(3/12/21)

- Starting point is $\hat{n}_\lambda^\tau(\vec{q})$ after evaluation of contractions. ($\vec{q}$ is not a relative momentum here!)

$$\hat{n}_\lambda^\tau(\vec{q}) = V\Big\{\underbrace{\sum_\sigma \Theta(k_F^\tau - q)}_{\text{Term 1}}$$

$$+ \sum_{\sigma\sigma'\tau'}\int\frac{d^3k}{(2\pi)^3}\Big[\underbrace{\langle\vec{k}\sigma\tau\sigma'\tau'|\delta U|\vec{k}\sigma\tau\sigma'\tau'\rangle}_{\text{Term 2}}$$

$$+ \underbrace{\langle\vec{k}\sigma\tau\sigma'\tau'|\delta U^\dagger|\vec{k}\sigma\tau\sigma'\tau'\rangle}_{\text{Term 3}}\Big]\Theta(k_F^\tau - q)\,\Theta(k_F^{\tau'} - |\vec{q}-2\vec{k}|)$$

$$+ \frac{1}{2}\sum_{\sigma\sigma'\sigma''\sigma'''}\sum_{\tau'\tau''\tau'''}\int\frac{d^3k}{(2\pi)^3}\frac{d^3K}{(2\pi)^3}\langle\vec{k}\sigma'\tau'\sigma''\tau''|\delta U|\vec{q}-\tfrac{1}{2}\vec{K}\,\sigma\tau\sigma'''\tau'''\rangle\times$$

$$\langle\vec{q}-\tfrac{1}{2}\vec{K}\,\sigma\tau\sigma'''\tau'''|\delta U^\dagger|\vec{k}\sigma'\tau'\sigma''\tau''\rangle\,\Theta(k_F^{\tau'} - |\tfrac{1}{2}\vec{K}+\vec{k}|)\,\Theta(k_F^{\tau''} - |\tfrac{1}{2}\vec{K}-\vec{k}|)\Big\} \quad (1)$$

Term 1 is simple. Contribution is $2\,\Theta(k_F^\tau - q)$.

Do 2nd + 3rd term (multiply 2nd term by 2 for full 2+3 contribution).

$$2\sum_{\sigma\sigma'\tau'}\int\frac{d^3k}{(2\pi)^3}\langle\vec{k}\sigma\tau\sigma'\tau'|\delta U|\vec{k}\sigma\tau\sigma'\tau'\rangle\,\Theta(k_F^\tau - q)\,\Theta(k_F^{\tau'} - |\vec{q}-2\vec{k}|)$$

$$= 2\sum_{\sigma\sigma'\tau'}\sum_{SM_S}\sum_{S'M_S'}\sum_{TM_T}\sum_{TM_T'}\sum_{LM_L}\sum_{L'M_L'}\sum_{JM_J}\sum_{JM_J'}\int_0^\infty dk\,k^2\,d\Omega_{\hat{k}}$$

$$\langle \sigma\sigma' | S M_S \rangle \langle \tau\tau' | T M_T \rangle \langle \hat{k} | k L M_L \rangle \langle M_S M_L | J M_J \rangle \times$$
$$\langle k J L S T | \delta\tilde{U} | k J' L' S' T' \rangle \langle J' M_J' | M_S' M_L' \rangle \langle k L' M_L' | \hat{k} \rangle \times$$
$$\langle T' M_T' | \tau\tau' \rangle \langle S' M_S' | \sigma\sigma' \rangle \, \Theta(k_F^{\tau} - q) \, \Theta(k_F^{\tau'} - |\vec{q} - 2\vec{k}|)$$

Apply $\sum_{\sigma\sigma'} \langle \sigma\sigma' | S M_S \rangle \langle S' M_S' | \sigma\sigma' \rangle = \delta_{S,S'} \delta_{M_S, M_S'}$ $\qquad \cancel{\sum_{S' M_S'}}$

Apply $\int d\Omega_{\hat{k}} \langle \hat{k} | k L M_L \rangle \langle k L' M_L' | \hat{k} \rangle = \frac{2}{\pi} \delta_{L,L'} \delta_{M_L M_L'}$ $\qquad \cancel{\sum_{L' M_L'}}$

Apply $\sum_{M_S M_L} \langle M_S M_L | J M_J \rangle \langle J' M_J' | M_S M_L \rangle = \delta_{JJ'} \delta_{M_J M_J'}$ $\qquad \cancel{\sum_{J' M_J}}$

$$= 2 \, \frac{2}{\pi} \sum_{\tau'} \sum_{S} \sum_{T M_T} \sum_{T' M_T'} \sum_{L} \sum_{J M_J} \int_0^\infty dk \, k^2 \langle \tau\tau' | T M_T \rangle \times$$
$$\langle k J L S T | \delta\tilde{U} | k J L S T' \rangle \langle T' M_T' | \tau\tau' \rangle \Theta(k_F^{\tau} - q) \Theta(k_F^{\tau'} - |\vec{q} - 2\vec{k}|)$$

