Learning, Changing Perceptions and the Anchoring of Expectations *

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Abstract

I explain inflation and unemployment dynamics in the post-war U.S. by estimating a model in which policymakers and private agents update their beliefs under structural learning, observing realizations of macroeconomic aggregates: inflation, unemployment rate and the nominal interest rate. Monetary policy is set optimally under discretion but private agents suspect that the policymakers' stabilization preferences are evolving, which the agents learn about based on observed policy behavior. Crucially, this framework generates an endogenous channel through which systematic monetary policy anchors expectations. The model demonstrates how Federal Reserve policy over the past forty years has stabilized long-run inflation expectations while rendering short-run expectations more susceptible to shocks by steadily putting more emphasis on real-side stabilization. The learning mechanism offers a new explanation for the recent inflation surge, and why it differed markedly from the crises of the 1970s and 1980s.

Keywords: Inflation Stabilization, Monetary Policy, Interest Rates, Lucas Critique, New Keynesian
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1 Introduction

The post-pandemic surge in inflation—which saw U.S. CPI inflation rise to a 40-year high of 9.1% in June 2022—caught both the private sector and policymakers off-guard. As inflation remained elevated, comparisons were drawn to the "Great Inflation" of the 1970s–80s. Both periods were accompanied by a sharp rise in oil prices and supply disruptions, yet they yielded very different macroeconomic outcomes: a protracted inflationary spiral culminating in a painful disinflation in the 1980s versus a relatively swift "soft landing" that has characterized the US economy's recovery from the pandemic shock. A particularly striking feature of the recent surge has been the stability of long-run inflation expectations - despite repeated upside surprises in inflation data, long-term expectations remained remarkably well-anchored—peaking around 3%.

To explain these divergent outcomes, I develop a model in which neither the private-agents (firms and households) nor the central bank policymakers know the structural parameters and shocks describing aggregate economic dynamics. Instead, they learn from realizations of macroeconomic aggregates and update their beliefs based on a forward looking model reflecting *structural* reasoning: internalizing the feedback beliefs to macroeconomic outcomes. Private agents also infer policymakers' time-varying stabilization preferences in real time from observed policy actions, creating an endogenous channel through which expectations become anchored. I estimate the model using monthly U.S. data on inflation, unemployment, and the federal funds rate over the postwar period, together with 10-year-ahead inflation expectations from the Survey of Professional Forecasters (SPF).

From the lens of the model, the contrasting dynamics of the two episodes emanated in large part from changes in the Federal Reserve's systematic conduct of monetary policy. During the Great Inflation of the 1970s and 1980s, the Fed was perceived as relatively tolerant of unemployment gaps. Combined with a prolonged period of accommodative policy, this perception led private agents to revise upward their beliefs about the Fed's inflation target, resulting in a de-anchoring of expectations. By contrast, in the post-Volcker era — particularly after the Global Financial Crisis — agents have perceived the Fed as placing greater weight on keeping the gap closed. This change in agents' perceptions regarding the policy environment they face, based on observed policy behavior has directly led to the anchoring of long-run inflation expectations over the past forty years.

This channel is also key to understanding the seemingly costless disinflation ("soft landing") that followed the recent inflation surge. When the central bank has a strong preference to stabilize unemployment-gaps, monetary policy acquires greater predictability: interest rates are less responsive to idiosyncratic shifts in the policymakers' inflation target. As a result, agents do not interpret short-run policy "mistakes" (the Fed falling "behind the curve" for instance) as signaling changes in the long-run target, making long-run expectations more resilient. At the same time, however, agents anticipate that policymakers will accommodate persistent shocks in the Phillips-curve — since offsetting them would entail costly real adjustments — thereby amplifying their pass-through to short-run inflation expectations. Thus under learning, short-run and long-run inflation stabilization are in tension. This mechanism is found to explain the peculiar discord between the behavior of realized inflation and long-run inflation expectations during the recent surge. In contrast with Taylor-rule type specifications,[1] this model directly accounts for the role of anchoring, which endogenously results from the interaction between systematic monetary policy and private agents learning.

The behavior of firms, households, and policymakers depends on their beliefs about the current and future state of the economy. Beliefs are shaped by how agents interpret observed outcomes through their perceived model of economic dynamics. While much of the literature assumes that agents rely on atheoretical, backward-looking learning rules, this paper instead emphasizes *structural* learning: agents, guided by economic reasoning, recognize the forward-looking aggregate dynamics implied by their own optimizing behavior (see Williams (2003) for the distinction between *structural* and *adaptive* learning). This framework models expectations under imperfect knowledge while preserving the discipline of rational expectations, ensuring that beliefs remain consistent with the structural dynamics generated by agents' decisions.

The central bank is assumed to operate under a dual mandate - weighing between stabilizing inflation around its long-run target and unemployment around its "natural" level. Central bank policymakers pursue these economic goals by setting nominal interest rate policy. In the absence of a commitment device, agents might reasonably suspect that monetary policy is subject to the whims of the policymakers, who may operate with different policy targets (as documented in Cogley, Primiceri and Sargent (2010), Ireland (2007), Del Negro and Eusepi (2011) and Lakdawala (2016) for the U.S) and weigh their

[1] Instrument-rules such as the Taylor-rule are opaque regarding the trade-offs that policymakers actually face while deliberating about the conduct of monetary policy. It also causes an inconsistent asymmetry regarding how policy behavior is modeled, departing from the forward-looking optimizing framework modeling household and firm behavior. This critique is outlined in Svensson (2003)

mandates differently.[2]

The central bank's policy decisions reveal information to the private-sector regarding the economic goals that policymakers seeks to achieve (represented by a loss function). Hence, there is a non-trivial role for a "learning-channel"[3] of monetary policy transmission in this environment informing agents about the direction of monetary policy. Through this channel, policy influences agents expectations an infinite horizon into the future: there is some *forward guidance* baked into policy itself. When private agents observe a policy response different from what they expect, they attribute part of it to be resulting from idiosyncratic policy errors which will get corrected in the future as policy catches up (policy falling "behind the curve" in some sense), but attribute the rest to be coming from a shift in policymakers' stabilization preferences and hence indicative of how policy will be conducted in the future (in this case, the agents are behind the curve and need to catch up).

Importantly, macroeconomic shocks by themselves do not cause expectations to dis-anchor in this framework so long as policy responds in the way that the agents expect. This makes the description of anchoring here distinct from papers such as Carvalho, Eusepi, Moench and Preston (2023) and Gati (2023) where forecast errors in inflation can directly cause dis-anchoring. Agents in the model reason and distinguish between outcomes resulting from factors that are under the policymakers' control and those outside their direct control.

The model also sheds light on the state-dependent nature of agents' inference: informative signals about structural parameters arrive only in particular states of the world, often in discrete "lumps". In the model, agents revise beliefs about policymakers' preference for unemployment stabilization primarily when large, persistent cost-push shocks generate visible policy trade-offs. Similarly, learning about the slope of the structural Phillips curve is exceptionally slow due to a particularly adverse signal-to-noise ratio posing identification problems, with beliefs getting updated almost exclusively around

[2]Time-variation in policymaker preferences preclude commitment even if a commitment mechanism did exist since the optimal commitment instrument-rule would be changing every period. In contrast, the optimal commitment solution discussed in Kydland and Prescott (1977) assumes a setting where there is an agreed-upon time-invariant social welfare function.

[3]Melosi (2016) studies signaling effects of policy in a DSGE framework, with the key distinction being that central bank policy is assumed to carry information about the aggregate shocks themselves and not about policymakers' objectives. In this paper, I assume that neither type of agent (private-sector or central bank) has an information advantage regarding the policy-invariant latent structural variables describing economic dynamics. The only source of information asymmetry results from firms and households in the model not being able to observe the central bank policymakers' stabilization preferences.

demand-driven recessions. Incorporating structural learning allows the model to speak to these dynamics which would otherwise remain entirely obscure.

Related literature: Under optimizing behavior, firms' pricing decisions, households' consumption and savings behavior, and policymakers' policy choices are all forward-looking. The New-Keynesian framework describes macroeconomic dynamics as arising from strategic behavior by individual agents who inform their decisions based on their expectations of the future. Hence the modeling of expectations is key - any economic model seeking to explain macroeconomic dynamics must also explain how expectations are formed and how agents inform their beliefs. This view heralded the rational expectations revolution in the 1970s. Lucas (1972) warned against economic analysis based on statistical relationships without a firm understanding of their structural foundations. This became all too clear when the apparent tradeoff between inflation and unemployment, first described by Phillips (1958) and central to policy doctrine at the time collapsed as both inflation and unemployment rose simultaneously during the "stagflation" of the 1970s.

The rational expectations revolution in economics emphasized deriving macroeconomic dynamics from the optimizing behavior of agents and structural reasoning, eschewing models based on purely statistical relationships in data. Even as rational expectations became the mainstay of macroeconomic theory, empirical success was mixed. DSGE models imposing full-information rational expectations have trouble matching the dynamics seen in data without resorting to *ad-hoc* adjustments (such as price-indexation and habit formation in consumption), while also being at odds with sluggish response of expectations to macroeconomic news as evidenced by survey data (studied in Coibion and Gorodnichenko (2015a)). The "full-information" assumption abstracts from the information structure of the economy. Agents have somehow solved econometric challenges that even economists continue to struggle with. The deep structural variables describing economic dynamics are known to agents at all times.

The classical literature on learning in macroeconomics studies how agents *arrive* at rational expectations via learning in real-time (Bray and Savin (1986), Marcet and Sargent (1989)). An off-shoot of this literature has explored the implications of learning in macroeconomic dynamics (Marcet and Nicolini (2003), Bullard and Cho (2005), Bullard and Eusepi (2005)). Agents' imperfect knowledge and real-time learning has been proposed as an alternate explanation of the persistence in economic dynamics (see Milani (2007) and Erceg and Levin (2003)). Primiceri (2006) and Sargent, Williams and Zha (2006) explain policy behavior during the Great Inflation period of the 80s and the subsequent disinflation as arising from policymakers' belief dynamics under learning. Other papers have emphasized on how private agents learn about the central bank's policy targets based on a Taylor-rule (Kozicki and Tinsley (2005), Erceg and Levin (2003)). The literature on learning in studies macroeconomic dynamics in a setting where economic agents do not possess full-information. However, most papers have studied learning under the assumption of bounded rationality - where agents use simple, mis-specified models of the economy (reasoning based on purely statistical relationships in the history of data) to inform their beliefs and forecasts, not accounting for feedback from their expectations into observed outcomes. This reduced-form approach to learning, while steering clear of imposing implausibly large cognitive demands of its agents renders their learning rules inconsistent with the actual behavior of macroeconomic aggregates that would arise under forward-looking optimizing behavior. In contrast, this paper studies structural learning: where agents reason based on a structural model of how these macroeconomic aggregates are determined in equilibrium as a result of agents' decisions. But the information structure of the economy only allows macroeconomic aggregates to be directly observed: the structural variables in their model must be inferred via learning. This allows for the modeling of learning dynamics within the discipline of rationality, by imposing that individual agents' beliefs are consistent with the resulting economic dynamics at all times.

The causes of the Great Inflation and the subsequent Great Moderation have been the subject of long-standing debate. One school attributes the volatility of the 1970s–80s era to "bad luck": a sequence of large, adverse non-policy shocks (e.g., Sims (1980); Bernanke and Mihov (1998)). Under this view, the structure of the economy and policy rules remained largely stable; it was the volatility of exogenous shocks that changed. This view implies that monetary policy made little fundamental progress—if similar shocks were to recur, so would similar outcomes.

An alternative view emphasizes policy errors. Clarida, Gali and Gertler (2000), Taylor (1993) and Lubik and Schorfheide (2004) argue that the shift to systematic anti-inflation policy by the Fed under the chairmanship of Paul Volcker was central to the subsequent stability. DeLong (1996) and Romer and Romer (2002) provide narrative evidence that policymakers in the 1970s were misled by a perceived long-run tradeoff between inflation and unemployment.

Anchoring of expectations in the recent years has been proposed as another explanation for the quiescence of the Great Moderation period. Recent papers have studied anchoring as resulting from agents' learning behavior. Carvalho, Eusepi, Moench and Preston (2023) describe how anchoring has from agents endogenously switching from a decreasing-gain to a constant-gain model to forecast inflation in response to realized forecast errors. Gati (2023) extends the framework to allow for continuous adjustment of the gain in response to forecast-errors. This paper adopts a different perspective: anchoring or de-anchoring results not from macroeconomic surprises themselves, but from how monetary policy responds to them and *signals* policymakers' incentive to stabilize. A period of sustained inflation alone is not sufficient to de-anchor expectations—so long as policy reacts in a way that credibly signals a strong incentive to restore stability. Hence, it is the interaction between macroeconomic surprises and the policy response that determines whether expectations are anchored and whether they remain anchored, rather than macroeconomic shocks in isolation.

This paper contributes to the debate by offering a framework that disentangles these three explanations and sheds light on how each of these contributed to shaping the two prominent inflationary episodes of the post-war U.S. The goal of the paper is to analyze the role of structural innovations to the economic environment, the conduct of monetary policy and their interaction with agents' learning in shaping inflation and unemployment dynamics.

In Section 2, I derive the equations describing the structural model of the economy as perceived by the private agents (firms and households) arising from their optimizing behavior. In Section 3, I describe how agents recursively update their beliefs conditional on their observations every period based on their perceived structural model of the economy. In Section 4, I describe the conduct of monetary policy from the perspective of the agents and describe how they update beliefs regarding the policymakers' stabilization preferences conditional on observed nominal interest rates. In Section 5, I discuss the estimation methodology and the evolution of agents' beliefs under learning followed by the concluding remarks in Section 6.

2 A forward looking model of the economy

2.1 Information Structure and Learning

In order to specify the the structural dynamics of the economy under learning, it is important to discuss how agents form expectations and what information they condition their beliefs upon. Time is discrete in the model ($t = 0, 1, 2, 3, \ldots$). I assume an information structure where neither the realizations of the latent structural parameters or the structural shocks are directly observed by the agents. The only relevant signals available to them comes in the form of the realizations of inflation, unemployment rate and the nominal interest rate. At the beginning of each period, firms and households make their decisions conditional on the history of signals observed until that point. The period-t inflation π_t and unemployment rate u_t realize resulting from their decisions and economy-wide structural innovations. The central bank then sets the interest rate i_{t+1} in anticipation of the future, conditional on its policy preferences in period-t. [4]

The entire history of aggregate signals observed by agents up to (and including) period t_0 is denoted by:

$$\mathcal{H}^{t_0} = \{i_{t+1}\}_{t=-\infty}^{t_0} \cup \{z_t\}_{t=-\infty}^{t_0}$$

where $z_t = [\pi_t, u_t]'$ is the vector of realizations of inflation and unemployment, and i_{t+1} denotes the interest-rate policy set in period-t.

2.2 The agents' perceived model of aggregate dynamics

I derive the perceived laws of motion of the macroeconomic aggregates from the perspective of private agents arising from their forward-looking decisions (households' consumption-savings decisions and firms' pricing decisions) every period. I assume that all private-agents (firms and households) have common expectations and beliefs[5] formed conditional on the same data and model. I denote private-agents beliefs formed based on $t-1$ history $\mathcal{H}^{t-1}$ as E^P_{t-1}.

[4] Such an assumption implies that policy-behavior must be forward-looking. Policymakers set period-$t+1$ interest rates without knowing period-$t+1$ macroeconomic outcomes z_{t+1}.

[5] I abstract from heterogeneity in agents' belief-formation process.

2.2.1 Aggregate demand: Households' saving problem

Consider the problem of a representative household who derive utility from consumption and save in nominal bonds B_t in every period t conditional on their beliefs formed based on observed history $\mathcal{H}^{t-1}$:

$$\max \ E^P_{t-1}\left[\sum_{k=0}^{\infty}\beta^k \frac{C_{t+k}^{1-\sigma}}{1-\sigma}\right] \tag{1}$$

where C_t is consumption, β is a discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution

The household's budget constraint in nominal terms is:

$$P_tC_t + B_t = (1 + i_{t-1})B_{t-1} + W_t + T_t + \Pi_t \tag{2}$$

where P_t is the aggregate price level, B_t are nominal bond holdings, i_{t-1} is the nominal interest rate in period $t-1$ (set at $t-2$), W_t denotes their nominal wage-income, T_t denote lump-sum government transfers and Π_t denotes profits from the ownership of firms.

In this setting, the only dynamic decision that the representative household makes is the savings decision:

$$C_t^{-\sigma} = \beta E^P_{t-1}\left[C_{t+1}^{-\sigma}\frac{1+i_t}{1+\pi_{t+1}}\right] \tag{3}$$

where $1 + \pi_{t+1} = P_{t+1}/P_t$.

Since there is no investment or government expenditure in the model, all output is consumed every period $Y_t = C_t$, taking logs we get the aggregate-demand IS-curve:

$$y_t = E^P_{t-1}[y_{t+1}] - \frac{1}{\sigma}(i_t - E^P_{t-1}[\pi_{t+1}] - \rho) \tag{4}$$

where $\rho = -\log\beta$. Defining the *ex-ante* real interest rate as $r_t = i_t - E^P_{t-1}[\pi_{t+1}]$, the IS curve may be more compactly written as:

$$y_t = E^P_{t-1}[y_{t+1}] - \frac{1}{\sigma}(r_t - \rho) \tag{5}$$

At some hypothetical real interest rate, output would continue along the trend path, y_t^n (or the "natural" rate of output) absent cyclical fluctuations. y_t^n corresponds to the output under efficient (flexible) prices. We call this the natural rate of interest and denote it by r_t^n.

$$y_t^n = E_{t-1}^P[y_{t+1}^n] - \frac{1}{\sigma}(r_t^n - \rho) \tag{6}$$

$$y_t - y_t^n = E_{t-1}^P[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(r_t - r_t^n) \tag{7}$$

Denoting $y_t - y_t^n$ as output-gap $\hat{y}_t$, we may re-write:

$$\hat{y}_t = E_{t-1}^P[\hat{y}_{t+1}] - \sigma^{-1}(i_t - E_{t-1}^P[\pi_{t+1}] - r_t^n) \tag{8}$$

Agents understand that due to nominal rigidities, macroeconomic shocks and monetary policy can cause cyclical deviations in output, away from its natural rate, causing it to be inefficiently high or low at times. Further, they assume a linear relationship between output-gap and cyclical fluctuations in the unemployment rate, or the unemployment-gap[6] defined as the difference between the unemployment rate and the natural rate of unemployment: $x_t = u_t - u_t^n$, we have

$$\hat{y}_t = -\eta x_t$$

This yields the dynamic IS-curve representing the law of motion of the unemployment-gap:

$$x_t = E_{t-1}^P x_{t+1} + \tau(i_{t,t-1} - E_{t-1}^P \pi_{t+1} - r_t^n) \tag{9}$$

where $\tau = 1/(\sigma\eta)$ represents the ratio of the intertemporal elasticity of substitution and the Okun's law coefficient. τ is assumed to be constant and known to all agents.

