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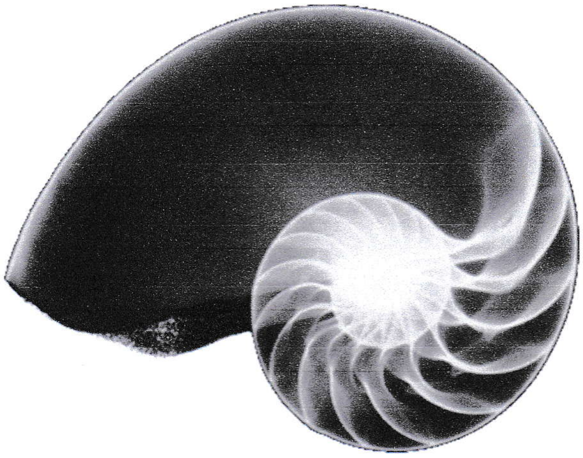

Calculus

Assignment for Paper Code 32355101

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- For the following functions,
  (a) find all critical numbers
  (b) find where the function is increasing and decreasing
  (c) find the critical points and identify each as a relative maximum, relative minimum, or neither.
  (d) find the second order critical numbers and tell where the graph is concave up and where its concave down.
  (e) Sketch the graph.

Q1. $f(x) = \frac{x^3}{3} - 9x + 2$

Ans. for $f(x)$,

$f'(x) = x^2 - 9$

$f''(x) = 2x$

for first-order critical points,

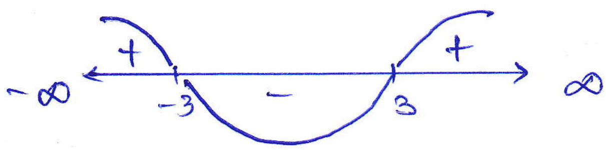
$f'(x) = 0$

$\Rightarrow x^2 - 9 = 0$

$\Rightarrow x = \pm 3$ — (a) (critical numbers)

$\therefore$ critical points are $(3, -16)$ and $(-3, 20)$ — (c)-(i)

for increasing / decreasing,
examining the sign of $f'(x)$



$\left.\begin{array}{l}\therefore f(x) \text{ is increasing on } (-\infty, -3) \cup (3, \infty). \\ \text{and, } f(x) \text{ is decreasing on } (-3, 3)\end{array}\right\}$ (b)

Applying the first derivative test,

$\rightarrow$ $f'(x)$ changes from +ve to −ve at $x = -3$,
$\therefore$ relative maximum at $x = -3$, where $f(x) = +20$. — (c)-(ii)

$\rightarrow$ $f'(x)$ changes from −ve to +ve at $x = 3$,
$\therefore$ relative minimum at $x = 3$, where $f(x) = -16$

for second order critical points,

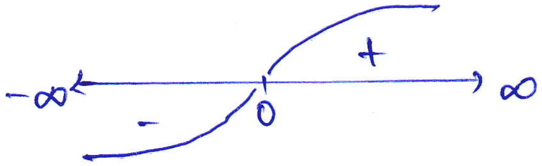
$f''(x) = 0$

$\Rightarrow 2x = 0$

$\Rightarrow x = 0$ — (d) – (i) (critical number)

for concavity,

examining the sign of $f''(x)$,

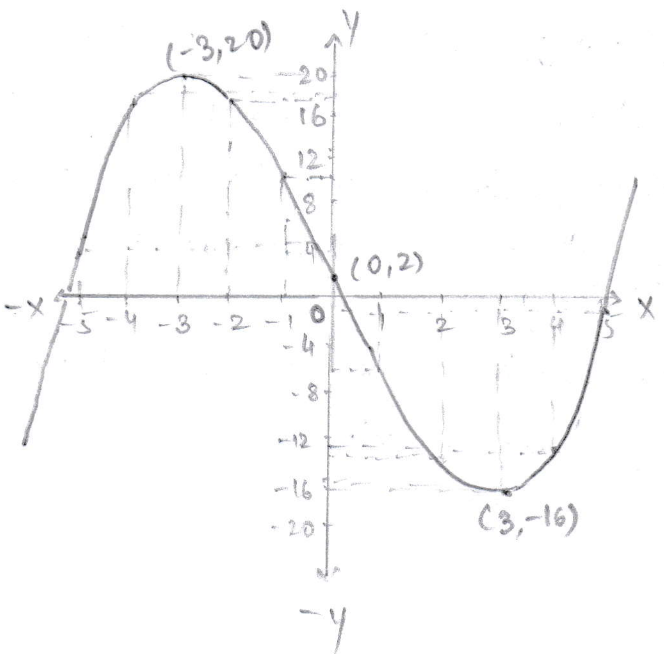


$\therefore$ $f(x)$ is concave down in $(-\infty, 0)$ and, concave up in $(0, \infty)$ and inflection pt. at $(0,2)$. } (d) – (ii)

sketching the graph of $f(x)$,

x	$f(x)$	
0	2	$\rightarrow$ inflection point
-3	20	$\rightarrow$ relative maximum
3	-16	$\rightarrow$ relative minimum
-1	$32/3 \approx 10.\overline{6}$	
1	$-20/3 \approx -6.\overline{6}$	
2	$-40/3 \approx -13.\overline{3}$	
-2	$52/3 \approx 17.\overline{3}$	

x	$f(x)$
-4	$50/3 \approx 16.\overline{6}$
4	$-38/3 \approx -12.\overline{6}$
-5	$16/3 \approx 5.\overline{3}$
5	$-4/3 \approx -1.\overline{3}$



Q2. $f(t) = (t+1)^2(t-5)$

Ans. for $f(t)$,

$f'(t) = 2(t+1)(t-5) + (t+1)^2$

or, $f'(t) = 3(t^2-2t-3) = 3[t^2+t-3t-3]$

or, $f'(t) = 3(t+1)(t-3)$

$f''(t) = 3(2t-2) = 6(t-1)$

for first order critical points,

$f'(t) = 0$

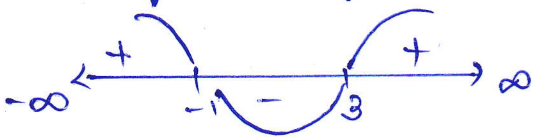
$\Rightarrow 3(t+1)(t-3) = 0$

$\Rightarrow t = -1$ and $t = 3$ – (a) (critical numbers)

$\therefore$ critical points are $(-1,0)$ and $(3,-32)$ – (c)(i)

for increasing / decreasing,

examining the sign of $f'(t)$,



$\therefore f(t)$ is increasing on $(-\infty,-1)\cup(3,\infty)$ (b)
and decreasing on $(-1,3)$

Applying the first derivative test,

→ $f'(t)$ changes from +ve to −ve at $t=-1$
$\therefore$ relative maximum at $t=-1$, where
$f(t) = 0$

