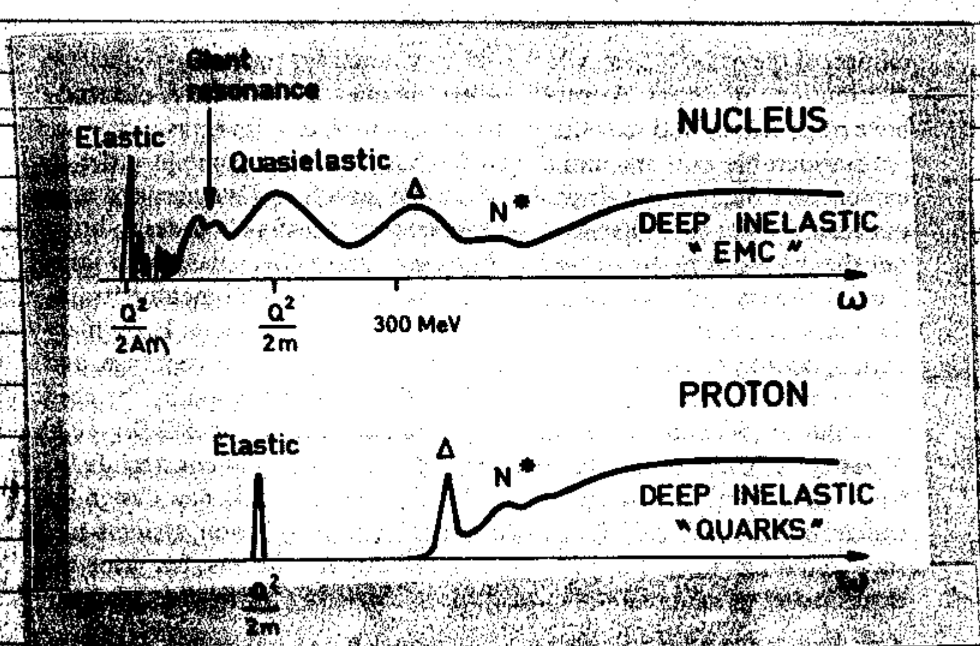
Loose ends on linear response...

- This is a large topic and we've only touched on some of the basics.
- Here we'll add a few notes to tie off some loose ends.

- The review article by Frois and Papanicolas on "Electron Scattering and Nuclear Structure" [Ann. Rev. Nucl. Part. Sci. 37 (1987) 133] has some useful pictures.

- First is the "big picture" of scattering from both a nucleus and a nucleon (a proton) as we hold the momentum transfer $\vec{q}$ fixed (it is $Q \equiv |\vec{q}|$ in the figure) and vary ω:



[Note: This is a schematic picture!]

So think of this as the absorption probability for a virtual photon at fixed wavelength as a function of the photon frequency (recall that we can vary them independently).

Let's look at the single nucleon response first.

- There is a sharp peak at $Q^2/2m$, with m the proton mass. This corresponds to the absorption of the photon without exciting the proton. It simply recoils, conserving energy and momentum, as in the discussion on 364
- Then there is no response until there is enough energy to reach excited states of the nucleon. If you look at the Particle Data Group compilations, you'll find the proton at 938 MeV, the neutron at 939.6 MeV (so we often take 939 MeV as the "nucleon mass").
  - a neutron decays to $p e^- \bar{\nu}_e$ via the weak interaction with a mean lifetime of 889 seconds
  - does a proton decay?
- After the proton and neutron, the next state is the $\Delta(1232)$, which comes in 4 charge states.
  - The quark content of the proton is uud; that of the neutron is udd.
  - The Δ quark content is:

    Δ^{++} – uuu Δ^{+} – uud Δ^{0} – udd Δ^{-} – ddd
  - The Δ is a strangeness 0, isospin 3/2 particle. It has spin-3/2 and positive parity.
  - It's decay width is about 115 MeV.
- The Δ shows up in the response as a well defined peak about 300 MeV above the elastic proton peak.
- The next state is the so-called "Roper resonance" at 1440 MeV (referred to as the N(1440). It has quantum numbers: $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$.

