A comparison of Credit Risk Models

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Contents

Abstract

We offer an in depth view into credit risk modelling by analysing structural and reduced-form approaches to quantitative credit analysis. Within structural models, we specifically focus on CreditGrades, an industry standard in assessing single name credit risk, by analysing how it improved on the existing structural approach at the time and exploring possible extensions to address its limitations. We contrast it to Zhou's jump-diffusion approach and the time inhomogeneous Poisson Process intensity model to provide a mathematical and qualitative discussion of the best way to model credit risk today.

1 Introduction

Credit risk arises from the possibility that the borrowing counterparty in a debt or derivatives contract will default and be unable to repay its promised payments, either partially or in full. The recent financial crisis highlighted the importance of measuring credit risk and including it in pricing models for credit instruments [2]. Credit-risk sensitive securities range from corporate bonds to simple credit derivatives to complex structured products. A single-name CDS is a credit derivative where the buyer, a particular company, pays a premium for protection against default and is insured in case it occurs. In this paper, we will use single-name CDS spreads as a risk measure to analyse a firm's credit quality.

Many different approaches to pricing credit risk exist but can be primarily divided into structural and reduced-form models. Structural models, pioneered by Black, Scholes and Merton, price firm-specific credit risk by building on the Black-Scholes option pricing framework. Default is defined as an option on the firm's equity which occurs when the firm value falls below its debt value. CreditGrades belongs to this class of models and became the industry standard in assessing quantitative credit risk as it provided an explicit link between equity options and credit risk. The more recent reduced-form or intensity models use the risk-neutral pricing theory to price credit instruments where the value of a risky instrument equals its present cash flows discounted at the risk-free rate. They model default as an unpredictable exogenous process that is not linked to firm-value or capital structure.

In this paper, we will compare different credit risk models and analyse how each approach contributes to providing a fuller picture of the firm's credit quality and default probabilities. Section 2 explores structural models, focusing particularly on CreditGrades, its architecture and its limitations in pricing credit risk and contrasts it with Zhou's jump-diffusion approach. Section 3 expands on reduced-form models and provides a more thorough comparison against the structural approach while Section 4 concludes the essay.

2 Structural Models

Structural models provide a view of default as an endogenous process by explicitly linking a firm's credit risk to its capital structure. The simplest structural model is introduced in Merton's seminal paper where a default occurs if the firm's asset value process falls below its debt level at maturity. The firm value, V_t is expressed as a sum of its equity, S_t, and debt, D_t processes. For any debt issued with a maturity of T, the equity is expressed as a call option on the underlying firm value:

$$S_T = \max(V_T - K, 0) \tag{1}$$

where K is the face value of the debt at maturity. If $V_T > K$, the debt-holders can be repaid the full amount and the shareholders get $V_T - K$. On the other hand, if $V_T < K$, the debt-holders can claim the assets leaving the shareholders with nothing and leading to default [1].

This model was further extended by Black and Cox (1976) to account for the possibility of default prior to maturity, giving rise to the family of first passage time models. These models follow the mathematics of barrier option pricing where a default event occurs the first time a firm's asset value hits a deterministic barrier from above [2]. Other extensions to the original Merton model include stochastic interest rates, stochastic default barriers and jump diffusion dynamics in the firm's asset value [3].

Structural models make strong assumptions regarding the firm's asset value process, capital and debt structures. The firm value dynamics typically follow a continuous diffusion process similar to stocks in the Black-Scholes framework. They assume complete information about a firm's balance-sheet, its default barrier and 'nearness to default'. If the firm value is far from the barrier, it cannot default in the short term as the probability of a sudden drop in a continuous diffusion process is zero. This produces artificially low credit spreads with short maturities and gives the illusion of default being a predictable event. This may seem reasonable as firms that are performing well do not suddenly default; however, it is contradicted by empirical market data with non-zero short-term credit spreads.

2.1 CreditGrades

2.1.1 Origins

CreditGrades belongs to the class of first-passage time structural models and provides an explicit connection between credit risk and the equity options markets. It differs from other structural models in that instead of predicting default probabilities, its goal is to track credit spreads and identify how they change with market conditions and the firm's credit quality [5]. It uses market spreads rather than propreitary default probabilities as training data. It also takes a more practical approach to parameter estimation; instead of modelling unobservable parameters like the firm value and volatility, it provides a closed-form formula with purely market observable inputs with easily ascertainable sensitivities and impacts. This made it the industry standard for calculating credit spreads and survival probabilities [4]. It uses a stochastic default barrier to address the unrealistically low short-term credit spreads in a pure diffusion and fixed default barrier model. More than credit pricing, CreditGrades aimed to start a discussion on the firm's credit quality; it can also be used to assess the level of risk in a portfolio, manage the credit quality of a firm and to analyse risk in trading and hedging opportunities [4].

