# Short-range correlation physics from operator evolution

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*ajt, S.K. Bogner, and R.J. Furnstahl, arXiv:2006.11186*

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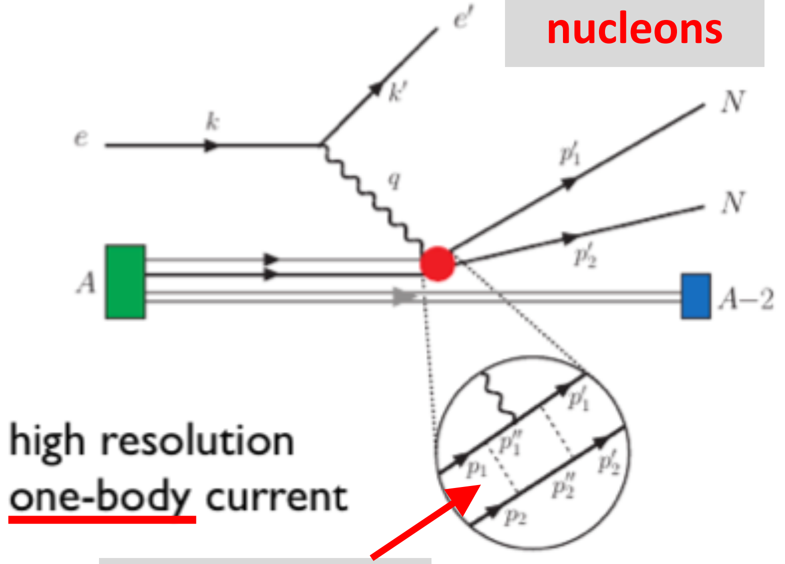
# Motivation

- Recent experiments have been able to isolate processes where short-range correlation (SRC) physics is dominant and well described by SRC phenomenology
- High RG resolution description of SRC physics
  - SRC pairs are components in the nuclear wave function with relative momenta above the Fermi momentum

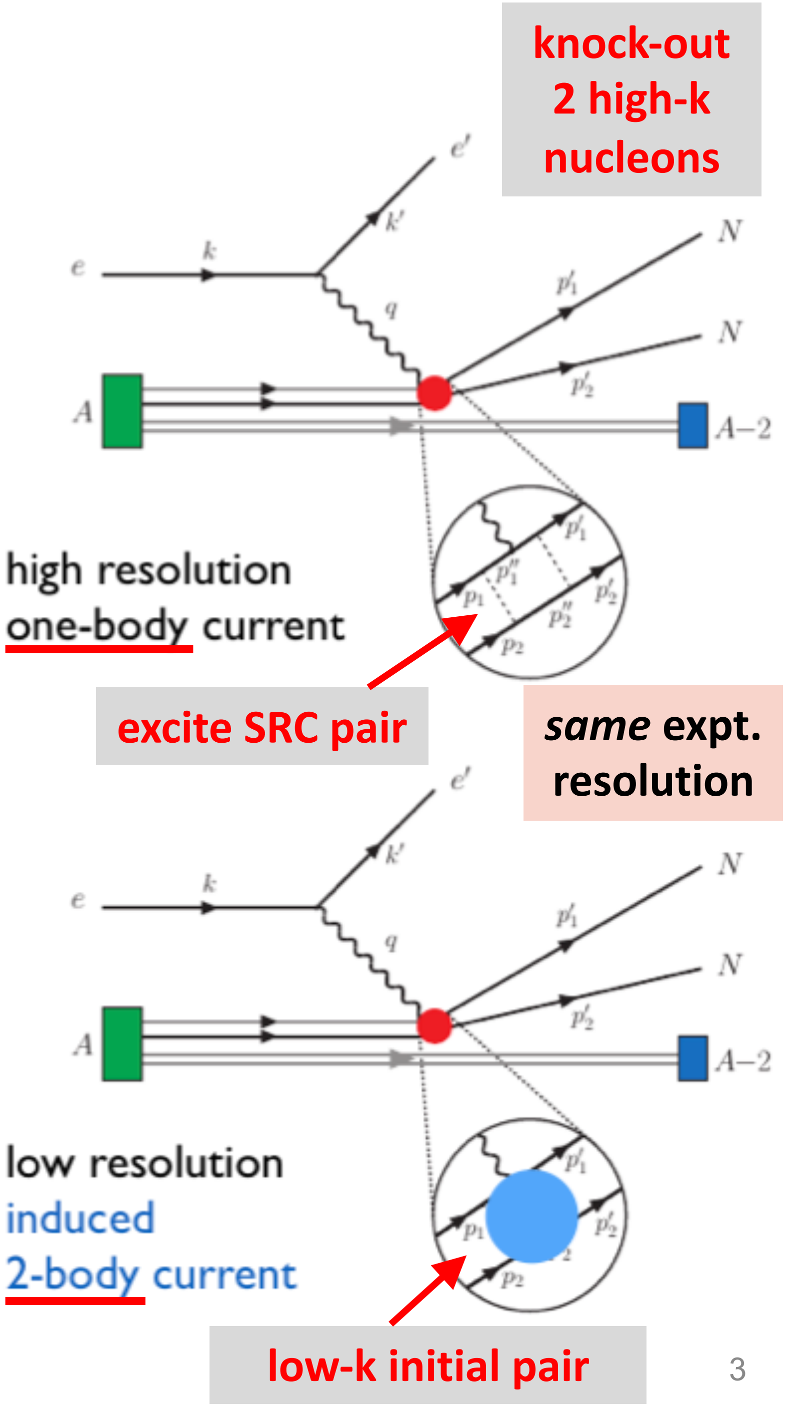
knock-out
2 high-k
nucleons




excite SRC pair

# Motivation

- Recent experiments have been able to isolate processes where short-range correlation (SRC) physics is dominant and well described by SRC phenomenology
- High RG resolution description of SRC physics
  - SRC pairs are components in the nuclear wave function with relative momenta above the Fermi momentum
- Alternative viewpoint
  - Using renormalization group (RG) methods we can tune the scale to low RG resolution
  - The SRC *physics* is shifted into the reaction operators from the nuclear wave function (which becomes soft)



# Motivation

- Experiments often rely on soft nuclear structure components (e.g., nuclear shell model) but mismatch scales by using high RG resolution reaction operators
- One can use low RG resolution operators to consistently match scales in structure and reaction components

# SRG formalism

- We use the similarity renormalization group (SRG) to evolve operators to low RG resolution

$$O(s) = U(s)O(0)U^{\dagger}(s)$$

where $s = 0 \to \infty$ and $U(s)$ is unitary

- In practice, solve differential flow equation

$$\frac{dO(s)}{ds} = [\eta(s), O(s)]$$

with SRG generator $\eta(s) \equiv \frac{dU(s)}{ds}U^{\dagger}(s) = [G, H(s)]$ and Hamiltonian $H(s)$

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- $G$ gives the scheme and $s$ gives the scale

