New Edge Detection Algorithms Based on Adaptive Estimation Filters

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Abstract

In this paper we describe new edge detection algorithms based on adaptive two-dimensional estimation filters. Examples with the image of Lena image are used to illustrate our concepts.

1. Introduction

A variety of classical edge detection masks have been used for years to remove noise and improve image quality. Adaptive filters using the Wiener algorithm are also used. In this paper we combine the classical concepts of edge detection with Wiener estimation filters to come up with edge estimation filters.

2. Classical Edge Detection Masks

A variety of classical edge detection masks exist. Of the edge detectors without incorporating smoothing, we will consider the Euler and bilinear edge detector masks. The Euler filter is defined as

$$h_{\frac{\partial}{\partial x}}(n,m) = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, h_{\frac{\partial}{\partial y}}(n,m) = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

The bilinear filter is defined as (multiply these by 1/2)

$$h_{\frac{\partial}{\partial x}}(n,m) = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, h_{\frac{\partial}{\partial y}}(n,m) = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{2}$$

Of the edge detectors incorporating smoothing, we will consider the Prewitt and Sobel edge detector masks. The Prewitt filter is defined as

$$\begin{aligned} h_{\frac{\partial}{\partial x}}(n,m) &= \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} * \frac{1}{3}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= \frac{1}{6}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned} \tag{3}$$

$$\begin{aligned} h_{\frac{\partial}{\partial y}}(n,m) &= \frac{1}{2}\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} * \frac{1}{3}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \frac{1}{6}\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{aligned} \tag{4}$$

The Sobel filter is defined as

$$\begin{aligned} h_{\frac{\partial}{\partial x}}(n,m) &= \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} * \frac{1}{4}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= \frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned} \tag{5}$$

$$\begin{aligned} h_{\frac{\partial}{\partial y}}(n,m) &= \frac{1}{2}\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} * \frac{1}{4}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \frac{1}{8}\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \end{aligned} \tag{6}$$

What's not intuitively obvious in the above filter representations is that the edge detection masks are the result of the convolution of a smoothing filter and a gradient filter such as

$$h_{classical}(x,y) = h_{smooth}(x,y) * h_{gradient}(x,y) \tag{7}$$

Thus the edge detection operators provide both a gradient operation and smoothing operation in the same mask. The Prewitt filter is the convolution of a bilinear gradient filter with a rectangular averaging filter. It takes three gradients in a row and averages them to get the gradient at the center point. The Sobel filter is the convolution of a bilinear gradient filter with a weighted smoothing filter. It takes three gradients in a row and performs a weighted average on them, giving twice as much weight to the middle gradient as to the adjacent gradients.

Once we separate these operators, we realize that we may use any smoothing and any gradient filter that we feel is appropriate. In this way we can draw on the vast knowledge about these two individual filter types. The complete block diagram for a general gradient estimation system is shown in Figure 1.

3. Adaptive Estimation Filters

Adaptive filters tailor their masks to the signal and noise being processed. Wiener filters minimize the integral squared error between the estimated output and the desired output. The general two-dimensional ideal Wiener filter is

$$H(\omega_1,\omega_2) = \frac{E\{D(\omega_1,\omega_2)R^*(\omega_1,\omega_2)\}}{E\{R(\omega_1,\omega_2)R^*(\omega_1,\omega_2)\}} \tag{8}$$

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When the input spectra $R = S + N$ and desired output spectra $D = S$, where S and N are the signal and noise spectra respectively, the optimal filter equals the familiar result

$$H_3(\omega_1, \omega_2) = \frac{E\{|S(\omega_1,\omega_2)|^2\}}{E\{|S(\omega_1,\omega_2)|^2 + |N(\omega_1,\omega_2)|^2\}} \quad (9)$$

when the signal and noise are uncorrelated.

