

Operator evolution from the similarity renormalization group

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Motivation

- Explosion of new NN interactions from chiral effective field theory (χ^{EFT}) in the last few years
- Previous SRG studies of operators were limited to phenomenological models or one χ^{EFT} interaction
- Revisit the question of how different potentials (regulator functions, cutoff, order, etc.) change under SRG transformations and how these transformations affect other operators

SRG formalism

- SRG transformations decouple low- and high-momenta in Hamiltonian

$$H(s) = U(s)H(0)U^\dagger(s)$$

where $s = 0 \rightarrow \infty$

- In practice, solve differential flow equation

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

with SRG generator $\eta(s) \equiv \frac{dU(s)}{ds}U^\dagger(s) = [G, H(s)]$

SRG formalism

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling

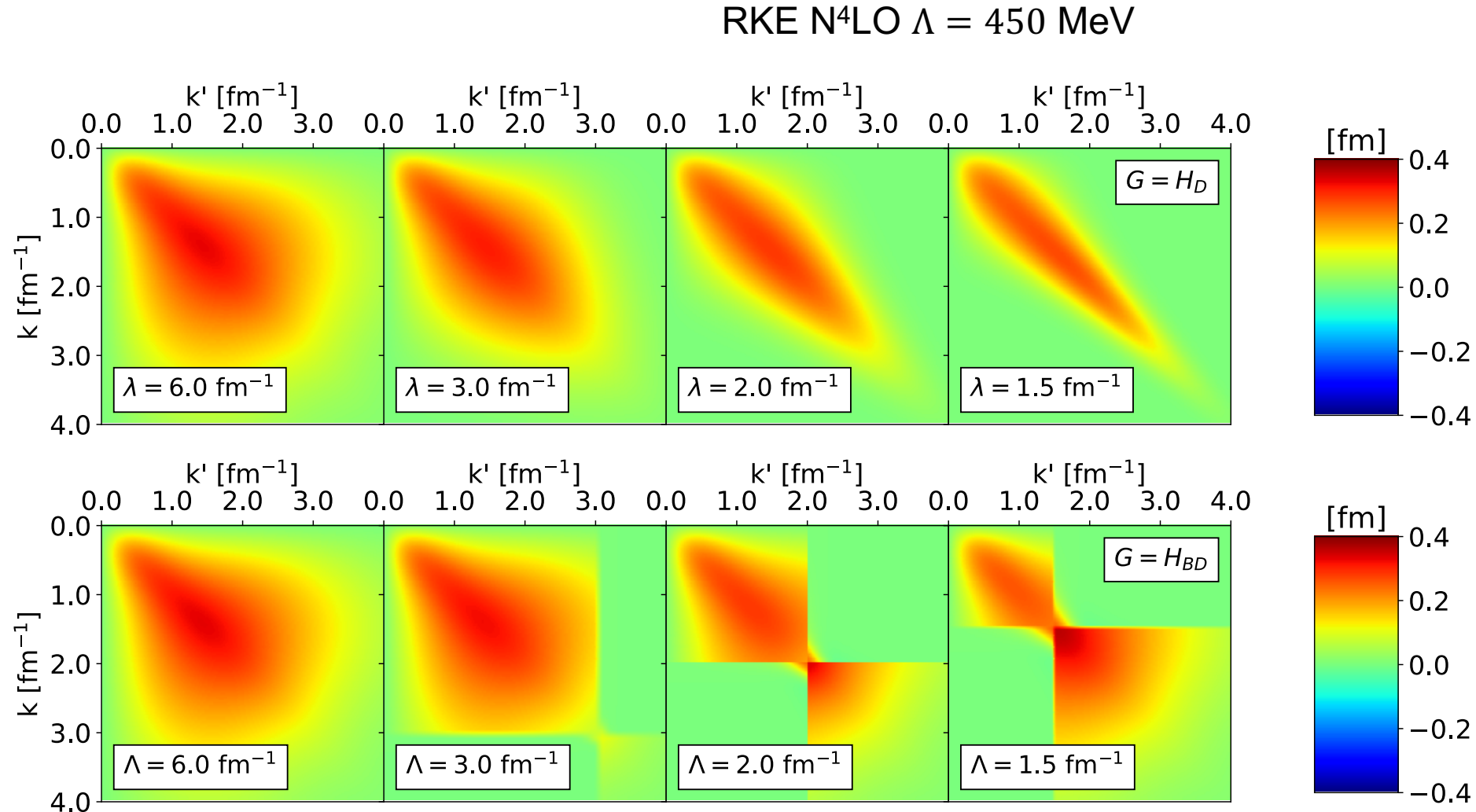


Fig. 1: SRG evolution of $V_\lambda(k, k')$ for several values of λ and Λ in the 1P_1 channel. Potentials from P. Reinert et al., Eur. Phys. J. A **54**, 86 (2018) which will be referred to as the RKE potentials.

SRG formalism

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling
- Parameters $\lambda = s^{-1/4}$ and Λ describe the decoupling of the evolved Hamiltonian

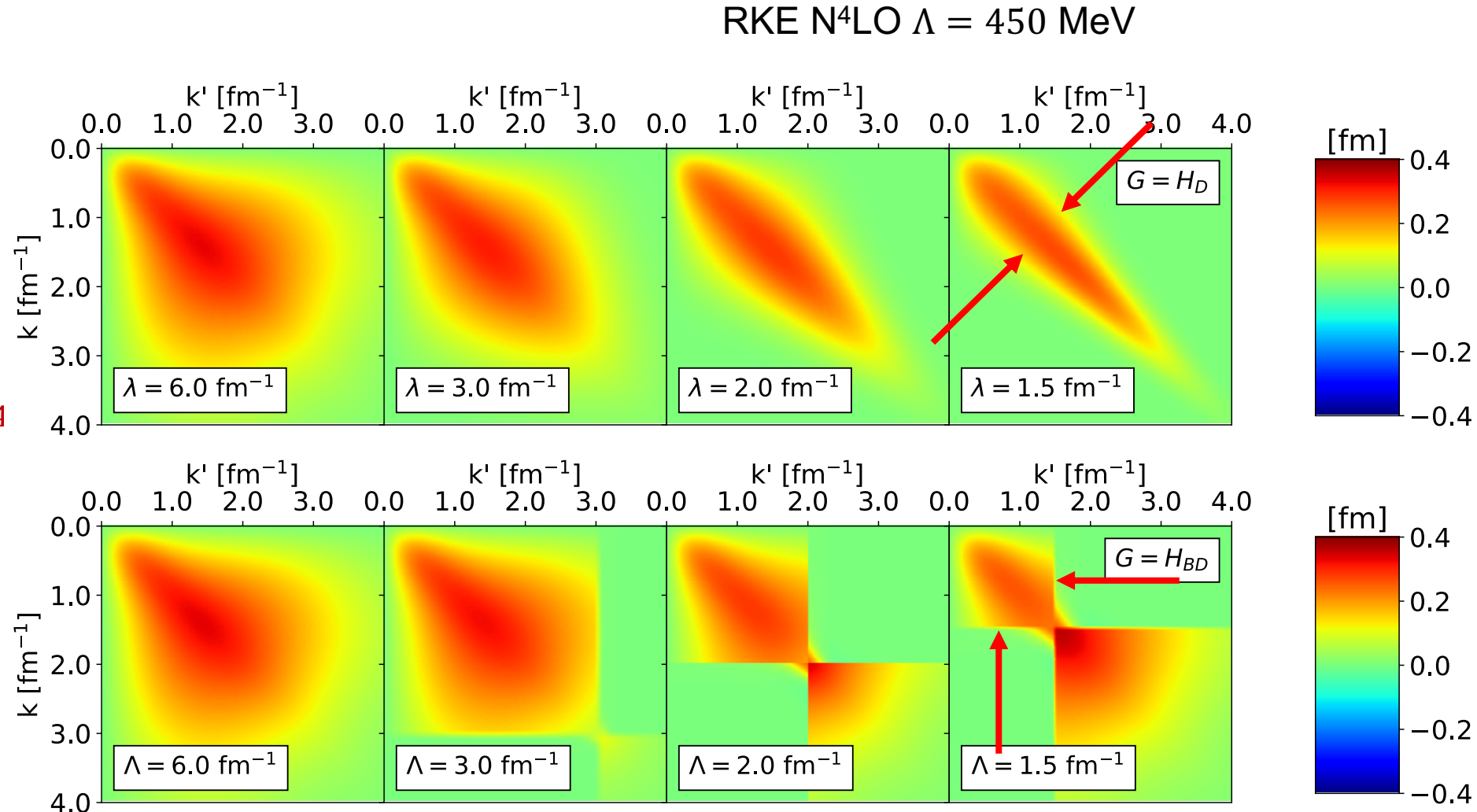


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SRG evolution of modern chiral potentials

- How do different potentials change under SRG transformations?