Θ depends on $|\vec{q} - 2\vec{k}| = \sqrt{q^2 + 4k^2 - 4\vec{q}\cdot\vec{k}}$

Average over angles $\int_{-1}^{1} d(\hat{q}\cdot\hat{k}) \, [\ldots] \, / \, 2$

$$= \frac{2}{\pi} (2J+1) \underbrace{\sum_{\tau'} \sum_{T M_T} \overset{\text{diagonal!}}{\cancel{\sum_{T' M_T'}}} \sum_{S L J}} \int_0^\infty dk \, k^2 \int_{-1}^{1} d(\hat{q}\cdot\hat{k}) \times$$
$$\langle \tau\tau' | T M_T \rangle \langle k J L S T | \delta\tilde{U} | k J L S T \rangle \langle T M_T | \tau\tau' \rangle \times$$
$$\Theta(k_F^{\tau} - q) \, \Theta(k_F^{\tau'} - |\vec{q} - 2\vec{k}|) \qquad (2)$$

$\tau = +\frac{1}{2}$

$\tau' = +\frac{1}{2} \rightarrow T = 1 \quad M_T = 1 \quad CG's = 1$

$\Rightarrow {}^1S_0 \rightarrow S = 0, L = 0, J = 0$

$\tau' = -\frac{1}{2} \rightarrow T = 1 \quad M_T = 0 \quad CG's = \frac{1}{\sqrt{2}} \qquad \langle \tfrac{1}{2} \, {-\tfrac{1}{2}} | 1 0 \rangle = \frac{1}{\sqrt{2}}$

$\Rightarrow {}^1S_0 \rightarrow S = 0 \quad L = 0 \quad J = 0$

$\rightarrow T = 0 \quad M_T = 0 \quad CG's = \frac{1}{\sqrt{2}}$

$\Rightarrow {}^3S_1 \rightarrow S = 1 \quad L = 0 \quad J = 1$

Do we want $L > 0$?

$$\hat{n}_\lambda^p(q) \approx \frac{3}{\pi} \int_0^\infty dk\, k^2 \int_{-1}^{1} d(\hat{q}\cdot\hat{k}) \Big\{ \delta U_{{}^1S_0}(k,k) \Big[\theta\big(k_F^p - |\vec{q} - 2\vec{k}|\big) + \frac{1}{2}\theta\big(k_F^n - |\vec{q} - 2\vec{k}|\big) \Big] + \frac{3}{2}\delta U_{{}^3S_1 - {}^3S_1}(k,k)\, \theta\big(k_F^n - |\vec{q} - 2\vec{k}|\big) \Big\} \times \theta\big(k_F^p - q\big) \tag{3}$$

(S-waves only)

Likewise

$$\hat{n}_\lambda^n(q) \approx \frac{3}{\pi} \int_0^\infty dk\, k^2 \int_{-1}^{1} d(\hat{q}\cdot\hat{k}) \Big\{ \delta U_{{}^1S_0}(k,k) \Big[\theta\big(k_F^n - |\vec{q} - 2\vec{k}|\big) + \frac{1}{2}\theta\big(k_F^p - |\vec{q} - 2\vec{k}|\big) \Big] + \frac{3}{2}\delta U_{{}^3S_1 - {}^3S_1}(k,k)\, \theta\big(k_F^p - |\vec{q} - 2\vec{k}|\big) \Big\} \times \theta\big(k_F^n - q\big) \tag{4}$$

To do LDA $\quad 4\pi \int_0^\infty dr\, r^2\, \hat{n}_\lambda^p(q; k_F^p, k_F^n) \,/\, Z$

$\hookrightarrow$ Determined by ρ functions!