2.1.2 Model Setup

Let us consider a continuous time model with maturity T over a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_{0\leq t\leq T}, Q)$ where Q is a risk-neutral probability measure. The following key variables are used to define the model [4]:

1. V_t, the asset value process per share. It is modelled as a lognormal random variable driven

by geometric brownian motion.

$$\frac{dV_t}{V_t} = \sigma dW_t + \mu_D dt \tag{2}$$

where W_t is a standard brownian motion, σ is the asset volatility and μ_D is the asset's drift relative to the default barrier. The assumption of $\mu_D = 0$ implies a stationary leverage where the barrier grows at the same drift as the firm value [5]. For initial asset value V_0,

$$V_t = V_0 e^{\sigma W_t - \frac{1}{2}\sigma^2 t} \tag{3}$$

2. L, average recovery rate on all the firm's liabilities. It follows a lognormal distribution with mean $\bar{L}$ and percentage standard deviation λ.

$$L = \bar{L} e^{\lambda Z - \frac{1}{2}\lambda^2} \tag{4}$$

where $Z \sim N(0,1)$ and Z is independent of the brownian motion W_t. Z introduces uncertainty in the default barrier which makes it possible for the firm value to default suddenly in the short-term despite evolving by pure diffusion. This makes default seem unpredictable and also produces higher and more realistic short-term credit spreads. It reflects the reality that the firm value might be much closer to default than it might seem and that the exact level of leverage cannot be known until default due to loans that are off the balance sheet.

3. D, debt per share. It is the ratio of the value of the liabilities to the equivalent number of shares and is a strictly positive constant. It is calculated using all financial liabilities that contribute to its leverage ratio; non-financial liabilities such as accounts payable and deferred taxes are not included.

4. $L \cdot D$, default barrier. This is defined as the amount of the firm's assets that remain in case of default.

$$LD = \bar{L} D e^{\lambda Z - \frac{1}{2}\lambda^2} \tag{5}$$

5. S_t, the stock price per share. Similar to (1), equity can be expressed a call option on the underlying assets; however, we use the default barrier as the strike instead of the face value of the debt at maturity. Thus it follows a shifted lognormal distribution:

$$S_t = \max(V_t - LD, 0) \tag{6}$$

$$\frac{dS_t}{S_t} = \sigma_S dW_t + \mu_S dt \tag{7}$$

where σ_S is the stock volatility and μ_S is the stock price drift [5].

CreditGrades follows a down-and-out random barrier model; default is avoided until the asset value crosses the default barrier [5]. For an initial asset value of V_0, the time τ of default is defined as [3]:

$$\tau = \inf(t \in (0,T] : V_0 e^{\sigma W_t - \frac{1}{2}\sigma^2 t} \leq \bar{L} D e^{\lambda Z - \frac{1}{2}\lambda^2})$$

where τ is a predictable $\mathcal{F}$-stopping time.
Following the derivation in [4], we can state the cumulative distribution function of survival probabilities of a firm up to time t as:

$$P(t) = \Phi(-\frac{A_t}{2} + \frac{\log d}{A_t}) - d\Phi(\frac{A_t}{2} - \frac{\log d}{A_t}) \tag{8}$$

$$d = \frac{V_0 e^{\lambda^2}}{\bar{L}D} \tag{9}$$

$$A_t^2 = \sigma^2 t + \lambda^2 \tag{10}$$

where Φ is the CDF of the normal distribution.
The par spread for a CDS with maturity t is:

$$c^* = r(1-R)\frac{1 - P(0) + e^{r\xi}(G(t+\xi) - G(\xi))}{P(0) - P(t)e^{-rt} - e^{r\xi}(G(t+\xi) - G(\xi))} \tag{11}$$

$$G(u) = d^{z+1/2}\Phi(-\frac{\log d}{\sigma\sqrt{u}} - z\sigma\sqrt{u}) + d^{-z+1/2}\Phi(-\frac{\log d}{\sigma\sqrt{u}} + z\sigma\sqrt{u}) \tag{12}$$

where r is the risk-free interest rate, R is the expected recovery rate of a specific liability underlying the CDS to be priced, $z = \sqrt{\frac{1}{4} + \frac{2r}{\sigma^2}}$ and $\xi = \frac{\lambda^2}{\sigma^2}$.

Market calibration Unlike previous structural models, CreditGrades aims to use observable market parameters to calculate credit spreads so it can better capture the dynamics of market data and is easy to implement.
For the initial asset value V_0 at time $t = 0$, we have:

$$V_0 = S + \bar{L}D \tag{13}$$

The equity and asset volatilities are related by:

$$\sigma_s = \sigma \frac{V}{S}\frac{\partial S}{\partial V} \tag{14}$$

Using (13), this gives us:

$$\sigma = \sigma_s \frac{S}{S + \bar{L}D} \tag{15}$$

Using historical data to estimate the stock price and volatility, we can rewrite (8) and (9) as:

$$d = \frac{S_0 + \bar{L}D}{\bar{L}D} e^{\lambda^2} \tag{16}$$

$$A_t^2 = (\sigma_S^* \frac{S^*}{S^* + \bar{L}D})^2 t + \lambda^2 \tag{17}$$

where S_0 is the initial stock price, S^* is a historical stock price and σ_S^* is the historical equity volatility.

2.1.3 Model Limitations

Some of the limitations of the CreditGrades model are:

1. Rearranging 15 shows us that the local equity volatility is a function of the stock price and the firm's leverage ratio.
$$\sigma_s = \sigma(1 + \frac{\bar{L}D}{S_t})$$
Hence, although CreditGrades is able to produce the implied volatility skew naturally, it cannot produce the volatility smile [3]. Furthermore, different firms with the same debt leverage ratio and asset volatility have the same skew shapes which suggests that it does not reflect new credit events in the future [3]. Sepp uses a stochastic variance model in [7] to address this and incorporate the volatility smile. However, this extension increases the model's complexity due to numerical integration and makes parameter estimation difficult as all input parameters are not market observables.

