COMPUTER SCIENCE 20, SPRING 2014

Module #12 (Graph Coloring)

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Last modified: March 30, 2014

Executive Summary

1. Graph Coloring is the assignment of colors to all vertices in a graph such that no two adjacent vertices are share the same color. The *chromatic number* of a graph (denoted $\chi(G)$) is the minimum number of colors required to color it.

2. Important Chromatic Numbers. Trees are 2-colorable. Cycles of even length are 2-colorable. Cycles of odd length are 3-colorable. Any *planar graph*, a graph that can be drawn so that no edges are crossing, is 4-colorable. In general, if the maximum degree of any vertex in a graph is k, the chromatic number of the graph is less or equal to $k + 1$.

3. A graph G that has at least one edge is bipartite if and only if $\chi(G) = 2$.

Small group problems

1. There are 5 senators serving on 6 committees as shown below. The committees are meeting once every week. What is the minimum number of weekly meeting times needed to ensure there are no scheduling conflicts for any of the senators? (Multiple meetings can be run at the same meeting time as long as there aren't any senators who would need to be in two meetings at once.)

   Athletics: amy, bob, cal

   Budget: bob, dan, eva

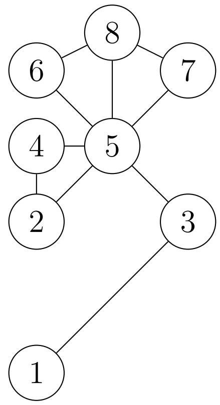
   Compensation: amy, cal, eva

   Diversity: cal, dan, eva

   Education: amy, bob

   Football: bob, cal, eva

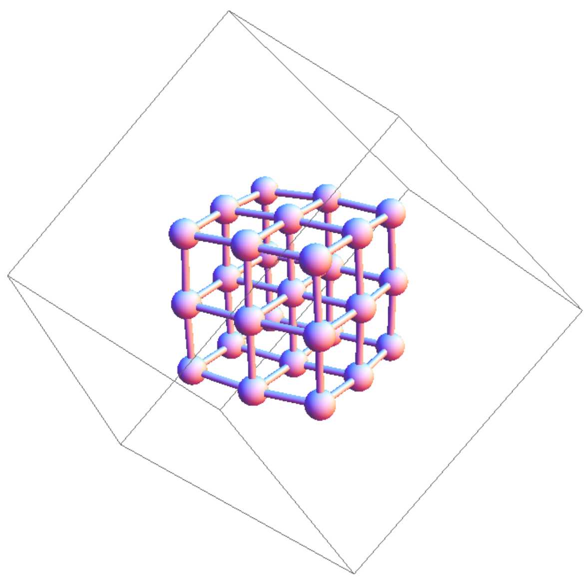
2. Find a four coloring for the graph below. Find a three coloring, or explain why no three coloring can exist. Find a two coloring, or explain why no two coloring can exist.



3. Given a $3 \times 3 \times 3$ lattice (like a cube) can you make a path for each of the following conditions that visits every vertex exactly once? An image of a lattice is shown below (the wireframe cube surrounding the lattice is not a part of the problem).

   (a) The walk must start one of the corners or the center of a face.

   (b) The walk must start at an edge but not a corner.



4. During CS 20 staff meeting, each of five TFs fell asleep exactly twice. For each pair of these teaching fellows, there was some moment when both were sleeping simultaneously. Prove that, at some moment, some three of them were sleeping simultaneously. [1]

   Hints:

   - Consider a graph with 10 vertices, one for each time a TF slept. Draw edges between two vertices if those two naps overlapped.
   - What is the significance of a cycle in this graph?

[1] Adapted from USAMO 1986 Problem 2