

## 76강 보충강의

Thm 가역행렬  $B$ 에 대해,  $\text{rank}(AB) = \text{rank}(A)$   
 $\hookrightarrow n \times n$  행렬  $\hookrightarrow m \times n$

Thm. 행렬  $A, B$ 가 서로 곱해질 수 있는 크기이지만 하면.  
 ( $AB$ 를 할 수 있으면)

$$\text{rank}(AB) \leq \text{rank}(A)$$

pf) rank? col의 차원.  $\text{col}(AB) = \{ABx \mid x \in \mathbb{R}^k\}$

$$\text{col}(A) = \{Ax \mid x \in \mathbb{R}^m\}$$

$$A = [a_1 \ a_2 \ \dots \ a_m] \quad \text{col}(A) = \text{span} \{a_1, a_2, \dots, a_m\}$$

$$= \{c_1 a_1 + c_2 a_2 + \dots + c_m a_m \mid (c_1, c_2, \dots, c_m) \in \mathbb{R}^m\}$$

$$= \{Ax \mid x \in \mathbb{R}^m\}$$

$$\text{col}(AB) \subseteq \text{col}(A)$$

$$\forall y \in \text{col}(AB) \Rightarrow y = ABx, \exists x \in \mathbb{R}^k$$

$$\begin{matrix} (m \times k) & (k \times 1) & \Rightarrow & m \times 1 \\ z = Bx \in \mathbb{R}^m & \leftarrow & \end{matrix} \quad \left. \vphantom{\begin{matrix} y = ABx \\ z = Bx \end{matrix}} \right\} y = Az \Rightarrow y \in \text{col}(A)$$

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pf)  $\text{rank}(A) \geq \text{rank}(AB) \geq \text{rank}(AB B^{-1}) = \text{rank}(A)$

$$\text{rank}(A) = \text{rank}(AB).$$