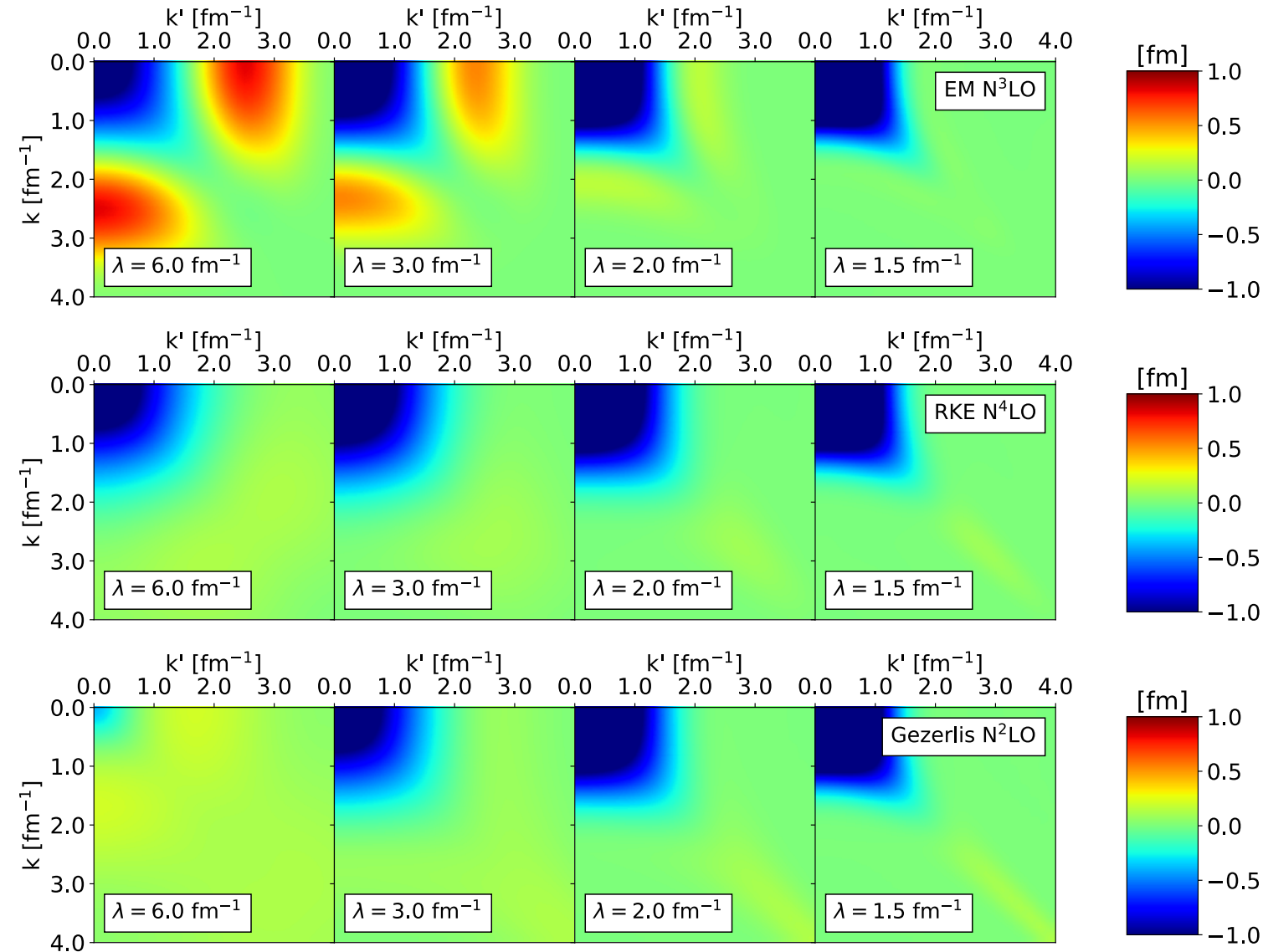


Fig. 2: SRG evolution of $V_\lambda(k, k')$ for several chiral potentials.

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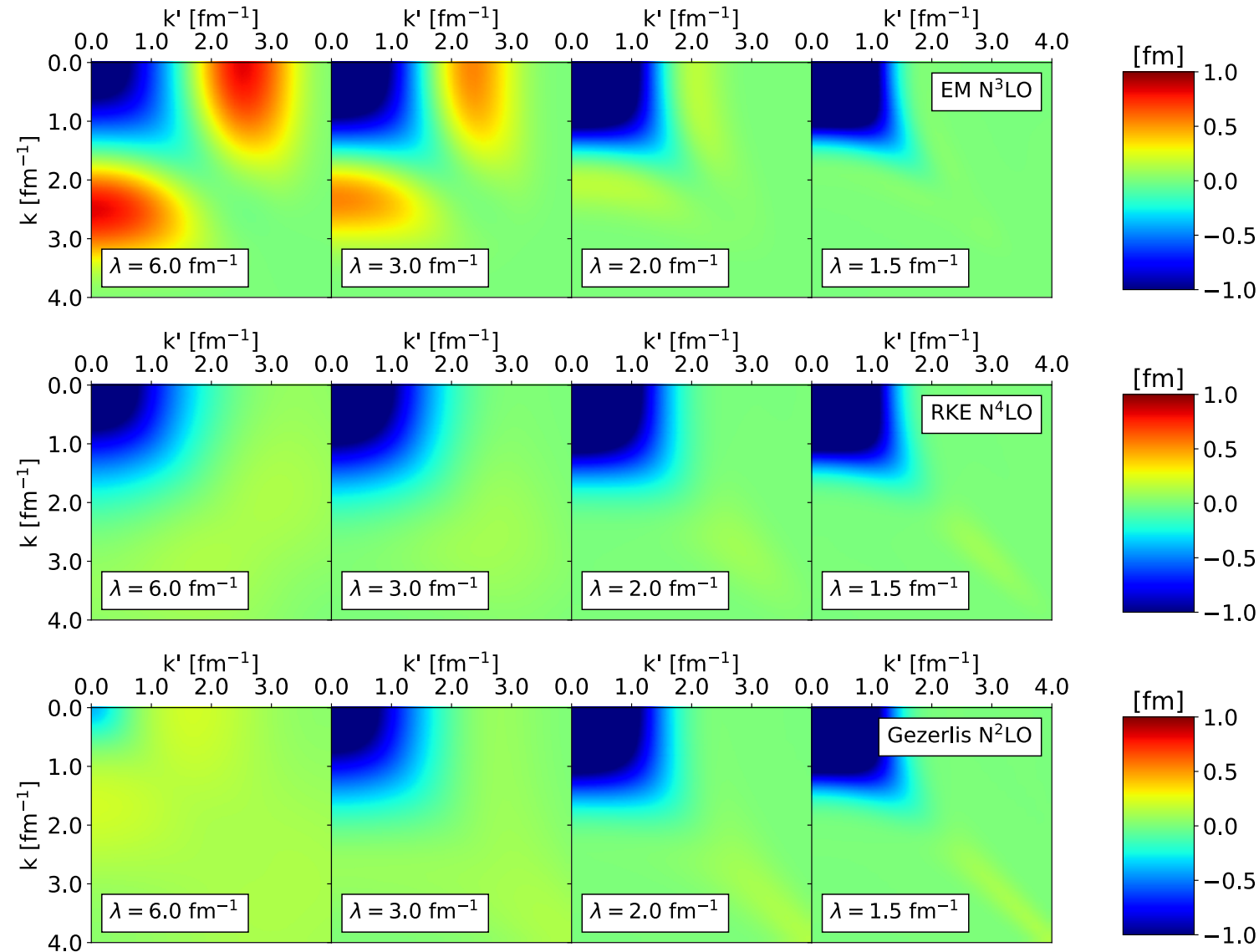
- How do different potentials change under SRG transformations?
- Use **non-local Entem-Machleidt¹** (500 MeV), **semi-local RKE²** (450 MeV), and **local Gezerlis et al.³** (1 fm) potentials as examples

