Advent of Code - Day 6

While reading the task of Advent of Code Day 6[1], the reader may swiftly realize that this problem is obviously nothing else then an inequality of a quadratic equation. Let t_0 be the time a race r takes and d_0 the distance one has to overcome. By holding down the button for a time t_c the boat accelerates without changing its position with unit acceleration $a = 1\mathrm{m/s}$. Thus, the velocity after a charging time of t_c is just $v = a \cdot t_\mathrm{c} = t_\mathrm{c}{}^2$. Using this we can find the relation

$$d_0 < t_0 \cdot t_\mathrm{c} - t_\mathrm{c}^2 \tag{1}$$

Solving this inequality for t_c yields the time-interval that one can target in order to win the race. The solution is formally given by

$$(t_\mathrm{c} - t_+)(t_\mathrm{c} - t_-) < 0 \tag{2}$$

with

$$t_\pm = \frac{t_0}{2} \pm \sqrt{\frac{t_0^2}{4} - d_0}. \tag{3}$$

Since $t_+ > t_-$ there is only one case for which this inequality holds, namely $t_\mathrm{c} \in I := (t_-, t_+)$. However since the problem is formulated for times $t_\mathrm{c} \in \mathbb{N}$, we have to round the interval I to the nearest integer. Define $I_N := (\lceil t_- \rceil, \lfloor t_+ \rfloor)$. All $t_\mathrm{c} \in I_N$ are valid solutions to the problem. The number of possible strategies s for a race r is given by the cardinality of I_N, i.e.

$$s = |I_N| = \lfloor t_+ \rfloor - \lceil t_- \rceil + 1. \tag{4}$$

Problem 1

The input for the first problem was given as displayed in Table 1. Using the

Race r	**Time** t_0	**Distance** d_0
1	54	302
2	94	1476
3	65	1029
4	92	1404

Table 1: Time and Distance

equations 3 together with 4 yields the following results: Multiplying all the results s together yields the number 1195150, which is the correct results.

[1] `https://adventofcode.com/2023/day/6`

[2] If not stated otherwise, all lengths are in millimeter and times in milliseconds.

Race r	t_+	t_-	s
1	47.66	6.34	41
2	74.07	19.93	55
3	37.72	27.30	10
4	72.68	19.32	53

Table 2: Time and Distance. Interval bounds are rounded to second decimal place.

Problem 2

This input was just one race with time $t_0 = 54946592$ and distance $d_0 = 302147610291404$. This results in

$$t_+ = 48748501.82302818 \quad , \quad t_- = 6198090.176971823 \quad \text{and} \quad s = 42550411.$$