A Simple Equation for Rapid Estimation of Rocket Nozzle Convective Heat Transfer Coefficients[1]

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Although recent analyses of the heat transfer in convergent-divergent nozzles based on considerations of the turbulent boundary layer have appeared in the literature (References 1, 2), the methods require considerable computational time for each new nozzle and have not as yet been confirmed (or denied) by reliable experimental data. This note was prepared because there still appears to be a need for a simple, yet reasonably accurate, approximation equation for making rapid preliminary estimates of the convective heat transfer coefficients in rocket nozzles.

Nomenclature

A = local cross-sectional area of flow
C_p = specific heat at constant pressure, Btu/lb, °F
C = constant coefficient, Equation [6], dimensionless
C^* = characteristic velocity, fps
D = diameter, in.
g = gravitational acceleration, fps²
h_g = heat transfer coefficient, Btu/in.² sec, °F
k = thermal conductivity
m = average molecular weight of combustion gases
M = Mach number
Nu = Nusselt number = h_gD/k
p_c = chamber pressure, lb/in.²
Pr = Prandtl number = $\mu C_p/k$
q = heat flux
r_c = throat radius of curvature
Re = Reynolds number = $\rho' UD/\mu$
T = static temperature, °R
T_c = chamber or flame temperature, °R
T_0 = stagnation temperature, °R
x = distance from inlet measured along wall
x_n = distance through nozzle measured along wall
U = free stream value of local gas velocity
γ = ratio of specific heats, dimensionless
μ = viscosity, lb/in. sec
ρ' = free stream value of local gas density
σ = dimensionless factor accounting for variation of ρ and μ values across boundary layer
ω = temperature exponent of viscosity equation

Subscripts

am = arithmetic mean
0 = stagnation conditions
* = conditions at nozzle throat
w = wall

EXPERIENCE gained from the turbulent boundary layer calculation methods (1, 2)[3] has shown that under certain conditions the dominant variable factor is the mass flow rate per unit area and that variations in velocity and temperature boundary layer thicknesses exert only a secondary, although not negligible, effect. This suggests that an equation of the form

$$h_g \sim (\rho' U)^m \quad [1]$$

might serve as the desired rough approximation equation. Equation [1] can account for mass flow rate variations by proper selection of m, while it ignores the effect of boundary layer development. (This is not to say that assumption of fully developed pipe flow has been made.) Equation [1] can be put into the more familiar nondimensional form

$$Nu = C(Re)^m(Pr)^n \quad [2]$$

which happens to be the same form as the equation that has been used for correlation of fully developed turbulent pipe flow heat transfer data.

From the equation for h_g in (2) the proper value for m can be shown to be 0.8. (Note that $(1/\theta)^{1/4}$ in Equation [31] of (2) is proportional to $Re^{0.05}$.) The value for n is arbitrarily selected as that frequently used for modifications of Reynolds analogy for Pr near unity 0.4. The problem of where to evaluate the physical properties must be answered, since in general large temperature differences are present in rocket nozzles. Since the procedure of evaluating properties at the arithmetic mean (am) between bulk temperature T and wall temperature T_w has been successful for low speed, high temperature difference problems (3) and high speed, low temperature difference problems (4), it is employed here. With these assumptions, Equation [2] can be solved for h_g to give

$$h_g = \frac{C}{D^{0.2}}\left(\frac{\mu^{0.2}C_p}{Pr^{0.6}}\right)_{am}(\rho_{am}U)^{0.8} \quad [3]$$

If it is allowed that C_p and Pr do not vary appreciably with temperature, they can be assumed constant at stagnation temperature values, while μ_{am} and ρ_{am} can be evaluated in terms of the stagnation and static temperature values, respectively. Thus Equation [3] can be written

$$h_g = \frac{C}{D^{0.2}}\left(\frac{\mu^{0.2}C_p}{Pr^{0.6}}\right)_0(\rho' U)^{0.8}\sigma \quad [4]$$

where $\sigma \equiv (\rho_{am}/\rho')^{0.8}(\mu_{am}/\mu_0)^{0.2}$. The factor σ contains all the corrections for property variation across the boundary layer. Noting that $T_{am} = 1/2(T + T_w)$, that $\rho \sim (1/T)$, and that $\mu \sim T^{\omega}$, the value of σ can be evaluated in terms of T_0, T_w, and M. (T_0 is selected since T_0 is constant through the nozzle.)