→ $f'(t)$ changes from −ve to +ve at $t=3$
$\therefore$ relative minimum at $t=3$, where
$f(t) = -32$. C-(ii)

for second order critical points,

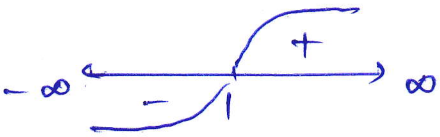
$f''(t) = 0$

$\Rightarrow 6(t-1) = 0$

$\Rightarrow t = 1$ – (d)-(i) (critical number)

for concavity,

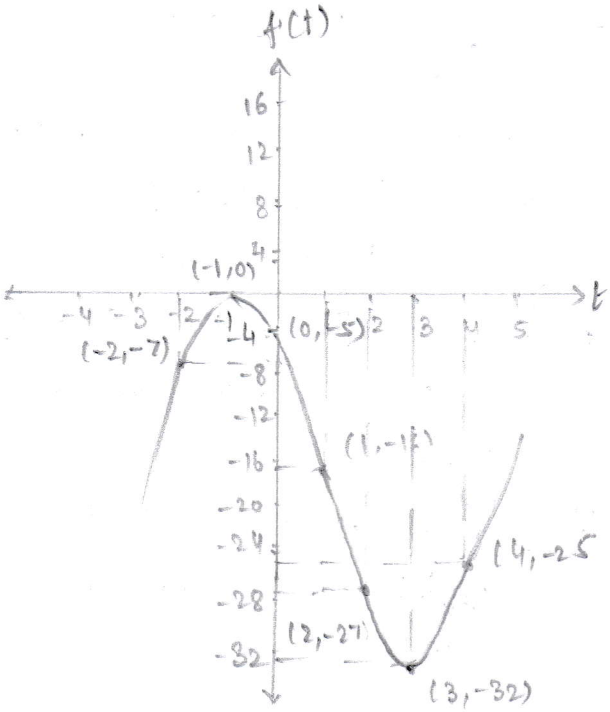
examining the sign of $f''(t)$,



∴ f(t) is concave up in $(1, \infty)$ and concave down in $(-\infty, 1)$ with inflection pt. at $(1, -16)$ } (d)-(ii)

Sketching the graph of f(t),

t	f(t)	
−2	−7	
−1	0	← relative maximum
0	−5	
1	−16	← inflection point.
2	−27	
3	−32	← relative minimum
4	−25	



- Find all vertical and horizontal asymptotes of the graph of the functions. Find where each graph is rising and where it is falling, determine concavity, and locate all critical points and points of inflection. Finally sketch the graph. Show any special features like cusps or vertical tangents.

Q29. $f(x) = \frac{x^3+1}{x^3-8}$

Ans. for $f(x)$,

$$f'(x) = \frac{(x^3-8)(3x^2)-(x^3+1)(3x^2)}{(x^3-8)^2} = \frac{-27x^2}{(x^3-8)^2}$$

$$f''(x) = \frac{(-54x)(x^3-8)^2-(-27x^2)(6x^2)(x^3-8)}{(x^3-8)^4}$$

$$\text{or, } f''(x) = \frac{-54x(x^3-8)(x^3-8-3x^3)}{(x^3-8)^4} = \frac{108x(x^3+4)}{(x^3-8)^3}$$

for critical points,

$f'(x) = 0$

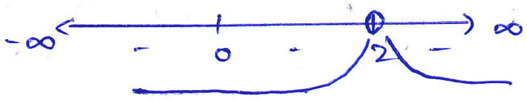
$\Rightarrow \frac{-27x^2}{(x^3-8)^2} = 0 \Rightarrow x = 0$

Also, $f'(x)$ is undefined at $x=2$!

$\therefore$ critical points are at $x=0$, ~~$(x=2)$~~ → point of discontinuity

for increasing / decreasing

examining the sign of $f'(x)$,



as there is no change in the sign of $f'(x)$, there exists no relative extrema.

$\therefore$ $f(x)$ is decreasing in $(-\infty, 2) \cup (2, \infty)$.

for second order critical points

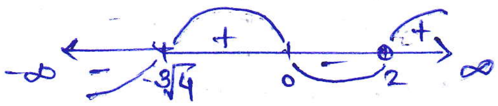
$f''(x) = 0$

$\Rightarrow \frac{108x(x^3+4)}{(x^3-8)^3} = 0 \Rightarrow x = 0, -\sqrt[3]{4}$

Also, $f''(x)$ is undefined at $x=2$!

$\therefore$ second order critical points are at $x=0$, ~~$(x=2)$~~ → point of discontinuity, $x=-\sqrt[3]{4}$

for concavity,
examining the sign of $f''(x)$



points of inflection are at $x = 0$, $x = -\sqrt[3]{4}$

$\therefore f(x)$ is concave down on $(-\infty, -\sqrt[3]{4}) \cup (0, 2)$ and concave up on $(-\sqrt[3]{4}, 0) \cup (2, \infty)$

for vertical tangents and cusps,
as $f(x)$ is not defined at $x = 2$ where $f(x) \to \infty$, $f(x)$ is therefore discontinuous!
$\therefore$ no vertical tangents or cusps exist for $f(x)$.

for asymptotes,
→ vertical asymptotes: $f(x) = +\infty$ at $x = 2$.
$\left(\lim_{x \to a} f(x) = \pm\infty\right)$

$\therefore$ vertical ~~tangent~~ asymptote exists at $x = 2$ and $\lim_{x \to 2^-} f(x) = -\infty$, $\lim_{x \to 2^+} f(x) = +\infty$

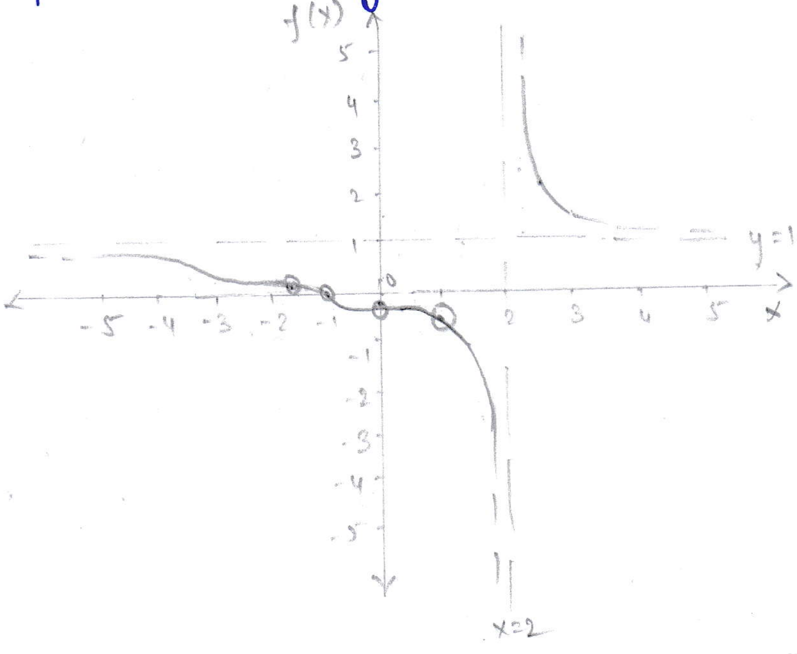
→ horizontal asymptotes: $\lim_{x \to \pm\infty} f(x) = \frac{x^3+1}{x^3-8} = \frac{\left(1+\frac{1}{x^3}\right)}{\left(1-\frac{8}{x^3}\right)}$
$\left(\lim_{x \to \pm\infty} f(x) = L\right)$

$$\Rightarrow \frac{\lim_{x \to \pm\infty} \left(1+\frac{1}{x^3}\right)}{\lim_{x \to \pm\infty} \left(1-\frac{8}{x^3}\right)} = \frac{1}{1} = 1$$