Term 4:

$$\frac{1}{2}\sum_{\sigma\sigma'\sigma''\sigma'''}\sum_{\tau\tau'\tau''\tau'''}\int\frac{d^3k}{(2\pi)^3}\frac{d^3K}{(2\pi)^3}\langle\vec{k}\,\sigma'\tau'\sigma''\tau''|\delta\tilde{U}|\vec{q}-\tfrac{1}{2}\vec{K}\,\sigma\tau\sigma'''\tau'''\rangle\times$$

$$\langle\vec{q}-\tfrac{1}{2}\vec{K}\,\sigma\tau\sigma'''\tau'''|\delta\tilde{U}^{\dagger}|\vec{k}\,\sigma'\tau'\sigma''\tau''\rangle\,\Theta(k_F^{\tau'}-|\tfrac{1}{2}\vec{K}+\vec{k}|)\times$$
$$\Theta(k_F^{\tau''}-|\tfrac{1}{2}\vec{K}-\vec{k}|)$$

$$=\frac{1}{2}\sum_{\sigma\sigma'\sigma''\sigma'''}\sum_{\tau\tau'\tau''\tau'''}\sum_{SM_S\cdots S'''M_S'''}\sum_{TM_T\cdots T'''M_T'''}\sum_{LM_L\cdots L'''M_L'''}\sum_{JM_J\cdots J'''M_J'''}\times$$

$$\int_0^{\infty}dk\,k^2\int_0^{\infty}dK\,K^2\int d\Omega_{\hat{k}}\int d\Omega_{\vec{K}}\,\langle\sigma'\sigma''|SM_S\rangle\langle\tau'\tau''|TM_T\rangle\times$$
$$\langle\hat{k}|k\,L\,M_L\rangle\langle M_S M_L|J M_J\rangle\langle k\,J\,L\,S\,T|\delta\tilde{U}||\vec{q}-\tfrac{1}{2}\vec{K}|\,J'L'S'T'\rangle\times$$
$$\langle J'M_J'|M_S'M_L'\rangle\langle|\vec{q}-\tfrac{1}{2}\vec{K}|\,L'M_L'|\vec{q}-\tfrac{1}{2}\vec{K}\rangle\langle T'M_T'|\tau\tau'''\rangle\langle S'M_S'|\sigma\sigma'''\rangle\times$$
$$\langle\sigma\sigma'''|S''M_S''\rangle\langle\tau\tau'''|T''M_T''\rangle\langle\vec{q}-\tfrac{1}{2}\vec{K}\,||\vec{q}-\tfrac{1}{2}\vec{K}|\,L''M_L''\rangle\langle M_S''M_L''|J''M_J''\rangle\times$$
$$\langle|\vec{q}-\tfrac{1}{2}\vec{K}|\,J''L''S''T''|\delta\tilde{U}^{\dagger}|\vec{k}\,J'''L'''S'''T'''\rangle\langle J'''M_J'''|M_S'''M_L'''\rangle\times$$
$$\langle k\,L'''M_L'''|\hat{k}\rangle\langle T'''M_T'''|\tau'\tau''\rangle\langle S'''M_S'''|\sigma'\sigma''\rangle\,\Theta(k_F^{\tau'}-|\tfrac{1}{2}\vec{K}+\vec{k}|)\times$$
$$\Theta(k_F^{\tau''}-|\tfrac{1}{2}\vec{K}-\vec{k}|)\qquad(5)$$

- Apply $\displaystyle\sum_{\sigma'\sigma''}\langle\sigma'\sigma''|SM_S\rangle\langle S'''M_S'''|\sigma'\sigma''\rangle=\delta_{SS'''}\delta_{M_SM_S'''}\qquad\cancel{\sum_{S'''M_S'''}}$

$$\sum_{\sigma\sigma'''}\langle\sigma\sigma'''|S'M_S'\rangle\langle S''M_S''|\sigma\sigma'''\rangle=\delta_{S'S''}\delta_{M_S'M_S''}\qquad\cancel{\sum_{S''M_S''}}$$

- Apply $\displaystyle\int d\Omega_{\hat{k}}\,\langle\hat{k}|k\,L\,M_L\rangle\langle k\,L'''M_L'''|\hat{k}\rangle=\frac{2}{\pi}\delta_{LL'''}\delta_{M_LM_L'''}$

$$\cancel{\sum_{L'''M_L'''}}$$

- Approximate $|\vec{q}-\frac{1}{2}\vec{K}| \approx q^2+\frac{K^2}{4}$

- Then $\int \frac{d\Omega_{\hat{q}\cdot\hat{K}}}{4\pi} \langle |\vec{q}-\frac{1}{2}\vec{K}|\, L'M_L' | \vec{q}-\frac{1}{2}\vec{K} \rangle \langle \vec{q}-\frac{1}{2}\vec{K} | |\vec{q}-\frac{1}{2}\vec{K}|\, L''M_L'' \rangle$

$$= \frac{1}{4\pi}\frac{2}{\pi}\delta_{L'L''}\delta_{M_L'M_L''} \qquad \cancel{\sum_{L''M_L''}}$$