$$\sigma = \frac{1}{\left[\frac{1}{2}\frac{T_w}{T_0}\left(1 + \frac{\gamma - 1}{2}M^2\right) + \frac{1}{2}\right]^{0.8-(\omega/5)}\left[1 + \frac{\gamma - 1}{2}M^2\right]^{\omega/5}} \quad [5]$$

The equation for h_g can be put in a form more easily used for rocket nozzle computations by evaluating $\rho' U$ in terms of C^*, and A_*/A

$$h_g = \left[\frac{C}{D_*^{0.2}}\left(\frac{\mu^{0.2}C_p}{Pr^{0.6}}\right)_0\left(\frac{p_cg}{C^*}\right)^{0.8}\right]\left(\frac{A_*}{A}\right)^{0.9}\sigma \quad [6]$$

Note that the factor in the brackets is a constant through a nozzle leaving only A_*/A and σ to be evaluated at each station.

The value of C was determined by equating the value of h_g computed from Equation [6] at the throat of a particular rocket nozzle with the value computed from the turbulent

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boundary layer analysis (see sample calculation, Reference 2, case 2a) for the same values of mass flow and gas properties. The resulting value of C was found to be 0.026 which is, coincidentally, quite close to the value usually used in heat transfer correlation for turbulent flow in pipes. The agreement between the results of Equation [6] and those of the turbulent boundary layer method over the rest of the nozzle is exhibited in Fig. 1.

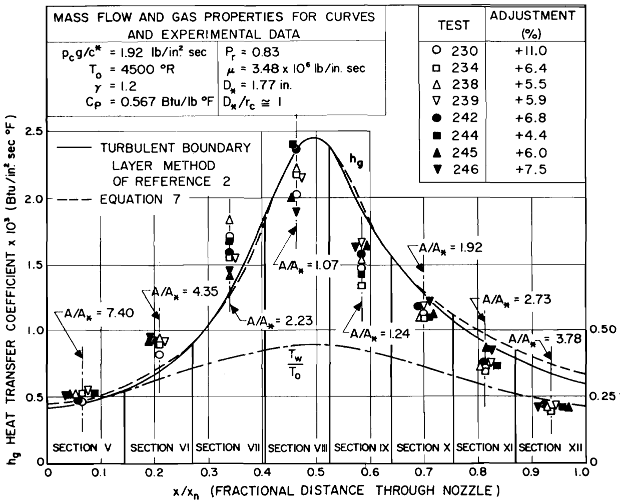

Fig. 1 Distribution of heat transfer coefficient for nozzle of Fig. 2, Ref. 2

In writing the equation in its final form, a factor to account for the effect of radius of curvature of the nozzle throat r_c is added. This factor, $(D_*/r_c)^{0.1}$ was suggested by the nozzle similarity considerations of (2)

$$h_g = \left[\frac{0.026}{D_*^{0.2}}\left(\frac{\mu^{0.2}C_p}{Pr^{0.6}}\right)_0\left(\frac{p_c g}{C^*}\right)^{0.8}\left(\frac{D_*}{r_c}\right)^{0.1}\right]\left(\frac{A_*}{A}\right)^{0.9}\sigma \quad\quad [7]$$

The particular nozzle contour for which the comparison between the simplified equation of this paper and the turbulent boundary layer method was made was one with D_*/r_c equal to about unity and having contraction and expansion half angles of 30 and 15 deg, respectively. Although nozzles with different angles will probably not show the same agreement between methods, the simplified equation will probably be sufficiently accurate for its intended purpose if the contraction and expansion angles are not changed by more than 50 per cent and the value of D_*/r_c is not greater than about 3.

As a step toward the goal of a rapid calculation method, values of σ have been computed for γ of 1.2, 1.3, 1.4; for ω of 0.6; and for various values of T_w/T_0. These are plotted in Fig. 2 vs. A/A_* on a log scale increasing on both sides of the minimum value of unity.

