Уилкинсон Дж.Х., Райнш С. Справочник алгоритмов на языке Алгол. Линейная алгебра

М.: Машиностроение, 1976. — 390 с.

В книге приведены алгоритмы решения всех основных задач линейной алгебры, реализованные в виде процедур на языке Алгол-

60. Для специалистов по теории управления представляют интерес алгоритмы решения проблемы собственных значений для произвольных матриц

$\sin(3) = 0.141$

page 37, (53)

$$A53\_o := \begin{pmatrix} 360360 & 180180 & 120120 & 90090 & 72072 & 60060 & 51480 \\ 180180 & 120120 & 90090 & 72072 & 60060 & 51480 & 45045 \\ 120120 & 90090 & 72072 & 60060 & 51480 & 45045 & 40040 \\ 90090 & 72072 & 60060 & 51480 & 45045 & 40040 & 36036 \\ 72072 & 60060 & 51480 & 45045 & 40040 & 36036 & 32760 \\ 60060 & 51480 & 45045 & 40040 & 36036 & 32760 & 30030 \\ 51480 & 45045 & 40040 & 36036 & 32760 & 30030 & 27720 \end{pmatrix} \qquad Gilbert7 := A53\_o$$

WRITEPRN("c:\Sci\tmp\linAlg\Wilk

B := READPRN("c:\Sci\tı

$det\_A53\_o := 381614277072600 \qquad det\_A53\_o2 := 8.47353913 \cdot 10^{-2} \cdot 2^{52}$

$det\_A53\_o2 - det\_A53\_o = -3.888 \times 10^{5}$

$$det\_A53 := |A53\_o| \qquad \delta\_A53 := \frac{|det\_A53 - det\_A53\_o|}{det\_A53\_o} \qquad \delta\_A53 = 5.289 \times 10^{-10}$$

$$invA53\_t := \left(A53\_o^{-1}\right)$$

$$invA53\_t = \begin{pmatrix} 1.36 \times 10^{-4} & -3.263 \times 10^{-3} & 0.024 & -0.082 & 0.135 & -0.108 & 0.033 \\ -3.263 \times 10^{-3} & 0.104 & -0.881 & 3.133 & -5.385 & 4.431 & -1.4 \\ 0.024 & -0.881 & 7.93 & -29.371 & 51.923 & -43.615 & 14 \\ -0.082 & 3.133 & -29.371 & 111.888 & -201.923 & 172.308 & -56 \\ 0.135 & -5.385 & 51.923 & -201.923 & 370.192 & -319.846 & 105 \\ -0.108 & 4.431 & -43.615 & 172.308 & -319.846 & 279.138 & -92.4 \\ 0.033 & -1.4 & 14 & -56 & 105 & -92.4 & 30.8 \end{pmatrix}$$

$$
invA53\_o := \begin{pmatrix}
1.35971314283\cdot 10^{-4} & -3.26325113753\cdot 10^{-3} & 2.44740603295\cdot 10^{-2} & -8.15793917156\cdot 10^{-2} & 1 \\
-3.26325113738\cdot 10^{-3} & 1.04422850066\cdot 10^{-1} & -8.81060620311\cdot 10^{-1} & 3.13264074649\cdot 10^{0} & - \\
2.44740603601\cdot 10^{-2} & -8.81060620282\cdot 10^{-1} & 7.92950934930\cdot 10^{0} & -2.93684512777\cdot 10^{1} & 5 \\
-8.15793917875\cdot 10^{-2} & 3.13264074692\cdot 10^{0} & -2.93684512768\cdot 10^{1} & 1.11879649598\cdot 10^{2} & - \\
1.346049544738\cdot 10^{-1} & -5.38420043662\cdot 10^{0} & 5.19190848981\cdot 10^{1} & -2.01907567906\cdot 10^{2} & 3 \\
-1.07683295031\cdot 10^{-1} & 4.43041069139\cdot 10^{0} & -4.36119354458\cdot 10^{1} & 1.72294292795\cdot 10^{2} & - \\
3.33303738252\cdot 10^{-2} & -1.39988227242\cdot 10^{0} & 1.39988675007\cdot 10^{1} & -5.59956005459\cdot 10^{1} & 1
\end{pmatrix}
$$

$$
invA53\_o - invA53\_t = \begin{pmatrix}
-3.822\times 10^{-9} & 1.521\times 10^{-7} & -1.464\times 10^{-6} & 5.69\times 10^{-6} & -1.043\times 10^{-5} & 9.013\times 1 \\
1.521\times 10^{-7} & -6.054\times 10^{-6} & 5.826\times 10^{-5} & -2.264\times 10^{-4} & 4.15\times 10^{-4} & -3.585\times \\
-1.464\times 10^{-6} & 5.826\times 10^{-5} & -5.606\times 10^{-4} & 2.178\times 10^{-3} & -3.992\times 10^{-3} & 3.449\times 1 \\
5.69\times 10^{-6} & -2.264\times 10^{-4} & 2.178\times 10^{-3} & -8.462\times 10^{-3} & 0.016 & -0.01 \\
-1.043\times 10^{-5} & 4.15\times 10^{-4} & -3.992\times 10^{-3} & 0.016 & -0.028 & 0.02 \\
9.013\times 10^{-6} & -3.585\times 10^{-4} & 3.449\times 10^{-3} & -0.013 & 0.025 & -0.02 \\
-2.96\times 10^{-6} & 1.177\times 10^{-4} & -1.133\times 10^{-3} & 4.399\times 10^{-3} & -8.062\times 10^{-3} & 6.965\times 1
\end{pmatrix}
$$