[6]This relationship is based on Okun's law which has been relatively stable across the sample period (see Ball et al. (2017)). I assume that the agents take this as a stylized fact.

2.2.2 Aggregate supply: Firms' pricing problem

Firm price-setting under dispersed information Every period, all firms are assumed automatically adjust prices by the perceived trend inflation rate conditional on their $t-1$ information set, given by π^{*P}_{t-1}.[7] The perceived level of trend inflation does not enter firms' pricing problem since all firms have the same beliefs and adjust to the same perceived trend rate every period. However, relative price adjustments are subject to Calvo frictions.

Consider the problem of price-setting by monopolistic firms in the standard New-Keynesian setting, where they solve for their optimal reset price while subject to Calvo frictions. Only a fraction $(1-\theta)$[8] of firms are able to adjust their price every period. Every period t, the firms solve for their optimal reset price conditional on observed history of signals $\mathcal{H}^{t-1}$.

Additionally, I assume that each firm receives a private-signal perfectly informing it about its current real marginal cost $mc_t(j)$ (based on Nimark (2008)'s model of dispersed information). This signal is comprised of the economy-wide real-marginal cost mc_t plus an idiosyncratic firm-specific $i.i.d$ (across firms and across time) component with *large* variance.[9]

$$mc_t(j) = mc_t + \varepsilon_t(j) \quad \varepsilon_t(j) \sim N(0, \sigma^2_\varepsilon)\ \forall\ j \in (0,1) \tag{10}$$

Markups are assumed to be zero for simplicity, but the model is agnostic to the presence of non-zero markups as long as they are constant.

Expressing prices in log-terms, firm j resetting its price at time t, under knowledge that it can only reset with probability $1-\theta$ any period in the future would reset to:

$$p^*_t(j) = (1-\beta\theta)E^j\Big[\sum_{k=0}^{\infty}(\beta\theta)^k(p_{t+k} + mc_{t+k}(j))|\mathcal{H}_{t-1} \cup mc_t(j)\Big] \tag{11}$$

[7] This assumption permits the model to be log-linearized around the zero inflation steady state despite non-zero trend inflation. Inflation here refers to inflation beyond this "normal" level.

[8] The frequency of price-adjustment is constant and known to all agents.

[9] This assumption is motivated by the observation that firm-specific price changes are much more volatile as compared to fluctuations in the aggregate price-level.

where:

- $mc_{t+k}(j)$: firm j's real marginal cost in period $t+k$.
- β: discount factor, $0 < \beta \leq 1$.
- θ: Calvo probability of price non-adjustment
- $E^j[\cdot]$: expectation operator conditional on firm j's information set
- p_{t+k} log price level in period $t+k$

Dispersed information assumption: The firm is atomistic, its own marginal cost $mc_t(j)$ signal reveals negligible information about the economy-wide real marginal cost mc_t.

$$E^j[mc_t(j)|\mathcal{H}^{t-1} \cup mc_t(j)] = mc_t(j) \tag{12}$$

$$E^j[mc_{t+k}|\mathcal{H}^{t-1} \cup mc_t(j)] \approx E^j[mc_{t+k}|\mathcal{H}^{t-1}] \; \forall \; k \geq 0 \tag{13}$$

Expectations regarding future marginal costs is the same across firms since their private signal is uninformative.

Firm j's optimal reset price

$$p_t^*(j) = (1-\beta\theta)(E_{t-1}^P p_t + mc_t(j)) + \beta\theta E_{t-1}^P p_{t+1}^* \tag{14}$$

Aggregate Optimal Reset Price

All firms face the same price-setting problem, just observe a different marginal cost signal. Aggregating across all firms yields:

$$p_t^* = (1-\beta\theta)(E_{t-1}^P p_t + mc_t) + \beta\theta E_{t-1}^P p_{t+1}^* \tag{15}$$

Under a Calvo-probability of price adjustment of $(1-\theta)$, evolution of the aggregate price index satisfies:

$$p_t = \theta p_{t-1} + (1-\theta)p_t^* \tag{16}$$

Combining Equation 15 and Equation 16 yields the aggregate supply condition in terms of inflation $\pi_t = p_t - p_{t-1}$:

$$\pi_t = \beta E^P_{t-1}\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} mc_t + \frac{(1-\theta)^2(1-\beta\theta)}{\theta}(E^P_{t-1}mc_t - mc_t) \quad (17)$$

The expectational errors manifest due to the fact that the economy-wide real marginal cost is not perfectly forecast-able. If the firms' private signal revealed mc_t perfectly, we would go back to the standard New-Keynesian Phillips curve formulation and the expectational error term goes away. Also, it must be noted that the smaller the degree of price-stickiness (θ), the larger the contribution of these errors to actual inflation.

Pro-cyclicality of marginal cost Agents assume that the economy-wide real marginal costs have a pro-cyclical component. When output is inefficiently high, or unemployment is inefficiently low, marginal costs rise across firms. However, supply shocks can cause shifts in the economy-wide real marginal costs independently of the pro-cyclical component. Hence,

$$mc_t = -\gamma x_t + s_t$$

where s_t denotes the supply shock, assumed to follow a mean-zero stationary process and independently of the unemployment-gap x_t. γ represents the link between economy-wide real marginal cost and the cyclical fluctuations in unemployment. It is assumed to be constant, but unknown.

Forward-looking Phillips Curve

The resulting law of motion describing the dynamics of inflation may be compactly summarized as:

$$\pi_t = \beta E^P_{t-1}[\pi_{t+1}] - \kappa x_t + \xi_t \quad (18)$$

where:

- $E^P_{t-1}[\pi_{t+1}]$ denotes the private-sector's expectation of inflation in period $t+1$ conditional on history of signals $\mathcal{H}^{t-1}$
- κ represents the sensitivity of inflation to current unemployment-gap. Agents assume κ is constant but unknown and hence learn about it every period.

- ξ_t is a cost-push shock composed of the supply-shock s_t and the expectational error.

The $-\kappa x_t$ term describes the contemporaneous link between inflation and the "real" side of the economy. The unemployment-gap, x_t contemporaneously affects inflation through the slope parameter κ. The negative relationship implies that unemployment, when its below the natural rate exerts inflationary pressure on the economy.

The cost-push shock ξ_t captures the component of inflationary pressures arising from factors over and beyond the unemployment gap (*aggregate demand* channel). Agents allow for the possibility that the cost-push shocks have a persistent component. Hence they decompose the cost-push shock, ξ_t to into a *persistent* AR(1) component denoted by ξ_t^P, with large persistence ρ and a *transitory* i.i.d normal component ξ_t^T.

$$\xi_t = \xi_t^P + \xi_t^T$$

$$\xi_t^T \sim iid\, N(0, \sigma^2_{\xi,T})$$

The persistent component is assumed to evolve as:

$$\xi_t^P = \rho \xi_{t-1}^P + \varepsilon_t^P$$

$$\varepsilon_t^P \sim iid\, N(0, \sigma^2_{\xi,P})$$

From the private-agents' perspective, the persistent component ξ_t^P affects future outcomes and is hence relevant for forming expectations. It is however, confounded by the transitory component ξ_t^T and hence must be inferred as part of a signal extraction problem faced by all the agents in the economy.

Natural rate hypothesis: Agents in the model a-priori assume that monetary policy cannot systematically drive real variables once prices and expectations fully adjust. Under the natural rate hypothesis, the unemployment rate and the real interest rate attain their natural levels in the long-run, beyond the control of monetary policy. This is equivalent to agents assuming that the discount-factor $\beta \to 1$[10] so that expectations-augmented Phillips curve is vertical in the long-run. This hypothesis serves as a transversality condition in the agents' model.

[10]Since otherwise, the long-run unemployment-gap must be non-zero from Equation 18.

3 Learning: Latent structure of the economy

3.1 Learning and the Anticipated Utility assumption

This paper takes the anticipated utility approach to learning described in Kreps (1998). Under this assumption, agents form "certainty-equivalent" estimates of the relevant state-variables every period and treat their best estimates as if they were constants known with certainty when making decisions. Agents disregard the possibility of updating their current estimates in the future. This assumption is widely used in the literature on learning in macroeconomics (see Evans and Honkapohja (2001)). It allows us to surmount the computational challenges associated with solving the dynamic problem in the full Bayesian setting. Cogley and Sargent (2008) demonstrate how the anticipated utility assumption provides an excellent approximation to Bayesian decision-making in a wide class of problems.

3.2 Outline

In order to characterize the learning-dynamics in the model, we must explicitly spell out how information arrives and how agents process signals to update their beliefs. Denote agents beliefs regarding the policymakers' stabilization preferences conditional on $\mathcal{H}^t$ by the vector $\mu_t^P = [\pi_t^{*P}, \lambda_t^P]$, and the true preferences as $\mu_t = [\pi_t^*, \lambda_t]$.

We may view each time period as consisting of two stages. In the first stage, the period-t aggregates $z_t = [\pi_t, u_t]$ are observed by the agents. Conditional on these observations, they update beliefs about $[u_t^n, r_t^n, \xi_t^P, \kappa]$. Based on these updated beliefs and μ_{t-1}^P, agents form expectations over an infinite horizon. Then the Fed's period-t interest rate decision $i_{t+1,t}$ is observed and the agents update their beliefs regarding the policymakers' preferences, arriving at μ_t^P.

The timeline can be summarized:

- $\{E_{t-1}^P x_{t+1}, E_{t-1}^P x_{t+2}, ...\}$,$\{E_{t-1}^P \pi_{t+1}, E_{t-1}^P \pi_{t+2}, ...\}$ formed based on $\mathcal{H}^{t-1}$
- Firms and households make their period-t decisions.
- $z_t = [\pi_t, u_t]$ observed, $u_{t|t}^n, r_{t|t}^n, \xi_{t|t}^P, \kappa_{t|t}$ updated

- Expectations $\{E_t^{P-}x_{t+1}, E_t^{P-}x_{t+2}, ...\}$,$\{E_t^{P-}\pi_{t+1}, E_t^{P-}\pi_{t+2}, ...\}$ conditional on $\mathcal{H}^{t-1}\cup z_t$ are formed

- Fed sets i_{t+1}

- Private agents observe i_{t+1} update μ_t^P

3.3 Recursive Learning

Measuring the unemployment-gap: To facilitate a description of the learning dynamics, we must first describe how agents infer the *cyclical gap* component x_t based on observing the aggregate unemployment rate u_t. The unemployment gap is defined as the deviation of unemployment-rate from its trend or "natural" rate u_t^n.

$$x_t = u_t - u_t^n$$

Conditional on observing the unemployment rate u_t, agents update their estimates of the natural rate $u_{t|t}^n$ according to the univariate algorithm[11]:

$$u_{t|t}^n = u_{t-1|t-1}^n + g_u(u_t - u_{t-1|t-1}^n) \tag{19}$$

From the natural rate hypothesis,

$$\lim_{h\to\infty} E_t x_{t+h} = 0$$

Hence, the unemployment rate is always expected to converge to the natural rate u_t^n in the long-run equilibrium, consistent with the update-rule Equation 19. Staiger, Stock and Watson (1997) have shown that such univariate algorithms deliver estimates that are essentially indistinguishable from the ones derived under more sophisticated procedures. Similarly, Kamber, Morley and Wong (2018) demonstrate that economic slack-measures derived from univariate procedures can be quite robust to revisions and are in line with more refined measures of real-time economic activity (such as the Chicago Fed National Economic Activity Index).

[11]This is essentially an exponentially weighted moving-average of the unemployment-rate, with past observations receiving exponentially smaller weights.

Agents' estimate of the contemporaneous unemployment gap is:

$$\hat{x}_t = u_t - u^n_{t|t}$$

In keeping with the anticipated utility assumption, I assume that the agents treat their real-time estimate of the unemployment-gap constructed based on this de-trending procedure as if it's known with certainty. I abstract from issues of real-time mis-measurement and data revisions. I do however find that the agents' implied measure lines up quite well with series that are revised retroactively (the CFNAI index as an example, see Appendix C) suggesting that these issues are not of large consequence.

Agents' perceived model of aggregate dynamics: The equations defining the aggregate laws of motion of the economy from the perspective of the private-agents are:

$$\pi_t = E^P_{t-1}[\pi_{t+1}] - \kappa\hat{x}_t + \xi_t$$

$$\xi_t = \xi^P_t + \xi^T_t; \quad \xi^T_t \sim iid\ N(0, \sigma^2_{\xi,T})$$

$$\xi^P_t = \rho\xi^P_{t-1} + \varepsilon^P_t\ ; \quad \varepsilon^P_t \sim iid\ N(0, \sigma^2_{\xi,P})$$

$$\hat{x}_t = u_t - u^n_t$$

$$u^n_t = u^n_{t-1} + g_u(u_t - u^n_{t-1})$$

$$\hat{x}_t = E^P_{t-1}x_{t+1} + \tau(i_t - E^P_{t-1}\pi_{t+1} - r^n_t)$$

Once the period-t aggregates $[\pi_t, u_t]$ are observed, agents update their beliefs regarding κ, ξ^p_t and r^n_t recursively every period treating $[\sigma^2_{\xi,T}, \sigma^2_{\xi,P}, g_u, \tau, \rho]$ as known constants.

Learning the natural rate of interest r^n_t: Conditional on the estimate of the current period unemployment-gap $\hat{x}_t$, the contemporaneous estimate of the natural rate of interest $r^n_{t|t}$ [12] is given by the dynamic IS curve (Equation 9):

[12] As we shall see, $r^n_{t|t}$ is irrelevant for expectations since stabilizing against demand shocks is costless and poses no policy trade-off.

$$r^n_{t|t} = \tau^{-1}(E^P_{t-1}x_{t+1} - \hat{x}_t) + i_t - E^P_{t-1}\pi_{t+1}$$

Further, agents assume that r^n_t follows a Martingale, so that $r^n_{t+1|t} = r^n_{t|t}$

Learning ξ_t, κ_t: Based on the estimated unemployment-gap $\hat{x}_t$, the persistent component of the cost-push shock ($\xi^P_{t|t}$) and the slope of the Phillips curve ($\kappa_{t|t}$) are recursively updated every period conditional on realized inflation π_t. We can observe that any inflation surprise will be perceived to result from three components:

$$\underbrace{\pi_t - E^P_{t-1}\pi_t}_{\text{inflation surprise}} = \underbrace{(\kappa - \kappa_{t|t-1})(-\hat{x}_t)}_{\text{mis-perceived slope of PC}} + \underbrace{\xi^P_t - \xi^P_{t|t-1}}_{\text{surprise in persistent supply shock}} + \underbrace{\xi^T_t}_{\text{transitory supply shock}}$$

Hence, agents face a fundamental signal-extraction problem in real time: they cannot immediately distinguish whether an observed shock originates from the demand channel or the supply channel. Even when attributing part of the disturbance to aggregate-supply shocks, they must still disentangle the persistent component—relevant for expectations—from the transitory one, which is just noise. Accordingly, agents revise their beliefs about ξ_t and κ_t according to the following equations:

Observation Equation:

$$\pi_t - E^P_{t-1}\pi_{t+1} = \begin{pmatrix} -\hat{x}_t & 1 \end{pmatrix} \begin{pmatrix} \kappa_t \\ \xi^P_t \end{pmatrix} + \xi^T_t, \quad \xi^T_t \sim N(0, \sigma^2_{\xi,T}).$$

State Transition:

$$\begin{pmatrix} \kappa_t \\ \xi^P_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} \kappa_{t-1} \\ \xi^P_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon^\kappa_t \\ \epsilon^P_t \end{pmatrix}, \qquad \begin{pmatrix} \epsilon^\kappa_t \\ \epsilon^P_t \end{pmatrix} \sim N(\mathbf{0}, \begin{pmatrix} 0 & 0 \\ 0 & \sigma^2_{\xi,P} \end{pmatrix})$$

Note that the dynamics equation implies that innovations to κ_t are zero, reflecting agents' prior that the true slope κ is time-invariant.

4 Monetary Policy

The private-agents assume that the central bank conducts monetary policy optimally under discretion, having the same beliefs about the policy-invariant latent structural variables as them formed based on the same economic model and data. The central bank is assumed to have a dual mandate characterized by the following loss function[13]:

$$\mathcal{L}_t = \sum_{j=1}^{\infty} [(\pi_{t+j|t} - \pi_t^*)^2 + \lambda_t x_{t+j|t}^2] \tag{20}$$

where $\pi_{t+j|t}$ and $x_{t+j|t}$ denote the central bank's forecasts of inflation and the unemployment-gap for period $t+j$ formed in period t after the contemporaneous π_t, x_t have been realized. Here, π_t^* and λ_t represent the central bank's policy preferences: its long-run inflation target and the relative weight it places on unemployment-gap stabilization. Private agents do not observe these and assume that they are subject to slow innovations over time. The only information asymmetry arises from private agents not being able to observe the policy preferences directly.

The central bank in its period-t decision cycle chooses the one-period ahead unemployment-gap plan $x_{t+1|t}$. Private-sector's expectations regarding future outcomes $E_t^{P-}\pi_{t+j}$ and $E_t^{P-}x_{t+j}$ formed prior to observing the period-t policy decision are assumed to be observed by the central bank and taken as given.[14]

From the agents' perspective, the central bank's period-t optimization problem is subject to the structural constraint implied by the Phillips curve:

$$\pi_{t+1|t} = E_t^{P-}\pi_{t+2} - \kappa_{t|t} x_{t+1|t} + \xi_{t+1|t} \tag{21}$$

Every period, the central bank sets $x_{t+1|t}$ which solves its static period-t minimization problem taking the observed private-sector expectations as given.

[13] I assume that the central bank's policy preferences are not fundamentally incompatible. Hence policymakers have no incentive to sustain a level of unemployment below the natural non-inflationary rate.

[14] E_t^{P-} denote expectations formed after the realization of the contemporaneous aggregates $z_t = [\pi_t, u_t]$ but before the period-t policy decision, and are thus formed conditional on beliefs $\kappa_{t|t}, \xi_{t|t}^p, \pi_{t-1}^{*P}, \lambda_{t-1}^P$.

The optimal unemployment gap plan $x_{t+1|t}$ is given by the targeting rule:

$$x_{t+1|t} = \frac{\kappa_{t|t}}{\lambda_t}(\pi_{t+1|t} - \pi_t^*) \tag{22}$$

Equation 22 describes the textbook "lean against the wind" policy solution where the central bank sets the one-period ahead unemployment-gap plan in response to expected deviations of inflation from its target.

