64강 정사영 정리

Thm. $\text{null}(A^TA) = \text{null}(A)$ (단, A : $m \times n$ 행렬)

$\quad \hookrightarrow A^TA\underline{x}=0 \quad \hookrightarrow A\underline{x}=0$

pf)

① $\text{null}(A^TA) \subseteq \text{null}(A)$

$\forall a \in \text{null}(A^TA) \Rightarrow a \in \text{null}(A)$

$\text{null}(A^TA) = \text{row}(A^TA)^\perp = \text{col}(A^TA)^\perp \ni a$

(A^TA) : 대칭행렬 $\quad (A^TA)^T = A^TA$

$\quad \hookrightarrow n \times n \qquad\qquad A^T(A^T)^T = A^TA$

$\forall v \in \text{col}(A^TA), \quad v \cdot a = 0 \qquad A^TA = [c_1 \, c_2 \cdots c_n]$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad Ax=0$

$\curvearrowleft a \cdot (A^TA a) = Aa \cdot Aa = 0 \Rightarrow Aa = 0$

② $\text{null}(A) \subseteq \text{null}(A^TA)$

$\forall a \in \text{null}(A) \Rightarrow a \in \text{null}(A^TA)$

$Aa = 0 \Rightarrow A^TA a = A^T 0 = 0$

$\qquad\qquad \underset{A^TAx=0}{\underline{}}$

Thm. $\text{rank}(A^TA) = \text{rank}(A) \qquad A : m \times n$

pf) $\text{rank}(A^TA) + \text{nullity}(A^TA) = n$

$\quad - \; \text{rank}(A) + \text{nullity}(A) = n$

$\overline{\qquad\qquad \text{rank}(A^TA) = \text{rank}(A) \qquad}$

Thm. A가 full column rank 이면 $\Rightarrow$ A가 가역

$A : m \times n$

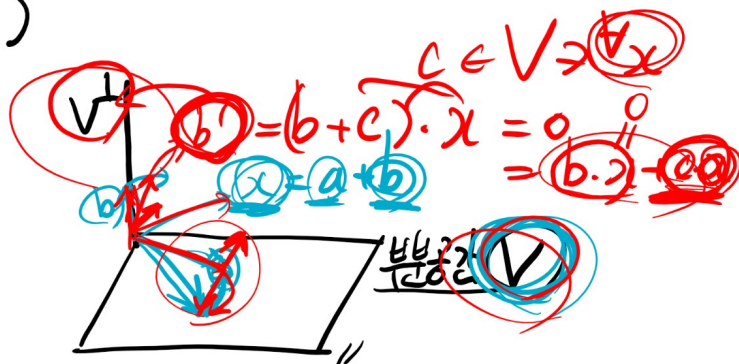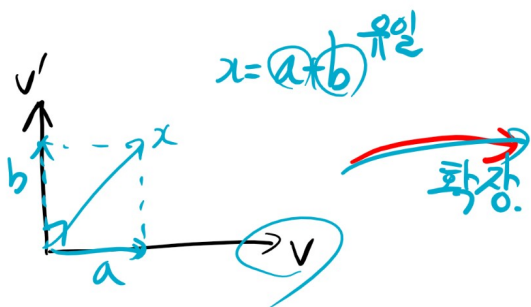$\qquad\qquad\qquad\qquad \Rightarrow \underline{A^TA \text{도 가역}}$

$\text{rank}(A) = n$

$= \text{rank}(A^TA) \Rightarrow A^TA :$ full column rank

$A^TA : n \times n$

Thm 정사영 정리

$V \subseteq \mathbb{R}^n$, V는 부분공간 일때.

$\forall x \in \mathbb{R}^n$, $x = x_1 + x_2$ 로 유일하게 표현된다.

(단, $x_1 \in V$, $x_2 \in V^\perp$)

$x = a+b$ 유일

확장.

$(b')=(b+c)\cdot x = 0$
$= (b\cdot c) = 0$
$c \in V$, $\forall x$

부분공간 V

pf) $\forall x \in \mathbb{R}^n$, $x = x_1 + x_2$, $x_1 \in V$, $x_2 \in V^\perp$, $V \subseteq \mathbb{R}^n$

ⅰ) $V = \{0\}$ $x_1 = 0$ $x_2 = (x)$ $x_2 \in V^\perp$, $\{0\}^\perp = \mathbb{R}^n$
   $\in \mathbb{R}^n$

$\{0\}^\perp \ni a$ $a \cdot 0 = 0$ 유일

ⅱ) $V \neq \{0\} \Rightarrow B = \{v_1, v_2, \cdots, v_m\}$ $\forall x \in \mathbb{R}^n$, $x = x_1 + x_2$

$x_1 \in V$, $x_2 \in V^\perp$, $x_1 = a_1 v_1 + a_2 v_2 + \cdots + a_m v_m = Ta$

$[v_1\ v_2 \cdots v_m] = T$ $x_2 = x - x_1 = \boxed{x - Ta} \in V^\perp$

$\forall y \in V$, $y \cdot (x - Ta) = 0$ $y = b_1 v_1 + b_2 v_2 + \cdots + b_m v_m$
                                              $= Tb$
$\forall b \in \mathbb{R}^m$ $Tb \cdot (x - Ta) = 0$ $Aa = T^T x$

$b \cdot T^T (x - Ta) = 0 \Rightarrow T^T(x - Ta) = 0$ 가역 x 유

$T^T x - T^T Ta = 0$ $T^T x = T^T Ta$ 존재

T가 full column rank $T^T T$ 가역 $a = (T^T T)^{-1} T^T x$ 유일