72강 블록행렬 (block matrix, partitioned matrix)

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4} = \begin{bmatrix} A_{11}_{2\times 1} & A_{12}_{2\times 3} \\ A_{21}_{1\times 1} & A_{22}_{1\times 3} \end{bmatrix}_{2\times 2}$$

* 블록행렬의 성질
같은 열(혹은 행)에 있는 행렬은 열(혹은 행)의 개수가 같다.

$$AB \quad (3\times 4)(4\times 2) = \begin{bmatrix} A_{11}_{2\times 1} & A_{12}_{2\times 3} \\ A_{21}_{1\times 1} & A_{22}_{1\times 3} \end{bmatrix}_{2\times 2} \begin{bmatrix} B_{11}_{1\times 2} \\ B_{12}_{3\times 2} \end{bmatrix}_{2\times 1} = \begin{bmatrix} A_{11}B_{11}+A_{12}B_{12} \\ A_{21}B_{11}+A_{22}B_{12} \end{bmatrix}_{2\times 1}$$

(2×2)(2×2) → (1×2)

* 블록 행렬 끼리의 곱은 크기만 잘 맞춰준다면 일반적인 행렬곱처럼 수행할 수 있다.

ex) $A=\left[\begin{array}{cc|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right] \quad B=\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}$

$$AB=\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} \end{bmatrix}$$

$A_{11}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, A_{12}=\begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$

$A_{21}=\begin{bmatrix} a_{31} & a_{32} \end{bmatrix}, A_{22}=\begin{bmatrix} a_{33} \end{bmatrix}$

$B_{11}=\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \quad B_{21}=\begin{bmatrix} b_{31} \end{bmatrix}$

$$A=\left[\begin{array}{cc} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \quad B=\begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}$$

$$AB=\begin{bmatrix} A_{11}B_{11}+A_{12}B_{21} \\ A_{21}B_{11}+A_{22}B_{21} \end{bmatrix}$$

$$A_{11}B_{11}+A_{12}B_{21}=\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} \\ a_{21}b_{11}+a_{22}b_{21} \end{bmatrix}+\begin{bmatrix} a_{13}b_{31} \\ a_{23}b_{31} \end{bmatrix}=\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} \end{bmatrix}$$