# AV18 at low RG resolution

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling scheme

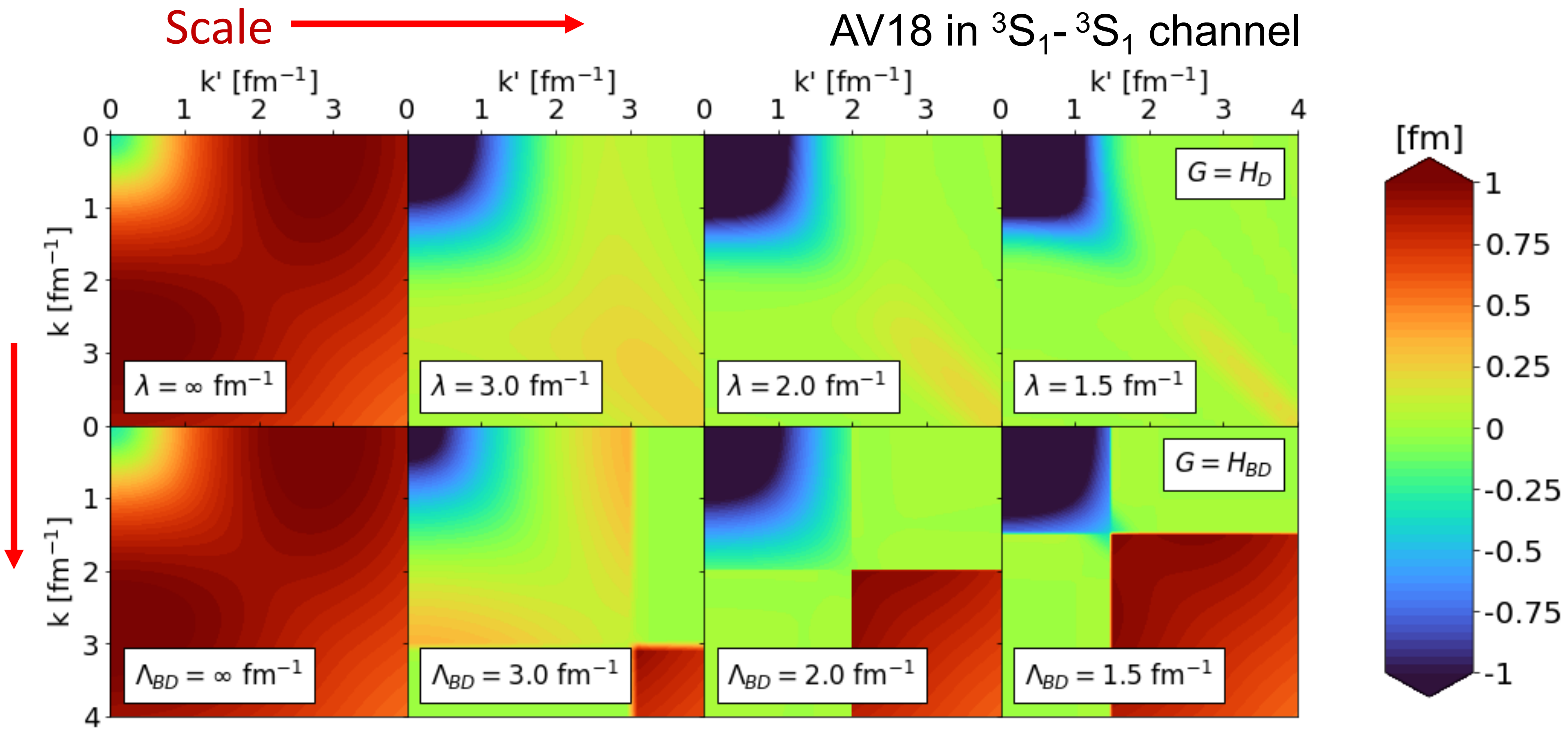
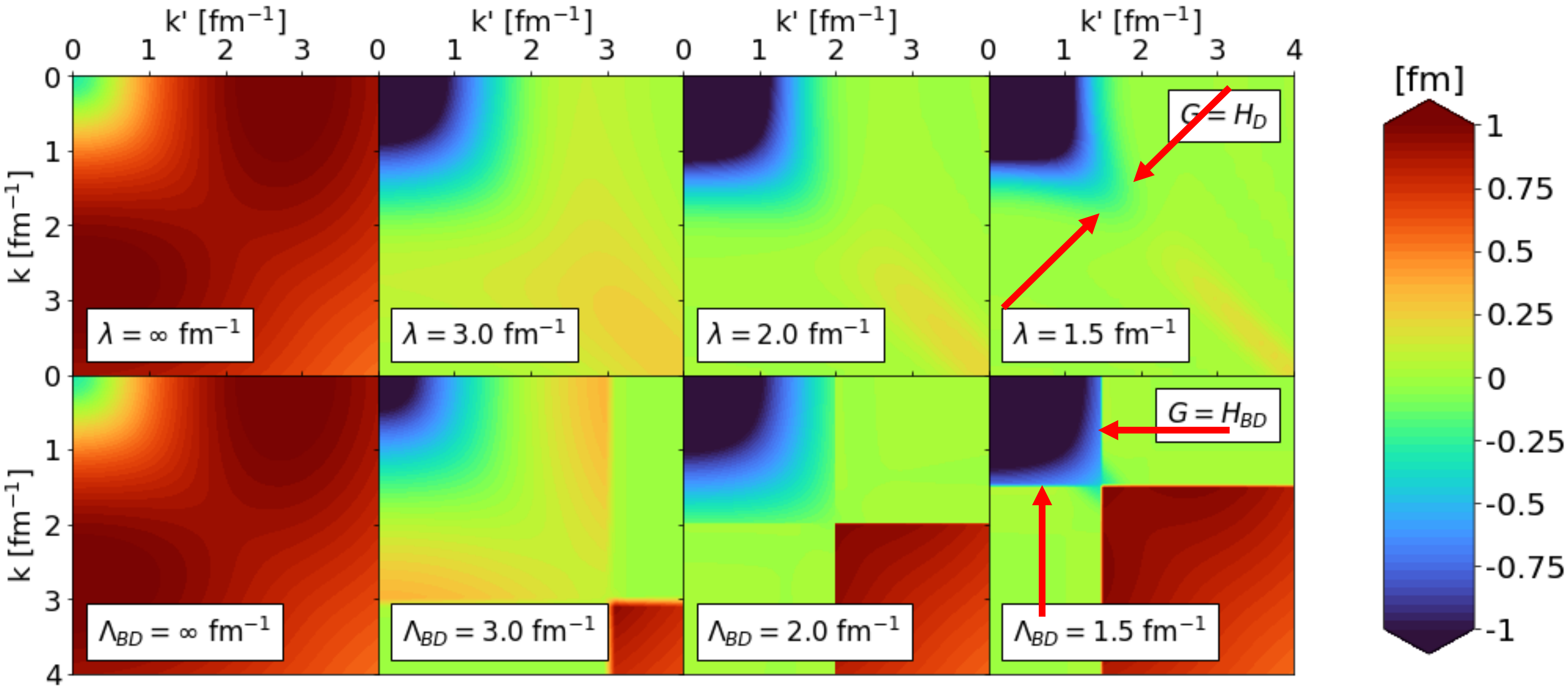



Fig. 1: SRG evolution of $V(k, k')$ for several values of $\lambda$ and $\Lambda$.

# AV18 at low RG resolution

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling scheme
- Parameters $\lambda = s^{-1/4}$ and $\Lambda_{BD}$ describe the decoupling scale of the evolved Hamiltonian

AV18 in $^3S_1$-$^3S_1$ channel

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# Deuteron wave function at low RG resolution

- AV18 wave function has significant SRC
- What happens to the wave function at low RG resolution?