2. CreditGrades uses a long-horizon 1000 day historical volatility estimator for the equity volatility, σ^*. Although spread predictions are robust in stable times, they lag true market levels in times of change causing credit curves to diverge from the market. In [6], CreditGrades is extended to use the option implied volatility instead and its performance is better, especially for firms with volatile credit spreads, lower credit ratings and high option volumes. This is because implied volatility reflects new market information in the credit curves in a timely manner. While short-term historical estimators are more attuned to the market as well, they are too noisy to beat the information content advantage of implied volatility.

3. The stochastic default barrier is meant to addresses the problem of artificially low short-term credit spreads. CreditGrades is able to produce high short-term spreads, especially for high volatilities and low equity-to-debt ratios.

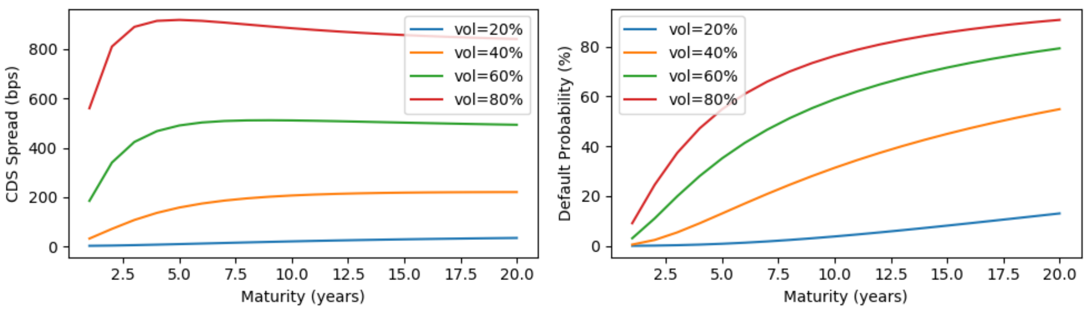


Figure 1: σ_s vs CDS spread and default probability with $R = 0.4, \bar{L} = 0.5, S^* = 100, \lambda = 0.3, r = 0.03, S_0/D = 1$

As shown in figure 1, higher volatilities signal higher risk and uncertainty in asset value which increases the odds of the firm value hitting the default barrier in the short-term; for long-term maturities, if default does not occur the credit spread tends to decrease producing a slightly downward sloping curve.

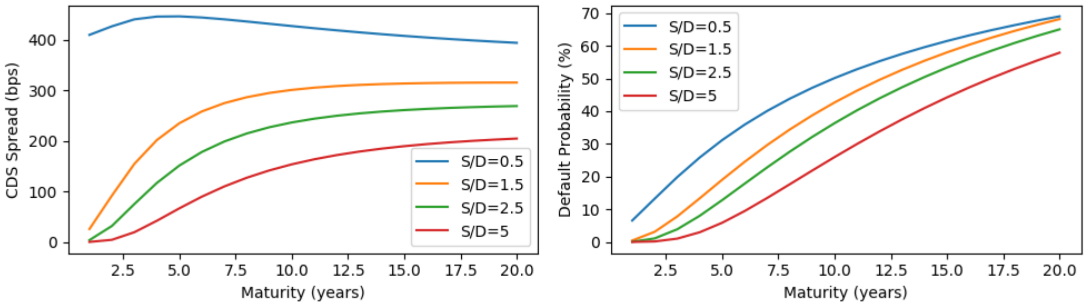


Figure 2: Equity-to-debt ratio vs CDS spread and default probability with $R = 0.4, \bar{L} = 0.5, S^* = 100, \lambda = 0.3, r = 0.03, \sigma_s^* = 0.5$

Figure 2 demonstrates that increasing the equity-to-debt ratio, S_0/D, decreases credit spreads and default probabilities over all maturities as it reduces the level of risk taken on by the firm. The short-term CDS spread increases considerably for very low equity-to-debt ratios as a high premium is required to cover the increased default risk.

Since the default barrier is unobservable, it is difficult to choose the best distribution to model it. CreditGrades models the global recovery rate L lognormally with standard deviation λ.

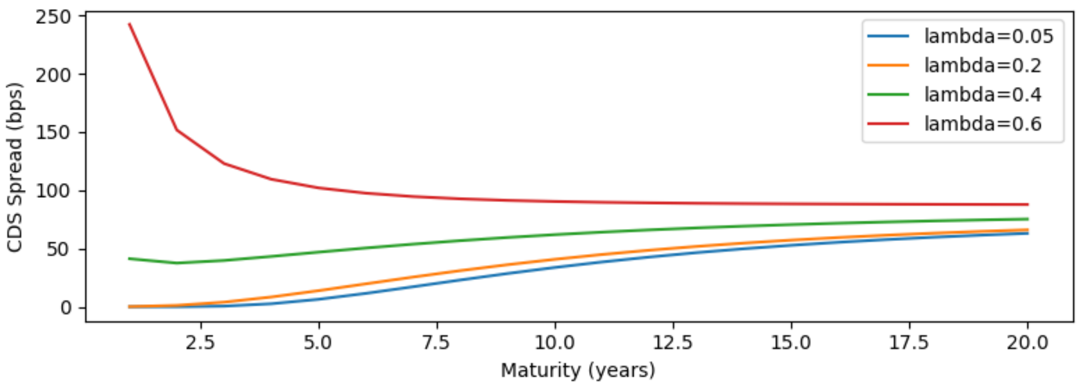


Figure 3: λ vs CDS term structure with $R = 0.4, \bar{L} = 0.5, S^* = 100, r = 0.03, \sigma_s^* = 0.25, S_0/D = 1$