A standard definition for edges is the gradient. Using this model, we may view edge detection as an estimation problem - estimating the gradient. We will use the gradient of the signal as the desired signal. The gradient of $f(x,y)$ is notated as $\nabla f(x,y)$ and is defined as

$$\nabla f(x,y) = \frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} \quad (10)$$

The magnitude of the gradient is

$$|\nabla f(x,y)| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \quad (11)$$

The Fourier transform of a two-dimensional signal is

$$\mathcal{F}\{f(x,y)\} = F(\omega_1,\omega_2) \quad (12)$$

From the general properties of the two-dimensional transform, we know that the Fourier transform of the partial derivative of a two-dimensional function is

$$\mathcal{F}\{\frac{\partial^n f(x,y)}{\partial x^n} + j\frac{\partial^n f(x,y)}{\partial y^n}\} = \omega_1^n F(\omega_1,\omega_2) + j\omega_2^n F(\omega_1,\omega_2) \quad (13)$$

The two-dimensional Fourier transform of the gradient is thus

$$\mathcal{F}\{\frac{\partial f(x,y)}{\partial x} + j\frac{\partial f(x,y)}{\partial y}\} = \omega_1 F(\omega_1,\omega_2) + j\omega_2 F(\omega_1,\omega_2) = (\omega_1 + j\omega_2)F(\omega_1,\omega_2) \quad (14)$$

The closed form solution for the adaptive edge estimation filter is

$$H(\omega_1,\omega_2) = (\omega_1 + j\omega_2)\frac{|S(\omega_1,\omega_2)|^2}{|S(\omega_1,\omega_2)|^2 + |N(\omega_1,\omega_2)|^2} \quad (15)$$

We see that (15) is made up of an adaptive Wiener estimation filter and an ideal gradient filter.

To estimate the magnitude of the edges, we may first estimate the edges using (15) then use (11) to get the magnitude of the edges.

4. Experimental Results

Figure 2a is adaptive, Figure 2b is smoothed adaptive Figure 2c is Prewitt, Figure 2d is Sobel Figure 2e is Euler, Figure 2f is bilinear The performance measures for the classical and adaptive filters are summarized in Table 1.

Another adjustment we make is to smooth the adaptive mask. It is clear from Figure 2 that the adaptive mask has considerable fine structure compared to the classical masks. To reduce this fine structure and produce a suboptimum but more interpretable mask, we smooth using

$$H(\omega_1,\omega_2) = H_{est}(\omega_1,\omega_2) * w(\omega_1,\omega_2) \quad (16)$$

where w denotes some smoothing mask. For simplicity we set w to be a 7×7 averaging mask set to all ones. The smoothed adaptive mask can be seen in Figure 2b.

To test the edge detection-estimation filters we used a 128×128 image of Lena corrupted with white Gaussian noise shown in Figures 2a, with the noise adjusted to obtain an input SNR of 20 dB.

The non-smoothing classical edge detectors give crisp edges that are not smoothed or deformed. Since the high frequency noise level is increased, these classical edge detectors tend to degrade in performance as they allow more of this high frequency noise to pass and degrade the output edge image. The adaptive filter does not suffer from this drawback.

The edges of the classical edge detectors are very thick and heavy, while the adaptive output has thinner edges. Even though the classical outputs look good having bold edges, they are a distortion of the desired signal which is the true edges which are thin.

The smoothed adaptive filters show the general filter shape that is required for the Lena image at the various SNR values. The smoothed adaptive filters give basically the same performance as the exact adaptive filters since the filters have the same basic shape.

The classical edge detectors trade off edge signal distortion for less additive noise distortion. The adaptive filter does not suffer from this drawback. The adaptive filter gives true edges that are not shifted or widened from the true edge. The smoothing classical filters tend to widen the edges due to their smoothing filters incorporated within them.

To our eye, the edge detectors which incorporate smoothing tended to out perform the edge detectors that do not incorporate smoothing in the presence of noise. To the performance criteria, when little or no noise is present, then the edge detectors without smoothing out perform the smoothing versions.

The drawback to the smoothing versions is that they tend to smear the edge, widening it and making it larger. Their main advantage is that they work well in the presence of noise because they smooth out the high frequency noise and do not respond to it. The drawback to the non-smoothing versions is that they tend to respond to high frequency noise causing a noisy edge response. If the noise is strong, the filter may respond much stronger to the noise than the actual edges. Their main advantage is that they do not smear the edges and work well when their is little or no noise.

Thus in a noisy environment we should use an edge detector with smoothing, while in a low noise environment we should use the non-smoothing edge detector. The adaptive filter on the other hand works well whether in a noisy or noise free environment. Because the adaptive filter uses the information about the signal it will adapt to the varying noise conditions to give a strong response. One nice feature is that it does not widen the edge.