¹D.R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003)

²P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A **54**, 86 (2018)

³A. Gezerlis, et al., Phys. Rev. C **90**, 054323 (2014)

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SRG evolution of modern chiral potentials

- How do different potentials change under SRG transformations?
- Use non-local Entem-Machleidt¹ (500 MeV), semi-local RKE² (450 MeV), and local Gezerlis et al.³ (1 fm) potentials as examples
- Different potentials evolve to the same low-momentum matrix elements!

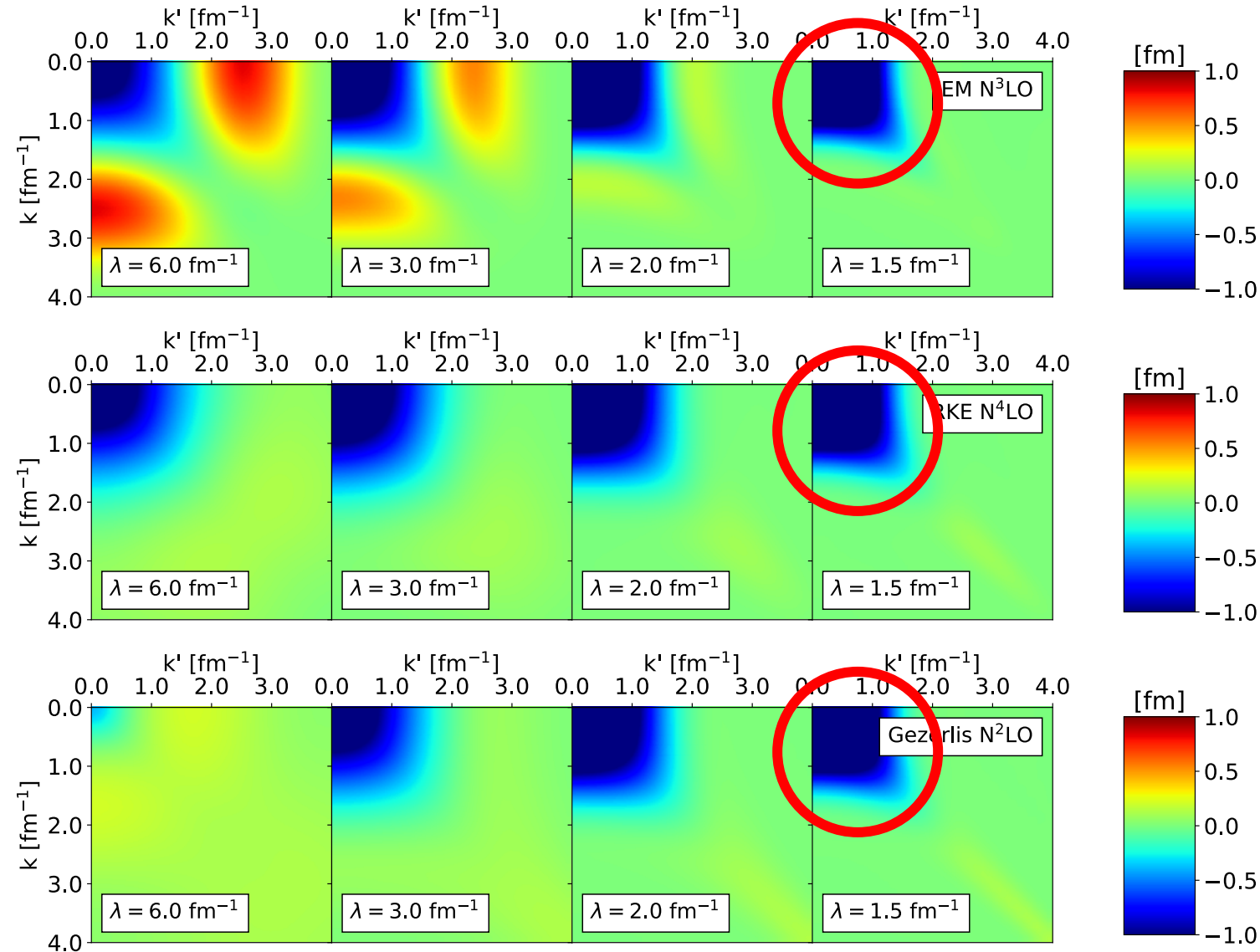


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Universality in SRG-evolved potentials

