COMPUTER SCIENCE 20, SPRING 2014
Homework Problems
Strong Induction, Propositional Logic

Due Wednesday February 12, 2014. CS 20 students should bring a hard copy to class. CSCI E-120 students should submit an electronic copy.

1. Suppose you are given a real number x such that $x + \frac{1}{x}$ is an integer. Use Strong Induction to show that $x^n + \frac{1}{x^n}$ is an integer for all positive integers n.

   Solution: For $n = 1$, the result holds. For $n = 2$, we know that $(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$, so the result holds for $n = 2$. Suppose the result holds for all positive integers $n < k$. The induction hypothesis tells us $x^{k-1} + \frac{1}{x^{k-1}}$ is an integer. Moreover, we know $x + \frac{1}{x}$ is an integer, so we conclude:

$$\begin{aligned}(x^{k-1} + \frac{1}{x^{k-1}})(x + \frac{1}{x}) &= x^k + \frac{x^{k-1}}{x} + \frac{x}{x^{k-1}} + \frac{1}{x^k} \\ &= x^k + \frac{1}{x^k} + x^{k-2} + \frac{1}{x^{k-2}}\end{aligned}$$

   Given that the induction hypothesis guarantees $x^{k-2} + \frac{1}{x^{k-2}}$ is an integer, it follows that $x^k + \frac{1}{x^k}$ is an integer.

   Question:
   The solution is a little bit unclear to me if we plug 2 into this equation, this is what I see as the result. $x^2 + \frac{1}{x^2}$ In your solution I see that you have an extra +2, not sure where that is comming from as it does not seem obvious to me. Also I noticed that you used $k - 1$ in ypur inductive hypothesis why is that? I thought that were always trying to prove for $n + 1$