

AJT notes (11/12/21)

(1)

$$\langle \Psi_{\alpha}^{q-1} | \hat{U}_1^+ \hat{U}_2 a_{\vec{p}} \hat{U}_3^+ \hat{U}_4 | \Psi_0^q \rangle$$

$$= \langle \Psi | a_{\alpha}^{\dagger} \hat{U}_2 a_{\vec{p}} \hat{U}_3^+ | \Psi \rangle \quad (1)$$

$$a_{\alpha}^{\dagger} \hat{U}_2 a_{\vec{p}} \hat{U}_3^+ = a_{\alpha}^{\dagger} a_{\vec{p}} + \frac{1}{4} \sum_{1234} \delta \tilde{U}_{1234} a_{\alpha}^{\dagger} a_1^{\dagger} a_2^{\dagger} a_4 a_3 a_{\vec{p}} \quad \text{--- 6}$$

$$+ \frac{1}{36} \sum_{5678910} \delta \tilde{U}_{5678910} a_{\alpha}^{\dagger} a_5^{\dagger} a_6^{\dagger} a_7^{\dagger} a_9 a_8 a_{\vec{p}} \quad \text{--- 8}$$

$$+ \frac{1}{4} \sum_{abcd} \delta \tilde{U}_{abcd} a_{\alpha}^{\dagger} a_{\vec{p}} a_a^{\dagger} a_b^{\dagger} a_c^{\dagger} a_d a_c \quad \text{--- 6}$$

$$+ \frac{1}{4} \frac{1}{4} \sum_{1234} \sum_{abcd} \delta \tilde{U}_{1234} \delta \tilde{U}_{abcd} a_{\alpha}^{\dagger} a_1^{\dagger} a_2^{\dagger} a_4 a_3 a_{\vec{p}} a_a^{\dagger} a_b^{\dagger} a_d a_c \quad \text{--- 10}$$

$$+ \frac{1}{36} \frac{1}{4} \sum_{5678910} \sum_{abcd} \delta \tilde{U}_{5678910} \delta \tilde{U}_{abcd} a_{\alpha}^{\dagger} a_5^{\dagger} a_6^{\dagger} a_7^{\dagger} a_{10} a_9 a_8 a_{\vec{p}} a_a^{\dagger} a_b^{\dagger} a_d a_c \quad \text{--- 12}$$

$$+ \frac{1}{36} \sum_{mnpqr} \delta \tilde{U}_{mnpqr} a_{\alpha}^{\dagger} a_{\vec{p}} a_m^{\dagger} a_n^{\dagger} a_o^{\dagger} a_r a_q a_p \quad \text{--- 8}$$

$$+ \frac{1}{4} \frac{1}{36} \sum_{1234} \delta \tilde{U}_{1234} \delta \tilde{U}_{mnpqr} a_{\alpha}^{\dagger} a_1^{\dagger} a_2^{\dagger} a_4 a_3 a_{\vec{p}} a_m^{\dagger} a_n^{\dagger} a_o^{\dagger} a_r a_q a_p \quad \text{--- 12}$$

$$+ \frac{1}{26} \frac{1}{36} \sum_{5678910} \sum_{mnpqr} \delta \tilde{U}_{5678910} \delta \tilde{U}_{mnpqr} a_{\alpha}^{\dagger} a_5^{\dagger} a_6^{\dagger} a_7^{\dagger} a_{10} a_9 a_8 a_{\vec{p}} a_m^{\dagger} a_n^{\dagger} a_o^{\dagger} a_r a_q a_p \quad \text{--- 14}$$

\* Split this up into terms organizing by total number of  $a$ 's

(2)

$$\textcircled{A} \quad a_\alpha^\dagger \hat{I} a_\beta^\dagger \hat{I} \quad [2]$$

(2)

$$a_\alpha^\dagger a_\beta^\dagger$$

(3)

\* Already normal-ordered.

$$\textcircled{B} \quad a_\alpha^\dagger \hat{\delta} U_1 a_\beta^\dagger \hat{I} \quad [6]$$

$$a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4^\dagger a_3^\dagger a_5^\dagger$$

(4)

\* Already normal-ordered.

