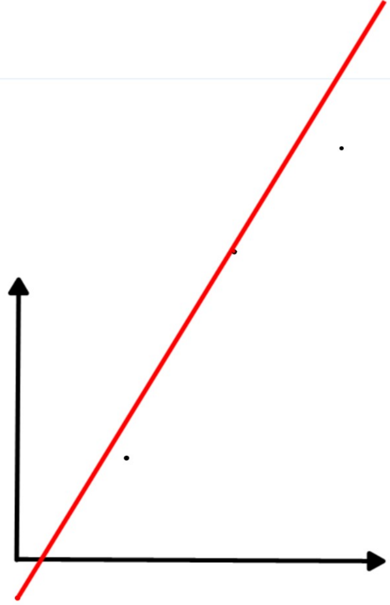
65강 정사영 정리 (2)



Thm $(V^\perp)^\perp = V$ (단, V는 벡터공간)

그냥 $S^\perp \Rightarrow$ 공간

pf)

i) $(V^\perp)^\perp \subseteq V$

$\forall a \in (V^\perp)^\perp \Rightarrow a \in V$, $a = a_1 + \cancel{a_2}$

단, $a_1 \in \boxed{V}$, $a_2 \in \boxed{V}^\perp$, 유일. ↳ $a \in V$

$\Rightarrow a_2 = 0$ 임을 보이자. $a \cdot a_2 = 0$

$= (a_1+a_2)\cdot a_2 = \cancel{a_1 \cdot a_2} + a_2 \cdot a_2$

$= a_2 \cdot a_2 = 0 \Rightarrow a_2 = 0$

ii) $V \subseteq (V^\perp)^\perp$

$\forall \boxed{a} \in V \Rightarrow a \in (V^\perp)^\perp$

$\forall x \in V^\perp$, $\boxed{a \cdot x} = 0$ 임을 보이자.

↳ $x \in V^\perp$ $\forall \boxed{y} \in V$, $y \cdot x = 0$

* 정사영 정리 공식

$X = X_1 + X_2$ 유일, $X_1 \in V$, $X_2 \in V^\perp$

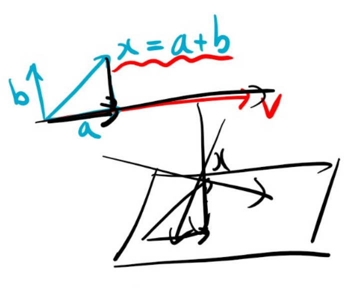
$X_1 = Ta$

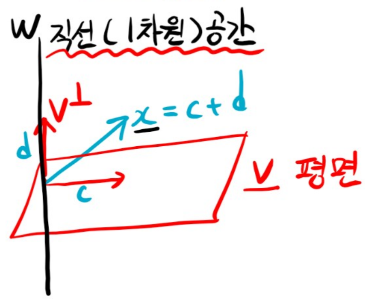
$X_2 = X - Ta$ $\quad a = (T^TT)^{-1}T^TX$

(단, T는 벡터공간 V의 기저를 열벡터로 하는 행렬)

$X_1 = T(T^TT)^{-1}T^TX = \boxed{\mathrm{Proj}_V X}$

$V \ni v_1$





$\mathrm{Proj}_V X = X_1 = T(T^TT)^{-1}T^TX.$

$\mathrm{Proj}_{v_1} X = \frac{X \cdot v_1}{\|v_1\|^2} v_1$

* V : 벡터공간 , $x \in \mathbb{R}^n$, $V \subseteq \mathbb{R}^n$

$x = x_1 + x_2$, $x_1 \in V$, $x_2 \in V^\perp$

$\checkmark$ $x = x_3 + x_4$, $x_3 \in V^\perp$, $x_4 \in (V^\perp)^\perp = V$

$\Rightarrow$ $x_1 = x_4$, $x_2 = x_3$

$x = x_1 + x_2 = \mathrm{Proj}_V x + x_2$ $\qquad x_2 = x - \mathrm{Proj}_V x$

$x = x_3 + x_4 = \boxed{\mathrm{Proj}_{V^\perp} x} + x_4$

$x = \mathrm{Proj}_V x + \mathrm{Proj}_{V^\perp} x$