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[illegible] [1] [illegible] [2], [3] [illegible] a_0 [illegible] a_0 [illegible] [4] [illegible] [5] [illegible] [6] [illegible] [1].

[illegible] [7] [illegible] [8] [illegible] [9] [illegible] [10] [illegible]

[illegible] $f(R)$ [illegible] [11] [illegible] [12] [illegible] [13], [14].

2. [illegible]

TeVeS contains extra gravitational degrees of freedom, which are carried by a vector field A_μ and a scalar field ϕ. We emphasize that TeVeS involves two frames: the Bekenstein frame for the gravitational fields and the matter frame for the matter fields. The Bekenstein frame has the metric $\tilde{g}_{\mu\nu}$ with the connection $\tilde{\nabla}_\mu$. The action for all matter fields is written using a physical metric $g_{\mu\nu}$ with the connection ∇_μ, which is related to the three gravitational fields $\tilde{g}_{\mu\nu}$, A_μ and ϕ as

$$g_{\mu\nu} = e^{-2\phi}\tilde{g}_{\mu\nu} - 2\sinh(2\phi)A_\mu A_\nu. \tag{1}$$

The fact that all matter fields couple to the physical metric means that the Einstein equivalence principle is obeyed. The vector field is normalized to be timelike and normalized with respect to the Bekenstein metric,

$$A_\mu A^\mu = -1, \tag{2}$$

where the contravariant index is raised with the Bekenstein metric, $A^\mu = \tilde{g}^{\mu\nu}A_\nu$. The inverse of the physical metric is obtained as

$$g^{\mu\nu} = e^{2\phi}\tilde{g}^{\mu\nu} + \frac{2\sinh(2\phi)e^{2\phi}}{e^{2\phi} - 2\sinh(2\phi)(A_\mu A^\mu + 1)}A^\mu A^\nu. \tag{3}$$

The action of the theory,

$$S = S_{\tilde{g}} + S_A + S_\phi + S_m, \tag{4}$$

consists of the actions for the metric $\tilde{g}_{\mu\nu}$, the vector field A_μ, the scalar field ϕ and matter, respectively. The action for $\tilde{g}_{\mu\nu}$ is defined as the standard Einstein–Hilbert action,

$$S_{\tilde{g}} = \frac{1}{16\pi G}\int_{\mathscr{M}} d^4x\sqrt{-\tilde{g}}\tilde{R} + \frac{1}{8\pi G}\oint_{\partial\mathscr{M}} d^3x\sqrt{|\gamma|}\tilde{K}, \tag{5}$$

where G is the bare gravitational constant, $\tilde{g} = \det\,\tilde{g}_{\mu\nu}$, and $\tilde{R}$ is the scalar curvature defined by the connection $\tilde{\nabla}$ which is compatible with the metric $\tilde{g}_{\mu\nu}$. The second integral over the boundary $\partial\mathscr{M}$ of the manifold $\mathscr{M}$ is included so that only the variation of the metric $\delta\tilde{g}_{\mu\nu}$ (and not its derivatives) needs to be imposed to vanish on the boundary, when obtaining the Einstein field equations for $\tilde{g}_{\mu\nu}$. In the second term, γ is the determinant of the induced metric on $\partial\mathscr{M}$ and $\tilde{K}$ is the trace of the extrinsic curvature of $\partial\mathscr{M}$.

The action for the vector field A_μ is given by

$$S_A = -\frac{1}{32\pi G}\int_{\mathscr{M}} d^4x\sqrt{-\tilde{g}}\left[\kappa F_{\mu\nu}F^{\mu\nu} - 2\lambda\left(A_\mu A^\mu + 1\right)\right], \tag{6}$$

where $F_{\mu\nu} = \tilde{\nabla}_\mu A_\nu - \tilde{\nabla}_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$, κ is a dimensionless constant and λ is a Lagrange multiplier ensuring (2). The original action (6) has since been extended with three extra terms which are quadratic in $\tilde{\nabla}_\mu A_\nu$, see [15], in order to cure certain dynamical instabilities, see e.g. [11]. Here we consider the original TeVeS for simplicity. A detailed Hamiltonian analysis of the extended TeVeS model will be presented in a future communication.

