86강 특잇값 분해를 이용한 손실 이미지 압축 시뮬레이션 (SVD)

*고유값 분해 : 정사각행렬, 대칭행렬

A : $m \times n$

$$\underset{m\times n}{A} = \underset{m\times m}{U}\ \underset{m\times n}{\Sigma}\ \underset{n\times n}{V^T}$$

basis 변환

$$A^TA = VDV^T$$

* 축소된 특잇값 분해

$k = \text{rank}(A) = \text{rank}(A^TA) = \text{rank}(D)$

↳ $\lambda_1 \sim \lambda_k$ $\quad \sqrt{\lambda_1} \sim \sqrt{\lambda_k}$ $\quad v_1 \sim v_k$: 정규직교화된 상태.

$U = [u_1\ u_2 \cdots u_k\ \underbrace{u_{k+1} \cdots u_m}_{\text{0벡터}}]$ $\quad u_i = \frac{1}{\sqrt{\lambda_i}} A v_i$ $\quad i > k \Rightarrow$ 0벡터

$$A = U\Sigma V^T = \underset{m\times m}{\begin{bmatrix} u_1 u_2 \cdots u_k\ u_{k+1} \cdots u_m \end{bmatrix}} \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \ddots & & \\ & & \sqrt{\lambda_k} & \\ & & & 0 \cdots 0 \end{bmatrix} \underset{n\times n}{\begin{bmatrix} v_1 v_2 \cdots v_k\ v_{k+1} \cdots v_n \end{bmatrix}^T}$$

$$= \underset{m\times k}{\underbrace{\begin{bmatrix} u_1 \cdots u_k \end{bmatrix}}} \underset{k\times k}{\underbrace{\begin{bmatrix} \sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_k} \end{bmatrix}}} \underset{k\times n}{\underbrace{\begin{bmatrix} v_1 \cdots v_k \end{bmatrix}^T}}$$

$$= \left[\begin{array}{c|c} U' & u_{k+1} \cdots u_m \end{array}\right] \left[\begin{array}{c|c} \Sigma' & 0 \\ \hline 0 & 0 \end{array}\right] \left[\begin{array}{c} V'^T \\ \hline v_n \end{array}\right]$$

$$= \left[\begin{array}{c|c} U'\Sigma' & 0 \end{array}\right] \left[\begin{array}{c} V'^T \\ \hline ? \end{array}\right] = \left[U'\Sigma'V'^T\right]$$

$$= \sqrt{\lambda_1} u_1 v_1^T + \sqrt{\lambda_2} u_2 v_2^T + \cdots + \sqrt{\lambda_k} u_k v_k^T$$

$$A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_k u_k v_k^T$$