Keeping up with the Curve: Learning, Evolving Beliefs and the Anchoring of Expectations *

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Abstract

I explain the behavior of inflation, nominal interest rates and inflation expectations in the post-war U.S. by estimating a forward-looking model in which private agents learn about macroeconomic fundamentals and the policymakers' stabilization preferences in real-time using a *structural* model: recognizing the feedback from their expectations under imperfect knowledge to observed aggregate dynamics. Monetary policy is conducted optimally but private agents suspect that the policymakers' stabilization preferences are evolving, which they learn from observed policy behavior. This gives rise to a nonlinear filtering problem. The model provides a novel, information-theoretic explanation of how systematic monetary policy shapes expectations when agents are learning: a demonstrated emphasis on stabilizing the real economy by minimizing cyclical fluctuations in unemployment has stabilized long-run inflation expectations while at the same time rendering short-run expectations *more* susceptible to supply shocks. The model offers a new explanation for the recent post-Covid inflation surge and the "costless disinflation" that followed, shedding light on why it differed markedly from the crises of the 1970s and the 80s.

Keywords: Monetary Policy, Interest Rates, Expectation Formation, New Keynesian
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1 Introduction

The post-pandemic surge in inflation—which saw U.S. CPI inflation rise to a 40-year high of 9.1% in June 2022—caught both the private sector and policymakers off-guard. As inflation remained elevated, comparisons were drawn to the *Great Inflation* of the 1970s–80s. Both periods were accompanied by a sharp rise in oil prices and supply disruptions, yet they yielded very different macroeconomic outcomes: a protracted inflationary spiral culminating in a painful disinflation in the 1980s versus a relatively swift "soft landing" that has characterized the US economy's recovery from the pandemic shock. A particularly striking feature of the recent surge has been the stability of long-run inflation expectations - even as private agents expected inflation to remain elevated in the short-run, their long-run expectations remained remarkably well-anchored (as seen in Figure 1).

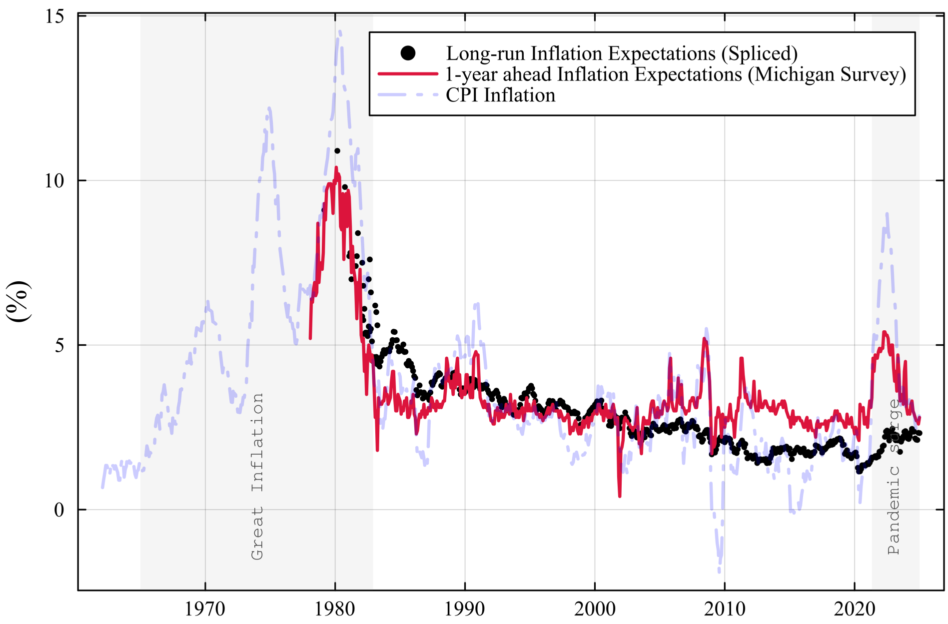


Figure 1: **Growing discord between short-run and long-run inflation expectations**

Note: Long-run Inflation Expectations series is constructed by splicing together inflation expectations over the 5-to-10-year horizon from Michigan Survey before 1982 and 10-year ahead inflation expectations from the Cleveland Fed afterwards (due to a shorter sample).

To explain these divergent outcomes, I develop a model with three key features. First, neither the private agents—households and firms nor central bank policymakers possess full information - their behavior is shaped by their *perceptions* regarding the fundamental state of the economy; which are formed based on interpreting macroeconomic outcomes through the lens of a forward-looking structural model, under *perpetual learning*. Second, monetary policy is conducted optimally under discretion by the central bank policymakers reflecting a dual mandate, weighing between stabilizing inflation around a target and unemployment around its *natural* level.[1] Third, agents do not know the central bank's objectives and suspect that they evolve over time, and hence they continually update their beliefs about them based on observed policy behavior.[2] Expectations are thus shaped by beliefs about both the structural features of the economy and the time-varying objectives of policymakers. Anchoring or de-anchoring of expectations thus results endogenously from learning. I estimate the model using monthly data on inflation, unemployment and the federal funds rate over the postwar period in the U.S. together with 10-year ahead inflation expectations from the Survey of Professional Forecasters (SPF). The model offers a narrative of the postwar U.S economy as seen through the eyes of private agents engaged in structural reasoning using a New-Keynesian model.

The estimated model suggests that the contrasting outcomes of the two inflationary episodes can be largely attributed to perceived shifts in the Federal Reserve's policy paradigm. In the run up to the Great Inflation, policy appeared to be guided almost entirely by a nominal target, with little concern for unemployment gaps effectively tying monetary policy to an unobservable and discretionary objective. In the late 1960s, leading up to the the Great Inflation the goalposts began to shift. A sustained period of expansionary monetary policy conveyed that the inflation target was drifting upward. Even as supply pressures mounted in the form of two great oil shocks and real activity softened, policy appeared to double down on accommodation. The agents took notice and came to interpret elevated inflation as a systematic feature of policy rather than a result of fundamental shocks, resulting in the de-anchoring of long-run inflation expectations. Short-run and long-run inflation expectations rose in unison.

The early 80s saw sustained hawkish policy by the Fed under Paul Volcker, even as supply pressures abated. This signaled the Fed's resolve to return to low long-run inflation: a swift, yet costly disinflation followed. In the aftermath of the Volcker disinflation, termed the *Great Moderation*, policy behavior was consistent with an increasing emphasis on minimizing unemployment gaps, over and beyond maintaining inflation close to their target. An aversion to unemployment gaps would restrain policymakers from driving non-fundamental inflation through expansionary policy, thus enabling an environment of stable long-run inflation expectations despite an inability of policymakers to explicitly commit.

Evolving perceptions about the Fed's policy are key to understanding the recent "soft-landing". Even as the Fed demonstrated an unwillingness to respond to the inflation crisis for over a year

[1] Time-variation in policymakers' stabilization preferences precludes commitment. The setup here is different from Kydland and Prescott (1977) where a time-invariant, commonly known social welfare function is assumed to exist. Here, policy is based on a legislative dual mandate, open to subjective interpretations by the policymakers.

[2] Bernanke (2007) stresses upon the need for incorporating all three of these features into the New Keynesian framework.

(prompting concerns that the Fed was falling "behind the curve" for instance), both firms and households' long-run inflation expectations remained remarkably stable: agents were slower to interpret the delayed policy response as evidence of a shift in the Fed's target. All the while, they still anticipated elevated inflation in the short-run owing to persistent supply shocks coupled with little incentive from policymakers to dampen the effects through adjustments on the real-side. The mechanism explains the peculiar discord between the behavior of short- and long-horizon inflation expectations over the recent years. Even as inflation resurfaced after remaining dormant for four decades, the agents anticipated policy to behave very differently this time, stemming from starkly different incentives underlying the Fed's behavior. The model delivers a key insight: when policymakers are seen as being guided by an inflation target unobserved by private agents, their long-run inflation expectations are fragile. Rather, when policy behavior is perceived to be guided by economic fundamentals, such as the natural rates, long-run expectations are stable.

The central bank's conduct of monetary policy is guided by the changing economic goals of the policymakers. In the model, monetary policy does not conform to any *instrument rule*, but rather.[3] Their policy decisions *reveal* information regarding their policy objectives. Hence, there is a non-trivial role for a "learning-channel" of monetary policy transmission in this environment, which operates by revealing information regarding the policymakers' stabilization preferences to the agents. Through this channel, policy influences agents expectations over all future horizons: there is *forward guidance* embedded into policy itself. When private agents observe discrepancies in policy behavior relative to their ex-ante expectations, they attribute them partly to idiosyncratic policy errors which will get corrected in the future, and the rest to shifts in the policymakers' stabilization preferences and hence indicative of how policy will be conducted in the future. Rational agents hence feel the need to constantly learn from policy behavior. Anchoring or de-anchoring of private agents expectations is an endogenous result of this learning process.

The literature on learning in macroeconomics highlights how agents update their beliefs in response to data *in real-time*, emphasizing the feedback from macroeconomic outcomes to agents' beliefs and decisions, something that full-information rational expectations abstracts from completely. However, much of the existing literature focuses on adaptive learning, where agents estimate reduced-form forecasting rules based on past data. In this setting, agents do not internalize the general equilibrium consequences of their beliefs, treating aggregate dynam-

[3] Instrument-rules such as the Taylor-rule are opaque regarding the trade-offs that policymakers actually face while deliberating about the conduct of monetary policy. It also causes an inconsistent asymmetry regarding how policy behavior is modeled, departing from the forward-looking optimizing framework modeling household and firm behavior. This critique is outlined in Svensson (2003).

ics as exogenous to their own behavior. This paper models *structural* learning (see Williams (2003)), in which agents form expectations based on a forward-looking New-Keynesian model that describes aggregate dynamics as arising from their behavior. While knowledge is imperfect, expectations remain disciplined by rational forward-looking behavior, ensuring that beliefs are consistent with the underlying structural dynamics implied by optimization.

Importantly, macroeconomic shocks by themselves do not cause agents' expectations to de-anchor so long as policy responds in the way that the agents expect, given the macroeconomic environment. Hence, it is the interaction between monetary policy and their perceptions about the state of the macroeconomy as well as their *ex-ante beliefs* regarding the incentives driving policymakers' behavior that determine whether expectations are anchored and whether they remain anchored, rather than macroeconomic shocks in isolation.

More generally, the rate at which agents receive information and update their beliefs is itself conditional on their perceived state of the economy, imparting a great deal of nonlinearity to their expectations-formation process. This is particularly relevant for learning about the slope of the Phillips curve, a key variable acting as the structural link between aggregate inflation and economic slack. Learning is exceptionally slow due to an adverse signal-to-noise ratio: the magnitude of slack is typically small in comparison with cost-push disturbances, making inference challenging. The slope is updated primarily around demand-driven recessions driving large unemployment gaps.

Related literature: The New-Keynesian model emphasizes the importance of expectations for understanding the structure of aggregate dynamics and individual economic behavior. The literature has highlighted the importance of explicitly accounting for agents' expectations-formation process, using survey data for understanding macroeconomic phenomena (see Coibion et al. (2018), Adam and Padula (2011), Milani (2023), Del-Negro and Eusepi (2011)). This paper attempts to model how agents form expectations in real-time, employing survey data and New-Keynesian principles to get a disciplined perspective on how their evolving beliefs have shaped the postwar U.S. economy.

The literature on learning in macroeconomics departs from the standard full-information benchmark and seeks to model how economic agents acquire information and form expectations. The literature has largely focused on adaptive learning (see Evans and Honkapohja (2001) for overview and applications) where agents use simple forecasting rules, typically reduced-form VARs to form expectations. Adaptive learning by private agents has been proposed as an explanation of the observed persistence in macroeconomic series that does not rely on ad-hoc frictions such as indexation to past prices and habit formation in consumption (see Milani

(2007) and Erceg and Levin (2003)).

This paper is an empirical application of learning to study macroeconomic behavior in the postwar U.S., and is closely related to the literature examining how private agents' and policy-makers' learning contributed to the Great Inflation of the 1970s and 1980s (e.g., Bullard and Eusepi (2005); Primiceri (2006); Sargent, Williams and Zha (2006),Orphanides and Williams (2005)). More broadly, it connects to the literature on how the private sector learns from monetary policy behavior. For instance, Melosi (2016) examines the signaling role of policy in a DSGE framework, where monetary policy is informative about aggregate shocks. By contrast, in this paper, the only source of information asymmetry arises from the private sector's inability to observe the central bank's stabilization preferences. Other related work has explored how agents infer policy targets from macroeconomic data (e.g., Kozicki and Tinsley (2005); Erceg and Levin (2003); Orphanides and Williams (2006)). However, much of this literature relies on adaptive learning, in which agents form beliefs using reduced-form forecasting rules based on historical correlations. In contrast, the learning behavior in this paper is grounded in a structural, forward-looking framework rooted in New Keynesian principles, where agents understand and internalize the effects of their expectations on macroeconomic outcomes.

This paper is also broadly related to the literature examining time-variation in the Fed's policy targets such as Ireland (2007), Cogley, Primiceri and Sargent (2010), Del-Negro and Eusepi (2011), and Lakdawala (2016). However, the focus in this paper is on private agents' *beliefs* about the Fed's objectives rather than the true policy objectives themselves.

The causes of the Great Inflation and the subsequent Great Moderation have been the subject of long-standing debate. One school attributes the volatility of the 1970s–80s era to "bad luck": a sequence of large, adverse non-policy shocks (e.g., Sims and Zha (2006); Bernanke and Mihov (1998)). Under this view, the structure of the economy and policy rules remained largely stable; it was the volatility of exogenous shocks that changed. This view implies that monetary policy made little fundamental progress—if similar shocks were to recur, so would similar outcomes. However, I find that how agents perceive the Fed's stabilization policy based on observed policy behavior has changed drastically which has had a huge bearing on the macroeconomic outcomes ever since the Great Moderation and beyond.

An alternative view emphasizes policy errors. Clarida, Gali and Gertler (2000), Taylor (1993) and Lubik and Schorfheide (2004) argue that the shift to systematic anti-inflation policy by the Fed under the chairmanship of Paul Volcker was central to the subsequent stability. De-Long (1996) and Romer and Romer (2002) provide narrative evidence that policymakers in the 1970s were misled by a perceived long-run tradeoff between inflation and unemployment.

Some papers study whether these policy mis-steps by the Fed were a result of "wrong" kind of incentives. For example Christiano and Gust (2000) invokes the idea that the Fed fell into an "expectations trap", lacking the incentives to take decisive action to rein in inflation during the 70s. At least from the private agents' perspective, I find that policy errors were made. The model I present does not take a stance on the *source* of these errors, but rather, the focus here is on the *consequences*: how these errors shaped the agents' views regarding the Fed's incentives and led to inflation through the expectations channel.

A related strand of literature has studied the mechanisms through which long-run inflation expectations get *anchored*. Carvalho, Eusepi, Moench and Preston (2023) and Gati (2023) describe anchoring in terms of how agents weight recent inflation data when updating their beliefs about the long-run mean of inflation based on past forecast errors (forecast-switching).[4] In this paper, anchoring directly results from agents learning about the central bank's stabilization preferences based on the *policy response* to inflation rather than the behavior of inflation itself.

This paper contributes to the debate by offering a framework that disentangles these explanations and sheds light on how each of these contributed to shaping the two prominent inflationary episodes in the post-war period in the U.S.

Outline: In Section 2, I motivate the agents' perceived structural model describing the economy as perceived by private agents which they use to reason about the macroeconomic environment that they face. In Section 3, I describe how agents recursively update their beliefs regarding the policy-invariant structural variables in their model conditional on macroeconomic observations. In Section 4, I describe the conduct of monetary policy from the perspective of the agents and describe how they update beliefs regarding the policymakers' stabilization preferences conditional on observed nominal interest rates. In Section 5, I describe the estimation methodology and the data. In Section 6, I discuss how the agents' beliefs have evolved during the postwar period in the U.S. under learning followed by concluding remarks.

2 A forward looking model under imperfect knowledge

Here I begin by describing the information structure of the economy, defining the set of signals that agents observe at any time, based on which they form their beliefs and make decisions. Then, I derive the agents' perceived law of motion describing the structural dynamics of the

[4] A related line of research (King and Lu (2022) and Debortoli and Lakdawala (2016)) models anchoring in the context of the ability of policymakers to commit.

macroeconomic aggregates under imperfect knowledge.

2.1 Information structure and learning

Time is discrete ($t = 0, 1, 2, 3, \dots$). Under the information structure in the model, neither the latent structural parameters or the structural shocks are directly observed by the agents. The only relevant signals available to them comes in the form of the realizations of the aggregate inflation rate, unemployment rate and the nominal interest rate. At the beginning of each period t, firms and households make their decisions conditional on the history of signals observed until that point. The period-t inflation π_t and unemployment rate u_t then realize and are observed by the agents. The central bank then sets the interest rate i_{t+1} in anticipation of future outcomes. The agents interpret the policy decision and form beliefs regarding the policymakers' stabilization preferences. Thus, at the end of period-t, π_t, u_t and i_{t+1} are realized and observed by all agents.

The entire history of aggregate signals observed by agents up to (and including) period t_0 is denoted by:

$$\mathcal{H}^{t_0} = \{i_{t+1}\}_{t=-\infty}^{t_0} \cup \{z_t\}_{t=-\infty}^{t_0}$$

where $z_t = [\pi_t, u_t]'$ is the vector of realizations of inflation and unemployment, and i_{t+1} denotes the interest-rate policy set in period-t.

2.2 The agents' perceived model of aggregate dynamics

Following New Keynesian principles, I derive the log-linearized equations describing the structural dynamics of the economy from the perspective of the agents, based on a model of optimizing behavior by firms and households under imperfect knowledge. I assume that all private agents have common beliefs, formed conditional on the same data and model.[5] I denote the private agents' expectations, formed conditional on history $\mathcal{H}^t$ as E_t^P.