Figure 3 shows that short-term credit spreads are almost unrealistically high for increasing values of λ. This could be because it significantly increases default risk in the short-term but in the event of no default over the long-term, the risk reduces causing CDS premiums to slope downwards. Estimating λ is tricky as it varies considerably between sectors; for example, financial firms might have lower values as they are highly regulated [4]. Another potential drawback of the lognormal distribution is that there is a positive probability of $L > 1$ in which case the default barrier would be greater than the firm's debt which is intuitively unrealistic [7].
Sepp [7] proposes the beta distribution instead which can be defined on a suitable interval $[a, b]$. Another alternative would be Brigo's scenario barrier time varying volatility model where the default barrier is a discrete random variable assuming different values by scenario, each with a different probability calibrated using market data [11]. A jump-diffusion model would be able to produce non-zero short-term credit-spreads without a stochastic default barrier by modelling a more realistic firm value process with sudden drops. This is covered in section 2.1.4.

4. CreditGrades assumes a constant risk-free interest rate instead of adopting a stochastic interest rate model as is observed in the market. As seen in figure 4, credit spreads are negatively correlated with the risk-free interest rate r and the specific recovery rate R. Intuitively, an increase in r implies an increase in equity drift which increases the distance from the firm value to the default barrier and decreases credit spreads. Stochastic interest rates would help to explore the correlation between credit risk and interest rate risk but would increase the model's complexity and compromise ease of implementation.

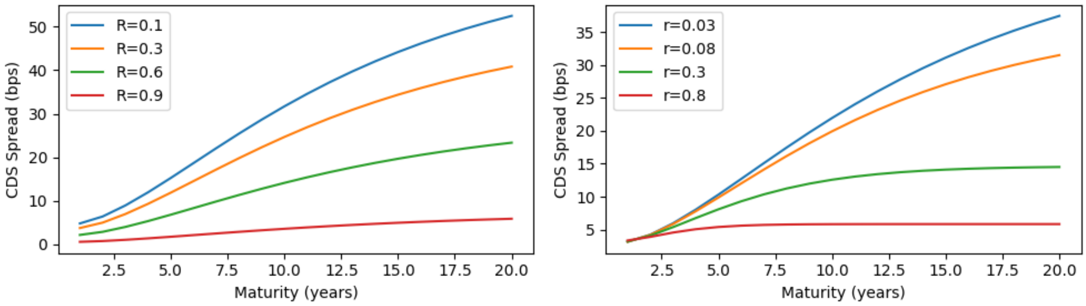


Figure 4: Effect of specific recovery rate (R) and risk-free interest rate (r) on the CDS spread term structure with $R = 0.4, \bar{L} = 0.5, S^* = 100, \lambda = 0.3, r = 0.03, \sigma_s^* = 0.5$

5. CreditGrades adopts a rather simplistic capital structure for the firm and does not attempt to model coupons and more exotic options. Defaults due to liquidity shortages, low cash flows and a lack of external financing are not accounted for. A possible solution, proposed

by Leland and Toft, could be to include an endogenous default barrier decided on by shareholders[13].

Incorporating stochastic interest rates and variance, a sophisticated capital structure for the firm and a more realistic firm value process with jumps would increase the complexity of the model and make it more difficult to implement and use. CreditGrades became the industry standard largely because it used market observable input parameters that were easily estimated. Some of its limitations arise because of this compromise in complexity and its emphasis on practicality and computational speed.

2.1.4 Comparison with jump diffusion models

In Zhou's jump diffusion model [8], the asset value at time t, V_t, has the following dynamics:

$$\frac{dV_t}{V_t} = (\mu - \lambda\nu)dt + \sigma dW_t + (\Pi - 1)dJ_t \tag{18}$$

$$\Pi = \begin{cases} 1, & \text{for no jump} \\ < 0, & \text{for downward jump} \\ > 0, & \text{for upward jump} \end{cases} \tag{19}$$

where μ is the asset drift excluding jumps, λ is the intensity of the Poisson process producing the jumps, ν is the expected jump value, σ is the asset volatility excluding jumps, W_t is a standard brownian motion, Π is the jump amplitude with expected value $\nu + 1$ and J_t is a Poisson process with jump intensity λ. dW_t, dJ_t and Π are mutually independent.
Zhou models lognormal jump sizes so that:

$$\ln(\Pi) \sim \phi(\mu_\pi, \sigma_\pi^2) \tag{20}$$

$$\nu = \mathbf{E}[\Pi - 1] = \exp\big(\mu_\pi + \sigma_\pi^2\big) - 1 \tag{21}$$

Advantages of jump-diffusion models, according to [8], are:

1. Sudden drops in the firm value can occur around the time of default as substantial accounting information is revealed to the market. At other times, drops in firm value occur as investors become aware of lawsuits or financial turmoil. As shown in Figure 5, jumps in the diffusion process can model these unexpected changes in firm value unlike the continuous diffusion process which can only cover marginal fluctuations in asset value. The existence of jump risk helps to produce higher credit spreads and default probabilities for short-term maturities [3], removing the need for a stochastic default barrier. Sepp extends CreditGrades to include double exponential jump diffusion and shows that it produces more accurate short-term credit spreads [7].