The action for the scalar field ϕ is given by

$$S_\phi = -\frac{1}{16\pi G}\int_{\mathscr{M}} d^4x\sqrt{-\tilde{g}}\left[\mu\hat{g}^{\mu\nu}\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\phi + V(\mu)\right], \tag{7}$$

where μ is a non-dynamical dimensionless scalar field and $\hat{g}$ is a new metric defined as

$$\hat{g}^{\mu\nu} = \tilde{g}^{\mu\nu} - A^{\mu}A^{\nu}. \tag{8}$$

The potential term $V(\mu)$ is an arbitrary function that typically depends on a scale. The metric $\hat{g}^{\mu\nu}$ is used in the scalar field action, rather than $\tilde{g}^{\mu\nu}$, in order to avoid superluminal propagation of perturbations. For the same reason, we assume that $\phi > 0$ [1].

All matter fields, denoted collectively by χ^A, are coupled to the physical metric $g_{\mu\nu}$ so that their action has the form

$$S_m = \int_{\mathscr{M}} d^4x\sqrt{-g}\mathscr{L}\left[g, \chi^A, \nabla\chi^A\right]. \tag{9}$$

For simplicity, we will consider a single matter field χ with the action

$$S_m = -\int_{\mathscr{M}} d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \mathscr{V}(\chi)\right]. \tag{10}$$

The determinant of the physical metric g is related to the determinant of the Bekenstein metric $\tilde{g}$ and the fields ϕ and A_μ as

$$g = e^{-4\phi}\left[1 - \left(1 - e^{-4\phi}\right)\left(A_\mu A^\mu + 1\right)\right]\tilde{g}. \tag{11}$$

3. Hamiltonian structure of TeVeS

The spacetime is assumed to admit a foliation into a family of nonintersecting spacelike hypersurfaces Σ_t, which are parameterized by the time t. The Bekenstein metric $\tilde{g}_{\mu\nu}$ induces a metric $h_{\mu\nu}$ on Σ_t, which is defined as

$$h_{\mu\nu} = \tilde{g}_{\mu\nu} + n_\mu n_\nu, \tag{12}$$

where n_μ is the future-directed unit normal to Σ_t. The unit normal can be written in terms of the lapse function N and the shift vector N^i $(i = 1, 2, 3)$ as

$$n_\mu = -\nabla_\mu t = (-N, 0, 0, 0), \qquad n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N}\right). \tag{13}$$

Consequently, the metric $\tilde{g}_{\mu\nu}$ is decomposed in terms of the ADM variables h_{ij}, N and N^i as

$$\tilde{g}_{00} = -N^2 + N^i h_{ij} N^j, \qquad \tilde{g}_{0i} = h_{ij}N^j, \qquad \tilde{g}_{ij} = h_{ij}. \tag{14}$$

The determinant of the metric $\tilde{g}_{\mu\nu}$ is written as

$$\sqrt{-\tilde{g}} = N\sqrt{h}, \quad h = \det\, h_{ij}. \tag{15}$$

The vector field A_μ is decomposed into components tangent and normal to Σ_t as

$${}_{\perp}A_\mu = h_\mu{}^\nu A_\nu, \qquad A_{\mathbf{n}} = n^\mu A_\mu, \tag{16}$$

respectively, where $h_\mu{}^\nu = h_{\mu\rho}\tilde{g}^{\rho\nu} = \delta_\mu{}^\nu + n_\mu n^\nu$ is the projection operator onto Σ_t. That is the components of [illegible] $A_0 = NA_{\mathbf{n}} + N^i A_i$ [illegible] $A_i =_\perp A_i$.

[illegible] h_{ij}, N, N^i, $A_{\mathbf{n}}$, A_i, λ, ϕ, μ [illegible] χ [illegible] π^{ij}, π_N, π_i, $p_{\mathbf{n}}$, p^i, p_λ, p_ϕ, p_μ [illegible] p_χ, [illegible] N, N^i, λ, $A_{\mathbf{n}}$ [illegible] μ, [illegible]

$$\pi_N \approx 0, \qquad \pi_i \approx 0, \qquad p_\lambda \approx 0, \qquad p_{\mathbf{n}} \approx 0, \qquad p_\mu \approx 0.$$

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[illegible]

$$H = \int_{\Sigma_t} d^3x \left(N\mathscr{H}_T + N^i \mathscr{H}_i + v_N \pi_N + v^i \pi_i + v_\lambda p_\lambda + v_{\mathbf{n}} p_{\mathbf{n}} + v_\mu p_\mu \right) + H_{\text{surf}}, \tag{18}$$

