COMPUTER SCIENCE E-20, SPRING 2014
Homework Problems
Quantificational Logic II
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1. Using the formula $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$ prove that

$$\neg p \Leftrightarrow q \equiv p \Leftrightarrow \neg q.$$

$$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p) \equiv (p \rightarrow \neg q) \wedge (\neg p \rightarrow p)$$

$$(p \vee q) \wedge (\neg q \vee \neg p) \equiv (\neg p \vee \neg q) \wedge (q \vee p)$$

$$\Big[(p \vee q) \wedge \neg q\Big] \vee \Big[(p \vee q) \wedge \neg p\Big] \equiv \Big[(\neg p \vee \neg q) \wedge q\Big] \vee \Big[(\neg p \vee \neg q) \wedge p\Big]$$

$$\Big[(p \wedge \neg q) \vee (q \wedge \neg q)\Big] \vee \Big[(p \wedge \neg p) \vee (q \wedge \neg p)\Big] \equiv \Big[(\neg p \wedge q) \vee (\neg p \wedge q)\Big] \vee \Big[(\neg p \wedge p) \vee (\neg q \wedge p)\Big]$$

$$(p \wedge \neg q) \vee (q \wedge \neg p) \equiv (\neg p \wedge q) \vee (\neg q \wedge p)$$

$$(\neg p \wedge q) \vee (\neg q \wedge p) \equiv (\neg p \wedge q) \vee (\neg q \wedge p)$$

2. For each of the following statements, determine whether they are true or false. If false, write their logical negation (distributing the $\neg$ across any expressions as necessary) and explain how this negation is true. In each of these, x and y are assumed to be integers.

   (a) $(\forall x)(\forall y)((y > x) \Rightarrow (x = 0))$
   This statement is false and its logical equivalence is $(\exists x)(\exists y)((y > x) \wedge (x \neq 0))$ The negation is true in the sense that we can claim that there exists some number x and y and also where $y > x$. The fact that x is less than y does not necessarily mean that $x = 0$.

   (b) $(\exists x)(\forall y)((y > x) \Rightarrow (x = 0))$
   This statement is true because there exists some number that can be 0, for all the $y's$. It does not really matter what the value of y is in order to make this statement true. From our truth table we know that $F \rightarrow T$ is $True$ Our implication is true so the statement must be true.

   (c) $(\forall x)(\exists y)((y > x) \Rightarrow (x = 0))$
   This statement is true because there can exists some y that will make $y > x$ be $False$ which will mean that our statement is $True$.

   (d) $(\forall x)(((\forall y)(y > x)) \Rightarrow (x = 0))$
   To test for the truthness of this statemnet we can first find what $(\forall y)((y > x) \Rightarrow (x = 0))$ evaluates to $False$. For all y there can exist a chance whereby it is less than x because the value is already known. This leaves us with the statement $(\forall x)(F \Rightarrow (x = 0))$ At this point it really does no matter whether or not $x = 0$ because we know that $False$ implies anything is $True$

3. After the first cabal's plot was discovered by the students of CS 20, the Teaching Fellows have formed a new cabal with the evil intent of making you prove the Four Color Theorem[1] on the next homework! The only way to stop them is to unravel their logic code and discover the identities of the new cabal members. The names of the possible members are

Keenan, Nick, Paul, Roger, Ruth, Yifan

Let $F(x)$ mean "x was in the first cabal" and $N(x)$ mean "x is in the new cabal." The code is the following:

(a) $F(\text{Keenan}) \wedge F(\text{Nick}) \wedge F(\text{Paul}) \wedge \neg(F(\text{Roger}) \vee F(\text{Ruth}) \vee F(\text{Yifan}))$

(b) $\exists x \exists y, (F(x) \wedge N(x) \wedge F(y) \wedge \neg N(y))$

(c) $\forall x, (F(x) \Rightarrow N(\text{Ruth}))$

(d) $N(\text{Keenan}) \vee N(\text{Roger}) \Rightarrow \forall x, (F(x) \Rightarrow N(x))$

(e) $N(\text{Yifan}) \Rightarrow \neg(N(\text{Paul}) \vee N(\text{Nick}))$

(f) $N(\text{Paul}) \Leftrightarrow N(\text{Nick})$

Solution:
From statement (a) we know that the first cabal was made up of Keenan, Nick and Paul. We also know from the second statement (b) that there exists someone who is in the first cabal and also in the new cabal, and also there exists someone who is in the first cabal but not in the new cabal. The third statement says that Ruth is in the new cabal because of the fact that $F(x)$ is true for some value x. Based on our conclusion from the second statement (b) we see that neither Keenan nor Roger are in the new cabal. From statement (e) and (b) we know that Yifan is not in the new cabal, if not it would mean that none of the old members being new members. We can conclude that the cabal is Ruth, Paul and Nick

[1]The proof of this required checking 1936 cases by computer, and its validity was questioned for years.