- Evolved matrix elements collapse to the same lines

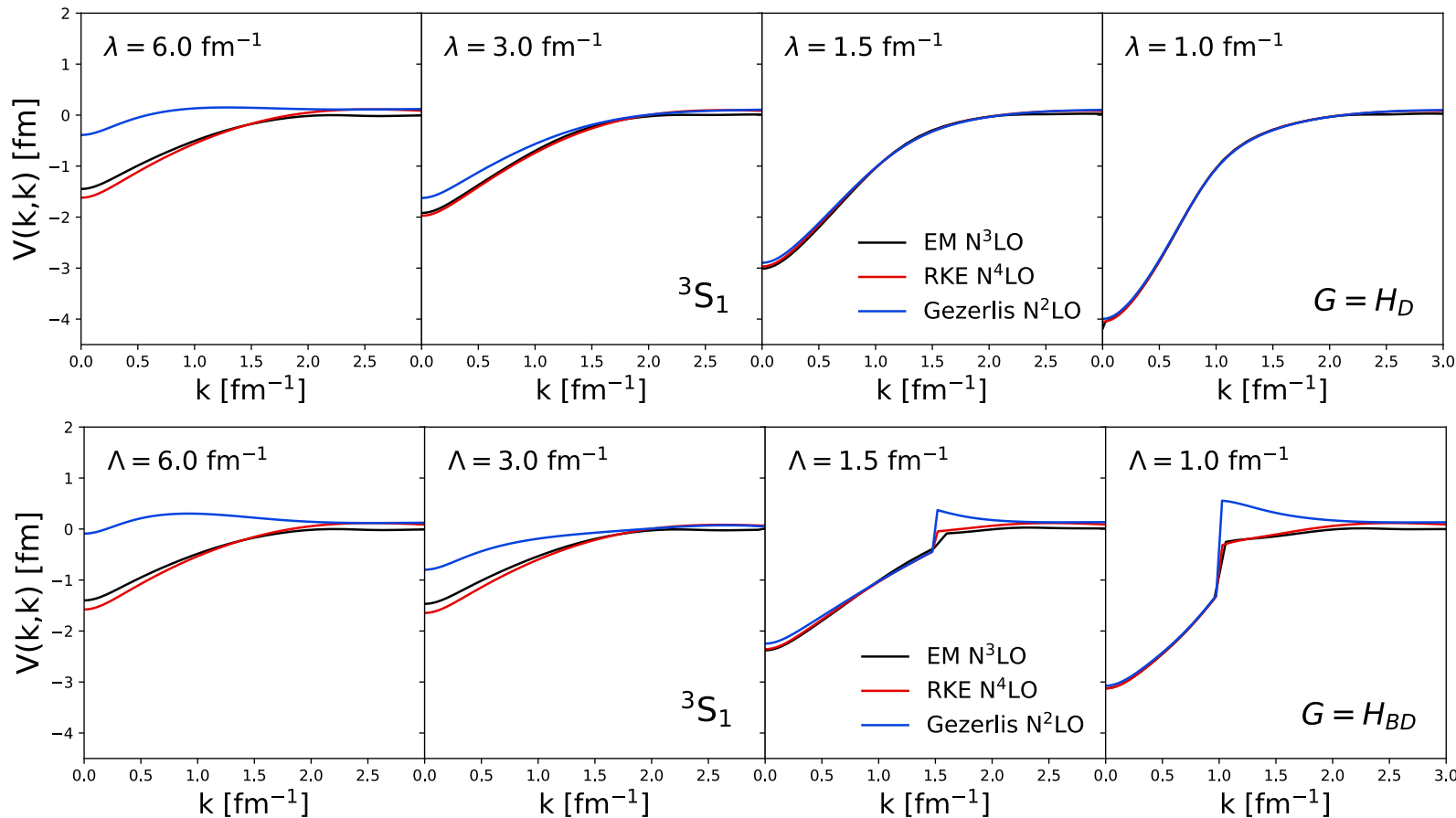
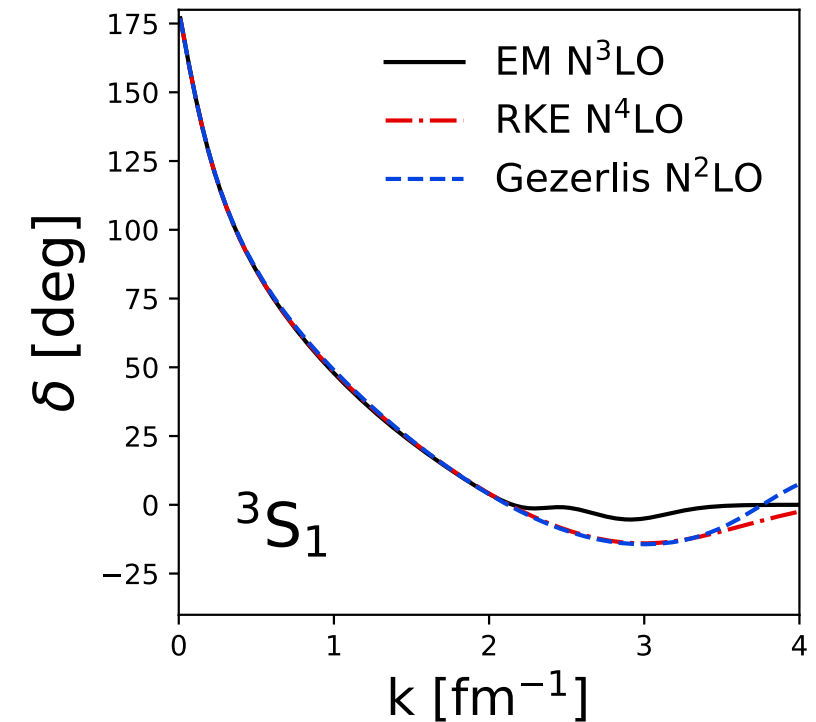
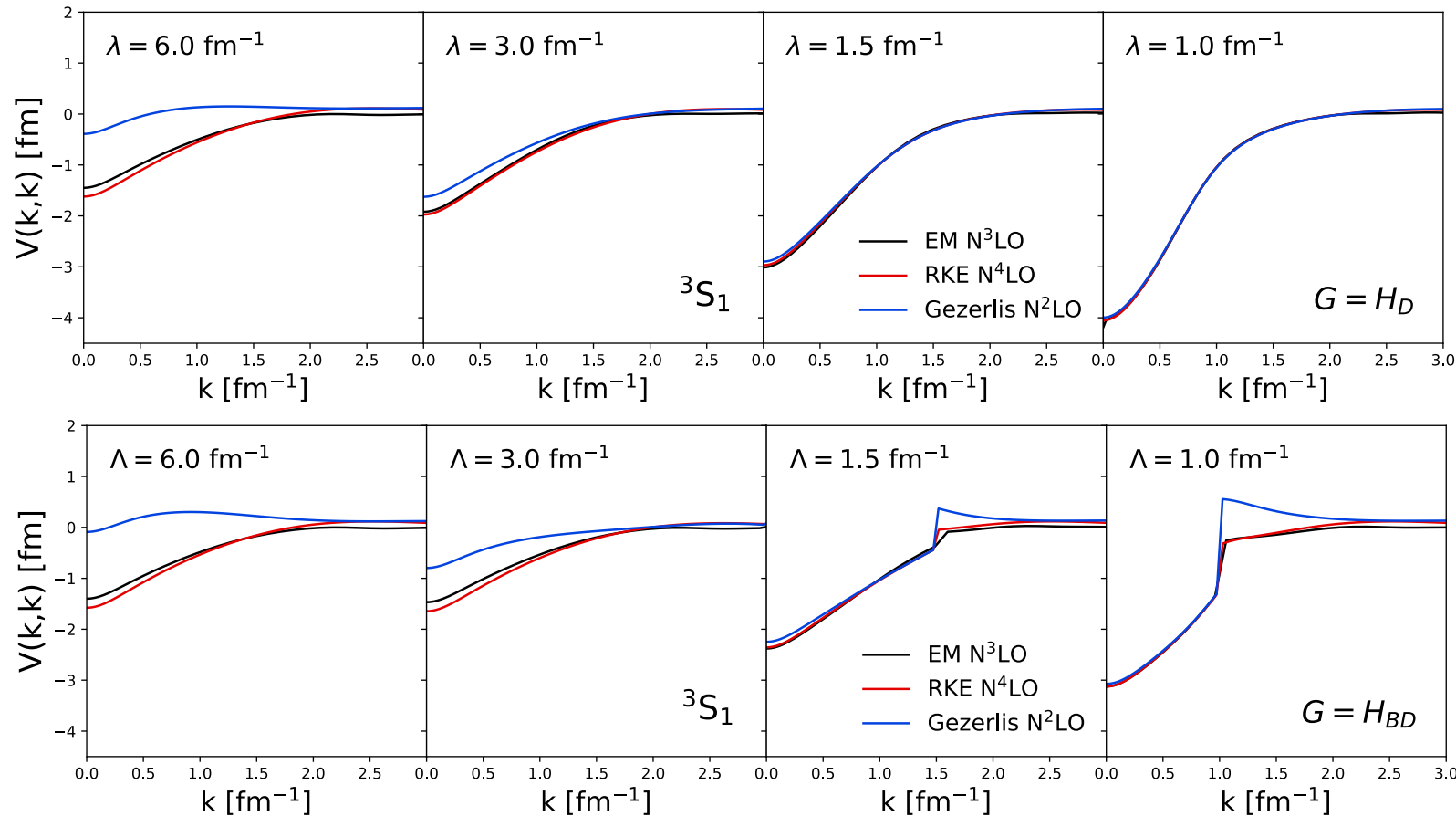


Fig. 3: Diagonal and far off-diagonal matrix elements of $V_\lambda(k, k')$ for several chiral potentials with band- and block-diagonal decoupling.

Universality in SRG-evolved potentials

- Universality in potential matrix elements is due to equivalent low-energy phase shifts¹



¹B. Dainton et al., Phys. Rev. C **89**, 014001 (2014)

SRG evolution for other operators

- SRG transformations will decouple the Hamiltonian but this behavior is not necessarily true for any operator

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- SRG transformations will decouple the Hamiltonian but this behavior is not necessarily true for any operator

$$\frac{dO(s)}{ds} = [\eta(s), O(s)]$$

- What are the characteristics of other evolved operators?
- Does universality hold for evolved operators?