If Pr and μ data are not available for the particular combustion-gas mixture under consideration, kinetic theory (5) can be used to get the approximate result

$$Pr = \frac{4\gamma}{9\gamma - 5} \quad\quad [8]$$

The NBS (6) data for the viscosity of air at high temperatures can be used to get a correlation equation which should be reasonably accurate for most mixtures consisting principally of diatomic gases.

$$\mu = (46.6 \times 10^{-10})\,(\bar{m})^{1/2}\,(T\ ^{\circ}\mathrm{R})^{\omega}\ \mathrm{lb/in.\ sec} \quad\quad [9]$$

where $\omega = 0.60$. The average molecular weight factor is suggested by the statistical mechanical transport property theory (7). The value of (C_p) is usually known from thermochemical calculations for the combustion gases under consideration.

For comparison with experimental h_g data, values measured[4] semilocally in a nozzle previously described in the

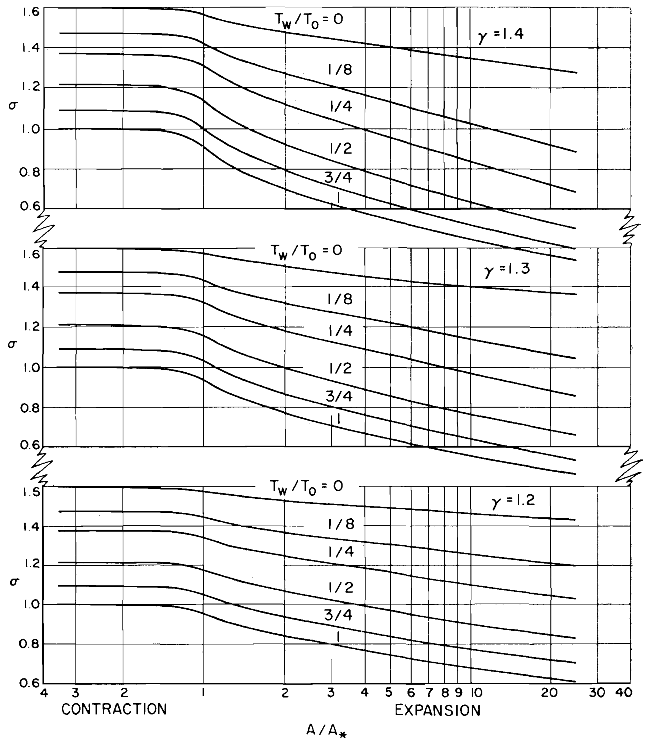

Fig. 2 Values of the properties variation parameter σ

literature (8) operating with the combustion gases of the RFNA-N_2H_4 system are also plotted in Fig. 1.

The vertical dashed lines about which the data are grouped represent the center lines of the sections defined by the vertical solid lines. Some points are plotted off the line to facilitate identification of individual data points. To achieve a common basis for comparison, these data were all adjusted to the values of mass flow and gas properties listed in the figure. The adjustments were made according to the dependence on gas flow properties indicated by Equation [7]. The maximum adjustment was 11 per cent. The experimental values of h_g were computed from calorimetrically measured semilocal values of q, computing T_0 from theoretical T_c values reduced by the square of the ratio of experimental to theoretical C^*. Wall temperatures T_w were calculated from considerations of water-cooling conditions and temperature drop through the wall.

These experimental data should not be construed as supporting or denying the results of either the equation presented or the turbulent boundary layer method. Additional reliable local data must be obtained for proper experimental evaluation of either method. Rather, it is significant (*a*) that the results of the simple correlation equation developed in this paper agree reasonably well with *a particular set* of calculations based on the turbulent boundary layer development in the nozzle, and (*b*) that both methods show reasonable agreement with the meager experimental data available from rocket motor tests. It must be remembered that the agreement between the two methods near the entrance is strongly dependent on the assumed entrance boundary layer conditions, which in this calculation may or may not have been typical of rocket-motor nozzles.

