SUPPLEMENTARY MATERIALS: Automatically Controlled Morphing of 2D Shapes with Textures[*]

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SM1. Automatic control of the Space-Time Blending: interval estimation for the coefficient a_0. For estimating coefficient a_0 we suggest to use interval arithmetic [SM1]. Let us assume that two input shapes S_1 and S_2 defined by the functions $f_1(x, y)$ and $f_2(x, y)$ are represented by their circumscribed circles for simplicity (to simplify further we will use f_1 and f_2). As the input data is represented by the images, all calculations are made in a normalised coordinate system: $x = x/I_w; y = y/I_h$, where I_w and I_h is the width and the height of an image. Instead of a bounded blending function we suggest to use the simple blending function described as

$$F_{blend} = f_1 + f_2 + \sqrt{f_1^2 + f_2^2} + \frac{a_0}{1 + \frac{1}{a_{1,2}^2}(f_1^2 + f_2^2)} \tag{SM1.1}$$

where $a_1 = a_2 = a_{1,2}$. We would like to bound the blending operation defined by equation (SM1.1) using two tangential lines m_1 and m_2, which can be written in the general case as

$$\begin{aligned} m_1(x, y) &= A_1 x + B_1 y + C_1; \\ m_2(x, y) &= A_2 x + B_2 y + C_2 \end{aligned}$$

where A_1, B_1, C_1 and A_2, B_2, C_2 can be obtained by solving system of the following equations

$$\begin{cases} A_i x_1 + B_i y_1 + C_i = R_1 \\ A_i x_2 + B_i y_2 + C_i = R_2 \\ A_i^2 + B_i^2 = 1 \end{cases}$$

where $i = 1, 2$, R_1 is the radius of the circumscribed circle around the initial shape with centre point $O_1(x_1, y_1)$ and R_2 is the radius of the circumscribed circle around the target shape with centre point $O_2(x_2, y_2)$. After solving this system the following coefficients are obtained:

$$\begin{aligned} A_{2,1} &= R_m X_m \pm Y_m \sqrt{1 - R_m^2}; \\ B_{2,1} &= R_m Y_m \mp X_m \sqrt{1 - R_m^2}; \\ C_{2,1} &= R_1 - (A_{2,1} x + B_{2,1} y), \end{aligned}$$

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[*]Submitted to the editors DATE.

and the following coefficients X_m, Y_m and R_m are defined as:

$$X_m = \frac{x_2 - x_1}{D_m}; \quad Y_m = \frac{y_2 - y_1}{D_m}; \quad R_m = \frac{R_2 - R_1}{D_m}; \quad D_m = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The indices for $A_{2,1}, B_{2,1}$ and $C_{2,1}$ are obtained using (SM1.2) and are inverted here because we use the normalised coordinate system in which all the calculations are conducted.

The final system of the inequality equations can be written as:

$$\begin{cases} a_0 \leq (A_1 x + B_1 y + C_1 - (f_1 + f_2 + \sqrt{f_1^2 + f_2^2}))(1 + \frac{1}{a_{1,2}^2}(f_1^2 + f_2^2)), \\ a_0 \geq (A_2 x + B_2 y + C_2 - (f_1 + f_2 + \sqrt{f_1^2 + f_2^2}))(1 + \frac{1}{a_{1,2}^2}(f_1^2 + f_2^2)) \end{cases} \tag{SM1.2}$$

For calculating the final interval this inequality equations system (SM1.2) should be converted to the interval representation. First let us define the minimum and maximum values for x and y taking into account that all the calculations are represented in a normalised coordinate system.

$$\begin{aligned} x_{min} &= \min(x_1, x_2) - R_i, \;\; x_{max} = \max(x_1, x_2) + R_i, \\ y_{min} &= \min(y_1, y_2) - R_i, \;\; y_{max} = \max(y_1, y_2) + R_i, \end{aligned} \tag{SM1.3}$$

The index i for the radius R_i is chosen through the index of the variable obtained using the min/max operation. The intervals for the following functions for the two circles, written in the normalised form

$$f_i = \sqrt{x^2 - 2x_i x + x_i^2} + \sqrt{y^2 - 2y_i y + y_i^2} - R_i; \quad i = 1, 2$$

can be written as

$$\begin{aligned} [f_1] = \Bigg[&\sqrt{x_{min}x_{max} - 2x_1 x_{max} + x_1^2} + \sqrt{y_{min}y_{max} - 2x_1 y_{max} + y_1^2} - R_1, \\ &\sqrt{x_{max}^2 - 2x_1 x_{min} + x_1^2} + \sqrt{y_{max}^2 - 2x_1 y_{min} + y_1^2} - R_1 \Bigg], \\ [f_2] = \Bigg[&\sqrt{x_{min}x_{max} - 2x_2 x_{max} + x_2^2} + \sqrt{y_{min}y_{max} - 2x_2 y_{max} + y_2^2} - R_2, \\ &\sqrt{x_{max}^2 - 2x_2 x_{min} + x_2^2} + \sqrt{y_{max}^2 - 2x_2 y_{min} + y_2^2} - R_2 \Bigg], \end{aligned} \tag{SM1.4}$$