[5]I abstract from any distinction in firms and households' belief-formation process.

2.2.1 Aggregate demand: Households' consumption smoothing problem

Consider the problem of a representative household, deriving utility from consumption of an aggregate good and saving in nominal bonds every period conditional on their beliefs formed based on the history of signals observed in the past. In period-t, the household solves:

$$\max_{\{C_{t+k}\}_{k=0}^{\infty}} E^P \left[\sum_{k=0}^{\infty} \beta^k \frac{C_{t+k}^{1-\sigma}}{1-\sigma} \Big| \mathcal{H}^{t-1} \right] \tag{1}$$

where C_t is consumption, β is a discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution and E^P denotes the representative household's expectations, formed conditional on its model.

The household's budget constraint in nominal terms is:

$$P_t C_t + B_t = (1 + i_{t-1}) B_{t-1} + W_t + T_t + \Pi_t \tag{2}$$

where P_t is the aggregate price level, B_t are nominal bond holdings, i_{t-1} is the nominal interest rate in period $t-1$, W_t denotes their nominal wage-income, T_t denote lump-sum government transfers and Π_t denotes profits from the ownership of firms.

In this setting, the only dynamic decision that the representative household makes is the savings decision, which leads to the following Euler equation:

$$C_t^{-\sigma} = \beta E^P \left[C_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \Big| \mathcal{H}^{t-1} \right] \tag{3}$$

where $1 + \pi_{t+1} = P_{t+1}/P_t$ is the aggregate inflation rate.

Since there is no investment or government expenditure in the model, all output is consumed every period $Y_t = C_t$, log-linearizing around the steady state, we get the aggregate-demand IS-curve:

$$\hat{y}_t = E_{t-1}^P[\hat{y}_{t+1}] - \frac{1}{\sigma}(i_t - E_{t-1}^P[\pi_{t+1}] - \rho) \tag{4}$$

where $\hat{y}_t$ denotes the log-deviation of output from its steady-state. $\rho = -\log \beta$. Defining the *ex-ante* real interest rate as $r_t = i_t - E_{t-1}^P[\pi_{t+1}]$, the IS curve may be more compactly written as:

$$\hat{y}_t = E_{t-1}^P[\hat{y}_{t+1}] - \frac{1}{\sigma}(r_t - \rho) \tag{5}$$

At some hypothetical real interest rate, output would continue along the trend path, y_t^n (or the "natural" level of output) absent cyclical fluctuations. We call this the natural rate of interest and denote it by r_t^n. Log-deviations of the natural level of output $\hat{y}_t^n$, follow:

$$\hat{y}_t^n = E_{t-1}^P[\hat{y}_{t+1}^n] - \frac{1}{\sigma}(r_t^n - \rho) \tag{6}$$

Subtracting Equation 6 from Equation 5 yields:

$$\hat{y}_t - \hat{y}_t^n = E_{t-1}^P[\hat{y}_{t+1} - \hat{y}_{t+1}^n] - \frac{1}{\sigma}(r_t - r_t^n) \tag{7}$$

Denoting $\hat{y}_t - \hat{y}_t^n$ as output gap $\tilde{y}_t$, we may re-write the dynamic IS curve in terms of output gap as:

$$\tilde{y}_t = E_{t-1}^P[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - E_{t-1}^P[\pi_{t+1}] - r_t^n) \tag{8}$$

2.2.2 Aggregate supply: Firms' pricing problem

Firm price-setting under dispersed information Every period t, all firms automatically adjust their prices by the perceived trend inflation rate conditional on their $t-1$ information set, given by π_{t-1}^{*P}.[6] The perceived level of trend inflation does not enter firms' pricing problem since all firms have the same beliefs and adjust to the same perceived trend rate every period. However, relative price adjustments are subject to Calvo frictions. Only a constant fraction $(1-\theta)$ of firms are able to adjust their price every period (adjustment frequency θ is common knowledge). All firms set prices in order to earn a constant markup over their nominal marginal cost. The steady-state relative price of each firm is unity. For concision, we express their optimal price setting problem in terms of log-deviations from steady-state.

In the absence of Calvo frictions, log-deviation in firm j's optimal price is equal to the sum of log-deviations in the aggregate price-level, denoted as $\hat{p}_t$ and log-deviations in their real marginal cost, $mc_t(j)$.

$$\hat{p}_t^*(j) = \hat{p}_t + \hat{mc}_t(j)$$

Under Calvo frictions, firms cannot set prices flexibly, so that each firm must set prices in order

[6] This assumption permits the model to be log-linearized around the zero inflation steady state despite non-zero trend inflation. Inflation here refers to inflation beyond this "normal" level.

to receive a constant markup *in expectation*. Their optimal price is given by:

$$\hat{p}_t^*(j) = (1-\beta\theta)E^j\Big[\sum_{k=0}^{\infty}(\beta\theta)^k(\hat{p}_{t+k} + \hat{mc}_{t+k}(j))|\mathcal{H}^{t-1}\Big] \tag{9}$$

where $E^j[\cdot]$ denotes the expectation operator conditional on firm j's information set. The firms' information set is the same as that of the representative households, and is denoted by $\mathcal{H}^{t-1}$

Additionally, each firm observes its own real marginal cost $\hat{mc}_t(j)$ at the beginning of the period. $\hat{mc}_t(j)$ is composed of an economy-wide component $\hat{mc}_t$ plus an idiosyncratic firm-specific *i.i.d* (across firms and across time) component, $\varepsilon_t(j)$ with *large* variance.[7]

$$\hat{mc}_t(j) = \hat{mc}_t + \varepsilon_t(j) \quad \varepsilon_t(j) \sim N(0, \sigma_\varepsilon^2)\ \forall\ j \in (0,1) \tag{10}$$

Under knowledge that it can only reset with probability $1-\theta$ any period in the future would, the optimal price is given by:

$$\hat{p}_t^*(j) = (1-\beta\theta)E^j\Big[\sum_{k=0}^{\infty}(\beta\theta)^k(\hat{p}_{t+k} + \hat{mc}_{t+k}(j))|\mathcal{H}^{t-1} \cup \hat{mc}_t(j)\Big] \tag{11}$$

Dispersed information assumption: The firm is atomistic, its own marginal cost reveals negligible information about innovations to the economy-wide real marginal costs or changes in the aggregate price level p_t. Thus, their expectations regarding these variables are unchanged upon observing $\hat{mc}_t(j)$ and common across all firms, denoted as E_{t-1}^P. Hence:

$$E^j[\hat{p}_t + \hat{mc}_t(j)|\mathcal{H}^{t-1} \cup \hat{mc}_t(j)] = E_{t-1}^P\hat{p}_t + \hat{mc}_t(j) \tag{12}$$

$$E^j[\hat{p}_{t+k} + \hat{mc}_{t+k}|\mathcal{H}^{t-1} \cup \hat{mc}_t(j)] \approx E_{t-1}^P\hat{p}_{t+k} + E_{t-1}^P\hat{mc}_{t+k}\ \forall\ k \geq 0 \tag{13}$$

Log-deviations in firm j's optimal reset price may be written as:

$$\hat{p}_t^*(j) = (1-\beta\theta)(E_{t-1}^P\hat{p}_t + \hat{mc}_t(j)) + \beta\theta E_{t-1}^P\hat{p}_{t+1}^* \tag{14}$$

All firms face the same price-setting problem, just observe a different marginal cost signal. Aggregating across all firms yields:

[7] This assumption is motivated by the observation that firm-specific price changes are much more volatile as compared to fluctuations in the aggregate price-level. This modeling framework is based on Nimark (2008).

$$\hat{p}_t^* = (1-\beta\theta)(E_{t-1}^P \hat{p}_t + \hat{mc}_t) + \beta\theta E_{t-1}^P \hat{p}_{t+1}^* \tag{15}$$

Under a Calvo-probability of price adjustment of $(1-\theta)$, evolution of the aggregate price index satisfies:

$$\hat{p}_t = \theta \hat{p}_{t-1} + (1-\theta)\hat{p}_t^* \tag{16}$$

Combining Equation 15 and Equation 16 yields the aggregate supply condition in terms of the aggregate inflation rate $\pi_t = \hat{p}_t - \hat{p}_{t-1}$:

$$\pi_t = \beta E_{t-1}^P \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{mc}_t + \frac{(1-\theta)^2(1-\beta\theta)}{\theta}(E_{t-1}^P \hat{mc}_t - \hat{mc}_t) \tag{17}$$

The expectational errors manifest due to the fact that the economy-wide real marginal cost is not perfectly forecast-able. If the firms' private signal revealed $\hat{mc}_t$ perfectly, we would go back to the standard New-Keynesian Phillips curve formulation and the expectational error term goes away. Also, it can be seen that the smaller the degree of price-stickiness (θ), the larger the contribution of these errors to actual inflation.

Pro-cyclicality of marginal cost The economy-wide real marginal cost mc_t is unobservable. Agents can only infer it from the realizations of macroeconomic aggregates. The structural dynamics driving the economy-wide component of real-marginal costs are unknown, but agents posit a pro-cyclical relationship: when output is above the flex-price level, it puts upward pressure on real marginal costs across all firms. Variation in $\hat{mc}_t$ above and beyond this pro-cyclical channel are attributed to economy-wide supply shocks, s_t. Hence, they perceive the following structural relationship:

$$\hat{mc}_t = \gamma \tilde{y}_t + s_t$$

where s_t denotes the supply shock, assumed to follow a mean-zero stationary process and independent of the unemployment-gap $\tilde{y}_t$. $\gamma > 0$ represents the link between economy-wide real marginal cost and the cyclical fluctuations in unemployment. It is assumed to be constant, but unknown.

The agents' reasoning motivates the following forward-looking Phillips curve:

$$\pi_t = \beta E_{t-1}^P[\pi_{t+1}] + \psi \tilde{y}_t + \xi_t \tag{18}$$

where (i) $E^P_{t-1}[\pi_{t+1}]$ denotes the private-sector's expectation of inflation in period $t+1$ conditional on history of signals $\mathcal{H}^{t-1}$ (ii) ψ represents the sensitivity of inflation to current output-gap. Agents assume κ is constant but unknown and hence learn about it every period. (iii) ξ_t is a cost-push shock composed of the supply-shock s_t and the expectational error.

The $\psi\tilde{y}_t$ term describes the contemporaneous link between inflation and the "real" side of the economy. The output gap, $\tilde{y}_t$ contemporaneously affects inflation through the slope parameter ψ. The relationship implies that output, when above its natural level ("potential") exerts inflationary pressure on the economy.

The cost-push shock ξ_t captures the component of inflationary pressures arising from factors over and beyond real economic activity. Agents allow for the possibility that the cost-push shocks have a persistent component. Hence they decompose the cost-push shock, ξ_t to into a *persistent* AR(1) component denoted by ξ^p_t, with large persistence ρ and a *transitory* i.i.d normal component ξ^T_t.

$$\xi_t = \xi^p_t + \xi^T_t, \quad \xi^T_t \sim iid\, N(0, \sigma^2_{\xi,T})$$

The persistent component is perceived to evolve as:

$$\xi^p_t = \rho\xi^P_{t-1} + \varepsilon^P_t, \quad \varepsilon^P_t \sim iid\, N(0, \sigma^2_{\xi,P})$$

Only the persistent component ξ^p_t affects future outcomes. Agents must learn about this shock to form expectations about the future. ξ^T_t is irrelevant for dynamics and just functions as noise.

Natural rate hypothesis: Agents in the model a-priori assume that monetary policy cannot systematically drive real variables once prices and expectations fully adjust. Under the natural rate hypothesis, the output and the real interest rate attain their natural levels in the long-run, beyond the control of monetary policy. This is equivalent to agents assuming that the discount-factor $\beta \to 1$ so that expectations-augmented Phillips curve is vertical in the long-run.[8] Agents reason that in the long run, monetary policy can only sustain non-zero inflation, not output gap.

2.2.3 Measuring real-economic activity

Economic slack, as measured by the *output gap* is an important variable driving aggregate inflation and forms the important structural link between interest-rate policy and aggregate inflation. However, it is not directly observable. It must be inferred based on real time indicators

[8] Since otherwise, the long-run unemployment-gap must be non-zero from Equation 20.

of economic activity. I assume that the only real indicator in this economy, informing agents about the state of aggregate demand is the unemployment rate. This is due to the unemployment rate being a salient variable, directly part of the dual mandate that most modern central banks in the world, including the Fed follow. For this reason, it is useful to re-frame the analysis in terms of unemployment gaps, defined as the difference between the realized unemployment rate and the one that would prevail under flexible prices, or the "natural" rate of unemployment.

In the model, the agents posit a linear relationship between the output gap and the unemployment gap[9] defined as the difference between the unemployment rate and the natural rate of unemployment (empirically, the "trend" component of unemployment rate): $x_t = u_t - u_t^n$, we have

$$\tilde{y}_t = -\eta x_t$$

where η is referred to as the Okun's law coefficient.

This yields the following dynamic IS-curve representing the law of motion of the unemployment-gap:

$$x_t = E^P_{t-1} x_{t+1} + \tau(i_t - E^P_{t-1}\pi_{t+1} - r_t^n) \tag{19}$$

where $\tau = 1/(\sigma\eta)$ represents the ratio of the intertemporal elasticity of substitution and the Okun's law coefficient. τ is assumed to be constant and known to all agents.

Similarly, the expectations-augmented Phillips curve in terms of unemployment gap becomes:

$$\pi_t = E^P_{t-1}\pi_{t+1} - \kappa x_t + \xi_t \tag{20}$$

where $\kappa = \psi\eta$ is assumed to be a constant that the agents learn about.

3 Learning: Latent structure of the economy

Every period, conditional on the observed unemployment rate u_t, all agents infer the contemporaneous unemployment gap, x_t and update their beliefs about the natural rate of interest r_t^n. Conditional on these, agents update their beliefs about the slope of the Phillips curve κ, and the persistent supply shock ξ_t^p based on the observed aggregate inflation rate π_t. Thus, they inform themselves regarding all the latent structural variables describing economic dynamics in their

[9]This relationship is sometimes referred to as Okun's law. Ball et al. (2017) find that it has been relatively stable across time.

model, every period. The rest of the section discusses the mechanics of their learning behavior.

3.1 Learning and the Anticipated Utility assumption

This paper takes the anticipated utility approach to learning described in Kreps (1998). Under this assumption, agents form "certainty-equivalent" estimates of the relevant state-variables every period and treat their best estimates as if they were constants known with certainty when making decisions. Agents disregard the possibility of updating their current estimates in the future. This assumption is widely used in the literature on learning in macroeconomics (see Evans and Honkapohja (2001)). It allows us to surmount the computational challenges associated with solving the dynamic problem in the full Bayesian setting. Cogley and Sargent (2008) demonstrate how the anticipated utility assumption provides an excellent approximation to Bayesian decision-making in a wide class of problems.

3.2 Inferring unemployment gap x_t

Conditional on the observed unemployment rate u_t, agents update their estimate of the natural rate u_t^n according to the univariate smoothing algorithm:

$$u_t^n = u_{t-1}^n + g_u(u_t - u_{t-1}^n) \tag{21}$$

The natural rate is inferred from a backward-looking filter based on the unemployment rate, putting more weight on recent observations. The gain coefficient g_u determines the degree to which shifts in the unemployment rate are attributed to shifts in the natural rate of unemployment. g_u is constant and known to all agents.

From the natural rate hypothesis, $\lim_{h\to\infty} x_{t+h} = 0$. Hence, the unemployment rate is always expected to converge to the natural rate u_t^n in the long-run equilibrium, consistent with the update-rule Equation 21.[10]

[10] Staiger, Stock and Watson (1997) and Orphanides and van Norden (2002) show that such univariate algorithms deliver estimates that are essentially indistinguishable from the ones derived under more sophisticated procedures. Similarly, Kamber, Morley and Wong (2018) demonstrate that economic slack-measures derived from similar univariate procedures can be robust to revisions and are found to be in line with more refined measures of real-time economic activity (such as the Chicago Fed National Economic Activity Index).

Agents' estimate of the contemporaneous unemployment gap is:

$$x_t = u_t - u_t^n$$

I abstract from issues of real-time mis-measurement and data revisions. I do however find that the agents' real-time measure inferred from data lines up quite well with series that are revised retroactively (the CFNAI index as an example, see Appendix D) suggesting that these issues are not of large consequence over the sample period.

3.3 Recursive updating

Agents' perceived model of aggregate dynamics: The equations describing the aggregate laws of motion of the economy from the perspective of the private agents are:

$$x_t = E_{t-1}^P[x_{t+1}] + \tau(i_t - E_{t-1}^P[\pi_{t+1}] - r_t^n)$$

$$\pi_t = E_{t-1}^P[\pi_{t+1}] - \kappa x_t + \xi_t^p + \xi_t^T, \quad \xi_t^T \sim iid\, N(0, \sigma_{\xi,T}^2)$$

$$\xi_t^p = \rho \xi_{t-1}^P + \varepsilon_t^P \ , \quad \varepsilon_t^P \sim iid\, N(0, \sigma_{\xi,P}^2)$$

Conditional on the observed aggregate inflation rate π_t and the unemployment gap x_t inferred from u_t based on the de-trending procedure, agents update their beliefs regarding κ, ξ_t^p and r_t^n recursively every period treating $[\sigma_{\xi,T}^2, \sigma_{\xi,P}^2, \tau, \rho]$ as known constants.

