89강 실수 내적공간 (real inner product space)
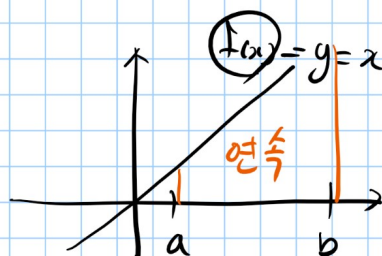
* 내적 일반화 ⇒ 벡터가 n-tuple이 아닐수도 있음.
  └→ 함수

* $u \cdot v$ $\langle u, v \rangle$

* 실수 내적 (스칼라가 실수 범위)

① $\langle u, v \rangle = \langle v, u \rangle$
② $\langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$
③ $\langle ku, v \rangle = k \langle u, v \rangle$
④ $\langle v, v \rangle \geq 0$, $\langle v, v \rangle = 0 \iff \vec{v} = \vec{0}$
  └→ 실수 내적공간

* Ⓥ$= \{$ 구간 $[a, b]$에서 연속인 함수$\}$
   $f(x) \in V$

① $\forall f, g \in V$ $f(x) + g(x) = h(x)$ $[a, b]$에서 연속.
                            └→ $h \in V$

② $f + g = g + f$
③ $f + (g + h) = (f + g) + h$
④ $f + 0 = f$
        $y = 0$
          ⋮

* V에서 내적을 정의내리자. (실수 내적)
$\langle f, g \rangle := \int_a^b f(x) g(x) dx$ $\langle 0, g \rangle = \int_a^b 0 \cdot g(x) dx = 0$
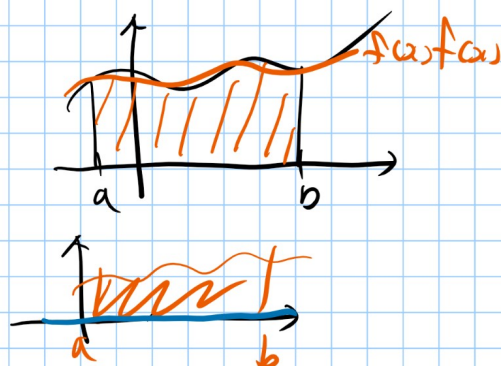
① $\langle f, g \rangle = \int_a^b f(x) g(x) dx = \int_a^b g(x) f(x) dx = \langle g, f \rangle$

② $\langle f+g, h \rangle = \int_a^b (f(x) + g(x)) h(x) dx = \int_a^b (f(x) h(x) + g(x) h(x)) dx$
                                                    $= \int_a^b f(x) h(x) dx + \int_a^b g(x) h(x) dx$
③ $\langle kf, g \rangle = \int_a^b k f(x) g(x) dx$
              $= k \int_a^b f(x) g(x) dx$ $= \langle f, h \rangle + \langle g, h \rangle$
              $= k \langle f, g \rangle$

④ $\langle f, f \rangle \geq 0$ → $\int_a^b f(x) f(x) dx$
        $\forall \alpha \in [a, b]$, $f(\alpha) \cdot f(\alpha) \geq 0$
   $\langle f, f \rangle = 0 \iff f(x) = 0$
                              └→ 늘

$$\vec{0} \cdot \vec{V} = \underline{0}$$

$\vec{0} = (0, 0, \cdots, 0)$
$\vec{V} = (u, v_2, \cdots, u_n)$ $= 0$

$\langle \vec{0}, \vec{V} \rangle = 0$

pf) 일반적으로 증명 ⇒ 공리만을 사용.
   내적 ①~④ ①~⑨ (23p)

$$\langle \vec{0}, \vec{V} \rangle = \langle \vec{0} + \vec{0}, \vec{V} \rangle = \langle \vec{0}, \vec{V} \rangle + \langle \vec{0}, \vec{V} \rangle$$

$$0 = \langle \vec{0}, \vec{V} \rangle$$

✸ $\langle u - v, w \rangle = \langle u, w \rangle - \langle v, w \rangle$

pf) $\langle u + (-v), w \rangle = \langle u, w \rangle + \langle -v, w \rangle$

$$= \langle u, w \rangle - \langle v, w \rangle$$

✸ $\langle u, v - w \rangle = \langle u, v \rangle - \langle u, w \rangle$

pf) $\langle u, v - w \rangle = \langle v - w, u \rangle = \langle v, u \rangle - \langle w, u \rangle$

$$= \langle u, v \rangle - \langle u, w \rangle$$

✸ $k \langle u, v \rangle = \langle u, kv \rangle$

pf) $k \langle u, v \rangle = k \langle v, u \rangle = \langle kv, u \rangle = \langle u, kv \rangle$

$k(u \cdot v) = (ku) \cdot v = u \cdot (kv)$ $a ✸ b ✸ c = 0 + 1 + 2 = ?$