Data Structures

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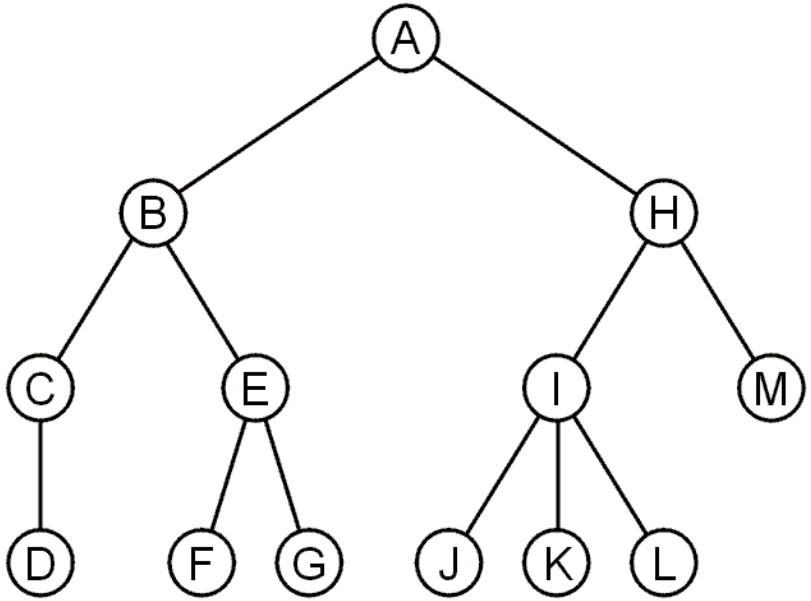
15. Tree Data Structure

Trees

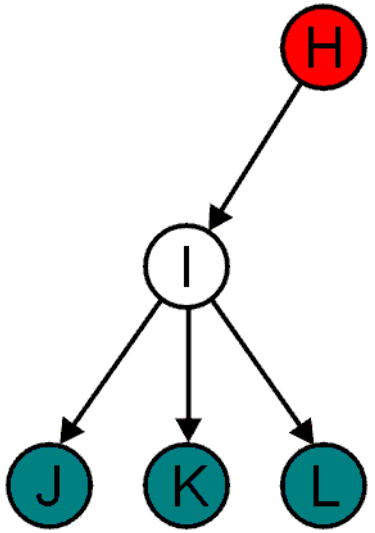
- A rooted tree data structure stores information in nodes
- Similar to linked lists:
  - There is a first node, or root
  - Each node has variable number of references to successors
  - Each node, other than the root, has exactly one node pointing to it



Terminology: Parent Child Relations

- All nodes have zero or more child nodes or children
  - I has three children: J, K and L

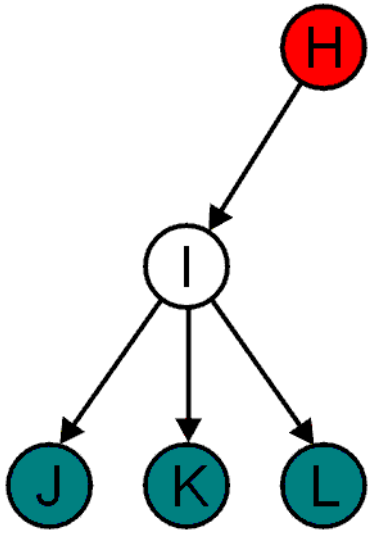
- For all nodes other than the root node, there is one parent node
  - H is the parent I



Terminology: Degree

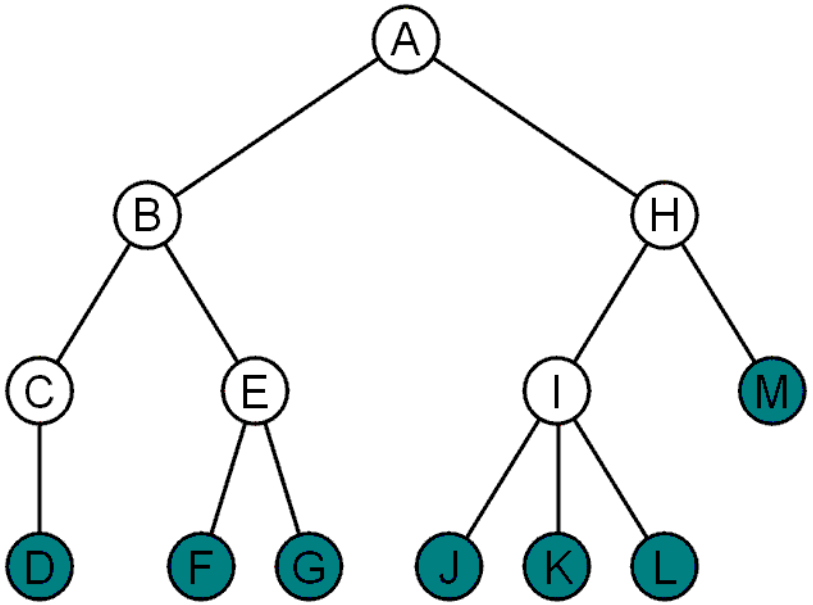
- The degree of a node is defined as the number of its children
  - `deg(I) = 3`