Using Equation 21 and Equation 22, the central bank's one-period ahead inflation and unemployment-gap plan satisfy:

$$\pi_{t+1|t} = \pi_t^* + \frac{\lambda_t}{\lambda_t + \kappa_{t|t}^2}(E_t^{P-}\pi_{t+2} + \xi_{t+1|t} - \pi_t^*) \tag{23}$$

$$x_{t+1|t} = \frac{\kappa_{t|t}}{\lambda_t + \kappa_{t|t}^2}(E_t^{P-}\pi_{t+2} + \xi_{t+1|t} - \pi_t^*) \tag{24}$$

Equation 23 shows that when $\lambda > 0$, the central bank allows inflation to rise above its target when faced with a persistent cost-push shock. Thus the central bank accommodates the effect of the shock to an extent. Aside from the cost-push shock itself, private-sector inflation expectations directly influence the central bank's anticipated level of inflation in the future. Equation 24 describes the central bank's optimal unemployment-gap plan from the private-agents' perspective, conditional on its information and policy preferences. We can see that the degree to which the central bank responds on the "real" side is decreasing in λ_t. This is intuitive since a larger value of λ_t implies that the central bank considers fluctuations on the real side to be more costly.

4.1 Private sector expectations in a learning equilibrium

Under rationality, private-agents' expectations for all future horizons must be consistent with their structural model of inflation dynamics:

$$E^P[\pi_{t+j}] = E^P[\pi_{t+j+1}] - E^P[\kappa x_{t+j}] + E^P[\xi_{t+j}] \; \forall \; j \geq 1$$

Given their beliefs $\kappa_{t|t}, \xi^{p}_{t|t}, \pi^{*P}_{t}, \lambda^{P}_{t}$ formed conditional on $\mathcal{H}^t$, from the perceived targeting rule Equation 24, we have:

$$E^P_t x_{t+j} = \frac{\kappa_{t|t}}{\lambda^P_t + \kappa^2_{t|t}}(E^P_t \pi_{t+j+1} + \rho^j \xi^P_{t|t} - \pi^{*P}_t) \tag{25}$$

Solving forward:

$$E^P_t \pi_{t+j} = \underbrace{\pi^{*P}_t}_{\text{permanent inflation}} + \underbrace{\frac{\lambda^P_t}{\lambda^P_t(1-\rho) + \kappa^2_{t|t}}\rho^j \xi^P_{t|t}}_{\text{cost-push driven}} \tag{26}$$

$$E^P_t x_{t+j} = \frac{\kappa_{t|t}}{\lambda^P_t(1-\rho) + \kappa^2_{t|t}}\rho^j \xi^P_{t|t} \tag{27}$$

According to Equation 26, from the private-sector's perspective, the path of inflation is ultimately determined by two forces: (i) π^{*P}_t representing the central bank's inflation target defines the level that the private-sector expects inflation to converge towards in the long run. Inflation expectations at all horizons move one-for-one with π^{*P}_t. Hence this belief is a crucial determinant of the low-frequency (slow-moving) dynamics of inflation; and (ii) persistent cost-push shocks $\xi^P_{t|t}$, which create involuntary inflation above and beyond this voluntary level in the short-run: this *pass-through*) is *decreasing* in the perceived slope of the Phillips curve $\kappa_{t|t}$ but *increasing* in λ^P_t, the perceived weight that the central bank puts on unemployment-gap stabilization.

In the absence of persistent cost-push shocks, agents expect inflation to remain at their perceived target at all future horizons and unemployment-gaps to be zero.

Perceived policy rule: The agents' perceived interest-rate rule is implied by the IS-curve as:

$$i^*_{t+1} = \tau^{-1}(x_{t+1|t} - E^{P-}_t x_{t+2}) + E^{P-}_t \pi_{t+2} + r^n_{t+1|t}$$

Substituting $x_{t+1|t}$ from Equation 24:

$$i_{t+1}^{*}=E_t^{P-}\pi_{t+2}+\frac{\tau^{-1}\kappa_{t|t}}{\lambda_t+\kappa_{t|t}^2}\left(E_t^{P-}\pi_{t+2}+\xi_{t+1|t}^{P}-\pi_t^{*}\right)-\tau^{-1}E_t^{P-}x_{t+2}+r_{t+1|t}^{n}. \quad (28)$$

When the central bank sets interest rates, it reveals information regarding its policy preferences. The second term represents the only component of systematic monetary policy that is unknown to agents, since all the other terms are common knowledge. Hence it is also the only component of the interest rate signal that contains any relevant information. Clearly, when $\lambda_t \to \infty$, this term shrinks to zero and interest rates carry no information. In this case, the central bank conducts monetary policy so that the ex-ante real interest rate tracks $-\tau^{-1}E_t^{P-}x_{t+2}+r_{t+1|t}^{n}$ closely: real interest rates only respond to demand conditions. The response to $E_t^{P-}x_{t+2}$ and $r_{t+1|t}^{n}$ does not depend on λ_t - since they don't pose a trade-off. It is only the fluctuations in the Phillips curve (due to private-sector expectations or persistent cost-push shocks) that pose a trade-off for the central-bank.

The implied policy rule resembles a "Taylor rule in expectations" which responds more than one-for-one to inflation expectations. The Taylor rule is a determinacy condition that rules out self-fulfilling dynamics in inflation. In the model, inflation determinacy is achieved via learning. If private agents are able to learn about the central bank's long-run inflation target, inflation would converge to this unique level in the long run. In the limiting case of $\lambda_t \to \infty$, private agents do not receive any information about the central bank's target. In this case, inflation would be self-fulfilling. Such preferences would also imply that there is no incentive by the central bank to stabilize inflation around a target and the long-run target becomes meaningless.

4.2 Monetary policy surprises

The central bank is assumed to set the nominal interest rate optimally every period up to some idiosyncratic policy error,[15] so that:

$$i_{t+1}=i_{t+1}^{*}+\epsilon_t^{i}, \quad \epsilon_t^{i}\sim N(0,\sigma_i^2)$$

[15]The source of this error could be motivated as coming from cognitive noise in decision-making. It could also be thought of as a "control error". The main point being that the private agents allow for idiosyncratic deviations from the optimal policy rule so that not all of the policy surprise reflects changes in the policymakers' stabilization preferences.

Whenever the nominal interest rate that the central bank sets is different from what the private agents expect-after observing the contemporaneous realizations of inflation and the unemployment rate—it constitutes a monetary policy surprise:

$$\underbrace{i_{t+1} - E^P[i_{t+1}|\mathcal{H}^{t-1}, z_t]}_{\text{monetary policy surprise}} = \underbrace{(i^*_{t+1} - E^P[i_{t+1}|\mathcal{H}^{t-1}, z_t])}_{\text{agents behind the curve}} + \underbrace{\epsilon^i_t}_{\text{policymakers behind the curve}}$$

where $\mathcal{H}^{t-1}$ represents the history of all observed signals up to $t-1$, and z_t captures the current period realizations of inflation and unemployment rate.

Agents interpret policy surprises as stemming from two sources: an evolution in policymakers' preferences, which they must learn about ('agents behind the curve'), or a transitory policy error, which policymakers must subsequently correct ('policymakers behind the curve').

4.3 Learning the central bank's objectives

Private-sector agents observe the nominal rate and update their beliefs using a nonlinear filter (due to $i^*_{t+1,t}$ being nonlinear in π^*_t and λ_t):

Observation Equation:

$$i_{t+1} = i^*_{t+1} + \epsilon^i_t, \quad \epsilon^i_t \sim N(0, \sigma^2_i)$$

Agents' perceived optimal interest rate rule is given by Equation 28:

$$i^*_{t+1} = E^{P-}_t \pi_{t+2} + \frac{\tau^{-1}\kappa_{t|t}}{\lambda_t + \kappa^2_{t|t}} \left(E^{P-}_t \pi_{t+2} + \xi^P_{t+1|t} - \pi^*_t\right) - \tau^{-1} E^{P-}_t x_{t+2} + r^n_{t+1|t}.$$

State transition:

$$\begin{pmatrix} log\ \pi^*_t \\ log\ \lambda_t \end{pmatrix} = \begin{pmatrix} log\ \pi^*_{t-1} \\ log\ \lambda_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon^{\pi^*}_t \\ \epsilon^{\lambda}_t \end{pmatrix}, \qquad \begin{pmatrix} \epsilon^{\pi^*}_t \\ \epsilon^{\lambda}_t \end{pmatrix} \sim N(\mathbf{0}, \begin{pmatrix} \sigma^2_{\pi^*} & 0 \\ 0 & \sigma^2_{\lambda} \end{pmatrix})$$

$$\pi_t^{*P} = \pi_{t|t}^*, \quad \lambda_t^P = \lambda_{t|t}$$

The observation equation relating $i_{t+1,t}^*$ to π_t^*, λ_t is nonlinear and thus, I use the Extended Kalman Filter (EKF) algorithm to approximate this part of the learning process.[16]

5 Estimation

To characterize the path of beliefs and expectations, we must pin down the vector of parameters:

$$\Phi = [\tau, g_u, \sigma_{\xi,P}^2/\sigma_{\xi,T}^2, \sigma_i^2, \sigma_{\pi^*}^2, \sigma_\lambda^2]$$

These parameters govern how private agents update beliefs in equilibrium conditional on their observations. Given a vector of parameters Φ and a history of realizations $\mathcal{H}^t$, the path of private-sector beliefs regarding the state variables $\{\kappa_t, \xi_t^P, u_t^n, r_t^n\}$ as well as the central bank's policy preferences $\{\pi_t^*, \lambda_t\}$ are determined under the learning scheme described in the previous sections.

I set the persistence parameter ρ of the cost-push shocks to 0.99, assuming that agents learn about the extremely low-frequency component of the cost-push process, since these would be the most relevant for forming expectations.

The learning algorithm is implemented with monthly data on CPI inflation, unemployment rate and the effective fed funds rate from 1958-2024 (available from FRED St Louis) as observations. For the zero-lower bound period, the Wu-Xia shadow interest rate is used as the measure of the nominal interest rate instead of the effective Fed funds rate. I initialize the state-variables to have large variance in order to allow beliefs to adjust freely in response to data. I also allow 48 periods for the filter to converge and the estimates from this "filter stabilization" period are discarded.

[16] The extended Kalman filter is based on a linearization of the observation equation around the most recent estimates $(\pi_{t-1}^{*P}, \lambda_{t-1}^P)$. Approximation using the extended Kalman filter involves the assumption that the true parameters π_t^*, λ_t remain close to agents' priors $\pi_{t-1}^{*P}, \lambda_{t-1}^P$. Thus agents assume that they are tracking changes in the policymakers' preferences efficiently at all times.

In order to estimate these parameters, I use the SPF long-run inflation forecasts (SPF 10-year inflation forecasts) as an empirical proxy for the private-agents' beliefs about the Fed's inflation target. I am implicitly assuming that these forecasts only reflect agents' beliefs about the inflation target and that the cost-push shock is assumed to die out completely at such a long-horizon. The SPF series is available at a quarterly frequency from 1991-2024. The goal of estimation is to pin down the vector Φ that generates a path of long-run expectations closest to the observed 10-year ahead SPF inflation expectations under the agents' learning mechanism given their observations. A least-squares minimum-distance estimator is employed, matching the path of model-implied private-sector beliefs regarding the inflation target π_t^{*P} with the observations from the SPF.

Denoting the targeted observable as $\upsilon_t = \pi_t^{10y,SPF}$, the point estimate Φ^* is given by:

$$\Phi^* = \underset{\Phi}{\operatorname{argmin}} \sum_{t=1991:01}^{2024:12} \left([\pi_t^{*P} | \mathcal{H}^t, \Phi] - \upsilon_t \right)^2$$

I discard values of Φ that generate explosive paths for beliefs. This problem arises because beliefs are a non-differentiable function of Φ: even small perturbations in the parameters can induce large changes in the updating process across all variables.[17]

The resulting point-estimates, Φ^* along with 95% credibility intervals generated using Sequential Monte Carlo (SMC) sampling are shown in Table 1.

[17] This also makes the use of gradient-based optimization algorithms infeasible. I use the Nelder-Mead direct search method that does not require the computation of gradients or derivatives of the objective function.

Table 1: Estimated parameters of the baseline model

Parameter	Description	Estimate (95% CI)
τ	Ratio of IES to Okun's law coefficient	$0.647\ (0.430,\ 1.184)$
g_u	Gain for updating u_t^n	$0.329\ (0.209,\ 0.351)$
σ_i^2	Variance of idiosyncratic monetary policy errors	$0.488\ (0.389,\ 1.334)$
$\sigma_{\pi^*}^2$	Variance of innovations to (log)inflation target	$6.696\ (5.186,\ 7.772) \times 10^{-4}$
σ_λ^2	Variance of innovations to (log)weight on unemployment-stabilization	$6.802\ (3.611,\ 9.173) \times 10^{-5}$
$\frac{\sigma_{\varepsilon^t}^2}{\sigma_{\varepsilon^p}^2}$	Ratio of the variance of innovations to the transitory component of cost-push shocks to that of the persistent component	$911.901\ (196.125,\ 5176.436)$

The point estimates imply that the variance of innovations in the policy preference parameters is very small compared with the idiosyncratic policy errors. The innovations to persistent cost-push shocks are also approximately 10^3 times smaller than i.i.d. innovations.

Note: Confidence intervals are constructed using Sequential Monte Carlo sampling. The likelihood function is not differentiable in the parameters Φ, so asymptotic standard errors cannot be used.

5.1 Evolution of Private-Sector Beliefs in the Post-War period

This section discusses time-paths of private-sector agents' beliefs regarding the structural variables describing the economy as implied by the point estimates given in Table 1. The section also provides context on how beliefs evolved and their bearing on the macroeconomic outcomes in the post-war period.

5.1.1 Beliefs about the Fed's inflation target

Figure 1 illustrates that under the model-implied path of beliefs, the private-sector perceives the Fed's inflation target to be close to 2% in the early 1960s. Towards the end of the decade these beliefs begin to exhibit sharp upward revisions.[18]

In the 1970s, the perceived inflation target rose sharply, peaking near 9% before subsid-

[18]Hence I find that the anchor had begun to shift in the late 1960s, before the appointment of Arthur Burns (in 1970), similarly to Reis (2021)

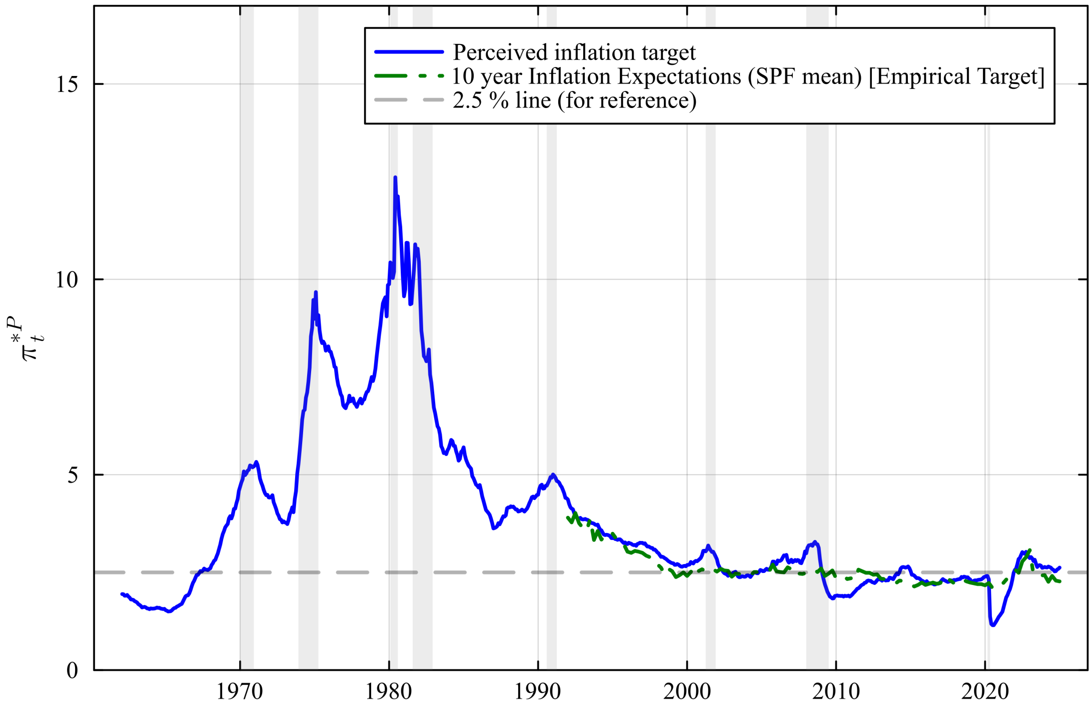


Figure 1: **Beliefs regarding inflation target** π^*:Model-implied *(solid)* vs SPF 10-year inflation expectations *(dashed)* [Empirical Target]
During the Great inflation, private-sector beliefs about the inflation target exhibit sharp upward revisions. This is not the case during the recent post-pandemic surge.
Note: Shaded regions indicate NBER recessions
SPF 10-year ahead inflation expectations begins in 1991

ing. Two pronounced upward revisions—in the early and mid-1970s—coincided with NBER recessions. Private agents inferred that the Fed's target oscillated, with a tendency to deliberately inflate during recessions followed by dis-inflationary pursuits soon after. As such patterns persisted, the perceived target climbed further, reaching nearly 12% by 1980.[19] The Volcker-era, beginning with the chairmanship of Paul Volcker in 1979 corresponds with the beginning of the turnaround. The following years saw a sharp decline in the perceived inflation target, again coinciding with two NBER-recessions. By the mid 1980s, the perceived target became much less volatile, falling below 5% after nearly a decade. Thus the U.S. economy entered the "Great Moderation". Beliefs were on a steady downward trajectory in the two decades that followed, under the chairman-

[19]The model does not take a stand on what caused the policymakers to acquire a taste for such exorbitant levels of inflation at the time, but Sargent (1999)'s thesis on the episode provides valuable clues: inflation was deliberately pursued under the false notion that unemployment could be artificially driven lower by inflationary surprises. From the model's perspective, the accommodative stance of policy could just as well be a result of a particularly bad sequence of idiosyncratic policy errors. However, why these errors caused such large revisions in the perceived target is the question we seek to answer here.

ship of Alan Greenspan (1987-2006) and Ben Bernanke (2006-2014). Only during the Great Financial crisis and the recent COVID-pandemic do we see the perceived inflation target exhibit some modest revisions (interestingly, breaking away from the SPF series).

The model generates a modest uptick in the perceived target during the post-pandemic episode that is seen in the SPF data as well, showing that there was some endogenous feedback from the post-pandemic policy to private-sector beliefs about the Fed's commitment to combat inflation causing the perceived target to be revised upwards, peaking at 3%. This is remarkable considering that CPI inflation itself reached a peak of 9.1% in June 2022. The same learning mechanism that drove the perceived target as high as 12% during the 80s remained largely muted during the recent surge. As we shall see, the reason has to do with changes in the agents' perceptions about the relative weight the Fed attaches to real-side stabilization.