Less direct experimental verification of the equation presented is offered by the excellent agreement between average values of heat flux over a large contraction ratio nozzle computed using Equation [7] and average heat flux measurements made over a wide range of chamber pressures with the RFNA-NH_3 system (9). This equation was notably unsuc-

[4] By of E. L. Wilson of the Jet Propulsion Laboratory.

cessful in predicting chamber heat fluxes for these tests, the values being only 40–50 per cent of the measured values. This is not altogether surprising since convection related to the average mass flow rate is not the only important mode of heat transfer in a chamber. However, in a nozzle, at least near the throat and beyond, heat fluxes should be successfully predicted by considering convection based on the average mass flow rate, except when (*a*) a substantial fraction of the gases are strong radiators, (*b*) there is substantial dissociation with subsequent recombination near the wall, or (*c*) there are strong high frequency flow instabilities, in which cases the predictions are expected to be too low. Such predictions will be too high when (*a*) the combustion gases deposit insulating solids on the walls and (*b*) the combustion reactions are not completed in the chamber.

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Note on "Hazards Associated With 90 Per Cent Hydrogen Peroxide Aerosols"[1]

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SOME concern has been expressed about the severe effects of exposure to aerosols (liquid airborne particles) of H_2O_2 reported in the article "Hazards Association With 90 Per Cent Hydrogen Peroxide Aerosols."

It should be pointed out that the aerosol concentrations used in the animal experiments reported are very high and are likely to persist under accidental conditions where a constant jet of material is fed into an enclosed space. The real hazard would exist when personnel were trapped in such a space without adequate protective equipment. Where escape is possible it is likely that the irritating effects to eyes and nose would preclude prolonged exposures to high aerosol concentrations which might be lethal. It is unlikely that high aerosol or liquid airborne particle concentrations would exist under normal handling conditions.

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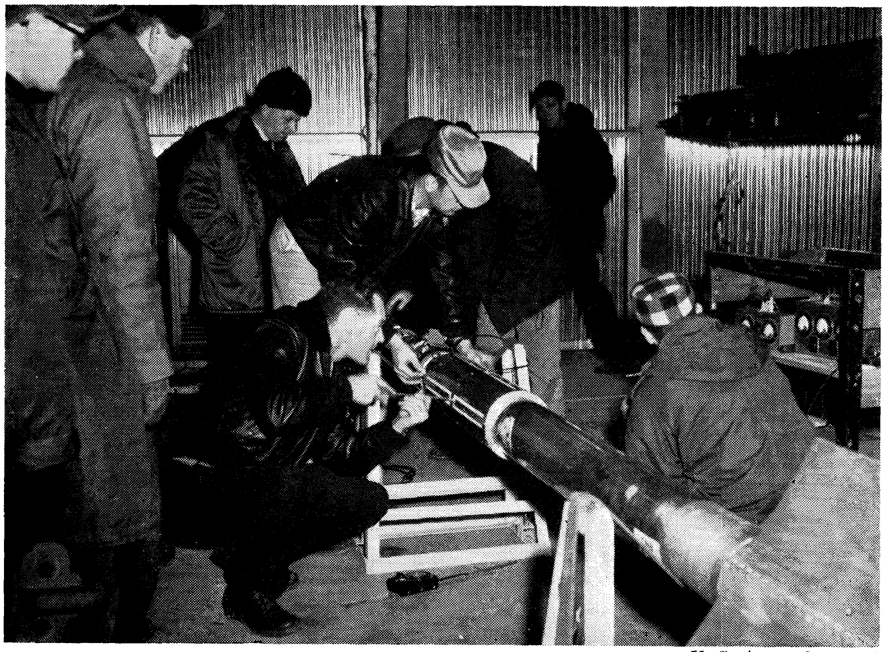
[1] By Charles L. Punte, Leon Z. Saunders, and Eugene H. Krackow. JET PROPULSION, vol. 26, June 1956, p. 500.



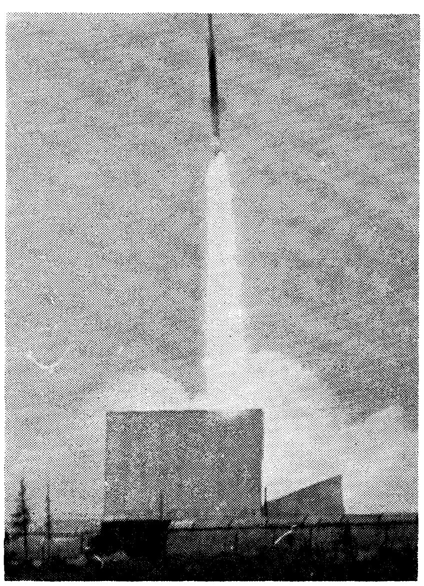
U. S. Army photograph

First to go at Fort Churchill, Nike-Cajun was fired for familiarization.