5. Conclusions

The adaptive edge estimators outperform the classical edge detectors because the adaptive estimation filter produces a better estimate of the image which allows for a more accurate estimation of the edges. The smoothing of the Prewitt and Sobel edge detectors enable them to operate in relatively noisy environments, yet the smoothing also tends to widen or blur the edge estimate. To our eye, this looks fine, but the blurring of the edge makes it difficult to identify the exact location of the edge. The Euler and bilinear edge detectors, which do not incorporate smoothing, produce accurate edge estimates, but can only operate in low noise environments because they amplify high frequency noise. The adaptive edge estimator, on the other hand, combines the best of both the smoothing and non-smoothing classical edge detectors. The adaptive filter is able to operate in both low and high noise environments and produce very accurate edge estimates. The smoothed adaptive edge detector filters show the general filter shape required for the Lena image at the various SNR values. The general adaptive edge detectors' shapes are clearly different from the classical edge detectors' shapes, which is why the adaptive filters produce such better performance. The edge estimators are summarized in Table 2.

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Table 1. Edge estimation filter performance measures - 20 dB

	Performance Criteria					
Filter	SNR_{dB}	$LSNR_{dB}$	ISE_{dB}	NBW_{dB}	ρ_{1x}	ρ_{2x}
Input	20.0	6.65	151.4	-	-	-
Input	-15.5	-4.1	12920	-	-	-
H_1	10.54	5.29	31.91	48.2	1.000	0.917
H_2	9.03	4.56	45.25	54.84	0.917	1.000
H_3	-13.2	-3.00	238	73.9	0.849	0.927
H_4	-16.4	-3.74	15940	78.4	0.859	0.939
H_5	3.53	2.54	160.4	81.5	0.676	0.741
H_6	6.18	3.15	87.1	90.5	0.825	0.903

Table 2. Comparison of edge detectors

Filter	Noise Tolerance	Edge Location
Euler	Low, > 20 dB SNR	Accurate
Bilinear	Low, > 20 dB SNR	Accurate
Prewitt	Medium, > 10 dB SNR	Blurred
Sobel	Medium, > 10 dB SNR	Blurred
Adaptive	High, > 0 dB SNR	Very accurate

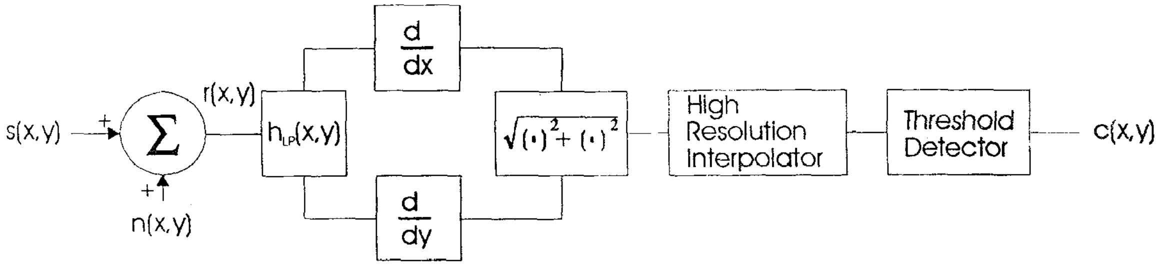

Fig. 1. General edge detection system.