Figure 2 further illustrates how the link between current inflation and inflation expectations has weakened following the Volcker disinflation. In the pre-Volcker period, the perceived inflation target moved closely with actual inflation. Afterward, however, while CPI inflation continued to fluctuate, agents' beliefs held relatively steady. In the mid-1980s, even as inflation fell below 2.5% for the first time in nearly two decades, the perceived target declined more gradually, hovering near 4%. Hence, for a time, agents viewed the benign inflation as a temporary reprieve driven by macroeconomic shocks, soon to be undone by policy.

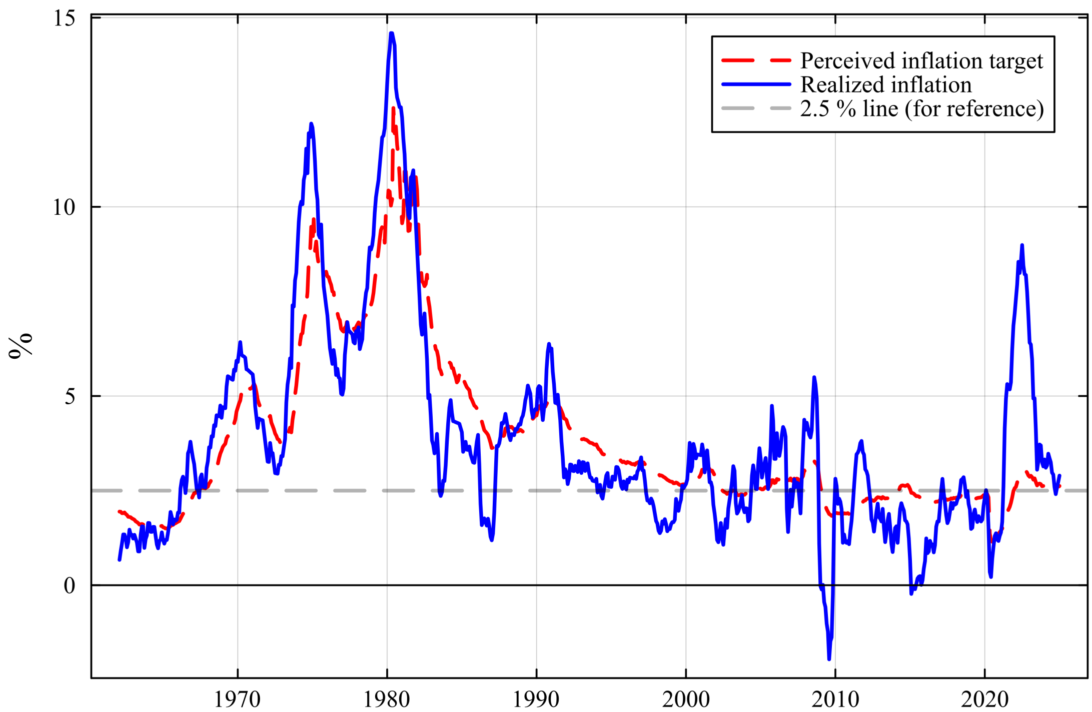


Figure 2: **Weakening link between the long-run and the short-run:** Realized CPI inflation *(solid)* vs Model-implied private-sector beliefs about the inflation target *(dashed)* *Co-movement between current CPI Inflation and the private-sector's beliefs regarding the long-run inflation target has weakened considerably since the Volcker disinflation.*

5.1.2 Beliefs about the Fed's preference for unemployment stabilization

Figure 3 plots the evolution of the private sector's estimates of λ_t: the relative weight the Fed puts on closing the unemployment-gap. The estimates suggest that, during the early 1960s, the private sector attributed relatively little concern from the Fed towards closing the gap.

After a modest rise following the Volcker disinflation, perceptions regarding λ remained relatively stable until sharp upward revisions occur following the Global Financial Crisis and again in the aftermath of the COVID-19 pandemic. As we shall explore later, this development has important implications for the anchoring of expectations. Concisely, adopting a high λ implies that the Fed "ties its hands"; unemployment-gaps are perceived as costly and hence policy emphasizes keeping the gap closed - policy behavior that is conducive to long-run stability of inflation. A high value of λ_t also has the effect of making future unemployment-gaps more predictable (since they expected to be closer to zero).

These findings are consistent with recent empirical literature using high-frequency identification utilizing changes in asset prices around Fed policy announcements to study how the private sector learns about the Fed's "policy rule" (e.g., Bauer and Swanson (2023), Pflueger (2025), Bauer, Pflueger and Sunderam (2024)). These studies find that monetary policy in the recent period has been perceived to be more focused on output stabilization, in line with a perceived rise in λ. A rise in λ, and the greater predictability of unemployment-gaps also complements earlier findings that output-gap volatility in the U.S. has declined significantly since the Volcker disinflation (see Blanchard and Simon (2001); McConnell and Perez-Quiros (2000)), underscoring the role of systematic monetary policy in engineering output-stability.

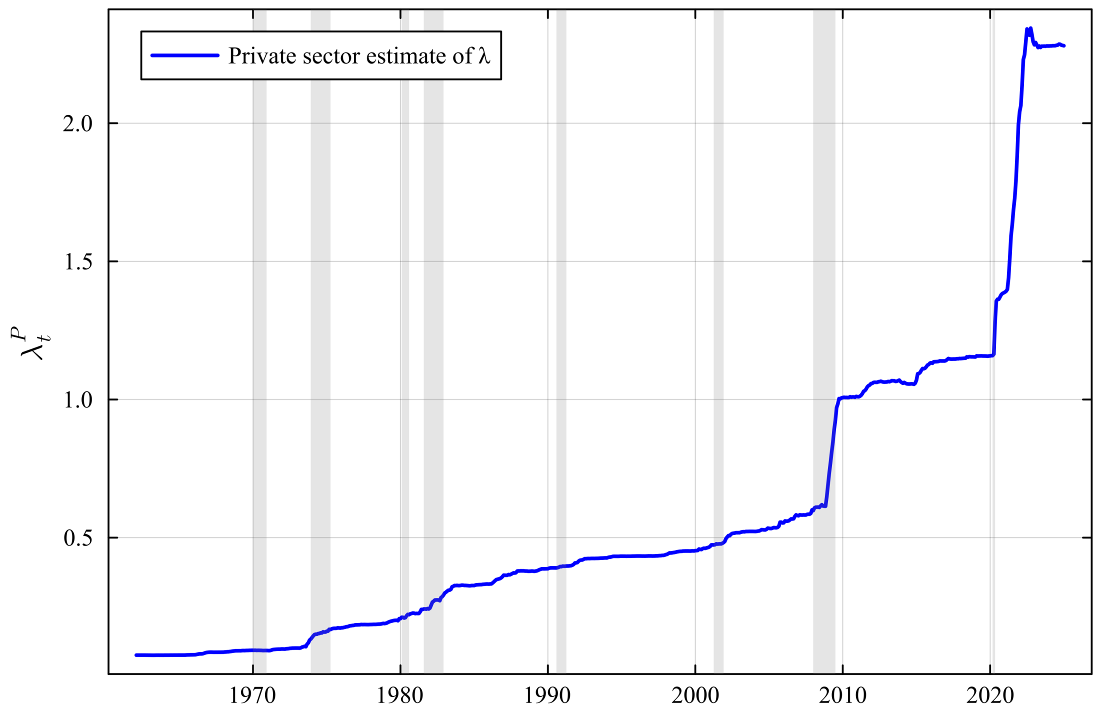


Figure 3: **Beliefs regarding the Fed's weight on unemployment-gap stabilization:** Evolution of the Private sector estimate of λ_t
The private sector perceives the Fed to put more emphasis on stabilizing the unemployment-gap
Note: Shaded regions indicate NBER recessions

5.1.3 What anchors long-run beliefs?

Anchoring reflects the entrenchment of expectations, where beliefs about the future remain resilient to surprises. In the model, cost-push shocks and a period of elevated inflation by itself would not lead agents to revise their beliefs about the policymakers' objectives (since the macroeconomic variables themselves would carry no additional information about the Fed's target). A revision is only warranted when policy is seen to not respond in the way that the agents expect. The following quote by Ben Bernanke summarizes what anchoring means in the context of monetary policy.

> "...I use the term "anchored" to mean relatively insensitive to incoming data. So, for example, if the public experiences a spell of inflation higher than their long-run expectation, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored..."
> —Bernanke (2007)

Whenever the private sector observes the nominal interest rate $i_{t+1,t}$, different from what it expects, it attributes a part of it to the Fed's systematic policy response and the rest to idiosyncratic policy errors unrelated to their stabilization preferences. Given a surprise delivered by the central bank's policy response, agents must infer how policymakers' preferences have evolved and by how much relative to their prior beliefs.

Anchoring is state-dependent

Belief-updates under rationality would follow Bayesian principles, with the weight placed on a signal determined by its perceived informativeness about the variables agents seek to learn about. From the perspective of private agents, the nominal interest rate is an observable outcome generated by a nonlinear optimal policy rule. How much information the interest-rate signal carries about changes in the Fed's policy preferences is given by the Jacobian (first-order partial derivatives) of the interest-rate rule Equation 28 with respect to π_t^*, λ_t (based on their prior beliefs π_{t-1}^{*P}, λ_{t-1}^P):

$$\frac{\partial i_{t+1,t}^*}{\partial \lambda_t}\Big|_{(\pi_{t-1}^{*P},t\lambda_{t-1}^P)} = -\frac{\tau^{-1}\kappa_{t|t}}{(\lambda_{t-1}^P + \kappa_{t|t}^2)^2}(E_t^{P-}\pi_{t+2} - \pi_{t-1}^{*P} + \xi_{t+1|t}^P) \tag{29}$$

$$\frac{\partial i^*_{t+1,t}}{\partial \pi^*_t}|_{(\pi^{*P}_{t-1}\lambda^P_{t-1})} = -\frac{\tau^{-1}\kappa_{t|t}}{\lambda^P_{t-1} + \kappa^2_{t|t}} \tag{30}$$

The Jacobian informs private-sector agents about how sensitive interest rates are to small changes in the policymakers' stabilization preferences. Given their prior beliefs, if interest rates are perceived to be highly sensitive to shifts in these parameters, it would warrant larger updates given a policy surprise (since policy changes are expected to carry a stronger signal regarding shifts in policymakers' preferences). Hence, the degree to which agents perceive policy to be informative - and hence how strongly their beliefs would respond given a policy surprise depends on their *ex-ante* beliefs.

Consider the economy in steady-state, with private-sector beliefs coinciding with the central bank's true preferences. Suddenly, a cost-push shock hits this economy and inflation expectations rise above the perceived target. Stabilization policy would entail raising interest rates and maintaining a positive real-interest rate gap (real interest rates above the natural rate) to "lean against the wind". Equation 29 tells us that the extent to which agents expect the nominal rate to be raised is *decreasing* in the agents' prior λ^P_{t-1} (as suggested by the negative sign). Hence a central bank with a large λ_t would be expected to accommodate the shock to a larger extent - conversely, when policymakers' response to shocks in the Phillips curve is perceived to be weaker than anticipated, λ^P_t is revised upwards. Intuitively, agents interpret this behavior as the policymakers signaling a distaste for unemployment-gap volatility (or intolerance towards generating real-side disturbances in response to cost-push shocks).

Similarly, Equation 30 implies that a higher inflation target would be signaled by a downward bias (due to negative sign) in interest rates. Hence a sustained period of interest rates undershooting expectations would lead to upward revisions in perceived inflation target. However, the magnitude of the updates would depend on the magnitude of the Jacobian. Importantly, when the private-sector agents perceive λ to be large, they anticipate little incentive from the Fed to pursue deliberately inflationary policy in response to exogenous shifts in the policymakers' long-run inflation target since such a pursuit would involve costly adjustments on the real-side (with a sustained period of negative unemployment gaps if the target moves up). In this scenario, interest rates would not be expected to carry a strong signal regarding changes in π^* and thus, revisions in response to perceived policy mistakes would be small. Thus, long-run expectations remain more firmly "anchored". A larger value of λ_t may also be interpreted as policymakers' adherence to the natural-rate theory under which they treat departures of the unemployment rate from its natural rate as costly.

Intuition regarding the role of λ : When λ is of small magnitude, like we see in during the late 1960s, the central bank is perceived to be mainly stabilizing inflation around its inflation-target. Movements in inflation are perceived to be coming in large part from shifts in the central bank's inflation-target itself in such an environment. When agents do not know the inflation target and suspect it to be time-varying, a perceived commitment to keeping inflation close to target is meaningless, since whimsical changes in the policymakers' target would cause unpredictable changes in policy behavior. But when λ is perceived to be large, the central bank is "pseudo-committing" to keeping unemployment close to the natural rate - a structural variable outside the direct control of policymakers. This kind of policy behavior is more predictable from the agents' perspective since the information-asymmetry posed by an unknown inflation target becomes less relevant. In a way, monetary policy acquires the kind of history-dependence that Woodford (1999) advocates for.

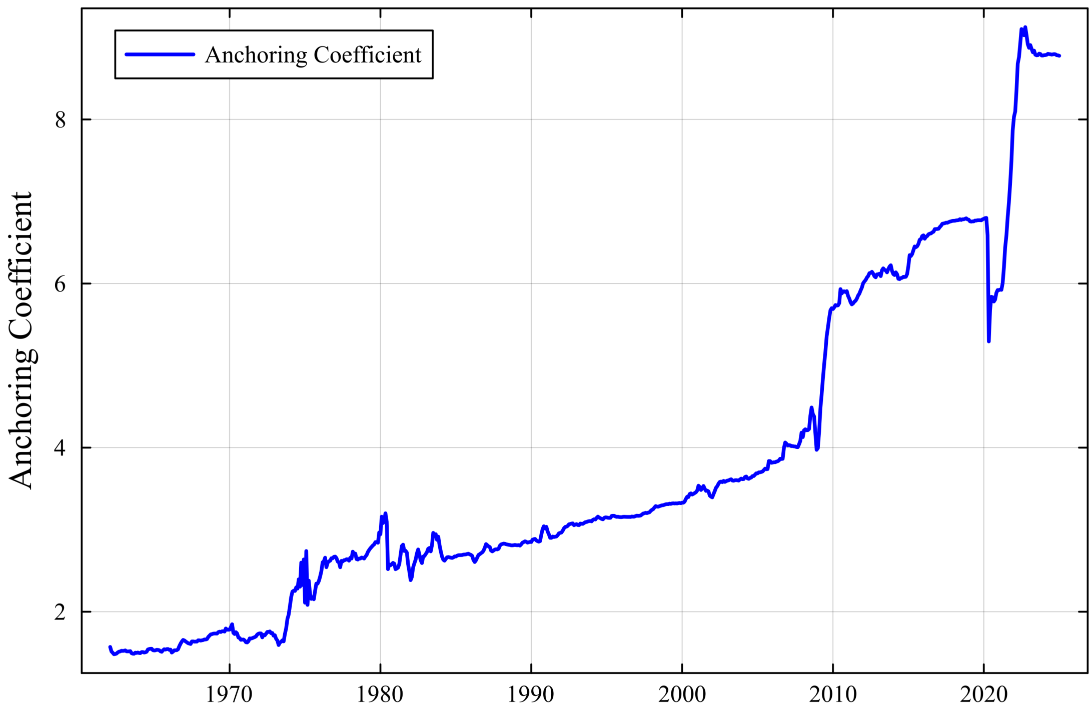


Figure 4: **Evolution of the anchoring coefficient** ($\frac{\lambda^P_{t-1}+\kappa^2_{t|t}}{\kappa_{t|t}}$): The coefficient measuring the "anchoring" of long-run inflation expectations has risen drastically

The magnitude of the term $\frac{\kappa_{t|t}}{\lambda^P_{t-1}+\kappa^2_{t|t}}$ is an important determinant of the degree of anchoring of long-run inflation expectations. Hence, I call its reciprocal the "anchoring coefficient" (large value corresponds to more anchored long-run inflation expectations) and plot the evolution of this variable in Figure 4. From the private-sector's vantage point, the policy environment has changed drastically in the 40 years separating the Great Inflation and the post-pandemic surge. Through its conduct of monetary policy, the Fed has signaled a distaste for unemployment-gaps. From the lens of the model, this credibility built by the policy behavior during the Volcker-era and beyond has played a crucial role in preventing the harsh outcomes of the 70s from repeating and has also given them the ability to "look-through" supply shocks without triggering large revisions in the agents' perceived inflation target.

Anchoring is robust to mis-measurement

It is also worth noting that in the model, mis-measurement of any of the latent structural variables ($\kappa, u_t^n, r_t^n, \xi_t^P$) is not relevant for anchoring as long as policymakers and private-agents share the same beliefs, conditional on the same macroeconomic data and model. Whether these common beliefs are different from the "truth" makes no difference for anchoring since what causes the private agents to revise their beliefs are "perceived" policy mistakes (and hence real-time mis-measurement would be irrelevant since both the private agents and policymakers would be equally wrong). All that matters is that the central bank act the way private-sector agents expect a central bank with stable preferences to act given their best-estimates at any given time. However, mis-measurement would certainly drive losses for the central bank since policy would be sub-optimal as compared to the hypothetical full-information benchmark.

5.2 "Bad luck" vs "Bad policy" revisited

The following section contributes to the debate regarding whether the economic outcomes of the post-Volcker period were a result of luck (favorable economic environment, benign shocks) or systematic policy (shifts in the Fed's implied policy objectives). As with most things, I find that we could benefit from a bit of nuance. Here, I discuss how the dynamic interaction between structural shocks, systematic monetary policy and private-agents' learning behavior contributed to shaping the macroeconomic outcomes during the post-war period in the U.S.

5.2.1 The role of policy in driving inflation

Figure 5 illustrates how from the perspective of the private agents, the 70s were characterized by a combination of persistent cost-pressures and expansionary policy characterized by the real rates being well below the perceived natural rate of interest.

