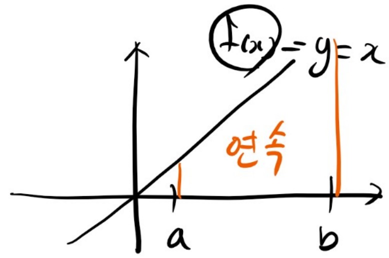
89강 실수 내적공간 (real inner product space)

* 내적 일반화 ⇒ 벡터가 n-tuple이 아닐수도 있음.
  ↳ 함수
* $u \cdot v$ $\quad \langle u, v \rangle$
* 실수 내적 (스칼라가 실수 범위)

① $\langle u, v \rangle = \langle v, u \rangle$
② $\langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$
③ $\langle ku, v \rangle = k \langle u, v \rangle$
④ $\langle v, v \rangle \geq 0$, $\langle v, v \rangle = 0 \iff \vec{v} = \vec{0}$
↳ 실수 내적공간



* V = { 구간 [a, b]에서 연속인 함수 }
  $f(x) \in V$

① $\forall f, g \in V$ $\quad f(x) + g(x) = h(x)$ [a, b]에서 연속.
↳ $h \in V$

② $f + g = g + f$
③ $f + (g + h) = (f + g) + h$
④ $f + 0 = f$
↳ $y = 0$

⋮

* V에서 내적을 정의내리자. (실수 내적)

$\langle f, g \rangle := \int_a^b f(x) g(x)\, dx$ $\quad \langle \vec{0}, g \rangle = \int_a^b 0 \cdot g(x)\, dx = 0$

① $\langle f, g \rangle = \int_a^b f(x) g(x)\, dx = \int_a^b g(x) f(x)\, dx = \langle g, f \rangle$

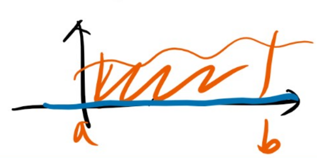
② $\langle f+g, h \rangle = \int_a^b (f(x) + g(x)) h(x)\, dx = \int_a^b (f(x)h(x) + g(x)h(x))\, dx$
$= \int_a^b f(x)h(x)\, dx + \int_a^b g(x)h(x)\, dx$
$= \langle f, h \rangle + \langle g, h \rangle$

③ $\langle kf, g \rangle = \int_a^b k f(x) g(x)\, dx$
$= k \int_a^b f(x) g(x)\, dx$
$= k \langle f, g \rangle$

④ $\langle f, f \rangle \geq 0 \rightarrow \boxed{\int_a^b f(x) f(x)\, dx}$

$\forall \alpha \in [a, b]$, $f(\alpha) \cdot f(\alpha) \geq 0$

$\langle f, f \rangle = 0 \longleftrightarrow f(x) = 0$
↳ $\vec{0}$



$\vec{0} \cdot \vec{v} = 0$

$\vec{0} = (0, 0, \cdots, 0)$
$\vec{v} = (v_1, v_2, \cdots, v_n) \quad = 0$

$\langle \vec{0}, \vec{v} \rangle = 0$

pf) 일반적으로 증명 ⇒ 공리만을 사용.
내적 ①~④ ①~⑨ (23강)

$\langle \vec{0}, \vec{v} \rangle = \langle \vec{0}+\vec{0}, \vec{v} \rangle = \langle \vec{0}, \vec{v} \rangle + \langle \vec{0}, \vec{v} \rangle$

$0 = \langle \vec{0}, \vec{v} \rangle$

✱ $\langle u-v, w \rangle = \langle u, w \rangle - \langle v, w \rangle$

pf) $\langle u+(-v), w \rangle = \langle u, w \rangle + \langle -v, w \rangle$

$= \langle u, w \rangle - \langle v, w \rangle$

✱ $\langle u, v-w \rangle = \langle u, v \rangle - \langle u, w \rangle$

pf) $\langle u, v-w \rangle = \langle v-w, u \rangle = \langle v, u \rangle - \langle w, u \rangle$

$= \langle u, v \rangle - \langle u, w \rangle$

✱ $k\langle u, v \rangle = \langle u, kv \rangle$

pf) $k\langle u, v \rangle = k\langle v, u \rangle = \langle kv, u \rangle = \langle u, kv \rangle$

$k(u \cdot v) = (ku) \cdot v = u \cdot (kv)$ $\quad a ✱ b ✱ c = 0+1+2 = ?$