normFrobenius(invA53_t) = 794.263   nFr(invA53_t) = 2.146

normFrobenius(invA53_o) = 794.202   nFr(invA53_o) = 2.146

distFrobenius(invA53_t, invA53_o) = 0.061   dFr(invA53_t, invA53_o) = 2.145

$$
\mathrm{minmax}(\mathrm{matVariation\_}(invA53\_t, invA53\_o)) = \begin{pmatrix} 1.032\times 10^{-11} & 7.678\times 10^{-5} \end{pmatrix}
$$

mkVec_e(size, index) :=

```
R[size-1] ← 0
R[index] ← 1
return R
```

page 49 $b\_p47\_o := mkVec\_e(7,0)$

$d\_p47\_t := lsolve(A53\_o, b\_p47\_o)$

$$d\_p47\_o := \begin{pmatrix} 1.35975135069 \cdot 10^{-4} \\ -3.26340327676 \cdot 10^{-3} \\ 2.44755247690 \cdot 10^{-2} \\ -8.1585082940 \cdot 10^{-2} \\ 1.34615387384 \cdot 10^{-1} \\ -1.07692310159 \cdot 10^{-1} \\ 3.33333341518 \cdot 10^{-2} \end{pmatrix}$$

b_p47_

$$d\_p47\_t^T = \begin{pmatrix} 1.36 \times 10^{-4} & -3.263 \times 10^{-3} & 0.024 & -0.082 & 0.135 & -0.108 & 0.033 \end{pmatrix}$$

WRITEPRN("c:\Sci\tmp\linAlg\Wilkinson\d_p47_o.prn") := d_p47_o

matVariation(d_p47_t, d_p47_o) = ▮

page56???LU-разложение LU decomposition

https://en.wikipedia.org/wiki/LU_decomposition

Substituting these values into the LU decomposition above yields

$$\begin{bmatrix} 4 & 3 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1.5 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 0 & -1.5 \end{bmatrix}.$$

$$M\_forLU1 := \begin{pmatrix} 4 & 3 \\ 6 & 3 \end{pmatrix}$$

WRITEPRN("c:\Sci\tmp\linAlg\Wilkinson\M_p56_o.prn") := M_forLU1

$$lu(M\_forLU1) = \begin{pmatrix} 0 & 1 & 1 & 0 & 6 & 3 \\ 1 & 0 & 0.667 & 1 & 0 & 1 \end{pmatrix}$$

$$\_\_digToVec(d, Vec) := \left| \begin{array}{l} n \leftarrow rows(Vec) - 1 \\ \text{for } i \in 0..n \\ \quad Vec_i \leftarrow d \\ \text{return } Vec \end{array} \right.$$

mkVecByDig(

page 61 Choleckij

$$genGdiag\_A61 := \left| \begin{array}{l} size \leftarrow 40 \\ R \leftarrow mkVecByDig(6, size) \\ R_0 \leftarrow 5 \\ R_{size-1} \leftarrow 5 \\ \text{return } R \end{array} \right.$$

$genGdiag\_A61^T =$

	0	1	2	3	4	5	6
0	5	6	6	6	6	6	

$$ftDiag\_A61 := \left| \begin{array}{l} size \leftarrow 40 - 1 \\ R \leftarrow mkVecByDig(-4, size) \\ \text{return } R \end{array} \right.$$

$ftDiag\_A61^T =$

	0	1	2	3	4	5	6
0	-4	-4	-4	-4	-4	-4	-

$$sdDiag\_A61 := \left| \begin{array}{l} size \leftarrow 40 - 2 \\ R \leftarrow mkVecByDig(1, size) \\ \text{return } R \end{array} \right.$$

$sdDiag\_A61^T =$

	0	1	2	3	4	5	6
0	1	1	1	1	1	1	

$fForGenDiag(i) := (i \quad i)$

$$fForFtDiag(i) := \begin{pmatrix} i & i+1 \\ i+1 & i \end{pmatrix}$$

fillMatr(Vec, Func

$$fForSdDiag(i) := \begin{pmatrix} i & i+2 \\ i+2 & i \end{pmatrix}$$

$$fForT\_rdDiag(i) := \begin{pmatrix} i & i+3 \\ i+3 & i \end{pmatrix}$$

$$mk3DiagMatr(genDiag, ftDiag, sdDiag) := \left| \begin{array}{l} n \leftarrow rows(genDiag) - 1 \\ R_{n,n} \leftarrow 0 \\ R \leftarrow fillMatr(genDiag, fForGenDiag, R) \\ R \leftarrow fillMatr(ftDiag, fForFtDiag, R) \\ R \leftarrow fillMatr(sdDiag, fForSdDiag, R) \\ \text{return } R \end{array} \right.$$

$A\_p61\_o := mk3DiagMatr(genGdiag\_A61, ftDiag\_A61, sdDiag\_A61)$

$det\_A\_p61\_o := 4.10400390 \cdot 10^{-1} \cdot 2^{12}$ $\qquad det\_A\_p61\_o = 1.681 \times 10^3$