Learning the natural rate of interest r_t^n: Conditional on the inferred unemployment-gap x_t, the contemporaneous estimate of the natural rate of interest $r_{t|t}^n$ is given by the dynamic IS curve (Equation 19):

$$r_{t|t}^n = \tau^{-1}(E_{t-1}^P x_{t+1} - x_t) + i_t - E_{t-1}^P \pi_{t+1}$$

Further, agents assume that r_t^n follows a Martingale, so that $r_{t+1|t}^n = r_{t|t}^n$

Learning ξ_t^p and κ: Given the unemployment-gap x_t, beliefs about the persistent component of the cost-push shock ($\xi_{t|t}^p$) and the slope of the Phillips curve ($\kappa_{t|t}$) are recursively updated every period based on observed aggregate inflation rate π_t.

Agents face a signal-extraction problem in real time: they cannot immediately distinguish whether an inflation surprise was driven by the unemployment-gap (due to mis-perception of the slope κ), persistent supply shocks—relevant for dynamics—or transitory supply shocks, irrelevant for dynamics.

The observation equation, relating aggregate inflation to ξ_t^p and κ in the agents' perceived model is given by:

$$\pi_t - E_{t-1}^P \pi_{t+1} = \begin{pmatrix} -x_t & 1 \end{pmatrix} \begin{pmatrix} \kappa \\ \xi_t^p \end{pmatrix} + \xi_t^T, \quad \xi_t^T \sim N(0, \sigma_{\xi,T}^2).$$

The state-transition equation specifying the agents' prior regarding how the state variables κ and ξ_t^p evolve over time is given by:

$$\begin{pmatrix} \kappa \\ \xi_t^p \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} \kappa \\ \xi_{t-1}^p \end{pmatrix} + \begin{pmatrix} 0 \\ \epsilon_t^P \end{pmatrix}, \qquad \epsilon_t^P \sim N(0, \sigma_{\xi,P}^2)$$

Note that the state-transition equation reflects the agents' prior that the true slope κ is time-invariant.

4 Learning: Policymakers' stabilization preferences

Given their beliefs regarding the latent structural variables - the slope of the Phillips curve $\kappa_{t|t}$, the persistent supply shock $\xi_{t|t}^p$ and the natural rate of interest $r_{t|t}^n$, the agents interpret the period-t policy decision i_{t+1} and learn about the policymakers' stabilization preferences underlying policy behavior. The next subsection describes the policymakers' optimal policy problem under a dual mandate, based on Svensson and Woodford (2004).

4.1 Monetary policy

The private-agents assume that the central bank conducts monetary policy optimally under discretion, having the same beliefs about the policy-invariant latent structural variables as them formed based on the same economic model and data. The central bank's *dual mandate* characterized by the following loss function[11]:

$$\mathcal{L}_t = \sum_{j=1}^{\infty} [(\pi_{t+j|t} - \pi_t^*)^2 + \lambda_t x_{t+j|t}^2] \tag{22}$$

where $\pi_{t+j|t}$ and $x_{t+j|t}$ denote the central bank's forecasts of inflation and the unemployment gap for period $t+j$ formed in period-t after the contemporaneous π_t, x_t have been realized. Here, π_t^* and λ_t represent the policymakers' period-t stabilization preferences: their inflation target and the relative weight they place on unemployment-gap stabilization. Private agents do not observe these and assume that they are subject to slow innovations over time. Their beliefs regarding these parameters conditional on $\mathcal{H}^t$ are denoted by π_t^{*P} and λ_t^P.

The central bank in its period-t decision cycle chooses the one-period ahead unemployment gap plan $x_{t+1|t}$. Private agents' period-t expectations regarding future inflation and unemployment gaps formed prior to observing the policy decision (denoted by $E_t^{P-}\pi_{t+j}$ and $E_t^{P-}x_{t+j}$) are observed by the central bank.[12] Every period, the central bank chooses $x_{t+1|t}$ which solves its period-t static optimization problem taking the observed private-sector expectations as given, conditional on the following structural constraint implied by the Phillips curve:

$$\pi_{t+1|t} = E_t^{P-}\pi_{t+2} - \kappa_{t|t} x_{t+1|t} + \xi_{t+1|t} \tag{23}$$

The optimal unemployment gap plan $x_{t+1|t}$ is given by the targeting rule:

$$x_{t+1|t} = \frac{\kappa_{t|t}}{\lambda_t}(\pi_{t+1|t} - \pi_t^*) \tag{24}$$

Equation 24 describes the textbook "lean against the wind" policy solution where the central bank reacts by creating a positive unemployment gap if it expects inflation to be above its target in the following period.

Using Equation 23 and Equation 24, the central bank's one-period ahead inflation and unemployment gap plan must satisfy:

$$\pi_{t+1|t} = \pi_t^* + \frac{\lambda_t}{\lambda_t + \kappa_{t|t}^2}(E_t^{P-}\pi_{t+2} + \xi_{t+1|t} - \pi_t^*) \tag{25}$$

[11] Agents assume that the central bank's policy preferences are not fundamentally incompatible. There is no stabilization bias.

[12] E_t^{P-} denote expectations formed after the realization of the contemporaneous aggregates $z_t = [\pi_t, u_t]$ but before the period-t policy decision, and are thus conditional on beliefs $\kappa_{t|t}, \xi_{t|t}^p, \pi_{t-1}^{*P}, \lambda_{t-1}^P$.

$$x_{t+1|t} = \frac{\kappa_{t|t}}{\lambda_t + \kappa_{t|t}^2}(E_t^{P-}\pi_{t+2} + \xi_{t+1|t} - \pi_t^*) \tag{26}$$

Equation 25 shows that when $\lambda_t > 0$, the central bank allows inflation to rise above its target when faced with a persistent cost-push shock. Deviations of inflation from the target, arising from private agents' inflation expectations and persistent cost-push shocks are accommodated to an extent. The larger λ_t is, the greater the accommodation, with $\lambda_t \to \infty$ characterized by the scenario where the central bank can no longer push back against private agents' expectations anymore. This represents an "expectations-trap" in the sense of Christiano and Gust (2000) that policymakers fall into where any real-side adjustment is deemed too costly. In the same vein, Equation 26 shows that the extent of unemployment gap adjustments that policymakers will engage in, is decreasing in λ_t.[13]

4.2 Private sector expectations in a learning equilibrium

Under rationality, private-agents' expectations for all future horizons must be consistent with their structural model of inflation dynamics[14]:

$$E^P[\pi_{t+j}] = E^P[\pi_{t+j+1}] - E^P[\kappa x_{t+j}] + E^P[\xi_{t+j}] \; \forall \, j \geq 1$$

The constraint above yields the following forward looking system of equations that their expectations at any period t must satisfy:

$$E_t^P \pi_{t+j} = E_t^P \pi_{t+j+1} - \kappa_{t|t} E_t^P x_{t+j} + \rho^j \xi_{t|t}^p$$

From the "lean against the wind" targeting rule, their unemployment gap expectations are:

$$E_t^P x_{t+j} = \frac{\kappa_{t|t}}{\lambda_t^P + \kappa_{t|t}^2}(E_t^P \pi_{t+j+1} + \rho^j \xi_{t|t}^p - \pi_t^{*P}) \tag{27}$$

Solving forward, we get an analytical expressions describing their inflation and unemployment gap expectations for all future horizons in terms of their beliefs $\kappa_{t|t}, \xi_{t|t}^p, \pi_t^{*P}, \lambda_t^P$ formed condi-

[13]The expression $\frac{\kappa_{t|t}}{\lambda_t + \kappa_{t|t}^2}$ is bounded above by $\frac{1}{2\sqrt{\lambda_t}}$. So even though it also depends on κ_t, its limiting behavior is determined by λ_t.

[14]Here, I implicitly assume that agents' beliefs are consistent with the law of iterated expectations.

tional on $\mathcal{H}^t$:

$$E_t^P \pi_{t+j} = \underbrace{\pi_t^{*P}}_{\text{perceived target}} + \underbrace{\frac{\lambda_t^P}{\lambda_t^P(1-\rho)+\kappa_{t|t}^2}\rho^j \xi_{t|t}^p}_{\text{cost-push inflation}} \tag{28}$$

$$E_t^P x_{t+j} = \frac{\kappa_{t|t}}{\lambda_t^P(1-\rho)+\kappa_{t|t}^2}\rho^j \xi_{t|t}^p \tag{29}$$

Their expectations reflect simple economic intuition. According to Equation 28, from the private-agents' perspective, the path of inflation is ultimately determined by two forces: (i) π_t^{*P}, representing their beliefs about the central bank's inflation target, defines the level that the agents expect inflation to converge to in the long run. Inflation expectations at all horizons move one-for-one with π_t^{*P}. Hence this belief is a crucial determinant of the low-frequency (slow-moving) dynamics of inflation; and (ii) persistent supply shocks $\xi_{t|t}^p$, which create involuntary inflation above and beyond this voluntary level in the short-run: the extent of their *pass-through* to short-run inflation expectations is *decreasing* in the perceived slope of the Phillips curve $\kappa_{t|t}$ but *increasing* in λ_t^P, the perceived weight that the central bank puts on unemployment-gap stabilization. Thus, when agents perceive the central bank to prioritize real-side stabilization (λ_t^P is large), their short-run inflation expectations respond to persistent supply shocks strongly and causes them to depart further away from the long-run level, π_t^{*P}.

Perceived policy rule: The interest-rate i_{t+1}^* that implements the optimal unemployment gap plan is given by the IS-curve:

$$i_{t+1}^* = \tau^{-1}(x_{t+1|t} - E_t^{P-} x_{t+2}) + E_t^{P-}\pi_{t+2} + r_{t+1|t}^n$$

The agents' ex-ante period-t expectations $E_t^{P-}\pi_{t+j}$ and $E_t^{P-}x_{t+j}$ are given by:

$$E_t^{P-}\pi_{t+j} = \pi_{t-1}^{*P} + \frac{\lambda_{t-1}^P}{\lambda_{t-1}^P(1-\rho)+\kappa_{t|t}^2}\rho^j \xi_{t|t}^p$$

$$E_t^{P-}x_{t+j} = \frac{\kappa_{t|t}}{\lambda_{t-1}^P(1-\rho)+\kappa_{t|t}^2}\rho^j \xi_{t|t}^p$$

4.3 Monetary policy surprises

The agents assume that the central bank sets interest rates optimally every period up to some idiosyncratic policy error of constant variance so that[15]:

$$i_{t+1} = i^*_{t+1} + \epsilon^i_t, \quad \epsilon^i_t \sim N(0, \sigma^2_i)$$

Whenever the nominal interest rate that the central bank sets is different from what the private agents expect-given their beliefs about prevailing macroeconomic conditions—it constitutes a monetary policy surprise:

$$\underbrace{i_{t+1} - E^P[i_{t+1}|\mathcal{H}^{t-1}, z_t]}_{\text{monetary policy surprise}} = \underbrace{(i^*_{t+1} - E^P[i_{t+1}|\mathcal{H}^{t-1}, z_t])}_{\text{agents behind the curve}} + \underbrace{\epsilon^i_t}_{\text{policymakers behind the curve}}$$

where $\mathcal{H}^{t-1}$ represents the history of all observed signals up to $t-1$, and z_t captures the current period realizations of inflation and unemployment rate.

Agents interpret policy surprises as stemming from two sources: an evolution in policymakers' preferences, which they must learn about ('agents behind the curve'), or a transitory policy error, which policymakers must subsequently correct ('policymakers behind the curve').

From the IS-curve and the optimal unemployment gap plan, we obtain the following expression mapping the policymakers' period-t preferences to the optimal nominal interest rate:

$$i^*_{t+1}(\pi^*_t, \lambda_t) = \underbrace{-\tau^{-1} E^{P-}_t x_{t+2} + r^n_{t|t} + E^{P-}_t \pi_{t+2}}_{\text{ex-ante known}} + \underbrace{\frac{\tau^{-1}\kappa_{t|t}}{\lambda_t + \kappa^2_{t|t}} (E^{P-}_t \pi_{t+2} + \rho \xi^p_{t|t} - \pi^*_t)}_{\text{contains information about Fed objectives}}$$

When the central bank sets interest rates, it reveals information regarding its policy preferences. The second term represents the only component of systematic monetary policy that contains information regarding the "true" period-t policy preferences π^*_t and λ_t. Observe that as λ_t increases, the term carrying information regarding the policymakers' stabilization preferences shrinks.

[15]The source of this error could be motivated as coming from cognitive noise in decision-making. It could also be thought of as a "control error". The main idea being that the private agents allow for idiosyncratic deviations from optimal policy arising from factors unrelated to changes in the policymakers' stabilization preferences.

In the limiting case of $\lambda_t \to \infty$, this term shrinks to zero and interest rates carry no information: the "learning channel" of policy completely dies out. When agents perceive the central bank to have such preferences, they expect the unemployment gap to be zero at all future horizons and the central bank is seen to implement a Wicksellian policy: with the the ex-ante real interest rate $i^*_{t+1} - E^{P-}_t \pi_{t+2}$ tracking the perceived natural rate of interest $r^n_{t|t}$ closely. In this case, policymakers are expected to tolerate arbitrary deviations in inflation: there is no inflation-target and the level of inflation is indeterminate, leading to self-fulfilling dynamics. On the other hand, when $\lambda^P_t \to 0$, agents' always expect the central bank to bring inflation back to its target π^*_t regardless of the unemployment gap costs. Hence their inflation expectations at all horizons are equal to the perceived inflation target, irrespective of supply shocks $\xi^p_{t|t}$. Policy is seen oriented only towards keeping inflation close to target. Interest rates provide a strong signal regarding the inflation target. Note that policy response to $E^{P-}_t x_{t+2}$ and $r^n_{t|t}$ does not depend on λ_t - since they don't pose a policy trade-off.

4.4 The nonlinear filter

The observed nominal interest rate functions as a noisy signal about the central bank's preferences $[\pi^*_t, \lambda_t]$, which describe the relevant state that the agents learn about:

$$i_{t+1} = i^*_{t+1}(\pi^*_t, \lambda_t) + \epsilon^i_t, \quad \epsilon^i_t \sim N(0, \sigma^2_i)$$

Linearizing around agents' prior estimates $\pi^P_{t-1}, \lambda^P_{t-1}$ assuming $\pi^*_t \approx \pi^P_{t-1}, \lambda_t \approx \lambda^P_{t-1}$[16]

$$i^*_{t+1}(\pi^*_t, \lambda_t) \approx i^*_{t+1}(\pi^{*P}_{t-1}, \lambda^P_{t-1}) + \frac{\partial i^*_{t+1}}{\partial \pi^*_t}|_{\pi^{*P}_{t-1}, \lambda^P_{t-1}}(\pi^*_t - \pi^{*P}_{t-1}) + \frac{\partial i^*_{t+1}}{\partial \lambda_t}|_{\pi^{*P}_{t-1}, \lambda^P_{t-1}}(\lambda_t - \lambda^P_{t-1})$$

The approximation above (called the Extended Kalman Filter) yields an expression for i^*_{t+1} that is linear in the latent states π^*_t and λ_t much like the simple Kalman filter. The key difference is that the coefficients mapping the observable i^*_{t+1} to the latent states are not constant, but state-dependent. They are determined by the partial derivatives of the interest rate rule with respect to the π^*_t and λ_t evaluated at the agents' prior estimates π^{*P}_{t-1} and λ^P_{t-1}. These determine how "informative" interest rates are perceived to be regarding the underlying stabilization preferences of the policymakers.

[16] This assumption is important for the filtering problem to be well-specified. Since interest-rate dynamics would be state-dependent due to the nonlinearity of i^*_{t+1}, large discrepancies can cause the filter to diverge. The assumption implicitly implies that under their own beliefs, agents track changes in the policymakers' preferences efficiently at all times. In Appendix G, I report evidence in support of this assumption based on the Fed's Greenbook forecasts.

The agents perceive that $\log \pi_t^*$ and $\log \lambda_t$ evolve exogenously as independent random walks, so that:

$$\begin{pmatrix} \log\ \pi_t^* \\ \log\ \lambda_t \end{pmatrix} = \begin{pmatrix} \log\ \pi_{t-1}^* \\ \log\ \lambda_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^{\pi^*} \\ \epsilon_t^{\lambda} \end{pmatrix}, \qquad \begin{pmatrix} \epsilon_t^{\pi^*} \\ \epsilon_t^{\lambda} \end{pmatrix} \sim N(\mathbf{0}, \begin{pmatrix} \sigma_{\pi^*}^2 & 0 \\ 0 & \sigma_{\lambda}^2 \end{pmatrix})$$

Every period, agents update their beliefs π_t^{*P} and λ_t^P based on their best-estimates as:

$$\pi_t^{*P} = \pi_{t|t}^*, \quad \lambda_t^P = \lambda_{t|t}$$

4.5 What anchors long-run beliefs?

Anchoring reflects the entrenchment of expectations, where beliefs about the future remain resilient to surprises. The following quote by Ben Bernanke summarizes what anchoring means in the context of monetary policy:

> "...I use the term "anchored" to mean relatively insensitive to incoming data. So, for example, if the public experiences a spell of inflation higher than their long-run expectation, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored..."

According to Equation 28, agents' long-run inflation expectations are completely pinned down by their beliefs regarding the central bank's inflation target. For inflation expectations to be "well-anchored" according to this definition, agents should not revise their beliefs regarding the target by much even as they anticipate inflation to remain elevated in the short-run on account of persistent supply shocks. In the model, revisions to agents' long-run inflation expectations occur only in response to interest rate surprises. The degree of *pass-through* from these surprises to their perceived inflation target depends on how informative they perceive policy actions to be regarding shifts in the central bank's inflation target π_t^*.