$\therefore$ horizontal asymptote exists at $y = 1$.

sketching the graph,

x	$f(x)$
0	$-1/8 \approx -0.125$
$-\sqrt[3]{4} \approx -1.587$	$1/4 \approx 0.25$
1	$-2/7 \approx -0.285$
-1	0
2.5	2.18
3	1.47



Q~~28~~ 4. $T(\theta) = \tan^{-1}(\theta) - \tan^{-1}\left(\frac{\theta}{3}\right)$

Ans. for $T(\theta)$,

$$T'(\theta) = \left(\frac{1}{1+\theta^2}\right) - \left(\frac{1}{3\left(1+\frac{\theta^2}{9}\right)}\right) = \frac{6-2\theta^2}{9+10\theta^2+\theta^4}$$

$$T''(\theta) = \frac{-2\theta}{(1+\theta^2)^2} + \frac{2\theta}{27\left(1+\frac{\theta^2}{9}\right)^2} = \frac{4\theta(\cancel{-20}\theta^4 - 6\theta^2 - 39)}{(\theta^2+1)^2(\theta^2+9)^2}$$

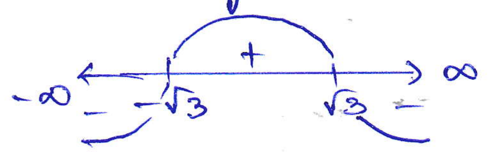
for critical points,

$T'(\theta) = 0$

$$\Rightarrow \frac{6-2\theta^2}{9+10\theta^2+\theta^4} = 0 \quad \Rightarrow \theta^2 = 3 \quad \Rightarrow \theta = \pm\sqrt{3}$$

$\therefore$ critical points are at $\theta = +\sqrt{3}, \theta = -\sqrt{3}$.

for increasing / decreasing,
examining the sign of $T'(\theta)$,



Applying first derivative test,

→ as $T'(\theta)$ changes from $-$ve to $+$ve at $\theta = -\sqrt{3}$,
$\therefore$ relative minimum at $\theta = -\sqrt{3}$, where
$T(\theta) = -\pi/6$

→ as $T'(\theta)$ changes from $+$ve to $-$ve at $\theta = +\sqrt{3}$,
$\therefore$ relative maximum at $\theta = +\sqrt{3}$, where
$T(\theta) = +\pi/6$

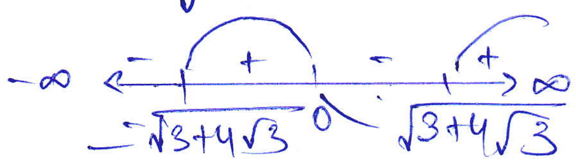
for second order critical points,

$T''(\theta) = 0$

$$\Rightarrow \frac{4\theta(\theta^4 - 6\theta^2 - 39)}{(\theta^2+1)^2(\theta^2+9)^2} = 0 \quad \Rightarrow \theta = 0, \pm\sqrt{3+4\sqrt{3}}$$

$\therefore$ second order critical points at $\theta = 0, \theta = \pm\sqrt{3+4\sqrt{3}}$

for concavity,
examining sign of $T''(\theta)$,



$\therefore T(\theta)$ is concave down on $(-\infty, -\sqrt{3+4\sqrt{3}}) \cup (0, \sqrt{3+4\sqrt{3}})$ and concave up on $(-\sqrt{3+4\sqrt{3}}, 0) \cup (\sqrt{3+4\sqrt{3}}, \infty)$

points of inflection are at $\theta = 0, \theta = \pm\sqrt{3+4\sqrt{3}}$

for asymptotes,

as $T(\theta)$ is continuous on $\mathbb{R}$, vertical asymptotes do not exist, Horizontal asymptote exists as $\lim_{\theta \to \pm\infty} T(\theta) = 0 \Rightarrow y = 0$ $\Rightarrow T(\theta) = 0$

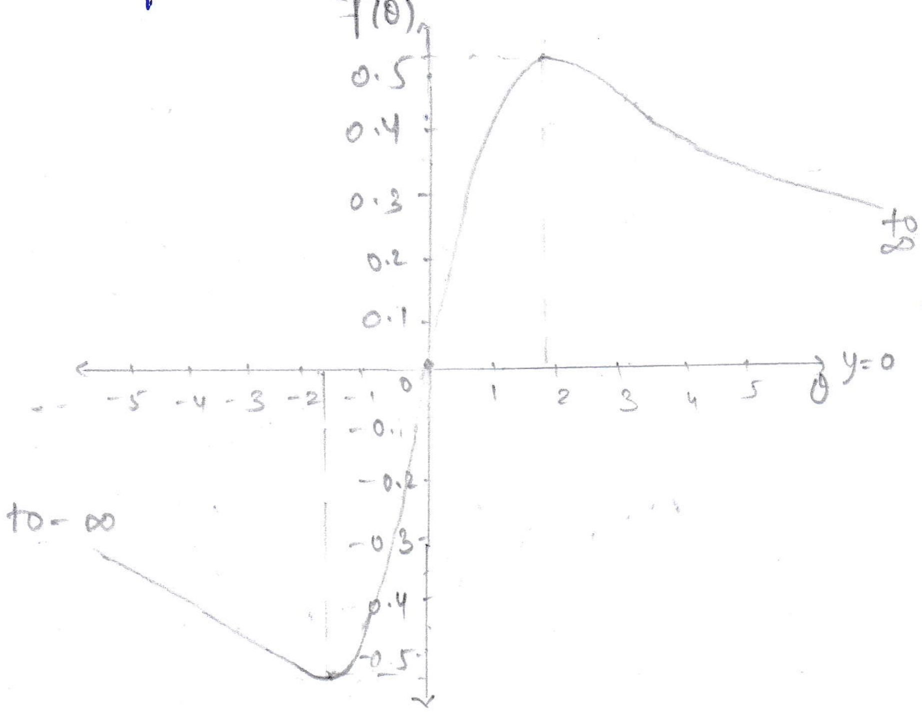
for vertical tangents / cusps,

$\nexists$ a value of θ where $T'(\theta)$ becomes infinite.

