AJT notes (4/1/21)

* In Weiss (2015), Eqn. (33-34):

they do not distinguish np

$$n_p(\vec{k}) \xrightarrow[k\to\infty]{} 2F_{pp}(\vec{k}) + F_{pn}(\vec{k}) \quad (1)$$

where $\int \frac{d^3k}{(2\pi)^3} n_p(\vec{k}) = Z$

$\int \frac{d^3k}{(2\pi)^3} F_{ij}(\vec{k}) = N$ ← number of pairs

- Let's verify (1) in our way taking only $^1S_0$ and $^3S_1$ contributions from the high-q $\delta U \delta U^\dagger$ term in single-nucleon and pair momentum distributions.

- We will start from final equations in "Single-nucleon momentum distribution (full derivation)" and "Pair momentum distribution (full derivation)".

- Single-nucleon $\delta U \delta U^\dagger$:

$$n_\lambda^p(q) = \frac{1}{2}\frac{1}{(2\pi)^6}\left(\frac{2}{\pi}\right)^2 \int_0^\infty dk\, k^2 \int_0^\infty dK\, K^2 \int_{-1}^{1} \frac{dx}{2} \times$$

$$\Big[\delta U^2_{^1S_0}(k, |\vec{q}-\tfrac{1}{2}\vec{K}|)\, \Theta_p^+ \Theta_p^- + \Big(\tfrac{1}{4}\, \delta U^2_{^1S_0}(k, |\vec{q}-\tfrac{1}{2}\vec{K}|) +$$

2

$$\frac{3}{4}\,\delta U^2_{^3S_1-^3D_1}\left(k,|\vec{q}-\tfrac{1}{2}\vec{K}|\right)\Big)\left(\Theta_p^+\Theta_n^- + \Theta_n^+\Theta_p^-\right)\Big] \tag{2}$$

where $\Theta_N^\pm \equiv \Theta\left(k_F^N - |\tfrac{1}{2}\vec{K}\pm\vec{k}|\right)$ and

$$|\tfrac{1}{2}\vec{K}\pm\vec{k}| = \sqrt{\frac{K^2}{4}+k^2\pm Kkx}$$

Average over x

- Pair $\delta U \delta U^\dagger$

$$\Lambda_\lambda^{pp}(q,\vec{Q}) = \frac{1}{4}\frac{1}{(2\pi)^3}\left(\frac{2}{\pi}\right)^2\frac{1}{4\pi}\int_0^\infty dk\,k^2 \times$$

$$\left[\delta U^2_{^1S_0}(k,q)\,\Theta_p^+\Theta_p^-\right] \tag{3}$$

$$\Lambda_\lambda^{pn}(q,\vec{Q}) = \frac{1}{4}\frac{1}{(2\pi)^3}\left(\frac{2}{\pi}\right)^2\frac{1}{4\pi}\int_0^\infty dk\,k^2 \times \Big[\frac{1}{4}\delta U^2_{^1S_0}(k,q)$$

$$+\frac{3}{4}\delta U^2_{^3S_1-^3D_1}(k,q)\Big]\left[\Theta_p^+\Theta_n + \Theta_n^+\Theta_p\right]$$

$$\Lambda_\lambda^{np}(q,\vec{Q}) = \Lambda_\lambda^{pn}(q,\vec{Q}) \tag{4}$$

- Average over x as in (2) and integrate at $\vec{Q}$:

$$\rightarrow \int\frac{d^3Q}{(2\pi)^3} = \frac{4\pi}{(2\pi)^3}\int_0^\infty dQ\,Q^2$$

$$\Rightarrow \quad \Lambda_\lambda^{pp}(q) = \frac{1}{4}\frac{1}{(2\pi)^6}\left(\frac{2}{\pi}\right)^2 \int_0^\infty dk\, k^2 \int_0^\infty dQ\, Q^2 \times \int_{-1}^{1}\frac{dx}{2}\left[\delta U^2_{^1S_0}(k,q)\, G_p^+ G_p^-\right] \tag{5}$$

$$\Lambda_\lambda^{pn}(q) = \frac{1}{4}\frac{1}{(2\pi)^6}\left(\frac{2}{\pi}\right)^2 \int_0^\infty dk\, k^2 \int_0^\infty dQ\, Q^2 \int_{-1}^{1}\frac{dx}{2} \times \left[\frac{1}{4}\delta U^2_{^1S_0}(k,q) + \frac{3}{4}\delta U^2_{^3S_1-^3D_1}(k,q)\right]\left[G_p^+ G_n^- + G_n^+ G_p^-\right] \tag{6}$$

Overall factors in front of integrals only differ by $\frac{1}{2} \to \frac{1}{4}$.

Then we see that if we assume

$\delta U(k, |\vec{q} - \frac{1}{2}\vec{K}|) \approx \delta U(k, q)$ for large q then

$$\Lambda_\lambda^{p}(q) \xrightarrow[q\to\infty]{} 2\Lambda_\lambda^{pp}(q) + \underbrace{\Lambda_\lambda^{pn}(q) + \Lambda_\lambda^{np}(q)}_{\text{Weiss calls this pn}} \tag{7}$$

Same holds for $\Lambda_\lambda^{n}(q)$.