$det\_A\_p61\_t := |A\_p61\_o|$ $\qquad det\_A\_p61\_t = 1.681 \times 10^3$

$A\_p61\_o =$

	0
0	5
1	-4
2	1
3	0
4	0
5	0
6	0
7	0

8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

$$d\_A\_p61 := \frac{|det\_A\_p61\_t - det\_A\_p61\_o|}{det\_A\_p61\_o} \qquad d\_A\_p61 = 1.524 \times 10^{-9}$$

b_p62 := mkVec_e(40, 0)

WRITEPRN("c:\Sci\tmp\linAlg\Wilkinson\d_A_p61_o.prn") := det_A_p61_o

x_p62_t := lsolve(A_p61_o, b_p62)

x_p62_t =

	0
0	13.171
1	25.366
2	36.61
3	46.927
4	56.341
5	64.878
6	72.561
7	79.415
8	85.463
9	90.732
10	95.244
11	99.024
12	102.098
13	104.488
14	106.22
15	...

$$x\_p62\_part1\_o := \begin{pmatrix} 1.3170731707\cdot 10^{1} \\ 2.5365853659\cdot 10^{1} \\ 3.6609756098\cdot 10^{1} \\ 4.6926829268\cdot 10^{1} \\ 5.6341463415\cdot 10^{1} \\ 6.4878048780\cdot 10^{1} \\ 7.2560975610\cdot 10^{1} \\ 7.9414634146\cdot 10^{1} \\ 8.5463414634\cdot 10^{1} \\ 9.0731707317\cdot 10^{1} \\ 9.5243902439\cdot 10^{1} \\ 9.9024390244\cdot 10^{1} \\ 1.0209756098\cdot 10^{2} \\ 1.0448780488\cdot 10^{2} \\ 1.0621951220\cdot 10^{2} \\ 1.0731707317\cdot 10^{2} \\ 1.07804878049\cdot 10^{2} \\ 1.0770731707\cdot 10^{2} \\ 1.0704878049\cdot 10^{2} \\ 1.0585365854\cdot 10^{2} \end{pmatrix}$$

$$x\_p62\_part2\_o := \begin{pmatrix} 1.0 \\ 1.0 \\ 9.9 \\ 9.6 \\ 9.2 \\ 8.8 \\ 8.4 \\ 7.9 \\ 7.4 \\ 6.9 \\ 6.4 \\ 5.85 \\ 5.2 \\ 4.6 \\ 4.0 \\ 3.3 \\ 2.7 \\ 2.0 \\ 1.3 \\ 6.8 \end{pmatrix}$$

x_p62_o := stack(x_p62_part1_o, x_p62_part2_o)

minmax(matVariation(x_p62_o, x_p62_t)) = ∎

page 88 size_p88 := 7

$$genGdiag\_A\_p88 := \left| \begin{array}{l} R \leftarrow mkVecByDig(6, size\_p88) \\ R_0 \leftarrow 5 \\ R_{size\_p88-1} \leftarrow 5 \\ return\ R \end{array} \right.$$

$$ftDiag\_A\_p88 := mkVecByDig(-4, size\_p88 - 1)$$

$$sdDiag\_A\_p88 := mkVecByDig(1, size\_p88 - 2)$$

$$A\_p88\_o := mk3DiagMatr(genGdiag\_A\_p88, ftDiag\_A\_p88, sdDiag\_A\_p88)$$

$$A\_p88\_o = \begin{pmatrix} 5 & -4 & 1 & 0 & 0 & 0 & 0 \\ -4 & 6 & -4 & 1 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 1 & -4 & 6 & -4 \\ 0 & 0 & 0 & 0 & 1 & -4 & 5 \end{pmatrix} \qquad b\_p88 := mkVec\_e(size\_p88, 4 - 1) \qquad b\_p88 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x\_p88\_t := lsolve(A\_p88\_o, b\_p88)$$

$$x\_p88\_t = \begin{pmatrix} 4 \\ 7.5 \\ 10 \\ 11 \\ 10 \\ 7.5 \\ 4 \end{pmatrix} \qquad x\_p88\_o := \begin{pmatrix} 4.0 \\ 7.5 \\ 10.0 \\ 11.0 \\ 10.0 \\ 7.5 \\ 4.0 \end{pmatrix}$$

$$minmax(matVariation(x\_p88\_t, x\_p88\_o)) = ▮$$

page104

$$b\_p105 := mkVec\_e(size\_p88, 5 - 1) \cdot 360360 \qquad b\_p105 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3.604 \times \\ 0 \\ 0 \end{pmatrix}$$

$$x\_p105\_t := lsolve(Gilbert7, b\_p105)$$

$$x\_p105\_t = \begin{pmatrix} 4.851\times10^{4} \\ -1.94\times10^{6} \\ 1.871\times10^{7} \\ -7.276\times10^{7} \\ 1.334\times10^{8} \\ -1.153\times10^{8} \\ 3.784\times10^{7} \end{pmatrix} \qquad x\_p105\_o := \begin{pmatrix} 4.85100315047\cdot10^{4} \\ -1.94040128440\cdot10^{6} \\ 1.87110125732\cdot10^{7} \\ -7.27650495356\cdot10^{7} \\ 1.33402591837\cdot10^{8} \\ -1.15259840116\cdot10^{8} \\ 3.78378265214\cdot10^{7} \end{pmatrix}$$