2. Jump-diffusion models can also produce more flexible credit spreads, including flat or downward sloping ones while typical structural models are only able to model upward

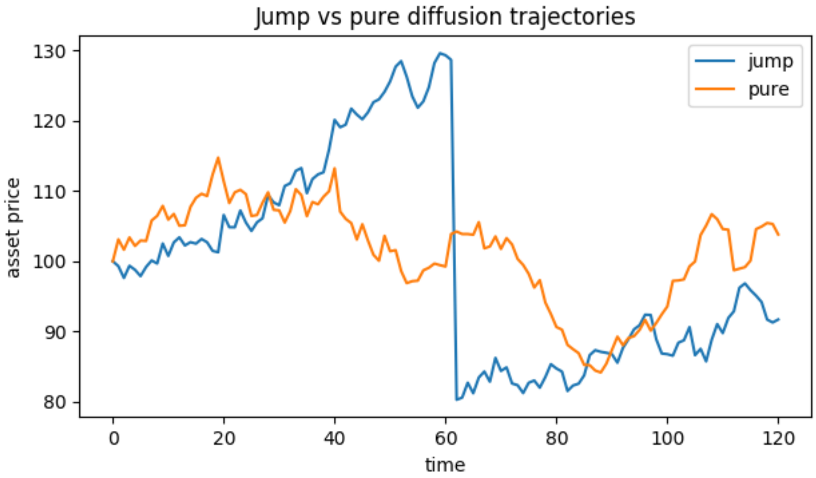


Figure 5: Asset value trajectories under a jump-diffusion vs a pure diffusion process.

sloping spread curves. This makes them more effective at modelling extreme events and pricing more exotic credit derivatives.

However, parameter estimation is more complex as we now have to examine the effect of the jump intensity λ and the jump distribution parameters $\mu_p i$ and $\sigma_p i$ on credit spreads and default probabilities. It is harder to achieve closed-form formulas expressed purely in terms of market observable parameters like in CreditGrades making the estimation of unobservable processes an inevitability.
Zhou's approach also isn't analytically tractable as first-exit time densities are difficult to obtain in closed-form. To address this, Levy processes, such as exponential and gamma distributions, are used in [3] to model jump distributions to produce tractable closed-form solutions for credit spread calculations.
Due to the discontinuity of jump diffusion processes, the default time in this approach is not a predictable but an inaccessible $\mathcal{F}$-stopping time; intuitively, this means default comes as a surprise which is more realistic compared to CreditGrades. These models can be described as hybrid or partial-information models as they combine the unpredictability of default from intensity models (discussed in section 3) while still linking default to firm value like structural models [12]. They realistically describe the two routes to default; a sudden drop in firm value due to external reasons or a consistent but slow decrease in asset value. With better parameter estimation techniques, they would be faster and easier to implement, making them a practical choice for credit risk modelling.

3 Intensity Models

Intensity or reduced-form models, pioneered by Jarrow and Turnbull (1995), assume that default occurs completely by surprise at an inaccessible time and aim to model the conditional law of this random time [10].

3.1 Model Setup

Let us consider a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_{0\leq t\leq T}, Q)$ where Q is a risk-neutral probability measure. In the simplest intensity model, time of default is modelled as the first jump of a time homogeneous Poisson process, $M_t, t \geq 0$. According to [2], M_t is defined as a unit-jump increasing, right continuous stochastic process with stationary independent increments. If we define the default time τ as the first jump of M, there exists a positive constant $\bar{\gamma}$ such that:

$$Q(M_t = 0) = Q(\tau > t) = e^{-\bar{\gamma}t} \tag{22}$$

where $Q(\tau > t) = e^{-\bar{\gamma}t}$ represents survival probabilities under the risk-neutral measure. Survival probabilities have the same structure as a discount factor with the default intensity replacing the interest rates. In order to model the term structure of credit spreads, we use a deterministic intensity function.

Given M_t, we can define N_t as a time in-homogeneous Poisson process with intensity λ and independent but not identically distributed jumps [2]:

$$N_t = M_{\Gamma(t)} \tag{23}$$

$$\Gamma(t) = \int_0^t \lambda(u)du \tag{24}$$

where $\lambda(t)$ is a deterministic time-varying intensity that is a positive and piecewise constant function. If N has its first jump at time τ, M has it at $\Gamma(\tau)$. Since M is a standard poisson process, the first jump is an exponential random variable so that

$$\Gamma(\tau) = \xi = \text{exponential}(1) \tag{25}$$

$$\tau = \Gamma^{-1}(\xi) \tag{26}$$

In this deterministic model, the default time τ is modelled as the first jump of N. This gives us the survival probability:

$$P(\tau > t) = P(\Gamma^{-1}(\xi) > t) = P(\xi > \Gamma(t)) = e^{-\Gamma(t)} \tag{27}$$