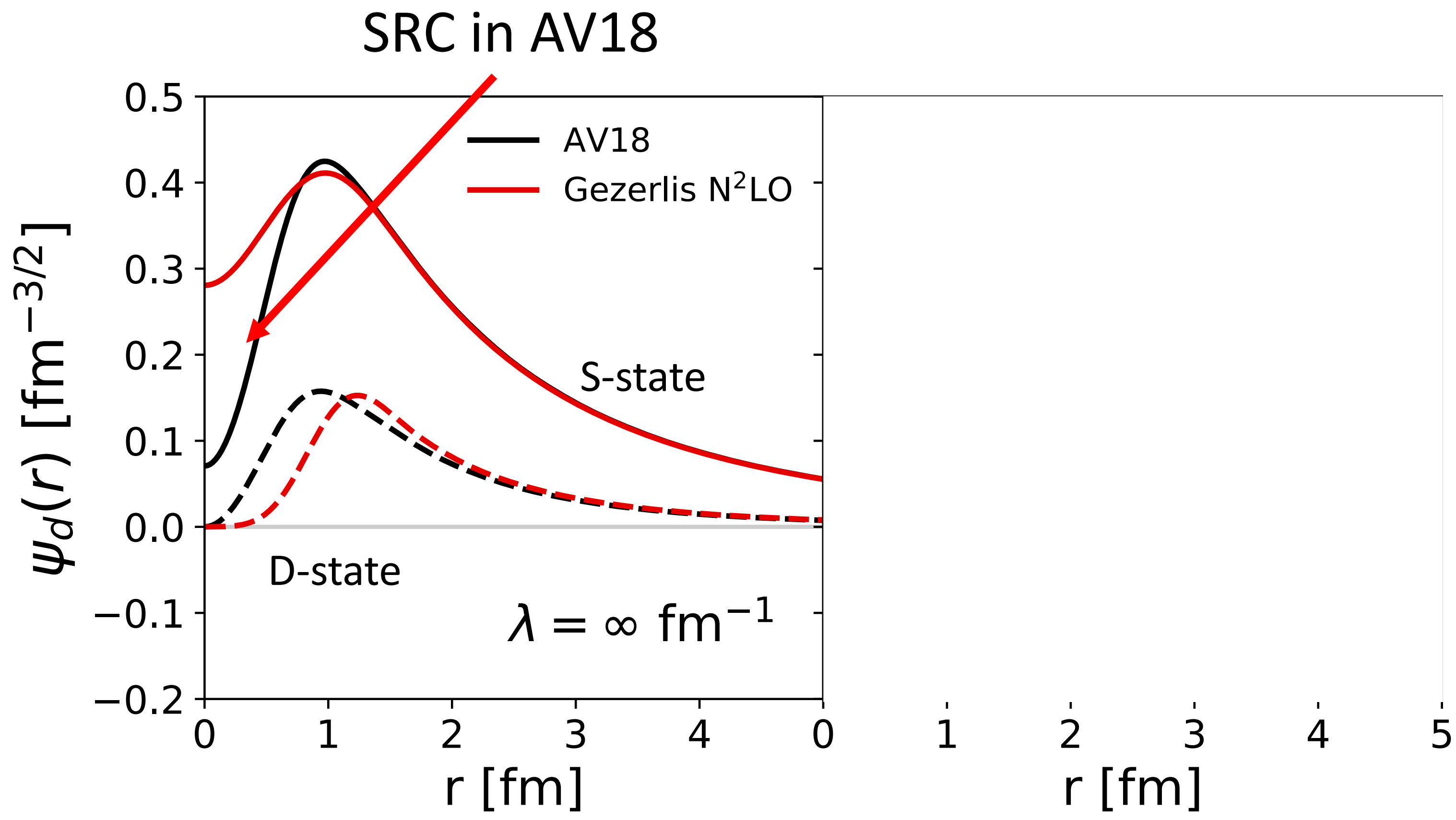
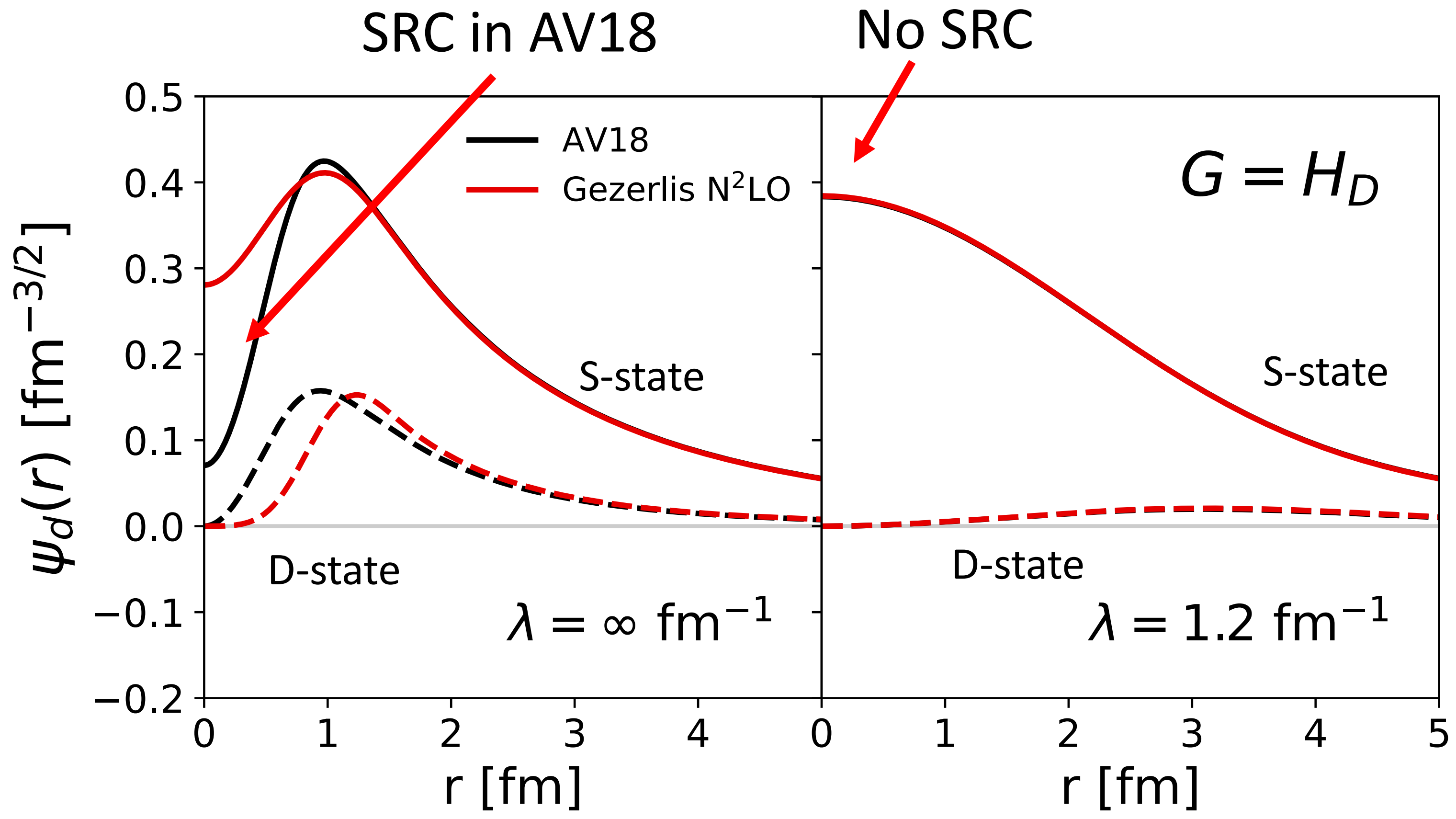



Fig. 2: SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis N2LO[1].

[1]A. Gezerlis, et al., Phys. Rev. C **90**, 054323 (2014)

# Deuteron wave function at low RG resolution

- SRC physics in AV18 (scheme dependent) is gone from wave function at low RG resolution
- Deuteron wave functions become soft and D-state probability goes down
- Observables such as asymptotic D-S ratio are the same

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# Connection to experiments

- In analyzing scattering observables, there is scale and scheme dependence in factorization of structure and reaction
- General problem for any matrix element $\langle \psi_f | O | \psi_i \rangle$

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- In analyzing scattering observables, there is scale and scheme dependence in factorization of structure and reaction
- General problem for any matrix element $\langle \psi_f | O | \psi_i \rangle$
- Use low RG resolution wave function to calculate high-energy reactions by consistently evolving the operator

$$\langle \psi_f(0) | O(0) | \psi_i(0) \rangle = \langle \psi_f(s) | O(s) | \psi_i(s) \rangle$$

- Mismatch of scales leads to incorrect observable (e.g., theory knock-out cross section compared to experiment)

# Deuteron momentum distribution

- Use simple operator $a_q^\dagger a_q$ which projects onto relative momentum $q$
  $a_q^\dagger a_q \sim \delta(k-q)\delta(k'-q)$