4.5.1 Anchoring: An information-theoretic perspective

The interest rates that the central bank sets serve as the only signal carrying information about the underlying π_t^* and λ_t that "generated" it. Based on their views about how policymakers operate, the volume of information regarding these parameters contained in a signal i_{t+1} would be given by the Fisher information increment. Fisher information tells us how well an observation is able to "locate" a parameter. The marginal increase in the Fisher information upon receiving a signal is the Fisher information increment, $\mathcal{I}_t$. It is given by:

$$\mathcal{I}_t = \frac{1}{\sigma_i^2} H_t H_t'$$

where H_t is the Jacobian of i_{t+1}^*, evaluated at their ex-ante beliefs:

$$H_t = \left[\frac{\partial i_{t+1}^*(\pi_t^*, \lambda_t)}{\partial \pi_t^*}, \frac{\partial i_{t+1}^*(\pi_t^*, \lambda_t)}{\partial \lambda_t} \right]_{\pi_{t-1}^{*P}, \lambda_{t-1}^P}$$

$\mathcal{I}_t$ is decreasing in σ_i^2, the variance of the idiosyncratic noise term. It also depends on the partial derivatives of the interest-rate function, evaluated at their ex-ante beliefs, given by:

$$\frac{\partial i_{t+1}^*}{\partial \pi_t^*}|_{(\pi_{t-1}^{*P} \lambda_{t-1}^P)} = -\frac{\tau^{-1}\kappa_{t|t}}{\lambda_{t-1}^P + \kappa_{t|t}^2} \tag{30}$$

Equation 30 implies that interest rates fall in response to an exogenous increase in the policymakers' inflation target. Thus when interest rates undershoot expectations, agents revise their beliefs regarding the inflation target upwards.

$$\frac{\partial i_{t+1}^*}{\partial \lambda_t}|_{(\pi_{t-1}^{*P}, t\lambda_{t-1}^P)} = -\frac{\tau^{-1}\kappa_{t|t}}{(\lambda_{t-1}^P + \kappa_{t|t}^2)[\lambda_{t-1}^P(1-\rho) + \kappa_{t|t}^2]} \rho \xi_{t|t}^p \tag{31}$$

Equation 31 tells us that the degree to which interest rates respond to persistent supply shocks is decreasing in λ_t. When interest rates are observed to co-move more strongly with supply shocks than agents anticipate, they lower their beliefs about λ_t.

The magnitude of shifts in agents' beliefs π_t^{*P}, λ_t^P: The factor $-\frac{\tau^{-1}\kappa_{t|t}}{\lambda_{t-1}^P + \kappa_{t|t}^2}$ is common to both the partial derivatives and determines the *perceived* sensitivity of interest rates to small changes in either π_t^* or λ_t. If agents believe this term to be small, the beliefs π_t^{*P} and λ_t^P move more inertially. The expression implies that the degree to which agents hold on to their beliefs about the Fed's behavior (or the degree of anchoring) is increasing in λ_{t-1}^P, i.e **the more weight the agents perceive policy to attach to real-side stabilization, the less volatile their beliefs about the inflation target would be.** The large disutility from creating unemployment-gaps serves to restrain policymakers from pursuing expansionary policy even if, for some exogenous reasons their inflation target π_t^* were to rise.

The direction of shifts in agents' beliefs π_t^{*P}, λ_t^P: The orientation of the vector H_t determines the direction in the $\pi^* - \lambda$ space that the observation i_{t+1} points toward, and hence, the direction that the agents' beliefs π_t^{*P}, λ_t^P would move in, upon observing i_{t+1}. We can represent the strength of the signaling channel with respect to λ_t as:

$$\chi_t = \frac{1}{\lambda_{t-1}^P + \frac{\kappa_{t|t}^2}{1-\rho}} \frac{\rho \xi_{t|t}^p}{1-\rho}$$

When χ_t is small, the interest rate surprise is perceived to mostly carry information regarding shifts in π_t^*. Clearly, this happens if either ρ or $\xi_{t|t}^p$ are small so that there is no policy trade-off. The larger and the more persistent[17] the perceived supply shocks are, the more the agents' beliefs move in the λ direction.

5 Estimation

To characterize the path of beliefs and expectations, we must pin down the vector of parameters:

$$\Phi = [\tau, g_u, \sigma_{\xi,P}^2 / \sigma_{\xi,T}^2, \sigma_i^2, \sigma_{\pi^*}^2, \sigma_\lambda^2]$$

These parameters govern how private agents update beliefs conditional on observations. Given a vector of parameters Φ and a history of realizations $\mathcal{H}^t$, the path of private-sector beliefs regarding the state of the economy $\{\kappa_t, \xi_t^p, u_t^n, r_t^n\}$ as well as the central bank's policy preferences $\{\pi_t^*, \lambda_t\}$ are determined under the learning algorithm described earlier.

I set the parameter ρ governing the persistence of the persistent component of the supply shocks as perceived by the agents to 0.99, so that it is meaningfully distinct from the transitory i.i.d component so that it doesn't pose identification problems. It also captures a particularly low-frequency component of the supply shocks which would be the most relevant for forming expectations.

5.1 Data

The learning algorithm is implemented with monthly data on Headline CPI Inflation (CPIAUCSL), Unemployment rate (UNRATE) and the Effective Fed Funds rate (FEDFUNDS)

[17]As $\rho \to 1$, for finite λ_{t-1}^P, $\chi_t \to \xi_{t|t}^p / \kappa_{t|t}^2$

from 1958:01-2024:12 (available from FRED St Louis) as observations. For the zero-lower bound period, the Wu-Xia shadow interest rate is used as the measure of the nominal interest rate instead of the effective Fed funds rate. The initial beliefs are set to have large variance in order to allow them to adjust freely in response to data. I allow 48 periods (4 years) for the filter to converge to steady-state and the estimates from this period are discarded.

To discipline the estimates, I use survey data on long-run inflation expectations from the Survey of Professional Forecasters (SPF)—specifically, the cross-sectional mean of the 10-year-ahead CPI inflation forecasts—as a proxy for private agents' beliefs about the Federal Reserve's inflation target.[18] The SPF series is available at a quarterly frequency over the period 1991–2024. The implicit assumption here is that professional forecasters' long-run inflation expectations are broadly representative of those held by firms and households. Given the low volatility of long-run inflation expectations during this period, this assumption is reasonable: discrepancies between types are unlikely to be large.[19]

Although alternative measures covering a longer sample period exist such as the Michigan Survey of Consumers and Cleveland Fed's 10-year ahead inflation expectations series, they are not used in the baseline estimation. However, they are employed in an out-of-sample validation test (see Appendix A) to assess the robustness of the results. Not explicitly including long-run inflation expectations data for the Great Inflation period allows us to test if the structural dynamics captured by the model can provide a consistent explanation for the expectations-formation process during this period without explicitly being fit to the data.

5.2 Estimation results

The goal of estimation is to pin down the vector Φ that generates a path of long-run expectations closest to the observed survey measure under the agents' learning mechanism given their observations. A least-squares minimum-distance estimator is used, matching the path of model-implied private-sector beliefs regarding the inflation target π_t^{*P} with the observations from the SPF. The minimum-distance estimate Φ^* is the solution of the resulting minimization problem:

[18] Short-run inflation forecasts depend upon a combination of π_t^{*P}, λ_t^P and $\xi_{t|t}^p$, which makes identification much more difficult. Long-run inflation expectations have the advantage that they isolate shifts in π_t^{*P} alone.

[19] For a relatively short sample covering the recent surge, data from Survey of Firms' Inflation Expectations (SoFIE) from the Cleveland Fed is available, which explicitly includes firm managers' perceptions about the Fed's target. This series is in extremely close agreement with the SPF series used.

$$\Phi^* = \underset{\Phi}{\text{argmin}} \sum_{t=1991:01}^{2024:12} ([\pi_t^{*P}|\mathcal{H}^t, \Phi] - \pi_t^{10y,SPF})^2$$

I discard values of Φ that generate explosive paths for beliefs. This problem arises because beliefs are a non-differentiable function of Φ: even small perturbations in the parameters can induce large changes in the updating process across all variables, setting off some beliefs on divergent paths.[20]

The point-estimate of Φ^* along with 95% credibility intervals generated using Sequential Monte Carlo (SMC) sampling are shown in Table 1. The point-estimate implies that the variance of innovations in the policy preference parameters is very small compared with the idiosyncratic policy errors. The innovations to persistent cost-push shocks are also approximately 10^3 times smaller than i.i.d. innovations.

Table 1: Estimated parameters of the baseline model

Parameter	**Description**	**Estimate (95% CI)**
τ	Ratio of IES to Okun's law coefficient	$0.647\ (0.430,\ 1.184)$
g_u	Gain for updating u_t^n	$0.329\ (0.209,\ 0.351)$
σ_i^2	Variance of idiosyncratic monetary policy errors	$0.488\ (0.389,\ 1.334)$
$\sigma_{\pi^*}^2$	Variance of innovations to (log)inflation target	$6.696\ (5.186,\ 7.772) \times 10^{-4}$
σ_λ^2	Variance of innovations to (log)weight on unemployment-stabilization	$6.802\ (3.611,\ 9.173) \times 10^{-5}$
$\frac{\sigma_{\varepsilon^t}^2}{\sigma_{\varepsilon^p}^2}$	Ratio of the variance of innovations to the transitory component of cost-push shocks to that of the persistent component	$911.901\ (196.125,\ 5176.436)$

Note: Confidence intervals are constructed using Sequential Monte Carlo sampling. The likelihood function is not differentiable in the parameters Φ, so asymptotic standard errors cannot be used.

Appendix B compares the agents' ex-ante forecasts of inflation, unemployment rate and the nominal interest rate informed by their model with the actual realizations to assess the fit. The agents' perceived model is able to track the target variables closely and in this sense, delivers a satisfactory "fit" to their observations.

[20] This also makes the use of gradient-based optimization algorithms infeasible. I use the Nelder-Mead direct search method that does not require the computation of gradients or derivatives of the objective function.

6 Evolution of the agents' beliefs

This section discusses time-paths of private-sector agents' beliefs regarding the structural variables describing the economy as implied by the point estimates given in Table 1. The section also provides context on how beliefs evolved and their bearing on the macroeconomic outcomes in the post-war period.

6.1 The Fed's inflation target

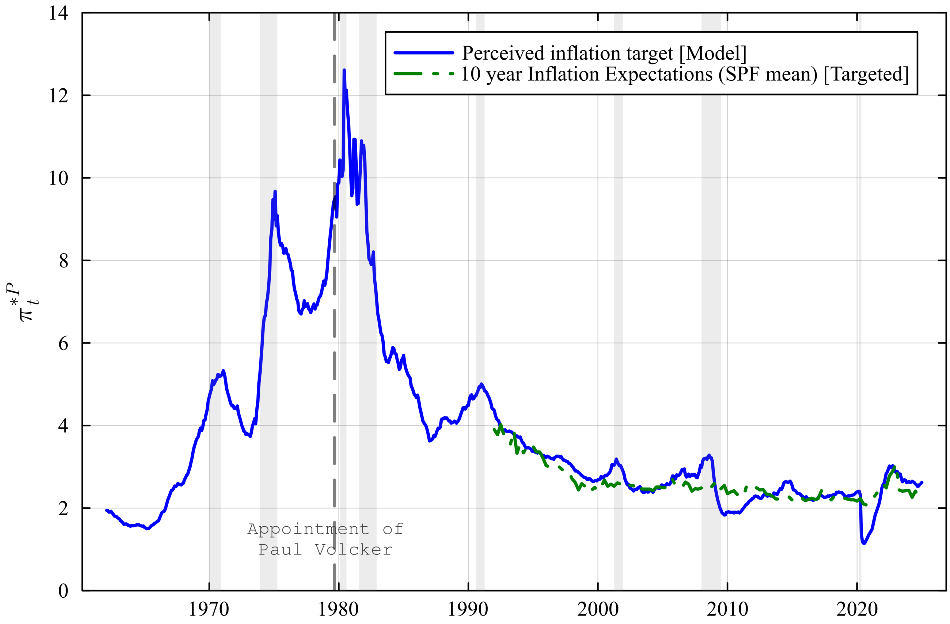


Figure 2: **Perception of the Fed's inflation target**

Note: Private agents' beliefs regarding the Fed's inflation target as implied by the model (π_t^{*P}) *(solid)* vs SPF 10-year inflation expectations *(dashed)* which serves as the empirical proxy. During the Great inflation, private-sector beliefs about the inflation target exhibit sharp upward revisions. This is not the case during the recent surge. The shaded regions indicate NBER recessions. The SPF 10-year ahead inflation expectations series begins in 1991:Q1 and is obtained from the Philadelphia Fed website.

Figure 2 illustrates the model-implied path of the agents' perceptions regarding the Fed's inflation target. The beliefs exhibit a significant degree of volatility prior to the mid-1980s, with sizable high-frequency movements.[21]

According to the model, the private agents perceive the Fed's inflation target to be close to 2%

[21]This is in contrast with papers such as Bhattarai, Lee and Park (2016) that rely on identifying the inflation target from low-frequency movements in inflation and identify smooth variation.

in the early 1960s. In the latter half of the decade, the perceived target begins to rise, sitting above 4% by 1970.[22]

From the eyes of the private sector, the 70s would be a period of great volatility in the Fed's inflation target. Their beliefs are sharply revised upwards on two occasions, each coinciding with an NBER recession—one during the beginning of the decade and the other around the time of the OPEC Embargo and the oil crisis of 1974. From the lens of the model, the private agents saw that the Fed had an *incentive* to inflate the economy, over and beyond any supply disturbances creating policy trade-offs. From their point of view, the Fed displayed a tendency to deliberately inflate the economy during recessions, operating with a higher inflation target, followed by dis-inflationary pursuits soon after. The Volcker-era, beginning with the chairmanship of Paul Volcker in 1979 is often cited as watershed moment in the Fed's eventual "conquest" of inflation. In October 1979, the Fed announced "new operating procedures" aimed at reining in inflation. But these do not immediately effect a disinflation (as also noted by Goodfriend and King (2005)). A recession follows and the perceived target shoots up yet again. It is not until 1982 that we see the perverse dynamics reverse course. The following years see a sharp decline in the perceived inflation target which coincides with a deep recession. By the mid 1980s, agents perceive the Fed's inflation target to be below 5% after nearly a decade. Thus the U.S. economy enters the "Great Moderation". Beliefs regarding the Fed's target are on a steady downward trajectory in the two decades that follow, under the chairmanship of Alan Greenspan (1987-2006). Only during the Great Financial crisis and the recent COVID-pandemic do we see the agents' perceptions exhibiting some modest revisions (interestingly, breaking away from the SPF series).

The model implies a modest uptick in the agents' perceptions regarding the Fed's inflation target during the post-pandemic episode, also observed in the SPF data, showing that there was some endogenous feedback from monetary policy in the post-pandemic period to private agents' long-run inflation expectations. Yet, beliefs about the target only rise to about 3%. This is remarkable considering that CPI inflation itself reached a peak of 9.1% in June 2022. The same learning mechanism that caused agents to anticipate inflation to remain put at levels as high as 11% during the 80s, told them that inflation was on course to settling around 3% - the inflation crisis left their long-run beliefs more or less unchallenged.

While the model implies that the conduct of policy created the perception among private agents

[22] Hence I find that the anchor had begun to loosen in the late 1960s, before the appointment of Arthur Burns (in 1970), similar to Reis (2021). In December 1965 a policy-tightening provoked an unprecedented backlash against Fed Chair Martin, marking what many observers regard as the beginning of a prolonged era of accommodative monetary policy.

that the inflation target had risen substantially during the 1970s, the model does not take a stand on what would cause the policymakers to acquire a taste for such exorbitant levels of inflation. Narrative evidence offered by Romer and Romer (2002), DeLong (1996) provide some valuable clues: inflation was deliberately pursued under the false notion that unemployment could be driven lower by inflationary surprises. Under this explanation, the policymakers operated with a disregard for the natural rate theory, instead viewing the Phillips curve as offering an exploitable long-run trade-off between inflation and unemployment. Indeed, counter-cyclical movements in the inflation target would be consistent with policymakers, misguided by a belief that the negative relationship between inflation and unemployment implied by the classical Phillips curve remains true in the long-run; and thus, in pursuit of lower unemployment they would be willing to tolerate higher inflation in the long-run. Another plausible story is that the policymakers simply ignored revisions in private sector inflation expectations, operating as if they remained more or less fixed or "anchored" while they were in fact highly responsive. The analysis in this paper does not however focus on *why* the policymakers set policy a certain way, but rather how their choices shaped the agents' beliefs and effected a self-fulfilling inflationary spiral.

Figure 3 plots the agents' short-run inflation expectations (one-year ahead, as implied by the model) along with their perceptions regarding the inflation target, or the level of inflation that they expected to prevail in the *long-run*. Owing to deflationary supply shocks corresponding to the oil glut in the mid-1980s, even as the agents expected inflation to be below 2.5% in the short-run, for the first time in nearly two decades, they still believed the Fed's inflation target to be near 4%. Hence, for a time, agents viewed the benign inflation as a temporary reprieve driven by the macroeconomic environment soon to be undone by policy. Over the recent period however, while short-run inflation expectations continued to be volatile, agents' beliefs about the target have held relatively stable, close to 2.5%. This shift has much to do with their views regarding the Fed's emphasis on minimizing real-side volatility, as we will see in the next subsection.

