EVD (고유값분해) → 대칭행렬

ex) $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

① $A^T A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 2 \end{bmatrix}$

$\begin{vmatrix} 5-\lambda & 0 & 2 \\ 0 & -\lambda & 0 \\ 2 & 0 & 2-\lambda \end{vmatrix} = (5-\lambda) \begin{vmatrix} -\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} + 2 \begin{vmatrix} 0 & -\lambda \\ 2 & 0 \end{vmatrix}$

$= (5-\lambda) \underbrace{\{ -\lambda(2-\lambda) \}}_{-2\lambda + \lambda^2} + 2 \underbrace{(-(-2\lambda))}_{+4\lambda}$

$= \lambda \{ (5-\lambda)(-2+\lambda) + 4 \}$

$= \lambda \{ \underbrace{-10 + 5\lambda + 2\lambda - \lambda^2 + 4}_{-6 + 7\lambda - \lambda^2} \}$

$= -\lambda(\lambda-6)(\lambda-1) = 0$

$\lambda_1 = 6, \quad \lambda_2 = 1, \quad \lambda_3 = 0$

② 고유벡터 구하기. $V_1' = (2,0,1), \quad V_2' = (-1,0,2), \quad V_3' = (0,1,0)$

⇒ 정규직교화. $V_1 = \frac{1}{\|V_1'\|} V_1' = (\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}})$

$V_2 = (-\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}})$

$V_3 = (0,1,0)$

③ $V = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sqrt{6} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\left\{ \underset{\underset{\frac{1}{\sqrt{6}} A V_1}{=}}{u_1}, \underset{\underset{A V_2}{=}}{u_2} \right\} \to \textcircled{B} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$

$u_1 = \frac{1}{\sqrt{6}} A V_1 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} \\ 0 \\ \frac{1}{\sqrt{5}} \end{bmatrix}$

$= \frac{1}{\sqrt{6}} \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{5}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}$

$$u_2 = AV_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{5}} \\ 0 \\ \frac{2}{\sqrt{5}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{5}} \\ 0 \\ \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$B = \begin{bmatrix} (AV_1)^T \\ (AV_2)^T \end{bmatrix} \quad Bx = 0 \quad \text{null space}$$

$$\begin{bmatrix} 2 & 5 & 1 \\ -1 & 0 & 2 \end{bmatrix}$$

↳ 기약행사다리꼴 (가우스조던소거법)

$$\begin{bmatrix} 2 & 5 & 1 \\ -1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x = 2z$$
$$y = -z$$
$$z = z$$

$$\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} t \;\middle|\; t \in \mathbb{R} \right\}$$

↳ 거리

$$u_3 = \left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

$$U = \begin{bmatrix} \frac{2}{\sqrt{30}} & -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{6}} \\ \frac{5}{\sqrt{30}} & 0 & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{30}} & \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

✗ $A = U \Sigma V^T$ "