$\mathrm{minmax}(\mathrm{matVariation}(x\_p105\_t, x\_p105\_o)) = ▮$

$complMul\_p105 := (1+i \;\; 1-i \;\; 1+2i \;\; 1-2i \;\; 1+3i \;\; 1-3i \;\; 1+4i)^{T}$

$G7complex\_p105 := Gilbert7\cdot\mathrm{diag}(complMul\_p105)$

$det\_p105\_cmplx\_t := |G7complex\_p105|$

$det\_p105\_cmplx\_t = 3.816\times10^{16} + 1.526i\times10^{17}$

$$G7complex\_p105 = \begin{pmatrix} 3.604\times10^{5} + 3.604i\times10^{5} & 1.8(\\ 1.802\times10^{5} + 1.802i\times10^{5} & 1.2(\\ 1.201\times10^{5} + 1.201i\times10^{5} & 9.0(\\ 9.009\times10^{4} + 9.009i\times10^{4} & 7.2(\\ 7.207\times10^{4} + 7.207i\times10^{4} & 6.0(\\ 6.006\times10^{4} + 6.006i\times10^{4} & 5.1 \\ 5.148\times10^{4} + 5.148i\times10^{4} & 4.5(\end{pmatrix}$$

$det\_p105\_cmplx\_o := \left(3.30997622\cdot10^{-2} + i\cdot1.32399048\cdot10^{-1}\right)\cdot2^{60}$

$det\_p105\_cmplx\_o - det\_p105\_cmplx\_t = -4.719\times10^{7} - 1.125i\times10^{9}$

$3.816\times10^{16} + 1.526i\times10^{17}$

$$\mathrm{lsolve}(G7complex\_p105, \mathrm{mkVec\_e}(size\_p88, 0)) = \begin{pmatrix} 6.799\times10^{-5} - 6.799i\times10^{-5} \\ -1.632\times10^{-3} - 1.632i\times10^{-3} \\ 4.895\times10^{-3} - 9.79i\times10^{-3} \\ -0.016 - 0.033i \\ 0.013 - 0.04i \\ -0.011 - 0.032i \\ 1.961\times10^{-3} - 7.843i\times10^{-3} \end{pmatrix}$$

????????

page 188 $size\_p188 := 44$

$$genGdiag\_B\_p188 := \left|\begin{array}{l} R \leftarrow mkVecByDig(6, size\_p188) \\ R_0 \leftarrow 5 \\ R_{size\_p188-1} \leftarrow 5 \\ \text{return } R \end{array}\right.$$

$$ftDiag\_B\_p188 := \left|\begin{array}{l} R \leftarrow mkVecByDig(3, size\_p188 - 1) \\ R_0 \leftarrow 2 \\ R_{size\_p188-2} \leftarrow 2 \\ \text{return } R \end{array}\right.$$

$$sdDiag\_B\_p188 := mkVecByDig(1, size\_p188 - 2)$$

$$trdDiag\_B\_p188 := mkVecByDig(1, size\_p188 - 3)$$

$$mk4DiagMatr(genDiag, ftDiag, sdDiag, trdDiag) := \left|\begin{array}{l} n \leftarrow rows(genDiag) - 1 \\ R_{n,n} \leftarrow 0 \\ R \leftarrow fillMatr(genDiag, fForGenDiag, R) \\ R \leftarrow fillMatr(ftDiag, fForFtDiag, R) \\ R \leftarrow fillMatr(sdDiag, fForSdDiag, R) \\ R \leftarrow fillMatr(trdDiag, fForTrdDiag, R) \\ \text{return } R \end{array}\right.$$

$$B\_p188\_o := mk4DiagMatr(genGdiag\_B\_p188, ftDiag\_B\_p188, sdDiag\_B\_p188, trdDiag\_B\_p188)$$

$$B\_p188\_o = \blacksquare$$

$eigenval\_B\_p188\_t := sort(eigenvals(B\_p188\_o))$

$eigenval\_B\_p188\_t = ▮$

$eigenvec\_for6 := eigenvec\left(B\_p188\_o, eigenval\_B\_p188\_t_{14}\right)$

$eigenvec\_for6 = ▮$

page 200

$$A\_p200 := \begin{pmatrix} 10 & 1 & 2 & 3 & 4 \\ 1 & 9 & -1 & 2 & -3 \\ 2 & -1 & 7 & 3 & -5 \\ 3 & 2 & 3 & 12 & -1 \\ 4 & -3 & -5 & -1 & 15 \end{pmatrix}$$

$$B\_p200 := \begin{pmatrix} 5 & 1 & -2 & 0 & -2 & 5 \\ 1 & 6 & -3 & 2 & 0 & 6 \\ -2 & -3 & 8 & -5 & -6 & 0 \\ 0 & 2 & -5 & 5 & 1 & -2 \\ -2 & 0 & -6 & 1 & 6 & -3 \\ 5 & 6 & 0 & -2 & -3 & 8 \end{pmatrix}$$

$eigenvals\_Ap200\_t1 := eigenvals(A\_p200)$

$$eigenvals\_Ap200\_t1 = \begin{pmatrix} 1.655 \\ 6.995 \\ 9.366 \\ 15.809 \\ 19.175 \end{pmatrix} \qquad vals\_A\_p200\_o := \begin{pmatrix} 1.65526620792 \cdot 10^{0} \\ 6.99483783061 \cdot 10^{0} \\ 9.36555492014 \cdot 10^{0} \\ 1.58089207644 \cdot 10^{1} \\ 1.91754202773 \cdot 10^{1} \end{pmatrix}$$