- Nodes with the same parent are siblings
  - J, K, and L are siblings



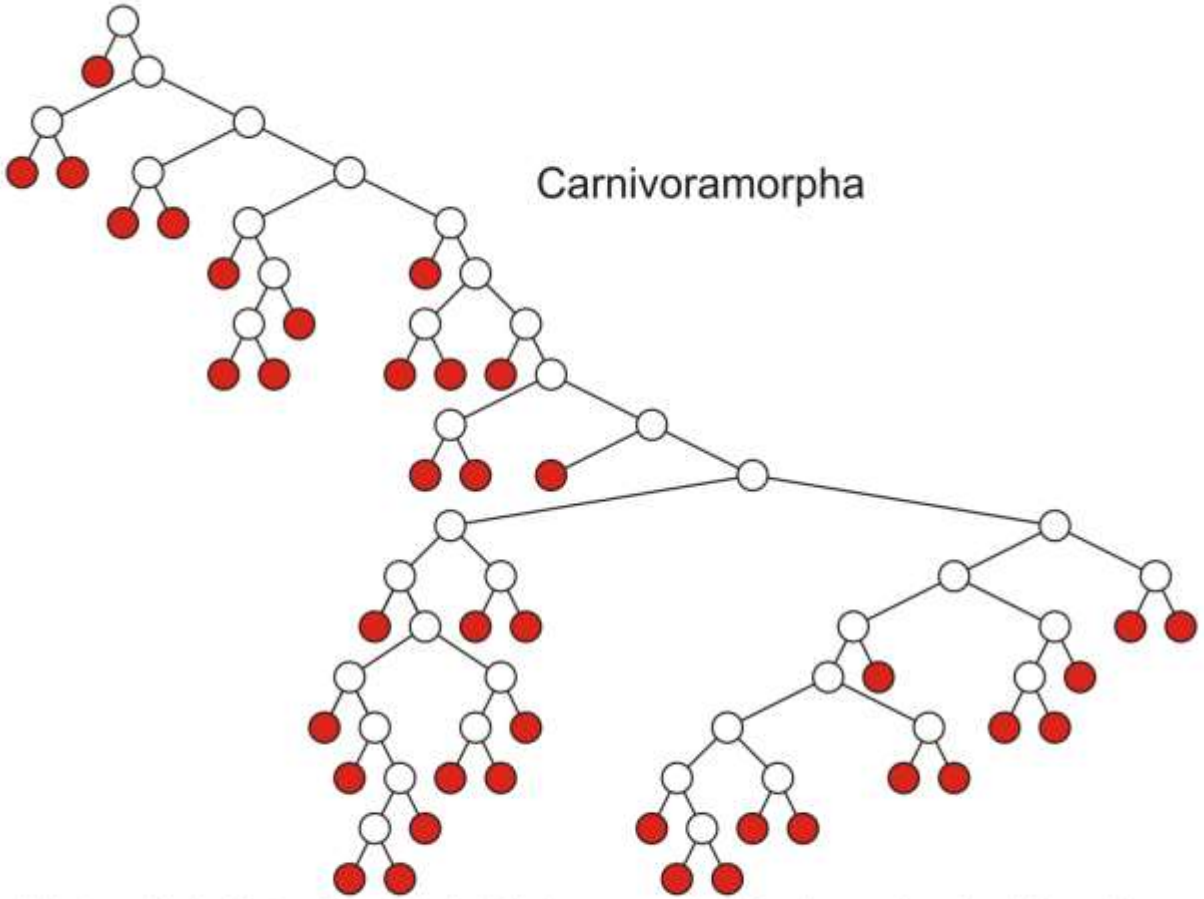
Terminology: Leaf And Internal Nodes

- Nodes with degree zero are also called leaf nodes
- All other nodes are said to be internal nodes, that is, they are internal to the tree



Terminology: Leaf Nodes Examples

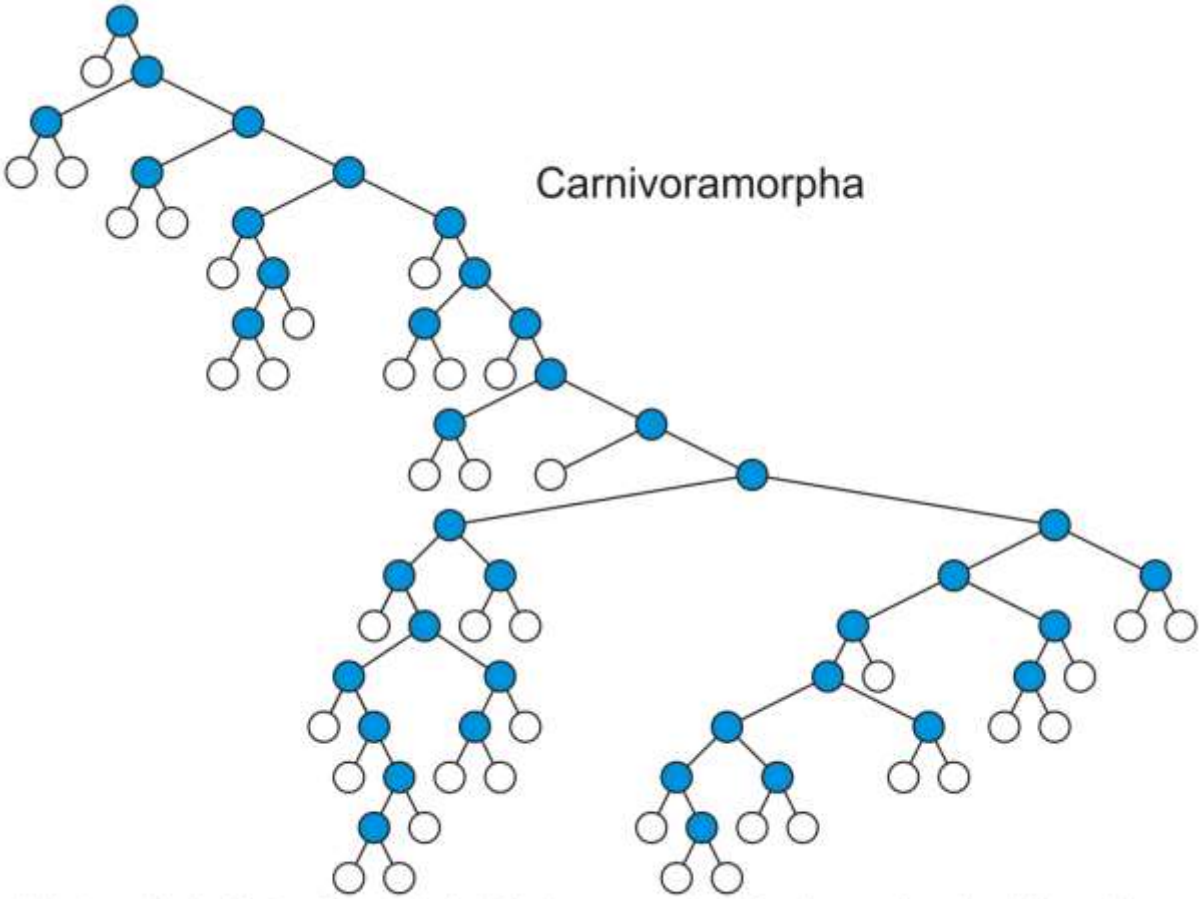
- Leaf nodes



Wesley-Hunt, G. D.; Flynn, J. J. "Phylogeny of the Carnivora: basal relationships among the Carnivoramorphans, and assessment of the position of 'Miacoidea'