$$\textcircled{C} \quad a_\alpha^\dagger \hat{I} a_\beta^\dagger \hat{\delta} U_2^\dagger \quad [6]$$

$$a_\alpha^\dagger a_\beta^\dagger a_\alpha^\dagger a_b^\dagger a_d a_c = :a_\alpha^\dagger a_\beta^\dagger a_\alpha^\dagger a_b^\dagger a_d a_c:$$

$$+ :a_\alpha^\dagger \overline{a_\beta^\dagger a_\alpha^\dagger a_b^\dagger a_d a_c}: - :a_\alpha^\dagger \overline{a_\beta^\dagger a_\alpha^\dagger a_b^\dagger a_d a_c}:$$

$$= a_\alpha^\dagger a_\alpha^\dagger a_b^\dagger a_\beta^\dagger a_d a_c + \delta_{\beta a} a_\alpha^\dagger a_\alpha^\dagger a_b^\dagger a_d a_c - \delta_{\beta b} a_\alpha^\dagger a_\alpha^\dagger a_d a_c$$

(5)

$$\textcircled{D} \quad a_\alpha^\dagger \hat{\delta} U_3 a_\beta^\dagger \hat{I} \quad [8]$$

*4-body*

~~$$a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10}^\dagger a_8 a_9 a_\beta^\dagger$$~~

\* Already normal ordered

(6)

(3)

$$\textcircled{E} \quad \hat{a}_\alpha^+ \hat{I} \hat{a}_p \hat{s} \hat{v}_3^+ \quad [8]$$

4-body

$$\begin{aligned} \hat{a}_\alpha^+ \hat{a}_p \hat{a}_m^+ \hat{a}_n^+ \hat{a}_o^+ \hat{a}_r \hat{a}_q \hat{a}_p &= : \hat{a}_\alpha^+ \hat{a}_p \hat{a}_m^+ \hat{a}_n^+ \cancel{\hat{a}_o^+} \hat{a}_r \hat{a}_q \hat{a}_p : \\ + : \hat{a}_\alpha^+ \hat{a}_p \hat{a}_m^+ \hat{a}_n^+ \hat{a}_o^+ \hat{a}_r \hat{a}_q \hat{a}_p : - : \hat{a}_\alpha^+ \hat{a}_p \hat{a}_m^+ \hat{a}_n^+ \cancel{\hat{a}_o^+} \hat{a}_r \hat{a}_q \hat{a}_p : \\ + : \hat{a}_\alpha^+ \hat{a}_p \hat{a}_m^+ \hat{a}_n^+ \cancel{\hat{a}_o^+} \hat{a}_r \hat{a}_q \hat{a}_p : \end{aligned}$$

$$= \delta_{\vec{p}m} \hat{a}_\alpha^+ \hat{a}_n^+ \hat{a}_o^+ \hat{a}_r \hat{a}_q \hat{a}_p - \delta_{\vec{p}n} \hat{a}_\alpha^+ \hat{a}_m^+ \hat{a}_o^+ \hat{a}_r \hat{a}_q \hat{a}_p$$

$$+ \delta_{\vec{p}o} \hat{a}_\alpha^+ \hat{a}_m^+ \hat{a}_n^+ \hat{a}_r \hat{a}_q \hat{a}_p \quad (7)$$

$$\textcircled{F} \quad \hat{a}_\alpha^+ \hat{s} \hat{v}_2 \hat{a}_p \hat{s} \hat{v}_2^+ \quad [10]$$

5-body

$$\hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \hat{a}_p \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c = : \hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \cancel{\hat{a}_p} \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c :$$

$$+ \sum_{\text{singlets}} (\dots) \quad 4\text{-body} \quad + : \hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \hat{a}_p \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c :$$

$$- : \hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \hat{a}_p \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c : + : \hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \hat{a}_p \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c :$$

$$- : \hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \hat{a}_p \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c : + : \hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \hat{a}_p \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c :$$

$$- : \hat{a}_\alpha^+ \hat{a}_1^+ \hat{a}_2^+ \hat{a}_y \hat{a}_3 \hat{a}_p \hat{a}_a^+ \hat{a}_b^+ \hat{a}_d \hat{a}_c :$$

(4)

$$= (\delta_{4b} \delta_{3a} - \delta_{4a} \delta_{3b}) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_p^\dagger a_d a_c$$

$$+ (\delta_{3b} \delta_{1a} - \delta_{3a} \delta_{1b}) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_y a_d a_c$$

$$+ (\delta_{ub} \delta_{1a} - \delta_{ua} \delta_{1b}) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_3 a_d a_c \quad (8)$$

$$\textcircled{G} \quad a_\alpha^\dagger \delta \hat{U}_3 a_p^\dagger \delta \hat{U}_2^\dagger \quad [12]$$

$$a_\alpha^\dagger a_s^\dagger a_b^\dagger a_7^\dagger a_{10} a_9 a_8 a_p^\dagger a_a^\dagger a_b^\dagger a_d a_c \quad (4\text{-body at least})$$

$$= :a_\alpha^\dagger a_s^\dagger a_b^\dagger a_7^\dagger a_{10} a_9 a_8 a_p^\dagger a_a^\dagger a_b^\dagger a_d a_c: + \sum_{\text{singles}} (...) \quad \begin{matrix} \text{6-body} \\ \text{5-body} \end{matrix}$$