Churchill Firings Prove It Can Be Done

As a preliminary to the upcoming International Geophysical Year rocket research program, the United States National Committee for IGY recently launched six high altitude sounding rockets from Fort Churchill, Canada, in an attempt to feel out the arctic atmosphere.

Termed a tremendous success, the firings also provided valuable scientific data. Most significant, for example, are the data from a firing instrumented to determine composition of the air and ion structure at high altitude. Although still not completely analyzed, say scientists connected with the project, this data will undoubtedly contribute greatly to the advancement of long-range communication and guidance.



Nike-Cajun on its way.

Not only will this new data extend knowledge of ionospheric penetration (i.e., getting through to rockets above the upper layers), but it is also expected to contribute to improvement in the use of ionospheric reflection, the method by which most signals are bounced over long distances. Conceivably knowledge gleaned from this and similar firings could lead to transoceanic television and the like. And, possibly, this data may have some import for interplanetary travel in regard to the possible ionization effect on space vehicles.

Readings from the other rockets fired at Fort Churchill—most of which were instrumented for pressure, temperature, and density experiments—are also expected to make significant contributions to the knowledge of the upper atmosphere, says Robert M. Slavin of the Air Force Cambridge Research Center which launched two rockets in the series. But, principally, the preliminary firings served to check out range and facilities at Fort Churchill for the forthcoming IGY. Secondarily, but almost as significant, this joint effort of the U. S. National Committee for IGY and the Department of Defense showed that with the high degree of cooperation achieved among the three services and group of civilian scientists a major effort in basic research could be made on a very limited budget.

The tests, of course, were conducted by the civilian scientists. But establishment and running of the over-all facility was carried out by the Army, Navy and Air Force under direction of a Defense Department Interservice Coordinating Group.

Trouble in the North: And, most important, says John W. Townsend who is in charge of the Naval Research Laboratory field group at Churchill, these firings proved it can be done. "We've learned that we can get payloads up to predicted altitudes under arctic conditions." This was no mean feat in itself.

Air and water lines froze. Power failures were frequent. Radar equipment was beset with cold weather problems. Holes were blasted through the specially designed, enclosed launching tower and it became impossible to maintain temperatures.

And, unlike White Sands and Holloman, weather conditions and terrain at Churchill made recovery next to impossible. Over 30 per cent of the area is water during the warm months. And if the rockets don't land in the water, they are likely to land in crevices just as inaccessible to spotting. Or if they do land on the ground, there are so many big rocks and short trees in the area that look like rockets, says Townsend, that you could drop an ICBM in there and never find it. Moreover, much of the land is so soft in the summer that rockets just disappear into the tundra with a

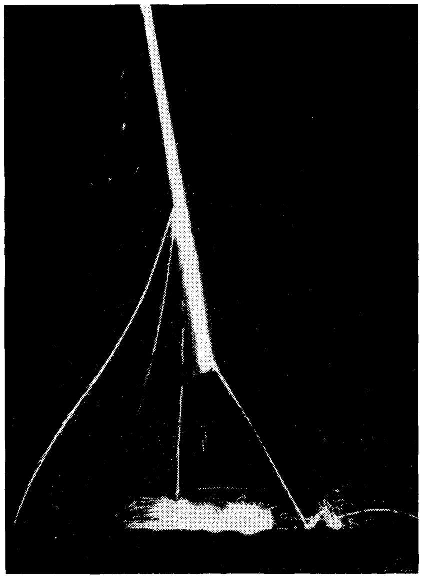
Tundra Test Site

Fort Churchill, site of the recent pre-IGY rocket launching tests, sits on the western edge of Hudson Bay in the province of Manitoba, Canada.

Most of its 500 nonmilitary inhabitants occupy themselves with fishing and gathering furs. A few hunt whales in Hudson Bay. For six weeks in late July and August, when the Bay is open to navigation, Fort Churchill serves as a port principally for the shipment of grain.