Terminology: Internal Nodes Example

- Internal nodes



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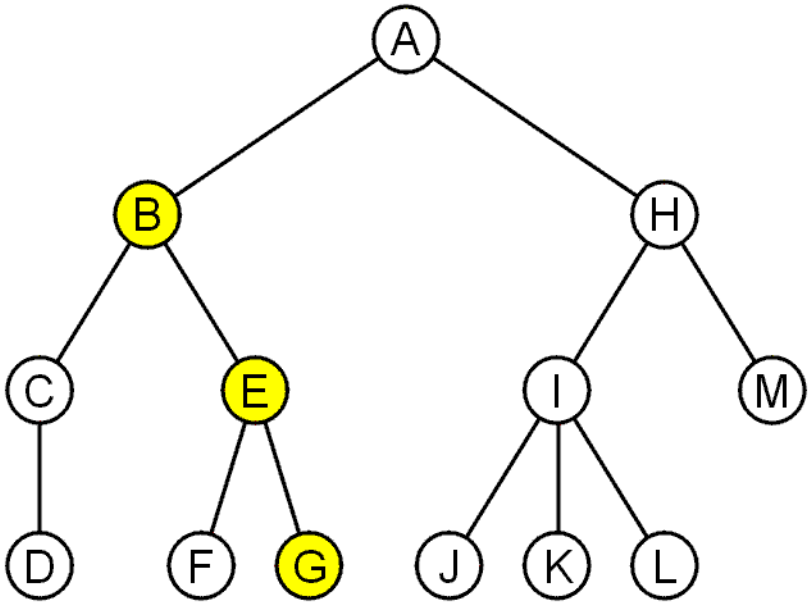
Terminology: Path

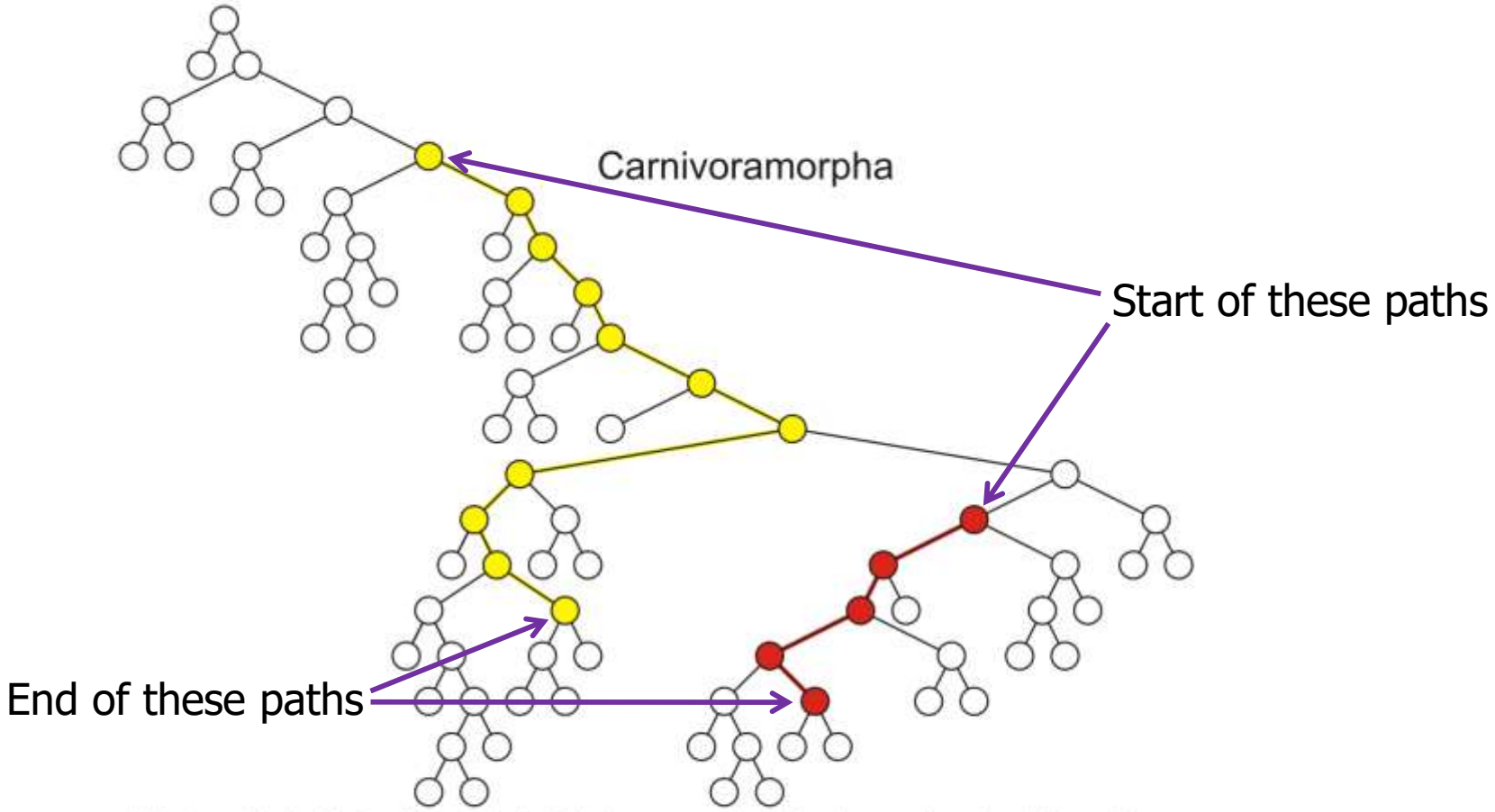
- A path is a sequence of nodes ($a_0, a_1, \ldots, a_n$)
  - Where $a_k + 1$ is a child of a_k

- The length of this path is: $n = |\text{nodes in the path}| - 1$
  - For example, the path (B, E, G) has length 2



Terminology: Path Example

- Paths of length 10 (11 nodes) and 4 (5 nodes)