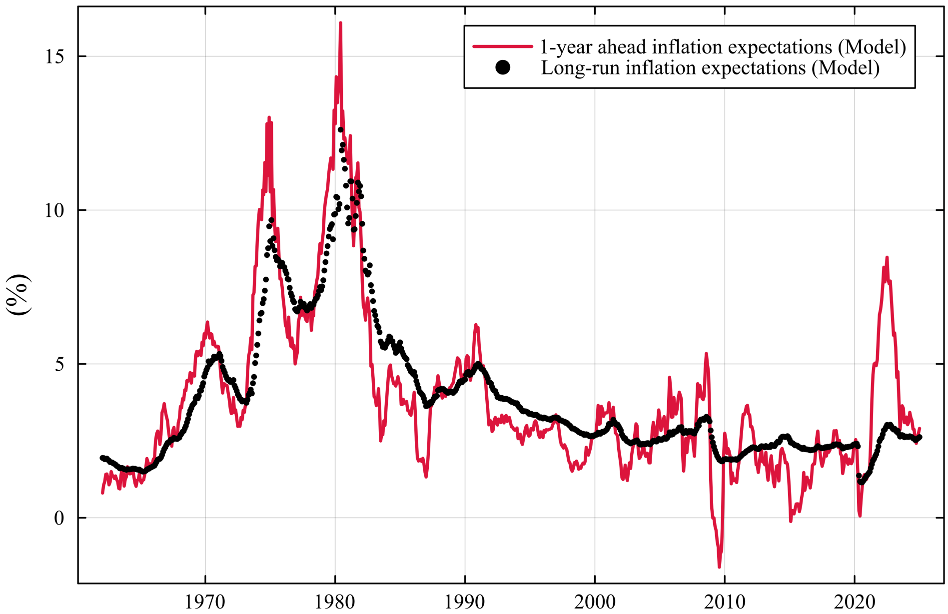


Figure 3: **Weakening link between short-run and long-run inflation expectations**

Note: The figure plots agents' 1-year ahead inflation expectations along with the long-run inflation expectations as implied by the model. This represents the model's reproduction of Figure 1 and highlights the same pattern. Agents' perceptions regarding the persistence of inflation have changed considerably over the postwar sample period.

6.2 The Fed's weight on unemployment gap stabilization

Figure 4 plots the evolution of the private sector's estimates of λ_t: the relative weight the Fed puts on closing the unemployment-gap. The estimates suggest that, during the early 1960s, the private sector attributed relatively little concern from the Fed towards closing the gap.

After a slow and steady rise following the Volcker disinflation, perceptions regarding λ were sharply revised upwards following the Global Financial Crisis and again in the aftermath of the COVID-19 pandemic: the emphasis of policy has shifted. As described in subsection 4.5, this development has had important implications for the anchoring of long-run inflation expectations. A large λ implies that the Fed effectively "ties its hands"; unemployment gaps are perceived as costly and hence policy emphasizes keeping the gap closed - constraining policymakers from responding to idiosyncratic shifts in their inflation target (owing to say, political pressure or short-term goals). A large λ_t also has the effect of making future unemployment gaps more predictable (since they are closer to zero) ultimately reducing real-side volatility.

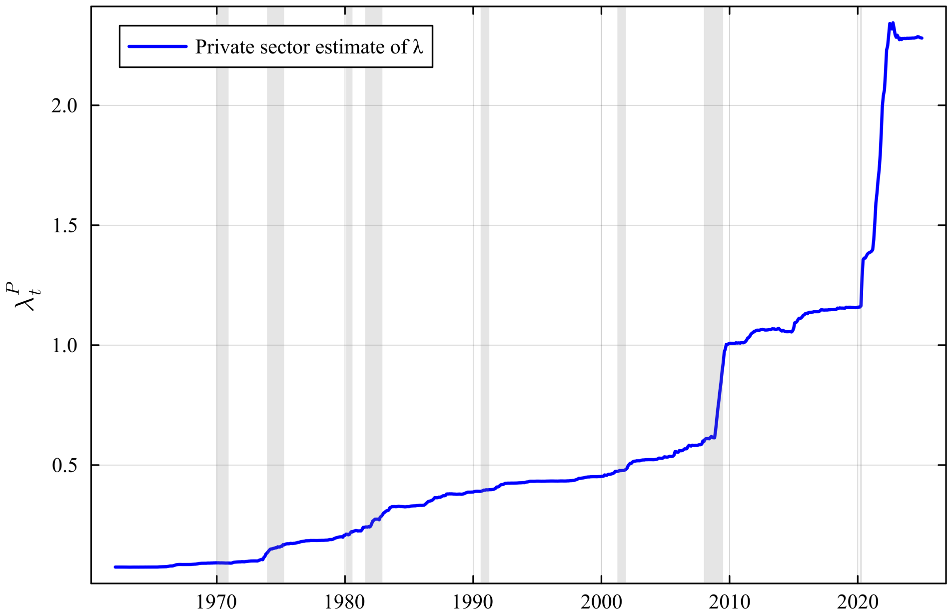


Figure 4: **Perception of the Fed's weight on unemployment-gap stabilization**

Note: Evolution of the Private sector estimate of λ_t. The private sector perceives the Fed to put more emphasis on stabilizing the unemployment-gap. The shaded regions indicate NBER recessions

These findings are consistent with recent empirical literature using high-frequency identification utilizing changes in asset prices around Fed policy announcements to study how the private sector learns about the Fed's "policy rule" (e.g., Bauer and Swanson (2023), Bauer, Pflueger and Sunderam (2024), Pflueger (2025)). These studies find that monetary policy in the recent period, following 2000, has been perceived to be increasingly responsive to output. A rise in λ_t, and the greater predictability of unemployment-gaps also complements earlier findings that output-gap volatility in the U.S. has declined significantly since the Volcker disinflation (see Blanchard and Simon (2001); McConnell and Perez-Quiros (2000)), underscoring the role of systematic monetary policy in engineering this phenomenon.

6.3 The slope of the expectations-augmented Phillips Curve

The parameter κ, represents the sensitivity of aggregate inflation to contemporaneous unemployment gap. Hence, it serves as the connective tissue between the real and nominal side of the economy and is a key parameter that agents must learn. Figure 5 plots the evolution of the agents' beliefs regarding the slope of the expectational Phillips curve under their learning mechanism. Under the agents' prior, κ is time-invariant. However, their real-time estimates under learning exhibit significant time-variation over the sample period.

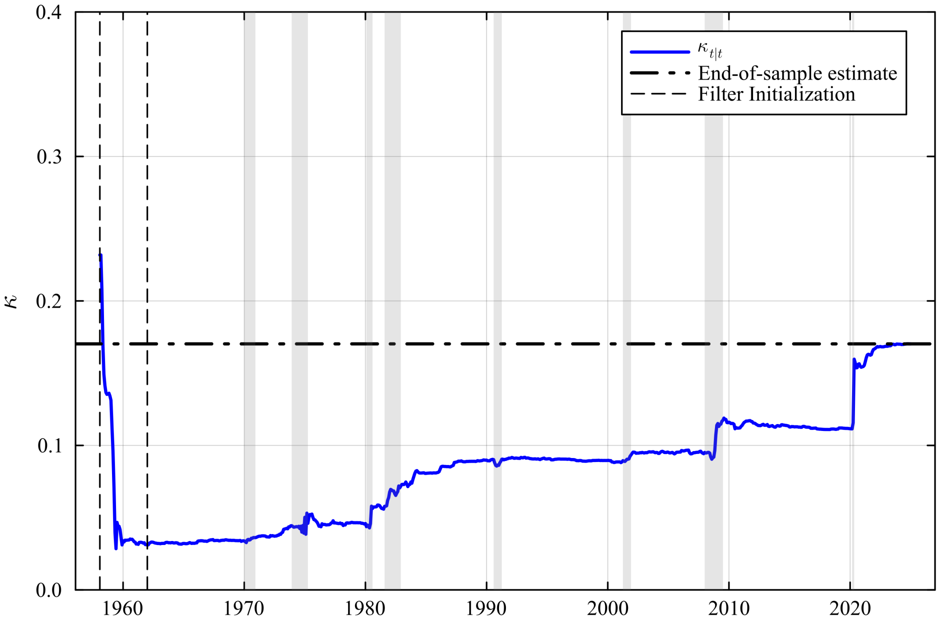


Figure 5: **Slope of the expectations-augmented Phillips curve**

Note: The figure plots the filtered estimates of the slope of the Phillips curve (*solid line*). From the private agents' perspective, the Phillips curve is steeper now than it was during the 70s and the 80s, meaning that inflation is *more* sensitive of real activity. Interestingly, though the agents learn under the assumption that the true slope of the structural Phillips curve is time-invariant, their beliefs are subject to constant revisions owing to the identification problems posed by the information structure of the economy. The shaded regions indicate NBER recessions

Since κ is not subject to innovations, the steady-state filter would be a decreasing-gain one. As new data is observed and beliefs are updated, each additional observation carries marginally less information and their beliefs should stabilize near a constant value. But it does not appear that any such convergence has attained within the sample period spanning about 60 years.

To understand why this is the case, let's revisit the observation equation through which agents update their beliefs regarding κ and ξ_t^p:

$$\pi_t - E_{t-1}^P \pi_{t+1} = -\kappa x_t + \xi_t^p + \xi_t^t$$

Note that current inflation, π_t delivers information about κ only through the unemployment gap x_t. It is not surprising then, that we see the bulk of the large updates in agents' estimates of κ around recessions (corresponding to large x_t). During "normal" times, the unemployment gap is small because nominal rigidities play a smaller role, and it is evident that learning about κ would slow down (since inflation carries little information regarding the true κ). The signal informing the agents about κ is susceptible to getting drowned out owing to: (i) the unemployment gap x_t being of small magnitude most of the time (Figure 14 in Appendix D plots the evolution of the variable) and (ii) aggregate supply shocks confounding the estimation. Supply shocks are known to confound the estimation of the Phillips curve and the literature has often relied on adding oil prices as proxies, for example. Inferring the slope of the structural Phillips curve has been among the most enduring and difficult problems in macroeconomics. It is hence no surprise that the agents in the model are not immune to the fundamental econometric challenges that it poses. Models assuming full-information mask such issues entirely.

The agents perceive the slope to be relatively "flat" during the 1970s. This belief would be reinforced by the experience of high inflation in the face of rising unemployment experienced during this period. The perception of a flat Phillips curve is one of the primary reasons cited by Primiceri (2006) and Sargent, Williams and Zha (2006) for the policymakers' aversion to act decisively against inflation during this period. The agents revise their beliefs over time and see the slope become steeper over time.[23] In the post-pandemic period, the large decline in inflation from its highs with little rise in unemployment leads the agents to update their beliefs about κ upwards. Overall, the results suggest that real-time identification of the Phillips curve is quite poor and agents' estimates would take an unreasonably long time to converge to a constant.

However, estimates during the entire sample period remain within an empirically plausible range (between 0.05-0.20) and the model does not rely on an excessively flat Phillips curve to rationalize the inflation outcomes of the past two decades. Some recent papers argue in favor of departing from the log-linearized Phillips curve to understand the behavior of inflation during the post-pandemic surge (see Harding, Lindé and Trabandt (2023) Benigno and Eggertsson (2024))[24]. Though the model considered here does not nest such nonlinearities, the *perceived* slope becomes substantially steeper following the pandemic-induced recession, consistent with the findings. However, the model shows that this can happen even in an environment where the true slope is fixed, owing to imperfect information.

6.4 Anchoring Coefficient

From Equation 30, the factor $\kappa_{t|t}/(\lambda^P_{t-1} + \kappa^2_{t|t})$ determines the sensitivity of interest rates to shifts in the policymakers' inflation target. This term is a crucial determinant of the anchoring

[23]This is in contrast with the literature finding a flattening Phillips curve over time. However, the estimates are not directly comparable to the one here since the empirical specification used is very different.

[24]However, Beaudry, Hou and Portier (2025)) find limited evidence in favor of nonlinear Phillips curve specifications once inflation expectations are explicitly taken into account.

of long-run inflation expectations. The larger this coefficient is, the more informative policy is perceived to be regarding the inflation target and the more π_t^{*P} gets revised upon receiving a policy surprise. Hence, I call its *reciprocal*, $(\lambda_{t-1}^P + \kappa_{t|t}^2)/\kappa_{t|t}$[25] the "anchoring coefficient" (larger value corresponds to stronger anchoring). I denote it by ζ_t and is given by

$$\zeta_t = \frac{\lambda_{t-1}^P + \kappa_{t|t}^2}{\kappa_{t|t}}$$

ζ_t depends both on the perceived slope of the Phillips curve $\kappa_{t|t}$ as well as ex-ante perception

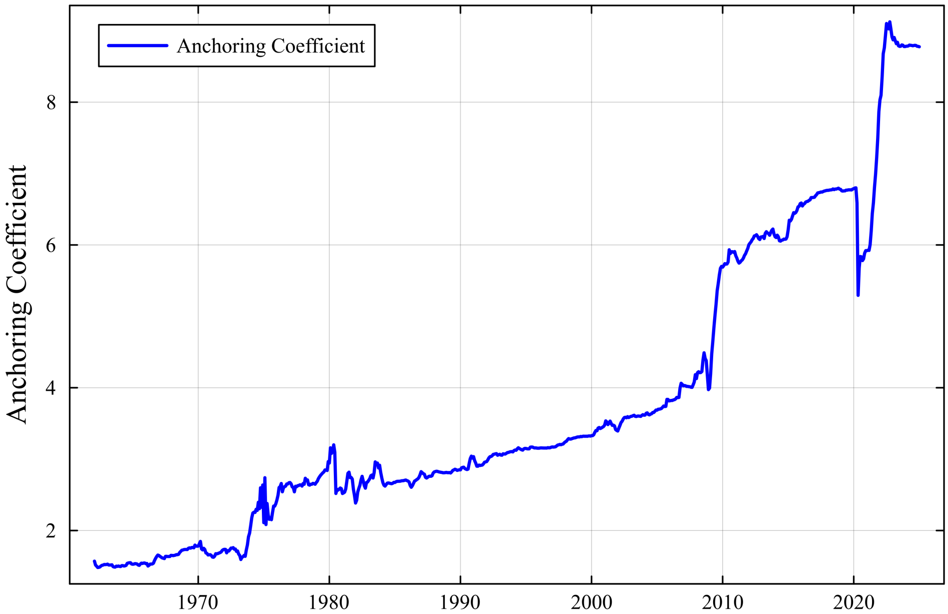


Figure 6: **Evolution of the anchoring coefficient** ($\frac{\lambda_{t-1}^P + \kappa_{t|t}^2}{\kappa_{t|t}}$)

Note: The coefficient measuring the "anchoring" of long-run inflation expectations has risen drastically. Even as the perceived slope of the Phillips curve has risen, the rise in the perceived weight on unemployment gap stabilization λ_t^P has outpaced it and made the coefficient larger.

about the weight the Fed puts on unemployment gap stabilization λ_{t-1}^P. Figure 6 plots the evolution of ζ_t over the postwar period. From the private agents' vantage point, the policy environment has changed drastically in the 40 years separating the Great Inflation and the post-pandemic surge. Through the lens of their model, interest rates are less responsive to idiosyncratic shifts in the inflation target than they were in the wake of the Great Inflation crisis. According to the model, long-run inflation expectations are more entrenched at this juncture than any other time in the post-war history of the United States. This development afforded the Fed enough room to to "look-through" supply shocks and adopt a wait-and-see

[25]This term is bounded below by $2\sqrt{\lambda_{t-1}^P}$

approach without triggering large revisions in the perceived inflation target. While this makes de-anchoring due to perceived mistakes unlikely, it also hampers the Fed's ability to guide these beliefs back to target *if* they were to de-anchor. An implication of the model is that the steadfast disinflation we saw in the 80s, in large part due to the Fed strongly signaling a low inflation target, would be much less likely today.

6.5 "Bad luck" vs "bad policy" revisited

The following section contributes to the debate regarding whether the economic outcomes of the post-Volcker period were a result of luck (favorable economic environment, benign shocks) or systematic policy (shifts in the Fed's implied policy objectives). I discuss how evolving perceptions regarding structural shocks and the Fed's systematic monetary policy contributed to shaping macroeconomic outcomes in the post-war period in the U.S.

6.5.1 The role of policy in shaping agents' expectations

In this section, I discuss the role of the Fed's systematic monetary policy in signaling the policymakers' preferences to the private agents. It is useful to consider the perceived real rate gap, defined as the difference between the ex-ante real interest rate and the perceived natural rate of interest, $r^n_{t|t}$. A positive real rate gap implies a contractionary policy stance and conversely, a negative real rate gap reflects an expansionary stance of policy.

Figure 7 plots the evolution of the perceived real rate gap along with the agents' real-time estimates of persistent supply shocks. As we can see, the late 1960s are characterized by moderate supply shocks. But the real rate gap tends to stay slightly negative throughout the latter half of the decade, signaling the policymakers' attempts at monetary expansion during this period.[26] The mid-1970s see mounting supply pressures and the Fed effectively doubling down on its stimulative efforts, presumably in an effort to quell the recession coinciding with the OPEC embargo in 1974. With interest rates being seen as strongly indicative of the Fed policymakers' inflation target, over-expansionary policy was interpreted as coming from a deliberate effort to achieve higher inflation. Long-run inflation expectations shot up.

[26] Narrative evidence from this period suggests that Chairman Martin's initial anti-inflation stance clashed with expansionary fiscal policy by the Congress. This ended up eroding the postwar monetary discipline of the Fed, with focus shifting from stabilizing inflation to accommodating the fiscal expansion.

