70강 직교기저의 증명

① 직교기저 ② → 정규직교기저

V : 벡터공간 , $B = \{v_1, v_2, \cdots, v_k\} \Rightarrow B' = \{w_1, \cdots, w_k\}$

$B'' = \{w_1/\|w_1\|, w_2/\|w_2\|, \cdots\}$

pf) ① $w_1 = v_1$ $\quad$ span(w_1)

그람슈미츠

그람슈미트.

잡 직 영

$w_2 = v_2 - \mathrm{proj}_{v_1} v_2$, $v_2 = \mathrm{proj}_{w_1} v_2 + \mathrm{proj}_{w_1^\perp} v_2$

$w_2 \cdot w_1 = (\mathrm{proj}_{w_1^\perp} v_2) \cdot w_1 = 0$

$w_2 \neq 0 \Rightarrow w_2 = 0 = v_2 - \mathrm{proj}_{w_1} v_2 \Rightarrow v_2 = \mathrm{proj}_{w_1} v_2$

$\Rightarrow v_2 = \mathrm{proj}_{v_1} v_2 = a v_1$ ($v_1, v_2 \Rightarrow$ dependent)

잡 직 영

$w_3 = v_3 - \mathrm{proj}_{w_2} v_3 - \mathrm{proj}_{w_1} v_3$, $v_3 = \mathrm{proj}_{w_1} v_3 + \mathrm{proj}_{w_1^\perp} v_3$

$= \mathrm{proj}_{w_2} v_3 + \mathrm{proj}_{w_2^\perp} v_3$

$w_3 \cdot w_1 = (\mathrm{proj}_{w_1^\perp} v_3 - \mathrm{proj}_{w_2} v_3) \cdot w_1 = 0$

$w_3 \cdot w_2 = (\mathrm{proj}_{w_2^\perp} v_3 - \mathrm{proj}_{w_1} v_3) \cdot w_2 = 0$

$$w_i = v_i - \sum_{j=1}^{i-1} \mathrm{proj}_{w_j} v_i$$

$w_3 \neq 0 \Rightarrow w_3 = 0 = v_3 - \mathrm{proj}_{w_2} v_3 - \mathrm{proj}_{w_1} v_3$

$\Rightarrow v_3 = \mathrm{proj}_{w_2} v_3 + \mathrm{proj}_{v_1} v_3 = \mathrm{proj}_{w_2} v_3 + a v_1$

$= (v_2 - \mathrm{proj}_{v_1} v_2) \cdot b + a v_1$

$= (\ ) v_1 + (\ ) v_2$

w_4

w_5 $\quad$ $w_1 \sim w_k$ 수직, $\neq 0$

$\vdots$

w_{i-1} , $w_1 \sim w_{i-1}$ 직교, $\neq 0$

$w_i = v_i - \mathrm{proj}_{w_{i-1}} v_i - \cdots - \mathrm{proj}_{w_1} v_i$

$v_i = \mathrm{proj}_{w_1} v_i + \mathrm{proj}_{w_1^\perp} v_i$

$= \mathrm{proj}_{w_2} v_i + \mathrm{proj}_{w_2^\perp} v_i$

$= \vdots$

$= \mathrm{proj}_{w_{i-1}} v_i + \mathrm{proj}_{w_{i-1}^\perp} v_i$

$w_i \cdot w_1 = (\mathrm{proj}_{w_1^\perp} v_i - \mathrm{proj}_{w_{i-1}} v_i - \cdots - \mathrm{proj}_{w_2} v_i) \cdot w_1 = 0$

$w_i \cdot w_2 = 0$

$\vdots$

$w_i \cdot w_{i-1} = 0$

$w_i \neq 0 \Rightarrow w_i = 0 = v_i - \mathrm{proj}_{w_{i-1}} v_i - \cdots - \mathrm{proj}_{w_1} v_i$

$v_i = \mathrm{proj}_{w_{i-1}} v_i + \cdots + \underbrace{\mathrm{proj}_{v_1} v_i}_{a v_1} = v_1 \sim v_{i-1}$ 선형결합

$\mathrm{proj}_{w_2} v_i = a v_1 + b v_2$

$\Rightarrow B' = \{w_1, \cdots, w_k\}$ $\quad w_i \neq 0$, $\quad w_i \cdot w_j = 0$

→ 기저 ① 선형독립

② span$\{B'\} = V$ $\quad$ $\dim(V) = k$.

① $c_1 w_1 + c_2 w_2 + \cdots + c_k w_k = 0$, $c_1 \sim c_k = 0$

$w_i \cdot (c_1 w_1 + c_2 w_2 + \cdots + c_k w_k) = 0$

$c_i w_i \cdot w_i = 0 \Rightarrow c_i \|w_i\|^2 = 0$

$c_1 w_1 \cdot w_1 = 0$

$c_2 w_2 \cdot w_2 = 0$