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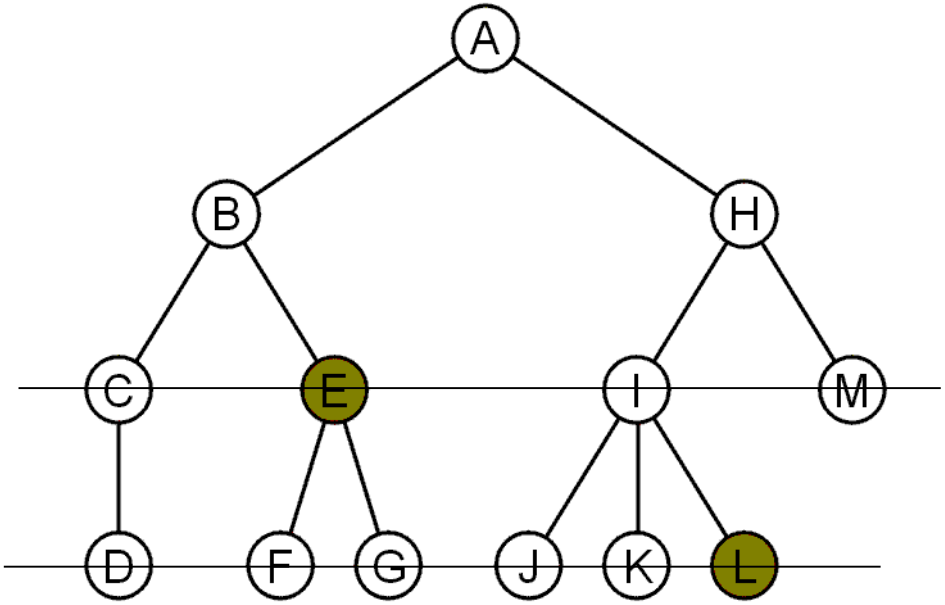
Terminology: Depth (or Level)

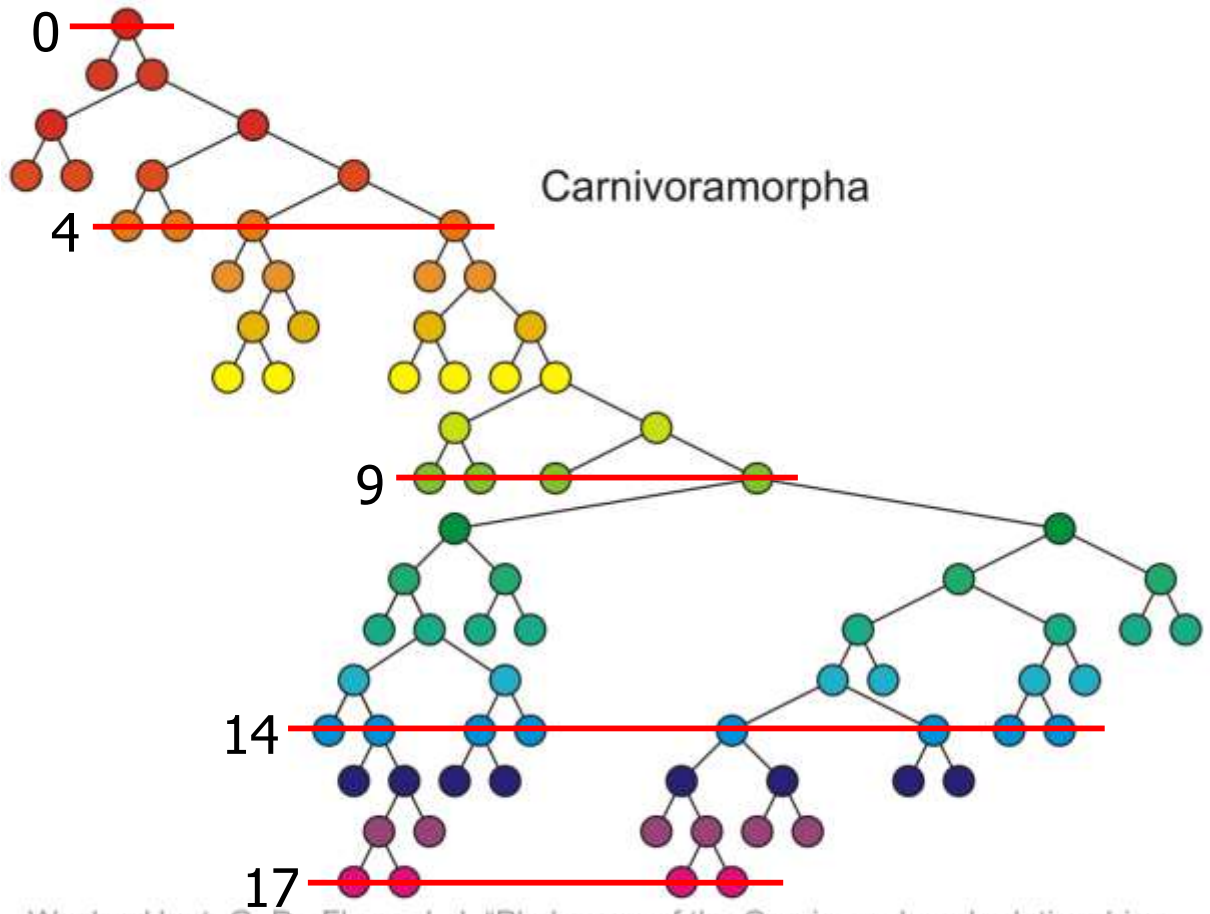
- For each node in a tree, there exists a unique path from the root node to that node

- The length of this path is the depth of the node, e.g.,
  - E has depth 2
  - L has depth 3



