

# Recursion

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## Advanced Sorting Algorithms

# Bubble Sort

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- ▶ Loop until no swap is done:

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- ▶ In each pass, loop over every index in list

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- ▶ Loop until no swap is done:
- ▶ In each pass, loop over every index in list
- ▶ Compare to the item to its right

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- ▶ In each pass, loop over every index in list
- ▶ Compare to the item to its right
- ▶ If bigger, swap

# Bubble Sort

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- ▶ In each pass, loop over every index in list
- ▶ Compare to the item to its right
- ▶ If bigger, swap
- ▶ Otherwise, do nothing



# Selection Sort

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# Selection Sort

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Modify Bubble Sort:

- ▶ Loop until no swap is done:
- ▶ In each pass, loop over every item in list
- ▶ Compare to the item to its left
- ▶ *If bigger, swap*
- ▶ Otherwise, do nothing

# Selection Sort

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Modify Bubble Sort:

- ▶ Loop until no swap is done:
- ▶ In each pass, loop over every item in list
- ▶ Compare to the item to its left
- ▶ **If bigger, save for later**
- ▶ Otherwise, do nothing

# Selection Sort

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Modify Bubble Sort:

- ▶ Loop until no swap is done:
- ▶ In each pass, loop over every item in list
- ▶ Compare to the item to its left
- ▶ **If bigger, save for later**
- ▶ Otherwise, do nothing
- ▶ **Swap biggest item to end of list**

# Insertion Sort

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*Insert each item into the already sorted sub-list*

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- ▶ Loop until all items have been inserted:
- ▶ In iteration  $i$ , insert the  $i$ th item into the sublist `myList[:i]`



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*Insert each item into the already sorted sub-list*

- ▶ Loop until all items have been inserted:
- ▶ In iteration  $i$ , insert the  $i$ th item into the sublist `myList[:i]`
  - ▶ Easy to insert into an already-sorted sub-list

# Insertion Sort

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*Insert each item into the already sorted sub-list*

- ▶ Loop until all items have been inserted:
- ▶ In iteration  $i$ , insert the  $i$ th item into the sublist `myList[:i]`
  - ▶ Easy to insert into an already-sorted sub-list
  - ▶ sub-list `myList[:i]` is already sorted

# Insertion Sort

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*Insert each item into the already sorted sub-list*

- ▶ Loop until all items have been inserted:
- ▶ In iteration  $i$ , insert the  $i$ th item into the sublist `myList[:i]`
  - ▶ Easy to insert into an already-sorted sub-list
  - ▶ sub-list `myList[:i]` is already sorted
  - ▶ so coding this insertion is *easy*

# Merge Sort

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- ▶ Break up `myList` into sub-lists of length 1

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- ▶ A sub-lists of length 1 is already sorted

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- ▶ Break up `myList` into sub-lists of length 1
- ▶ A sub-lists of length 1 is already sorted
- ▶ Merge together sorted sub-lists until everything has been merged

# Merge Sort

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# Merge Sort

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```
myList = [8, 5, 10, 1, 4]
```

# Merge Sort

---

```
myList = [8, 5, 10, 1, 4]
```

8

5

10

1

4

# Merge Sort

---

```
myList = [8, 5, 10, 1, 4]
```

8            5            10                    1            4

5, 8                                    1, 10                    4

# Merge Sort

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```
myList = [8, 5, 10, 1, 4]
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8            5            10                    1            4

5, 8                                    1, 10                    4

5, 8                                    1, 4, 10

# Merge Sort

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```
myList = [8, 5, 10, 1, 4]
```

8            5            10                    1            4

5, 8                                    1, 10                    4

5, 8                                    1, 4, 10

1, 4, 5, 8, 10

# Merge Sort

---

```
myList = [8, 5, 10, 1, 4]
```

8            5            10                    1            4

5, 8                                    1, 10                    4

5, 8                                    1, 4, 10

1, 4, 5, 8, 10

►  $\mathcal{O}(\log n)$  levels

# Merge Sort

---

`myList = [8, 5, 10, 1, 4]`

8            5            10                    1            4

5, 8                                    1, 10                    4

5, 8                                    1, 4, 10

1, 4, 5, 8, 10

- ▶  $\mathcal{O}(\log n)$  levels
- ▶  $\mathcal{O}(n)$  work to merge per level

# Merge Sort

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`myList = [8, 5, 10, 1, 4]`

8            5            10                    1            4

5, 8                                    1, 10                    4

5, 8                                    1, 4, 10

1, 4, 5, 8, 10

- ▶  $\mathcal{O}(\log n)$  levels
- ▶  $\mathcal{O}(n)$  work to merge per level
- ▶  $\mathcal{O}(n \log n)$  total work



# Coding Merge Sort

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Merge Sort is tricky to code...

# Coding Merge Sort

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Merge Sort is tricky to code...

We need to learn about *recursion*

# Recursion

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A **recursive** function is a function that calls itself:

# Recursion

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A **recursive** function is a function that calls itself:

```
def my_func(n):  
    if n > 0:  
        return (1 + my_func(n-1))  
    else:  
        return 0
```

# Recursion Loops

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# Recursion Loops

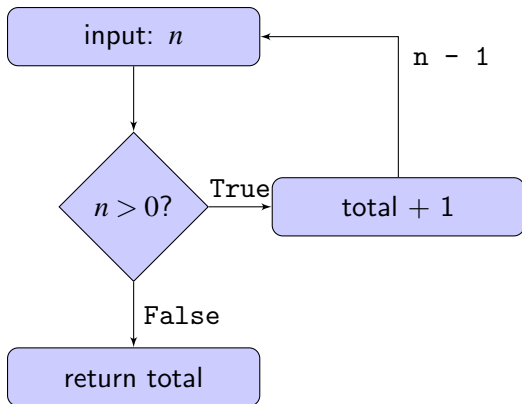
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This is an alternative way to create a loop:



# Recursion Loops

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# Anatomy of a Recursive Function

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        return 0
```

# Anatomy of a Recursive Function

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```
def my_func(n):  
    if n > 0:  
        return (1 + my_func(n-1))  
    else:  
        return 0
```

**Recursive Case/Recursive Call**

# Anatomy of a Recursive Function

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```
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    if n > 0:  
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        return 0
```

# Anatomy of a Recursive Function

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```
def my_func(n):  
    if n > 0:  
        return (1 + my_func(n-1))  
    else:  
        return 0
```

**Base Case**

# A Recursive Task

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$$n! = n \times (n-1) \times (n-2) \times (n-3) \times \dots \times 1$$

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$$n! = n \times (n-1)!$$

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► `SortedList = merge(firstHalf, secondHalf)`



# Merge Sort

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- ▶ `SortedList = merge(firstHalf, secondHalf)`
- ▶ `firstHalf = merge(firstQuarter, secondQuarter)`

# Merge Sort

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- ▶ `SortedList = merge(firstHalf, secondHalf)`
- ▶ `firstHalf = merge(firstQuarter, secondQuarter)`
- ▶ `secondHalf = merge(thirdQuarter, fourthQuarter)`