$minmax(matVariation(eigenvals\_Ap200\_t1, vals\_A\_p200\_o)) = ▮$

$eigenvals\_Bp200\_t1 := eigenvals(B\_p200)$

$$eigenvals\_Bp200\_t1 = \begin{pmatrix} -1.599 \\ -1.599 \\ 4.456 \\ 4.456 \\ 16.143 \\ 16.143 \end{pmatrix} \qquad vals\_B\_p200\_o := \begin{pmatrix} -1.59873429360 \cdot 10^{0} \\ -1.59873429360 \cdot 10^{0} \\ 4.45598963849 \cdot 10^{0} \\ 4.45598963849 \cdot 10^{0} \\ 1.61427446551 \cdot 10^{1} \\ 1.61427446551 \cdot 10^{1} \end{pmatrix}$$

$minmax(matVariation(eigenvals\_Bp200\_t1, vals\_B\_p200\_o)) = ▮$

$eigenVecss\_Ap200\_t1 := eigenvecs(A\_p200)$

$$eigenVecss\_Ap200\_t1 = \begin{pmatrix} -0.387 & -0.654 & 0.052 & 0.624 & 0.175 \\ 0.366 & -0.2 & -0.86 & 0.159 & -0.247 \\ 0.704 & -0.257 & 0.506 & 0.227 & -0.362 \\ -0.119 & 0.66 & 2.012 \times 10^{-4} & 0.693 & -0.264 \\ 0.453 & 0.174 & -0.046 & 0.233 & 0.841 \end{pmatrix}$$

$$vecs\_A\_p200\_o^{\langle 0 \rangle} := \begin{pmatrix} 3.87296874886 \cdot 10^{-1} \\ -3.66221021131 \cdot 10^{-1} \\ -7.04377266220 \cdot 10^{-1} \\ 1.18926222076 \cdot 10^{-1} \\ -4.53423108037 \cdot 10^{-1} \end{pmatrix} \qquad vecs\_A\_p200\_o^{\langle 1 \rangle} := \begin{pmatrix} 6.54082984085 \cdot 10^{-1} \\ 1.99681268959 \cdot 10^{-1} \\ 2.56510456336 \cdot 10^{-1} \\ -6.60402722389 \cdot 10^{-1} \\ -1.74279863500 \cdot 10^{-1} \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o}^{\langle 2\rangle} := \begin{pmatrix} 5.21511178463\cdot 10^{-2} \\ -8.59963866689\cdot 10^{-1} \\ 5.05575072575\cdot 10^{-1} \\ 2.01166650650\cdot 10^{-4} \\ -4.62191996239\cdot 10^{-2} \end{pmatrix} \qquad \text{vecs\_A\_p200\_o}^{\langle 3\rangle} := \begin{pmatrix} -6.23702499852\cdot 10^{-1} \\ -1.59101120870\cdot 10^{-1} \\ -2.27297494237\cdot 10^{-1} \\ -6.92684385756\cdot 10^{-1} \\ -2.32822283880\cdot 10^{-1} \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o}^{\langle 4\rangle} := \begin{pmatrix} 1.74505109459\cdot 10^{-1} \\ -2.47302518851\cdot 10^{-1} \\ -3.61641739446\cdot 10^{-1} \\ -2.64410853099\cdot 10^{-1} \\ 8.41244069212\cdot 10^{-1} \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o} = \begin{pmatrix} 0.387 & 0.654 & 0.052 & -0.624 & 0.175 \\ -0.366 & 0.2 & -0.86 & -0.159 & -0.247 \\ -0.704 & 0.257 & 0.506 & -0.227 & -0.362 \\ 0.119 & -0.66 & 2.012\times 10^{-4} & -0.693 & -0.264 \\ -0.453 & -0.174 & -0.046 & -0.233 & 0.841 \end{pmatrix}$$

$$\text{normVec}(v) := \text{return } \sum \overrightarrow{|v|}$$

$$\text{normVec}\left(\text{vecs\_A\_p200\_o}^{\langle 0\rangle}\right) = 2.03$$

$$\text{VariationEigenVecs}(A,B) := \left| \begin{array}{l} n \leftarrow \text{rows}(A) - 1 \\ \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} \text{mult} \leftarrow \dfrac{A_{0,i}}{B_{0,i}} \\ R^{\langle i\rangle} \leftarrow A^{\langle i\rangle} - \text{mult}\cdot B^{\langle i\rangle} \end{array} \right. \\ \text{return } R \end{array} \right.$$

$$\text{deltaEigenVecs\_A\_p200} := \text{VariationEigenVecs}(\text{vecs\_A\_p200\_o}, \text{eigenVecss\_Ap200\_t1})$$

$$\text{deltaEigenVecs\_A\_p200} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1.439\times 10^{-12} & -1.56\times 10^{-11} & -1.035\times 10^{-10} & -6.502\times 10^{-12} & 5.741\times 10^{-12} \\ 2.327\times 10^{-12} & 1.027\times 10^{-11} & 5.396\times 10^{-11} & 3.32\times 10^{-12} & 1.243\times 10^{-11} \\ 5.545\times 10^{-12} & -1.222\times 10^{-11} & 1.722\times 10^{-11} & 1.104\times 10^{-11} & 1.164\times 10^{-11} \\ -2.715\times 10^{-12} & -4.011\times 10^{-12} & -2.551\times 10^{-12} & 6.824\times 10^{-12} & -1.913\times 10^{-11} \end{pmatrix}$$

$$\text{minmax}(\text{deltaEigenVecs\_A\_p200}) = \begin{pmatrix} -1.035\times 10^{-10} & 5.396\times 10^{-11} \end{pmatrix}$$