Terminology: Depth Example

- Nodes of depth up to 17



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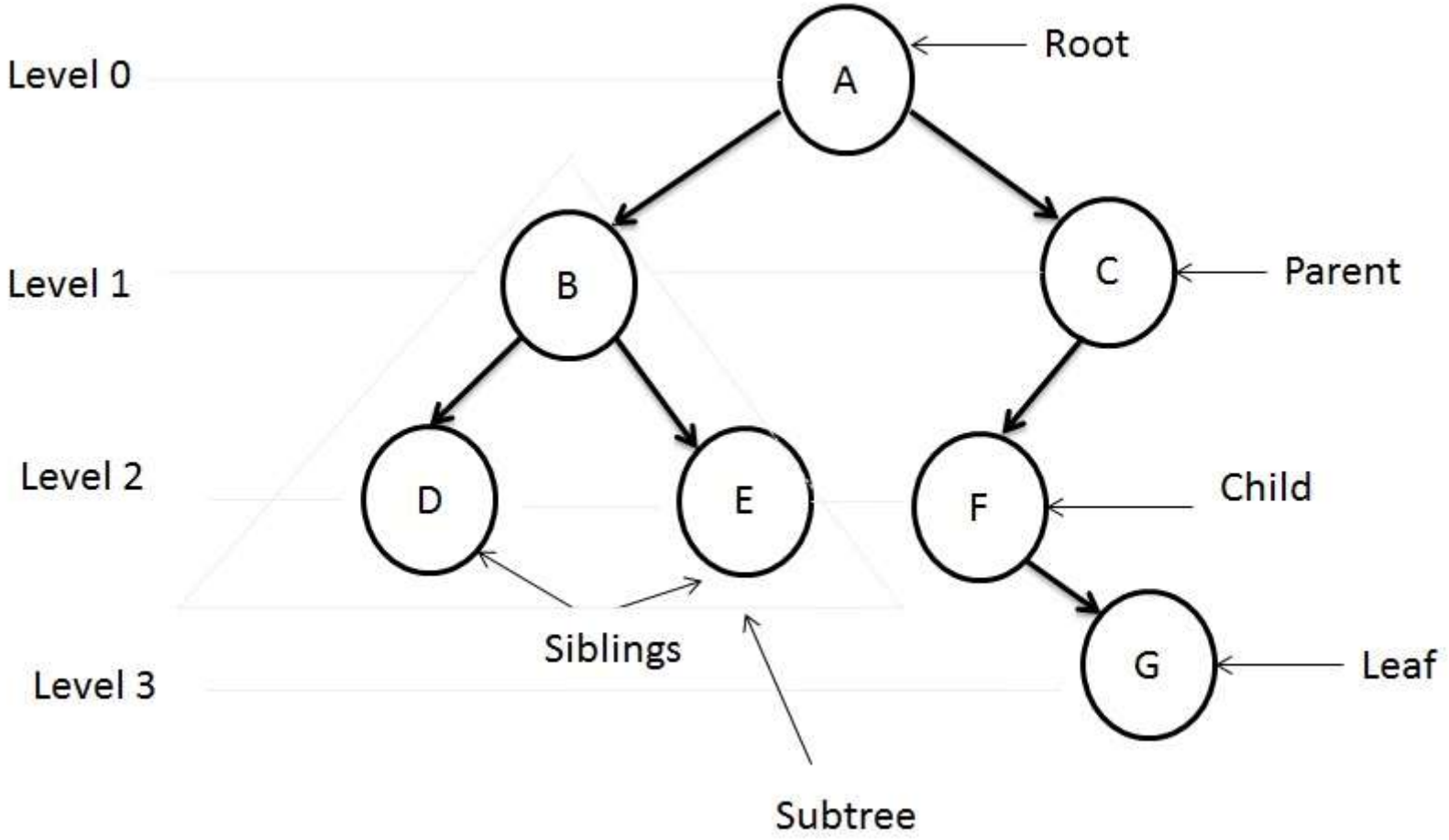
Example

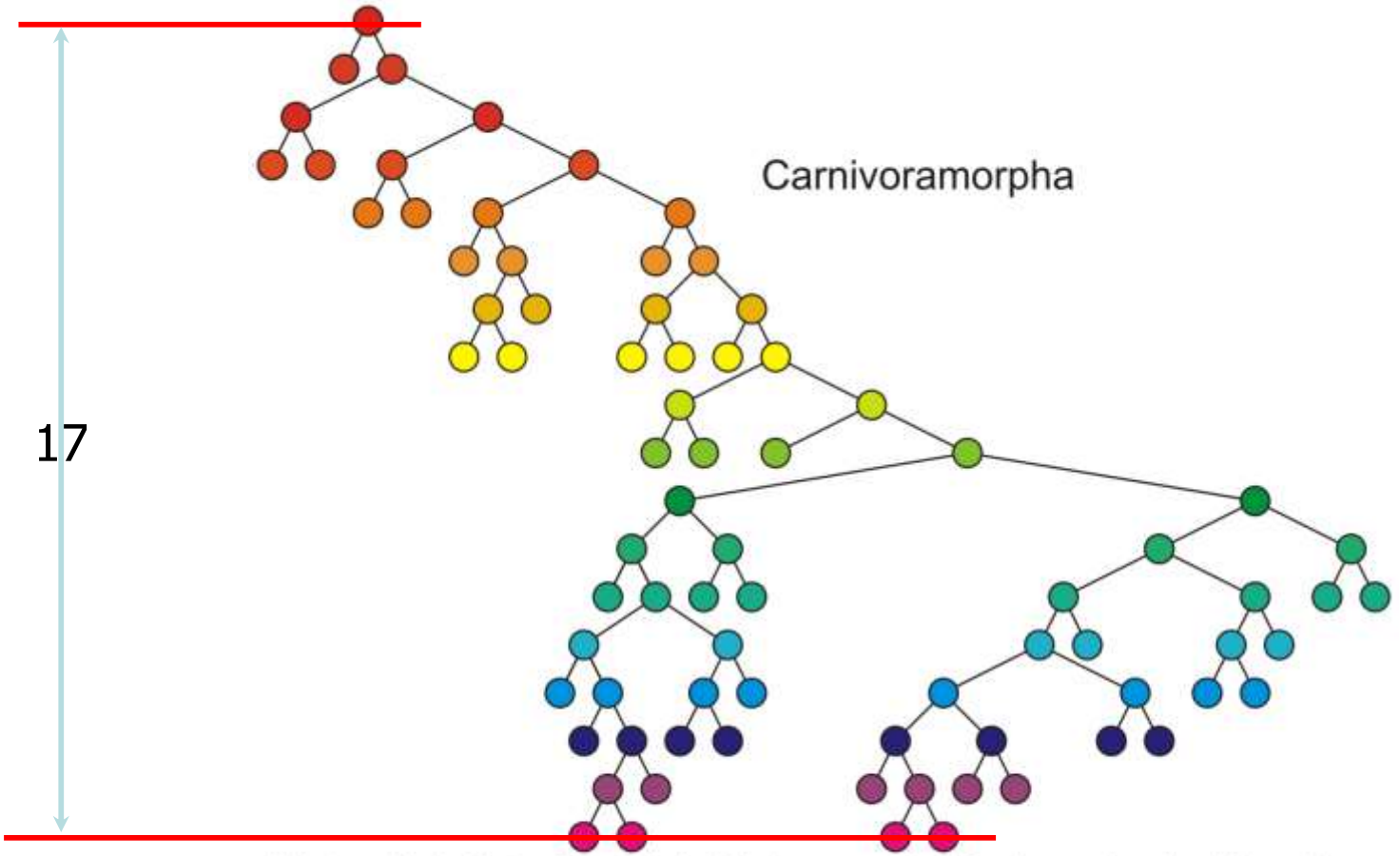

Level 0
Level 1
Level 2
Level 3
A
Root
B
C
Parent
D
E
F
Child
G
Leaf
Siblings
Subtree