Momentum projection operator

- We use the momentum projection operator $a_q^\dagger a_q$ as a test case

$$a_q^\dagger a_q \psi(k) = \int d^3k' \delta^3(\vec{k} - \vec{k}') \psi(k')$$

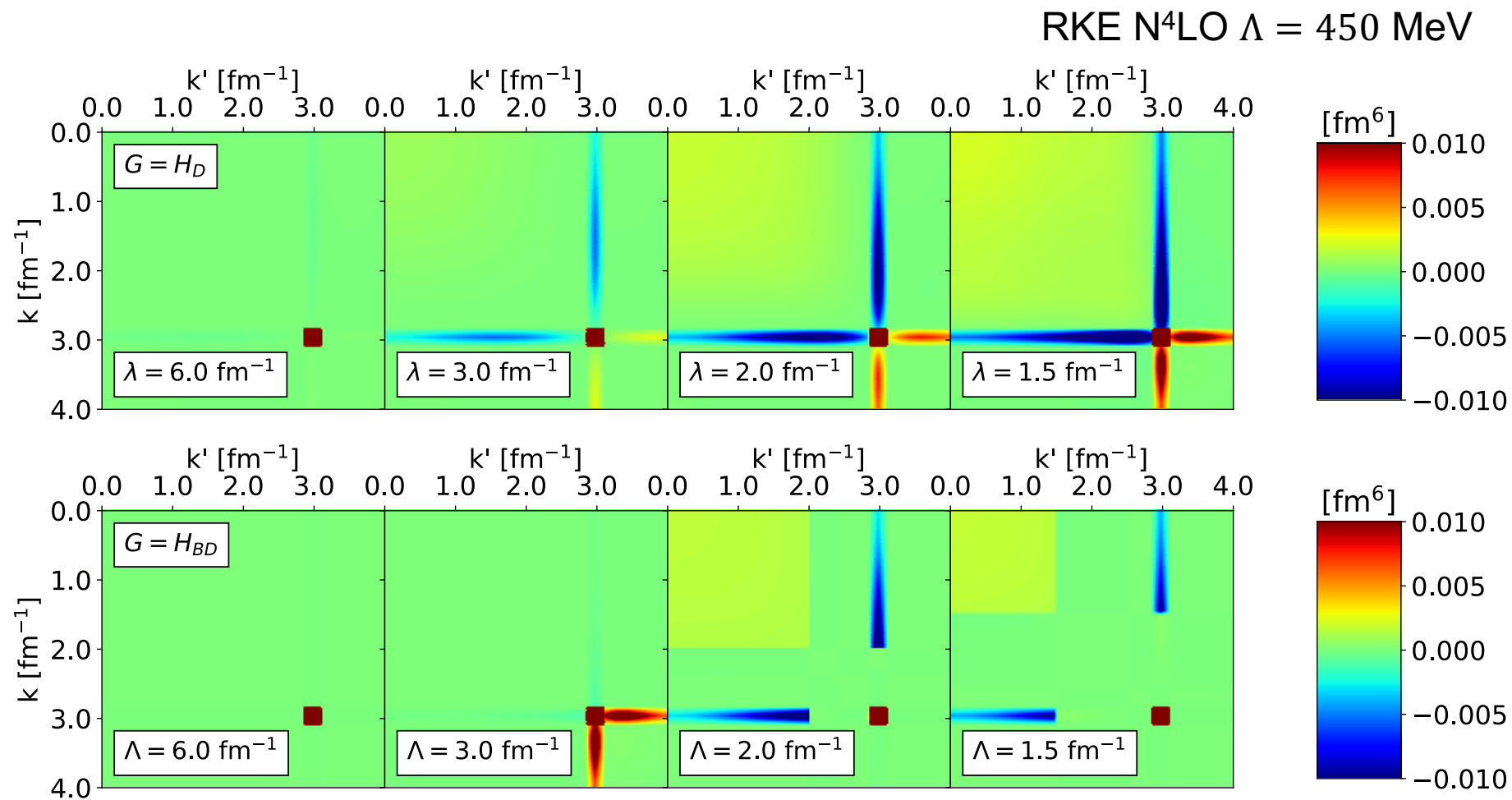


Fig. 4: SRG evolution of $a_q^\dagger a_q(k, k')$ for several values of λ and Λ where the transformations are done using the RKE N⁴LO potential. Here $q = 3$ fm⁻¹.

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- Initially starts out as a δ -function
- SRG transformations induce low-momentum contributions