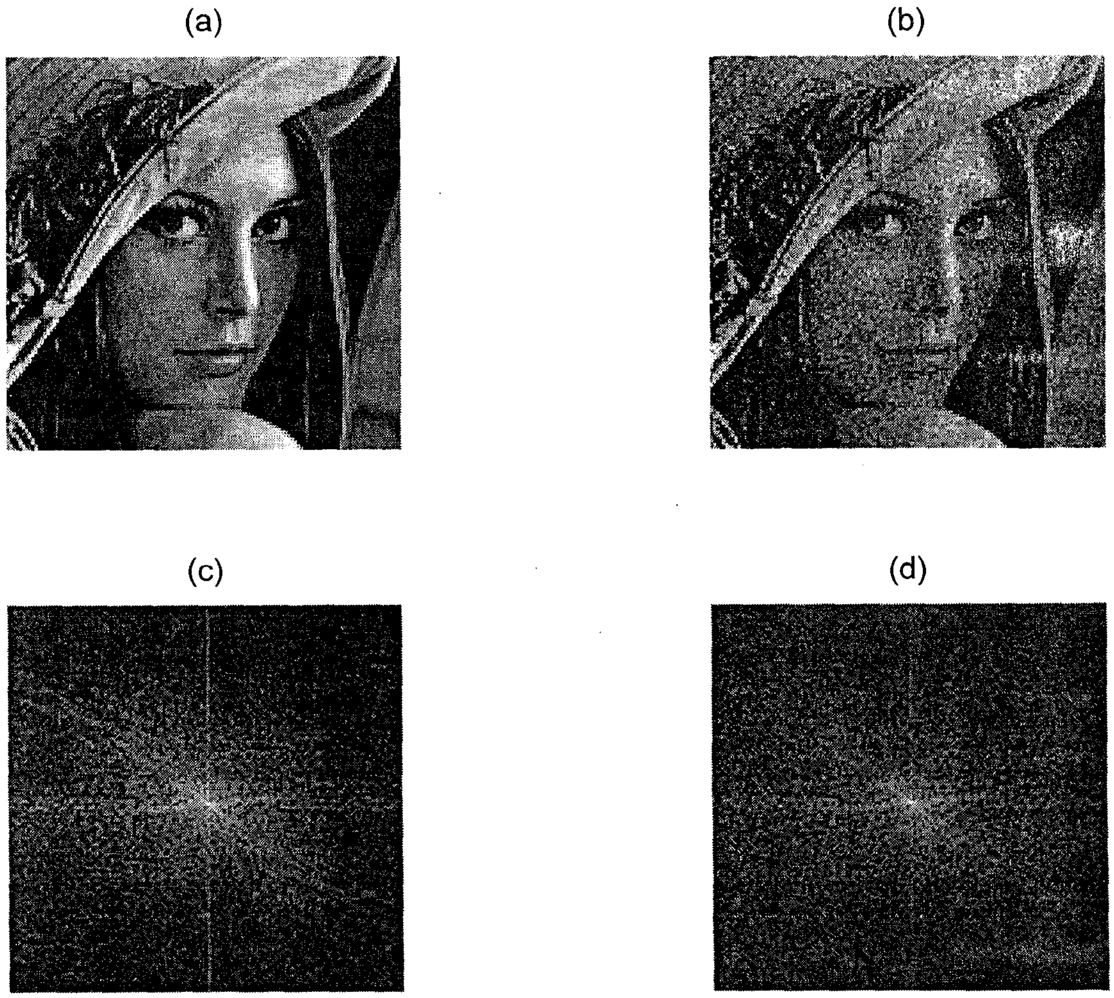

Fig. 2. (a) Lena image, (b) input image $SNR = 20$ dB, (c) Lena spectrum, and (d) input spectrum.