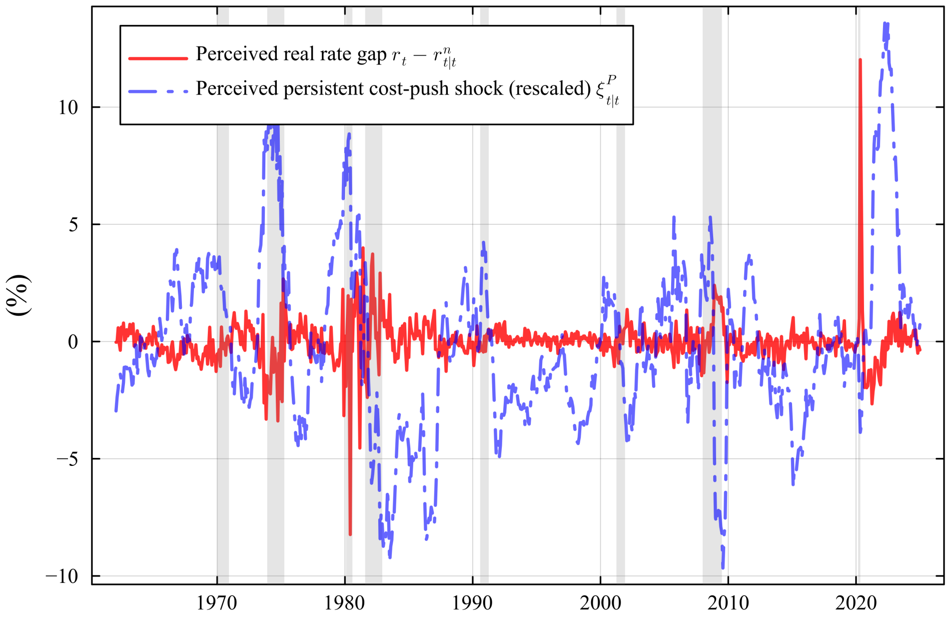


Figure 7: **Policy response to inflationary shocks:**

Note: The perceived real rate gap $r_t - r^n_{t|t}$ *(solid)* vs filtered estimates of persistent supply shocks $\xi^p_{t|t}$ *(dashed)*. We can see that supply-side pressures in the 70s and 80s were met with accommodative policy by the Fed. Policy was perceived to be similarly accommodative at the beginning of the post-pandemic surge. The shaded regions indicate NBER recessions

"Forever inflation" takes hold with no resolve from policymakers to combat it. It is not until 1980, under the chairmanship of Paul Volcker that we see a sustained positive real rate gap leading to the eventual "conquest over inflation". From the model's perspective, two aspects of policy behavior in the post-Volcker period were important for the onset of the "Great Moderation": (i) a positive real rate gap was maintained for a prolonged period signaling a lower inflation target and (ii) muted response of the real rate gap to supply shocks, signaling a steadily increasing weight on real-side stabilization represented by λ_t. These gradually led the agents to believe in a low, stable inflation target. Note that without (ii), the anchoring would be fragile and long-run beliefs would have been similarly volatile, shooting up at any hint that Fed policy was "too loose".

Turning to the post-pandemic period, the agents see the mistakes of the 70s being repeated: persistent supply shocks re-emerge and the Fed is nowhere close to contain the imminent crisis. Yet, we see long-run inflation expectations remain largely stable. This time however, after a period of "looking through" supply shocks, the Fed quickly catches up and the agents see the real-rate gap close. From the model's perspective, even as the agents perceived the policymakers to be making a similar kind of mistake at the beginning during the recent surge, their expectations responded differently: in large part due to a change in their perceptions regarding

λ_t - agents were slower to revise their beliefs about the inflation target despite the "mistake". Disciplined Fed policy has shaped agents' beliefs in a way that prevented the sudden inflationary spiral of the 70s despite a delayed policy response. Good policy of the past continues to bear fruit. Since the Fed acted decisively when it did and managed to raise the real rates above the perceived natural rate, it managed to signal that the inflation target was intact.

6.5.2 Supply shocks: The structural drivers of policy trade-offs

As the only variable driving a trade-off between inflation and unemployment gap stabilization, persistent supply shocks cause all "involuntary" inflation, over and beyond the Fed's targeted level. Agents' beliefs about these shocks are an important determinant of their short-run inflation expectations and hence of inflation dynamics themselves.

Figure 8 plots the time-series of the agents' beliefs (real-time as well as smoothed) regarding persistent supply shocks in the Phillips curve. One fact that emerges is that the agents perceived large persistent supply shocks, both during the Great Inflation of the 70s and in the more recent inflation surge. Moreover, the shocks were of roughly similar magnitude across the two episodes if not larger during the recent episode. Hence, the going by the size of the shocks alone, one would find it hard to explain why the inflation crisis was much more benign this time.

The perceived supply shocks line up well with documented episodes of oil price surges. There is ample evidence in the literature that fluctuations in relative prices of energy goods are significant drivers of inflation (see Afrouzi, Bhattarai and Wu (2024), Gagliardone and Gertler (2023) for recent work examining the link between crude oil prices and the post-pandemic inflation surge). Interestingly, the period of recovery from the Great Financial Crisis (GFC) also coincides with persistent cost-push shocks. This is consistent with the theory that oil price shocks in the aftermath of the crisis kept inflation expectations propped up, accounting for the "missing deflation" (Coibion and Gorodnichenko (2015)). According to the model, even as the agents' long-run inflation expectations remain stable during this period, their short-run expectations are quite responsive to inflationary supply shocks during this period.

To aid interpretation of these "persistent supply shocks", in Figure 9 I plot the series against food and energy price inflation derived as the difference between Headline CPI and Core CPI. The series are de-meaned and re-scaled for comparison. The two series display significant co-movement. Since Core CPI is not a series that the model has "seen" (only Headline CPI data is

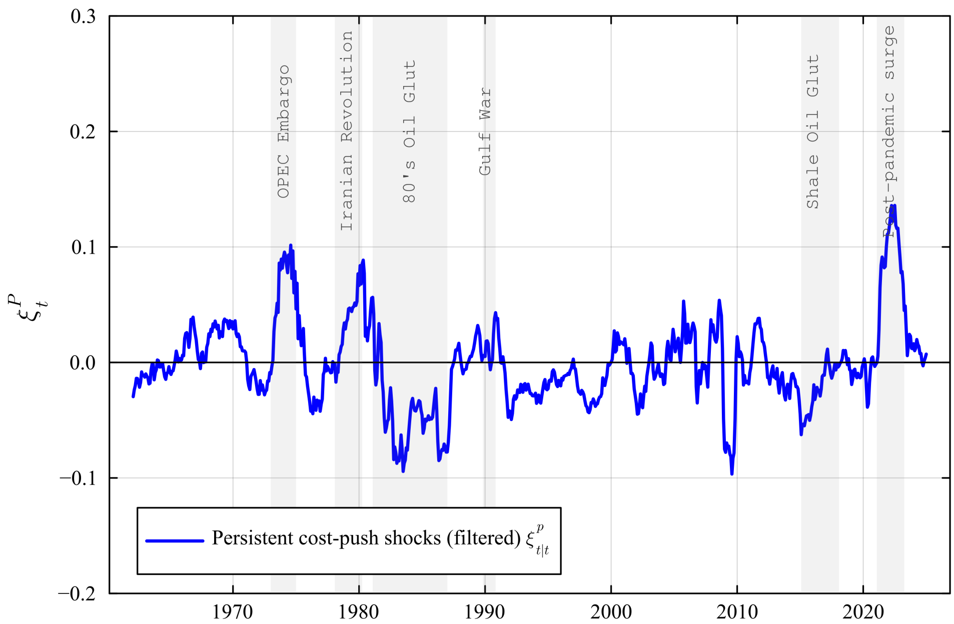


Figure 8: **Structural drivers of inflation**

Note: Private agents' estimates of the persistent component of the cost-push process ξ_t^P: Filtered *(solid)* vs smoothed *(dashed)* The magnitude of persistent cost-push shocks perceived by the agents during the post-COVID period were similar to those during the Great Inflation period. The shaded regions indicate known episodes of significant supply pressures.

used during estimation), it is remarkable that such a link emerges.

The post-pandemic episode was characterized by structural shifts over and above oil price surges, such as sectoral shocks shifting consumption between goods and services (see Guerrieri et al. (2021)) structural changes in the labor market (Blanchard and Bernanke (2023), Ball et al. (2022)), increasing attention to inflation (Pfauti (2023)) to name a few. However, shocks to relative food and energy prices appear to explain a large part of the variation in these persistent supply shocks perceived by the agents. This is intuitive considering how *salient* these prices are (as also argued by Coibion and Gorodnichenko (2015)). It makes sense that firms and households' perceptions about inflation in the short run would respond to these prices given their pervasive impact on the rest of the economy (as argued in Afrouzi et al. (2024) using a model of input-output linkages). Hence, the model adds support to the literature emphasizing the importance of relative price disturbances in driving aggregate inflation dynamics.

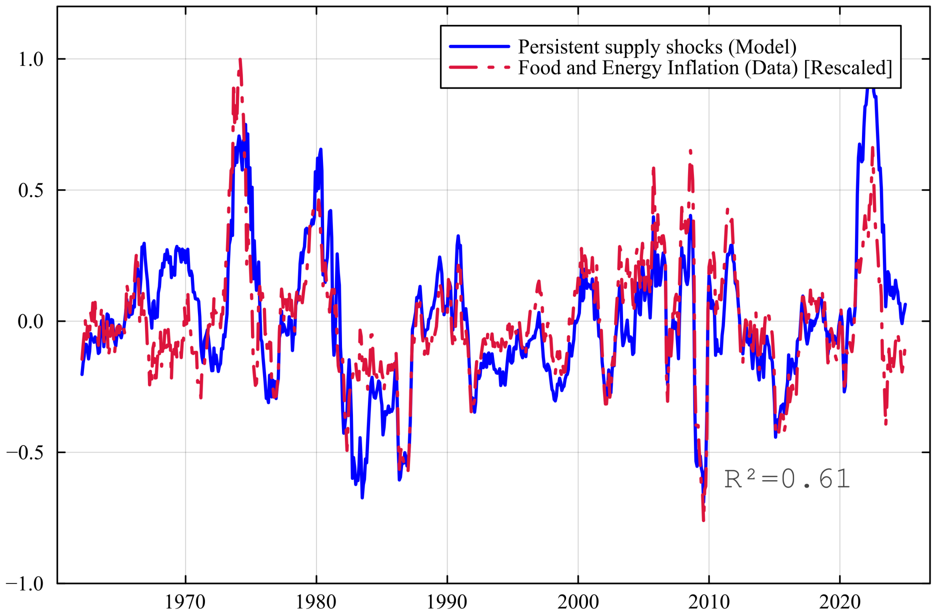


Figure 9: **Relative prices and perceptions about inflation**

Note: Persistent supply shocks as perceived by the agents vs inflation in food and energy prices (Untargeted). Measure of Food and Energy inflation is constructed as the difference between headline and core CPI inflation

7 Conclusions

The rational expectations revolution—spearheaded by the work of Lucas and Sargent—marked a profound shift in macroeconomic thought. New Keynesian macroeconomics incorporated this transformation by explicitly modeling how aggregate outcomes emerge from the optimizing behavior of individual agents. The model developed in this paper presents a natural extension of the standard New Keynesian framework to an environment characterized by imperfect knowledge where neither the true structure of the economy nor the objectives of the central bank policymakers are known to the agents. Their expectations reflect their changing beliefs as they continue to *learn*. In doing so, it describes postwar U.S. macroeconomic dynamics from the perspective of agents reasoning from New-Keynesian principles about observed macroeconomic outcomes in real-time.

A central implication of the model is that, because expectations are pivotal in shaping macroeconomic outcomes, the function of monetary policy extends beyond mechanical stabilization. Policy actions not only influence current economic conditions but also actively shape agents' expectations—a point emphasized by Kydland and Prescott (1977). The learning mechanism embedded in the model illustrates how the anchoring of expectations emerges endogenously from the systematic conduct of monetary policy. The evolution of postwar U.S. macroeconomic dynamics is thus closely linked to how Federal Reserve policymakers have signaled the policy objectives underlying their decisions. The model provides a structural interpretation of how monetary policy contributes to the anchoring of expectations, highlighting its dual role as both a stabilizing instrument and a signal relaying information and affecting expectations.

A demonstrated emphasis by the Fed on minimizing policy-driven fluctuations in unemployment has contributed to the anchoring of long-run inflation expectations, *cushioning* people's long-run beliefs against policy misbehavior; while simultaneously amplifying the inflationary impact of persistent supply shocks. From the lens of the model, these perceptions afforded the Fed the ability to "look-through" supply shocks without de-anchoring long-run expectations. Beliefs shaped by the disciplined conduct of monetary policy over the forty years following the Volcker disinflation have contributed greatly to the "soft-landing" that we saw recently. While recent work (e.g., Coibion and Gorodnichenko (2025)) has warned of inflationary risks resulting from de-anchoring of expectations from inflationary pressures in the short-run, the model presented here paints a less grim picture while recognizing the risks that an indecisive policy response to supply shocks could pose in the future.

This work also highlights the explanatory power of the textbook New-Keynesian model when augmented with imperfect information and learning. The model reconciles the empirically observed persistence in macroeconomic aggregates, delivering an excellent fit to macroeconomic data without resorting to backward-looking mechanisms such as indexation to past prices, habit formation in consumption or interest rate smoothing. Finally, the model highlights the *state-dependent* nature of agents' learning: the rate at which information arrives is neither uniform over time nor identical across structural parameters. One key parameter for which this distinction is particularly relevant is the slope of the expectations-augmented Phillips curve, which remains poorly identified under learning. This finding suggests that the full-information assumption commonly imposed in macroeconomic models may be far less innocuous than often presumed.

References

Adam, Klaus and Mario Padula, "Inflation Dynamics and Subjective Expectations in the United States," *Economic Inquiry*, 2011, *49* (1), 13–25. 5

Afrouzi, Hassan, Saroj Bhattarai, and Edson Wu, "Relative-price changes as aggregate supply shocks revisited: Theory and evidence," *Journal of Monetary Economics*, 2024, *148*, 103650. Inflation in the COVID Era and Beyond. 38, 39

Ball, Laurence, Daniel Leigh, and Prakash Loungani, "Okun's Law: Fit at 50?," *Journal of Money, Credit and Banking*, 2017, *49* (7), 1413–1441. 14

Ball, Laurence M, Daniel Leigh, and Prachi Mishra, "Understanding U.S. Inflation During the COVID Era," Working Paper 30613, National Bureau of Economic Research October 2022. 39

Bauer, Michael D. and Eric T. Swanson, "A Reassessment of Monetary Policy Surprises and High-Frequency Identification," *NBER Macroeconomics Annual*, 2023, *37* (1), 87–155. 32

Bauer, Michael D, Carolin E Pflueger, and Adi Sunderam, "Perceptions About Monetary Policy*," *The Quarterly Journal of Economics*, 06 2024, *139* (4), 2227–2278. 32

Beaudry, Paul, Chenyu Hou, and Franck Portier, "On the Fragility of the Nonlinear Phillips Curve View of Recent Inflation," Working Paper 33522, National Bureau of Economic Research February 2025. 34

Benigno, Pierpaolo and Gauti B. Eggertsson, "The Slanted-L Phillips Curve," *AEA Papers and Proceedings*, May 2024, *114*, 84–89. 34

Bernanke, Ben, "Inflation Expectations and Inflation Forecasting," July 10 2007. At the Monetary Economics Workshop of the National Bureau of Economic Research Summer Institute, Cambridge, Massachusetts. 3

Bernanke, Ben S. and Ilian Mihov, "Measuring Monetary Policy*," *The Quarterly Journal of Economics*, 08 1998, *113* (3), 869–902. 6

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park, "Policy Regimes, Policy Shifts, and U.S. Business Cycles," *The Review of Economics and Statistics*, 12 2016, *98* (5), 968–983. 28

Blanchard, Olivier and John Simon, "The Long and Large Decline in U.S. Output Volatility," *Brookings Papers on Economic Activity*, 2001, *2001* (1), 135–164. 32

Blanchard, Olivier J and Ben S Bernanke, "What Caused the US Pandemic-Era Inflation?," Working Paper 31417, National Bureau of Economic Research June 2023. 39

Bullard, James and Stefano Eusepi, "Did the great inflation occur despite policymaker commitment to a Taylor rule?," *Review of Economic Dynamics*, 2005, *8* (2), 324–359. Monetary Policy and Learning. 6

Carvalho, Carlos, Stefano Eusepi, Emanuel Moench, and Bruce Preston, "Anchored Inflation Expectations," *American Economic Journal: Macroeconomics*, January 2023, *15* (1), 1–47. 7

Christiano, Lawrence and Christopher Gust, "The expectations trap hypothesis," *Economic Perspectives*, 2000, *25* (Q II), 21–39. 7, 19

Clarida, Richard, Jordi Gali, and Mark Gertler, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory*," *Quarterly Journal of Economics*, February 2000, *115* (1), 147–180. 6

Cogley, Timothy and Thomas J. Sargent, "Anticipated Utility and Rational Expectations as approximations of Bayesian decision making*," *International Economic Review*, February 2008, *49* (1), 185–221. 15

_ , Giorgio E. Primiceri, and Thomas J. Sargent, "Inflation-Gap Persistence in the US," *American Economic Journal: Macroeconomics*, January 2010, *2* (1). 6

Coibion, Olivier and Yuriy Gorodnichenko, "Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation," *American Economic Journal: Macroeconomics*, January 2015, *7* (1), 197–232. 38, 39

_ and _ , "Inflation, Expectations and Monetary Policy: What Have We Learned and to What End?," Working Paper 33858, National Bureau of Economic Research May 2025. 41

_ , _ , and Rupal Kamdar, "The Formation of Expectations, Inflation, and the Phillips Curve," *Journal of Economic Literature*, 2018, *56* (4), 1447–1491. 5

Debortoli, Davide and Aeimit Lakdawala, "How Credible Is the Federal Reserve? A Structural Estimation of Policy Re-Optimizations," *American Economic Journal.Macroeconomics*, 07 2016, *8* (3), 42–76. 7

Del-Negro, Marco and Stefano Eusepi, "Fitting observed inflation expectations," *Journal of Economic Dynamics and Control*, 2011, *35* (12), 2105–2131. Frontiers in Structural Macroeconomic Modeling. 5, 6

DeLong, J. Bradford, "America's Only Peacetime Inflation: The 1970s," NBER Historical Working Papers 0084, National Bureau of Economic Research, Inc May 1996. 6, 30

Erceg, Christopher J. and Andrew T. Levin, "Imperfect credibility and inflation persistence," *Journal of Monetary Economics*, 2003, *50* (4), 915–944. 6, 49

Evans, George W. and Seppo Honkapohja, *Learning and Expectations in Macroeconomics*, Princeton University Press, 2001. 5, 15

