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The affine growth of hurricane speeds

Z.W.T. Mason

Sheffield, UK

Abstract

Hurricane strengths form a skewed distribution which is variously fit by the Weibull or log-normal distributions. For Bernoulli trials it has been recently shown affine returns may be approximated by an appropriately scaled logit-normal distribution. A reasonable fit for some of the hurricane strengths is performed by this latter distribution whose parameters are derived using a mixture of Maximum Likelihood Estimation and a grid search.

1. Introduction

Hurricane strength, often measured by wind speed or the Saffir-Simpson scale, is commonly modelled using a Weibull distribution or a log-normal distribution.

By way of example, if there were some upper scale applicable then the logit-normal distribution may be appropriate. The logit-normal distribution is much overlooked but important, for example it moderates the growth of a well-mixed epidemic from exponential to logistic as shown by the author [1].

The author [2] also showed, for stock prices, that affine returns on coin tosses can be approximated by a stretched-out logit-normal distribution for the case when the growth converges to a finite support. The same shapes were also evident in the case where the support grows exponentially with t when $\beta > 1$ in equation 1.

$$\left(0, \delta \frac{1-\beta^t}{1-\beta}\right) \tag{1}$$

The process itself being described by the random variable S_t growing from $S_0 = 0$ with probability p

$$S_{t+1} = \beta S_t + \delta,\ 0 < \beta \neq 1,\ \delta > 0 \tag{2}$$

and shrinking with probability $q = 1 - p$

$$S_{t+1} = \alpha S_t,\ 0 < \alpha < 1 \tag{3}$$

When $\beta < 1$ the probability distribution converges. Applying the equations to the distribution leads to shrinks and shifts which, at the tails, means only one copy is applied. The resulting equations then show that both tails are power laws. i.e. The logit-normal distribution may be appropriate for the extreme speeds. In fact, it should be expected that only the one distribution should cover all the natural cases rather than the mish-mash of the existing approach. Whether the model is appropriate though hinges on whether it fits the data.

Email address: papers@stochanswers.com (Z.W.T. Mason)

2. Parameter estimation

The Maximum Likelihood Estimators for the log-normal and logit-normal distributions are the same as for the normal distribution fitted to the values transformed by the appropriate function, so the fitting is done in a highly distorted space. The probability density function for the scaled logit-normal is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \frac{L}{x(L-x)} \exp\left(-\frac{(\log(x) - \log(L-x) - \mu)^2}{2\sigma^2}\right) \tag{4}$$

Taking logarithms

$$-\log(\sigma) - \frac{1}{2}\log(2\pi) + \log(L) - \log(x) - \log(L-x) - \frac{(\log(x) - \log(L-x) - \mu)^2}{2\sigma^2} \tag{5}$$

Differentiating with respect to μ, σ, L results in, respectively

$$\frac{\log(x) - \log(L-x) - \mu}{\sigma^2} \tag{6}$$

$$-\frac{1}{\sigma} + \frac{(\log(x) - \log(L-x) - \mu)^2}{\sigma^3} \tag{7}$$

$$\frac{1}{L} - \frac{1}{L-x} + \frac{\log(x) - \log(L-x) - \mu}{\sigma^2} \frac{1}{L-x} \tag{8}$$

Resulting in the usual, if L were equal to one, estimates of

$$\mu\Sigma_i 1 = \Sigma_i (\log(x_i) - \log(L-x_i)) \tag{9}$$

$$\sigma^2\Sigma_i 1 = \Sigma_i (\log(x_i) - \log(L-x_i) - \mu)^2 \tag{10}$$

The final derivative was not summed over the data and used to estimate L but was used to estimate the error. The mean square error between the formula and the histogram was also calculated. Finally, the result was checked to see whether it could be improved upon by searching through a grid of parameters.

3. Data analysis

3.1. Data from the NOAA

One hundred and sixty four years of hurricane and typhoon wind speeds, from 1851 to 2014, for the Atlantic and Pacific Oceans taken from the NOAA National Hurricane Center (NHC) - HURDAT2 Database were downloaded from Kaggle [3].

For the Atlantic Ocean the Maximum Likelihood Estimation was performed with L from 162 to 1000 with the mean square error decreasing to a minimum at 612. Subsequently a matrix of tests was performed with a minimum at $L = 990$, $\mu = -3.0$, $\sigma^2 = 0.316$ is shown in figure 1.

For the Pacific Ocean the Maximum Likelihood Estimation was performed with L from 181 to 1000 with the mean square error decreasing monotonically. Subsequently a matrix of tests was performed with a minimum at $L = 999$, $\mu = -3.16$, $\sigma^2 = 0.288$ is shown in figure 2.

4. Discussion

That the Maximum Likelihood Estimation wasn't sufficient to determine the parameters may be due to the support being semi-infinite. i.e. The length scale tends to infinity. A rough grid search, although ugly and a misuse of cheap computing power, seemed to be sufficient.

