integrals are performed).

The EFT Lagrangian for a nonrelativistic spin-1/2 fermion field with spin-independent interactions is the same as in Ref. [7], but converted to Euclidean form (which means $t \to -i\tau$ and an overall minus sign in defining $\mathcal{L}_E$ and the interaction Lagrangian $\mathcal{L}_E^{\rm int}$):

$$\begin{aligned}
\mathcal{L}_E \;&=\; \psi^\dagger\Big[\frac{\partial}{\partial\tau} - \frac{\overrightarrow{\nabla}^{\,2}}{2M}\Big]\psi + \frac{C_0}{2}(\psi^\dagger\psi)^2 - \frac{C_2}{16}\Big[(\psi\psi)^\dagger(\psi\overleftrightarrow{\nabla}^2\psi) + \text{ h.c.}\Big] \\
&\quad - \frac{C_2'}{8}(\psi\overleftrightarrow{\nabla}\psi)^\dagger\cdot(\psi\overleftrightarrow{\nabla}\psi) + \ldots \\
&\equiv\; \psi^\dagger\Big[\frac{\partial}{\partial\tau} - \frac{\overrightarrow{\nabla}^{\,2}}{2M}\Big]\psi + \mathcal{L}_E^{\rm int}(\psi^\dagger,\psi)\;, \qquad (1)
\end{aligned}$$

where $\overleftrightarrow{\nabla} = \overleftarrow{\nabla} - \overrightarrow{\nabla}$ is the Galilean invariant derivative and h.c. denotes the Hermitian conjugate.[1] The terms proportional to C_2 and C_2' contribute to s-wave and p-wave scattering, respectively, while the dots represent terms with more derivatives and/or more fields, as well as renormalization counterterms. The Lagrangian Eq. (1) is a particular canonical form, which can be reached via field redefinitions. For example, higher-order terms with time derivatives are omitted, as they can be eliminated in favor of terms with spatial derivatives (see, for example, Ref. [43]). We will restrict ourselves here to spin-1/2 (i.e., degeneracy $\nu = 2$) and spin-independent interactions, which is sufficient to illustrate the formalism and the renormalization issues; generalizing to isospin and spin-dependent interations is straightforward. We also consider only the C_0 vertex here, but comment at the end on the inclusion of vertices with derivatives.

We first consider a Euclidean generating functional with chemical potential μ and external sources $\eta(x)$ and $\eta^\dagger(x)$ [16, 44]:

$$Z[\eta,\eta^\dagger;\mu] \equiv e^{-W[\eta,\eta^\dagger;\mu]} = \int D\psi_\alpha D\psi_\alpha^\dagger\; e^{-\int d^4x\,[\mathcal{L}_E\,-\,\mu\,\psi_\alpha^\dagger(x)\psi_\alpha(x)+\eta_\alpha^\dagger(x)\psi_\alpha(x)+\psi_\alpha^\dagger(x)\eta_\alpha(x)]}\;, \qquad (2)$$

where $\int d^4x$ includes a $d\tau$ integration that runs from $-\beta/2$ to $\beta/2$ (to facilitate the $\beta\to\infty$ limit) and anti-periodic boundary conditions are imposed. For simplicity, normalization factors are considered to be implicit in the functional integration measure. (See Refs. [17, 18, 44] for more detailed treatments of similar path integrals.) We have written the spin index α explicitly in Eq. (2); we will interchangeably use $\alpha = \{1,2\}$ and $\alpha = \{\uparrow,\downarrow\}$ in the sequel to denote individual spins, and the spin indices will be left implicit where there is no chance of confusion. We have kept the chemical potential μ separate from the Lagrangian in Eq. (2) to emphasize its role as an external source. In the next section, we will add a corresponding source coupled to the pair density.

A conventional perturbative expansion is realized by removing the interaction terms from the path integral in (1) in favor of functional derivatives with respect to η and $\eta^\dagger$ and performing the remaining Gaussian integration over ψ and $\psi^\dagger$ [16, 44]:

$$Z[\eta,\eta^\dagger;\mu] = Z_0\, e^{-\int d^4x\,\mathcal{L}_E^{\rm int}[\delta/\delta\eta(x),-\delta/\delta\eta^\dagger(x)]}\, e^{\int d^4y\,d^4y'\,\eta^\dagger(y)\mathcal{G}_0(y,y')\eta(y')}\;, \qquad (3)$$

where the spin indices are implicit and we have introduced the noninteracting partition function Z_0. Explicit expressions for the Matusbara Green's function $\mathcal{G}_0$ in coordinate and

[1] We use units with $\hbar = 1$.