Gagliardone, Luca and Mark Gertler, "Oil Prices, Monetary Policy and Inflation Surges," Working Paper 31263, National Bureau of Economic Research May 2023. 38

Gati, Laura, "Monetary policy & anchored expectationsâ€"An endogenous gain learning model," *Journal of Monetary Economics*, None 2023, *140* (S), 37–47. 7

Goodfriend, Marvin and Robert G. King, "The incredible Volcker disinflation," *Journal of Monetary Economics*, July 2005, *52* (5), 981–1015. 29

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Ivan Werning, "Monetary Policy in Times of Structural Reallocation," 2021. Proceedings of the Jackson Hole Symposium, 2021. 39

Harding, Martín, Jesper Lindé, and Mathias Trabandt, "Understanding post-COVID inflation dynamics," *Journal of Monetary Economics*, 2023, *140*, S101–S118. Inflation: Drivers and Dynamics 2022. 34

Ireland, Peter N., "Changes in the Federal Reserve's Inflation Target: Causes and Consequences," *Journal of Money, Credit and Banking*, 2007, *39* (8), 1851–1882. 6, 51

Kamber, Güneş, James Morley, and Benjamin Wong, "Intuitive and Reliable Estimates of the Output Gap from a Beveridge-Nelson Filter," *The Review of Economics and Statistics*, 07 2018, *100* (3), 550–566. 15

King, Robert G. and Yang K. Lu, "Evolving Reputation for Commitment: The Rise, Fall and Stabilization of US Inflation," NBER Working Papers 30763, National Bureau of Economic Research, Inc Dec 2022. 7

Kozicki, Sharon and P.A. Tinsley, "Permanent and transitory policy shocks in an empirical macro model with asymmetric information," *Journal of Economic Dynamics and Control*, 2005, *29* (11), 1985–2015. Expectations, learning, and monetary policy. 6

Kreps, David M., "Anticipated Utility and Dynamic Choice (1997)," in Donald P. Jacobs, Ehud Kalai, Morton I. Kamien, and Nancy L. Schwartz, eds., *Frontiers of Research in Economic Theory: The Nancy L. Schwartz Memorial Lectures, 1983–1997*, Econometric Society Monographs, Cambridge: Cambridge University Press, 1998, pp. 242–274. 15

Kydland, Finn E. and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 1977, *85* (3), 473–491. 3, 40

Lakdawala, Aeimit, "Changes in Federal Reserve preferences," *Journal of Economic Dynamics and Control*, 2016, *70*, 124–143. 6

Lubik, Thomas A. and Frank Schorfheide, "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, March 2004, *94* (1), 190–217. 6

McConnell, Margaret M. and Gabriel Perez-Quiros, "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?," *The American Economic Review*, 2000, *90* (5), 1464–1476. 32

Melosi, Leonardo, "Signalling Effects of Monetary Policy," *The Review of Economic Studies*, 09 2016, *84* (2), 853–884. 6

Milani, Fabio, "Expectations, learning and macroeconomic persistence," *Journal of Monetary Economics*, 2007, *54* (7), 2065–2082. 5, 49

_ , "Chapter 18 - Expectational data in DSGE models," in Rüdiger Bachmann, Giorgio Topa, and Wilbert van der Klaauw, eds., *Handbook of Economic Expectations*, Academic Press, 2023, pp. 541–567. 5

Nimark, Kristoffer, "Dynamic pricing and imperfect common knowledge," *Journal of Monetary Economics*, 2008, *55* (2), 365–382. 11

Orphanides, Athanasios and John C. Williams, "The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations," *Journal of Economic Dynamics and Control*, 2005, *29* (11), 1927–1950. Expectations, learning, and monetary policy. 6

_ **and** _ , "Monetary Policy with Imperfect Knowledge," *Journal of the European Economic Association*, 2006, *4* (2/3), 366–375. 6

_ **and Simon van Norden**, "The Unreliability of Output-Gap Estimates in Real Time," *The Review of Economics and Statistics*, 2002, *84* (4), 569–583. 15

Pfauti, Oliver, "The Inflation Attention Threshold and Inflation Surges," Papers 2308.09480, arXiv.org Aug 2023. 39

Pflueger, Carolin, "Back to the 1980s or not? The drivers of inflation and real risks in Treasury bonds," *Journal of Financial Economics*, 2025, *167*, 104027. 32

Phillips, A. W., "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957," *Economica*, 1958, *25* (100), 283–299. 55

Primiceri, Giorgio E, "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U. S. Postwar Stabilization Policy*," *Quarterly Journal of Economics*, August 2006, *121* (3), 867–901. 6, 34

Reis, Ricardo, "Losing the Inflation Anchor," *Brookings Papers on Economic Activity*, 2021. 29

Romer, Christina D. and David H. Romer, "A Rehabilitation of Monetary Policy in the 1950's," *American Economic Review*, May 2002, *92* (2), 121–127. 6, 30

Sargent, Thomas, Noah Williams, and Tao Zha, "Shocks and Government Beliefs: The Rise and Fall of American Inflation," *American Economic Review*, September 2006, *96* (4), 1193–1224. 6, 34

Sims, Christopher A and Tao Zha, "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, February 2006, *96* (1), 54–81. 6

Staiger, Douglas, James H. Stock, and Mark W. Watson, "The NAIRU, Unemployment and Monetary Policy," *Journal of Economic Perspectives*, March 1997, *11* (1), 33–49. 15

Svensson, Lars E. O., "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules," *Journal of Economic Literature*, June 2003, *41* (2), 426–477. 4

_ **and Michael Woodford**, "Implementing Optimal Policy through Inflation-Forecast Targeting," in "The Inflation-Targeting Debate," University of Chicago Press, December 2004. 17

Taylor, John B., "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 1993, *39*, 195–214. 6

Williams, Noah, "Adaptive Learning and Business Cycles," *Unpublished*, 2003. 5

8 Appendix

A Long-run inflation expectations

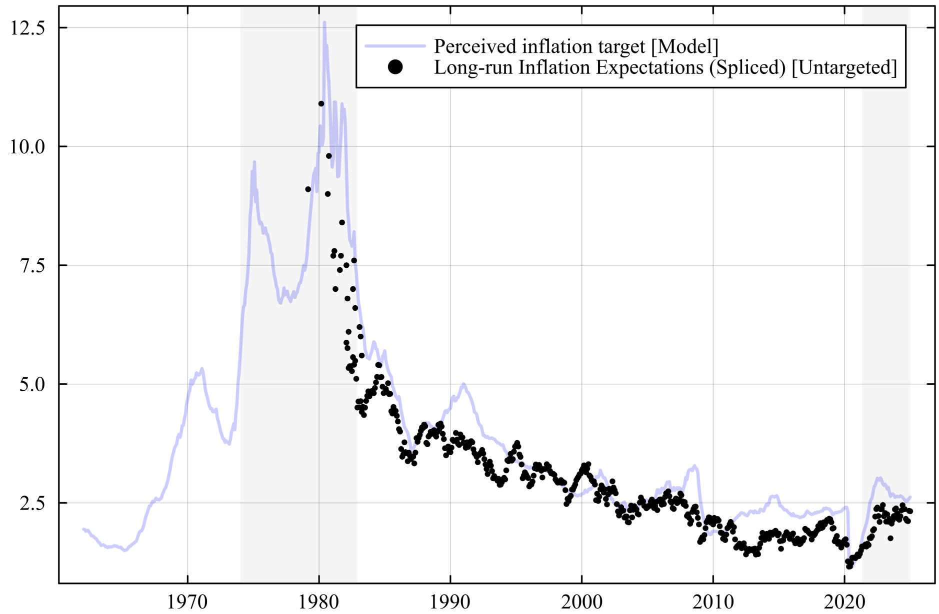
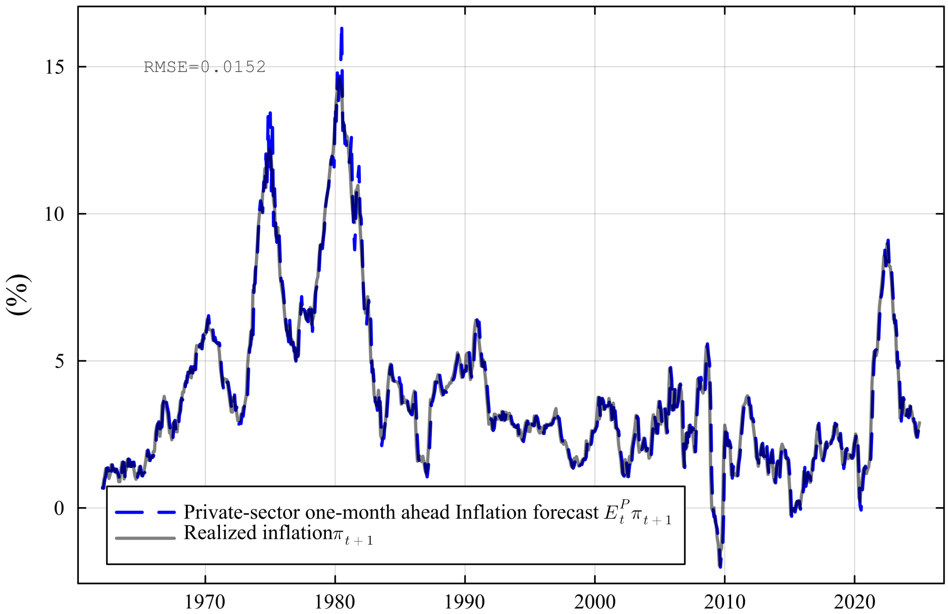


Figure 10: **Comparing with untargeted measures of long-run inflation expectations**

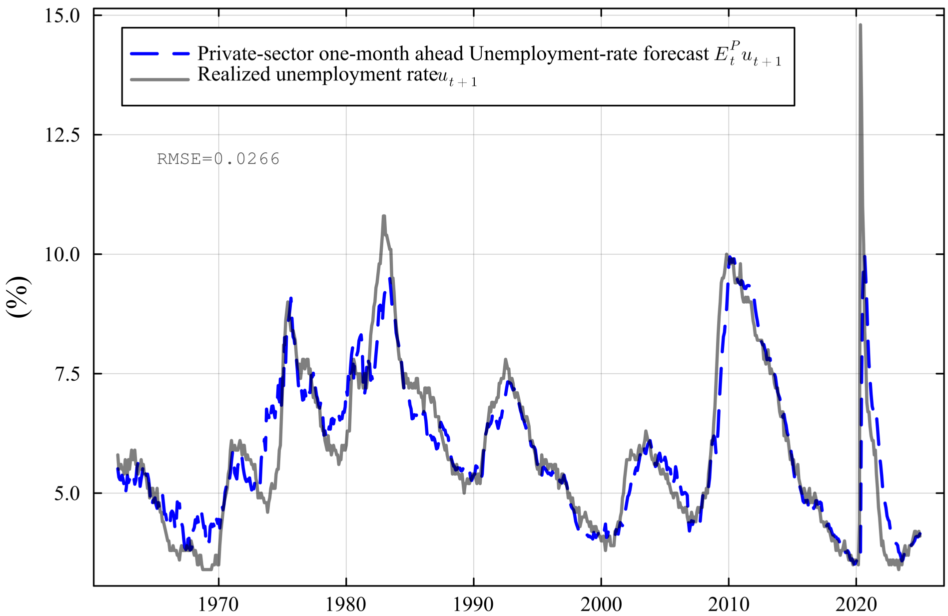
Note: Long-run Inflation Expectations series is constructed by splicing together 5-year ahead inflation expectations from Michigan Survey before 1982 and 10-year ahead inflation expectations from the Cleveland Fed afterwards (due to a shorter sample). The series is not used for estimation and is hence untargeted.

B Forecast performance of agents' model

If the learning algorithm arising from the agents' assumed model of economic dynamics implies forecasts that are persistently off-target, it wouldn't make sense to assume that the agents would continue using it. Rational agents would abandon the model. Hence, it is useful to check how well the agents' ex-ante forecasts agree with the realizations of the inflation, unemployment rate and interest rates or in other words if the agents' assumed model remains plausible given what they observe. Figure 12 plots the agents' ex-ante expectation of the policy rate (prior to announcement) and the realized effective Fed funds rate. Similarly, in Figure 11 I plot the agents' one-period ahead forecasts of inflation and unemployment rate against their corresponding realized values.



(a) Agents' one-month ahead forecast of inflation vs realized value



(b) Agents' one-month ahead forecast of unemployment rate vs realized value

Figure 11: **Forecast performance of agents' model**

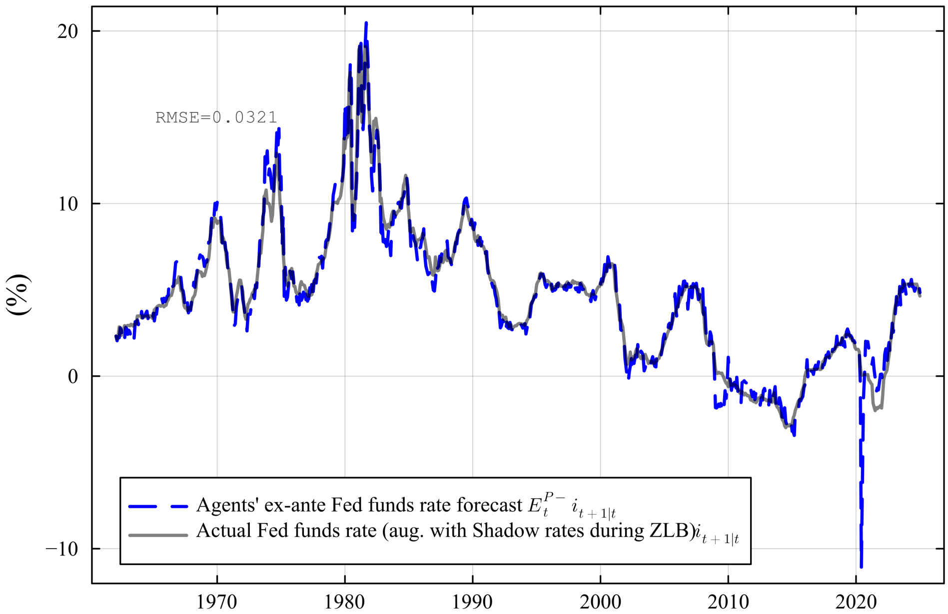


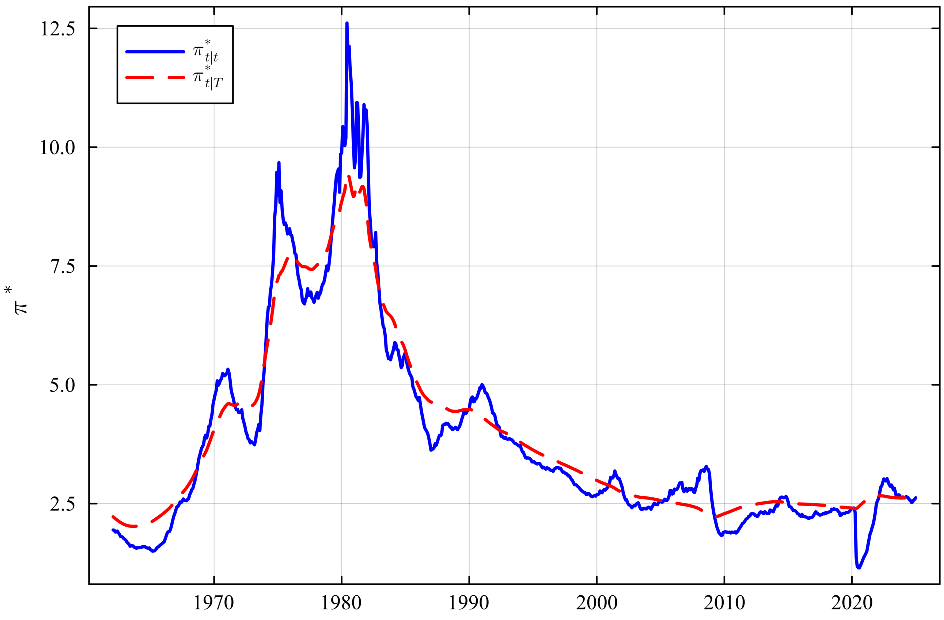
Figure 12: **How well do private agents track policy behavior?**

Note: The figure compares the private agents' expected nominal interest rates based on their ex-ante beliefs with the actual Fed funds rate (augmented with the shadow rate).

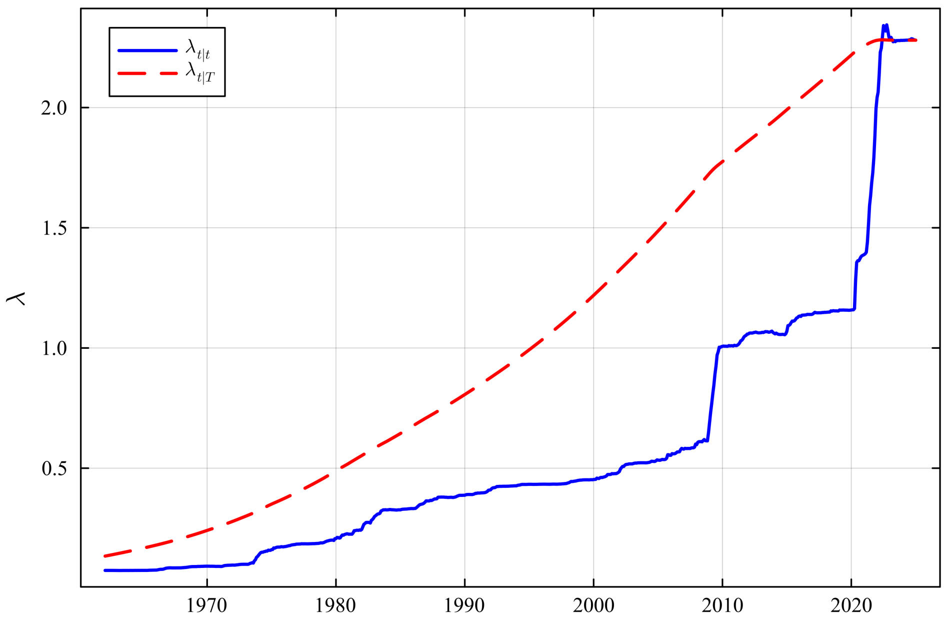
The learning model generates forecasts errors of small magnitude (in the RMSE sense). Thus, their model of the economy "works" - they wouldn't find a compelling reason to abandon learning. It is also worth noting that their expectations display a similar degree of persistence as the corresponding macroeconomic variables themselves. They are able to "match" the persistent behavior within a fully forward-looking model. Hence, forces such as price-indexation and habit-formation in consumption do not seem to be necessary to generate persistence in the macroeconomic aggregates once agents' real-time learning is taken into account, as also demonstrated by Milani (2007) and Erceg and Levin (2003).

