

74 장

(Algebraic multiplicity and geometric multiplicity (2))

Thm

고유값 λ 의 대수적 중복도를 a

기하적 중복도를 g 라고 하면

항상 $a \geq 9$ 이다.

$$(A - \lambda I)x = 0$$

pf) $g = \dim(S)$ $S = \{x \in \mathbb{R}^m \mid Ax = \lambda x\}$

$$B = \{ \underline{\lambda_1}, \underline{\lambda_2}, \dots, \underline{\lambda_q} \}$$

$$g \leq m$$

$$S \subseteq \mathbb{R}^m$$

$$\hookrightarrow E = \{x_1, x_2, \dots, x_n\}$$

$$T = [x_1 \ x_2 \ \dots \ x_g \ | \ y_1 \ \dots \ y_{m-g}] = [X]$$

$$T = [x_1 \ x_2 \ \dots \ x_g \ y_1 \ \dots \ y_{m-g}] = [X \ Y]$$

$$AT = \begin{bmatrix} A & 0 \end{bmatrix} [x_1 \ x_2 \ \dots \ x_g \ y_1 \ \dots \ y_{m-g}]_{1 \times m}$$

$$= [A x_1, A x_2, \dots, A x_g \mid \cancel{A y_1}, \cancel{A y_2}, \dots, \cancel{A y_{m-g}}]$$

$$= [\lambda x_1 \quad \lambda x_2 \quad \dots \quad \lambda x_m \quad \cancel{A y_1} \quad \cancel{A y_2} \quad \dots \quad \cancel{A y_{m-g}}]$$

$$= \begin{bmatrix} \lambda x_m \end{bmatrix} \textcircled{1} \textcircled{2} = \begin{bmatrix} x_{m \times g} \end{bmatrix} \begin{bmatrix} \lambda I_{1 \times 2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \textcircled{3} \textcircled{4}$$

Diagram illustrating the dimensions of the matrices in the block-matrix multiplication:

$g \times g$	$g \times (m-g)$
$(m-g) \times g$	$(m-g) \times (m-g)$

The overall dimensions are $m \times m$.

$$[x_1, \dots, x_g] = X, [y_1, \dots, y_{m-g}] = Y$$

$$X P + Y R = \lambda X, \quad X Q + Y S = \lambda Y$$

$$\begin{bmatrix} x_1 & x_2 & \dots & x_g \end{bmatrix} \begin{bmatrix} ? \\ \vdots \\ ? \end{bmatrix} + \begin{bmatrix} y_1 & y_2 & \dots & y_{m-g} \end{bmatrix} \mathbf{0}$$

$\hookrightarrow \lambda \mathbf{I}$ $\times: \lambda \mathbf{I} = \lambda$

$$AT = T \cdot \lambda I$$
$$= \lambda X \quad A = T \cdot \lambda I \cdot T^{-1}$$

$$[x_1, x_2, \dots, x_g]Q + [y_1, \dots, y_{m-g}]S = A[y_1, \dots, y_{m-g}]$$

$$\hookrightarrow [q_1, q_2, \dots, q_{m-g}]$$

$[s_1 s_2 \dots s_{m-g}]$

$$f(x) = Ax$$

$$\begin{aligned} \underline{XQ_1} + \underline{YS_1} &= Ay_1 = \underline{f(y_1)} \\ \underline{XQ_2} + \underline{YS_2} &= Ay_2 = \underline{f(y_2)} \end{aligned}$$

$$XQ_2 + YS_2 = Ay_2 = f(y_2)$$

$$xQ_{m-g} + rS_{m-g} = Ay_{m-g} = \underline{f(y_{m-g})}$$

$$A_T = T \begin{bmatrix} \lambda I & Q \\ 0 & S \end{bmatrix}$$

$$= \underline{[X \ Y]} \begin{bmatrix} \lambda I & Q \\ 0 & S \end{bmatrix}$$

$$A = T \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} T^{-1}$$

1 det

fctr

③ 고유값 \rightarrow 특성방정식

$$\det(cI - A) = \det(cI - \begin{bmatrix} \lambda & 0 \\ 0 & s \end{bmatrix}) = \det(cI - \lambda I)$$

$$= \det \begin{pmatrix} (C-\lambda)I & -Q \\ 0 & CI-S \end{pmatrix} = \underline{(C-\lambda)^n}$$

$$= \det((c-\lambda)I) \det(cI-S)$$

$$= (c-\lambda)^9 \det(CI - S)$$

$a \geq 9$.