The nature of the Roper is still under debate. In some models it is a "breathing mode" of the nucleon — that is, it is just like a giant monopole resonance. It is being studied on the lattice at present, which should give more definitive answers in the near future.

- There are many other resonances identified, including more with the quantum numbers of the nucleons (often called N^* resonances), others like the Δ. A major experimental effort at Jefferson lab is to study these resonances and how they connect to quark model and lattice gauge theory precision.
    - At Jlab one has the ability to use polarized beams and targets and to do coincidence experiments.
    - See Jlab.org website for more details.

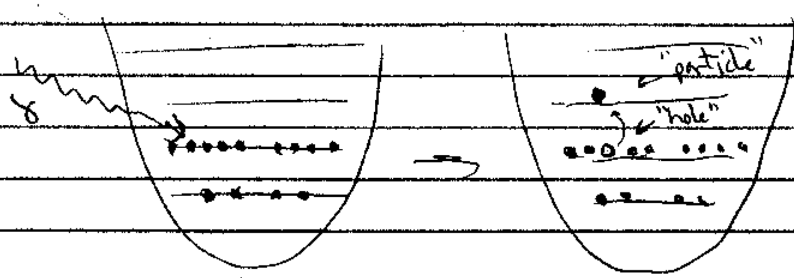
- There are also many baryons (3-quark particles) with at least one strange quark, starting with the Λ at 1116 MeV. When such particles are embedded in nuclei we have "hypernuclei" which are also studied at Jlab and elsewhere.

- As we turn the virtual photon frequency up further, we enter the analogous region to the nuclear quasi elastic region, where the scattering is from individual quarks rather that the nucleon as a whole.
    - This is referred to as the "deep inelastic" region.
    - For these kinematics, a description in terms of quark and gluon degrees of freedom is essential
    - Major efforts are underway at both particle and nuclear accelerators to measure the structure functions (which are analogous to the structure functions we've talked about) for quarks and gluons

Now return to the response function for the nucleus.

· The elastic peak is at the recoil energy for the nucleus as a whole to absorb the photon without getting excited $\Rightarrow$ all energy goes into kinetic energy. If there are A nucleons of mass m, this peak is at $Q^2/2Am$.

· Above the elastic peak (and much lower in reality) are sharp peaks from individual excitations, which are usually distinguished by whether they are "collective" or not. If we recall our shell model picture:



The simplest excitation would excite a particle to a higher shell model state (in a background potential due to the other nucleons) $\Rightarrow$ a "particle-hole" excitation. Angular momentum conservation means that we have to couple the angular momenta of the particle and whole appropriately!

- There will be both $T=0$ and $T=1$ states (isoscalar and isovector), depending on how proton and neutron states combine (add isospin $\frac{1}{2}+\frac{1}{2} \Rightarrow 0$ or 1, just like spin).
- If we consider oxygen, we can model it as filled $1s\frac{1}{2}$, $1p\frac{1}{2}$, and $1p\frac{3}{2}$ states. The low-lying particle-hole excitations will be from exciting $1p\frac{1}{2}$ or $1p\frac{3}{2}$ nucleons to the (unoccupied) $2s\frac{1}{2}$, $1d\frac{3}{2}$, or $1d\frac{5}{2}$ levels.
  - negative parity states (can you see why?)
  - total J can be from 0 to 4.

