COMPUTER SCIENCE 20, SPRING 2014

Module #20 (Graph Connectivity)

Author: Keenan Monks
Reviewer: Paul Handorff
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Executive Summary

1. Connected. Two vertices are *connected* if there is a path between them. *Being connected* is an equivalence relation. A *connected component* is a subgraph consisting of some vertex and all other vertices and edges connected to it. A graph is *connected* if it has only one connected component.

2. Edge connectivity. The edge connectivity of a graph is the smallest number of edges that must be removed to make it *disconnected*. An edge whose deletion increases the number of connected components is called a *bridge*.

3. Vertex connectivity. The vertex connectivity of a graph is the smallest number of vertices that must be removed to make it *disconnected*. A vertex whose deletion increases the number of connected components is called an *articulation point*.

Small group problems

1. What are the edge and vertex connectivities of the following graphs?

   (a) The complete graph K_n.

   (b) A cycle of length n, C_n.

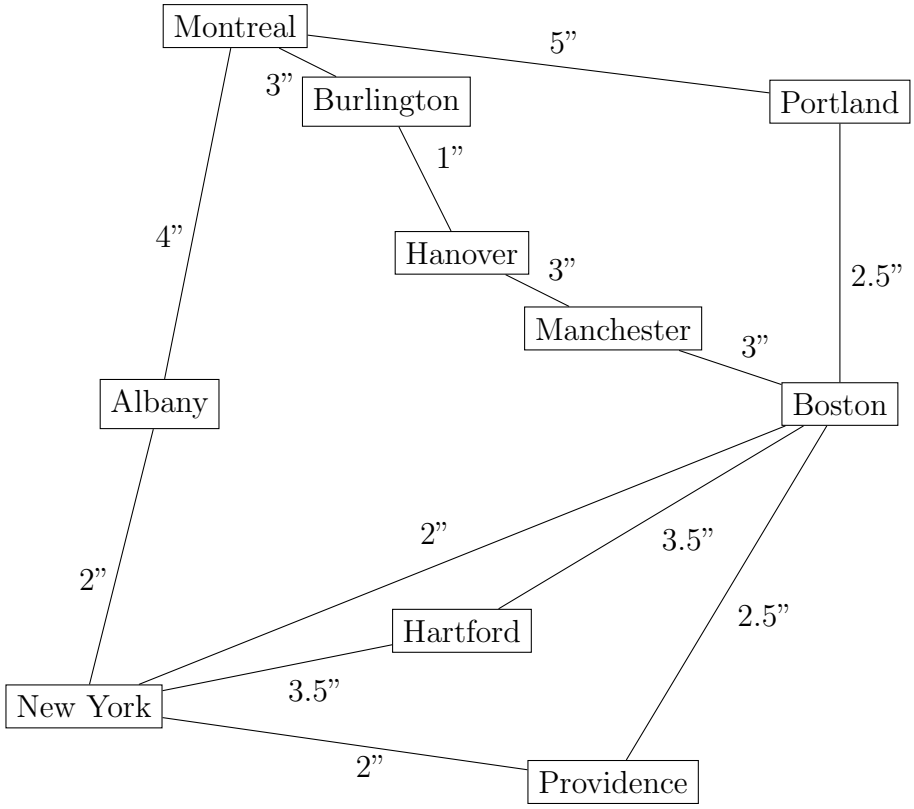
   (c) A path of length n (n vertices), P_n.

   (d) The complete bipartite graph $K_{n,n}$ (all possible edges are present).

2. Schedule the final exams for the 6 courses — CS50, MATH22, CS20, CS179, STAT110, and CS121 — using the fewest number of different time slots. Courses that have students in common cannot be scheduled at the same time, and the following 8 pairs share at least one student:

   (CS50, MATH22), (CS50, CS179), (MATH22, CS20), (MATH22, STAT110), (CS20, CS179),(CS20, STAT110), (CS121, CS179), (STAT110, CS121)

3. Below is a graph of some cities and the roads connecting them showing the maximum number of inches of snow that can fall before the roads become impassable. For example, if 2 inches of snow fall, the New York to Albany route remains operational, but if 2.25 inches of snow fall, the road is closed.



(a) Ignoring the inch labels, what is the edge connectivity of the region? What is the vertex connectivity?

(b) Now following the snowfall criterion for edge removal, what is the minimum amount of snowfall that would disconnect the region?

(c) Does the graph have any articulation points? Does it have any bridges? How would your answers change if 2 inches of snow fell on the region?