$$|\Phi_d\rangle = \psi_d(k)\,\frac{1}{\sqrt{V}}\,\underbrace{e^{i\vec{K}\cdot\vec{R}}}_{\hookrightarrow\ \Phi_d(\vec{K})}$$

$$\hat{\Lambda}_\lambda = \frac{1}{2}\sum_{m_s m_s'}\sum_{m_t m_t'}\Big[\big[1 + \langle \vec{q}\ m_s m_t\ m_s' m_t' | \delta\tilde{U} | \vec{q}\ m_s m_t m_s' m_t'\rangle +$$

$$\langle \vec{q}\ m_s m_t\ m_s' m_t' | \delta\tilde{U}^{+} | \vec{q}\ m_s m_t m_s' m_t'\rangle\big]\,|\psi_d^\lambda(\vec{q})|^2\,|\Phi_d(\vec{G})|^2\Big]$$

$$+\ \frac{1}{4}\sum_{\vec{k}}\sum_{m_{s_i}}\sum_{m_{t_j}}\langle \vec{k}\ m_{s_1} m_{t_1}\ m_{s_2} m_{t_2} | \delta\tilde{U} | \vec{q}\ m_s m_t m_s' m_t'\rangle \times$$

$$\langle \vec{q}\ m_s m_t m_s' m_t' | \delta\tilde{U}^{+} | \vec{k}\ m_{s_1} m_{t_1} m_{s_2} m_{t_2}\rangle\,|\psi_d^\lambda(\vec{k})|^2\,|\Phi_d(\vec{G})|^2$$

$$= \ldots + \frac{V}{4}\sum_{m_{s_i}}\sum_{m_{t_j}}\int\frac{d^3k}{(2\pi)^3}\langle \vec{k}\ m_{s_1} m_{t_1}\ m_{s_2} m_{t_2} | \delta\tilde{U} | \vec{q}\ m_s m_t m_s' m_t'\rangle \times$$

$$\langle \vec{q}\ m_s m_t m_s' m_t' | \delta\tilde{U}^{+} | \vec{k}\ m_{s_1} m_{t_1} m_{s_2} m_{t_2}\rangle\,|\psi_d^\lambda(\vec{k})|^2\,|\Phi_d(\vec{G})|^2$$

$\downarrow$ Partial waves, taking only S-waves

We know $\psi_d \to J=1,\ S=1,\ T=0 \qquad L=0,2$

$$= \frac{1}{2}\sum_{m_s m_s'}\sum_{m_t m_t'}\Big[1 + \sum_{PW's}\langle \vec{q}\,|\,q\,L M_L\rangle\langle m_s m_s'\,|\,S\,M_s\rangle\langle m_t m_t'\,|\,T M_T\rangle \times$$

$$\langle M_L M_s\,|\,J\,M_J\rangle\,\delta\tilde{U}_{^3S_1-^3S_1}(q,q)\,\langle J M_J\,|\,M_L M_s\rangle\langle T M_T\,|\,m_t m_t'\rangle\langle S\,M_s\,|\,m_s m_s'\rangle \times$$

$$\langle q\,L M_L\,|\,\vec{q}\rangle + h.c.\Big]\,|\psi^d_{^3S_1}(q)|^2\,|\Phi_d(\vec{G})|^2\ +$$

$$\frac{V}{4} \sum_{M_S M_S' M_S'' M_S'''} \sum_{M_t M_t' M_t'' M_t'''} \sum_{PW's} \int_0^\infty \frac{dk\,k^2}{(2\pi)^3} \overbrace{\langle \hat{k} | k L M_L \rangle}^{\int d\Omega_k} \langle M_S'' M_S''' | S M_S \rangle \times$$

$$\langle M_t'' M_t''' | T M_T \rangle \langle M_L M_S | J M_J \rangle \, \delta\tilde{U}_{3S_1 - X}(k, q) \langle J M_J | M_L' M_S \rangle \times$$