ξ is independent of all default free market quantities. It is an external source of randomness [?].
Extensions include stochastic intensity models that account for credit spread volatility.
Intensity models are used to extract default probabilities from the market quotes of bonds and

CDS. Following the CDS pricing derivation in [2], we get:

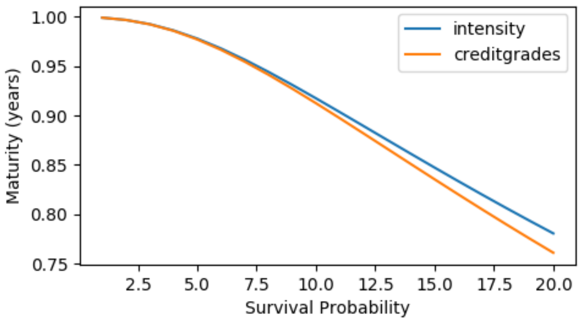
$$\begin{aligned}\mathrm{CDS}_{(0,b)}(0,R,LGD;\Gamma(.)) &= R\sum_{i=1}^{b}\gamma_i\int_{T_{i-1}}^{T_i}\exp(-\Gamma_{i-1}-\gamma_i(u-T_{i-1}))P(0,u)(u-T_{i-1})du \\ &+ R\sum_{i=1}^{b}P(0,T_i)\alpha_i e^{-\Gamma(T_i)} - L_{GD}\sum_{i=1}^{b}\gamma_i\int_{T_{i-1}}^{T_i}\exp(-\Gamma_{i-1}-\gamma_i(u-T_{i-1}))P(0,u)du \quad (28)\end{aligned}$$

where R is the periodic CDS spread quote, LGD is the loss given default, $\alpha_i = T_i - T_{i-1}$, b is the maturity and γ is a piecewise constant function $\gamma(t) = \gamma_i, t \in [T_{i-1}, T_i]$ where the T_i's span different maturities.

Thus we have to solve for the intensity parameters:

$$\mathrm{CDS}_{(0,b)}(0, R_{0,b}^{mkt}, LGD; \gamma^1, \gamma^2, \cdots, \gamma^b) = 0 \quad (29)$$

We use our implementation of CreditGrades (Appendix 6.1) to calculate CDS quotes for maturities over 20 years. The 'credule' library in R is used to bootstrap the CDS quotes and produce the intensity model curves. Figure 6 shows the survival probabilities calculated in both models to be similar with CreditGrades producing slightly higher default probabilities. We also see that the intensity increases as the survival probability decreases and subsequently the default risk increases.



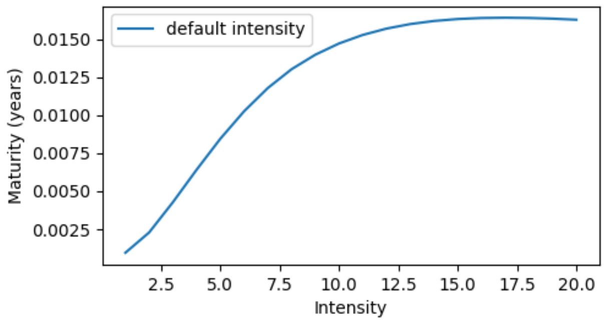


Figure 6: (Right) Comparison of survival probabilities in CreditGrades and the Intensity Model with $R = 0.4, \bar{L} = 0.5, S^* = 100, \lambda = 0.3, r = 0.03, \sigma_s^* = 0.25, S_0/D = 1$. (Left) Plot of default intensity vs maturity

3.2 Comparison with structural models

The key distinction between structural and intensity models is in the information set available to each model[12]. Structural models assume complete knowledge of the firm's assets, liabilities and balance sheet, similar to the firm's management. Default is defined as a predictable stopping time as it is a consequence of the firm value and default barrier, both of which are known.

Reduced-form models work with the less detailed information set available to the market and treat default as an unpredictable external event that may or may not be a consequence of the firm value [9]. Thus it is an inaccessible stopping time which intuitively means that it comes as a complete surprise, similar to the real world.
Intensity models are much easier to calibrate than structural models. In reality, firms publish complete accounting information quarterly at most; thus the asset value process and default barrier cannot be continuously observed which makes parameter estimation difficult. Although CreditGrades uses market observable inputs to address this issue, it still has to estimate equity prices, volatility and interest rates where it can face problems as discussed in section 2.1.3. Conversely, reduced-form models extract default probabilities directly from market data as shown in equation 29. Not only does this make calibration easier, but it also makes intensity models computationally faster and easier to implement. Brigo tested intensity models for firms that had faced bankruptcy such as Lehman Brothers and Parmalat and found them to be accurate reflections of the market conditions [2]. On the other hand, reduced form models can suffer from strong in-sample fitting properties but poor out-of-sample preditive abilities [14].
While intensity models better explain the link between default probabilities and credit risk, they are unable to offer theoretical insights into why the default event occurred. They do not connect default to the economy or the firm value but leave it as an external jump process. Structural models, meanwhile, provide a detailed economic rationale for the 'why' behind the default by linking it to market observables and asset value. Structural models can be transformed to intensity models simply by reducing the information available to the modeller. The default time τ is a stopping time in both approaches; as the information set reduces, the default time goes from predictable to completely inaccessible as it is projected onto a smaller filtration [12].
Intensity models may be better suited for pricing and hedging derivatives as prices are determined by the market based on the information it has available. Structural models are better for credit risk assessments within the firm such as evaluation of the impact on the firm's credit quality due to increased borrowing, share repurchases or the acquisition of another firm [14].