[illegible] $\mathscr{H}_T$ [illegible] $\mathscr{H}_i$ [illegible] H_{surf} [illegible]

[illegible]

$$\mathscr{H}_i = -2h_{ij} D_k \pi^{jk} - A_i \partial_j p^j + (\partial_i A_j - \partial_j A_i) p^j + \partial_i \phi p_\phi + \partial_i \chi p_\chi \approx 0, \tag{19}$$

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[illegible] D_k [illegible] h_{ij}. [illegible] Σ_t. [illegible]

$$\mathscr{H}_T = \mathscr{H}_T^{\text{GR}} + \mathscr{H}_T^{A} + \mathscr{H}_T^{\phi} + \mathscr{H}_T^{\chi} \approx 0, \tag{20}$$

[illegible]

$$\mathscr{H}_T^{\text{GR}} = \frac{16\pi G}{\sqrt{h}} \pi^{ij} \mathscr{G}_{ijkl} \pi^{kl} - \frac{\sqrt{h}}{16\pi G} {}^{(3)}R, \tag{21}$$

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[illegible]

$$\mathscr{G}_{ijkl} = \tfrac{1}{2}(h_{ik}h_{jl} + h_{il}h_{jk}) - \tfrac{1}{2} h_{ij} h_{kl} \tag{22}$$

[illegible] ${}^{(3)}R$ [illegible] D_i. [illegible]

$$\mathscr{H}_T^{A} = \frac{4\pi G}{\kappa\sqrt{h}} p^i h_{ij} p^j - A_{\mathbf{n}} D_i p^i + \frac{\kappa}{32\pi G} \sqrt{h} h^{ik} h^{jl} (D_i A_j - D_j A_i)(D_k A_l - D_l A_k) + \frac{\lambda}{16\pi G} \sqrt{h} \left(A_i A^i - A_{\mathbf{n}}^2 + 1 \right) \tag{23}$$

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[illegible]

$$\mathscr{H}_T^{\phi} = \frac{4\pi G}{\sqrt{h}\mu(1+A_{\mathbf{n}}^2)} p_\phi^2 + \frac{A_{\mathbf{n}}}{(1+A_{\mathbf{n}}^2)} p_\phi A^i \partial_i \phi - \frac{\mu\sqrt{h}}{16\pi G(1+A_{\mathbf{n}}^2)} \left(A^i \partial_i \phi \right)^2 + \frac{1}{16\pi G} \mu \sqrt{h} h^{ij} \partial_i \phi \partial_j \phi + V(\mu). \tag{24}$$

[illegible] (1). [illegible]

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$$\mathscr{H}_T^{\chi} = \frac{\sqrt{1-(1-e^{-4\phi})\mathscr{G}_\lambda}}{2\sqrt{h}(e^{-4\phi}-(1-e^{-4\phi})A_iA^i)}p_\chi^2 - \frac{(1-e^{-4\phi})A_{\mathbf{n}}}{e^{-4\phi}-(1-e^{-4\phi})A_iA^i}A^i\partial_i\chi p_\chi + \sqrt{h\left(1-\left(1-e^{-4\phi}\right)\mathscr{G}_\lambda\right)}$$
$$\times\left[\frac{1-e^{-4\phi}}{2(e^{-4\phi}-(1-e^{-4\phi})A_iA^i)}\left(A^i\partial_i\chi\right)^2 + \tfrac{1}{2}h^{ij}\partial_i\chi\partial_j\chi + e^{-2\phi}\mathscr{V}(\chi)\right]. \quad (25)$$

[illegible] (17) [illegible]

$$\mathscr{G}_\lambda = A_iA^i - A_{\mathbf{n}}^2 + 1 \approx 0, \quad (26)$$

$$\mathscr{G}_{\mathbf{n}} = D_ip^i + \frac{\lambda\sqrt{h}}{8\pi G}A_{\mathbf{n}} + \ldots \approx 0, \quad (27)$$

$$\mathscr{G}_\mu = \frac{4\pi G}{\sqrt{h}\mu^2(1+A_{\mathbf{n}}^2)}p_\phi^2 + \frac{\sqrt{h}}{16\pi G(1+A_{\mathbf{n}}^2)}\left(A^i\partial_ip_\phi\right)^2 - \sqrt{h}h^{ij}\partial_i\phi\partial_j\phi - \sqrt{h}\frac{\delta V(\mu)}{\delta\mu} \approx 0, \quad (28)$$

[illegible] (27) [illegible] (26) [illegible] (25).