$$\langle T M_T | M_t M_t' \rangle \langle S M_S | M_S M_S' \rangle \langle q L' M_L' | \vec{q} \rangle \langle \vec{q} | q L' M_L' \rangle \times$$

$$\langle M_S M_S' | S M_S' \rangle \langle M_t M_t' | T M_T \rangle \langle M_L' M_S' | J M_J' \rangle \, \delta\tilde{U}^{+}_{X - 3S_1}(q, k) \times$$

$$\langle J M_J' | M_L M_S' \rangle \langle T M_T | M_t'' M_t''' \rangle \langle S M_S' | M_S'' M_S''' \rangle \langle k L M_L | \hat{k} \rangle$$

Simplifications: $L = 0$, $M_L = 0$, $S = 1$, $T = 0$, $M_T = 0$

$$\sum_{M_S'' M_S'''} \langle M_S'' M_S''' | S M_S \rangle \langle S M_S' | M_S'' M_S''' \rangle = \delta_{M_S M_S'}$$

$\rightarrow$ same with other one

$$\sum_{M_t M_t'} \langle M_t M_t' | T M_T \rangle \langle T M_T | M_t M_t' \rangle = 1 \quad \text{(and other one too)}$$

$$\int d\Omega_k \langle \hat{k} | k L M_L \rangle \langle k L' M_L' | \hat{k} \rangle = \frac{?}{\pi} \delta_{LL'} \delta_{M_L M_L'}$$

Take S-waves: $\langle 0\, M_S \,|\, 1\, M_J \rangle$ factors are always $\delta_{J,S}\, \delta_{M_J, M_S}$! (No sum over M_J!)

The expression reduces to

$$= |\Phi_d(\vec{Q})|^2 \Big[\frac{1}{2} \sum_{M_S M_S'} \sum_{M_J M_J'} |\psi^d_{^3S_1}(q)|^2 + \frac{1}{2} \sum_{M_S} \langle \vec{q} \,|\, q\,0\,0 \rangle \big(\delta\tilde{U}_{^3S_1 - ^3S_1}(q,q)$$

$$+ \delta\tilde{U}^{\dagger}_{^3S_1 - ^3S_1}(q,q) \big) \langle q\,0\,0 \,|\, \vec{q} \rangle\, |\psi^d_{^3S_1}(q)|^2$$

$$+ \frac{V}{4} \sum_{M_S} \sum_{L' M_L'} \frac{2}{\pi} \int_0^\infty \frac{dk\,k^2}{(2\pi)^3}\, \delta\tilde{U}_{^3S_1 - x}(k,q) \langle 1\, M_S \,|\, M_L' M_S \rangle$$

$$\langle q\, L' M_L' \,|\, \vec{q} \rangle \langle \vec{q} \,|\, q\, L' M_L' \rangle \langle M_L' M_S \,|\, 1\, M_S \rangle\, \delta\tilde{U}^{\dagger}_{x - ^3S_1}(q,k)\, |\psi^d_{^3S_1}(k)|^2 \Big]$$

Only way for $\langle M_L' M_S \,|\, 1\, M_S \rangle \neq 0 \leftrightarrow M_L' = 0$

$\sum_{M_S} \to (2S+1) = 3$

sum over $\sum_{M_S M_S'} \sum_{M_J M_J'}$?

$$= \frac{1}{2} |\Phi_d(\vec{Q})|^2 \Big[1 + \langle \vec{q} \,|\, q\,0\,0 \rangle \big(\delta\tilde{U}_{^3S_1 - ^3S_1}(q,q) + \delta\tilde{U}^{\dagger}_{^3S_1 - ^3S_1}(q,q) \big) \times$$

$$\langle q\,0\,0 \,|\, \vec{q} \rangle \big) |\psi^d_{^3S_1}(q)|^2 + \frac{3V}{4} \sum_{L'} \frac{2}{\pi} \int_0^\infty \frac{dk\,k^2}{(2\pi)^3}\, \delta\tilde{U}_{^3S_1 - L'}(k,q)$$

$$\langle q\, L'\, 0 \,|\, \vec{q} \rangle \langle \vec{q} \,|\, q\, L'\, 0 \rangle\, \delta\tilde{U}^{\dagger}_{L' - ^3S_1}(q,k)\, |\psi^d_{^3S_1}(k)|^2 \Big]$$