$\therefore$ vertical tangents / cusps do not exist.

sketching the graph,

θ	$T(\theta)$
$\sqrt{3}$	0.5235
0	0
$-\sqrt{3}$	-0.5235
$\sqrt{3+4\sqrt{3}}$	0.4535
$-\sqrt{3+4\sqrt{3}}$	-0.4535
5	0.343
-5	-0.343



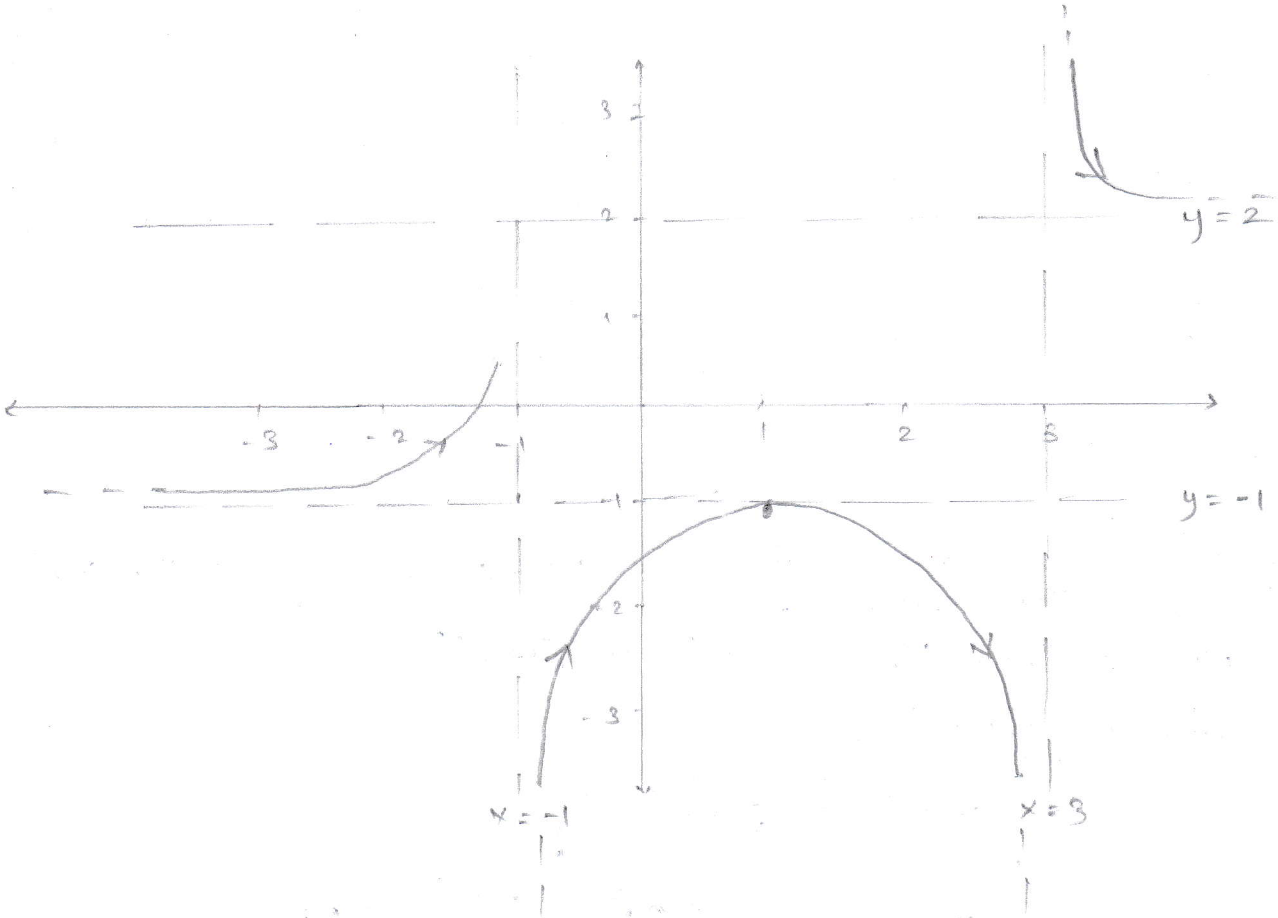
Q5. Sketch a graph of a function f with the following properties:

(i) f is increasing for $x < -1$ and $-1 < x < 1$ and decreasing for $1 < x < 3$ and $x > 3$;

(ii) the graph has only one critical point $(1, -1)$ and no inflection points;

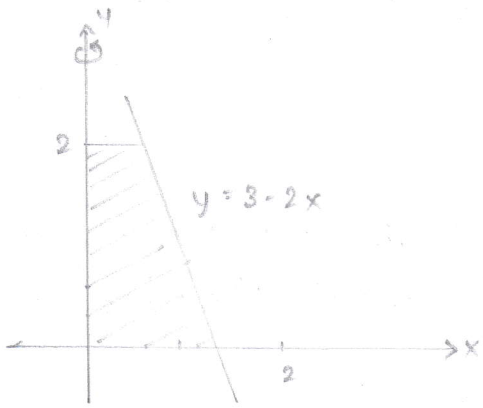
(iii) $\lim_{x \to \infty} f(x) = -1$; $\lim_{x \to +\infty} f(x) = 2$;

$\lim_{x \to -1^+} f(x) = \lim_{x \to 3^-} f(x) = -\infty$.



- Find the volume of the solid that results when the shaded region is revolved about the indicated axis.

Q6.



Ans. On revolving shaded region about y-axis, we get a solid which can be sliced into a disk with radius $g(y)$ and area of this disk is

$$A(y) = \pi (g(y))^2 = \pi \left(\frac{(3-y)}{2}\right)^2$$

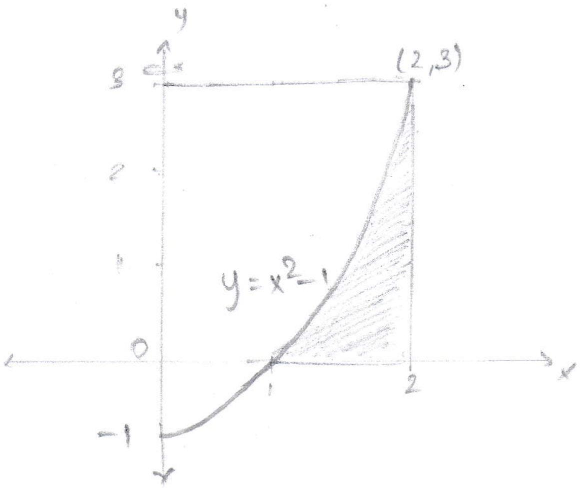
Volume of the solid,

$$V = \pi \int_0^2 \frac{(3-y)^2}{4}\, dy = \frac{\pi}{4}\int_0^2 (y^2 - 6y + 9)\, dy$$

or, $$V = \frac{\pi}{4}\left\{ \frac{y^3}{3} - 3y^2 + 9y \right\}\Big|_0^2 = \frac{13\pi}{6}$$

$\therefore$ Volume $= \dfrac{13\pi}{6}$ cubic units

Q7.



Ans. On revolving shaded region about y-axis, we get a solid which can be sliced to washer-like shape.

Area of the washer $= \pi(2)^2 - \pi(\sqrt{1+y})^2$

Volume of the solid,

$$V = \int_0^3 \pi(2)^2 - \pi(\sqrt{1+y})^2\, dy$$

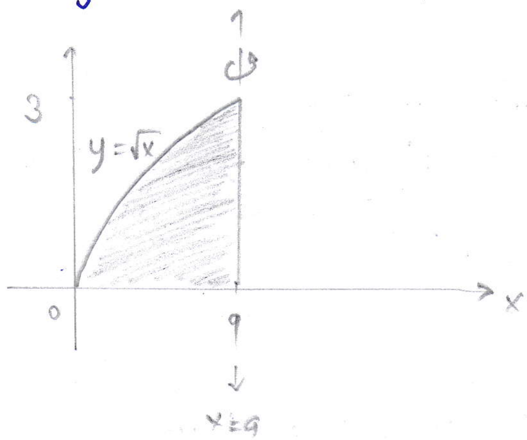
$$\text{or, } V = \left\{4\pi y - \pi y - \frac{\pi y^2}{2}\right\}\Bigg|_0^3 = \left\{3\pi y - \frac{\pi y^2}{2}\right\}\Bigg|_0^3$$

$$\text{or, } V = 3\pi(3-0) - \frac{\pi}{2}(3^2-0^2) = 9\pi - \frac{9\pi}{2} = \frac{9\pi}{2}$$

$\therefore$ Volume $= \frac{9\pi}{2}$ cubic units

Q8. Find the volume of the solid that results when the region enclosed by $y=\sqrt{x}$, $y=0$ and $x=9$ is revolved about the line $x=9$.