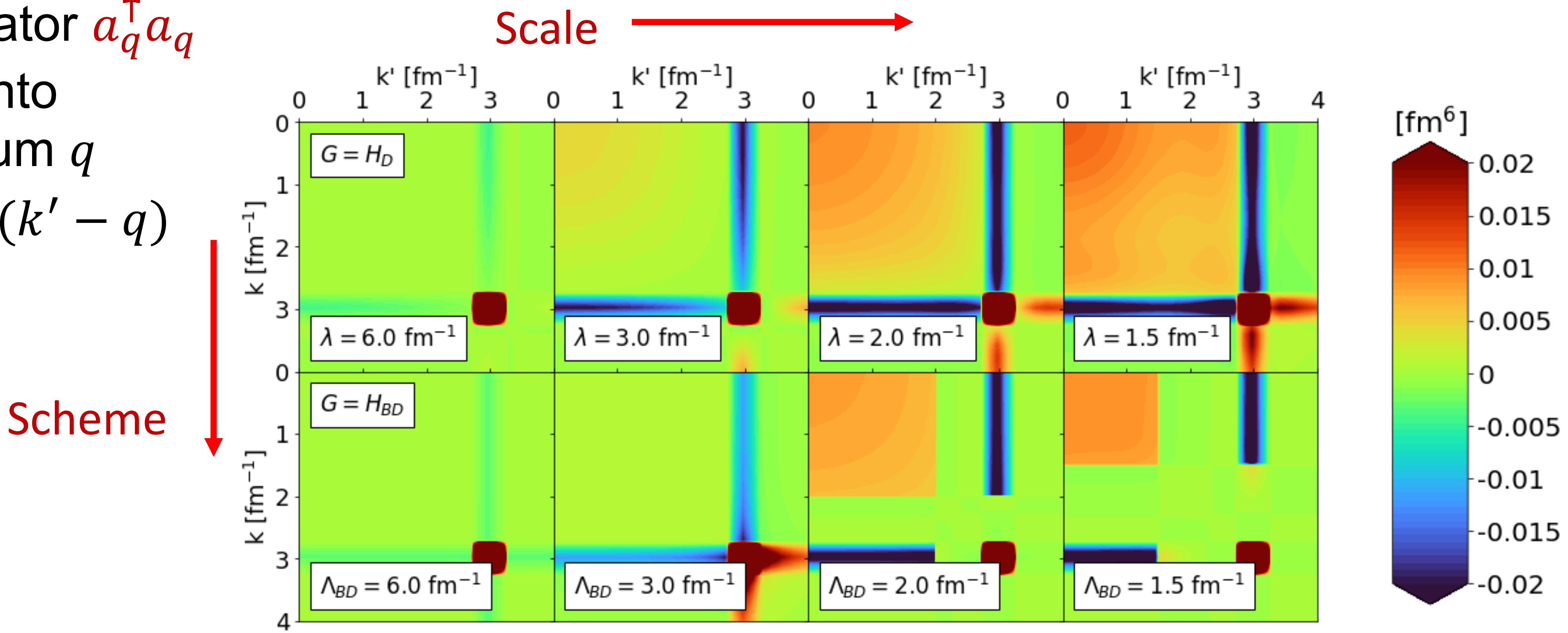
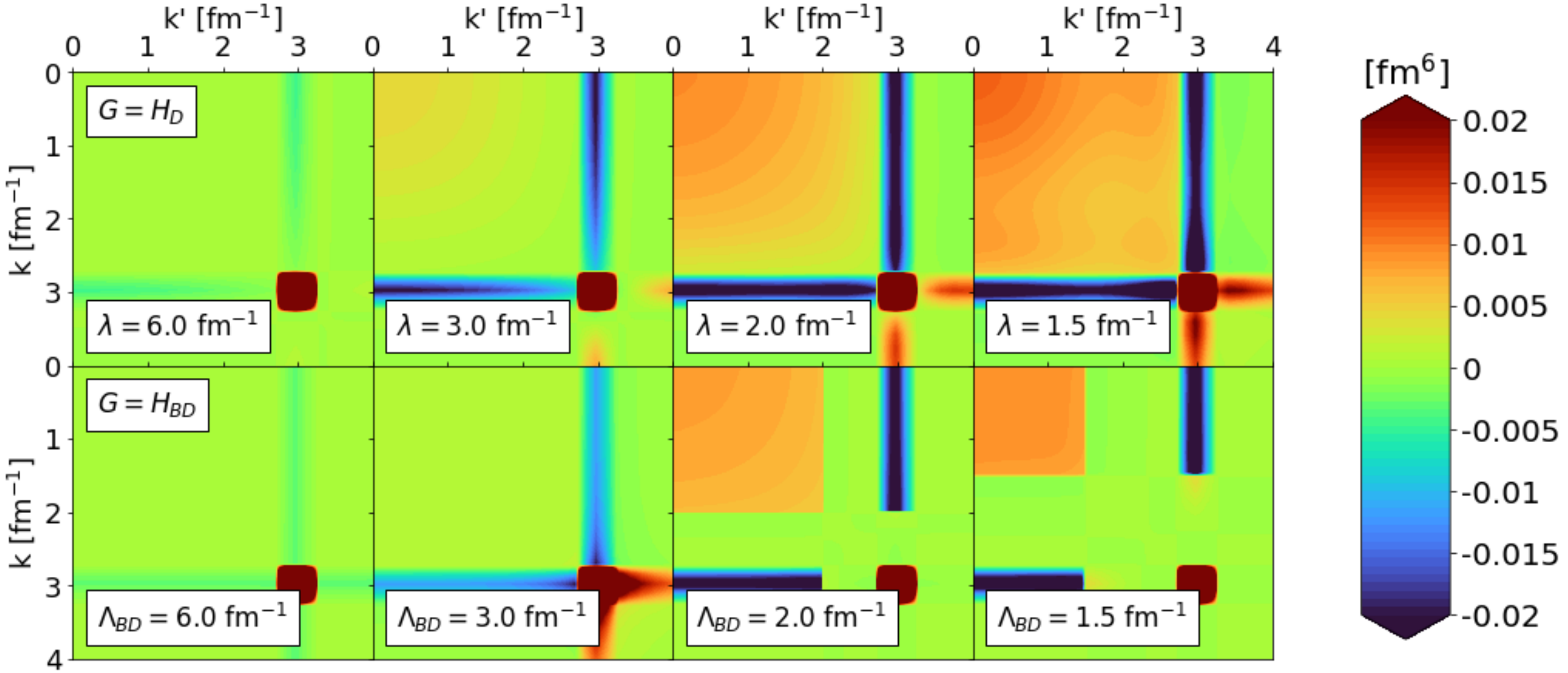



Fig. 3: SRG evolution of $a_q^\dagger a_q$ for $q = 3$ fm$^{-1}$ in the $^3S_1$- $^3S_1$ channel. Transformations done with AV18.

# Deuteron momentum distribution

- Use simple operator $a_q^\dagger a_q$ which projects onto relative momentum $q$
  $a_q^\dagger a_q \sim \delta(k-q)\delta(k'-q)$
- Smooth induced contributions at low momentum reproduce UV physics of the original NN potential

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# Deuteron momentum distribution

Consistently evolve the wave functions!