~~$a_\alpha^\dagger a_s^\dagger a_b^\dagger a_7^\dagger a_{10} a_9 a_8 a_p^\dagger a_a^\dagger a_b^\dagger a_d a_c$~~

$$+ \sum_{\text{doubles}} (...) \quad (9) \quad \begin{matrix} \text{4-body} \\ \text{3-body} \end{matrix}$$

$$\textcircled{H} \quad a_\alpha^\dagger \delta \hat{U}_2 a_p^\dagger \delta \hat{U}_3^\dagger \quad [12]$$

$$a_\alpha^\dagger a_1^\dagger a_2^\dagger a_u a_3 a_p^\dagger a_m a_n^\dagger a_o^\dagger a_r a_q a_p$$

$$= :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_u a_3 a_p^\dagger a_m a_n^\dagger a_o^\dagger a_r a_q a_p: + \sum_{\text{singles}} (...) + \sum_{\text{doubles}} (...) \quad \begin{matrix} \text{6-body} \\ \text{5-body} \\ \text{4-body} \end{matrix}$$

~~$a_\alpha^\dagger a_1^\dagger a_2^\dagger a_u a_3 a_p^\dagger a_m a_n^\dagger a_o^\dagger a_r a_q a_p$~~

$$+ :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_u a_3 a_p^\dagger a_m a_n^\dagger a_o^\dagger a_r a_q a_p:$$

$$- :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_u a_3 a_p^\dagger a_m a_n^\dagger a_o^\dagger a_r a_q a_p:$$

(5)

$$+ : \alpha^+ \beta^+ \gamma^+ \delta^+ \alpha_r \beta_j \beta_p \alpha_m \beta_n^+ \beta_o^+ \alpha_r \alpha_q \alpha_p :$$

$$- : \alpha^+ \beta^+ \gamma^+ \delta^+ \alpha_r \beta_j \beta_p \alpha_m^+ \beta_n^+ \beta_o^+ \alpha_r \alpha_q \alpha_p :$$

$$+ : \alpha^+ \beta^+ \gamma^+ \delta^+ \alpha_r \beta_j \beta_p^+ \alpha_m^+ \beta_n^+ \beta_o^+ \alpha_r \alpha_q \alpha_p :$$

$$- : \alpha^+ \beta^+ \gamma^+ \delta^+ \alpha_r \beta_j \beta_p^+ \alpha_m^+ \beta_n^+ \beta_o^+ \alpha_r \alpha_q \alpha_p :$$

$$= \left[ \delta_{uo} (\delta_{3n} \delta_{\tilde{p}m} - \delta_{3m} \delta_{\tilde{p}n}) + \delta_{qn} (\delta_{3m} \delta_{\tilde{p}o} - \delta_{3o} \delta_{\tilde{p}m}) \right. \\ \left. + \delta_{un} (\delta_{3n} \delta_{\tilde{p}o} - \delta_{3o} \delta_{\tilde{p}n}) \right] \alpha^+ \alpha^+ \alpha^+ \alpha_r \alpha_q \alpha_p \quad (10)$$

$$\textcircled{I} \quad \alpha^+ \delta \hat{U}_3 q^+ \delta \hat{U}_3^+ \quad [14]$$

$$\alpha^+ \alpha_s^+ \alpha_b^+ \alpha_7^+ \alpha_{10} \alpha_q \alpha_8 \alpha_p^+ \alpha_m^+ \alpha_n^+ \alpha_o^+ \alpha_r \alpha_q \alpha_p \quad (4\text{-body at least})$$

(11)

\* Recombining terms

- Maybe organize diagrams?

(6)

$$a_\alpha^\dagger \tilde{U}_\alpha c_{\vec{p}} \tilde{U}_\alpha^\dagger$$

3-body level

$$\approx a_\alpha^\dagger c_{\vec{p}} + \frac{1}{4} \sum_{abcd} \delta \tilde{U}_{abcd}^\dagger \left( (\delta_{\vec{p}a} a_\alpha^\dagger a_b^\dagger a_d a_c - \delta_{\vec{p}b} a_\alpha^\dagger a_a^\dagger a_d a_c) \right.$$

$$\left. + a_\alpha^\dagger a_a^\dagger a_b^\dagger a_{\vec{p}} a_d a_c \right] + \frac{1}{4} \sum_{1234} \delta \tilde{U}_{1234} a_\alpha^\dagger c_1^\dagger a_2^\dagger a_4 c_3 c_{\vec{p}}$$

