Q. Is set of odd numbers with binary operations (+), i.e., $\langle O, + \rangle$ an abelian group? If not explain the reason with necessary notations.

$\boxed{\text{Answer:}}$ No, the set of odd integers with addition is not an abelian group.

Let $O = \{2K+1 \mid K \in \mathbb{Z}\}$

Check the group axioms:

1. Closure: Take $a = 2m+1 \in O$ and $b = 2n+1 \in O$

$$a+b = (2m+1)+(2n+1)$$
$$= 2(m+n+1)$$

which is an even integer, so $a+b \notin O$ in general.

Example: $1, 3 \in O$ but $1+3 \notin O$.

2. Associativity: Addition is associative on $\mathbb{Z}$, so it is associative on O.

3. Identity element: The additive identity in $\mathbb{Z}$ is 0, but $0 \notin O$. So there is no identity element in O for addition.

4. Inverses: If $a = 2K+1 \in O$, then $-a = -2K-1 = 2(-K-1)+1 \in O$. So each element has a additive inverse inside O.

5. Commutativity: Addition is commutative on $\mathbb{Z}$, so it would be commutative on O.

Becaus closure (hence identity) fails, $\langle O, + \rangle$ is not a group – and therefor not an abelian group.