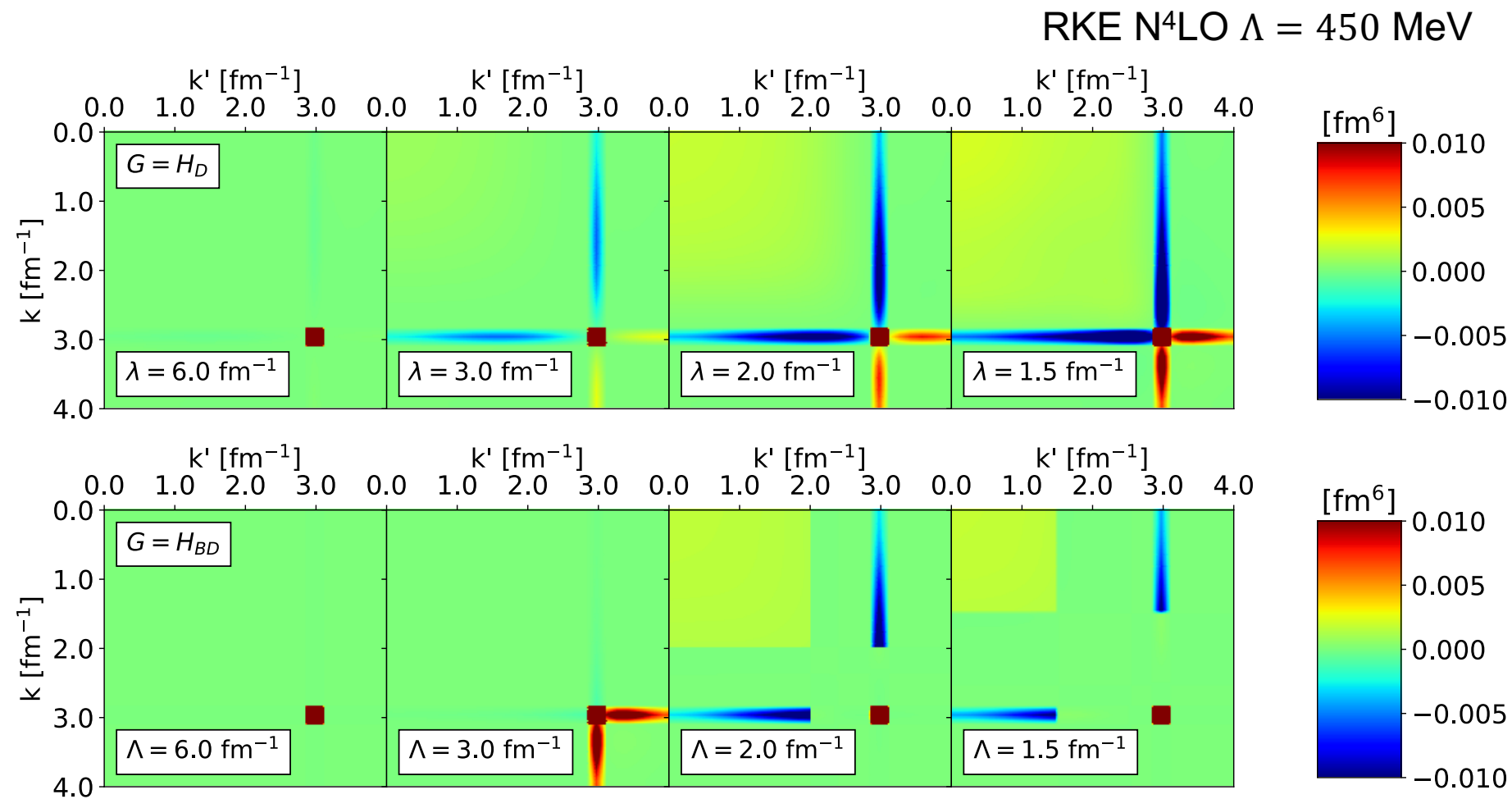


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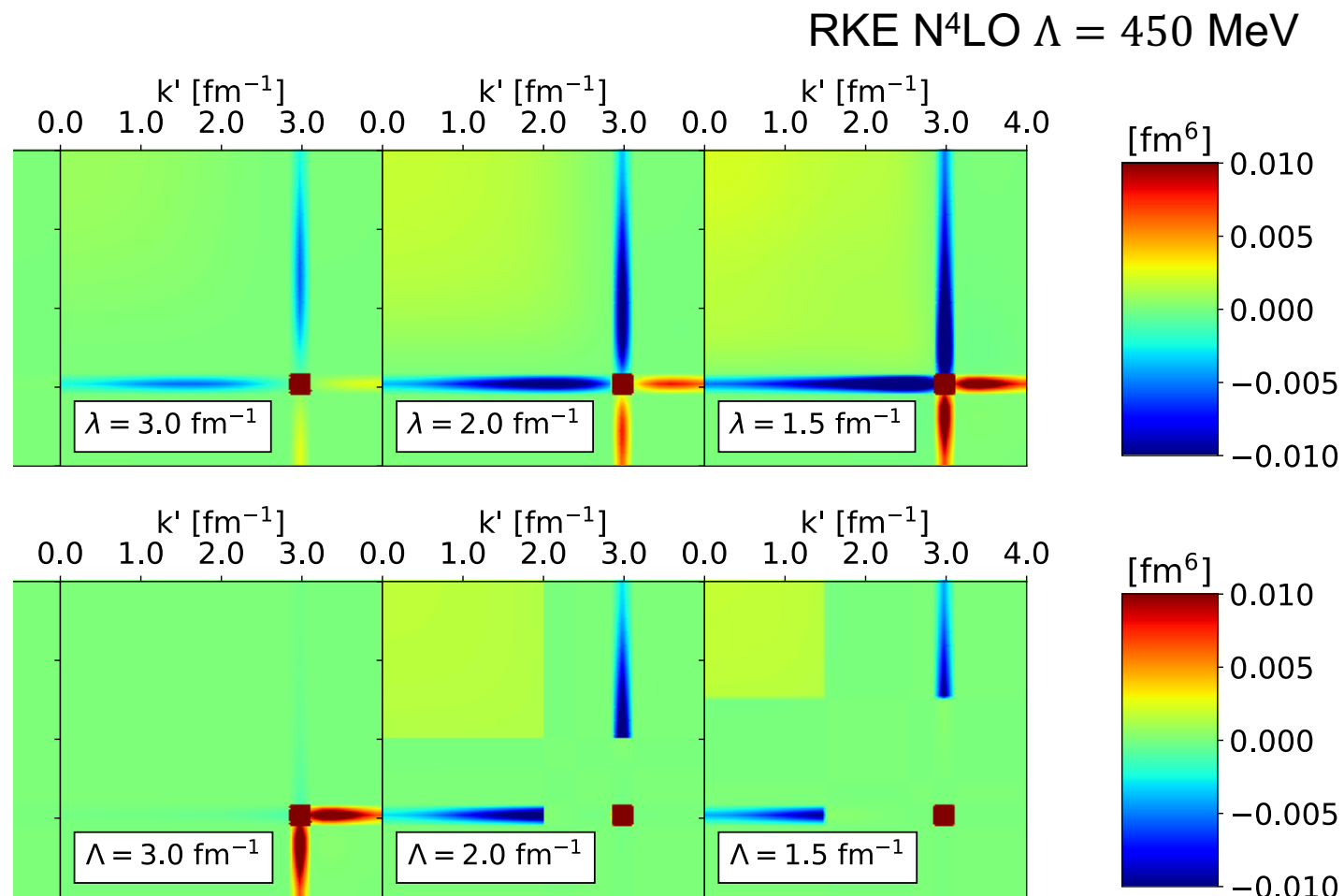
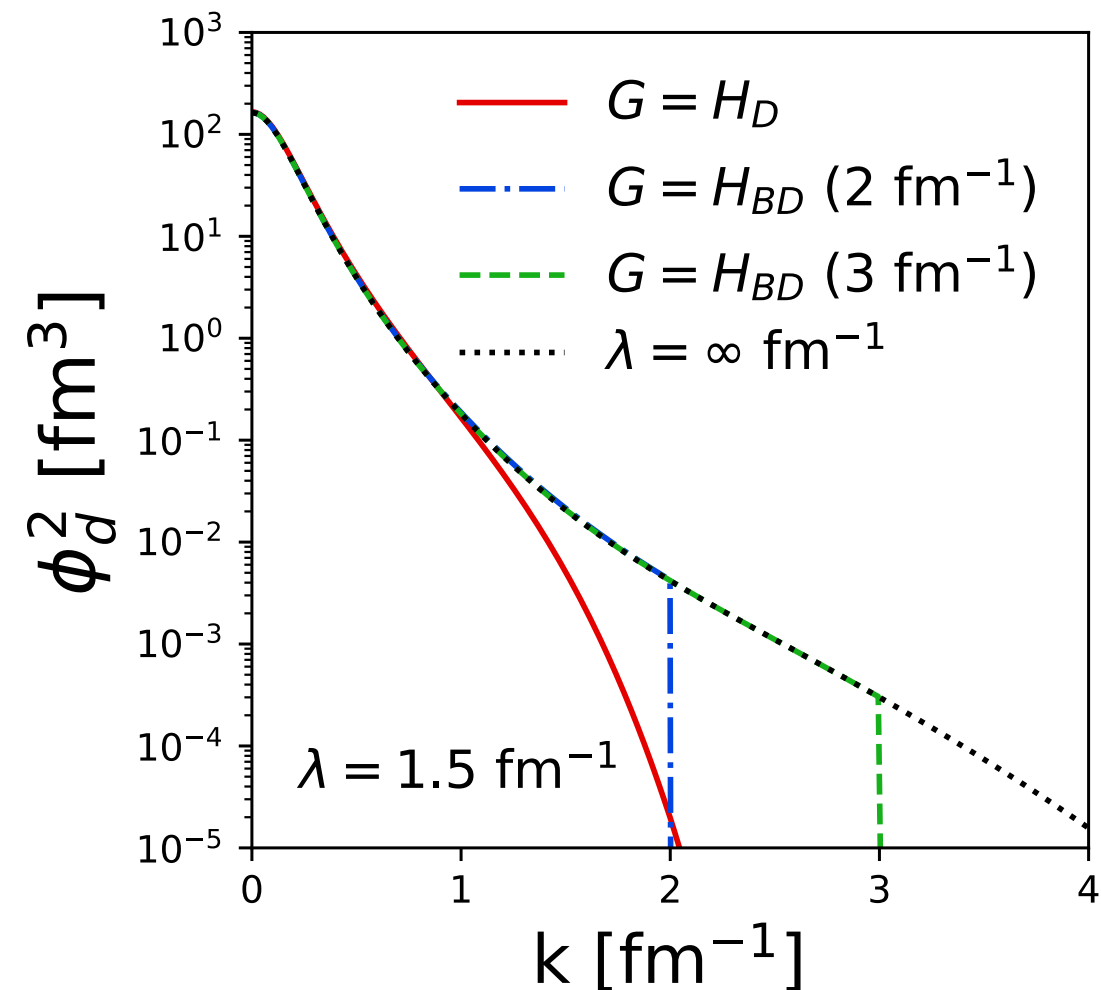