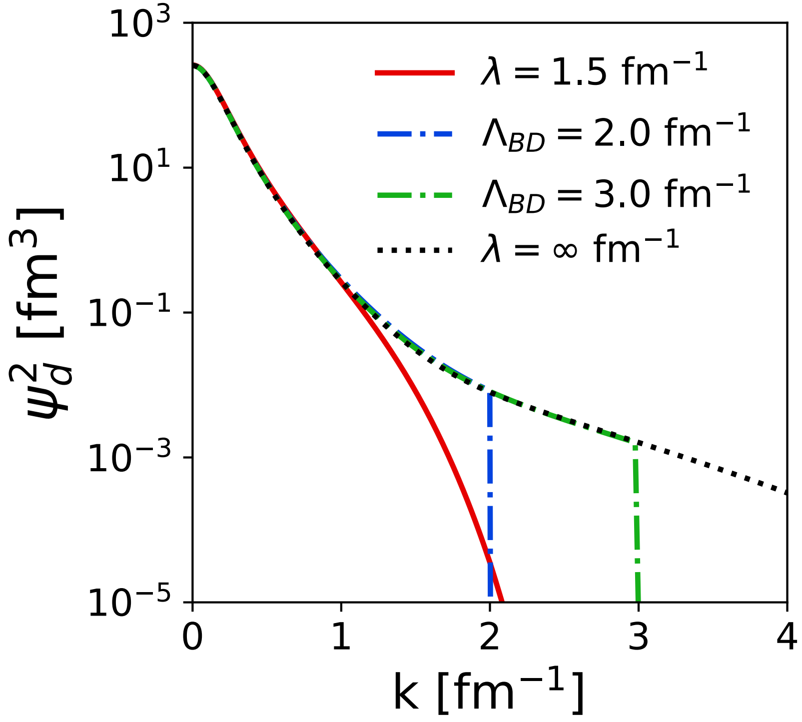
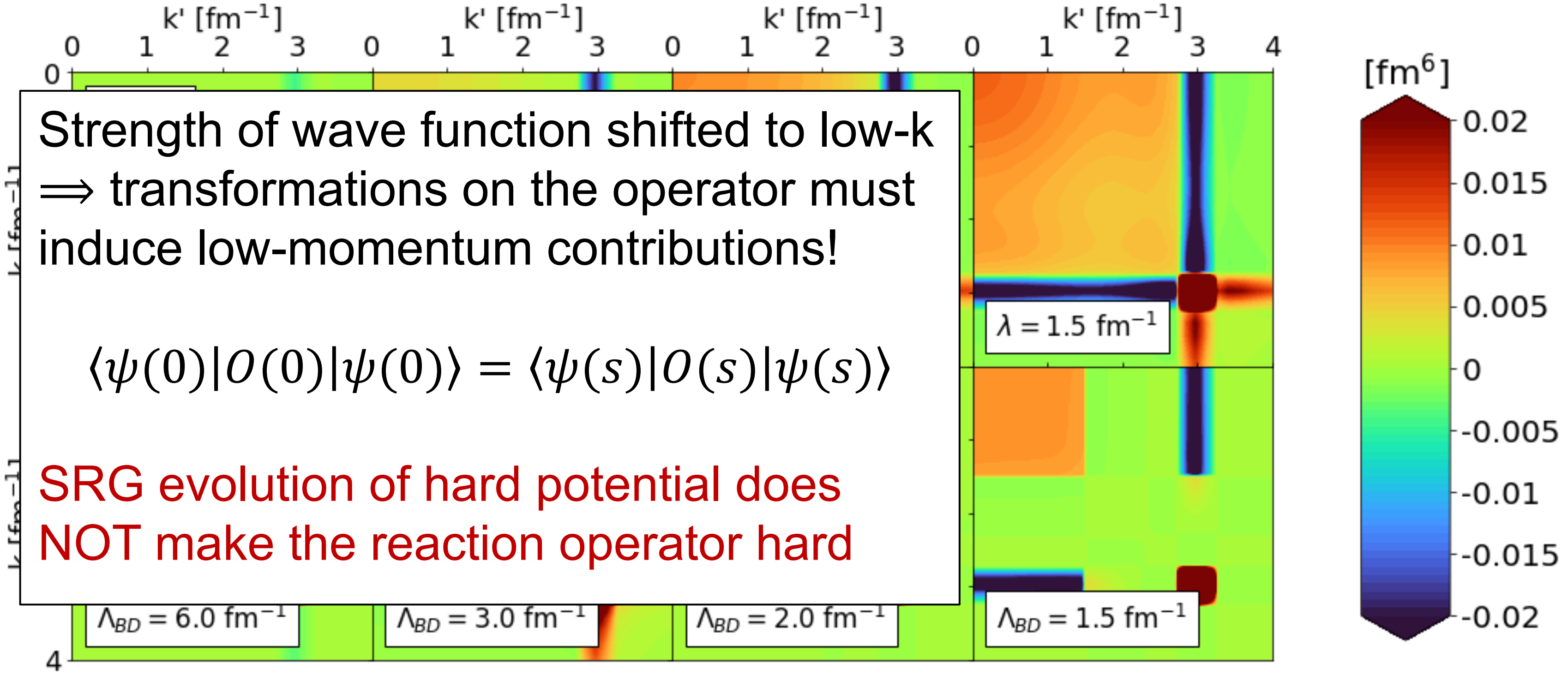



Fig. 4: SRG evolution of $\psi_d^2(k)$.




Strength of wave function shifted to low-k $\Longrightarrow$ transformations on the operator must induce low-momentum contributions!

$$\langle \psi(0) | O(0) | \psi(0) \rangle = \langle \psi(s) | O(s) | \psi(s) \rangle$$

SRG evolution of hard potential does NOT make the reaction operator hard

Fig. 3: SRG evolution of $a_q^\dagger a_q$ for $q = 3$ fm$^{-1}$ in the $^3S_1$-$^3S_1$ channel. Transformations done with AV18.

# Deuteron momentum distribution

- Expectation value $\langle \psi_d | a_q^\dagger a_q | \psi_d \rangle$ is driven to low-momentum
- Note, each panel gives the correct result from unitarity of transformation!

Scale

Scheme

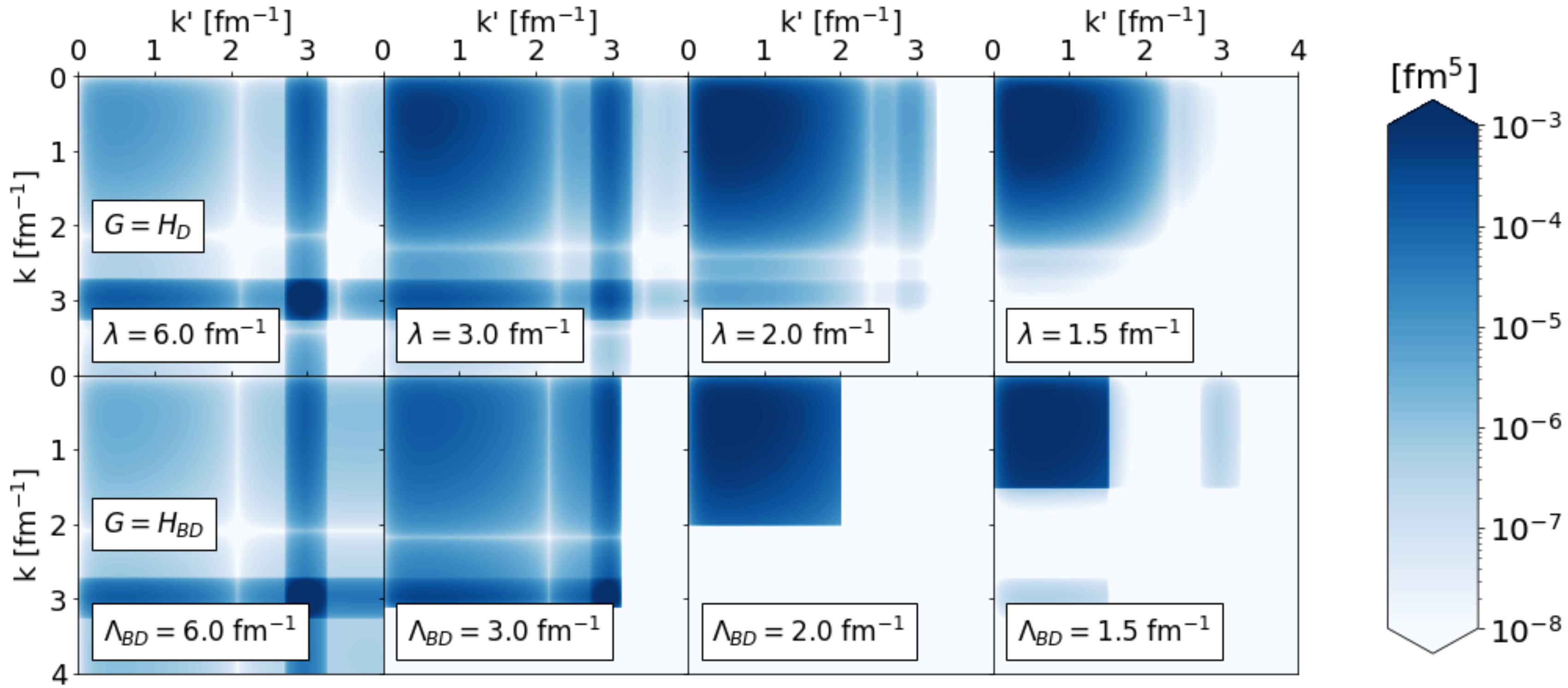
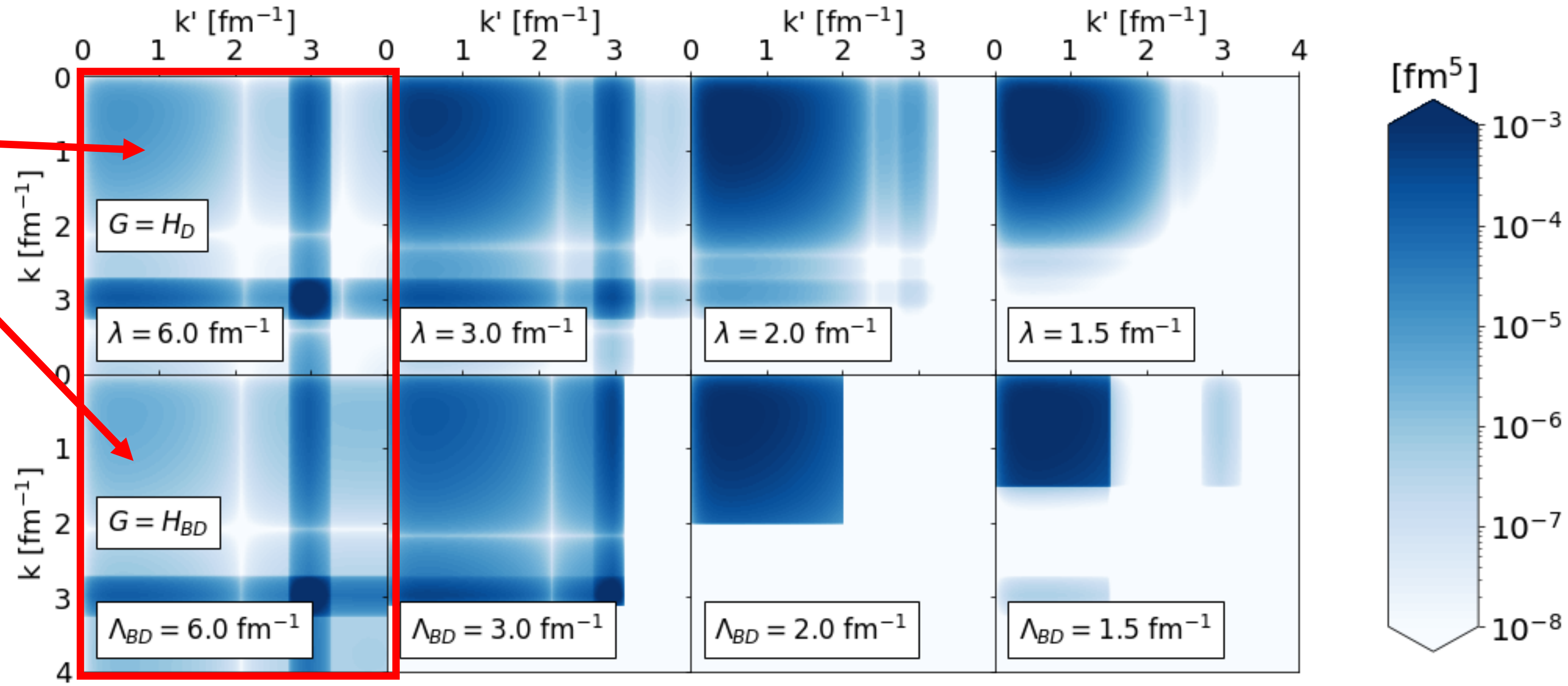