$$+ \frac{1}{4} \frac{1}{4} \sum_{1234} \sum_{abcd} \delta \tilde{U}_{1234} \delta \tilde{U}_{abcd}^\dagger \left( (\delta_{4b} \delta_{3a} - \delta_{4a} \delta_{3b}) a_\alpha^\dagger a_1^\dagger c_2^\dagger c_{\vec{p}} a_d a_c \right.$$

$$\left. + (\delta_{3b} \delta_{\vec{p}a} - \delta_{3a} \delta_{\vec{p}b}) a_\alpha^\dagger a_1^\dagger c_2^\dagger a_4 a_d a_c \right)$$

$$+ (\delta_{ab} \delta_{\vec{p}c} - \delta_{ac} \delta_{\vec{p}b}) a_\alpha^\dagger c_1^\dagger a_2^\dagger c_3 a_d a_c \Big]$$

$$+ \frac{1}{36} \sum_{mnpqr} \delta \tilde{U}_{mnpqr}^\dagger \left( \delta_{\vec{p}m} a_\alpha^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p \right.$$

$$\left. - \delta_{\vec{p}n} a_\alpha^\dagger a_m^\dagger a_o^\dagger a_r a_q a_p + \delta_{\vec{p}o} a_\alpha^\dagger a_m^\dagger a_n^\dagger a_r a_q a_p \right)$$

$$+ \frac{1}{4} \frac{1}{36} \sum_{1234} \sum_{mnpqr} \delta \tilde{U}_{1234} \delta \tilde{U}_{mnpqr}^\dagger \left[ \delta_{40} (\delta_{3n} \delta_{\vec{p}m} - \delta_{3m} \delta_{\vec{p}n}) \right.$$

$$\left. + \delta_{4n} (\delta_{3n} \delta_{\vec{p}0} - \delta_{30} \delta_{\vec{p}n}) + \delta_{4m} (\delta_{3n} \delta_{\vec{p}0} - \delta_{30} \delta_{\vec{p}n}) \right] a_\alpha^\dagger a_1^\dagger a_2^\dagger a_r a_q a_p$$

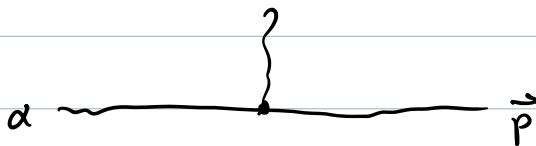
(12)

(11/30/21)

(7)

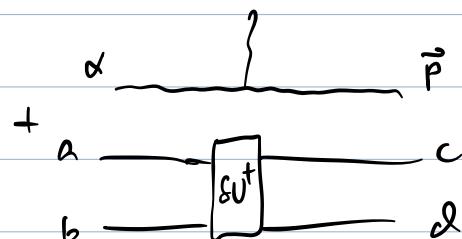
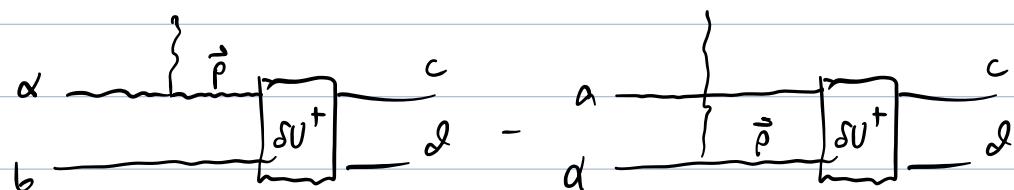
\* Consider diagrams of contributions in evolved operator

I)  $a_\alpha^+ \hat{I} a_{\bar{p}} \hat{I} \sim a_\alpha^+ a_{\bar{p}}$

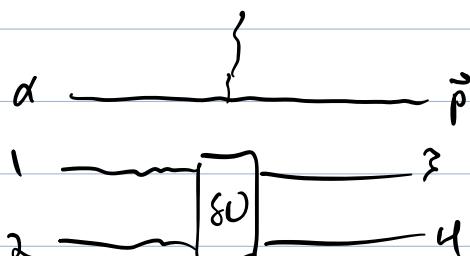


II)  $a_\alpha^+ \hat{I} a_{\bar{p}} \delta \tilde{U}_{(1)}^+$

$$\sim \delta \tilde{U}_{abcd}^+ \left( (\delta_{\bar{p}a} a_\alpha^+ a_b^+ a_\alpha a_c - \delta_{\bar{p}b} a_\alpha^+ a_b^+ a_\alpha a_c) + a_\alpha^+ a_a^+ a_b^+ a_{\bar{p}} a_\alpha a_c \right)$$