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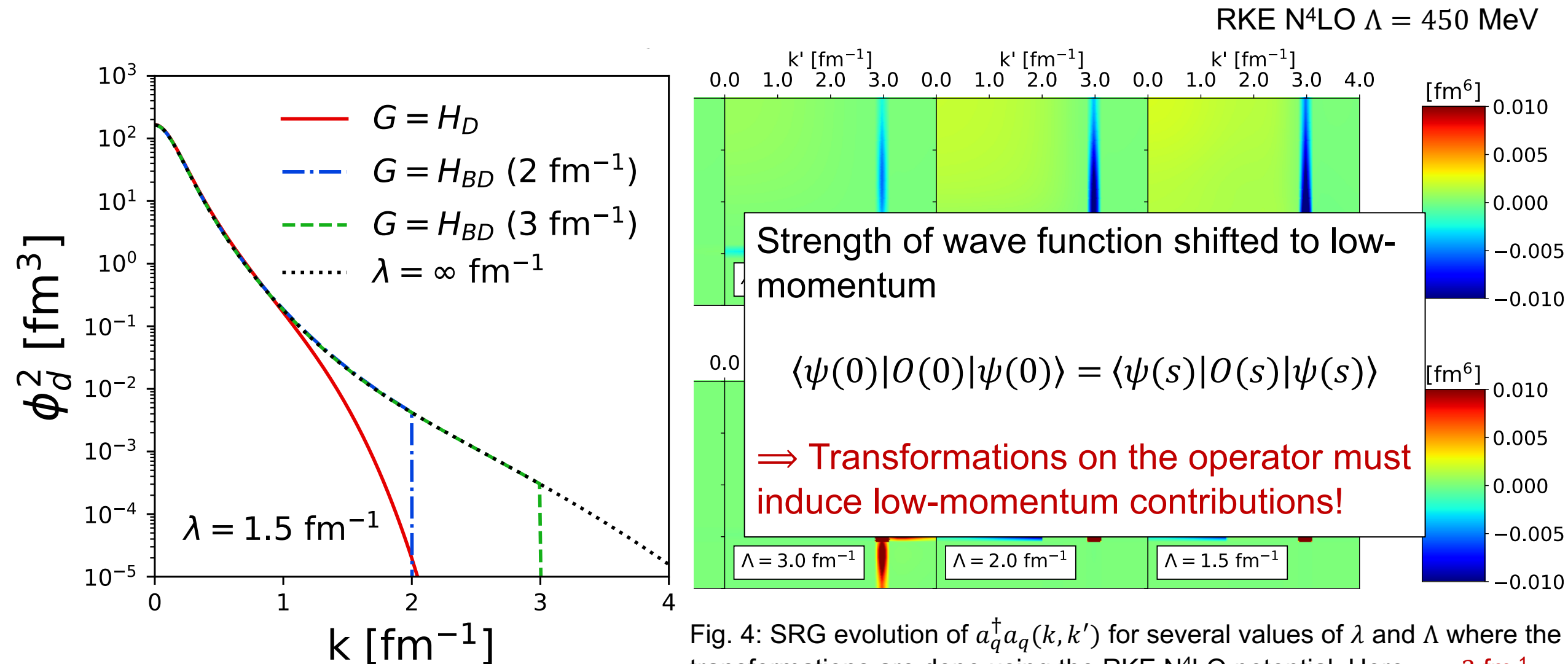


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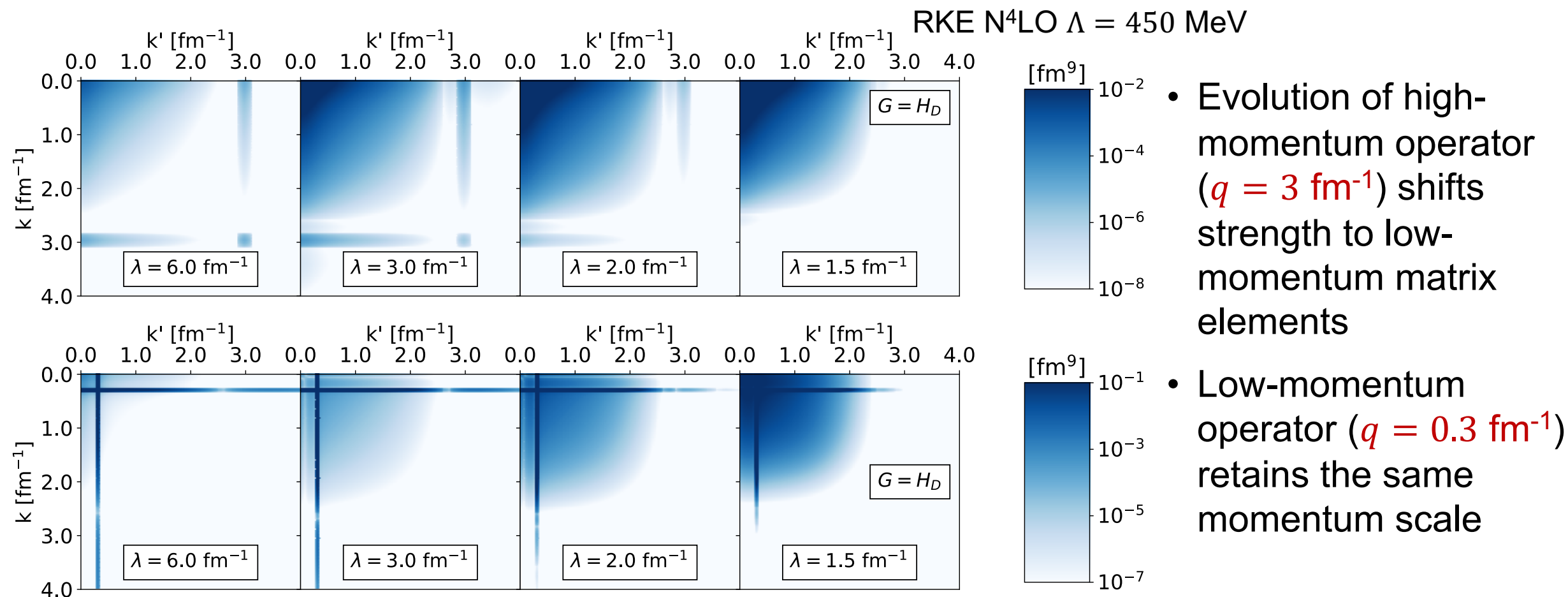
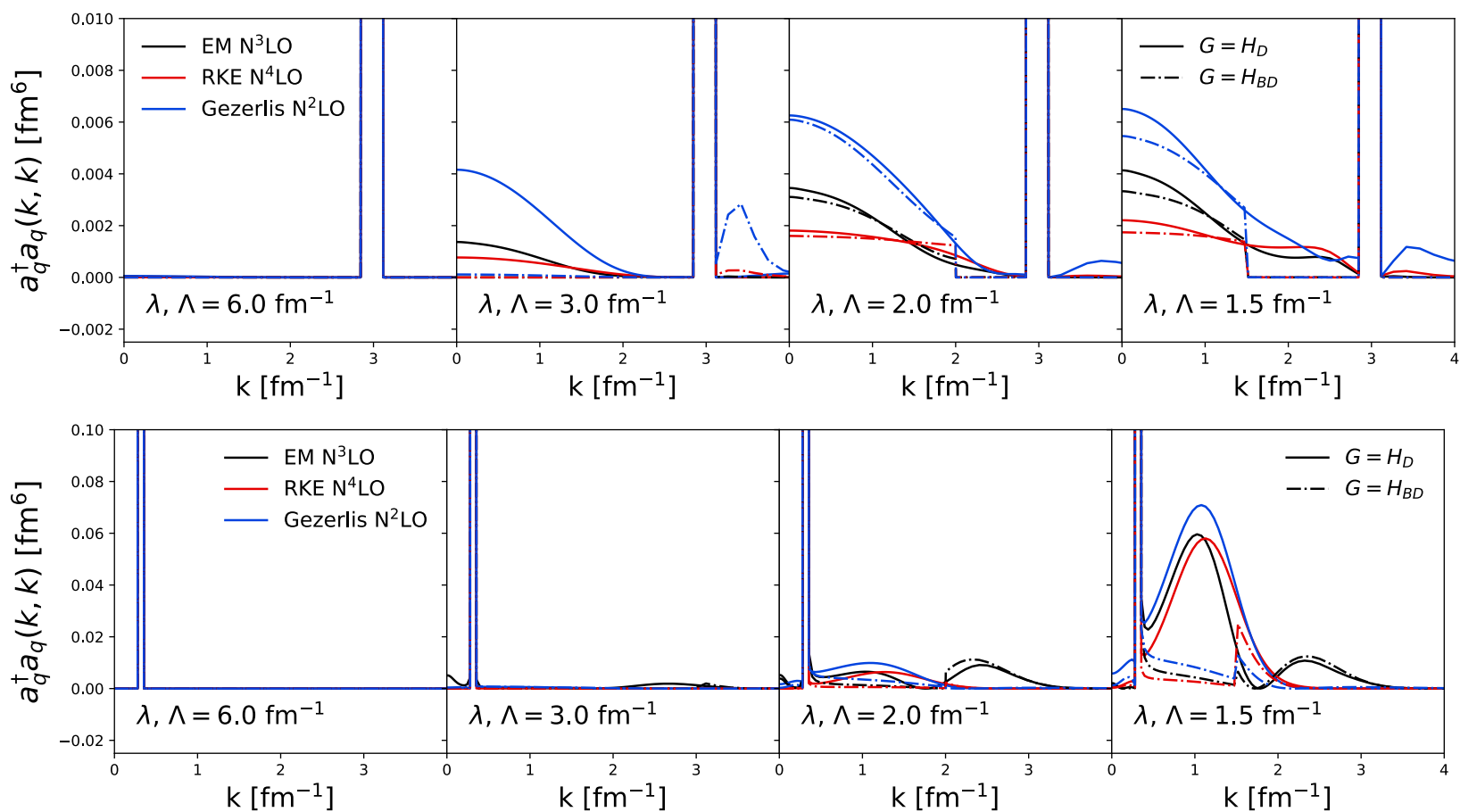