Fig. 5: SRG-evolved matrix elements of $\langle \psi | a_q^\dagger a_q | \psi \rangle$ with AV18 in the $^3S_1$-$^3S_1$ channel and $q = 3$ fm$^{-1}$.

# Deuteron momentum distribution

- At high RG resolution $^3S_1$- $^3S_1$ channel contributes to ~25% of the expectation value $\langle\psi_d|a_q^\dagger a_q|\psi_d\rangle$ (heavy contribution from tensor force)

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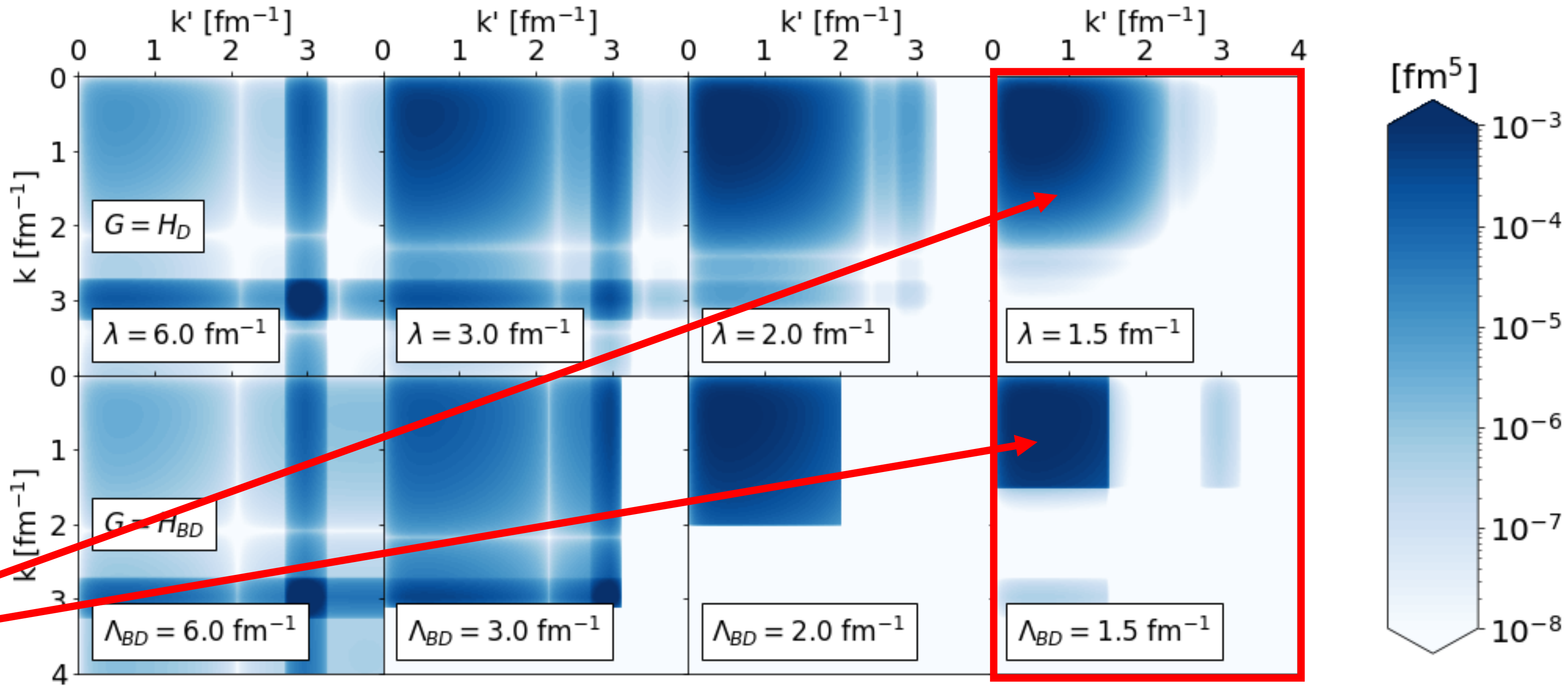
- At high RG resolution $^3S_1$- $^3S_1$ channel contributes to ~25% of the expectation value $\langle \psi_d | a_q^\dagger a_q | \psi_d \rangle$ (heavy contribution from tensor force)
- At low RG resolution $^3S_1$- $^3S_1$ channel contributes to ~95% of the expectation value $\langle \psi_d | a_q^\dagger a_q | \psi_d \rangle$

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# NN pair ratios

- At high RG resolution, the tensor force and the repulsive core of the NN interaction kicks nucleon pairs into SRCs
- Seen in the ratio of pairs produced where np dominates because the tensor force requires spin triplet pairs (pp are spin singlets)