The fits for the Atlantic and Pacific Oceans seem reasonable given the spikiness of the data. Obviously more data needs to be fitted and comparison needs to be made with the other distributions. However, using a scaled logit-normal distribution does appear to be a promising option.

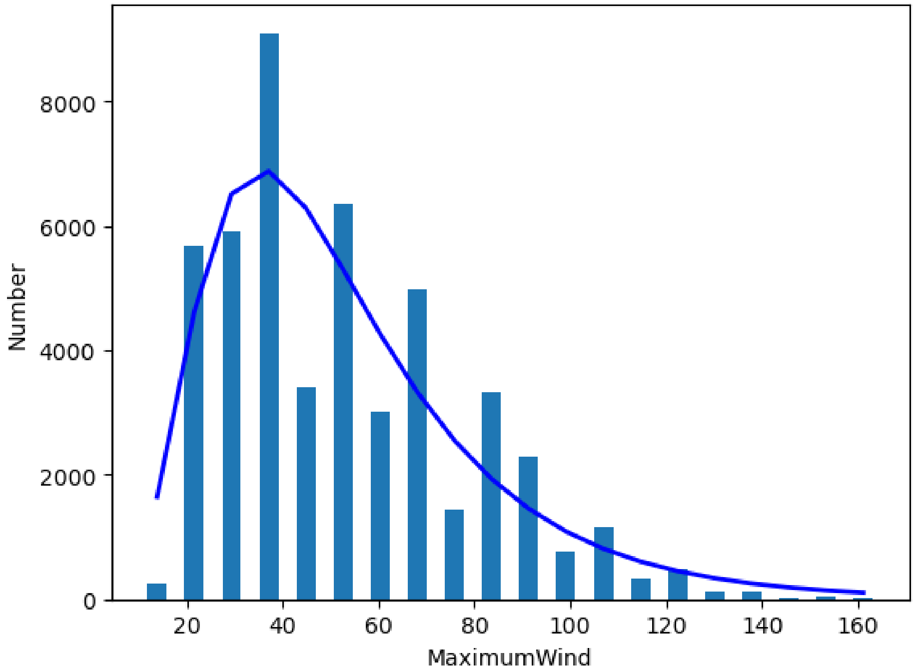

Figure 1. Fit of maximum wind speed in the Atlantic Ocean

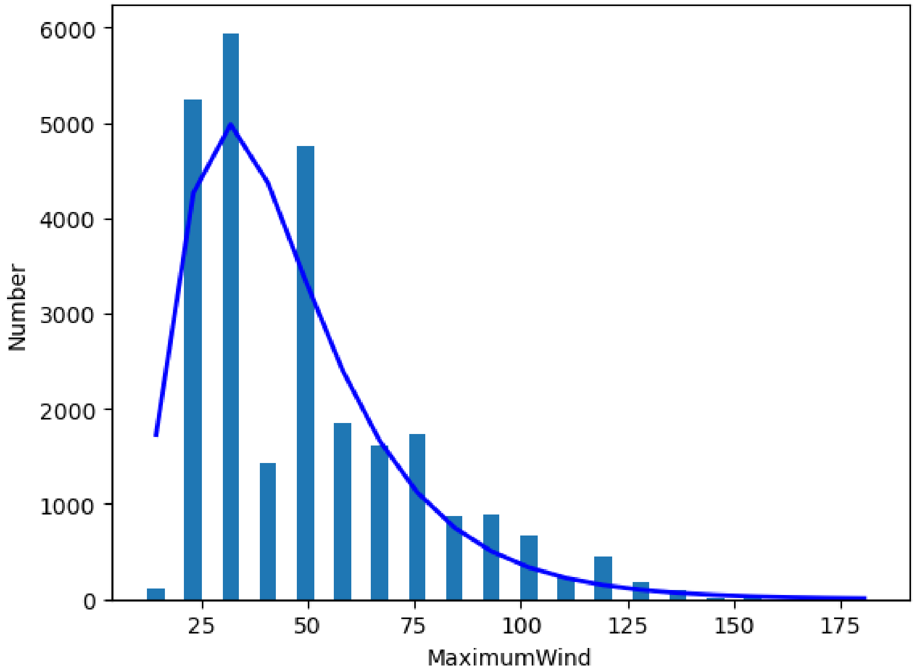

Figure 2. Fit of maximum wind speed in the Pacific Ocean

References

[1] Z. Mason, Naturally extending the standard sir model to stochastic growth, TBDdoi:10.31219/osf.io/y6ckv.

[2] Z. Mason, Affine returns on bernoulli trials in finance, TBD.
[3] A. Larion, Hurricanes and typhoons, 1851-2014, https://www.kaggle.com/datasets/noaa/hurricane-database (2015).