Terminology: Height

- The height of a tree is defined as the maximum depth of any node within the tree
- The height of a tree with one node is 0
  - Just the root node
- For convenience, we define the height of the empty tree to be –1

Terminology: Height Example

- Height of this tree is 17



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Terminology: Ancestors And Descendants

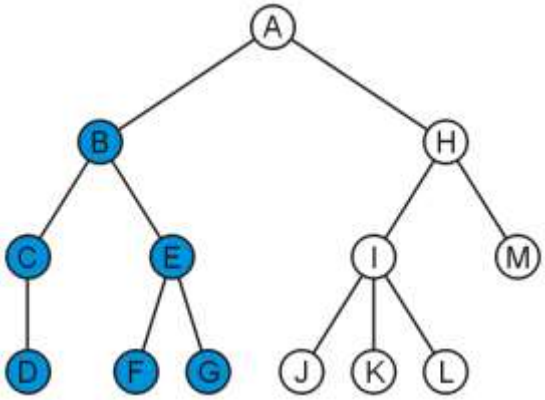
- If a path exists from node a to node b
  - a is an ancestor of b
  - b is a descendent of a

- Thus, a node is both an ancestor and a descendant of itself
  - We can add the adjective strict to exclude equality
  - a is a strict descendent of b if a is a descendant of b but $a \neq b$

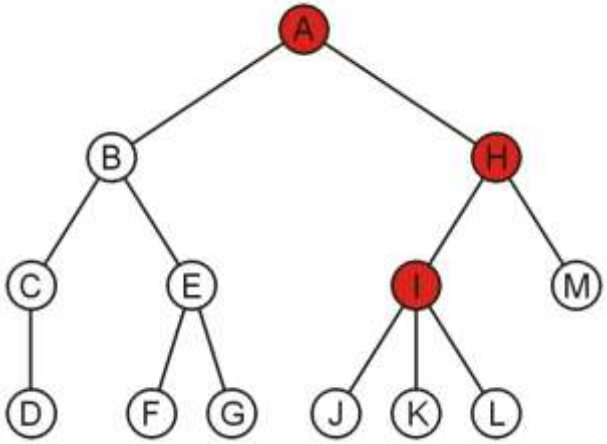
- The root node is an ancestor of all nodes

