COMPUTER SCIENCE E-20, SPRING 2014
Homework Problems
Quantificational Logic II
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1. Using the formula $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$ prove that

$$\neg p \Leftrightarrow q \equiv p \Leftrightarrow \neg q.$$

$$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p) \equiv (p \rightarrow \neg q) \wedge (\neg p \rightarrow p)$$

$$(p \vee q) \wedge (\neg q \vee \neg p) \equiv (\neg p \vee \neg q) \wedge (q \vee p)$$

$$\Big[(p \vee q) \wedge \neg q\Big] \vee \Big[(p \vee q) \wedge \neg p\Big] \equiv \Big[(\neg p \vee \neg q) \wedge q\Big] \vee \Big[(\neg p \vee \neg q) \wedge p\Big]$$

$$\Big[(p \wedge \neg q) \vee (q \wedge \neg q)\Big] \vee \Big[(p \wedge \neg p) \vee (q \wedge \neg p)\Big] \equiv \Big[(\neg pq) \vee (\neg p \wedge q)\Big] \vee \Big[(\neg p \wedge p) \vee (\neg q \wedge p)\Big]$$

$$(p \wedge \neg q) \vee (q \wedge \neg p) \equiv (\neg p \wedge q) \vee (\neg q \wedge p)$$

2. For each of the following statements, determine whether they are true or false. If false, write their logical negation (distributing the $\neg$ across any expressions as necessary) and explain how this negation is true. In each of these, x and y are assumed to be integers.

   (a) $(\forall x)(\forall y)((y > x) \Rightarrow (x = 0))$
   This statement is false and its logical equivalence is $(\exists x)(\exists y)((y > x) \wedge (x \neq 0))$

   (b) $(\exists x)(\forall y)((y > x) \Rightarrow (x = 0))$
   $(\forall x)(\exists y)((y > x) \Rightarrow (x \neq 0))$

   (c) $(\forall x)(\exists y)((y > x) \Rightarrow (x = 0))$
   $(\exists x)(\forall y)((y > x) \Rightarrow (x \neq 0))$

   (d) $(\forall x)(((\forall y)(y > x)) \Rightarrow (x = 0))$
   $(\exists x)(((\exists y)(y > x)) \Rightarrow (x \neq 0))$

3. After the first cabal's plot was discovered by the students of CS 20, the Teaching Fellows have formed a new cabal with the evil intent of making you prove the Four Color Theorem[1] on the next homework! The only way to stop them is to unravel their logic code and discover the identities of the new cabal members. The names of the possible members are

Keenan, Nick, Paul, Roger, Ruth, Yifan

[1] The proof of this required checking 1936 cases by computer, and its validity was questioned for years.

Let $F(x)$ mean "x was in the first cabal" and $N(x)$ mean "x is in the new cabal." The code is the following:

(a) $F(\text{Keenan}) \wedge F(\text{Nick}) \wedge F(\text{Paul}) \wedge \neg(F(\text{Roger}) \vee F(\text{Ruth}) \vee F(\text{Yifan}))$

(b) $\exists x \exists y, (F(x) \wedge N(x) \wedge F(y) \wedge \neg N(y))$

(c) $\forall x, (F(x) \Rightarrow N(\text{Ruth}))$

(d) $N(\text{Keenan}) \vee N(\text{Roger}) \Rightarrow \forall x, (F(x) \Rightarrow N(x))$

(e) $N(\text{Yifan}) \Rightarrow \neg(N(\text{Paul}) \vee N(\text{Nick}))$

(f) $N(\text{Paul}) \Leftrightarrow N(\text{Nick})$