Fig. 6: Integrand of $\langle \psi_a | a_q^\dagger a_q | \psi_a \rangle$ in momentum-space for $q = 3$ (top) and 0.3 (bottom) fm^{-1} with SRG transformations for several values of λ where the transformations are done using the RKE N⁴LO potential.

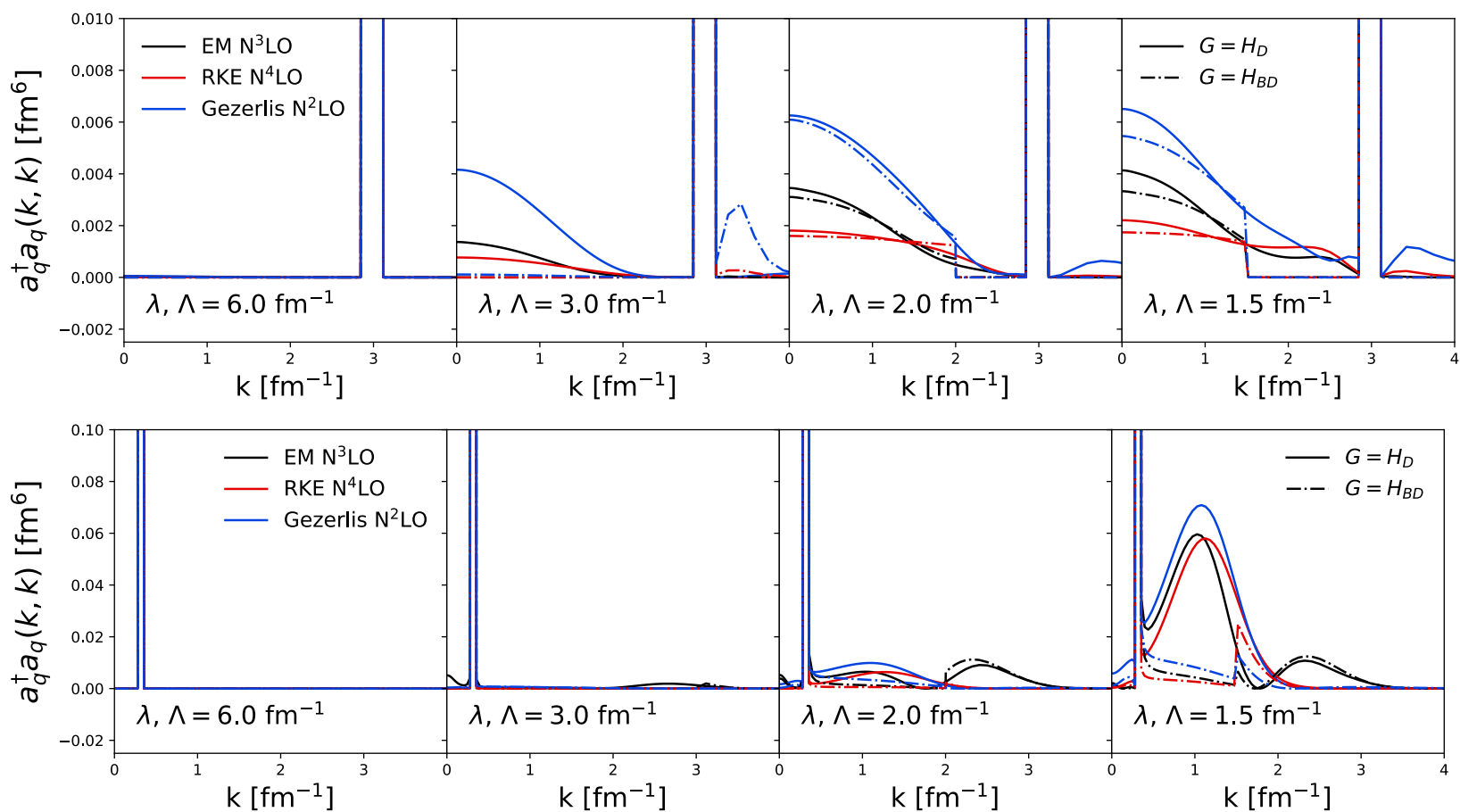
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- Matrix elements do not collapse for different cases

Fig. 5: Diagonal matrix elements of $a_q^\dagger a_q(k, k')$ for $q = 3$ (top) and 0.3 (bottom) fm^{-1} with SRG transformations from several chiral potentials with band- and block-diagonal decoupling.

Momentum projection operator



- Matrix elements do not collapse for different cases
- Initial $|\phi_d(q)|^2$ is not the same for various potentials

⇒ No universality in SRG-evolved $a_q^\dagger a_q(k, k')$ operator

Fig. 5: Diagonal matrix elements of $a_q^\dagger a_q(k, k')$ for $q = 3$ (top) and 0.3 (bottom) fm^{-1} with SRG transformations from several chiral potentials with band- and block-diagonal decoupling.

Summary

- Different SRG softened interactions collapse to universal form at low-energy if corresponding phase shifts are the same
- Universality depends on the pattern of SRG decoupling – the SRG generator (band- or block-diagonal)
- Other operators do not necessarily decouple like evolved potentials but shift strength to low-momentum to reflect changes in the evolved wave function

Outlook

- Further test SRG-evolution of other operators (\hat{r}^2 , \hat{Q} , etc.)

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- Recent interest in high cutoff potentials and spurious, deeply bound states¹
 - Spurious bound states can affect decoupling and universality (choice in $G = H_D$ or T_{rel} is important)²
 - How do other operators change with these potentials?

¹I. Tews et al., Phys. Rev. C **98**, 024001 (2018), ²K.A. Wendt et al., Phys. Rev. C **83**, 034005 (2011)

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- Further test SRG-evolution of other operators (\hat{r}^2 , \hat{Q} , etc.)
- Recent interest in high cutoff potentials and spurious, deeply bound states¹
 - Spurious bound states can affect decoupling and universality (choice in $G = H_D$ or T_{rel} is important)²
 - How do other operators change with these potentials?
- The Magnus expansion provides an improved approach to the SRG³
 - Do the same characteristics of operator evolution, universality, and generator dependence hold in this approach?
 - What does this imply for IMSRG calculations?

¹I. Tews et al., Phys. Rev. C **98**, 024001 (2018), ²K.A. Wendt et al., Phys. Rev. C **83**, 034005 (2011), ³T.D. Morris et al., Phys. Rev. C **92**, 034331 (2015)

Extras

