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Module #3 (Induction I)
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Use induction to prove that for all nonnegative integers n:

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Solution: We are going to use induction to prove

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

for all nonnegative integers n
base case: We prove when n is 0

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=0}^{n} 0^2 = \frac{0(0+1)(2(0)+1)}{6}$$

$$0^2 = \frac{0}{6}$$

Inductive step: We prove that it is true for $n+1$, first we will solve the left hand side of our equation

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=0}^{n+1} k^2 = \frac{(n+1)(n+2)(2(n+1)+1)}{6}$$

$$\sum_{k=0}^{n} k^2 + (n+1)^2 = \frac{(n+1)(n+2)(2(n+1)+1)}{6}$$

$$\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2n+2+1)}{6}$$

$$\frac{n(n+1)(2n+1)+6(n+1)^2}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\frac{(n+1)[(n)(2n+1)+6(n+1)]}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\frac{(n+1)[(2n^2+7n+6)]}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$
$$\frac{(n+1)(2n^2+7n+6)}{6} = \frac{(n+1)(2n^2+7n+6)}{6}$$

Then: $P(n)$ is true for all nonnegative integers n