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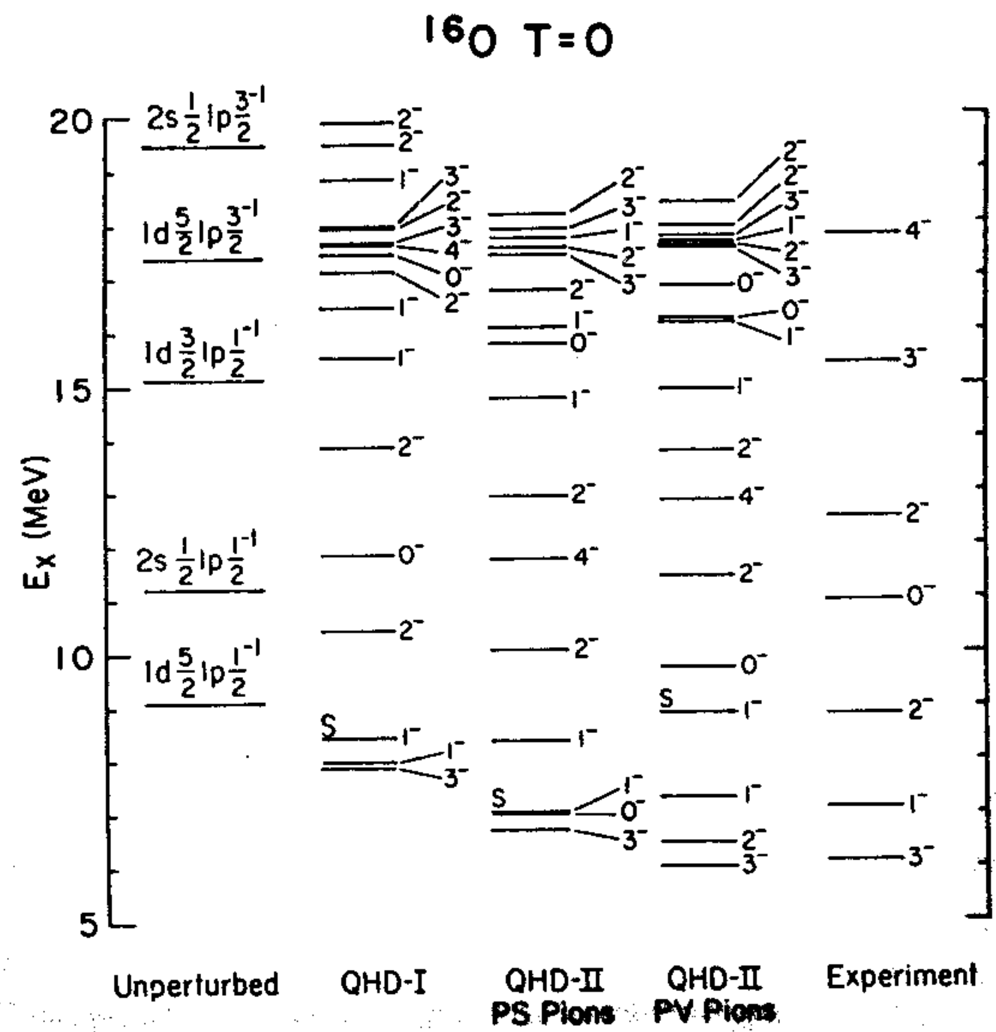

Here is a plot from my thesis of "unperturbed" particle hole states on the left, 3 calculations in the RPA (sum ring diagrams) and then the experimental negative parity spectrum.

· unperturbed would be the result from a single ring.

⇒ we see that the levels split and move up and down.

· S stands for "spurious" state. If the calculations were "correct" this would come out at zero energy (more later!).

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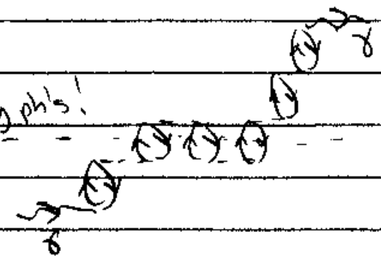
We see that it is essential to sum the rings.



If we have a Hartree-Fock ground state (direct plus exchange, or ○ - ○ + ○), then the consistent linear response is to sum rings and ph ladders ○ + ○

- If a state is mostly a particle-hole state, then most of the intermediate states are a single particle hole configuration, or the transition density, expanded in particle-hole unperturbed wave functions, is dominated by one (or a few) configurations.
  - e.g. in my calculations, the lowest 4^- state was mostly $1d^{5/2}\,(1p^{3/2})^{-1}$.

- A state is called collective if its expansion includes a large number of particle hole states, adding coherently with significant coefficients.
  - An example is the collective octupole vibration, which is the lowest 3^- state in the plot.
  - This is a common feature of the low-lying spectrum in all doubly magic nuclei.

$\Rightarrow$ see the transition charge densities on the next page.