$\text{eigenVecss\_Bp200\_t1} := \text{eigenvecs}(\text{B\_p200})$?????

$$\text{eigenVecss\_Bp200\_t1} = \begin{pmatrix} 0.44 & 0.259 & -0.557 & 0.499 & -0.075 & 0.419 \\ 0.265 & 0.499 & 0.509 & -0.352 & -0.304 & 0.453 \\ 0.621 & -0.185 & 0.224 & -0.092 & 0.722 & 0 \\ 0.259 & -0.44 & 0.499 & 0.557 & -0.419 & -0.075 \\ 0.499 & -0.265 & -0.352 & -0.509 & -0.453 & -0.304 \\ -0.185 & -0.621 & -0.092 & -0.224 & 0 & 0.722 \end{pmatrix}$$

$$\text{vecs\_B\_p200\_o}^{\langle 0 \rangle} := \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} \qquad \text{eigenvals\_Bp200\_t1} = \begin{pmatrix} -1.599 \\ -1.599 \\ 4.456 \\ 4.456 \\ 16.143 \\ 16.143 \end{pmatrix}$$

???????

$$\text{eigenvec}\left(\text{B\_p200}, \text{eigenvals\_Bp200\_t1}_1\right) = \begin{pmatrix} 0.259 \\ -0.011 \\ 0.633 \\ 0.44 \\ 0.565 \\ 0.14 \end{pmatrix}$$

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$$\text{genGdiag\_A\_p227} := \left| \begin{array}{l} \text{size} \leftarrow 30 \\ \text{for } i \in 0..\text{size}-1 \\ \quad R_i \leftarrow (i+1)^4 \\ \text{return } R \end{array} \right.$$

$$\text{ftDiag\_A\_p227} := \left| \begin{array}{l} \text{size} \leftarrow 30-1 \\ \text{for } i \in 0..\text{size}-1 \\ \quad R_i \leftarrow (i+1)-1 \\ \text{return } R \end{array} \right.$$

$$\text{mkDiagMatr}(\text{genDiag}, \text{ftDiag}) := \left| \begin{array}{l} n \leftarrow \text{rows}(\text{genDiag}) - 1 \\ R_{n,n} \leftarrow 0 \\ R \leftarrow \text{fillMatr}(\text{genDiag}, \text{fForGenDiag}, R) \\ R \leftarrow \text{fillMatr}(\text{ftDiag}, \text{fForFtDiag}, R) \\ \text{return } R \end{array} \right.$$

$A\_p227 := mkDiagMatr(genGdiag\_A\_p227, ftDiag\_A\_p227)$

$vals\_A\_p227\_t := eigenvals(A\_p227)$ $vals\_A\_p227\_approx\_o := genGdiag\_A\_p227$

vals_A_p227_t =

	0
0	1
1	15.985
2	80.993
3	255.998
4	625.001
5	$1.296 \cdot 10^3$
6	$2.401 \cdot 10^3$
7	$4.096 \cdot 10^3$
8	$6.561 \cdot 10^3$
9	$1 \cdot 10^4$
10	$1.464 \cdot 10^4$
11	$2.074 \cdot 10^4$
12	$2.856 \cdot 10^4$
13	$3.842 \cdot 10^4$
14	$5.063 \cdot 10^4$
15	...

chole

$minmax(matVariation(vals\_A\_p227\_t, vals\_A\_p227\_approx\_o)) =$ ▮

page_343

$$reA\_p343 := \begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix} \qquad imA\_p343 := \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$A\_p343 := reA\_p343 + i \cdot imA\_p343$

$$A\_p343 = \begin{pmatrix} 1+i & -1-i & 2+2i \\ 0 & i & 2 \\ 0 & -1 & 3+i \end{pmatrix}$$

$$eigenvals(A\_p343) = \begin{pmatrix} 1+i \\ 1+i \\ 2+i \end{pmatrix}$$

$$eigenvecs(A\_p343) = \begin{pmatrix} 1 & -0.236-0.236i & 0.707 \\ 0 & 0.843 & 0.354-0.354i \\ 0 & 0.422 & 0.354-0.354i \end{pmatrix}$$

?????

$$\text{matVariation\_}(A,B) := \left[\begin{array}{l} A2 \leftarrow \overrightarrow{|A|} \\ B2 \leftarrow \overrightarrow{|B|} \\ k \leftarrow \max(B2) \\ R \leftarrow \overrightarrow{\left(\frac{A2 - B2}{k}\right)} \\ \text{return } R \end{array}\right]$$

$$\text{matGonj}(A) := \overrightarrow{\overline{A}} \quad \text{matMod}$$

$$\text{normFrobenius}(A) := \left| \begin{array}{l} C \leftarrow \\ C\_h \\ \text{res} \\ \text{retu} \end{array}\right.$$

$$\text{nFr}(A) := \left| \begin{array}{l} F \leftarrow \text{normFrol} \\ \text{return } \frac{}{\text{matMa}} \end{array}\right.$$

