

정수론 10강 일차 합동식

$$ax = c \quad x = \frac{c}{a}$$

$ax \equiv c \pmod{m} \Rightarrow x \equiv \frac{c}{a} \pmod{m}$
정수환에서는 곱셈에 대한 역원이 항상 존재하지는 않는다.

$$AX = C \quad X = A^{-1}C$$

↪ 가역행렬

⇒ $ax \equiv c \pmod{m}$ 를 만족하는 x 를 찾자.

⇔ $m \mid (ax - c)$ 를 만족하는 x 를 찾자.

⇔ $ax - c = k \cdot m$ 가 되게 하는 k 와 x 를 찾자.

$$ax - mk = c$$

$$ax + m \cdot (-k) = c$$

i) $g \nmid c \Rightarrow$ 해가 없다.

$$\gcd(a, m) = g$$

$$ax + by = c \Rightarrow (x_0 + k \cdot \frac{b}{g}, y_0 + k \cdot \frac{a}{g})$$

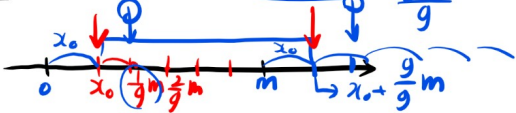
ii) $g \mid c \Rightarrow x_0, -k_0$
 $(x_0 + n \cdot \frac{m}{g}, -k_0 - n \cdot \frac{a}{g})$

$$x = x_0 + n \cdot \frac{m}{g}$$

$$ax \equiv c \pmod{m} \quad \text{합동식} \quad \text{mod } m \quad 0 \sim m-1$$

$$x = x_0 + n \cdot \frac{m}{g} \quad n = 0, 1, \dots, g-1$$

$$x \equiv \square \pmod{m}$$



$$x \equiv x_0 + n \cdot \frac{m}{g} \pmod{m}$$

$0 \sim g-1 \quad g > 1$

$$x_0 \equiv x_0 + \frac{g}{g}m \pmod{m}$$

$$ax \equiv c \pmod{m} \quad \gcd(a, m) \mid c$$

$$x \equiv x_0 + k \cdot \frac{m}{g} \pmod{m} \quad k = 0 \sim g-1$$

확장된 유클리드 알고리즘

ex) $7x \equiv 4 \pmod{14}$
 $\gcd(7, 14) = 7 \quad 7 \nmid 4 \Rightarrow$ 해가 없다.

ex) $12x \equiv 8 \pmod{4}$
 $\gcd(12, 4) = 4 \quad 4 \mid 8$

$$4 \mid 12x - 8 \Rightarrow 12x - 8 = 4k$$

$$12x - 4k = 8$$

$$12x + 4 \cdot (-k) = 8$$

-3x

x	-k	12x - 4k
1	0	12
0	1	4
1	-3	0

$$x = 0, k = -2$$

$$(x = 0, k = -1) \times 2$$

$$12x - 4k = 4$$

$$12x - 4k = 8$$

$$x_0 = 0$$

$$12x \equiv 8 \pmod{4}$$

$$x \equiv 0 + k \cdot \frac{4}{4} \pmod{4}$$

$$\equiv k \pmod{4} \quad k = 0 \sim g-1$$

$$\equiv 0, 1, 2, 3 \pmod{4} = 0 \sim 3$$

$$12 \cdot 0 = 0 \pmod{4}$$

$$0 \equiv 0$$

$$\underline{12} = \underline{8} \pmod{4} \quad (12 \cdot 2)$$