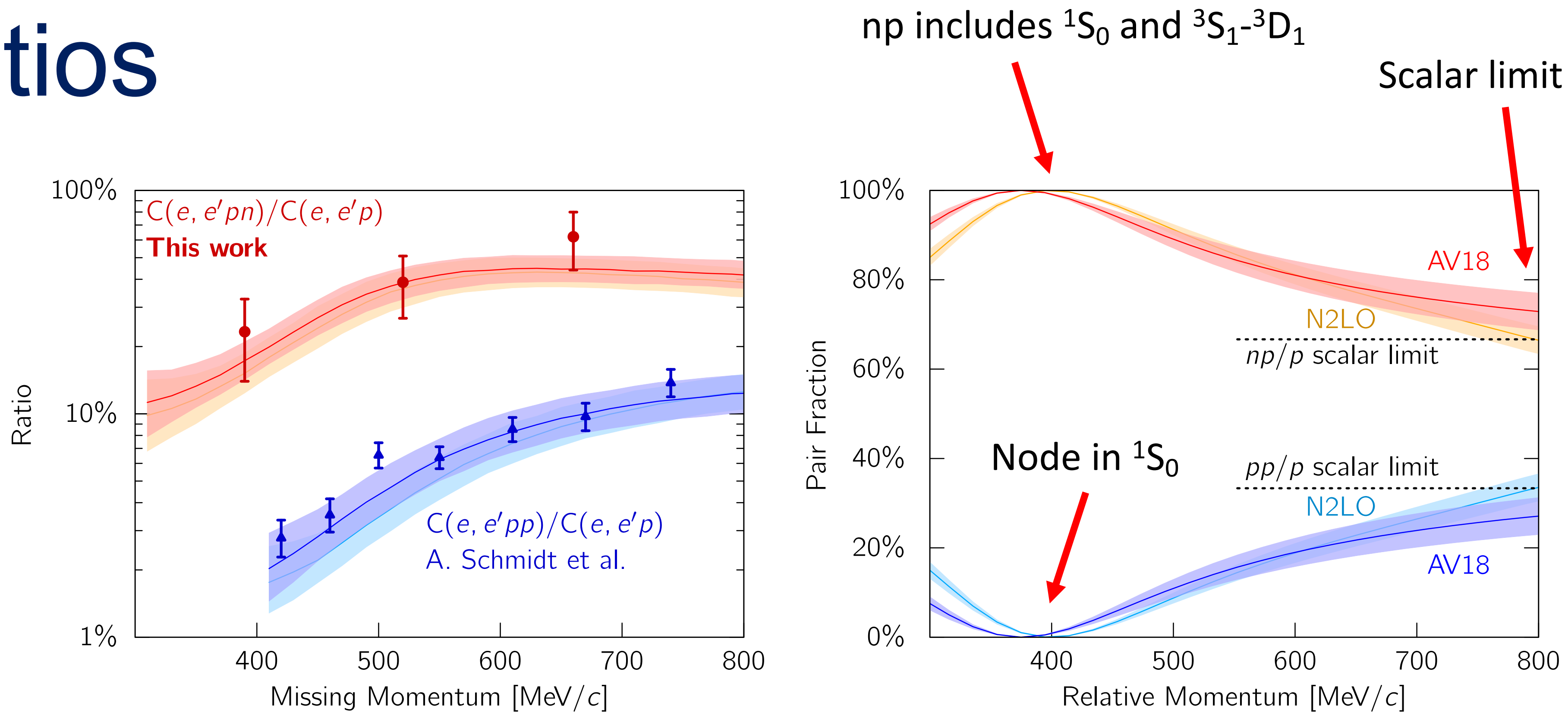



Fig. 6: (a) Ratio of two-nucleon to single-nucleon electron-scattering cross sections for carbon as a function of missing momentum. (b) Fraction of np to p and pp to p pairs versus the relative momentum. Figure from CLAS collaboration publication[1].

[1]I. Korover et al. (CLAS), arXiv:2004.07304 (2014)

# NN pair ratios

- At low RG resolution, SRCs are suppressed in the potential
- Consider the ratio of the momentum projection operator $a_q^\dagger a_q$ for $^3S_1/^1S_0$

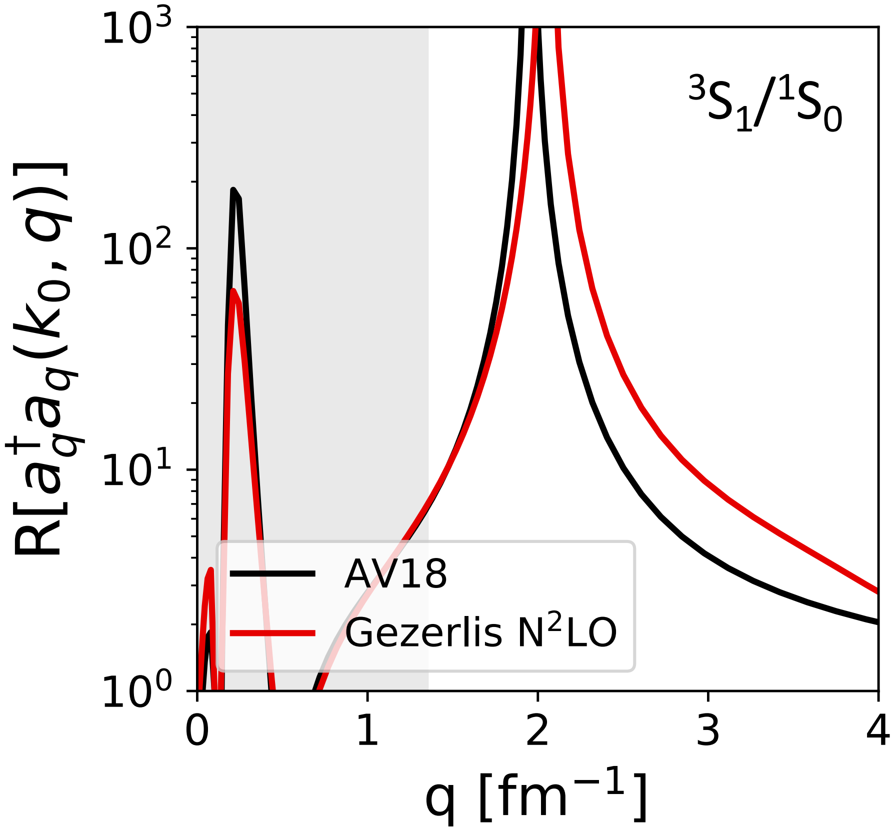
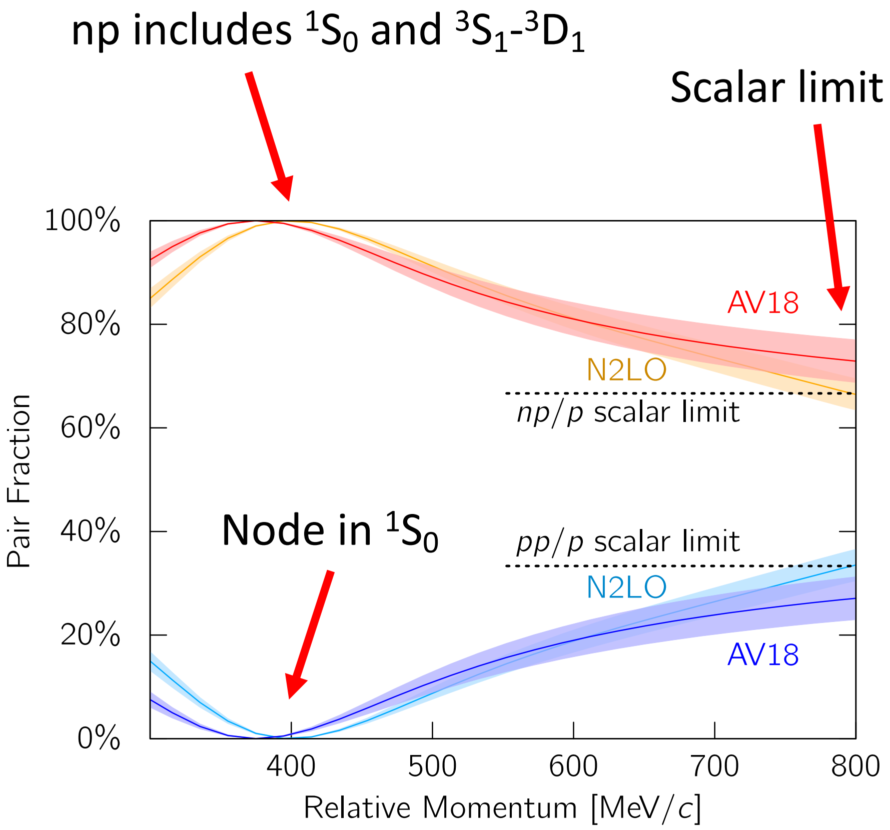
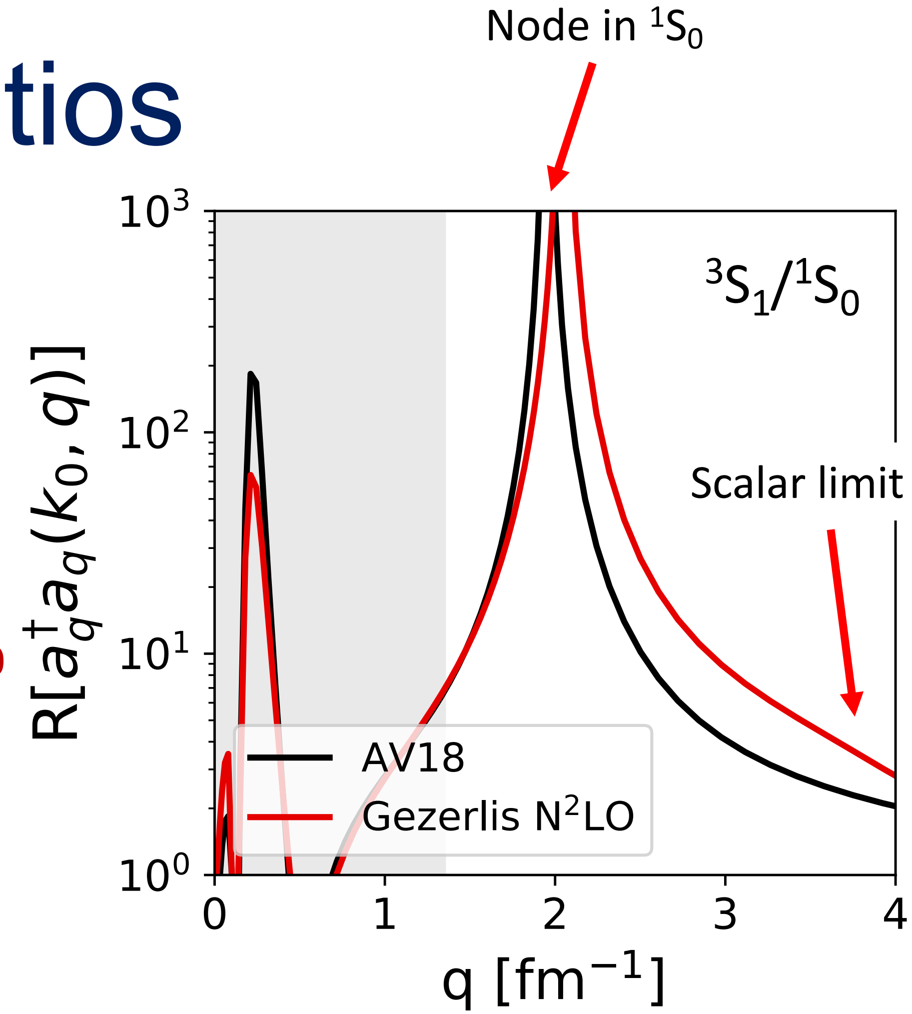



