

제 48 강 기저변환과 대각화

Thm. $E = \{e_1, e_2, \dots, e_n\}$, $B = \{v_1, v_2, \dots, v_n\}$

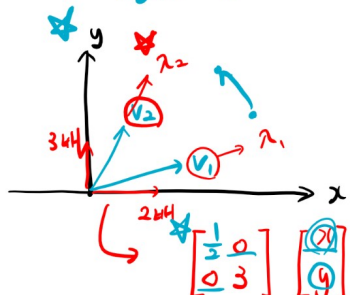
$$\vec{a} = (a_1, a_2, \dots, a_n) \times = (\lambda_1, \lambda_2, \dots, \lambda_n)_B$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = [v_1 \ v_2 \ \dots \ v_n] \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad (E) \rightarrow B$$

$$\star [e_1 \ e_2 \ \dots \ e_n] = I \quad \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = [v_1 \ \dots \ v_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$\star B \rightarrow B' \quad B' = \{v'_1, v'_2, \dots, v'_n\}$$

$$\star E \rightarrow B' \quad \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = [v'_1 \ v'_2 \ \dots \ v'_n]^{-1} [v_1 \ v_2 \ \dots \ v_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$



$\star A$: 회전변환, 대칭변환, 확대변환...
 허근 \times
 k 개 중근 $\rightarrow k$ 개의 고유벡터 \times
 유사 축...
 $\star D^n \quad D^{-1} \rightarrow$ 삼각

$$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 x \\ 3/4 y \end{bmatrix} \Rightarrow \text{축을 기준으로 상수배하는 변환은 대각행렬이다.}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A, \quad \lambda_1 = \frac{5 + \sqrt{33}}{2}, \quad v_1 = \left(\frac{-3 + \sqrt{33}}{6}, 1 \right)$$

$$\lambda_2 = \frac{5 - \sqrt{33}}{2}, \quad v_2 = \left(\frac{-3 - \sqrt{33}}{6}, 1 \right)$$

$$\star B = \{v_1, v_2\} \quad \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = A' \quad \star (1, 1)_B = v_1 + v_2 \quad \uparrow (\lambda_1, \lambda_2)_B$$

$$\text{기저변환 } E \rightarrow B \quad (a_1, a_2)_E = (b_1, b_2)_B \quad A' [1] = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = [v_1 \ v_2]^{-1} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \star A' \times_B : \mathbb{R}^2_B \rightarrow \mathbb{R}^2_B$$

$$\star B \rightarrow E \quad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = [v_1 \ v_2] \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad [v_1 \ v_2] (A' [v_1 \ v_2]^{-1}) \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\boxed{[v_1 \ v_2] A' [v_1 \ v_2]^{-1} = A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}$$

$$\begin{bmatrix} \frac{-3+\sqrt{33}}{6} & \frac{-3-\sqrt{33}}{6} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{5+\sqrt{33}}{2} & 0 \\ 0 & \frac{5-\sqrt{33}}{2} \end{bmatrix} \begin{bmatrix} \frac{-3+\sqrt{33}}{6} & \frac{-3-\sqrt{33}}{6} \\ 1 & 1 \end{bmatrix}^{-1}$$

$B \quad D \quad B^{-1}$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

\Rightarrow 대각화 \Rightarrow 다루기 쉽다.

\Rightarrow 고유값

\hookrightarrow 2차원 \rightarrow 2개



$$\boxed{A = B D B^{-1}} \text{ 대각화.}$$