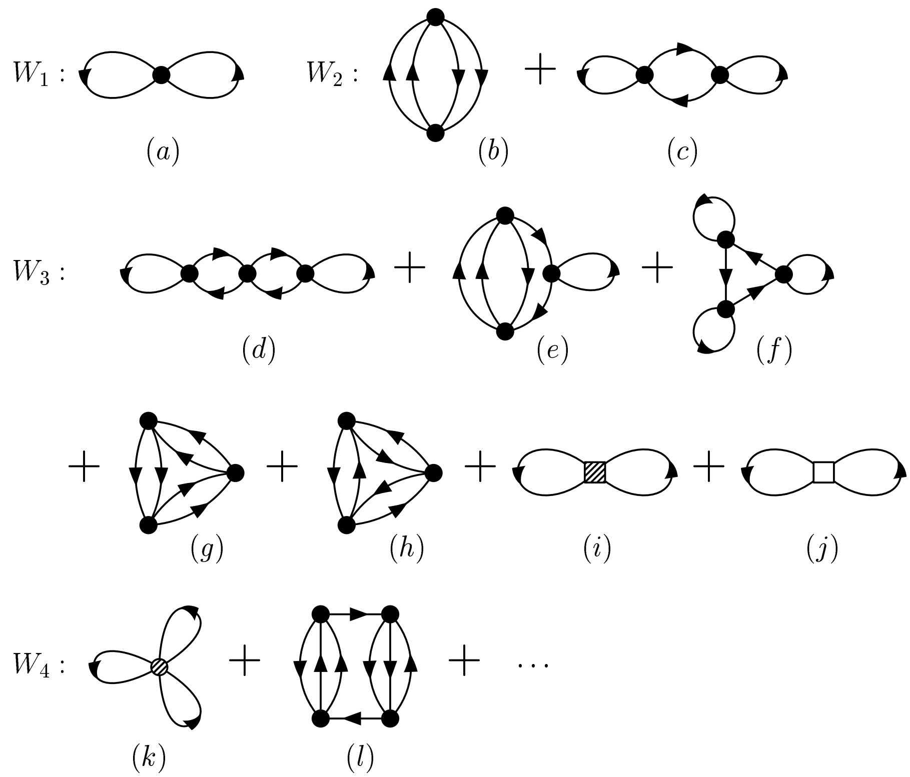

FIG. 1: Hugenholtz diagrams for the unrenormalized W_i functionals in a homogeneous, dilute Fermi system. The vertices are defined in Fig. 5.

momentum space can be found in Ref. [44]. The linked-cluster theorem [44] shows that the difference of the interacting and noninteracting thermodynamic potentials Ω and Ω_0 is given by the sum of connected diagrams from the expansion of Z, with the external sources $\eta^\dagger$ and η set to zero at the end:

$$\Omega(\mu, T, V) - \Omega_0(\mu, T, V) = \frac{1}{\beta}(W[0, 0; \mu] - W_0[0, 0; \mu]) \ . \tag{4}$$

The connected Feynman diagrams that sum (with appropriate symmetry factors) to $W - W_0$, which are labled W_i with $i \geq 1$, are shown in Fig. 1, with the fermion lines representing Matsubara propagators $\mathcal{G}_0$ with chemical potential μ [44]. The subscript i labels the leading order to which W_i contributes in the dilute EFT expansion. (In the DR/MS scheme, each diagram contributes to only one order but this is no longer true in DR/PDS.) The chemical potential can be determined for given N by inverting the thermodynamic expression for the particle number, $N = \partial\Omega/\partial\mu$. The regularization and renormalization of divergences arising in the evaluation of the W_i are described below.

B. Kohn-Luttinger-Ward Inversion

The Kohn-Luttinger-Ward (KLW) theorem [26, 27] relates the perturbative calculation of diagrams using the finite-temperature Matsubara formalism in the zero-temperature limit to