Terminology: Ancestors And Descendants Example

- The descendants of node B are C, D, E, F, and G

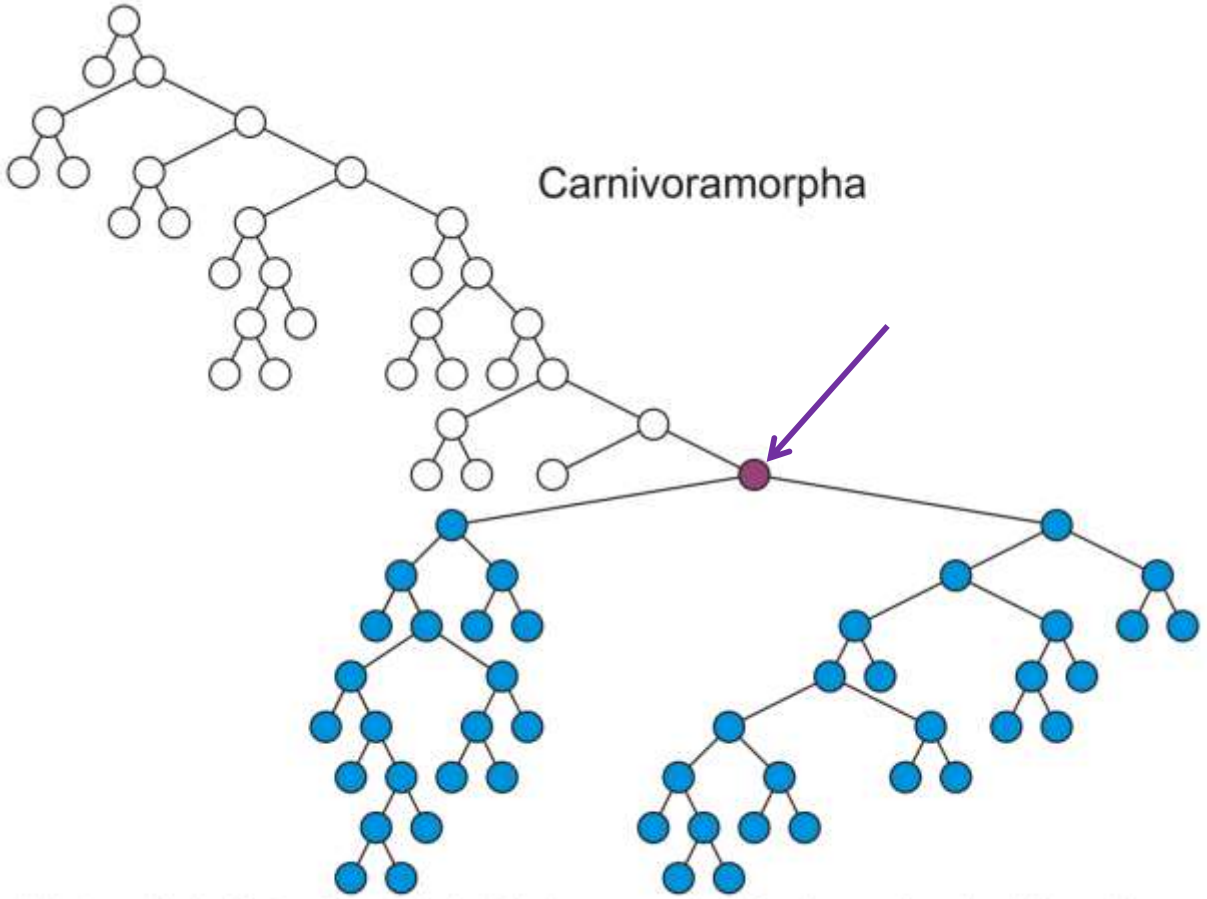


- The ancestors of node I are H and A



Terminology: Descendants Example

- All descendants (including itself) of the indicated node



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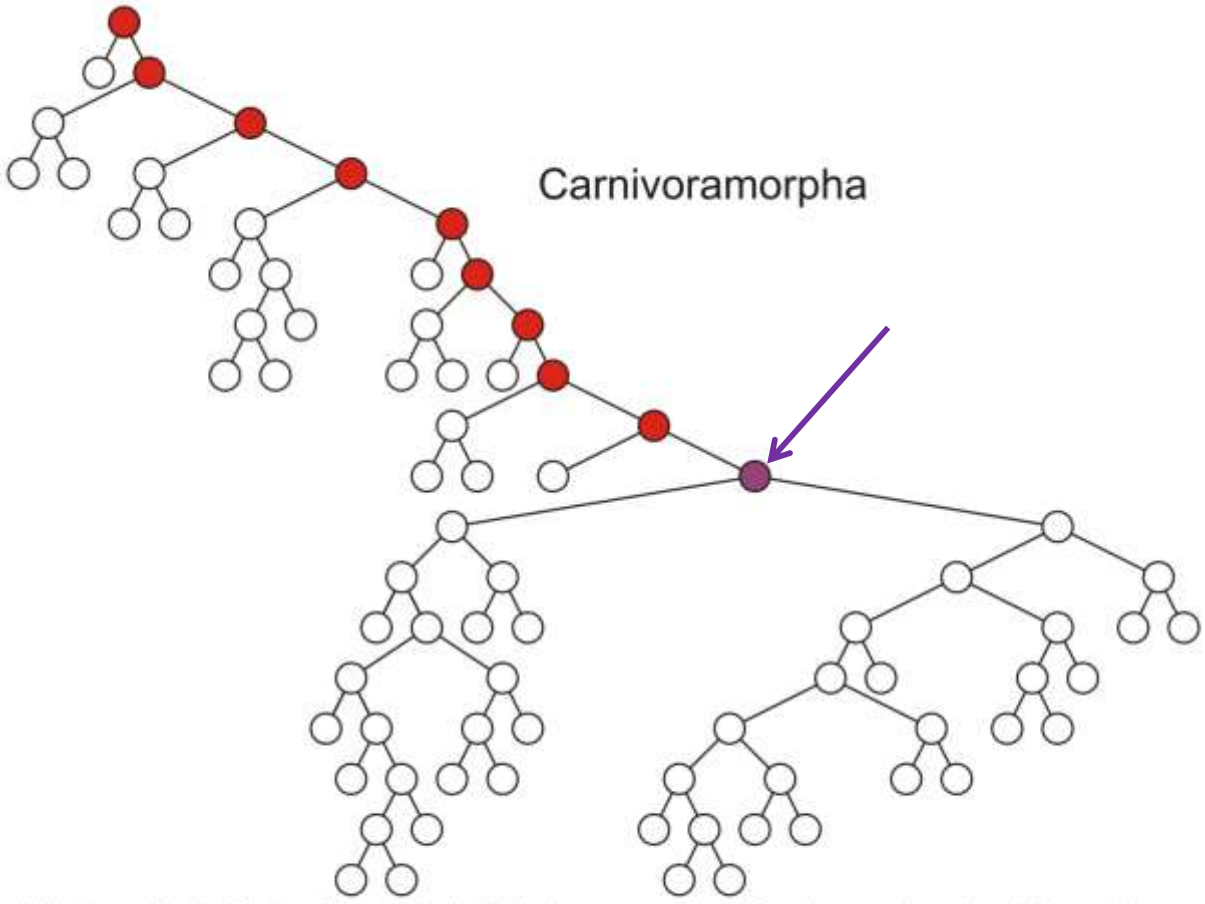
Terminology: Ancestors Example

- All ancestors (including itself) of the indicated node



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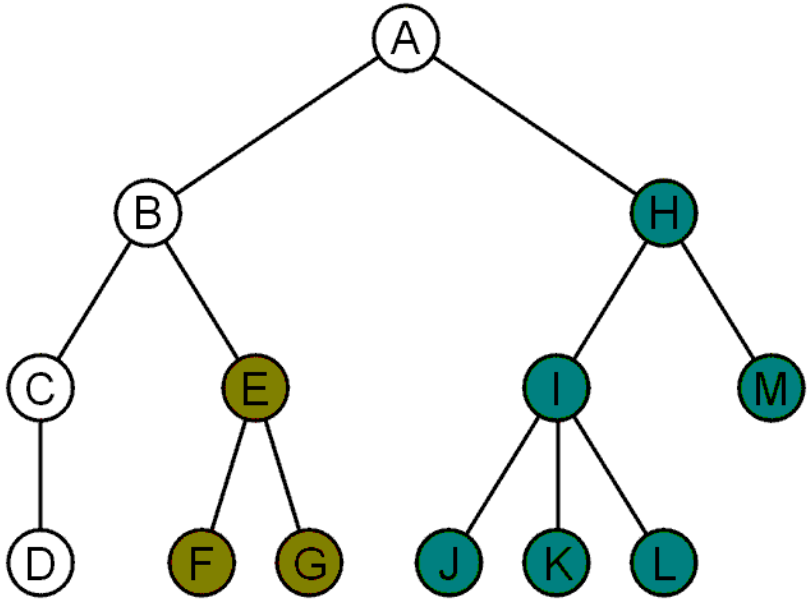
Terminology: Subtree