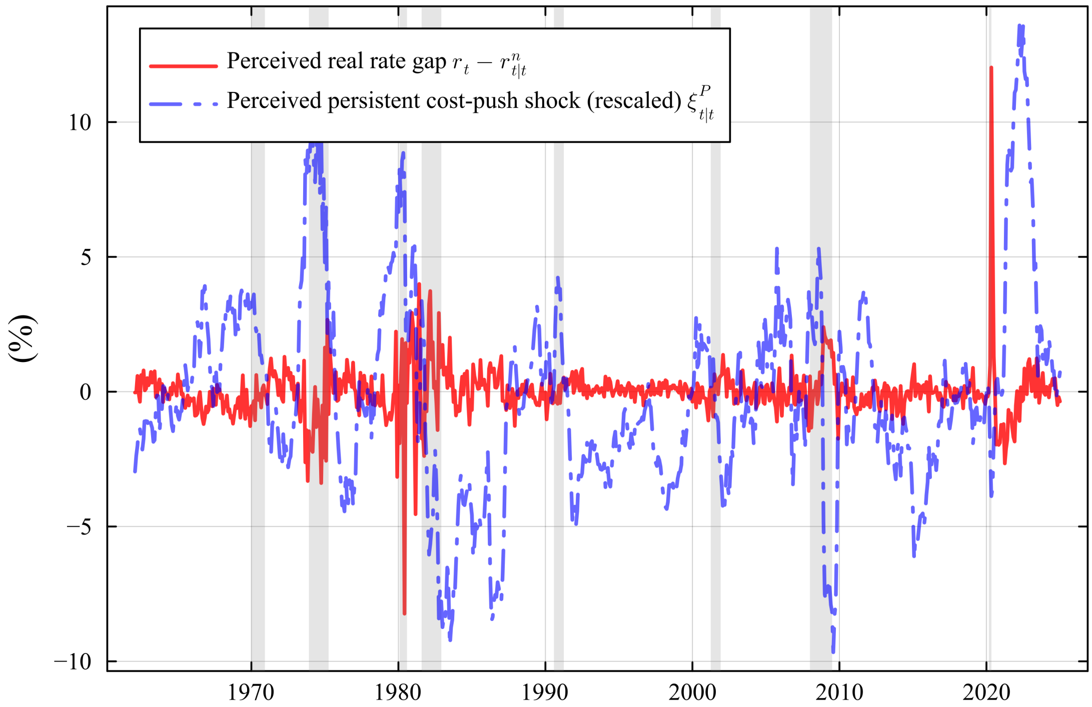


Figure 5: **Policy behavior in face of inflationary shocks:** The real rate gap *(solid)* vs filtered estimates of persistent cost-push shocks *(dashed)*
Cost-push shocks in the 70s and 80s were met with accommodative policy by the Fed. Policy was perceived to be similarly accommodative during the early stage of the post-pandemic inflation.
Note: Shaded regions indicate NBER recessions

Due to the perceived weight on unemployment-gap stabilization being low during this period, the policy accommodation even in the face of inflationary pressures was interpreted as resulting largely from Fed having a higher inflation target. The private-sector agents took note, sharply revising their beliefs. It is only during the chairmanship of Volcker that we see a sustained positive real rate gap. Two aspects of policy behavior in the post-Volcker period were important for the onset of the "Great Moderation": (i) a positive real rate gap was maintained for a prolonged period signaling a lower inflation target and (ii) muted response of policy to persistent shocks in the Phillips curve signaling a steadily increasing emphasis on unemployment-gap stabilization represented by λ_t.

The real rate gap becomes positive again during the Global Financial Crisis, reflecting that stance of policy during this period was interpreted to be contractionary by the private-sector. During the post-pandemic period, we see large cost-push shocks putting upwards pressure on inflation along with a policy that is actively expansionary (perceived real rate gap is negative). While the Fed's response lagged in the beginning of the crisis, it quickly picks up pace and the agents see the real-rate gap close. As the shocks die out, we see inflation fall steeply without the onset of a deep recession (the oft-cited "soft-landing"), in sharp contrast with the Great Inflation episode of the 70s. Thus, once we consider the role of policy in signaling the central bank's preferences, costless dis-inflations are possible if (i) the policymakers have previously demonstrated a distaste for unemployment-gaps, which arrest upward revisions in the perceived target, even as they continue to "look-through" supply shocks (ii) they act decisively to signal their inflation preferences as the anchor begins to loosen. From the lens of the model, the "soft landing" was precisely due to the credibility that the Fed has built through disciplined policy over the past forty years. This is also a cautionary tale for current policymakers: The current conduct of policy will shape not only current macroeconomic outcomes, but also how expectations behave in the future. The gains from disciplined policy continue to yield favorable outcomes.

5.2.2 The role of shocks: What shocks drive inflation?

As the only variable driving a trade-off between inflation and unemployment stabilization, persistent cost-push shocks cause "involuntary" inflation, over and beyond the Fed's targeted level. Agents' beliefs about these shocks are an important determinant of their short-run inflation expectations and hence of inflation itself.

Figure 6 plots the time-series of the agents' beliefs (real-time as well as smoothed) regarding persistent supply shocks in the Phillips curve. One fact that emerges is that the agents perceived large, persistent cost-shocks, both during the Great Inflation of the 70s-80s and in the more recent inflation surge. Moreover, the shocks were of roughly similar magnitude across the two episodes (if not larger during the recent episode).

The perceived supply shocks line up well with documented episodes of oil price surges. There is ample evidence in the literature that fluctuations in relative prices of energy goods are significant drivers of inflation (see Afrouzi, Bhattarai and Wu (2024), Gagliardone and Gertler (2023) for recent papers examining link between crude oil prices and

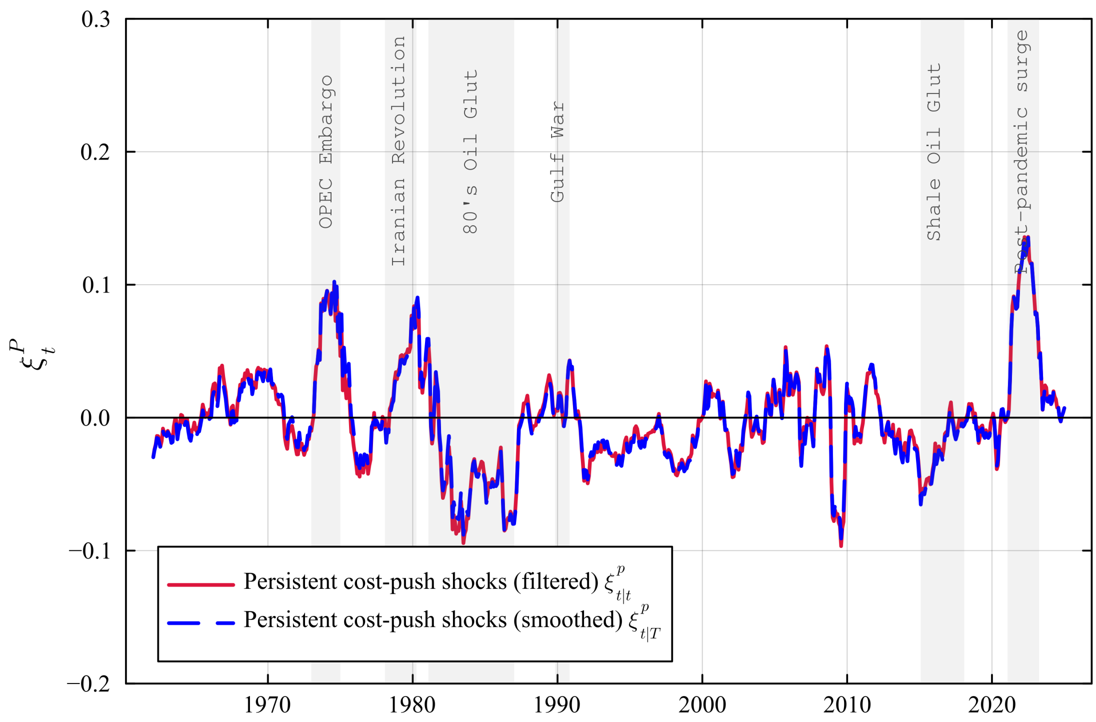


Figure 6: **Structural shocks that drive inflation:** Private agents' estimates of the persistent component of the cost-push process ξ_t^P: Filtered *(solid)* vs smoothed *(dashed)* *The magnitude of persistent cost-push shocks perceived by the agents during the post-COVID period were similar to those during the 70s-80s Great Inflation period.*

the post-pandemic inflation surge). Interestingly, the last time that the agents perceived inflationary supply shocks was during the recovery from the Great Financial Crisis (GFC). This is consistent with evidence that the oil price shocks in the aftermath of the crisis propped up inflation expectations accounting for the "missing deflation" (Coibion and Gorodnichenko (2015b)).

To interpret these "persistent supply shocks", in Figure 7 I plot the smoothed series against food and energy price inflation derived as the difference between Headline CPI and Core CPI. The series are de-meaned and re-scaled for ease of comparison. The two series display significant co-movement. Since Core-CPI is not a series that the model has "seen" (only Headline CPI data is used during estimation), it is remarkable that such a link emerges.

The post-pandemic episode was characterized by structural shifts over and above oil price surges, such as sectoral shocks shifting consumption between goods and services (see Guerrieri et al. (2021)) structural changes in the labor market (Blanchard and Bernanke (2023), Ball et al. (2022)), increasing attention to inflation (Pfauti (2023)) to name a few.

However, shocks to relative food and energy prices appear to explain a large part of the variation in these persistent "cost-push" shocks that the agents perceive. This is intuitive considering the pervasive impact of these shocks to the rest of the economy and how *salient* these prices are. Hence, the model adds support to the literature emphasizing the importance of relative price disturbances in driving aggregate inflation dynamics.

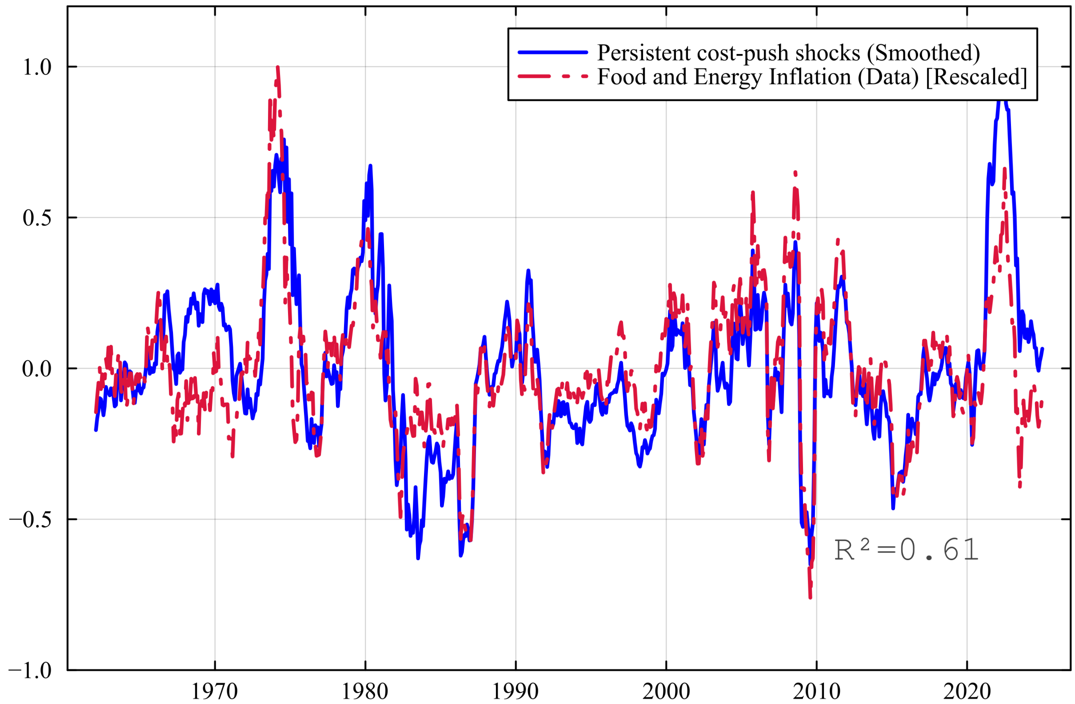


Figure 7: **Do relative prices drive inflation expectations?:** Persistent supply shocks perceived by the agents (smoothed estimate) vs relative price inflation in food and energy
Note: Measure of Food and Energy inflation constructed as difference between headline and core CPI inflation

5.3 The slope of the expectations-augmented Phillips Curve

The parameter κ, represents the contemporaneous response of inflation and unemployment-gap. Hence, it serves as the connective tissue between the real and nominal side of the economy and is a key parameter that agents learn in real-time. Figure 8 plots the filtered estimates representing the private agents's beliefs regarding the slope of the expectational Phillips curve under their learning mechanism. In the agents' own model, κ is time-invariant. However, their real-time estimates under learning exhibit significant time-variation over the sample period.

Since κ is not subject to innovations, the steady-state filter would be a decreasing-gain one. As new data is observed and beliefs are updated, each additional observation carries marginally less information and the estimates would be expected to attain a constant value under the learning dynamics described in the paper. But it does not appear that any such convergence has attained within the sample period of estimation (which spans about 60 years of macroeconomic data).

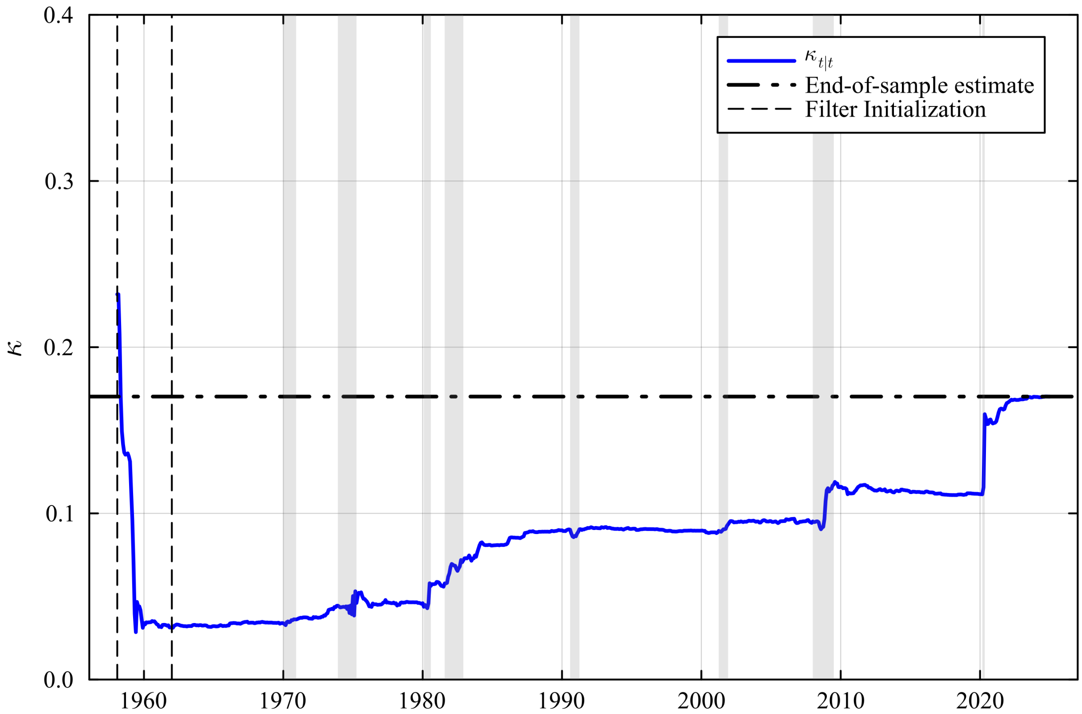


Figure 8: **Slope of the expectations-augmented Phillips curve:** Filtered (*solid*) estimates of the slope of the Phillips curve.
Note: From the private-sector's and policymakers' perspective, the Phillips curve is steeper now than it was perceived to be during the Great Inflation period. Even though the agents learn under the assumption that the true structural Phillips curve has time-invariant slope, their estimates are subject to large revisions over time.
Note: Shaded regions indicate NBER recessions

To understand why this is the case, let's revisit the observation equation through which agents update their beliefs regarding κ and ξ_t^P:

$$\pi_t - E_{t-1}^P \pi_{t+1} = \begin{pmatrix} -\hat{x}_t & 1 \end{pmatrix} \begin{pmatrix} \kappa \\ \xi_t^P \end{pmatrix} + \xi_t^T$$

Note that π_t delivers information regarding κ only through the measured unemployment gap $\hat{x}_t$. It is not surprising then, that we see the bulk of the large updates in agents' estimates of κ around recessions (corresponding to large variations in $\hat{x}_t$). During "normal" times, the unemployment rate is close to the trend natural rate, it is evident that learning would slow down (since inflation carries little information regarding the true κ). In totality, the signal informing the agents about κ gets drowned out due to (i) the estimated unemployment-gap $\hat{x}_t$ being of small magnitude most of the time (Figure 12 in Appendix C) and (ii) the presence of cost-push shocks confounding the estimation. The second of these reasons is a familiar one: Supply shocks are known to confound the estimation of the Phillips curve and the literature has often relied on adding oil prices or similar variables as controls in the regression specification as a proxy for supply shocks. Inferring the slope of the structural Phillips curve has been among the most enduring and difficult problems in the macroeconomics profession. It is hence no surprise that the agents in the model are not immune to the fundamental econometric challenge it poses. Models assuming full-information mask such issues entirely.

The model implies that the Phillips curve was perceived to be relatively "flat" during the 1970s. This belief would be reinforced by the experience of high inflation in the face of rising unemployment experienced during this period. The perception of a flat Phillips curve is one of the primary reasons cited by Primiceri (2006) and Sargent, Williams and Zha (2006) for the policymakers' aversion to act decisively against inflation during this period. The perceived expectations-augmented Phillips curve has steepened in the recent years. More so in the post-pandemic period where the large decline in inflation from its highs with little rise in unemployment would lead the agents to update their estimate of κ upwards. Overall, the results suggest that real-time identification of the Phillips curve is quite poor and agents' estimates would take an unreasonably long time to converge to the true κ.

However, estimates during the entire sample period remain within a plausible range (between 0.05-0.20) and the agents never perceive it to be excessively flat, which is reassuring. The estimates also shine a light on why the literature has invoked specifications involving a non-linear Phillips curve to explain the post-pandemic episode (see Harding, Lindé and Trabandt (2023) Benigno and Eggertsson (2024)). Though the model considered here does not nest a non-linear Phillips curve, agents record an apparent "steepening" during the post-pandemic period. Whether the true Phillips curve has become steeper is still up for academic debate. For example, recent work (Beaudry, Hou and Portier (2025)) finds limited evidence in favor of nonlinear Phillips curve specifications once expectations are explicitly controlled for.

6 Conclusions

The rational expectations revolution—spearheaded by the work of Lucas and Sargent among others—marked a profound shift in macroeconomic thought. New Keynesian macroeconomics internalized this transformation by explicitly modeling how aggregate macroeconomic outcomes result from optimizing behavior on part economic agents at the micro-level. The model presented in this paper serves as a natural extension of the forward-looking New-Keynesian framework to an environment with imperfect knowledge. In this paper, we take a structural view of the economy from the perspective of the agents operating within it who are themselves learning about its fundamentals.

A central implication of this model is that due to the principal role of expectations in shaping macroeconomic outcomes, the function of monetary policy goes beyond period-stabilization in the mechanical sense as it actively shapes private agents' expectations (a point first made in Kydland and Prescott (1977) in their criticism of the use of control theory in economics). The learning mechanism in the model offers valuable insights into how central bank credibility (and anchoring) results directly from its conduct of monetary policy and how it influences macroeconomic stability.