The transition charge density in coordinate space is proportional to $\langle \Psi^N_{3^-} | \hat{\rho}(\vec{x}) | \Psi^N_0 \rangle$ (projected on to the correct quantum numbers while the form factor $|F_L(q)|^2$ is proportional to the square of the Fourier transform of this quantity.

(see picture from my thesis on 374.)

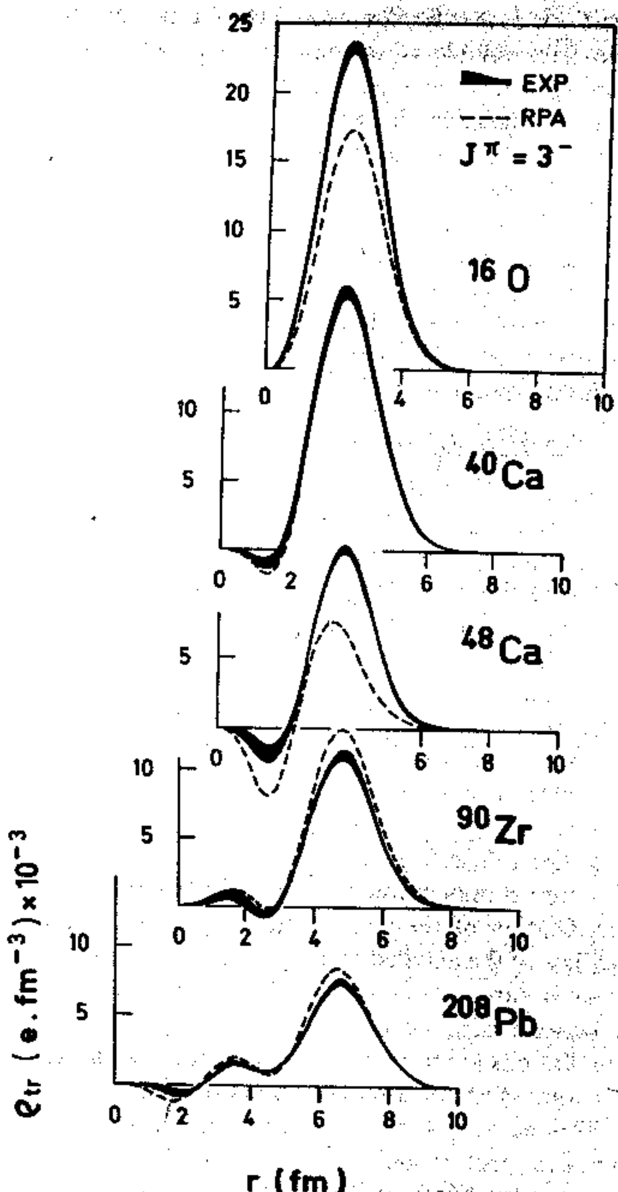


Figure 9 Transition charge densities for the first collective octupole vibrations of doubly closed-shell nuclei (56, 58, 60; K. Seth, private communication; J. Heisenberg, private communication). Same conventions as Figure 8. The theoretical predictions (53–55; Dechargé & Gogny, private communication) are obtained in a [illegible]

From The Frois, Papanicolas article.
(There are more recent calculations that do even better!)