Ans.



On revolving shaded region about $x=9$, we get a disk of radius $g(y) = 9-y^2$ and area of the disk $\Rightarrow A(y) = \pi(9-y^2)^2$

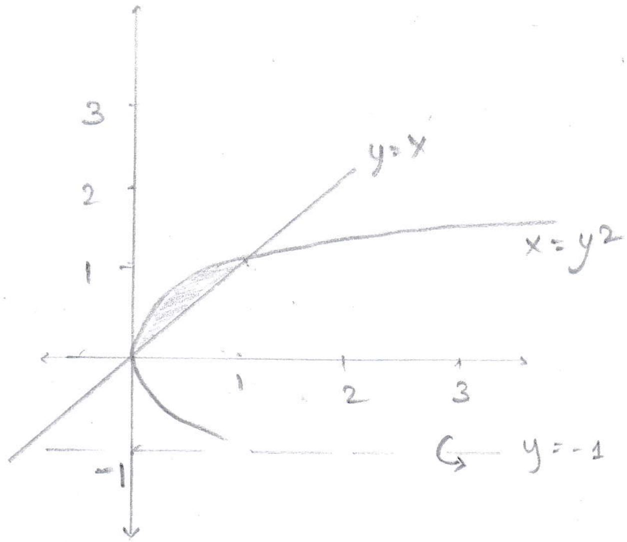
$$\text{Volume of the solid, } V = \pi\int_0^3 (9-y^2)^2\, dy = \pi\int_0^3 (y^4 - 18y^2 + 81)\, dy$$

$$\text{or, } V = \pi\left\{\frac{y^5}{5} - \frac{\cancel{18}^{\,6}\, y^3}{\cancel{3}} + 81y\right\}\Bigg|_0^3 = \frac{648\pi}{5}$$

$\therefore$ Volume $= \frac{648\pi}{5}$ cubic units

Q9. Find the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

Ans.



On revolving ~~the solid ab~~ shaded region about $y = -1$, we get a washer whose area is given by,

$$A(x) = \pi(1+\sqrt{x})^2 - \pi(1+x)^2$$

Volume of the solid,

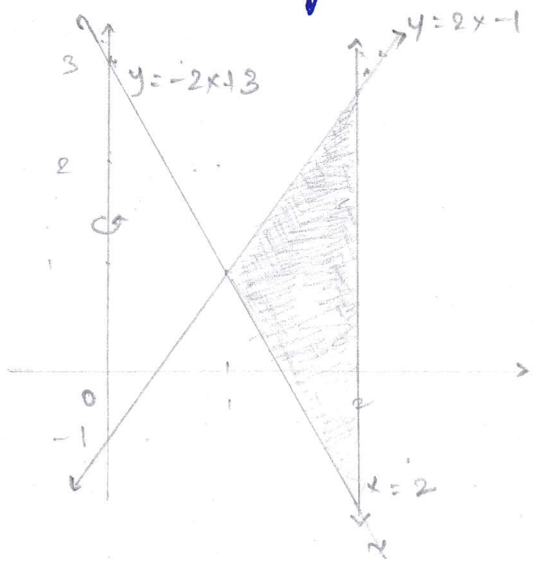
$$V = \pi\int_0^1 \left[(1+\sqrt{x})^2 - (1+x)^2\right] dx = \pi\int_0^1 \left[(x+2\sqrt{x}+1) - (1+x^2+2x)\right] dx$$

$$\text{or, } V = \pi\int_0^1 (2\sqrt{x} - x - x^2)\, dx = \pi\left\{\frac{2x^{3/2}}{3/2} - \frac{x^2}{2} - \frac{x^3}{3}\right\}\Bigg|_0^1 = \frac{\pi}{2}$$

$\therefore$ Volume $= \frac{\pi}{2}$ cubic units

Q10. Use cylindrical shells to find the volume of the solid generated when the region enclosed by $y = 2x - 1$, $y = -2x + 3$, $x = 2$ is revolved about the y-axis.

Ans.



For cylindrical shell method,

$$V = \int_a^b 2\pi x\,(f(x_1) - f(x_2))\, dx$$

Here $f(x_1) = 2x - 1$ and;
$f(x_2) = -2x + 3$; $a = 1$; $b = 2$;

Volume of the solid,

$$V = \int_1^2 2\pi x\,(2x - 1 + 2x - 3)$$

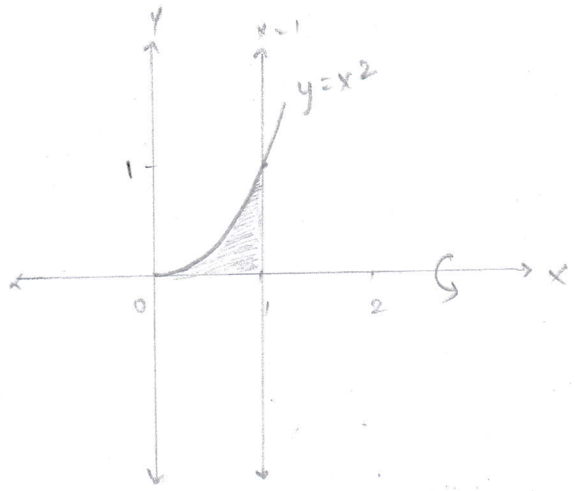
$$\text{or, } V = 8\pi\int_1^2 (x^2 - x)\,dx = 8\pi\left\{\frac{x^3}{3} - \frac{x^2}{2}\right\}\Bigg|_1^2$$

or, $V = 8\pi\left(\frac{5}{6}\right) = \frac{40\pi}{6} = \frac{20\pi}{3}$

$\therefore$ Volume $= \frac{20\pi}{3}$ cubic units

Q11. Use cylindrical shells to find the volume of the solid generated when the region enclosed by $y=x^2$, $x=1$, $y=0$ is revolved about the x-axis.

Ans.



For cylindrical shell method,

$$V = \int_c^d 2\pi y\,[g(y_1) - g(y_2)]\,dy$$

Here, $g(y_1) = 1$ and;
$g(y_2) = \sqrt{y}$; $c=0$; $d=1$;

Volume of the solid,

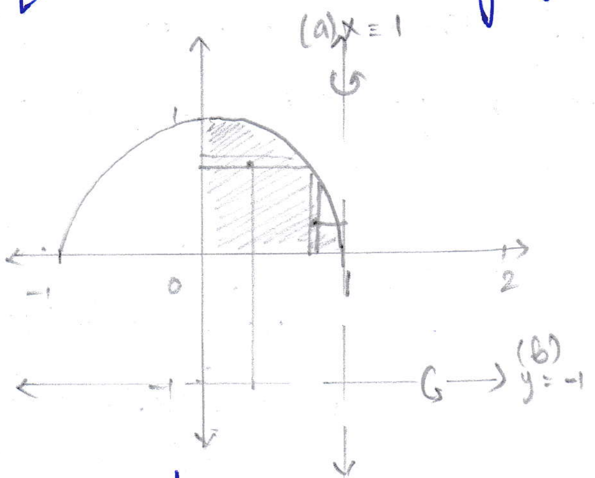
$$V = \int_0^1 2\pi y\,(1-\sqrt{y})\,dy$$

$$\Rightarrow V = 2\pi \int_0^1 (y - y^{3/2})\,dy = \pi/5$$

$\therefore$ Volume $= \frac{\pi}{5}$ cubic units

Q12. Using the method of cylindrical shells, set up an integral for the volume of the solid generated when the region R is revolved about (a) $x=1$ and (b) $y=-1$. R is the region in the first quadrant bounded by $y=\sqrt{1-x^2}$, $y=0$, $x=0$.