Only $\theta_{\vec{q}}$ dependence in CG's. Average over $\Omega_{\vec{q}}$

$$|\Phi_d(\vec{Q})|^2 \Bigg[\Big(1 + \frac{1}{2} \int \frac{d\Omega_{\vec{q}} \langle \hat{q} | q00 \rangle \langle q00 | \hat{q} \rangle}{4\pi} \big(\delta\tilde{U}_{^3S_1-^3S_1}(q,q) +$$

$$\delta\tilde{U}^{\dagger}_{^3S_1-^3S_1}(q,q) \big) \Big) |\psi^d_{^3S_1}(q)|^2 + \frac{3V}{4} \sum_{L'} \frac{2}{\pi} \int \frac{d\Omega_{\vec{q}} \langle q L'0 | \hat{q} \rangle \langle \hat{q} | q L'0 \rangle}{4\pi} \times$$

$$\int \frac{dk\, k^2}{(2\pi)^3} \delta\tilde{U}_{^3S_1-L'}(k,q) \delta\tilde{U}^{\dagger}_{L'-^3S_1}(q,k) |\psi^d_{^3S_1}(k)|^2 \Bigg]$$

$$= |\Phi_d(\vec{Q})|^2 \Bigg\{ \Big[1 + \frac{1}{2} \frac{2}{\pi} \frac{1}{4\pi} \big(\delta\tilde{U}_{^3S_1-^3S_1}(q,q) + \delta\tilde{U}^{\dagger}_{^3S_1-^3S_1}(q,q) \big) \Big] \times$$

$$|\psi^d_{^3S_1}(q)|^2 + \frac{3V}{4} \left(\frac{2}{\pi} \right)^2 \frac{1}{4\pi} \sum_{X = {^3S_1}, {^3D_1}} \int_0^\infty \frac{dk\, k^2}{(2\pi)^3} \delta\tilde{U}_{^3S_1-X}(k,q) \times$$

$$\delta\tilde{U}^{\dagger}_{X-^3S_1}(q,k) |\psi^d_{^3S_1}(k)|^2 \Bigg\}$$

* Extra factor of V?

* This is $\langle \hat{n}(q,\vec{Q}) \rangle$! $\int d^3Q \, |\Phi_d(\vec{Q})|^2 = 1$

$$\langle \hat{n}_d(q) \rangle_d = \Big[1 + \frac{1}{2} \frac{2}{\pi} \frac{1}{4\pi} \big(\delta\tilde{U}_{^3S_1-^3S_1}(q,q) + \delta\tilde{U}^{\dagger}_{^3S_1-^3S_1}(q,q) \big) \Big] |\psi^d_{^3S_1}(q)|^2$$

$$+ \frac{3V}{4} \left(\frac{2}{\pi} \right)^2 \frac{1}{4\pi} \sum_{X = {^3S_1}, {^3D_1}} \int_0^\infty \frac{dk\, k^2}{(2\pi)^3} \delta\tilde{U}_{^3S_1-X}(k,q) \delta\tilde{U}^{\dagger}_{X-^3S_1}(q,k) \times$$

$$|\psi^d_{^3S_1}(k)|^2$$

* Might be messing up $\sum_{m_s m_s'} \sum_{m_t m_t'} \mathbb{1}$ term.

* To test contributions, you could compare to $|\psi_d^\infty(q > \lambda)|^2$ and $|\psi_d^\infty(q < \lambda)|^2$