III)  $a_\alpha^+ \delta \tilde{U}_{(1)} a_{\bar{p}} \hat{I} \sim \delta \tilde{U}_{1234} a_\alpha^+ a_1^+ a_2^+ a_4 a_3 a_{\bar{p}}$



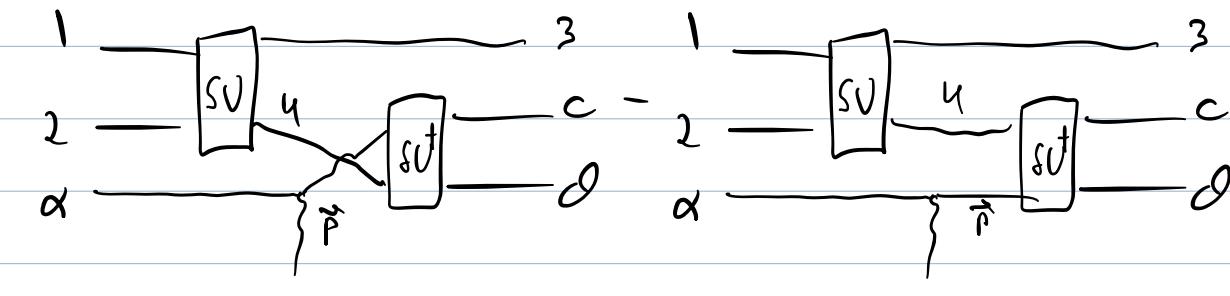
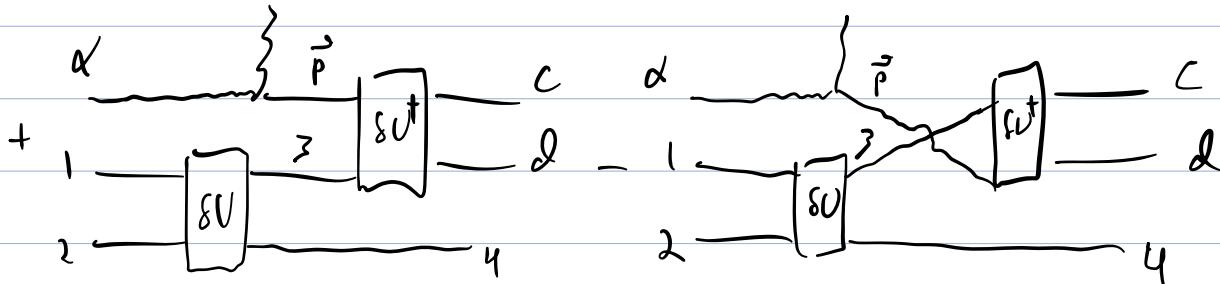
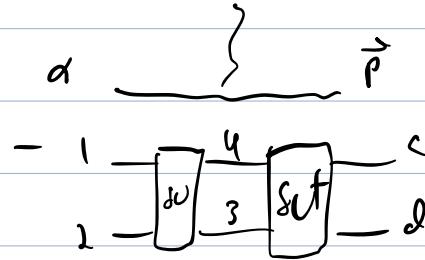
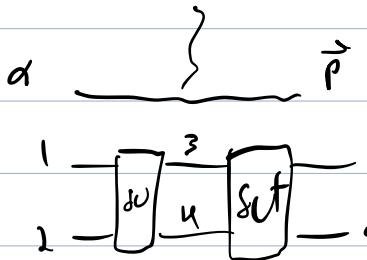
$$\text{IV) } a_\alpha^+ \delta \hat{U}_{(2)} a_{\vec{p}}^- \delta \hat{U}_{(2)}^+$$

(8)

$$\sim \delta \tilde{U}_{1234} \delta \tilde{U}_{abcd}^+ \left[ (\delta_{4b} \delta_{3a} - \delta_{4a} \delta_{3b}) a_\alpha^+ a_1^+ c_2^+ a_{\vec{p}}^- c_0 c_c \right.$$

$$+ (\delta_{3b} \delta_{\vec{p}a} - \delta_{3a} \delta_{\vec{p}b}) a_\alpha^+ a_1^+ c_2^+ a_{\vec{q}}^- c_d c_c$$