Ans.



(a) Distance of cross section from the axis of revolution is $(1-x)$ and the height of the cross section is $y = (\sqrt{1-x^2})$.

Using cylindrical shell method,

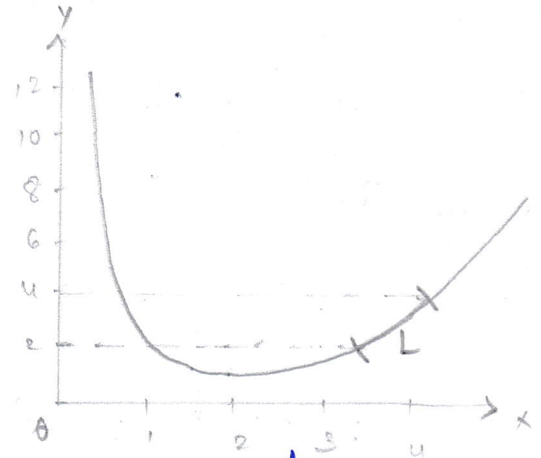
$$\Rightarrow V_1 = \int_0^1 2\pi\,(1-x)\sqrt{1-x^2}\,dx$$

(b) Distance of cross section from the axis of revolution is $(1+y)$ and the height of the cross section is $x = (\sqrt{1-y^2})$.

$$\Rightarrow V_2 = \int_0^1 2\pi(1+y)\sqrt{1-y^2}\,dy$$

Q13. Find the exact arc length of the curve $24xy = y^4 + 48$ over the interval $y=2$ to $y=4$.

Ans. $24xy = y^4+48 \Rightarrow x = \frac{y^3}{24} + \frac{2}{y} = g(y)$



$$\text{Arc Length } (L) = \int_c^d \sqrt{1+(g'(y))^2}\cdot dy$$

Now, $c=2$; $d=4$; $g'(y) = \frac{y^2}{8} - \frac{2}{y^2}$

$$L = \int_2^4 \sqrt{1+\left(\frac{y^2}{8}-\frac{2}{y^2}\right)^2}\cdot dy = \int_2^4 \sqrt{\frac{y^4}{64}+\frac{4}{y^4}+\frac{1}{2}}\cdot dy$$

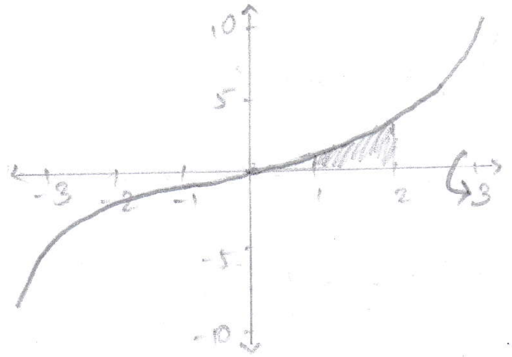
$$\text{or, } L = \int_2^4 \left(\frac{y^2}{8}+\frac{2}{y^2}\right) dy = \frac{17}{6}$$

$\therefore$ Length of arc $= \frac{17}{6}$ units

- Use a CAS to find the exact area of the surface generated by revolving the curves about the stated axis.

Q14. $y = \frac{x^3}{3} + \frac{1}{4x}$, $1 \le x \le 2$; x-axis

Ans.



Surface area of solid,

$\cancel{S = 2\pi \int_a^b f(x)\, f(x)\, dx \sqrt{1+f'(x)^2}\, dx}$

$$\Rightarrow S = 2\pi \int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$

or, $S = 2\pi \int_1^2 \left(\frac{x^3}{3}+\frac{1}{4x}\right)\sqrt{1+\cancel{\left(\frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{2}\right)}\left(x^2-\frac{1}{4x^2}\right)^2}\cdot dx$

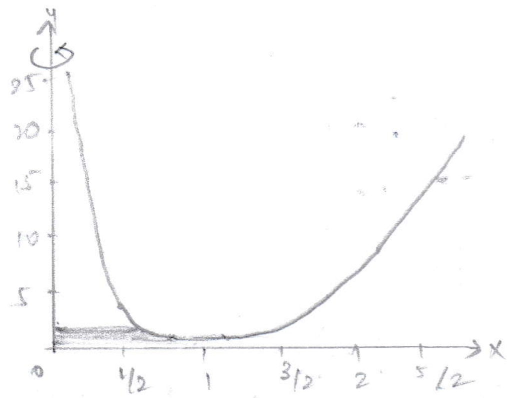
or, $S = 2\pi\int_1^2 \left(\frac{x^3}{3}+\frac{1}{4x}\right)\sqrt{\frac{1}{(4^2x^4)}+\frac{x^4}{\cancel{4}}+\frac{1}{2}}\cdot dx = 2\pi\int_1^2\left(\frac{x^3}{3}+\frac{1}{4x}\right)\left(x^2+\frac{1}{4x^2}\right)\cdot dx$

or, $S = 2\pi\int_1^2\left(\frac{x^5}{3}+\frac{x}{3}+\frac{1}{16x^3}\right)\cdot dx = \frac{515\pi}{64} = 25.28000338$

$\therefore$ Surface area $=$ 25.2800038 square units

Q15. $8xy^2 = 2y^6+1,\ 1 \leq y \leq 2$; y-axis

Ans. $8xy^2 = 2y^6+1 \Rightarrow x = \frac{y^4}{4}+\frac{1}{8y^2}$



Surface area of solid,

$S = \int_c^d 2\pi\, g(y)\sqrt{1+g'(y)^2}\, dy$

or, $S = 2\pi\int_1^2\left(\frac{y^4}{4}+\frac{1}{8y^2}\right)\sqrt{1+\left(y^3-\frac{1}{4y^3}\right)^2}\cdot dy$

or, $S = 2\pi\int_1^2\left(\frac{y^4}{4}+\frac{1}{8y^2}\right)\sqrt{y^6+\frac{1}{16y^6}+\frac{1}{2}}\cdot dy$

or, $S = 2\pi\int_1^2\left(\frac{y^4}{4}+\frac{1}{8y^2}\right)\left(y^3+\frac{1}{4y^3}\right)\cdot dy = 2\pi\int_1^2\left(\frac{y^7}{4}+\frac{y}{8}+\frac{y}{16}+\frac{1}{32y^5}\right)\cdot dy$

or, $S = \frac{\pi}{16}\int_1^2\left(8y^7+6y+y^{-5}\right)\cdot dy = 16911\pi/1024 = 51.88229$

$\therefore$ Surface area $=$ 51.88229 square units

- Sketch the parabola and label the focus, vertex and directrix.

