

COMPUTER SCIENCE 20, SPRING 2014
Homework Problems
Basic Counting, Counting Subsets, Basic Probability
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1. At each step a robot has to move one unit in the positive x direction or one unit in the positive y direction. Assuming that the robot starts at the origin (0,0), in how many different ways can the robot reach the point (4,5)?

$$\binom{9}{4} = \binom{9}{5} = 126$$

2. (a) How many odd, four-digit numbers are there with no repeated digits? Note that, for example, 0123 is not considered a four-digit number, but rather a three-digit number.

$$5 \cdot 9 \cdot 8 \cdot (6 + 7)$$

- (b) How many even, four-digit numbers are there with no repeated digits?

$$5 \cdot 9 \cdot 8 \cdot (6 + 7)$$

3. How many ways are there to order the 26 letters of the alphabet so that no two of the vowels a, e, i, o, u appear consecutively and the last letter in the ordering is not a vowel?

Solution: We have 26 letters in the alphabet and 5 of them are vowels, first we can arrange all the letters in the alphabet without the vowels. Now that we have 21 letters arranged, the first vowel can be placed in 21 different positions (i.e, before the first letter and the before the last letter). Now that the first vowel has taken a spot, there are 20 different places that we can place the next vowel. Next we have 19 for the next vowel, the next vowel will have 18 spots that it can fill. Finally there would be a total of 17 spots left for the last vowel. This leads us to have

$$21! \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17$$

4. You have two identical 10-sided dice numbered 1,2,...,10, with the strange property that, for each die, the probability of rolling n is *proportional* to n . In other words, the probability of rolling a 2 is twice the probability of rolling a 1, the probability of rolling a 3 is three times the probability of rolling a 1, and so on.

- (a) For one of these dice, what is the probability of rolling an n ? (Express your answer as some formula involving n .)

$$P(\text{rolling } n) = \frac{n}{55}$$

- (b) You roll these two dice at the same time. What is the probability that the sum of the two rolls is at least 18? (Please leave your answer as a fraction.)

$$P(\text{ at least } 18) = (\frac{9}{52} * \frac{9}{52}) + (\frac{8}{52} * \frac{10}{52}) + (\frac{10}{52} * \frac{8}{52}) + (\frac{10}{52} * \frac{10}{52}) + (\frac{9}{52} * \frac{10}{52}) + (\frac{10}{52} * \frac{10}{52})$$

- (c) Is the probability of rolling at least one 9 independent of the probability that the sum of the two rolls is at least 18? Justify your answer mathematically.

The probability of rolling at least one 9 is not independent of the probability that the sum of the two rolls is at least 18. The first 9 that we get from rolling our first could be influenced by the fact that our sum is 18, rolling a 9 on the next die will cause our sum to be 18 and will mean that the first 9 from the first die influenced the result.

$$P(18) = \frac{\text{rolling a 9}}{\text{sum of our roll}}$$