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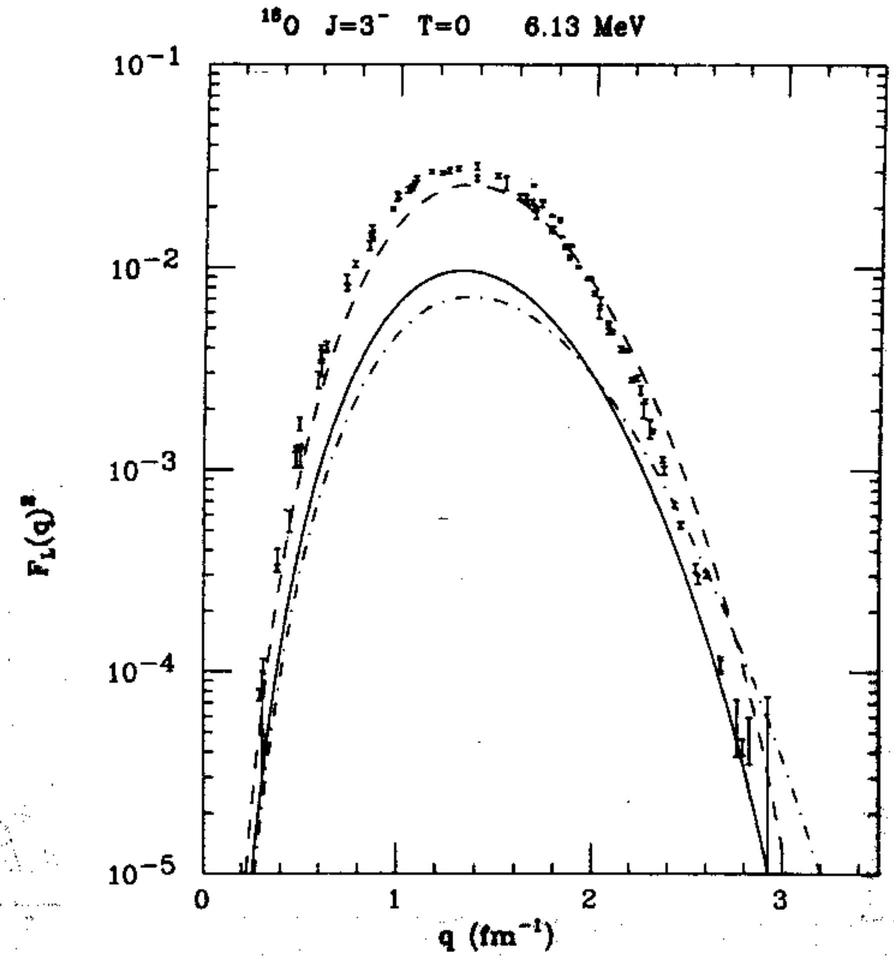

Longitudinal (charge) transition form factor for the 3^- state ($J=3^-$, $T=0$) at 6.13 MeV in ^{16}O.

- The data points were measured at the Bates accelerator (run by MIT)
- The calculations were from my thesis.
- Much better calculations exist today in more modern versions of the models I used.

6/2/03

Follow-up on quasielastic scattering...

If we take into account that virtual photons probe both the charge and current distributions in nuclei, we find two response functions we could measure:

$$\frac{\partial^2\sigma}{\partial\Omega\, dE'} = \sigma_m\left[\frac{|q_\mu^4|}{q^4} R_L(q,w) + \left(-\frac{1}{2}\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2}\right) R_T(q,w)\right]$$

longitudinal transverse

$$Q^2 = -q_\mu^2 = -4EE'\sin^2\frac{\theta}{2} \qquad w = E - E'$$

· Here are results from $^{40}Ca(e,e')$.

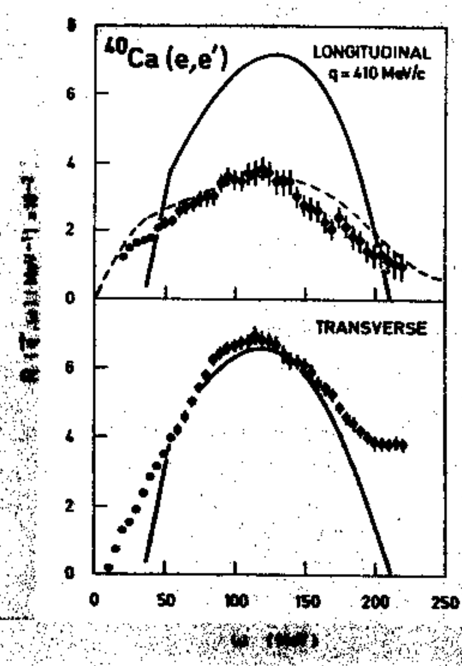

Figure 15 ^{40}Ca longitudinal and transverse response functions at $q = 410$ MeV/c (81). The solid curve is the Fermi gas prediction. The dashed curve is the result of a calculation for nuclear matter that treats correlations (97).

Whoops!