4 Conclusion

CreditGrades' popularity stemmed from the many improvements it offered over existing structural models at the time. It explicitly connected credit risk to the equity options market and addressed the problem of artificially low short-term credit spreads by introducing a stochastic default barrier. While previous models struggled with estimating unobservable parameters such as firm value and volatility, it linked survival probabilities and credit spreads to market observables making parameter estimation and model implementation much easier. However, while theoretically sound, it could not provide accurate credit spreads when compared with reduced-form models.
These models combine computational speed and mathematical tractability with more accurate

market calibration which has proved to be an attractive alternative over time. Since they extract default probabilities from market data, they do not face the same parameter estimation challenges that all structural models do and are much more flexible in fitting market spreads. They are also more realistic in their treatment of default as they operate on the same information set that is available to the market.

Despite this, structural models are still valuable as they offer an intuitive economic explanation for default which reduced-form models are unable to do. CreditGrades itself was more interested in expressing a view on the firm's credit quality than being a pricing model for credit instruments. A possible solution might be to use hybrid models which adopt the view of default as an inaccessible stopping time from reduced-form models and link it to the firm's equity and firm value processes, similar to structural models. Thus this would afford a realistic approach to modelling default with an insight into the firm-specific factors that led to it in the first place.

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5 Appendix

5.1 CreditGrades Model Implementation

```
import numpy as np
import pandas as pd
from scipy.stats import norm
from matplotlib import pyplot as plt

def calculate_survival_probability(L_mean, S0, S_ref, sigma_ref, D, lmbda, t):
    d = (S0 + L_mean*D)/(L_mean*D) * np.exp(lmbda**2)
    a_squared = (sigma_ref*S_ref/(S_ref+L_mean*D))**2*t + lmbda**2
    a_t = np.sqrt(a_squared)
    prob = norm.cdf(-a_t/2 + np.log(d)/a_t) - d*norm.cdf(-a_t/2 - np.log(d)/a_t)
    return prob

def G(u, d, sigma_at, r):
    z = np.sqrt(0.25 + (2 * r)/sigma_at**2)

    a = -np.log(d) / (sigma_at * np.sqrt(u))
    b = z * sigma_at * np.sqrt(u)

    param1 = d**(z+0.5) * norm.cdf(a-b)
    param2 = d**(-z+0.5) * norm.cdf(a+b)
    return param1 + param2

def creditgrades(L_mean, S0, S_ref, sigma_ref, D, lmbda, r, R, t):
    prob_0 = calculate_survival_probability(L_mean, S0, S_ref, sigma_ref, D, lmbda, 0)
    prob_t = calculate_survival_probability(L_mean, S0, S_ref, sigma_ref, D, lmbda, t)

    d = (S0 + L_mean*D)/(L_mean*D) * np.exp(lmbda**2)
    sigma_at = sigma_ref*S_ref/(S_ref+L_mean*D)
    eta = lmbda**2 / sigma_at**2

    x1 = np.exp(r*eta) * (G(t+eta, d, sigma_at, r) - G(eta, d, sigma_at, r))
    x2 = 1 - prob_0 + x1
    x3 = prob_0 - prob_t*np.exp(-r*t) - x1

    cds_spread = r*(1-R)*(x2/x3)*10000
    df = pd.DataFrame(data=[t, sigma_ref, S0, D, cds_spread, prob_t]).T
    df.columns = ['maturity', 'equity_vol', 'S0', 'debt', 'cds_spread',
        'survival_probability']
    return df
```