:inson\Gilbert7.prn" , Gilbert7)

$$\text{minmax}(M) := \left| \begin{array}{l} R \leftarrow (\min(M) \quad \max(M)) \\ \text{return } R \end{array}\right.$$

np\linAlg\Wilkinson\Gilbert7.prn")

https://math.stackexchar
matrices
http://mathworld.wolfram.
.html

$\text{matVariation}(A53_o, A53_o) = $

$$\begin{pmatrix} .34604954657\cdot10^{-1} & -1.07683294804\cdot10^{-1} & 3.33303738787\cdot10^{-2} \\ -5.38420043660\cdot10^{0} & 4.43041069058\cdot10^{0} & -1.39988227277\cdot10^{0} \\ 5.19190848985\cdot10^{1} & -4.36119354451\cdot10^{1} & 1.39988675014\cdot10^{1} \\ -2.01907567907\cdot10^{2} & 1.72294292794\cdot10^{2} & -5.59956005457\cdot10^{1} \\ 3.70163885040\cdot10^{2} & -3.19821597912\cdot10^{2} & 1.04991937762\cdot10^{2} \\ -3.19821597914\cdot10^{2} & 2.79117246791\cdot10^{2} & -9.23930348819\cdot10^{1} \\ 1.04991937763\cdot10^{2} & -9.23930348819\cdot10^{1} & 3.07977132849\cdot10^{1} \end{pmatrix}$$

$$\begin{pmatrix} 10^{-6} & -2.959\times10^{-6} \\ 10^{-4} & 1.177\times10^{-4} \\ 10^{-3} & -1.133\times10^{-3} \\ 3 & 4.399\times10^{-3} \\ 5 & -8.062\times10^{-3} \\ 21 & 6.965\times10^{-3} \\ 10^{-3} & -2.287\times10^{-3} \end{pmatrix}$$

WRITEPRN("c:\Sci\tmp\linAlg\Wilkinson\invA53.prn") := invA53_o

$$invA53\_o = \begin{pmatrix} 1.36\times10^{-4} & -3.263\times10^{-3} & 0.024 & -0.082 & 0.135 & -0.108 & 0.033 \\ -3.263\times10^{-3} & 0.104 & -0.881 & 3.133 & -5.384 & 4.43 & -1.4 \\ 0.024 & -0.881 & 7.93 & -29.368 & 51.919 & -43.612 & 13.999 \\ -0.082 & 3.133 & -29.368 & 111.88 & -201.908 & 172.294 & -55.996 \\ 0.135 & -5.384 & 51.919 & -201.908 & 370.164 & -319.822 & 104.992 \\ -0.108 & 4.43 & -43.612 & 172.294 & -319.822 & 279.117 & -92.393 \\ 0.033 & -1.4 & 13.999 & -55.996 & 104.992 & -92.393 & 30.798 \end{pmatrix}$$

$$ \cdot = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} $$

$$ m1 := \begin{pmatrix} 2 + 3 \cdot i & 1 - 4 \cdot i \\ 3 + 4 \cdot i & 5 + 6 \cdot i \end{pmatrix} \qquad m2 := m1 \cdot 2 $$

$$ \text{matGonj}(m1) = \begin{pmatrix} 2 - 3i & 1 + 4i \\ 3 - 4i & 5 - 6i \end{pmatrix} $$

$$ \text{tr}(m1) = 7 + 9i $$

nFr(invA53

$$ \text{matModulus}(m1) = \begin{pmatrix} 3.606 & 4.123 \\ 5 & 7.81 \end{pmatrix} \qquad \text{matMaxElem}(m1) = $$

$\text{normFrobenius}(m1) = 10.77$

$\text{normFrobenius}(m2) = 21.541$ $\qquad$ normFrobenius(invA53_o) = 794

$\text{distFrobenius}(m1, m2) = 6.928$

$\text{nFr}(m1) = 1.379$

$\text{distFrobenius}(m1 \cdot 0.5, m2 \cdot 0.5) = 3$

$\text{nFr}(m2) = 1.379$

$\text{dFr}(m1, m2) = 0.887$

$\text{dFr}(m1 \cdot 2, m2 \cdot 2) = 0.887$

distFrobenius(invA53_t, invA53_o) = 0.061

dFr(invA53_t, invA53_o) = 2.145

$(dig, size) := \left| \begin{array}{l} n \leftarrow size - 1 \\ R_n \leftarrow 0 \\ \text{for } i \in 0..n \\ \quad R_i \leftarrow dig \\ \text{return } R \end{array} \right.$

6	7	8	9
6	6	6	...

	7	8	9
4	-4	-4	...

	7	8	9
1	1	1	...