The model further provides a holistic picture of the economic dynamics in the post-war U.S, describing how the macro-policy environment has evolved and the role of monetary policy in shaping it. Perceived changes in the Federal Reserve's policy priorities — specifically, a heightened emphasis on unemployment stabilization has contributed to the anchoring of the long-run inflation expectations (cushioning against perceived policy mistakes), simultaneously amplifying the inflationary impact of persistent supply shocks. While recent work (e.g., Coibion and Gorodnichenko (2025)) has warned of inflationary risks resulting from dis-anchoring of expectations from short-run inflationary pressures going forward, the model presented here paints a less grim picture while recognizing the risks that cost-push shocks could pose in the future. Another implication of the model is that the Fed's response to one inflationary crisis shapes private agents' beliefs going into the next one: they have memory.

This work also highlights the explanatory power of the textbook three-equation New Keynesian model when augmented with imperfect information and learning. The model reconciles the empirically observed persistence in macroeconomic aggregates without resorting to backward-looking mechanisms such as price indexation, habit formation in consumption or interest rate smoothing. Finally, the model discusses the identification problems faced by agents when they are learning in real-time and their consequences - specifically regarding the slope of the structural Phillips curve which is poorly identified in real-time.

References

Afrouzi, Hassan, Saroj Bhattarai, and Edson Wu, "Relative-price changes as aggregate supply shocks revisited: Theory and evidence," *Journal of Monetary Economics*, 2024, *148*, 103650. Inflation in the COVID Era and Beyond. 36

Ball, Laurence, Daniel Leigh, and Prakash Loungani, "Okun's Law: Fit at 50?," *Journal of Money, Credit and Banking*, 2017, *49* (7), 1413–1441. 10

Ball, Laurence M, Daniel Leigh, and Prachi Mishra, "Understanding U.S. Inflation During the COVID Era," Working Paper 30613, National Bureau of Economic Research October 2022. 37

Bauer, Michael D. and Eric T. Swanson, "A Reassessment of Monetary Policy Surprises and High-Frequency Identification," *NBER Macroeconomics Annual*, 2023, *37* (1), 87–155. 30

Bauer, Michael D, Carolin E Pflueger, and Adi Sunderam, "Perceptions About Monetary Policy*," *The Quarterly Journal of Economics*, 06 2024, *139* (4), 2227–2278. 30

Beaudry, Paul, Chenyu Hou, and Franck Portier, "On the Fragility of the Nonlinear Phillips Curve View of Recent Inflation," Working Paper 33522, National Bureau of Economic Research February 2025. 40

Benigno, Pierpaolo and Gauti B. Eggertsson, "The Slanted-L Phillips Curve," *AEA Papers and Proceedings*, May 2024, *114*, 84–89. 40

Bernanke, Ben S. and Ilian Mihov, "Measuring Monetary Policy*," *The Quarterly Journal of Economics*, 08 1998, *113* (3), 869–902. 6

Blanchard, Olivier and John Simon, "The Long and Large Decline in U.S. Output Volatility," *Brookings Papers on Economic Activity*, 2001, *2001* (1), 135–164. 30

Blanchard, Olivier J and Ben S Bernanke, "What Caused the US Pandemic-Era Inflation?," Working Paper 31417, National Bureau of Economic Research June 2023. 37

Bray, M. M. and N. E. Savin, "Rational Expectations Equilibria, Learning, and Model Specification," *Econometrica*, 1986, *54* (5), 1129–1160. 5

Bullard, James and In-Koo Cho, "Escapist policy rules," *Journal of Economic Dynamics and Control*, November 2005, *29* (11), 1841–1865. 5

_ and Stefano Eusepi, "Did the great inflation occur despite policymaker commitment to a Taylor rule?," *Review of Economic Dynamics*, 2005, *8* (2), 324–359. Monetary Policy and Learning. 5

Carvalho, Carlos, Stefano Eusepi, Emanuel Moench, and Bruce Preston, "Anchored Inflation Expectations," *American Economic Journal: Macroeconomics*, January 2023, *15* (1), 1–47. 4, 7

Clarida, Richard, Jordi Gali, and Mark Gertler, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory*," *Quarterly Journal of Economics*, February 2000, *115* (1), 147–180. 6

Cogley, Timothy and Thomas J. Sargent, "ANTICIPATED UTILITY AND RATIONAL EXPECTATIONS AS APPROXIMATIONS OF BAYESIAN DECISION MAKING*," *International Economic Review*, February 2008, *49* (1), 185–221. 15

_ , Giorgio E. Primiceri, and Thomas J. Sargent, "Inflation-Gap Persistence in the US," *American Economic Journal: Macroeconomics*, January 2010, *2* (1), 43–69. 3

Coibion, Olivier and Yuriy Gorodnichenko, "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, August 2015, *105* (8), 2644–78. 5

_ and _ , "Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation," *American Economic Journal: Macroeconomics*, January 2015, *7* (1), 197–232. 37

_ and _ , "Inflation, Expectations and Monetary Policy: What Have We Learned and to What End?," Working Paper 33858, National Bureau of Economic Research May 2025. 41

Del Negro, Marco and Stefano Eusepi, "Fitting observed inflation expectations," *Journal of Economic Dynamics and Control*, 2011, *35* (12), 2105–2131. Frontiers in Structural Macroeconomic Modeling. 3

DeLong, J. Bradford, "America's Only Peacetime Inflation: The 1970s," NBER Historical Working Papers 0084, National Bureau of Economic Research, Inc May 1996. 6

Erceg, Christopher J. and Andrew T. Levin, "Imperfect credibility and inflation persistence," *Journal of Monetary Economics*, 2003, *50* (4), 915–944. 5, 6, 49

Evans, George W. and Seppo Honkapohja, *Learning and Expectations in Macroeconomics*, Princeton University Press, 2001. 15

Gagliardone, Luca and Mark Gertler, "Oil Prices, Monetary Policy and Inflation Surges," Working Paper 31263, National Bureau of Economic Research May 2023. 36

Gati, Laura, "Monetary policy & anchored expectationsâ€”An endogenous gain learning model," *Journal of Monetary Economics*, None 2023, *140* (S), 37–47. 4, 7

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Ivan Werning, "Monetary Policy in Times of Structural Reallocation," 2021. Proceedings of the Jackson Hole Symposium, 2021. 37

Harding, Martín, Jesper Lindé, and Mathias Trabandt, "Understanding post-COVID inflation dynamics," *Journal of Monetary Economics*, 2023, *140*, S101–S118. Inflation: Drivers and Dynamics 2022. 40

Ireland, Peter N., "Changes in the Federal Reserve's Inflation Target: Causes and Consequences," *Journal of Money, Credit and Banking*, 2007, *39* (8), 1851–1882. 3, 49

Kamber, Güneş, James Morley, and Benjamin Wong, "Intuitive and Reliable Estimates of the Output Gap from a Beveridge-Nelson Filter," *The Review of Economics and Statistics*, 07 2018, *100* (3), 550–566. 16

Kozicki, Sharon and P.A. Tinsley, "Permanent and transitory policy shocks in an empirical macro model with asymmetric information," *Journal of Economic Dynamics and Control*, 2005, *29* (11), 1985–2015. Expectations, learning, and monetary policy. 6

Kreps, David M., "Anticipated Utility and Dynamic Choice (1997)," in Donald P. Jacobs, Ehud Kalai, Morton I. Kamien, and Nancy L. Schwartz, eds., *Frontiers of Research in Economic Theory: The Nancy L. Schwartz Memorial Lectures, 1983–1997*, Econometric Society Monographs, Cambridge: Cambridge University Press, 1998, pp. 242–274. 15

Kydland, Finn E. and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 1977, *85* (3), 473–491. 4, 41

Lakdawala, Aeimit, "Changes in Federal Reserve preferences," *Journal of Economic Dynamics and Control*, 2016, *70*, 124–143. 3

Lubik, Thomas A. and Frank Schorfheide, "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, March 2004, *94* (1), 190–217. 6

Lucas, Robert E, "Expectations and the neutrality of money," *Journal of Economic Theory*, April 1972, *4* (2), 103–124. 5

Marcet, Albert and Juan P. Nicolini, "Recurrent Hyperinflations and Learning," *American Economic Review*, December 2003, *93* (5), 1476–1498. 5

_ **and Thomas J Sargent**, "Convergence of least squares learning mechanisms in self-referential linear stochastic models," *Journal of Economic Theory*, 1989, *48* (2), 337–368. 5

McConnell, Margaret M. and Gabriel Perez-Quiros, "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?," *The American Economic Review*, 2000, *90* (5), 1464–1476. 30

Melosi, Leonardo, "Signalling Effects of Monetary Policy," *The Review of Economic Studies*, 09 2016, *84* (2), 853–884. 4

Milani, Fabio, "Expectations, learning and macroeconomic persistence," *Journal of Monetary Economics*, 2007, *54* (7), 2065–2082. 5, 49

Nimark, Kristoffer, "Dynamic pricing and imperfect common knowledge," *Journal of Monetary Economics*, 2008, *55* (2), 365–382. 11

Pfauti, Oliver, "The Inflation Attention Threshold and Inflation Surges," Papers 2308.09480, arXiv.org Aug 2023. 37

Pflueger, Carolin, "Back to the 1980s or not? The drivers of inflation and real risks in Treasury bonds," *Journal of Financial Economics*, 2025, *167*, 104027. 30

Phillips, A. W., "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957," *Economica*, 1958, *25* (100), 283–299. 5, 62

Primiceri, Giorgio E, "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U. S. Postwar Stabilization Policy*," *Quarterly Journal of Economics*, August 2006, *121* (3), 867–901. 5, 40

Reis, Ricardo, "Losing the Inflation Anchor," *Brookings Papers on Economic Activity*, 2021. 26

Romer, Christina D. and David H. Romer, "A Rehabilitation of Monetary Policy in the 1950's," *American Economic Review*, May 2002, *92* (2), 121–127. 6

Sargent, Thomas J., *The Conquest of American Inflation*, Princeton University Press, 1999. 27

Sargent, Thomas, Noah Williams, and Tao Zha, "Shocks and Government Beliefs: The Rise and Fall of American Inflation," *American Economic Review*, September 2006, *96* (4), 1193–1224. 5, 40

Sims, Christopher A., "Macroeconomics and Reality," *Econometrica*, January 1980, *48* (1), 1. 6

Staiger, Douglas, James H. Stock, and Mark W. Watson, "The NAIRU, Unemployment and Monetary Policy," *Journal of Economic Perspectives*, March 1997, *11* (1), 33–49. 16

Svensson, Lars E. O., "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules," *Journal of Economic Literature*, June 2003, *41* (2), 426–477. 3

Taylor, John B., "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 1993, *39*, 195–214. 6

Williams, Noah, "Adaptive Learning and Business Cycles," *Unpublished*, 2003. 3

Woodford, Michael, "Commentary : how should monetary policy be conducted in an era of price stability?," *Proceedings - Economic Policy Symposium - Jackson Hole*, None 1999, *None* (None), 277–316. 33

7 Appendix

A Performance of agents' model

If the learning algorithm arising from the agents' assumed model of economic dynamics implies forecasts that are persistently off-target, it wouldn't make sense to assume that the agents would continue using it. Rational agents would find a reason to abandon the model. Hence, it is useful to verify if the agents' ex-ante forecasts agree with the realizations of the target variables π_t, u_t and $i_{t+1,t}$, or in other words if the agents' assumed model remains plausible given what they observe. Figure 9 plots the agents' ex-ante expectation of the policy rate (prior to announcement) and the realized effective Fed funds rate. Similarly, in Figure 10 I plot the agents' one-period ahead forecasts of inflation and unemployment rate against their corresponding realized values.

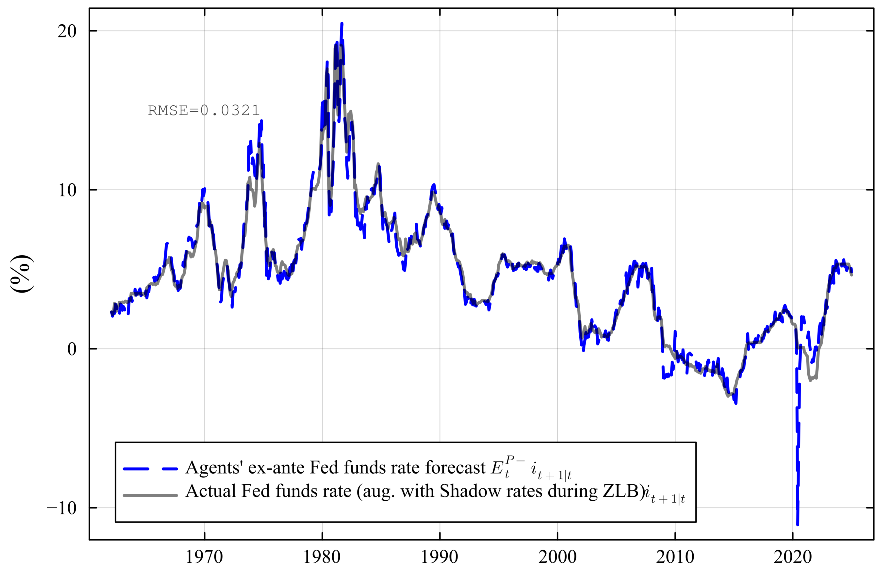
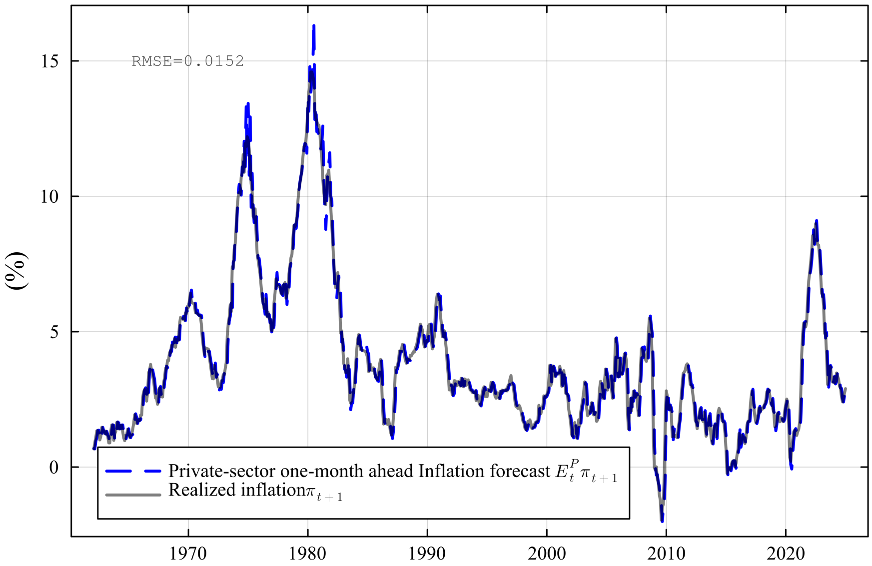
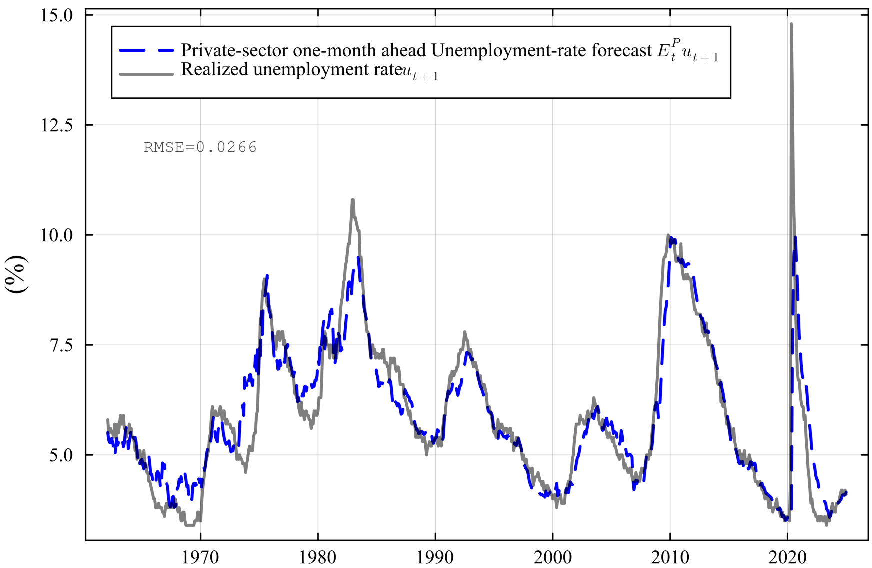


Figure 9: **How well has private-sector tracked policy behavior?:** The figure compares the private agents' expected policy rate conditional on observed inflation and unemployment rate with the actual path of the Fed funds rate (augmented with the shadow rate) *Agents' learning algorithm allows them to track policy behavior quite closely.*



(a) Agents' one-month ahead forecast of inflation vs realized value



(b) Agents' one-month ahead forecast of unemployment rate vs realized value

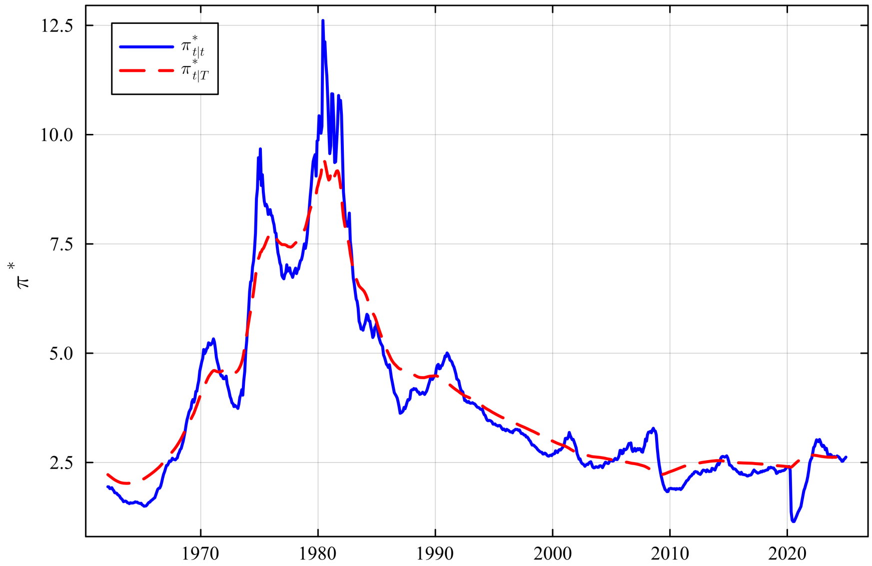
Figure 10: **Performance of agents' learning algorithm at tracking the macroeconomic targets**: Agents' 1-month ahead forecasts of inflation and unemployment rate plotted against actual realizations

The learning model would generate forecasts errors of small magnitude (in the RMSE sense). Thus, their model of the economy "works" - they wouldn't find a compelling reason to abandon learning. What is also interesting is the fact that the time-series of their expectations displays a similar degree of persistence as the corresponding macroeconomic variables themselves. The fact that this happens despite their model not having mechanical sources of persistence is noteworthy. Hence, forces such as price-indexation and habit-formation in consumption do not seem to be necessary to generate persistence in the macroeconomic aggregates once the role of learning in agents' expectations-formation process is modeled, as also demonstrated in Milani (2007) and Erceg and Levin (2003).