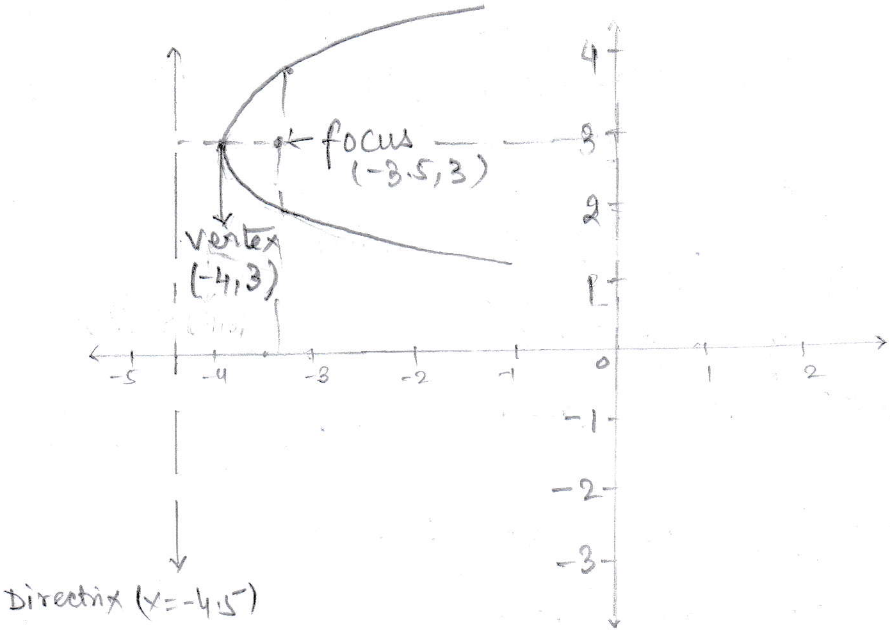
Q16. (a) $y^2-6y-2x+1=0$

(a) $\Rightarrow y^2-6y+1 = 2x$

$\Rightarrow y^2-6y+9 = 2x+8$

$\Rightarrow (y-3)^2 = 2(x+4) = 4\cdot\frac{1}{2}\cdot(x+4)$

- Vertex at $(-4,3)$
- Focus $(-4+1/2, 3)$ at $(-3.5, 3)$
- Directrix is $x = -4-1/2 = -4.5$.



Q16 (b) $y = 4x^2 + 8x + 5$

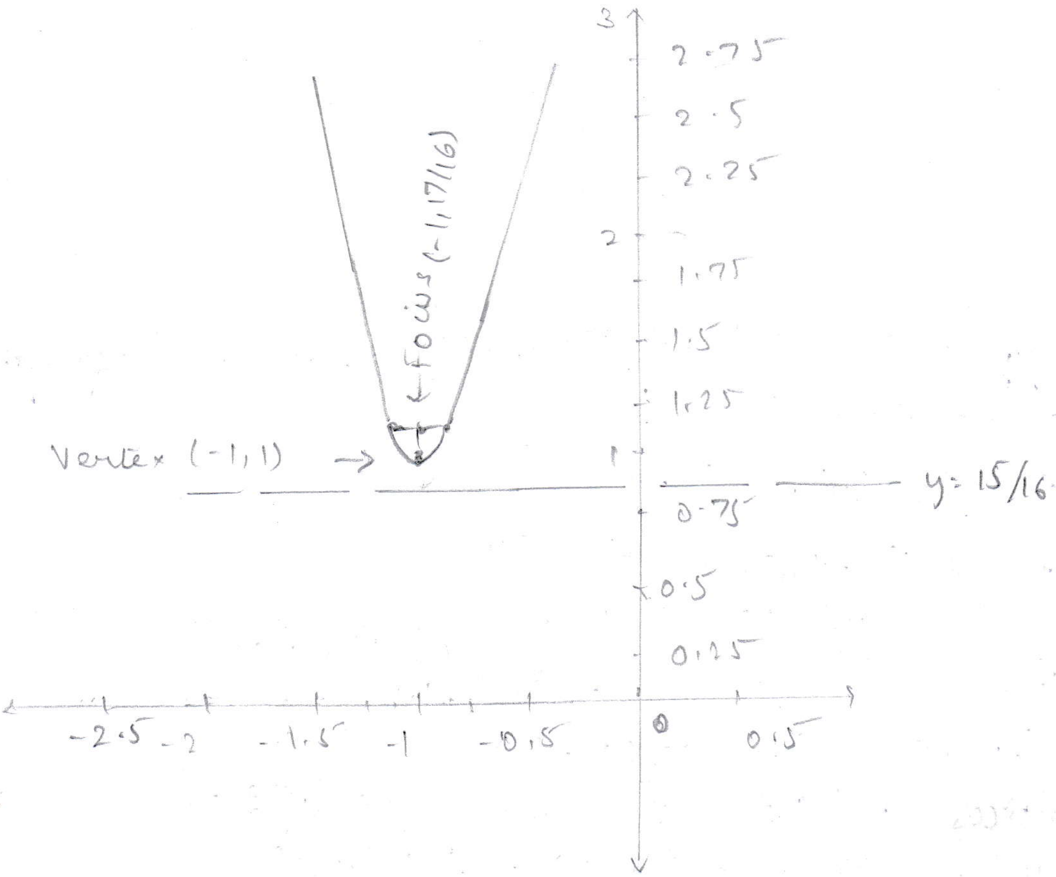
(b) $\Rightarrow y - 5 = 4x^2 + 8x = 4(x^2 + 2x)$

$\Rightarrow y - 1 = 4(x^2 + 2x + 1)$

$\Rightarrow y - 1 = 4(x + 1)^2$

$\Rightarrow (x + 1)^2 = 4 \cdot \frac{1}{16} \cdot (y - 1)$

- Vertex at $\underline{(-1, 1)}$
- Focus at $\underline{(-1, 17/16)}$
- Directrix is $\underline{y = 15/16}$



∘ Sketch the ellipse and label the vertices, foci and the ends of the minor axis.