C What were the Fed's "true" policy preferences?

While the parameters representing the "true" policy preferences of the Fed over the post-war period remain latent (it evolves as an exogenous structural variable as far as the model is concerned), it is possible to arrive at a best-guess using the model by obtaining the smoothed (backward-looking) estimates of the time-t policy preferences (Figure 13). These estimates represent the evolution of Fed's policy preferences in the post-war period from the perspective of a private agent in the model looking back at the end of the sample period (Dec 2024).



(a) π_t^*: Filtered *(solid)* vs Smoothed *(dashed)* estimates



(b) λ_t: Filtered *(solid)* vs Smoothed *(dashed)* estimates

Figure 13: **The Fed's "true" policy preferences**

Note: The figure plots the smoothed estimates of π_t^* and λ_t along with the filtered estimates. The smoothed estimates represent the agents' best-estimates of the parameters representing the Fed's policy preferences conditional on the entire sample period.

The estimates suggest that the Fed's true inflation target was upwards of 8 percent during the peak of the 80s-era inflation. Notably, the estimates of the time-varying inflation target are also in line with Ireland (2007) who uses a specification that is very different from the one used here.

Taken at face value, the smoothed estimates of λ suggest that Fed was hardly concerned with keeping the unemployment-gap closed during the 70s-80s. Over time however, the Fed has steadily emphasized closing the gap as part of its monetary policy strategy: which brings us to present-day. The estimated weight on unemployment-stabilization is almost twice as large as that on inflation-stabilization. The backward-looking estimates of λ_t display periods of departure from the real-time ones. This is because meaningful information regarding λ only arrives during periods where agents perceive large cost-push shocks driving policy tradeoffs (since λ is only relevant to policy when such a trade-off exists). Thus from the private agents' perspective, though λ has been evolving continuously, information regarding λ only arrives in clumps during episodes involving large persistent cost-push shocks driving policy trade-offs. Not much learning regarding λ takes place during "normal" times.

D Unemployment gap

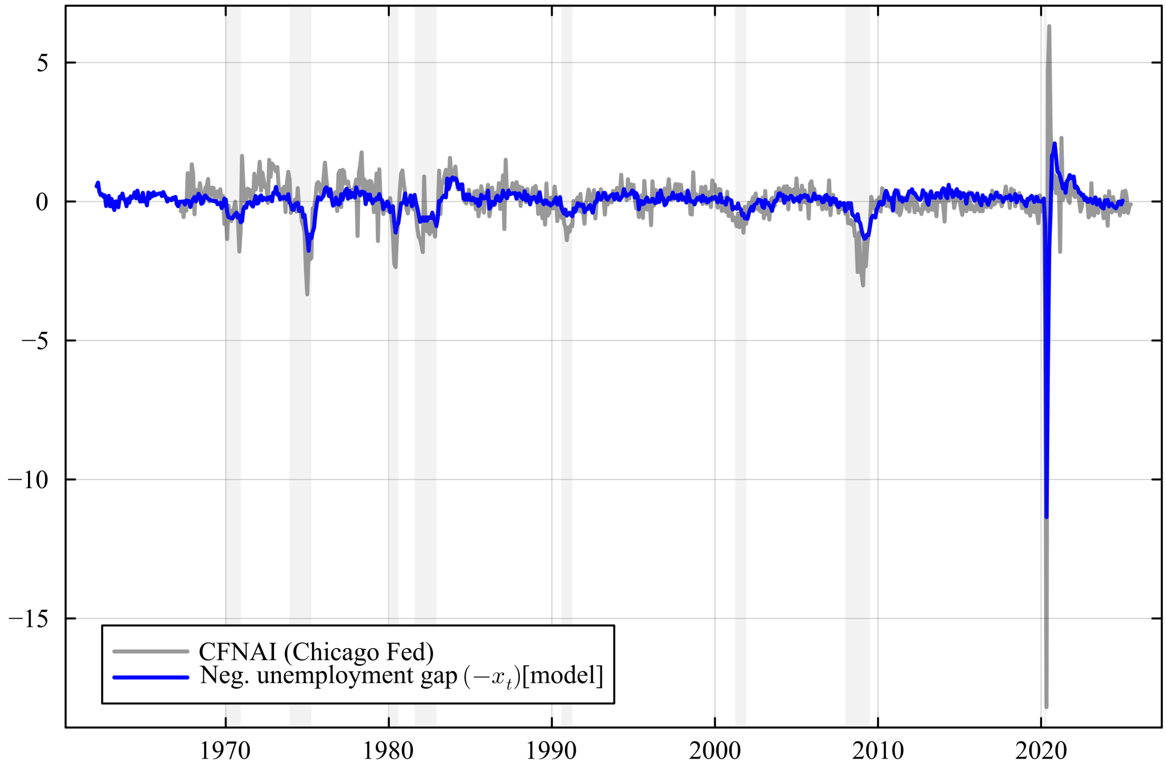


Figure 14: **Estimated unemployment-gap**

Note: The figure plots the agents' real-time estimates of the (negative of) unemployment gap with the CFNAI (Chicago Fed National Activity Index), a composite measure of real activity based on 81 series. The shaded regions indicate NBER recessions. CFNAI is obtained from the Chicago Fed website.

E Perceived natural rate of interest $r^n_{t|t}$

Figure 15 plots the evolution of the natural rate of interest as inferred by agents in real-time. This is a useful measure and serves as a barometer to gauge the stance of policy.

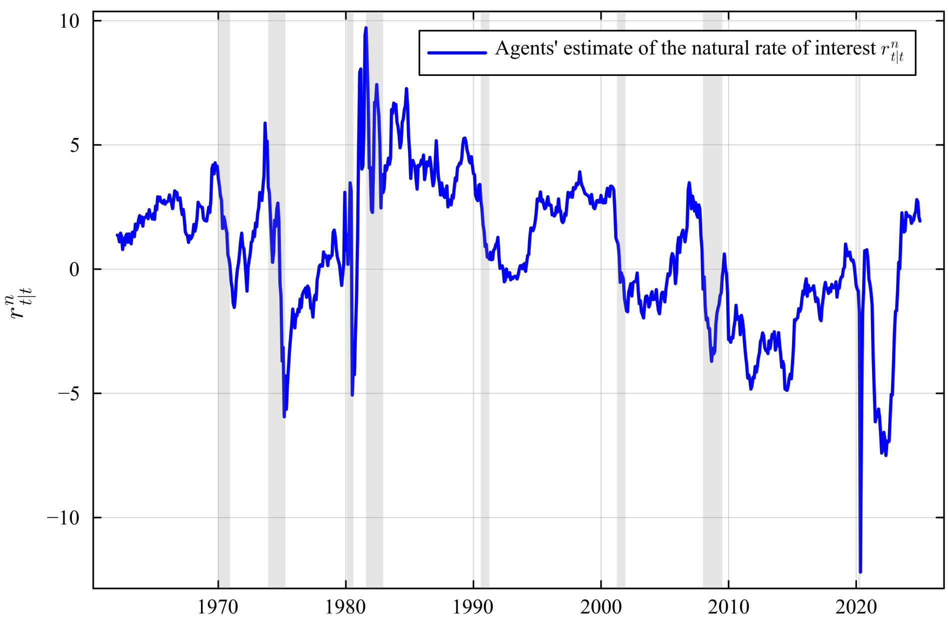


Figure 15: **Estimates of the natural rate of interest**

Note: Shaded regions indicate NBER recessions

The estimates are broadly in line with the secular decline reported in the literature. They typically fall with NBER recessions, which is rather intuitive based on the IS-curve.

F Has the "true" slope of the Phillips curve changed over time?

In the baseline model, we proceeded with the assumption of a time-invariant Phillips curve in order to keep the model parsimonious and avoid over-reliance on time-variation Phillips curve to explain the observations. However, one can statistically test if there has been a change in the true relationship between unemployment-gap and inflation during the pandemic (motivated by literature arguing that structural shifts in the Phillips curve during the pandemic period were an important factor driving the inflation surge).

In order to do this, we can construct an expectation-adjusted measure of inflation π_t^{adj} as:

$$\pi_t^{adj} = \pi_t - E_{t-1}^P \pi_{t+1}$$

This expectations-adjusted measure of inflation is regressed against the unemployment-gap variable x_t. The OLS result would not recover the true underlying κ since we would expect x_t to be correlated with the supply shock ξ_t. It will only be able to recover the reduced-form relationship between inflation and unemployment gap, not the structural one.

Table 2 shows that once expectations are taken into account, the relationship between inflation and the unemployment-gap is statistically significant - implying a 1% increase in the unemployment-gap associated with a roughly 10 bp decrease in inflation. It is worth noting that the estimate is quite close to the agents' beliefs about κ at the end of the sample shown in Figure 5 (≈ -0.11). However, as the second column shows, the dummy representing the change in slope post 2020 is not found to be statistically significant at the 5% level. The regression does not give convincing evidence of a change in the reduced-form relationship during the recent surge.

Table 2: Expectations-augmented Phillips curve: Examining change in slope

	Model 1 Baseline	Model 2 Change in slope post-2020
$\hat{x}_t$	**-0.106****	**-0.193****
	(0.037)	(0.063)
$\hat{x}_t \times \mathbf{1}_{\{t>2020\}}$	—	0.119
		(0.067)
Observations	756	756
R^2 (uncentered)	0.023	0.028
F-stat	8.322	10.42
p-value	0.004	3.44e-05

Notes: Dependent variable is π_t^{adj}. Model 2 augments Model 1 with the interaction $x_t \times \mathbf{1}_{\{t>2020\}}$, where $\mathbf{1}_{\{t>2020\}}$ equals one for dates from 2020 onward. HAC (Newey–West) robust standard errors (Bartlett kernel, maxlags = 6). R^2 is computed without centering because the model has no constant. Significance: * indicates that the coefficient is significant at the 5% level.

Another way to shed light on whether time-variation in the Phillips curve slope is an important feature of economic dynamics necessary to explain the data is to directly test for time-variation in the κ_t against the null of a constant-κ_t in the relationship:

$$\pi_t^{adj} = -\kappa_t x_t + \xi_t$$

If the data supports time-variation in κ_t, then relaxing the restriction of a constant κ should result in a significantly better fit, as measured by log-likelihood. A likelihood-ratio (LR) test is conducted, testing the alternate hypothesis of time-variation in κ against the null of a constant

κ. The LR test is not statistically significant with $p \approx 1.00$. This suggests that the model finds no evidence in favor of time-variation slope of the Phillips curve once expectations are accounted for. The results of the test are reported in Table 3. Re-estimating the model with a time-varying slope κ (allowing for innovations with a reasonably small variance 10^{-3} over time) produces nearly identical paths of beliefs as the baseline. Overall, the results suggest that changes in the slope of the structural Phillips curve are not a necessary feature to explain post-war U.S. economic dynamics.

Table 3: Results of the Likelihood Ratio Test

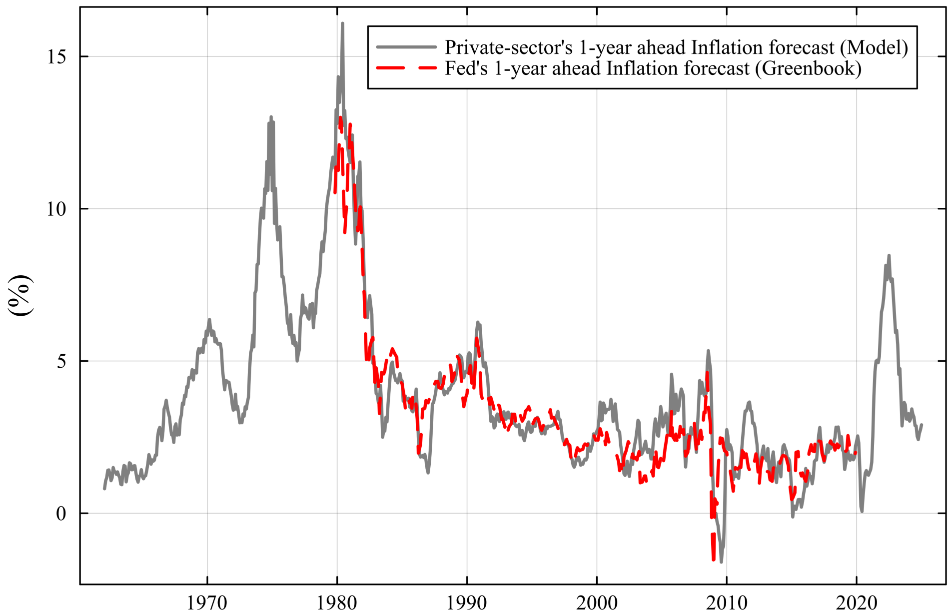
Model	**Log-Likelihood**	**Parameters**	χ^2	**df**	**p-value**
Restricted (Constant κ)	-402.460	3	3.1×10^{-7}	1	1.000
Full (Time-Varying κ)	-402.460	4			

Note: The restricted model assumes a constant κ coefficient, while the full model allows it to vary over time. The constant parameter case is nested within the full time-varying parameter model. The Likelihood Ratio Test (LRT) statistic (χ^2) is calculated as $2 \times (\log L_{\text{Full}} - \log L_{\text{Restricted}})$, with degrees of freedom (df) equal to the difference in the number of parameters between the two models.

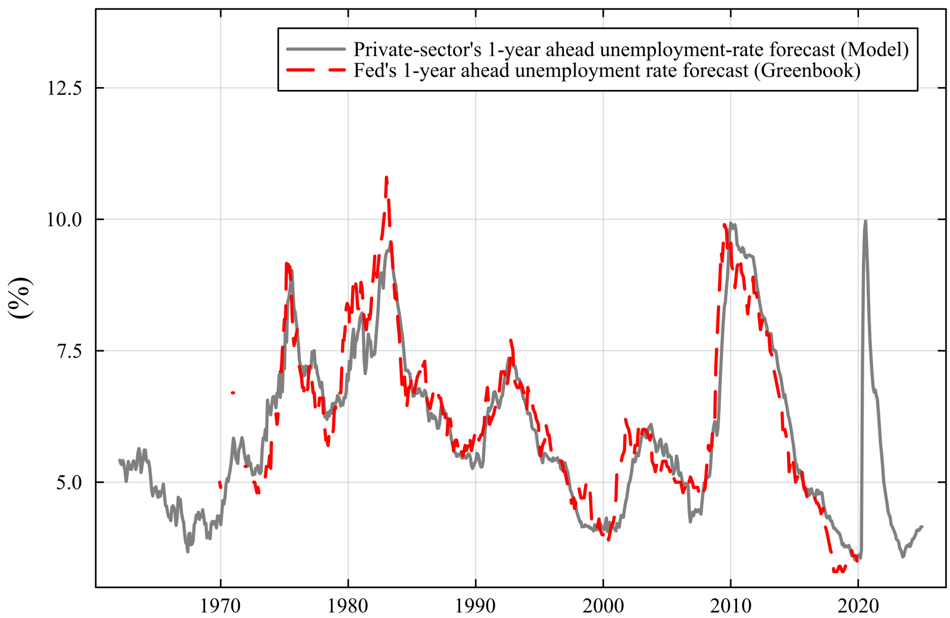
Overall, these results favor the time-invariant view of the Phillips curve adopted by the private agents in the model. Since the modeling framework in this paper explicitly accounts for endogenous formation of expectations, κ should not be interpreted as a purely a statistical relationship between inflation and unemployment described by Phillips (1958).

G Is the degree of information asymmetry small or large?

The analysis in this paper assumes very little information asymmetry between the Fed and the private-sector: (i) their beliefs about the structural parameters and shocks in the economy are shaped by the same data and the same economic model (ii) the filtering problem assumes that private-agents are able to track the true policy preferences efficiently, meaning that their tracked estimates never stray far away from the "true" underlying parameters describing the Fed policymakers' stabilization preferences. One way to test whether these assumptions hold well during the sample period is to compare the agents' forecasts as implied by the model with those from the Greenbook: the Fed's internal forecasts prepared by the staff before each FOMC meeting. If the Fed policymakers do hold significantly different beliefs compared to the private agents, then it would be reflected as a discrepancy in their forecasts. From Figure 16, we observe that Fed's internal forecasts as seen in the Greenbook seem to be in line with those of the private agents in the model. This suggests that any information advantage that the Fed possessed was of small order, at least to the extent that it influenced their outlook.



(a) **Inflation forecasts:** Comparing the Fed's 1-year ahead Headline CPI inflation forecast from Greenbook with 1-year ahead private-agents' forecasts from the baseline model



(b) **Unemployment rate forecasts:** Comparing the Fed's 1-year ahead unemployment rate forecast from Greenbook with 1-year ahead private agents' forecasts from the baseline model

Figure 16: **Macroeconomic outlook: Fed (Greenbook) vs Private Agents (Model)**