

# 89강 실수 내적공간 (real inner product space)

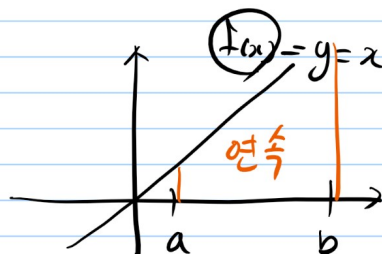
\* 내적 일반화  $\Rightarrow$  벡터가 n-tuple 이 아닐 수도 있음.

\*  $u \odot v \quad \langle u, v \rangle$

\* 실수 내적 (스칼라가 실수 범위)

- ①  $\langle u, v \rangle = \langle v, u \rangle$
- ②  $\langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$
- ③  $\langle ku, v \rangle = k \langle u, v \rangle$
- ④  $\langle v, v \rangle \geq 0, \quad \langle v, v \rangle = 0 \iff \vec{v} = \vec{0}$

$\hookrightarrow$  실수 내적공간



\*  $V = \{ \text{구간 } [a, b] \text{에서 연속인 함수} \}$

$f(x) \in V$

- ①  $\forall f, g \in V \quad f(x) + g(x) = h(x) \quad [a, b] \text{에서 연속.}$   
 $\hookrightarrow h \in V$

②  $f+g = g+f$

③  $f+(g+h) = (f+g)+h$

④  $f+0 = f$   
 $\hookrightarrow \underline{y=0}$

\*  $V$ 에서 내적을 정의내리자. (실수 내적)

$\langle f, g \rangle := \int_a^b f(x)g(x)dx \quad \langle \vec{0}, g \rangle = \int_a^b 0 \cdot g(x)dx = 0$

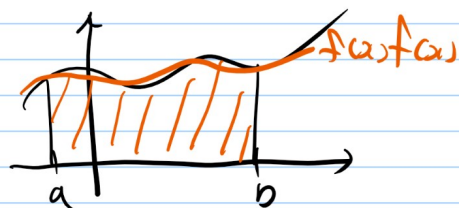
①  $\langle f, g \rangle = \int_a^b f(x)g(x)dx = \int_a^b g(x)f(x)dx = \langle g, f \rangle$

②  $\langle f+g, h \rangle = \int_a^b (f(x)+g(x))h(x)dx = \int_a^b (f(x)h(x) + g(x)h(x))dx$

③  $\langle kf, g \rangle = \int_a^b k f(x)g(x)dx = k \int_a^b f(x)g(x)dx = k \langle f, g \rangle$

$= \int_a^b f(x)h(x)dx + \int_a^b g(x)h(x)dx = \langle f, h \rangle + \langle g, h \rangle$

④  $\langle f, f \rangle \geq 0 \rightarrow \int_a^b f(x)f(x)dx$   
 $\forall x \in [a, b], \quad f(x) \cdot f(x) \geq 0$   
 $\langle f, f \rangle = 0 \iff f(x) = 0$   
 $\hookrightarrow \vec{0}$



$$\vec{0} \cdot \vec{v} = 0$$

$$\vec{0} = (0, 0, \dots, 0)$$

$$\vec{v} = (v_1, v_2, \dots, v_n) = 0$$

$$\langle \vec{0}, \vec{v} \rangle = 0$$

pf) 일반적으로 증명  $\Rightarrow$  공리만을 사용.

내적 ①~④ ①~⑨ (23장)

$$\langle \vec{0}, \vec{v} \rangle = \langle \vec{0} + \vec{0}, \vec{v} \rangle = \langle \vec{0}, \vec{v} \rangle + \langle \vec{0}, \vec{v} \rangle$$

$$0 = \langle \vec{0}, \vec{v} \rangle$$

$$\star \langle u - v, w \rangle = \langle u, w \rangle - \langle v, w \rangle$$

pf)  $\langle u + (-v), w \rangle = \langle u, w \rangle + \langle -v, w \rangle$

$$= \langle u, w \rangle - \langle v, w \rangle$$

$$\star \langle u, v - w \rangle = \langle u, v \rangle - \langle u, w \rangle$$

pf)  $\langle u, v - w \rangle = \langle v - w, u \rangle = \langle v, u \rangle - \langle w, u \rangle$

$$= \langle u, v \rangle - \langle u, w \rangle$$

$$\star k \langle u, v \rangle = \langle u, kv \rangle$$

pf)  $k \langle u, v \rangle = k \langle v, u \rangle = \langle kv, u \rangle = \langle u, kv \rangle$

$$k(u \cdot v) = (kv) \cdot v = u \cdot (kv)$$

$$a \star b \star c = 0 + 1 + 2 = ?$$