Because of its location in the subarctic and in the middle of the aurora zone, Churchill appeared to be a choice site for investigation of the upper atmosphere. Up to this time, upper air research had been confined essentially to desert regions in the southwest United States; the arctic atmosphere and auroral phenomena were practically unknown.

Thus, the National Committee for the International Geophysical Year eagerly seized upon Canada's invitation to use Fort Churchill as an IGY test site. And, starting next July, the base will serve as one of the principal launching areas for high altitude research rockets.



Taxi calls meant night firings.

"squoosh." In the winter months, cold weather and blizzards make it dangerous to send recovery crews out, and the snow quickly covers the rockets.

As a result, says Townsend, there is very little shooting for recovery; most data must be telemetered. Recovery might be feasible if the rockets were equipped with parachutes and electronic locators, in which case other interesting experiments could be run. But, as now planned, most tests will be run without hope of recovery.

The use of telemetering and tracking devices, however, was not without its share of problems too. In addition to the usual cold weather malfunctioning, the systems were plagued by interference. Using a doppler frequency of about 40 megacycles, for example, Fort Churchill technicians found that signals from Patrick AFB (Fla.) and White Sands Proving Grounds (N. Mex.) were mixing with ground-to-rocket signals from their own tracking system.

Even Charlotte (N. C.) taxicabs were cutting in on the Churchill transmissions. Consequently, most firings were rescheduled for nights and weekends when interference was at a minimum.

Count-down: Despite these numerous and vexing problems, scientists at Fort Churchill experienced only one major mishap, were able to get off six of the seven rockets that were scheduled for this preliminary run-through of the IGY. Of the six firings, all were considered eminently successful.

Here, in some detail, is a playback of the recently concluded check-out program at Fort Churchill:

• October 20. An Air Force Nike-Cajun, instrumented for pressure, temperature, and density readings, was fired principally for familiarization and check-out of the whole facility (see picture). Rocket carried approximately a 100-lb payload to an altitude of about 70 miles.

• October 23. An Air Force Aerobee, intermediate model, reached an altitude of about 90 miles with a 200-lb payload. This vehicle was also instrumented for pressure, temperature, and density readings. Although considered a good firing, pressure from the booster blast started to spread the metal sheets covering the launching tower.

• November 5. A full-blown RV-N-13B, Naval Research Laboratory Aerobee-Hi exploded while holding X-minus-15 min, tossed 700 lb of acid around inside the tower, and threw shrapnel through the sides. The rocket carried 220 lb of auroral particle instrumentation. Damage to the launching tower resulting from the explosion (and fire) was repaired.

• November 11. Signal Corps Engineering Laboratory fired an old model Aerobee in a grenade experiment (see picture) to measure temperature and winds. The rocket reached an altitude of about 45 miles with a 220-lb payload. Because this firing was so successful, the Signal Corps called off the other launching they had scheduled. But this firing really blew out the sides of the recently repaired launching tower, made it extremely difficult to maintain temperatures.

• November 15. Naval Research Laboratory group launched another RV-N-13B Aerobee-Hi instrumented for ionosphere exploration. Carrying a 180-lb payload, the rocket reached an altitude of over 80 miles. Although this was considered underperformance, it was adequate for the experiment.

• November 17. Another NRL RV-N-13B Aerobee-Hi, instrumented for pressure, temperature, and density readings, achieved near-perfect performance, reaching 130-mile altitude with 190-lb payload.

• November 20. The last rocket in the test program, a RV-N-13C Aerobee-Hi, actually hit peak performance—160-lb payload to 157-mile altitude. Unlike the other models, this Aerobee-Hi was equipped with proper tankage for its thrust chamber; i.e., there was no fuel left over at the end of its run. The vehicle was instrumented with three mass spectrometers to measure composition of air and ions at high altitudes.

Full significance of these preliminary tests, of course, must await complete analysis of the collected data. But it is already evident that the program fulfilled its primary purpose of equipping members of SCIGY (Special Committee for the International Geophysical Year) with invaluable background for the forthcoming IGY rocket program, a major part of which (over half the solid rocket launchings) will take place at Fort Churchill starting next July.