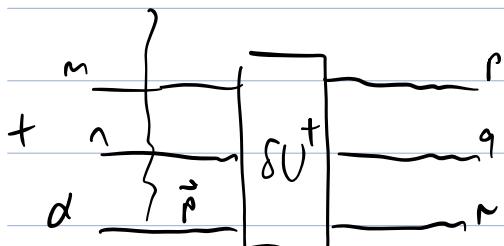
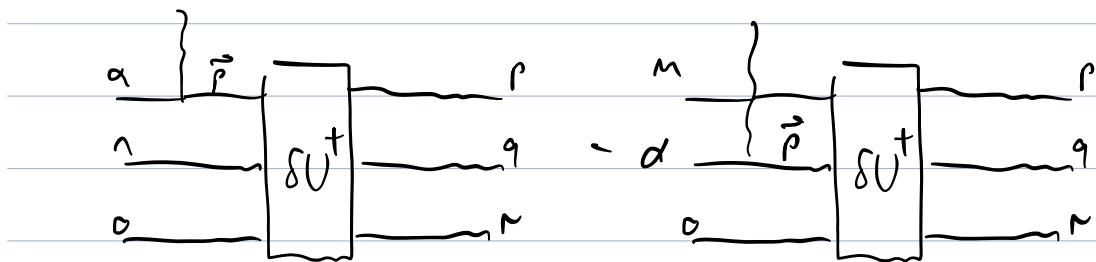
$$\left. + (\delta_{ab} \delta_{\vec{p}c} - \delta_{ac} \delta_{\vec{p}b}) a_\alpha^+ c_1^+ c_2^+ c_3^- c_d c_c \right]$$



$$\text{V) } a_\alpha^+ \hat{I} a_{\vec{p}}^- \delta \hat{U}_{(3)}^+ \sim \delta \tilde{U}_{\text{unpair}}^+ (\delta_{\vec{p}m} a_\alpha^+ a_n^+ a_o^+ a_r a_q a_p$$

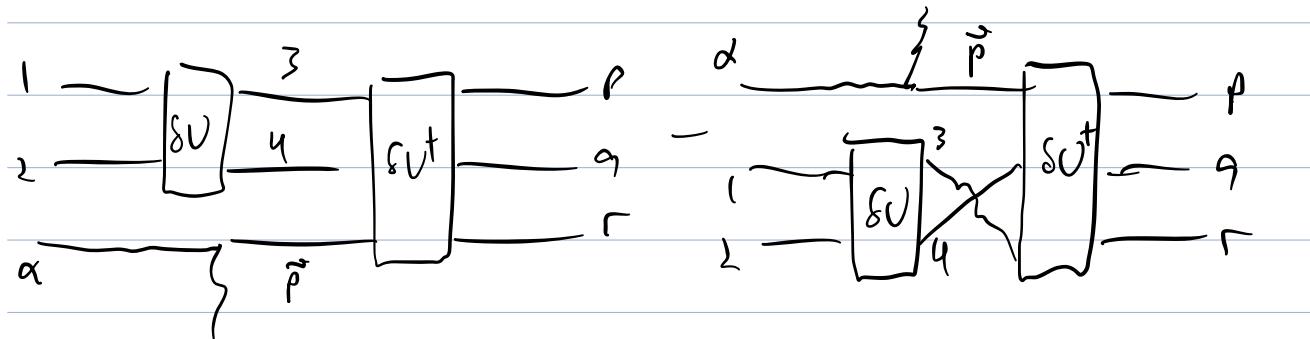
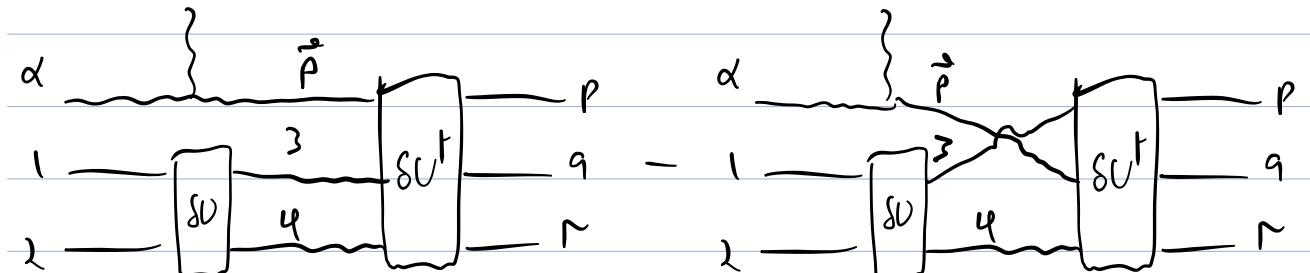
$$- \delta_{\vec{p}n} a_\alpha^+ a_m^+ a_o^+ a_r a_q a_p + \delta_{\vec{p}o} a_\alpha^+ a_m^+ a_n^+ a_r a_q a_p)$$