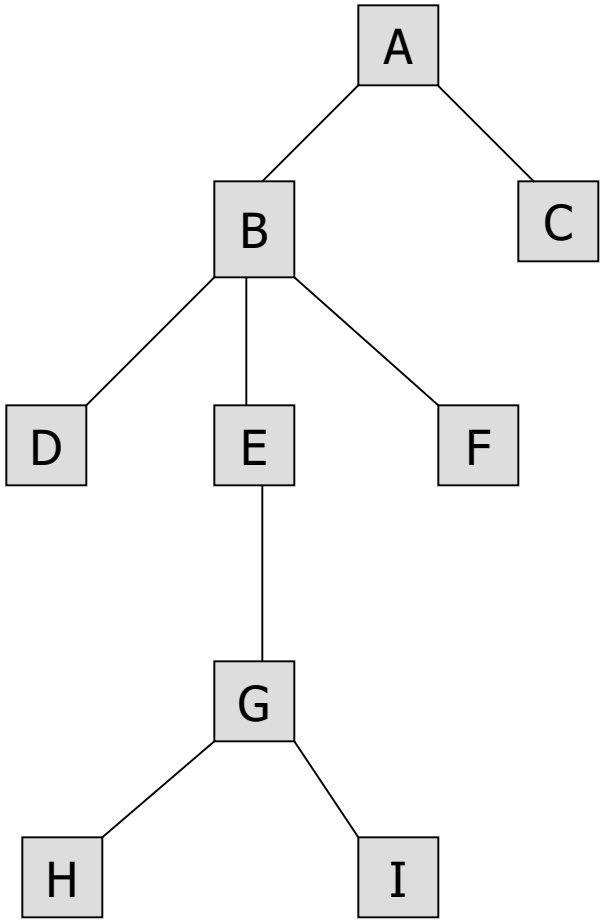
- Another approach to a tree is to define the tree recursively
  - A degree-0 node is a tree

- A node with degree n is a tree if it has n children
  - All of its children are disjoint trees (i.e., with no intersecting nodes)

- Given any node a within a tree with root r, the collection of a and all of its descendants is said to be a subtree of the tree with root a



Tree Properties



Property	Value
Number of nodes	
Height	
Root Node	
Leaves	
Ancestors of H	
Descendants of B	
Siblings of E	
Left subtree	

Example: HTML (1)

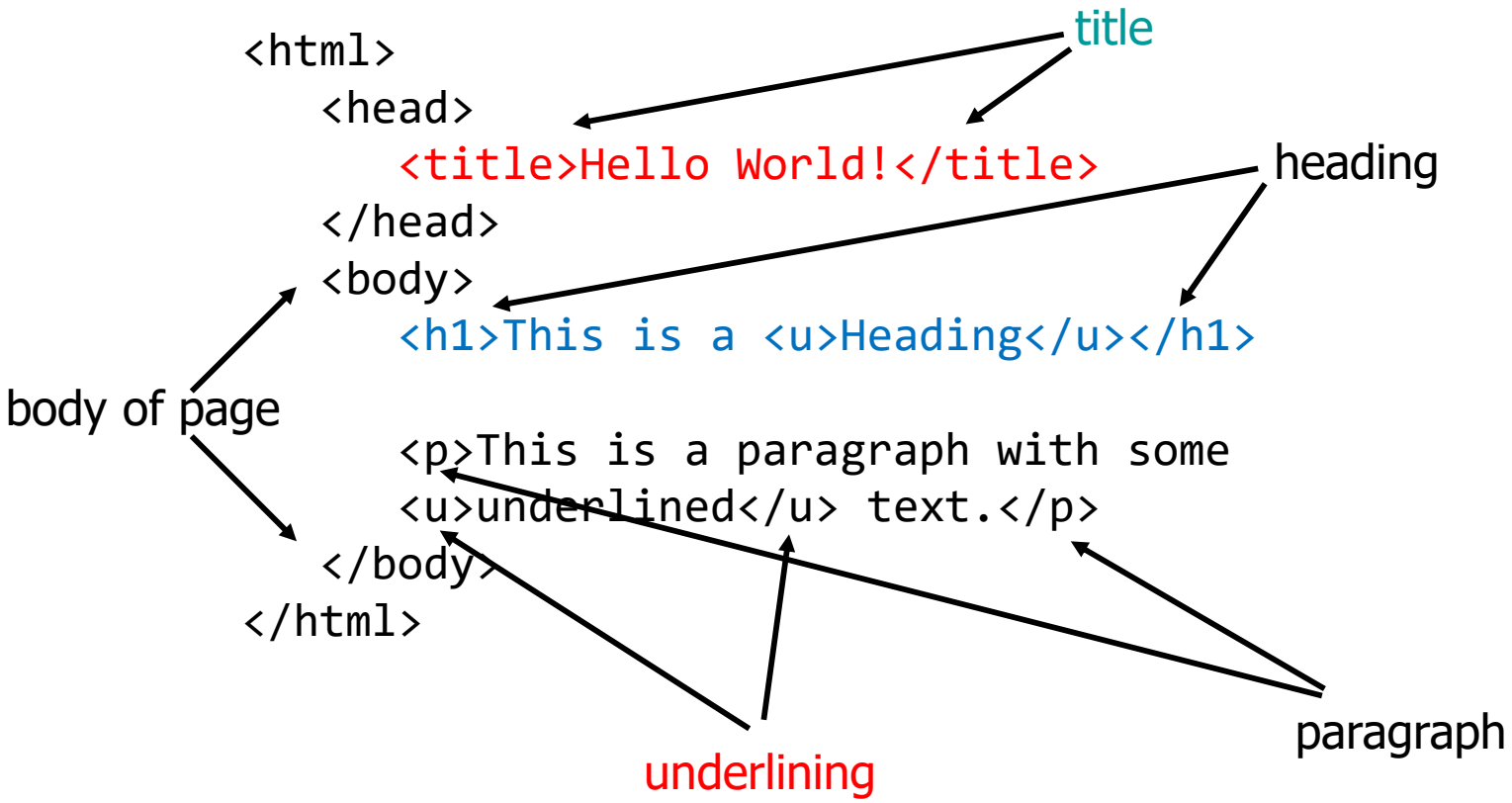
- HTML document has a tree structure

```
<html>
   <head>
      <title>Hello World!</title>
   </head>
   <body>
      <h1>This is a <u>Heading</u></h1>

      <p>This is a paragraph with some
      <u>underlined</u> text.</p>
   </body>
</html>
```

Example: HTML (2)

- HTML document has a tree structure




```
<html>
   <head>
      <title>Hello World!</title>
   </head>
   <body>
      <h1>This is a <u>Heading</u></h1>

      <p>This is a paragraph with some
      <u>underlined</u> text.</p>
   </body>
</html>
```

Example: HTML (3)

- The nested tags define a tree rooted at the HTML tag

```
<html>
   <head>
      <title>Hello World!</title>
   </head>
   <body>
      <h1>This is a <u>Heading</u></h1>

      <p>This is a paragraph with some
      <u>underlined</u> text.</p>
   </body>
</html>
```