[illegible] (1), [illegible] $g_{ij} = e^{-2\phi}h_{ij} - 2\sinh(2\phi)A_iA_j$, [illegible] ϕ [illegible] $\frac{1}{4}\ln(1+\left(A_iA^i\right)^{-1})$. [illegible]

$$\det(g_{ij}) = e^{-2\phi}\left(e^{-4\phi} - \left(1-e^{-4\phi}\right)A_iA^i\right)h. \quad (29)$$

[illegible] (25), [illegible] (29), $e^{-4\phi} - (1-e^{-4\phi})A_iA^i$. [illegible] $\phi = \frac{1}{4}\ln(1+\left(A_iA^i\right)^{-1})$ [illegible] p_χ^2 [illegible] ϕ [illegible] Σ_t [illegible] $\phi > 0$, [illegible]

$$0 < \phi < \tfrac{1}{4}\ln\left(1+\frac{1}{A_iA^i}\right). \quad (30)$$

[illegible] A_μ [illegible] $A_{\mathbf{n}}$, [illegible] A_i, $0 \le A_iA^i \ll 1$, [illegible] (30) [illegible] ϕ [illegible] $A_iA^i \gg 1$, [illegible] ϕ [illegible] $1/(4A_iA^i)$.

[illegible] $\pi_N, \pi_i, \mathscr{H}_T, \mathscr{H}_i$ [illegible] $p_\lambda, p_{\mathbf{n}}, p_\mu, \mathscr{G}_\lambda, \mathscr{G}_{\mathbf{n}}, \mathscr{G}_\mu$ [illegible] $\lambda, A_{\mathbf{n}}, \mu$ [illegible] h_{ij}, A_i, ϕ [illegible]

[illegible] H_{surf} [illegible] (18) [illegible] $H_{\mathrm{phys}} = H - H_{\mathrm{b}}$, [illegible] H_{b} [illegible] $t^\mu = Nn^\mu + N^\mu$ [illegible]

$$E = -\oint_{\partial\Sigma_t} d^2x\left(\frac{1}{8\pi G}N\sqrt{\sigma}\left({}^{(2)}K - {}^{(2)}K_{\mathrm{b}}\right) - 2N^ih_{ij}r_k\pi^{jk} - r_ip^i\left(NA_{\mathbf{n}} + N^jA_j\right)\right), \quad (31)$$

[illegible] σ, ${}^{(2)}K$ [illegible] r_i [illegible] Σ_t, [illegible] ${}^{(2)}K_{\mathrm{b}}$ [illegible] A_n [illegible] $\mathscr{G}_\lambda = 0$ [illegible] $A_n = \pm\sqrt{A_i A^i + 1}$. [illegible] (31) [illegible] Σ_t [illegible] $\tilde g_{\mu\nu}$ [illegible] $g_{\mu\nu}$) [illegible] $\oint_{\partial\Sigma_t} d^2 x r_i p^i A_0$. [illegible] [16].

[illegible] (31) [illegible] [17], [18]. [illegible] (31) [illegible] ($A^r = 0$) [illegible] [19] [illegible]

$$\tilde g_{tt} = -\left(\frac{r-r_c}{r+r_c}\right)^{r_g/2r_c}, \qquad \tilde g_{rr} = \frac{(r^2-r_c^2)^2}{r^4}\left(\frac{r-r_c}{r+r_c}\right)^{-r_g/2r_c}, \tag{32}$$

[illegible]

$$r_c = \frac{r_g}{4}\sqrt{1+\frac{k}{\pi}\left(\frac{Gm_s}{r_g}\right)^2 - \frac{\kappa}{2}}, \tag{33}$$

[illegible] m_s [illegible] r_g [illegible] [1], [illegible] (7). [illegible] (32) [illegible]

$$E_{\mathrm{ADM}} = -\frac{1}{2G}\lim_{r\to\infty} r^2\frac{\partial h_{rr}}{\partial r} = \frac{r_g}{2G}. \tag{34}$$

[illegible] r_g [illegible] (33) [illegible] $r_g = 2Gm$.

4. [illegible]

[illegible] [1]. [illegible] $\tilde g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$, $A^\mu = (1,0,0,0)$ [illegible] $\phi = \phi_c = \mathrm{constant}$, [illegible] $e^{-2\phi_c}$ [11]. [illegible] (6) [illegible] [11]). [illegible]

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