To represent the intervals for the final function in a simplified form let us assume that $[f_1] = [f_{1_{min}}, f_{2_{max}}]$ and that $[f_2] = [f_{2_{min}}, f_{2_{max}}]$. Then the inequality system of equations

(SM1.2) can be rewritten using interval arithmetic as

$$\begin{aligned}
a_0 &\leq [min[g_{1_{min}} b_{1_{max}}, g_{1_{max}} b_{1_{min}}, g_{1_{min}} b_{1_{min}}, g_{1_{max}} b_{1_{max}}],\\
&\quad max[g_{1_{min}} b_{1_{max}}, g_{1_{max}} b_{1_{min}}, g_{1_{min}} b_{1_{min}}, g_{1_{max}} b_{1_{max}}]]\\
a_0 &\geq [min[g_{2_{min}} b_{1_{max}}, g_{2_{max}} b_{1_{min}}, g_{2_{min}} b_{1_{min}}, g_{2_{max}} b_{1_{max}}],\\
&\quad max[g_{2_{min}} b_{1_{max}}, g_{2_{max}} b_{1_{min}}, g_{2_{min}} b_{1_{min}}, g_{2_{max}} b_{1_{max}}]]\\
[g_1] &= [A_1 x_{min} + B_1 y_{min} + C_1 - f_{s_{max}}, A_1 x_{max} + B_1 y_{max} + C_1 - f_{s_{min}}]\\
[g_2] &= [A_2 x_{min} + B_2 y_{min} + C_2 - f_{s_{max}}, A_2 x_{max} + B_2 y_{max} + C_2 - f_{s_{min}}]\\
[b_1] &= \left[\left(f_{1_{min}} f_{1_{max}} + f_{2_{min}} f_{2_{max}}\right)\frac{1}{a_{1,2}^2} + 1, \left(f_{1_{max}}^2 + f_{2_{max}}^2\right)\frac{1}{a_{1,2}^2} + 1\right]\\
[f_{s_{min}}, f_{s_{max}}] &= [f_{1_{min}} + f_{2_{min}} + \sqrt{f_{1_{min}} f_{1_{max}} + f_{2_{min}} f_{2_{max}}},\\
&\quad f_{2_{max}} + f_{2_{max}} + \sqrt{f_{1_{max}}^2 + f_{2_{max}}^2}]
\end{aligned} \tag{SM1.5}$$

If circumscribed circles are not overlapping and their centres coincide the estimation procedure for a_0 should be changed. The coefficients $a_1 = a_2 = a_{1,2}$ should be set to 1 and inequality (SM1.2) rewritten as

$$a_0 \leq ((x - x_i)^2 + (y - y_i)^2 - R_i^2 - (f_1 + f_2 + \sqrt{f_1^2 + f_2^2}))\Big(1 + \frac{1}{a_{1,2}^2}\Big(f_1^2 + f_2^2\Big)\Big),$$

where $i = 1, 2$ and depending on the chosen radius of the biggest circumscribed circle $max(R_1, R_2)$. In this case the blending area is restricted by the circumscribed circle with the biggest radius. The interval for the function f_c defining this circle can be written as

$$\begin{aligned}
[f_c] &= [(x_{min} - x_i)(x_{max} - x_i) + (y_{min} - y_i)(y_{max} - y_i) - max(R_1^2, R_2^2),\\
&\quad (x_{max} - x_i)^2 + (y_{max} - y_i)^2 - max(R_1^2, R_2^2)], \quad i = \begin{cases} 1, & if\ max(R_1^2, R_2^2) = R_1^2 \\ 2, & if\ max(R_1^2, R_2^2) = R_2^2 \end{cases}.
\end{aligned} \tag{SM1.6}$$

Then using interval arithmetic we can obtain the final interval

$$\begin{aligned}
a_0 &\leq [min[f_{c_{min}} b_{1_{max}}, f_{c_{max}} b_{1_{min}}, f_{cmin} b_{1_{min}}, f_{c_{max}} b_{1_{max}}],\\
&\quad max[f_{c_{min}} b_{1_{max}}, f_{c_{max}} b_{1_{min}}, f_{c_{min}} b_{1_{min}}, f_{c_{max}} b_{1_{max}}]]
\end{aligned} \tag{SM1.7}$$

where the interval for $[b_1]$ was introduced in (SM1.5).

REFERENCES

[1] R. E. Moore, R. B. Kearfott, and M. J. Cloud, *Introduction to Interval Analysis*, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2009.