1 \right) + \epsilon} \tag{4}$$

Evaluated:

$$s(\boldsymbol{a}, \boldsymbol{b}, t) = -1.8751299 \tag{5}$$

3 ConvertParams0ToParamsGaussLayer

- Layer constructor:

$$\begin{aligned} \text{Freqs} = \boldsymbol{f} = \begin{pmatrix}1 & 2 & 3\end{pmatrix} \\ \text{No freqs} = L = 3 \\ \boldsymbol{b}_{\mu h,\text{init}} = \begin{pmatrix}3 & 6 & 1\end{pmatrix} \\ \boldsymbol{a}_{\mu h,\text{init}} = \begin{pmatrix}2 & 5 & 3\end{pmatrix} \\ \boldsymbol{b}_{Lhv,\text{init}} = \begin{pmatrix}1 & 8 & 4\end{pmatrix} \\ \boldsymbol{a}_{Lhv,\text{init}} = \begin{pmatrix}4 & 5 & 4\end{pmatrix} \\ \boldsymbol{b}_{Lh,\text{init}} = \begin{pmatrix}8 & 10 & 9\end{pmatrix} \\ \boldsymbol{a}_{Lh,\text{init}} = \begin{pmatrix}3 & 7 & 8\end{pmatrix} \end{aligned} \tag{6}$$

  Here Lh and Lhv refer to decomposing the Cholesky decomposition into block matrices:

$$L = \begin{pmatrix} Lv & 0 \\ Lhv & Lh \end{pmatrix} \tag{7}$$

  The Fourier equations are

$$s(\boldsymbol{a}_{\mu h}, \boldsymbol{b}_{\mu h}, t) = \frac{\sum_{l=1}^{L} \left(a_{\mu h,l} \cos(f_l t) + b_{\mu h,l} \sin(f_l t)\right)}{\max\left(\sum_{l=1}^{L} (|a_{\mu h,l}| + |b_{\mu h,l}|), 1\right) + \epsilon} \tag{8}$$

  and similarly for Lhv, Lh (offsets are zero).

- Layer inputs:

$$\begin{aligned} \text{Timepoint} = 2 \\ \mu_v = \begin{pmatrix}10 & 8 & 4\end{pmatrix} \\ L_v = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 8 & 0 \\ 0 & 4 & 7 \end{pmatrix} \end{aligned} \tag{9}$$

- Layer actions:

  1. The layer first constructs the Cholesky decomposition in the standard space:

$$\begin{aligned} \hat{L}_v = (...) \\ \hat{L}_{hv} = 0 \\ \hat{L}_h = I \\ \hat{L} = \begin{pmatrix} \hat{L}_v & 0 \\ \hat{L}_{hv} & \hat{L}_h \end{pmatrix} \end{aligned} \tag{10}$$

     Also, $\hat{\mu}_h = 0$.
  2. Next, from the Fourier equations, L_h and $L_h v$ and μ_h are constructed.
  3. Next, L_v is calculated to keep the observed covariances Σ_v the same.
  4. Finally, $\mu = (\mu_v, \mu_h)$ and L are returned.