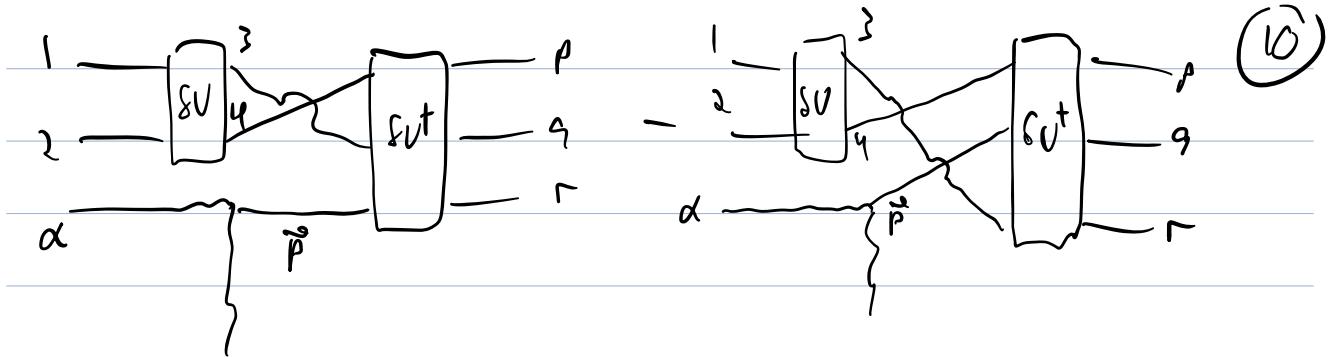
(9)



$$\text{VI) } a_\alpha^\dagger \delta U_{(1)} a_p^\dagger \delta U_{(3)}^\dagger \sim \delta U_{1234} \delta U_{mnpq}^{24} \left[ \delta_{40} (\delta_{3n}\delta_{\tilde{p}m} - \delta_{3m}\delta_{\tilde{p}n}) \right.$$

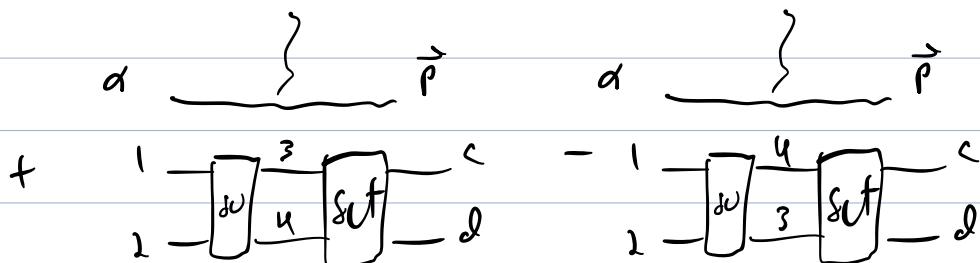
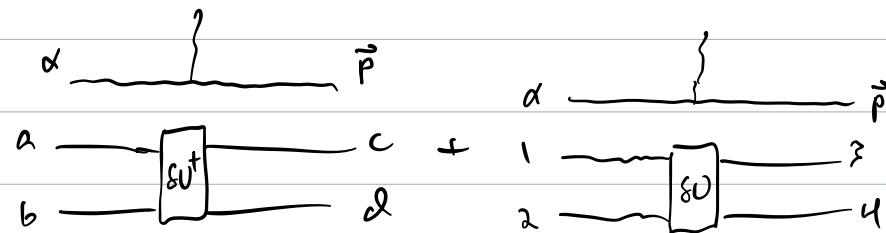
$$\left. + \delta_{4n} (\delta_{3n}\delta_{\tilde{p}0} - \delta_{30}\delta_{\tilde{p}n}) + \delta_{4m} (\delta_{3n}\delta_{\tilde{p}0} - \delta_{30}\delta_{\tilde{p}n}) \right] a_\alpha^\dagger a_1^\dagger a_2^\dagger a_r a_q a_p$$





(12/18/21) Update on disconnected diagrams

\* Consider the disconnected diagrams :



$$\text{Recall } \hat{U}_\lambda^+ \hat{U}_\lambda = \hat{I}$$

$$[\hat{I} + \frac{1}{4} \sum \delta U_\lambda^+] [\hat{I} + \frac{1}{4} \sum \delta U_\lambda^-]$$

$$= \hat{I} + \frac{1}{4} \sum_{K, K'} \left[ \delta U_\lambda^+(k, k') + \delta U_\lambda(k, k') + \frac{1}{2} \sum_{k''} \delta U_\lambda^+(k, k'') \delta U_\lambda(k'', k') \right]$$

$$\times \alpha_{\frac{K}{2}+k}^+ \alpha_{\frac{K}{2}-k}^+ \alpha_{\frac{K}{2}-k'}^- \alpha_{\frac{K}{2}+k'}^-$$

