# 64강 정사영 정리

Thm. $null(A^TA) = null(A)$ (단, $A : m\times n$ 행렬)

↳ $A^TA\underline{x}=0$ ↳ $A\underline{x}=0$

pf)

i) $null(A^TA) \subseteq null(A)$

$\forall a \in null(A^TA) \Rightarrow a \in null(A)$

$null(A^TA) = row(A^TA)^{\perp} = col(A^TA)^{\perp} \ni a$

$(A^TA)$ : 대칭행렬 $(A^TA)^T = A^TA$

↳ $n\times n$ $A^T(A^T)^T = A^TA$

$\forall v \in col(A^TA)$, $v \cdot a = 0$ $A^TA = [c_1\ c_2 \cdots c_n]$

$Ax=0$

$a\cdot(A^TAa) = Aa\cdot Aa = 0 \Rightarrow Aa = 0$

ii) $null(A) \subseteq null(A^TA)$

$\forall a \in null(A) \Rightarrow a \in null(A^TA)$

$Aa=0 \Rightarrow A^TAa = A^T0 = 0$ ┘

$A^TAx=0$

Thm. $rank(A^TA) = rank(A)$ $A : m\times n$

pf) $rank(A^TA) + \cancel{nullity}(A^TA) = \cancel{n}$

$-\ rank(A) + \cancel{nullity}(A) = \cancel{n}$

$rank(A^TA) = rank(A)$

Thm. A가 full column rank 이면 ⇒ A가 가역

$A : m\times n$

⇒ $A^TA$도 가역

$rank(A) = n$

$= rank(A^TA) \Rightarrow A^TA$ : full column rank

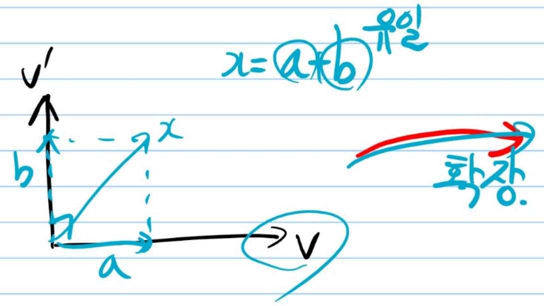
$A^TA : n\times n$

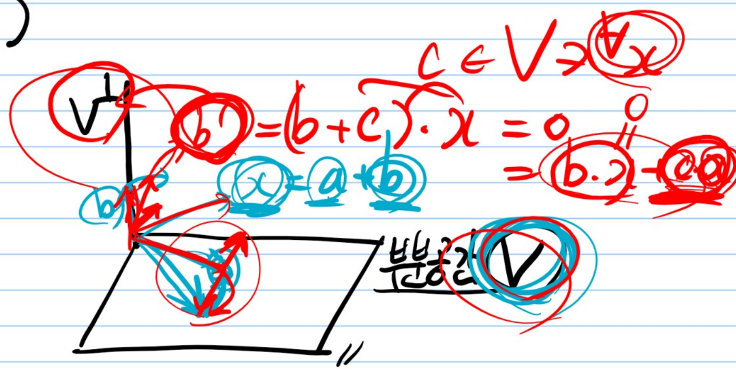
Thm 정사영 정리

$V \subseteq \mathbb{R}^n$, V는 부분공간 일때.

$\forall x \in \mathbb{R}^n$, $x = x_1 + x_2$ 로 유일하게 표현된다.

(단, $x_1 \in V$, $x_2 \in V^{\perp}$)




pf) $\forall x \in \mathbb{R}^n$, $x = x_1 + x_2$, $x_1 \in V$, $x_2 \in V^{\perp}$, $V \subseteq \mathbb{R}^n$

i) $V = \{0\}$ $\quad x_1 = 0 \quad x_2 = x \quad x_2 \in V^{\perp}$, $\in \mathbb{R}^n$, $\{0\}^{\perp} = \mathbb{R}^n$

$\{0\}^{\perp} \ni a \quad a \cdot 0 = 0$ 유일

ii) $V \neq \{0\} \Rightarrow B = \{v_1, v_2, \cdots, v_m\}$ $\forall x \in \mathbb{R}^n$, $x = x_1 + x_2$

$x_1 \in V$, $x_2 \in V^{\perp}$, $\quad x_1 = a_1 v_1 + a_2 v_2 + \cdots + a_m v_m = Ta$

$[v_1\ v_2 \cdots v_m] = T \quad x_2 = x - x_1 = \boxed{x - Ta} \in V^{\perp}$

$\forall y \in V \quad y \cdot (x - Ta) = 0 \quad y = b_1 v_1 + b_2 v_2 + \cdots + b_m v_m = Tb$

$\forall b \in \mathbb{R}^m \quad Tb \cdot (x - Ta) = 0 \qquad Aa = T^T x$ → 가역 → x 무

$b \cdot T^T (x - Ta) = 0 \Rightarrow T^T (x - Ta) = 0$

$T^T x - T^T T a = 0 \qquad T^T x = T^T T a$ 존재

T가 full colum rank $T^T T$ 가역 $\quad a = (T^T T)^{-1} T^T x$ 유일