85강 특잇값 분해 일반화

$A : n \times n$ (symmetric 일 필요는 없다.)

$m \times n$

$Ax = U \Sigma V^T x$ 단, U, V는 직교 행렬

Σ는 대각 행렬

↳ 각 축에 상수배하는 변환

$A : m \times n$ $\qquad \underset{m\times n}{A} = \underset{m\times m}{U} \; \underset{m\times n}{\Sigma} \; \underset{n\times n}{V^T}$

① V, $\quad \underset{n\times m \;\; m\times n}{A^TA} = VDV^T$: n차 정사각

→ symmetric $\quad (A^TA)^T = A^TA$

② 특잇값. $\quad A^TA$의 고유값들의 양의 제곱근.

$n \times n$ ↳ 실수

↳ 고유값이 모두 양수 ⇒ $\sqrt{\lambda_1} \; \sqrt{\lambda_2} \; \sqrt{\lambda_3} \cdots \sqrt{\lambda_k}$

$$\Sigma = \begin{array}{c} k \\ \\ m-k \end{array} \left[\begin{array}{cccc|c} \sqrt{\lambda_1} & & 0 & & \\ & \sqrt{\lambda_2} & & & O_{k\times(n-k)} \\ 0 & & \ddots & & \\ & & & \sqrt{\lambda_k} & \\ \hline & O_{(m-k)\times k} & & & O_{(m-k)\times(n-k)} \end{array}\right]$$

$\qquad\qquad k \qquad\qquad n-k$

③ $U\Sigma = AV$ $\qquad V^{-1} = V^T \qquad V = [v_1 \; v_2 \cdots v_n]$

$$\underset{1\times 2}{\left[\underset{m\times k}{\alpha} \,\middle|\, \underset{m\times(m-k)}{\beta}\right]} \underset{2\times 2}{\left[\begin{array}{c|c} \underset{k\times k}{\sqrt{\lambda_i}} & 0 \\ \hline 0 & 0 \end{array}\right]} = [\,Av_1 \; Av_2 \; \cdots \; Av_k \mid 0 \; 0 \; 0 \cdots 0\,] \;\; 1\times 2$$

↳ $m \times m$

$k = \text{rank}(A) = \text{rank}(A^TA) = \text{rank}(D)$

$\|Av_{k+1}\| = \sqrt{\lambda_{k+1}} = 0$

$\alpha = [u_1 \cdots u_k]$ $\qquad u_i = \dfrac{1}{\sqrt{\lambda_i}} Av_i$

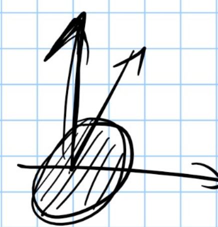
$\beta = [u_{k+1} \cdots u_n]$ → 71강 null space

$$\left[\; m \times k \;\middle|\; m \times (m-k) \;\right] \left[\begin{array}{c|c} k \times k & k \times (n-k) \\ \hline (m-k) \times k & (m-k) \times (n-k) \end{array}\right]$$

$$= \left[\begin{array}{c|c} m\times k & m\times (n-k) \end{array}\right]_{m\times n}$$

$A : m\times n$

$U_{m\times m} = [\, u_1 \cdots u_k \mid u_{k+1} \cdots u_n \,]$



m차원 $\Sigma_{m\times n} = \left[\begin{array}{cc|c} \sqrt{\lambda_1} & 0 & \\ 0 & \ddots\ \sqrt{\lambda_k} & 0 \\ \hline 0 & & 0 \end{array}\right] \begin{bmatrix} y_1 \\ \vdots \\ y_n \\ 0 \\ 0 \end{bmatrix}$

$V_{n\times n}$ ⇒ m개

$A^TA = VDV^T$

차원 확장

$$\underbrace{A}_{m\times n}x = \underbrace{U}_{m차}\underbrace{\Sigma}_{m\times n}\underbrace{V^T}_{n\times n}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$