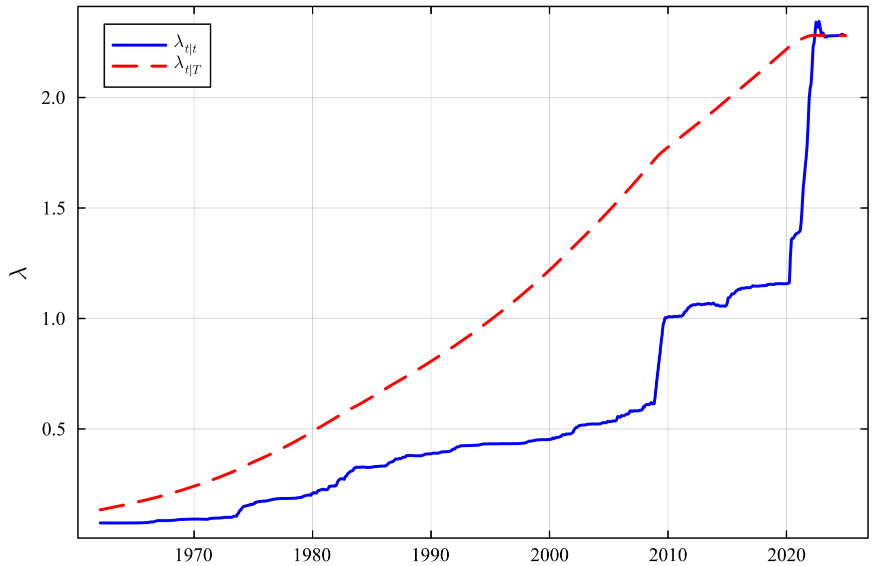
B What were the Fed's "true" policy preferences?

While the parameters representing the "true" policy preferences of the Fed over the post-war period remain latent (it evolves as an exogenous structural variable as far as the model is concerned), it is possible to arrive at a best-guess using the model by obtaining the smoothed (backward-looking) estimates of the time-t policy preferences (Figure 11). These estimates represent the evolution of Fed's policy preferences in the post-war period from the perspective of a private agent in the model looking back at the end of the sample period (Dec 2024). The estimates suggest that the Fed's true inflation target was upwards of 8 percent during the peak of the 80s-era inflation. Notably, the estimates of the time-varying inflation target are also in line with Ireland (2007) who uses a specification that is very different from the one used here.

Taken at face value, the smoothed estimates of λ suggest that Fed was hardly concerned with keeping the unemployment-gap closed during the 70s-80s. Over time however, the Fed has steadily emphasized closing the gap as part of its monetary policy strategy: which brings us to present-day. The estimated weight on unemployment-stabilization is almost twice as large as that on inflation-stabilization. The backward-looking estimates of λ_t display periods of departure from the real-time ones. This is because meaningful information regarding λ only arrives during periods where agents perceive large cost-push shocks driving policy tradeoffs (since λ is only relevant to policy when such a trade-off exists). Thus from the private agents' perspective, though λ has been evolving continuously, information regarding λ only arrives in clumps during episodes involving large persistent cost-push shocks driving policy trade-offs. Not much learning regarding λ takes place during "normal" times.



(a) π^*_t: Filtered *(solid)* vs Smoothed *(dashed)* estimates



(b) λ_t: Filtered *(solid)* vs Smoothed *(dashed)* estimates

Figure 11: **The Fed's "true" policy preferences:** Smoothed estimates of π^*_t and λ_t

C Estimates of the unemployment-gap

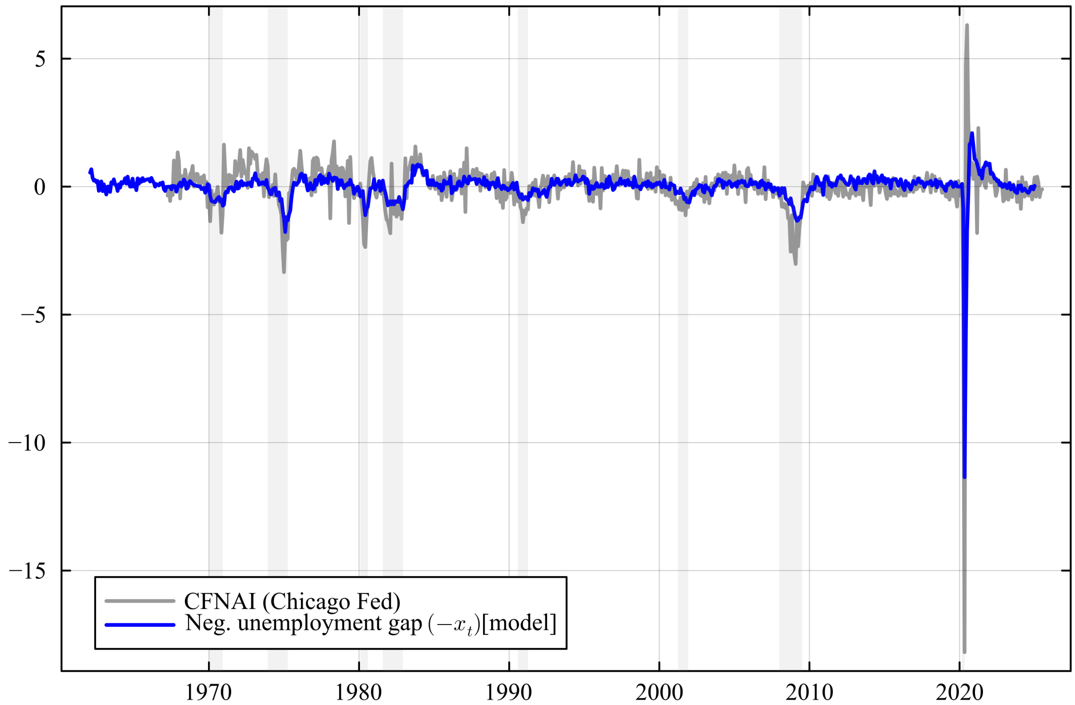


Figure 12: **Estimated unemployment-gap**: Agents' real-time estimates of the (negative of) unemployment gap compared with the CFNAI (Chicago Fed National Activity Index), a composite measure of real activity based on 81 series.
Note: Shaded regions indicate NBER recessions

Source of CFNAI: Chicago Fed

Table 2: OLS Regression: $CFNAI_t \sim Constant + \hat{x}_t$ on agents' estimated unemployment-gap (x_t)

	$CFNAI_t$
Constant	0.0023
	(0.036)
$\hat{x}_t$	**-1.0302*** **
	(0.142)
Observations	694
R-squared	0.394
F-statistic	52.55

Notes: Robust standard errors (HAC, 12 lags) in parentheses.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

D Perceived natural rate of interest $r^n_{t|t}$

Figure 13 plots the evolution of the natural rate of interest as inferred by agents in real-time. This is a useful measure and serves as a barometer to gauge the stance of policy.

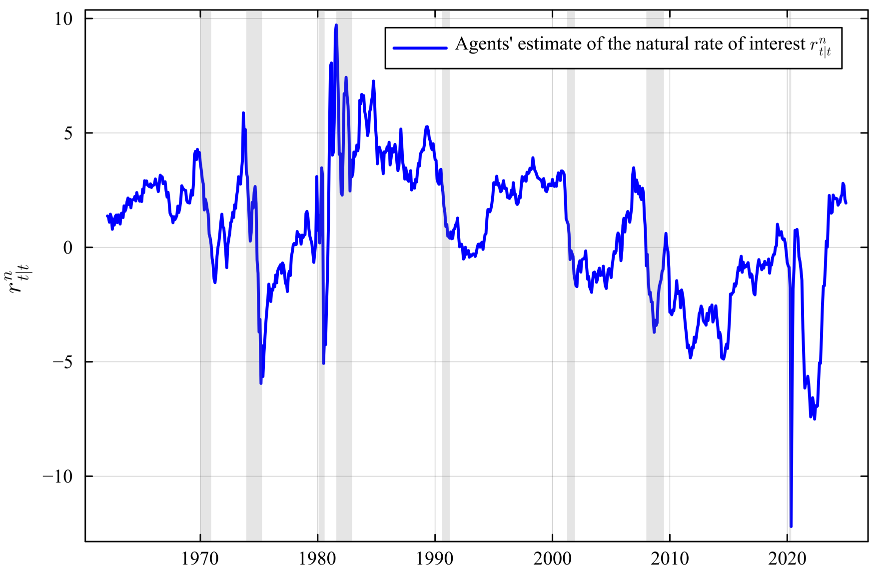


Figure 13: **Estimates of the natural rate of interest**: Model-implied path of the natural rate of interest r^n_t

Note: Shaded regions indicate NBER recessions

E Inferring the Fed's preferences from Greenbook forecasts

The Fed "Greenbook" contains projections of certain macroeconomic variables, including CPI inflation and the unemployment rate up to a few quarters into the future made at the time of the Federal Reserve Open Market Committee (FOMC) meetings. These internal forecasts are informative of the policymakers' views regarding the evolution of inflation and unemployment rate under the intended path of policy at the time. Hence, these could provide important clues to learn about their underlying policy objectives (π^* and λ). In this subsection, I estimate the Fed's "true" policy preferences based on the Greenbook forecasts, as opposed to using SPF forecasts to infer them.

Estimation is conducted using the minimum-distance approach, minimizing the least-squared distance between the model-implied one-year ahead Fed forecasts of CPI inflation and unemployment rate and the corresponding measure constructed from the Greenbook. The Greenbook provides quarterly forecasts, indexed with reference to the quarter that the forecasts were prepared in. The CPI forecasts give annualized quarterly growth rate (with respect to the previous quarter) in the Headline price index. In the Greenbook, Headline CPI inflation forecasts are denoted by the "gPCPI" series. The horizon of the forecast is indicated by the suffix "Fx" (denoting x-quarters ahead forecast measured with reference to the current quarter that the Greenbook is constructed in). Similarly, projections regarding the unemployment rate are given as "UNEMPFx" where x is the horizon of the forecast with reference to current quarter.

A coarse measure of one-year ahead inflation and unemployment rate expectations is constructed from Greenbook forecasts in the following way:

- Fed's one-year ahead forecast of inflation as implied by Greenbook data is constructed by averaging over the quarter-over-quarter annualized measures reported for up to 5 quarters ahead:

$$\pi^{GB}_{t+1y|t} = (gPCPIF0 + 2\times gPCPIF1 + 2\times gPCPIF2 + 2\times gPCPIF3 + gPCPIF4)/8$$

- Fed's one-year ahead unemployment-rate forecast is constructed by simply taking considering the average of the three- and four-quarter ahead forecast from Greenbook:

$$u^{GB}_{t+1y|t} = (UNEMPF3 + UNEMPF4)/2$$

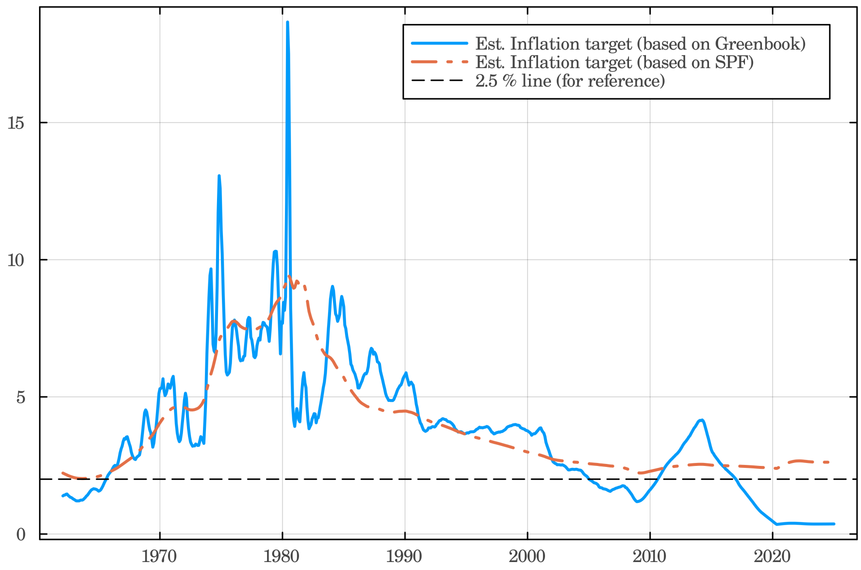
The measure is coarse because the the exact one-year ahead forecasts from the date of publication are not known. Only the end-of-quarter projections are reported. If the forecasts were made towards the beginning of the current quarter (call this Q0), the horizon ending at end of Q3 would constitute the correct forecast-horizon. If it were constructed toward the end of the quarter however, the appropriate horizon to consider would be Q1 through Q4. Although it doesn't make much of a difference for the analysis, for the sake of concreteness, I take the average of these two as the implied one-year forecast for estimation.

The parameters relevant to the Fed's true policy preferences are collected into a vector Φ and re-estimated. This includes: (i) the variance of structural innovations to (logarithm of) inflation target, (ii) the variance of structural innovations to (logarithm of) unemployment-stabilization weight and (iii) the variance of the idiosyncratic monetary policy shocks. Private-sector beliefs are kept the same as in the baseline model (since their learning behavior is still based on observed macroeconomic data) and the Fed's beliefs regarding the structural parameters are also fixed to be the same as under the baseline model, since they are learned based on commonly observed macroeconomic realizations.

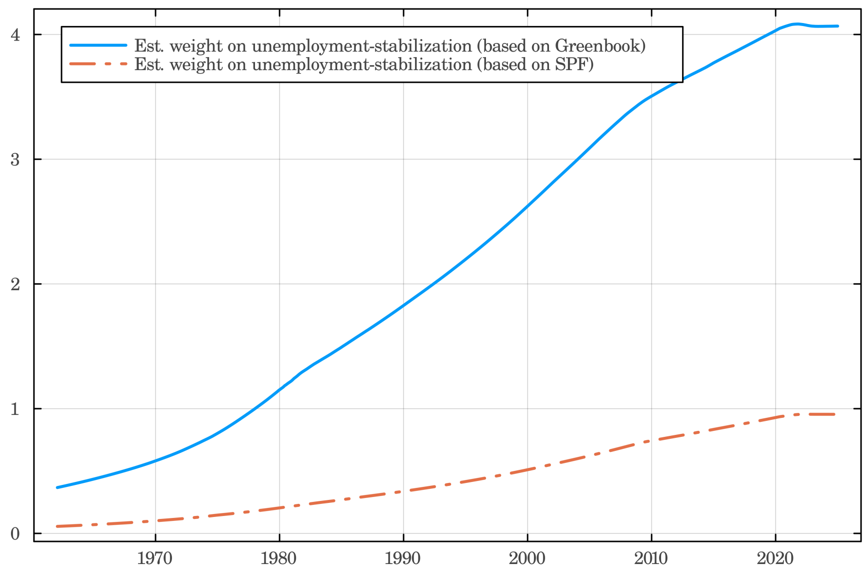
$$\Phi^* = \underset{\Phi}{\operatorname{argmin}} \sum_{t=1979:04}^{2019:04} ([\pi_{t+1y|t}^{Fed,MI}|\mathcal{H}^t, \Phi] - \pi_{t+1y|t}^{GB})^2 + \sum_{t=1967:01}^{2019:04} ([u_{t+1y|t}^{Fed,MI}|\mathcal{H}^t, \Phi] - u_{t+1y|t}^{GB})^2$$

where $[\pi_{t+1y|t}^{Fed,MI}|\mathcal{H}^t, \Phi]$ denotes the model-implied one-year ahead inflation forecast by the Fed conditional on the vector of parameters Φ and observed history of signals given by $\mathcal{H}^t$. Similarly $u_{t+1y|t}^{Fed,MI}$ denote the corresponding unemployment-rate forecasts. The Headline CPI inflation forecasts become available after October 12, 1979 while the unemployment rate forecasts are available since March 29, 1967. The Greenbook data is released after a 5-year lag, so the current vintage ends in the fourth quarter of 2019, giving us no information about the policymakers' beliefs during the post-COVID inflation surge which certainly limits its relevance. However, the broad strokes remain the same as the baseline model.

The resulting smoothed mean-estimates (conditional on the entire sample period) are shown in Figure 14. The estimates are much noisier than the baseline due to the Greenbook forecasts themselves being much more volatile. Since Greenbook releases forecasts over a shorter horizon, it limits the information the model can glean regarding the policymakers' long-run targets as they are less likely to influence outcomes in the short-run. This is one of the reasons why the SPF long-run inflation expectations series seems better suited to serve as the empirical target for estimation. The estimates do however exhibit a similar pattern as those based on SPF forecasts.

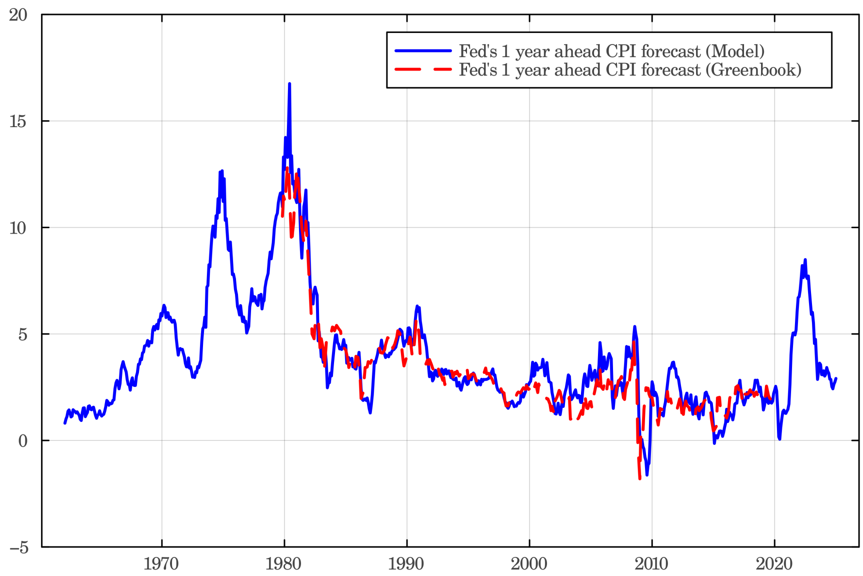


(a) π_t^*: Smoothed estimates inferred based on Greenbook forecasts

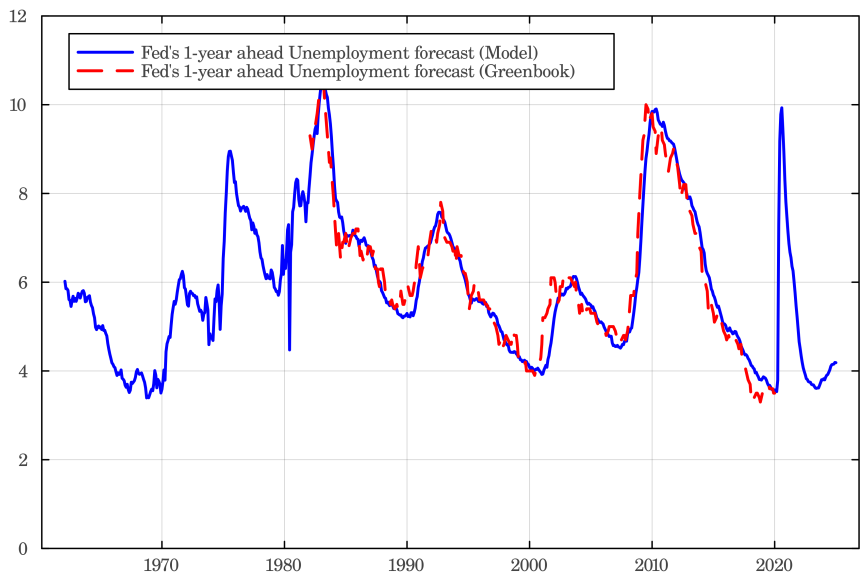


(b) λ_t: Smoothed estimates inferred based on Greenbook forecasts

Figure 14: **Evolution of Fed's policy preferences (estimated based on Greenbook forecasts):** Smoothed estimates of π_t^* and λ_t
Estimates of the Fed's inflation target are much more volatile than in the baseline model.