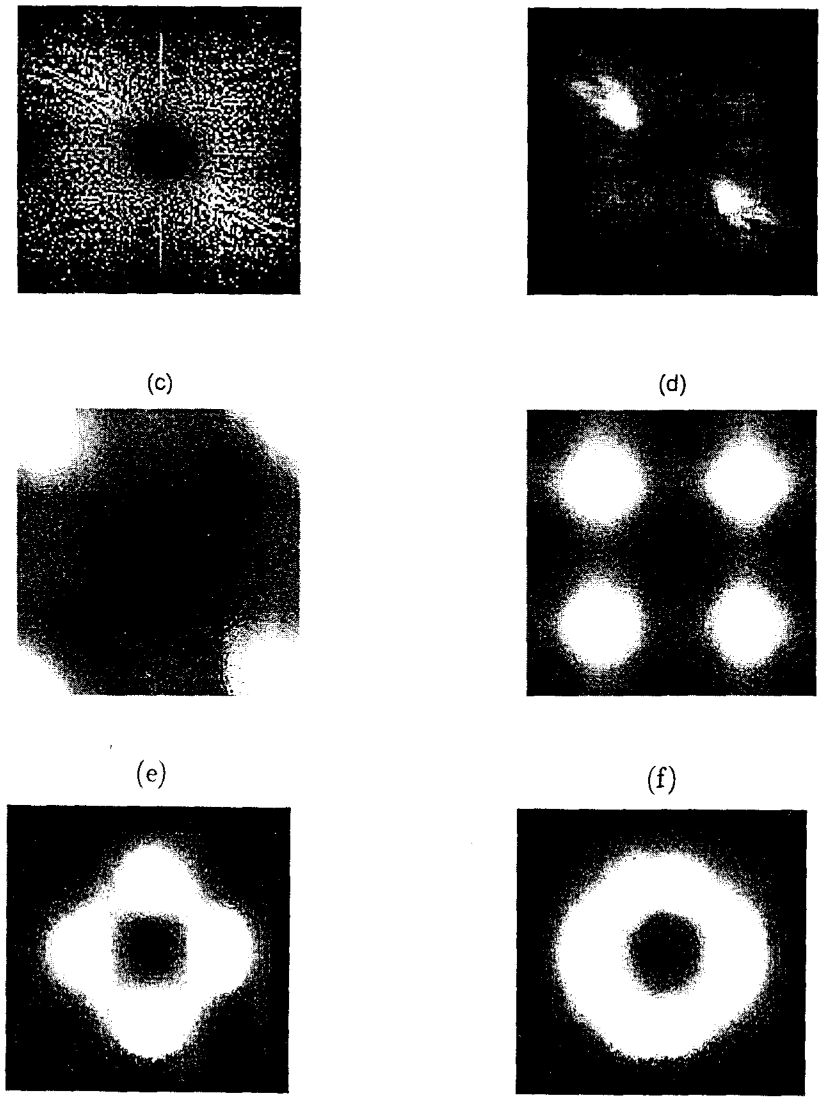

Fig. 3. Gains for (a) adaptive, (b) smoothed adaptive, (c) backward Euler, (d) bilinear, (e) Prewitt, and (f) Sobel filters.

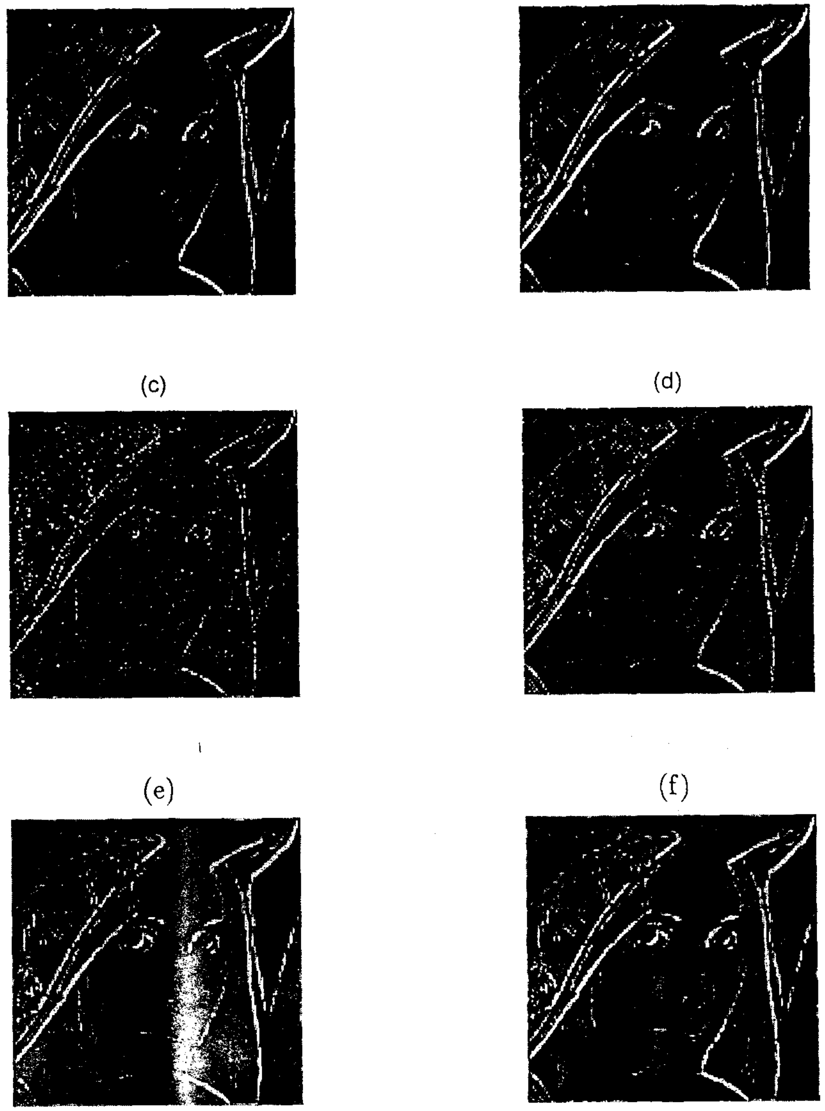

Fig. 4. Output images of (a) adaptive, (b) smoothed adaptive, (c) backward Euler, (d) bilinear, (e) Prewitt, and (f) Sobel filters.