$t, Matr) := \left| \begin{array}{l} n \leftarrow rows(Vec) - 1 \\ \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} indx \leftarrow Funct(i) \\ \text{for } j \in 0..rows(indx) - 1 \\ \quad \left| \begin{array}{l} r \leftarrow indx_{j,0} \\ c \leftarrow indx_{j,1} \\ Matr_{r,c} \leftarrow Vec_i \end{array} \right. \end{array} \right. \\ \text{return } Matr \end{array} \right.$

1	2	3	4	5	6	7	8	9
-4	1	0	0	0	0	0	0	0
6	-4	1	0	0	0	0	0	0
-4	6	-4	1	0	0	0	0	0
1	-4	6	-4	1	0	0	0	0
0	1	-4	6	-4	1	0	0	0
0	0	1	-4	6	-4	1	0	0
0	0	0	1	-4	6	-4	1	0
0	0	0	0	1	-4	6	-4	1

	0
0	1
1	0
2	0

0	0	0	0	0	1	-4	6	-4
0	0	0	0	0	0	1	-4	6
0	0	0	0	0	0	0	1	-4
0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	...

b_p62 =

3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	...

$$\begin{pmatrix} 414634146\cdot 10^{2} \\ 195121951\cdot 10^{2} \\ 292682927\cdot 10^{1} \\ 195121951\cdot 10^{1} \\ 682926829\cdot 10^{1} \\ 780487805\cdot 10^{1} \\ 512195122\cdot 10^{1} \\ 902439024\cdot 10^{1} \\ 975609756\cdot 10^{1} \\ 756097561\cdot 10^{1} \\ 268292683\cdot 10^{1} \\ 36585365854\cdot 10^{1} \\ 585365854\cdot 10^{1} \\ 439024390\cdot 10^{1} \\ 121951220\cdot 10^{1} \\ 658536585\cdot 10^{1} \\ 073170732\cdot 10^{1} \\ 390243902\cdot 10^{1} \\ 634146341\cdot 10^{1} \\ 292682927\cdot 10^{0} \end{pmatrix}$$

10^5

$02 \times 10^{5} - 1.802i \times 10^{5}$	$1.201 \times 10^{5} + 2.402i \times 10^{5}$	$9.009 \times 10^{4} - 1.802i \times 10^{5}$	$7.207 \times 10^{4} + 2.162i \times 10^{5}$	$6.006 \times$
$01 \times 10^{5} - 1.201i \times 10^{5}$	$9.009 \times 10^{4} + 1.802i \times 10^{5}$	$7.207 \times 10^{4} - 1.441i \times 10^{5}$	$6.006 \times 10^{4} + 1.802i \times 10^{5}$	$5.148 \times$
$09 \times 10^{4} - 9.009i \times 10^{4}$	$7.207 \times 10^{4} + 1.441i \times 10^{5}$	$6.006 \times 10^{4} - 1.201i \times 10^{5}$	$5.148 \times 10^{4} + 1.544i \times 10^{5}$	$4.505 \times$
$07 \times 10^{4} - 7.207i \times 10^{4}$	$6.006 \times 10^{4} + 1.201i \times 10^{5}$	$5.148 \times 10^{4} - 1.03i \times 10^{5}$	$4.505 \times 10^{4} + 1.351i \times 10^{5}$	$4.004 \times$
$06 \times 10^{4} - 6.006i \times 10^{4}$	$5.148 \times 10^{4} + 1.03i \times 10^{5}$	$4.505 \times 10^{4} - 9.009i \times 10^{4}$	$4.004 \times 10^{4} + 1.201i \times 10^{5}$	$3.604 \times$
$48 \times 10^{4} - 5.148i \times 10^{4}$	$4.505 \times 10^{4} + 9.009i \times 10^{4}$	$4.004 \times 10^{4} - 8.008i \times 10^{4}$	$3.604 \times 10^{4} + 1.081i \times 10^{5}$	$3.276 \times$
$05 \times 10^{4} - 4.505i \times 10^{4}$	$4.004 \times 10^{4} + 8.008i \times 10^{4}$	$3.604 \times 10^{4} - 7.207i \times 10^{4}$	$3.276 \times 10^{4} + 9.828i \times 10^{4}$	$3.003 \times$

ulus(A) := $\overrightarrow{|A|}$

matMaxElem(A) := max(matModulus(A))

- matGonj(A)

$\leftarrow C^T$

$\leftarrow \sqrt{tr(A \cdot C\_h)}$

rn res

distFrobenius(A, B) := | C ← A − B
| D ← C·matGonj(C)
| return $\sqrt{tr(D)}$

enius(A)

$\frac{F}{\text{xElem(A)}}$

dFr(A, B) := | F ← distFrobenius(A, B)
| dA ← matMaxElem(A)
| dB ← matMaxElem(B)
| return $\frac{F}{\text{matMaxElem}(A - B)}$

nge.com/questions/507742/distance-similarity-between-two-

.com/FrobeniusNorm

_o) = 2.146

7.81

ı.202

.464

$$
\begin{matrix}
10^4 - 1.802i \times 10^5 & 5.148 \times 10^4 + 2.059i \times 10^5 \\
10^4 - 1.544i \times 10^5 & 4.505 \times 10^4 + 1.802i \times 10^5 \\
10^4 - 1.351i \times 10^5 & 4.004 \times 10^4 + 1.602i \times 10^5 \\
10^4 - 1.201i \times 10^5 & 3.604 \times 10^4 + 1.441i \times 10^5 \\
10^4 - 1.081i \times 10^5 & 3.276 \times 10^4 + 1.31i \times 10^5 \\
10^4 - 9.828i \times 10^4 & 3.003 \times 10^4 + 1.201i \times 10^5 \\
10^4 - 9.009i \times 10^4 & 2.772 \times 10^4 + 1.109i \times 10^5
\end{matrix}\right)
$$