(a) **The Fed's inflation forecasts:** Comparing the Fed's 1-year ahead Headline CPI inflation forecasts as implied by the model vs those implied by the Greenbook



(b) **The Fed's unemployment rate forecasts:** Comparing the Fed's 1-year ahead unemployment-rate forecasts as implied by the model vs those implied by the Greenbook

Figure 15: **Comparison of fit: Model vs Greenbook:** Model-implied path of the Fed's 1y-ahead forecasts for the target variables inflation and unemployment are plotted against the corresponding Greenbook forecasts (empirical target used for estimation)
The model-implied forecasts are constructed based on the smoothed estimates of the Fed's true preferences estimated from the Greenbook forecasts, while maintaining the same paths for all other variables as in the baseline.

F Private-sector inflation expectations: Model vs Surveys

For any description of the private-sector's expectations formation process to be satisfactory, it should be able to reconcile with dynamics observed in survey data. To this end, I plot the 1-year ahead (real-time) inflation expectations generated based on private-sector beliefs in the model against the corresponding series observed in the SPF in Figure 16. Unfortunately, the Cleveland Fed's Survey of Firms Inflation Expectations (SoFIE) and the New York Fed's Survey of Consumer Expectations (SCE) series cover a relatively short sample only going back a few years and we only have the Michigan Survey going back far enough to give us a picture of the Great Inflation period of the 80s.

Since this data was not part of the model estimation, it provides a good validation test for the model. Reassuringly, the model replicates the dynamics observed in survey forecasts well. Interestingly, firms' inflation expectations seem to be even more volatile than consumers during the recent inflation surge. However, in the model presented here, no distinction is made between them and they are assumed to share identical beliefs.

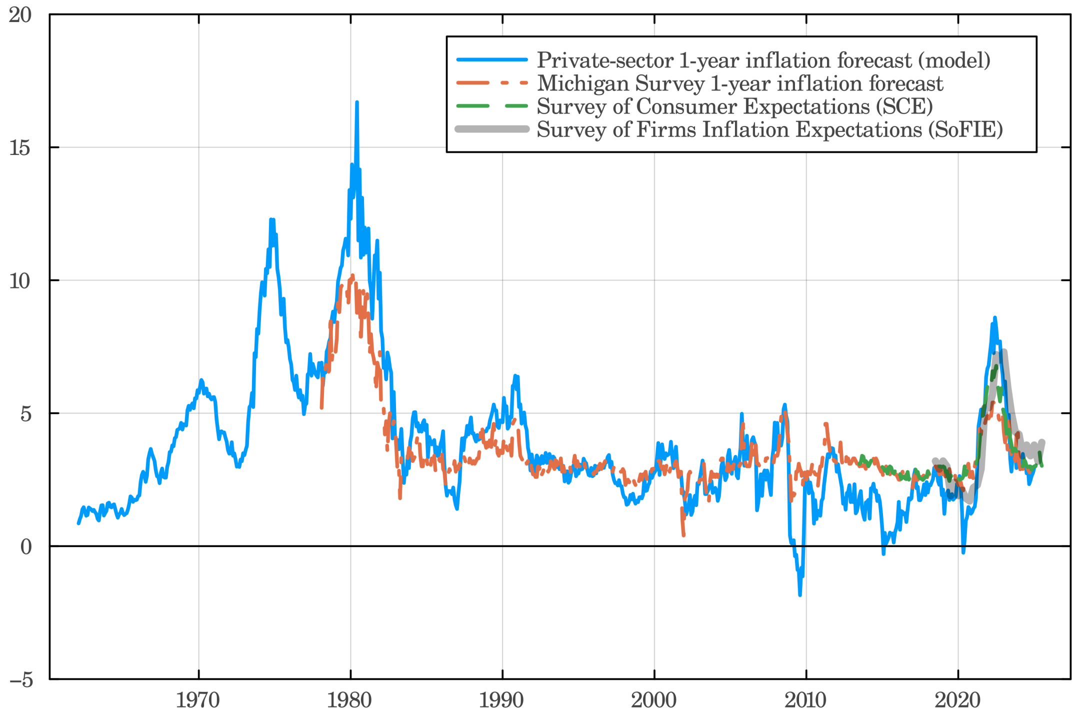


Figure 16: **Inflation Expectations compared to survey measures:** Model-implied 1-year ahead private-sector inflation expectations vs various survey measures: Survey of Firms Inflation Expectations, Survey of Consumer Expectations, Michigan Survey of Consumers

Note: Shaded regions indicate NBER recessions

Sources:

Survey of Firms Inflation Expectations: Cleveland Fed

Survey of Consumer Expectations: New York Fed

Michigan Survey of Consumers-University of Michigan

G Simulated IRFs

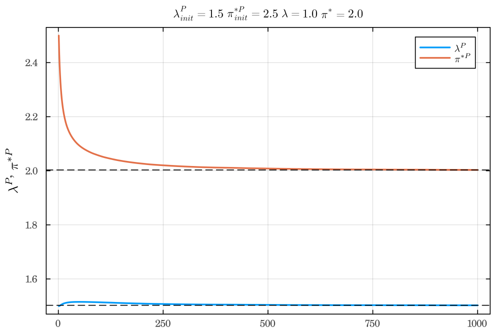
At time t, the economy experiences a sudden policy-preference shock (a Central Bank pivot to new policy preferences), moving the preference parameters away from the private-sector estimates. $\pi_t^* = 2.0\%$ $\lambda_t = 1.0\,\forall t \geq t$ whereas $\pi_t^{*P} = 2.5\%$, $\lambda_t^P = 1.5$. Figure 17 describes the dynamics of private agents' beliefs in different scenarios:

- **Scenario a:** No monetary policy shock (ϵ_t^i) or cost-push shock (ξ_t^p) - λ_t^P does not converge to the truth, slow convergence of π_t^{*P}

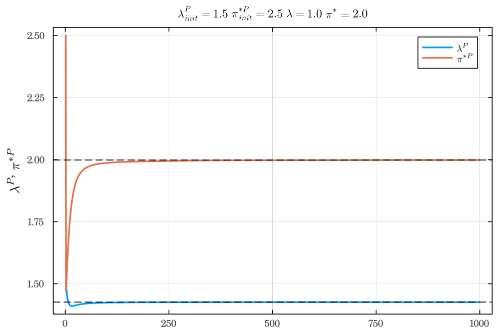
- **Scenario b:** A +100 bp monetary policy shock ($\epsilon_t^i = 1.00$) - λ_t^P does not converge to the truth, policy shock leads to idiosyncratic increase in the nominal rates and aids the convergence of $\pi_t^{*,P}$ to truth.

- **Scenario c:** Both λ_t^P and $\pi_t^{*,P}$ converge to the truth since the cost-push shock creates a trade-off and aids private agents learning about λ_t

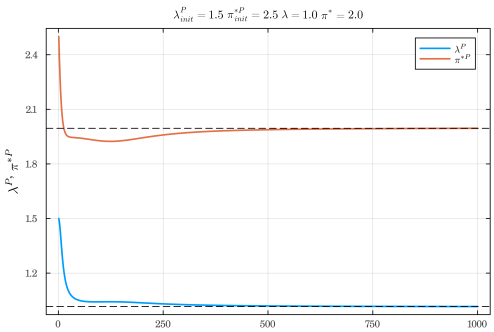
- **Scenario d:** Both λ_t^P and $\pi_t^{*,P}$ converge to the truth but follow complex dynamics due to the interaction of the shocks.



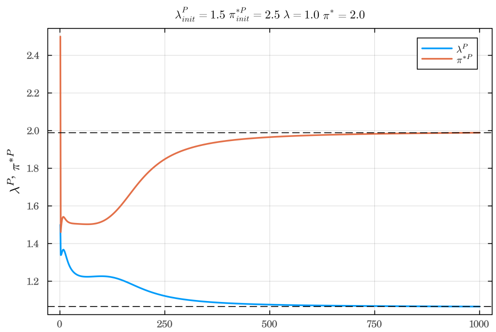
(a) No monetary policy or cost-push shocks



(b) Monetary Policy shock (100 bp)



(c) Persistent cost-push shock (10 bp)



(d) Both shocks coincide

Figure 17: **Shock-interactions are important under learning:** Private-sector beliefs under learning

This figure illustrates how private-sector beliefs regarding the central bank's policy preferences evolve as they learn from policy. The various shocks interact with policy behavior and have dynamic effects on agents' beliefs (hence necessarily on macroeconomic outcomes). The shocks are found to also have multiplicative interactions as opposed to additive ones which cannot be captured in the VAR or SVAR frameworks. Under these models, the effects of shocks are captured by their linear combinations. This is not true when agents are learning.

H Testing the assumptions

H.1 Has the "true" slope of the Phillips curve changed over time?

In the baseline model, we proceeded with the assumption of a time-invariant Phillips curve in order to keep the model parsimonious and avoid over-reliance on time-variation Phillips curve to explain the observations. However, one can statistically test if there has been a change in the true relationship between unemployment-gap and inflation during the pandemic (motivated by literature arguing that structural shifts in the Phillips curve during the pandemic period were an important factor driving the inflation surge).

In order to do this, we can construct an expectation-adjusted measure of inflation π_t^{adj} as:

$$\pi_t^{adj} = \pi_t - E_{t-1}^P \pi_{t+1}$$

This expectations-adjusted measure of inflation is regressed against the measured unemployment-gap. The results are reported in Table 3. The first specification (baseline) assumes a time-invariant Phillips curve relationship. The results of the baseline regression show that once expectations are taken into account, the relationship between inflation and the unemployment-gap is statistically significant - implying a 1% increase in the unemployment-gap associated with a roughly 10 bp decrease in inflation. It is worth noting that the estimate is quite close to the model-implied private-sector and policymaker beliefs at the end of the sample shown in Figure 8 (≈ -0.11). However, the second column shows that the dummy representing the difference in slope during the post-2020 period is not found to be statistically significant at the 5% confidence level. This implies that the data does not find enough support for a structural shift in the true Phillips curve after the pandemic.

Another way to shed light on whether time-variation in the Phillips curve slope is an important feature of economic dynamics necessary to explain the data is to directly test for time-variation in the κ_t against the null of a constant-κ_t in the relationship:

$$\pi_t^{adj} = -\kappa_t x_t + \xi_t$$

Table 3: Expectations-augmented Phillips curve: Examining change in slope

	Model 1 Baseline	Model 2 Change in slope post-2020
$\hat{x}_t$	**-0.106****	**-0.193****
	(0.037)	(0.063)
$\hat{x}_t \times \mathbf{1}_{\{t>2020\}}$	—	0.119
		(0.067)
Observations	756	756
R^2 (uncentered)	0.023	0.028
F-stat	8.322	10.42
p-value	0.004	3.44e-05

Notes: Dependent variable is π_t^{adj}. Model 2 augments Model 1 with the interaction $x_t \times \mathbf{1}_{\{t>2020\}}$, where $\mathbf{1}_{\{t>2020\}}$ equals one for dates from 2020 onward. HAC (Newey–West) robust standard errors (Bartlett kernel, maxlags = 6). R^2 is computed without centering because the model has no constant. Significance: * indicates that the coefficient is significant at the 5% level.

If the data supports time-variation in κ_t, then relaxing the restriction of a constant κ should result in a significantly better fit, as measured by log-likelihood. A likelihood-ratio (LR) test is conducted, testing the alternate hypothesis of time-variation in κ against the null of a constant κ. The LR test is not statistically significant with $p \approx 1.00$. This suggests that the model finds no evidence in favor of time-variation slope of the Phillips curve once expectations are accounted for. The results of the test are reported in Table 4.

Table 4: Results of the Likelihood Ratio Test

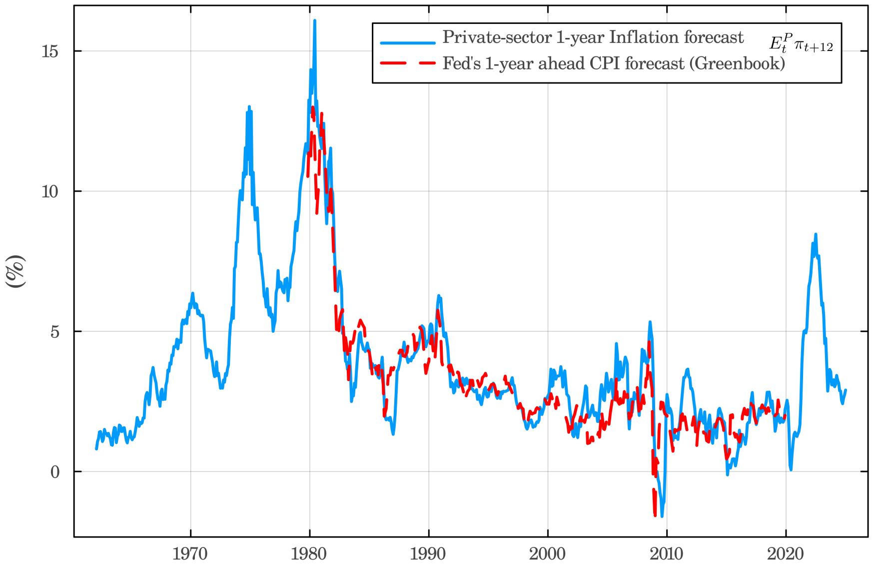
Model	**Log-Likelihood**	**Parameters**	χ^2	**df**	**p-value**
Restricted (Constant κ)	-402.460	3	3.1×10^{-7}	1	1.000
Full (Time-Varying κ)	-402.460	4			

Note: The restricted model assumes a constant κ coefficient, while the full model allows it to vary over time. The constant parameter case is nested within the full time-varying parameter model. The Likelihood Ratio Test (LRT) statistic (χ^2) is calculated as $2 \times (\log L_{\text{Full}} - \log L_{\text{Restricted}})$, with degrees of freedom (df) equal to the difference in the number of parameters between the two models.

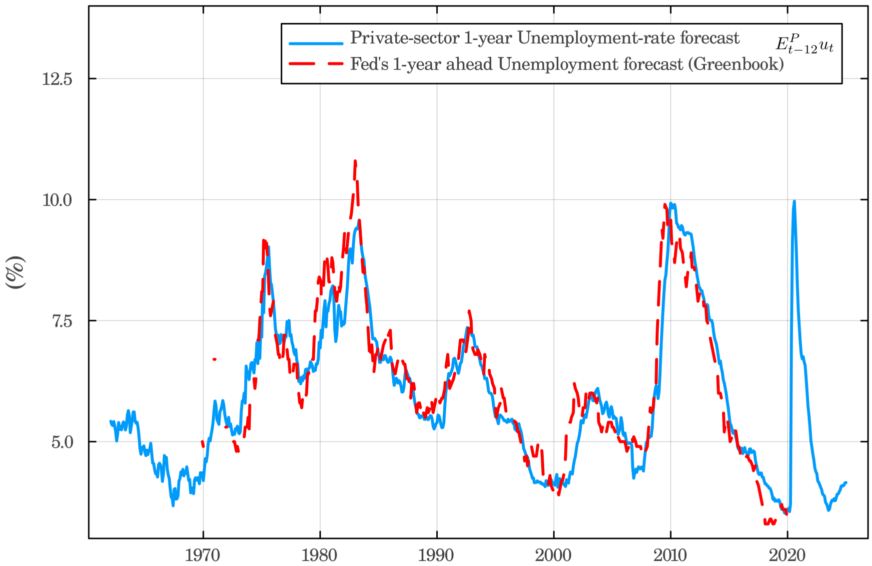
Overall, these results favor the view that time-invariant Phillips curve view adopted by the private agents in the model. Since the modeling framework in this paper explicitly accounts for endogenous formation of expectations, κ should not be interpreted as a purely a statistical relationship between inflation and unemployment, differently from the classical Keynesian specification of the Phillips curve described by Phillips (1958).

H.2 Is the degree of information asymmetry small or large?

This paper assumes little information asymmetry between the Fed and the private-sector: (i) their estimates of the structural parameters and shocks are shaped by the same learning process and data - hence they are the same throughout the sample (ii) the filtering problem assumes that private-agents are able to track the true policy preferences efficiently, meaning that tracked state never strays away from the "true" parameters governing the Fed's policy stance. Violation of these assumptions would have implications for how agents' beliefs evolve and can make the filtering problem ill-posed. One way to test whether these assumptions hold well during the sample period is to compare the forecasts that agents would generate under learning with the Fed's Greenbook forecasts: the Fed's internal forecasts of its outlook on the economy prepared by the Fed staff before each FOMC meeting. If the Fed policymakers do possess significantly different beliefs compared to the private agents, then their forecasts would reflect that and be different.



(a) **Inflation forecasts:** Comparing the Fed's 1-year ahead Headline CPI inflation forecast from Greenbook with 1-year ahead private-agents' forecasts from the model



(b) **Unemployment rate forecasts:** Comparing the Fed's 1-year ahead unemployment rate forecast from Greenbook with 1-year ahead private agents' forecasts from the model

Figure 18: **Macroeconomic outlook (1 year-ahead):** The Fed's 1 year-ahead forecasts for the target variables inflation and unemployment are plotted against the corresponding Greenbook forecasts (empirical target used for estimation)
The model-implied forecasts are constructed based on the smoothed estimates of the Fed's true preferences estimated from the Greenbook forecasts, while maintaining the same paths for all other variables as in the baseline.

To the extent that asymmetric information is large enough to drive a wedge between private agents' and the Fed's forecasts, it does not seem to be important during the sample period. Private-sector forecasts as implied by the model are very close to the corresponding internal forecasts of the Fed (as observed in the Greenbook).