Fig. 7: Ratio of $^3S_1$ to $^1S_0$ SRG transformations for low momentum $k_0$ and high momentum $q$



# NN pair ratios

- At low RG resolution, SRCs are suppressed in the potential
- Consider the ratio of the momentum projection operator $a_q^\dagger a_q$ for ${}^3S_1/{}^1S_0$
- Reproduces the characteristics of the cross section ratios with low RG resolution operators

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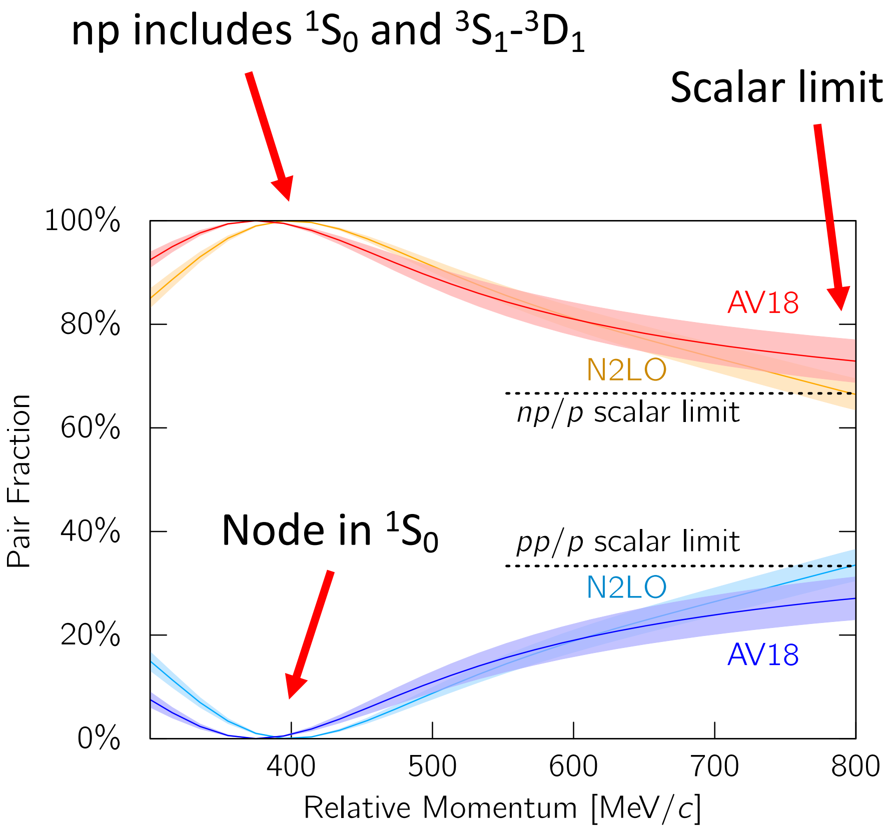

# Summary and outlook

- Results suggest that we can analyze high-energy nuclear reactions with low RG resolution structure (e.g., shell model) and evolved operator (and correct initial operator)
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- Results suggest that we can analyze high-energy nuclear reactions with low RG resolution structure (e.g., shell model) and evolved operator (and correct initial operator)
  - Matching resolution scale between structure and reactions is crucial!
- Ongoing work:
  - Calculate pair distributions in nuclei (N=Z, N>Z) using local density approximation
  - Relate to quenching in knock-out reactions by applying to different processes with factorization

# Back up slides

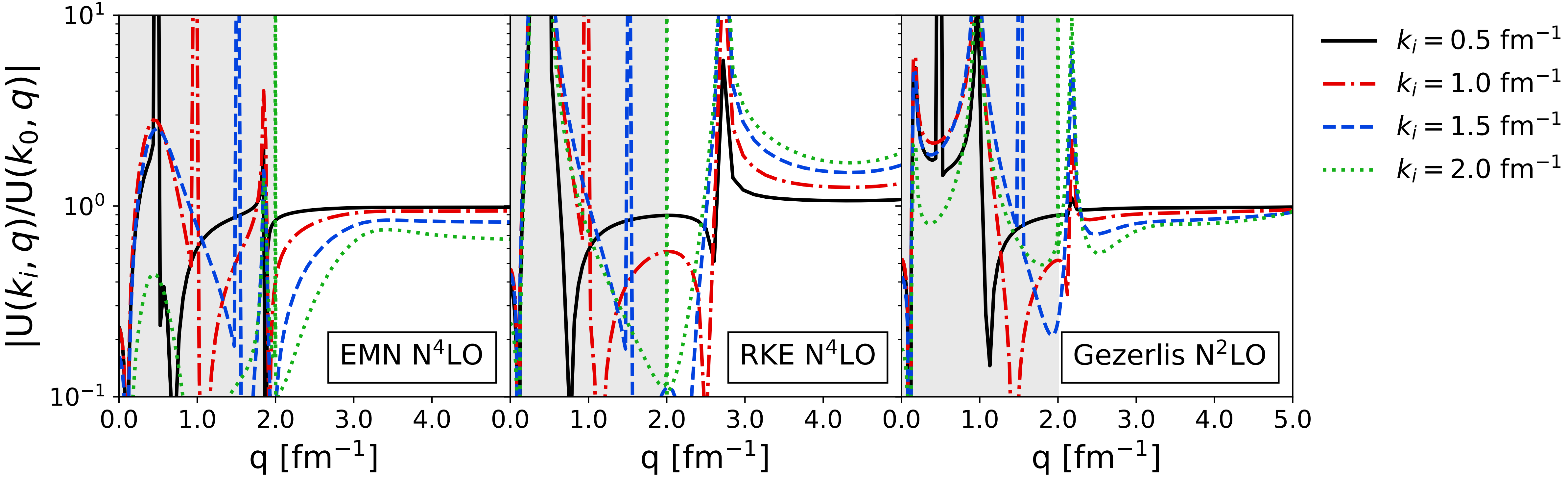


Fig. 8: Ratio of SRG transformations $U(k,q)$ at low- and high-momentum values with respect to high-momentum $q$, and fixing the low-momentum of the denominator $k_0$ and varying the low-momentum of the numerator $k_i$.