

65강 정사영 정리 (2)

Thm $(V^\perp)^\perp = V$ (단, V 는 벡터공간)

그냥 $S^\perp \Rightarrow$ 공간

pf)

$$i) (V^\perp)^\perp \subseteq V$$

$$\forall a \in (V^\perp)^\perp \Rightarrow a \in V, a = a_1 + a_2$$

$$\text{단, } a_1 \in V, a_2 \in V^\perp, \text{ 유일. } \hookrightarrow a \in V$$

$$\Rightarrow a_2 = 0 \text{ 임을 보자. } a \cdot a_2 = 0$$

$$= (a_1 + a_2) \cdot a_2 = a_1 \cdot a_2 + a_2 \cdot a_2$$

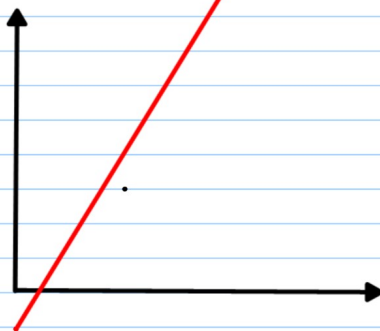
$$= a_2 \cdot a_2 = 0 \Rightarrow a_2 = 0$$

$$ii) V \subseteq (V^\perp)^\perp$$

$$\forall a \in V \Rightarrow a \in (V^\perp)^\perp$$

$$\forall x \in V^\perp, a \cdot x = 0 \text{ 임을 보자.}$$

$$\hookrightarrow x \in V^\perp \quad \forall y \in V, y \cdot x = 0$$



* 정사영 정리 공식 유일

$$X = X_1 + X_2, X_1 \in V, X_2 \in V^\perp$$

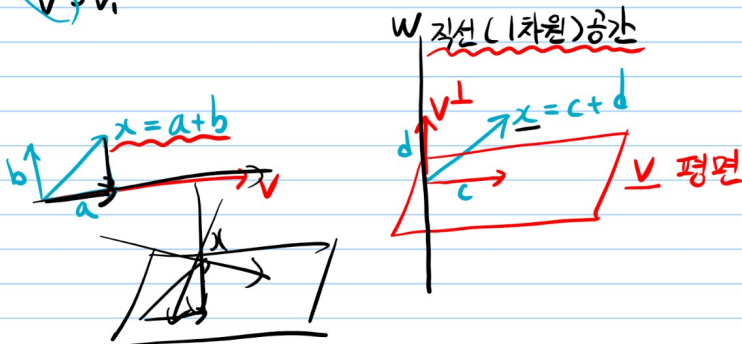
$$X_1 = T a$$

$$X_2 = X - T a$$

$$a = (T^T T)^{-1} T^T X$$

(단, T 는 벡터공간 V 의 기저를 열벡터로 하는 행렬)

$$X_1 = T(T^T T)^{-1} T^T X = \text{Proj}_V X$$



$$\text{Proj}_V X = X_1 = T(T^T T)^{-1} T^T X$$

$$\text{Proj}_{v_1} X = \frac{X \cdot v_1}{\|v_1\|^2} v_1$$

* $V: \text{ 벡터공간 } , x \in \mathbb{R}^n , V \subseteq \mathbb{R}^n$

$$x = x_1 + x_2, \quad x_1 \in V, \quad x_2 \in V^\perp$$

$$Vx = x_3 + x_4, \quad x_3 \in V^\perp, \quad x_4 \in (V^\perp)^\perp = V$$

$$\Rightarrow x_1 = x_4, \quad x_2 = x_3$$

$$x = x_1 + x_2 = \text{proj}_V x + x_2 \quad x_2 = x - \text{proj}_V x$$

$$x = x_3 + x_4 = \text{proj}_{V^\perp} x + x_4$$

$$x = \text{proj}_V x + \text{proj}_{V^\perp} x$$