(13)

$$\Rightarrow \delta U_{\lambda}^+(\vec{k}, \vec{k}') + S_{\lambda}(\vec{k}, \vec{k}') + \frac{1}{2} \sum_{\vec{k}''} \delta U_{\lambda}^+(\vec{k}, \vec{k}'') S_{\lambda}(\vec{k}'', \vec{k}') = 0$$

(11)  
(14)

- The diagrams correspond to

$$= \frac{1}{4} \sum_{1234} (\delta \tilde{U}_{1234}^+ a_{\alpha}^+ a_1^+ a_2^+ a_{\vec{p}}^- a_u a_3 + \delta \tilde{U}_{1234}^- a_{\alpha}^+ a_1^+ a_2^+ a_u a_3^- a_{\vec{p}})$$

$$+ \frac{1}{4} \frac{1}{2} \sum_{1234} \sum_{ab} \delta \tilde{U}_{12ab} \delta \tilde{U}_{ab34}^+ a_{\alpha}^+ a_1^+ a_2^+ a_{\vec{p}}^- a_u a_3$$

$$\{a_{\vec{p}}, a_i\} = 0 \Rightarrow a_{\alpha}^+ a_1^+ a_2^+ a_u a_3^- a_{\vec{p}} = a_{\alpha}^+ a_1^+ a_2^+ a_{\vec{p}}^- a_u a_3$$

$$= \frac{1}{4} \sum_{1234} \left[ \delta \tilde{U}_{1234}^+ + \delta \tilde{U}_{1234}^- + \cancel{\frac{1}{2} \sum_{ab} \delta \tilde{U}_{12ab} \delta \tilde{U}_{ab34}^+} \right] a_{\alpha}^+ a_1^+ a_2^+ a_{\vec{p}}^- a_u a_3$$

$= 0$

$\Rightarrow$  These disconnected diagrams cancel

Update formula for  $a_{\alpha}^+ \tilde{U}_{\lambda} a_{\vec{p}}^- \tilde{U}_{\lambda}^+$  and diagrams

(12)

$$a_\alpha^+ \hat{U}_\alpha c_\beta \hat{U}_\alpha^+$$

3-body level

$$\approx a_\alpha^+ h_\beta + \frac{1}{4} \sum_{abcd} \delta \tilde{U}_{abcd}^+ ( \delta_{\tilde{p}a} a_\alpha^+ a_b^+ a_d a_c - \delta_{\tilde{p}b} a_\alpha^+ a_a^+ a_d a_c )$$

$$+ \frac{1}{4} \frac{1}{4} \sum_{1234} \sum_{abcd} \delta \tilde{U}_{1234} \delta \tilde{U}_{abcd}^+ \left[ (\delta_{3b} \delta_{\tilde{p}a} - \delta_{3a} \delta_{\tilde{p}b}) a_\alpha^+ a_1^+ a_2^+ a_4 a_d a_c \right. \\ \left. + (\delta_{4b} \delta_{\tilde{p}a} - \delta_{4a} \delta_{\tilde{p}b}) a_\alpha^+ a_1^+ a_2^+ a_3 a_d a_c \right]$$

$$+ \frac{1}{36} \sum_{mnpqr} \delta \tilde{U}_{mnpqr}^+ ( \delta_{\tilde{p}m} a_\alpha^+ a_n^+ a_o^+ a_r a_q a_p \\ - \delta_{\tilde{p}n} a_\alpha^+ a_m^+ a_o^+ a_r a_q a_p + \delta_{\tilde{p}o} a_\alpha^+ a_m^+ a_n^+ a_r a_q a_p )$$

$$+ \frac{1}{4} \frac{1}{36} \sum_{1234} \sum_{mnpqr} \delta \tilde{U}_{1234} \delta \tilde{U}_{mnpqr}^+ \left[ \delta_{40} (\delta_{3n} \delta_{\tilde{p}m} - \delta_{3m} \delta_{\tilde{p}n}) \right. \\ \left. + \delta_{4n} (\delta_{3n} \delta_{\tilde{p}0} - \delta_{30} \delta_{\tilde{p}n}) + \delta_{4m} (\delta_{3n} \delta_{\tilde{p}0} - \delta_{30} \delta_{\tilde{p}n}) \right] a_\alpha^+ a_1^+ a_2^+ a_r a_q a_p$$

(15)

(13)

$$a_\alpha^+ \hat{U}_\alpha \hat{G}_\beta \hat{U}_\beta^+$$