5.2 Plotting

```
# plotting
R = 0.4
L_mean = 0.5
S_ref = 100
lmbda = 0.3
r = 0.05

def plot_cds_and_pd(df_cds, df_sp):
    fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(10,3))
    df_cds.plot(ax=ax1)
    df_sp.plot(ax=ax2)
    ax1.set_ylabel('CDS Spread (bps)')
    ax1.set_xlabel('Maturity (years)')
    ax1.legend()
    ax2.set_ylabel('Default Probability (%)')
    ax2.set_xlabel('Maturity (years)')
    ax2.legend()
    plt.tight_layout()
    plt.show()

# plot of cds spread vs vol
def plot_cds_spread_term_structure_vs_vol():
    n = 20
    t = pd.Series(list(range(1,21)))
    S0 = pd.Series([100]*n)
    D = pd.Series([100]*n)
    df_30 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref,
        sigma_ref=pd.Series([.20]*n), D=D, lmbda=lmbda, r=r, R=R, t=t)
    df_45 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref,
        sigma_ref=pd.Series([.40]*n), D=D, lmbda=lmbda, r=r, R=R, t=t)
    df_60 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref,
        sigma_ref=pd.Series([.60]*n), D=D, lmbda=lmbda, r=r, R=R, t=t)
    df_75 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref,
        sigma_ref=pd.Series([.80]*n), D=D, lmbda=lmbda, r=r, R=R, t=t)
    df_cds = pd.DataFrame()
    df_cds['vol=20%'] = df_30['cds_spread']
    df_cds['vol=40%'] = df_45['cds_spread']
    df_cds['vol=60%'] = df_60['cds_spread']
```

```
    df_cds['vol=80%'] = df_75['cds_spread']
    df_cds.index = df_30.maturity
    df_sp = pd.DataFrame()
    df_sp['vol=20%'] = (1-df_30['survival_probability'])*100
    df_sp['vol=40%'] = (1-df_45['survival_probability'])*100
    df_sp['vol=60%'] = (1-df_60['survival_probability'])*100
    df_sp['vol=80%'] = (1-df_75['survival_probability'])*100
    df_sp.index = df_30.maturity
    plot_cds_and_pd(df_cds, df_sp)

# plot of cds spread vs L
def plot_cds_spread_term_structure_vs_leverage():
    n = 20
    t = pd.Series(list(range(1,21)))
    S0 = pd.Series([100]*n)
    sigma_ref=pd.Series([.80]*n)
    df_30 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref,
        D=pd.Series([100/0.5]*n), lmbda=lmbda, r=r, R=R, t=t)
    df_45 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref,
        D=pd.Series([100/1.5]*n), lmbda=lmbda, r=r, R=R, t=t)
    df_60 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref,
        D=pd.Series([100/2.5]*n), lmbda=lmbda, r=r, R=R, t=t)
    df_75 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref,
        D=pd.Series([100/5.0]*n), lmbda=lmbda, r=r, R=R, t=t)
    df_cds = pd.DataFrame()
    df_cds['S/D=0.5'] = df_30['cds_spread']
    df_cds['S/D=1.5'] = df_45['cds_spread']
    df_cds['S/D=2.5'] = df_60['cds_spread']
    df_cds['S/D=5'] = df_75['cds_spread']
    df_cds.index = df_30.maturity
    df_sp = pd.DataFrame()
    df_sp['S/D=0.5'] = (1-df_30['survival_probability'])*100
    df_sp['S/D=1.5'] = (1-df_45['survival_probability'])*100
    df_sp['S/D=2.5'] = (1-df_60['survival_probability'])*100
    df_sp['S/D=5'] = (1-df_75['survival_probability'])*100
    df_sp.index = df_30.maturity
    plot_cds_and_pd(df_cds, df_sp)

# plot of cds spread vs lambda
def plot_cds_spread_term_structure_vs_lambda():
    n = 20
    t = pd.Series(list(range(1,21)))
    S0 = pd.Series([100]*n)
```

```
    D = pd.Series([100]*n)
    sigma_ref=pd.Series([.25]*n)
    df_30 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=0.05, r=r, R=R, t=t)
    df_60 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=0.2, r=r, R=R, t=t)
    df_75 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=0.4, r=r, R=R, t=t)
    df_76 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=0.6, r=r, R=R, t=t)
    df_cds = pd.DataFrame()
    df_cds['lambda=0.05'] = df_30['cds_spread']
    df_cds['lambda=0.2'] = df_60['cds_spread']
    df_cds['lambda=0.4'] = df_75['cds_spread']
    df_cds['lambda=0.6'] = df_76['cds_spread']
    df_cds.index = df_30.maturity
    fig = df_cds.plot(figsize=(6,3))
    fig.set_xlabel('Maturity (years)')
    fig.set_ylabel('CDS Spread (bps)')
    plt.tight_layout()
    plt.show()

# plot of cds spread vs lambda
def plot_cds_spread_term_structure_vs_recovery_rate():
    n = 20
    t = pd.Series(list(range(1,21)))
    S0 = pd.Series([100]*n)
    D = pd.Series([100]*n)
    sigma_ref=pd.Series([.25]*n)
    df_30 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=lmbda, r=r, R=0.1, t=t)
    df_60 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=lmbda, r=r, R=0.3, t=t)
    df_75 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=lmbda, r=r, R=0.6, t=t)
    df_76 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
        lmbda=lmbda, r=r, R=0.9, t=t)
    df_R = pd.DataFrame()
    df_R['R=0.1'] = df_30['cds_spread']
    df_R['R=0.3'] = df_60['cds_spread']
    df_R['R=0.6'] = df_75['cds_spread']
    df_R['R=0.9'] = df_76['cds_spread']
    df_R.index = df_30.maturity
```

```
df_30 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
    lmbda=lmbda, r=0.03, R=R, t=t)
df_60 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
    lmbda=lmbda, r=0.08, R=R, t=t)
df_75 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
    lmbda=lmbda, r=0.3, R=R, t=t)
df_76 = creditgrades(L_mean=L_mean, S0=S0, S_ref=S_ref, sigma_ref=sigma_ref, D=D,
    lmbda=lmbda, r=0.8, R=R, t=t)
df_r = pd.DataFrame()
df_r['r=0.03'] = df_30['cds_spread']
df_r['r=0.08'] = df_60['cds_spread']
df_r['r=0.3'] = df_75['cds_spread']
df_r['r=0.8'] = df_76['cds_spread']
df_r.index = df_30.maturity
plot_cds_and_pd(df_R, df_r)
```