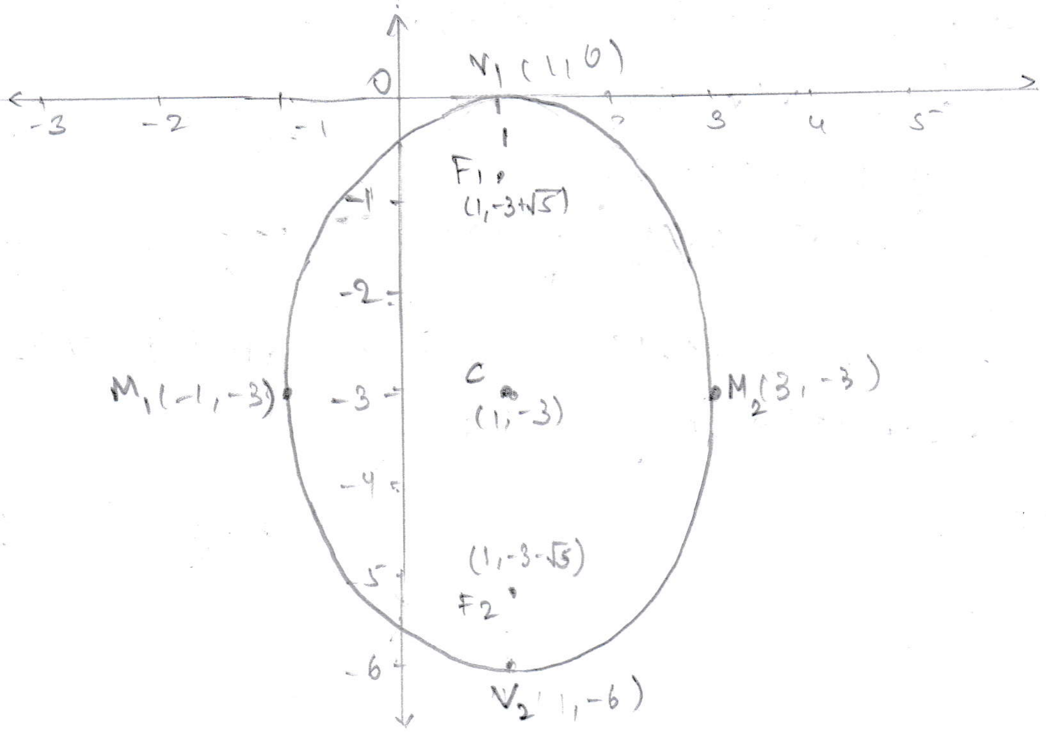
Q17. $9x^2+4y^2-18x+24y+9=0$

Ans. $\Rightarrow 9(x^2-2x)+4(y^2+6y)=-9$

$\Rightarrow 9(x^2-2x+1)+4(y^2+6y+9)=36$

$\Rightarrow \dfrac{(x-1)^2}{4}+\dfrac{(y+3)^2}{9}=1 \quad \Big\} \begin{matrix} a=2 \\ b=3 \end{matrix} \Big\} \quad a<b \Rightarrow \underline{c=\sqrt{b^2-a^2}=\sqrt{5}}$

- center at $\underline{(1,-3)}$ ∘ foci at $(1,-3\pm\sqrt{5})$
- vertices at $(1,0)$ and $(1,-6)$
- ends of minor axis at $(3,-3)$ and $(-1,-3)$



$\{F_1, F_2\}$ = foci ; $\{V_1, V_2\}$ = vertices ; $\{M_1, M_2\}$ = ends of minor axis

∘ Sketch the hyperbola, and label the vertices, foci and asymptotes.

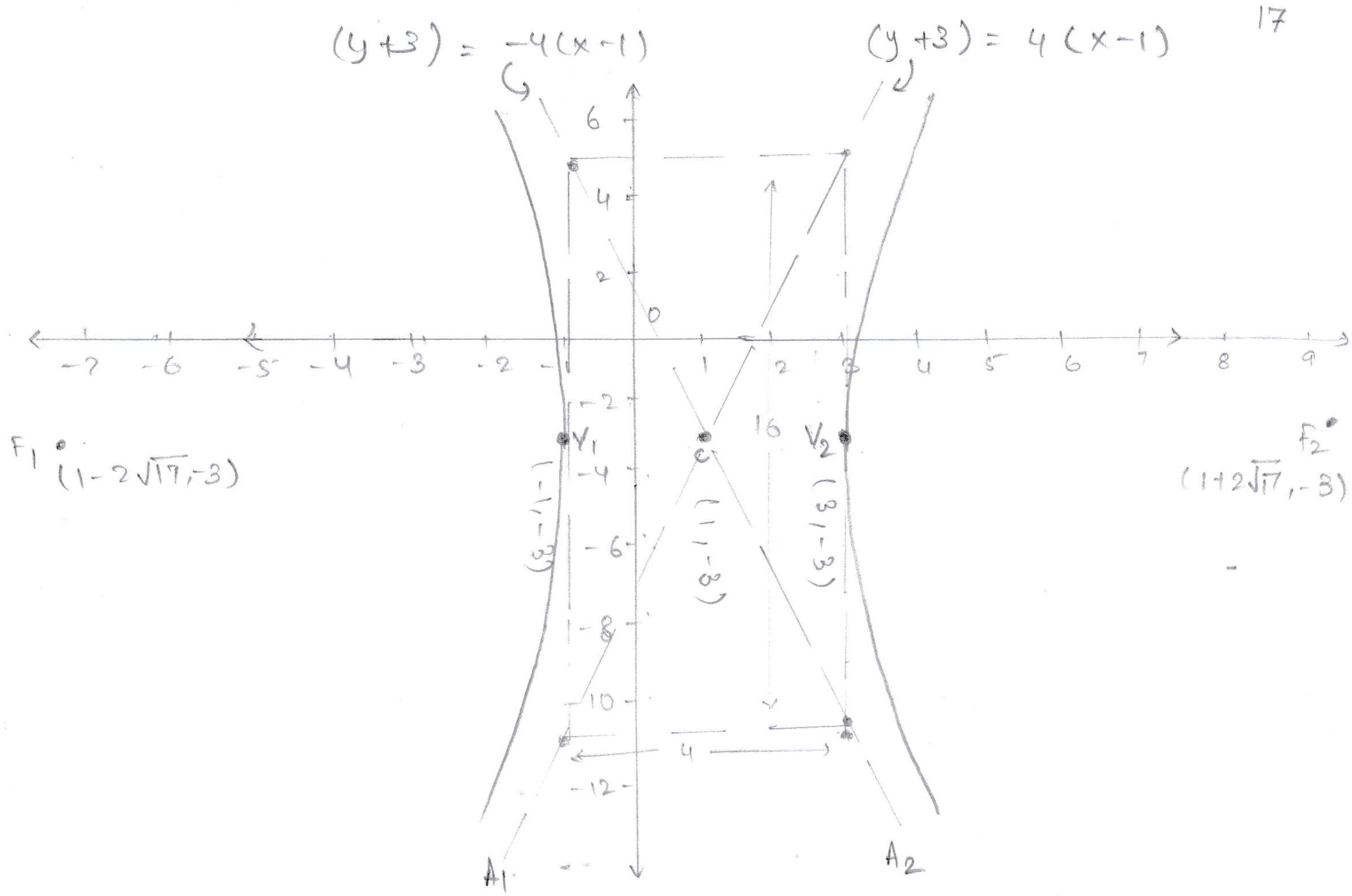
Q18. $16x^2-y^2-32x-6y=57$

Ans. $\Rightarrow 16(x^2-2x)-(y^2+6y)=57$

$\Rightarrow 16(x^2-2x+1)-(y^2+6y+9)=64$

$\Rightarrow \dfrac{(x-1)^2}{4}-\dfrac{(y+3)^2}{64}=1 \quad \Big\} \begin{matrix} a=2 \\ b=8 \end{matrix} \Big\} \quad c=\sqrt{a^2+b^2}=2\sqrt{17}$

- ~~center at~~ center at $\underline{(1,-3)}$
- vertices at $\underline{(1\pm2,-3)}$ ∘ foci at $\underline{(1\pm2\sqrt{17},-3)}$
- asymptotes are $\underline{(y+3)=\pm4(x-1)}$



{V1, V2} = vertices ; {F1, F2} = foci ; asymptotes = {A1, A2}