- Apply $\sum_{M_SM_L} \langle M_SM_L | JM_J \rangle \langle J'''M_J''' | M_SM_L \rangle = \delta_{JJ'''}\delta_{M_JM_J'''} \qquad \cancel{\sum_{J'''M_J'''}}$

$$\sum_{M_S'M_L'} \langle J'M_J' | M_S'M_L' \rangle \langle M_S'M_L' | J''M_J'' \rangle = \delta_{J'J''}\delta_{M_J'M_J''} \qquad \cancel{\sum_{J''M_J''}}$$

$$= \frac{1}{2}\frac{1}{4\pi}\left(\frac{2}{\pi}\right)^2 \sum_{\tau\tau''\tau'} \sum_{TM_T\ldots T'''M_T'''} \sum_{SS'} \sum_{LL'} \sum_{JM_J} \sum_{J'M_J'} \int_0^\infty dk\,k^2 \int_0^\infty dK\,K^2 \int d\Omega_{\vec{K}}$$

$$\langle \tau'\tau'' | TM_T \rangle \langle k\,JLST | \delta\tilde{U} | |\vec{q}-\tfrac{1}{2}\vec{K}|\, J'L'S'T' \rangle \langle T'M_T' | \tau\tau''' \rangle$$
$$\times \langle \tau\tau''' | T''M_T'' \rangle \langle |\vec{q}-\tfrac{1}{2}\vec{K}|\, J'L'S'T'' | \delta\tilde{U}^\dagger | k\,JLST''' \rangle \langle T'''M_T''' | \tau'\tau'' \rangle$$
$$\times\ \theta(k_F^{\tau'} - |\tfrac{1}{2}\vec{K}+\vec{k}|)\ \theta(k_F^{\tau''} - |\tfrac{1}{2}\vec{K}-\vec{k}|)$$

- Now apply $\delta\tilde{U}$, $\delta\tilde{U}^\dagger$ being diagonal in $J\,M_J\,S\,M_S\,T\,M_T$

- Average $\int_{-1}^{1} \frac{d(\hat{K}\cdot\hat{k})}{2}$

- $\int d\Omega_{\vec{K}} = 4\pi$ $\qquad$ - $\sum_{M_J} \rightarrow (2J+1)$

$$= \frac{1}{2}\frac{1}{\cancel{4\pi}}\cancel{4\pi}\left(\frac{2}{\pi}\right)^2(2J+1)\sum_{\tau'\tau''\tau'''}\sum_{TM_TT'M_T'}\sum_{S}\sum_{LL'}\sum_{J} \times$$

$$\int_0^\infty dk\,k^2\int_0^\infty dK\,K^2\int_{-1}^{1}\frac{d(\hat{K}\cdot\hat{k})}{2}\langle\tau\tau''|TM_T\rangle \times$$

$$\langle kJLST|\delta U||\vec{q}-\tfrac{1}{2}\vec{K}|JL'ST\rangle\langle TM_T|\tau\tau'''\rangle\langle\tau\tau'''|T'M_T'\rangle \times$$

$$\langle|\vec{q}-\tfrac{1}{2}\vec{K}|JL'ST'|\delta U^\dagger|kJLST'\rangle\langle T'M_T'|\tau'\tau''\rangle \times$$

$$\theta\left(k_F^{\tau'}-|\tfrac{1}{2}\vec{K}+\vec{k}|\right)\theta\left(k_F^{\tau''}-|\tfrac{1}{2}\vec{K}-\vec{k}|\right) \qquad (6)$$

Step through cases:

$S=0 \Rightarrow T=1$ and $T'=1$
$S=1 \Rightarrow T=0$ and $T'=0$

So no point in summing over $T'M_T'$

Fix $\tau = +\frac{1}{2}$

$\tau''' = +\frac{1}{2} \Rightarrow T=1 \quad M_T=1 \Rightarrow {}^1S_0,\ CG's = 1$

$\rightarrow \tau' = +\frac{1}{2} \quad \tau'' = +\frac{1}{2} \qquad CG's = 1$

$\tau''' = -\frac{1}{2} \Rightarrow T=1 \quad M_T = 0 \Rightarrow {}^1S_0,\ CG's = \frac{1}{\sqrt{2}}$

$\tau' = \pm\frac{1}{2} \quad \tau'' = \mp\frac{1}{2} \Rightarrow CG's = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow T=0 \quad M_T=0 \Rightarrow {}^3S_1,\ CG's = \frac{1}{\sqrt{2}}$

$\tau' = \pm\frac{1}{2} \quad \tau'' = \mp\frac{1}{2} \Rightarrow CG's = \frac{1}{\sqrt{2}}$

Do schematic of terms for $\tau = +\frac{1}{2} \qquad L=0$

$$\langle +\tfrac{1}{2}\, +\tfrac{1}{2} \,|\, 1\, 1 \rangle\, \delta\tilde{U}_{^1S_0}\, \langle 1\, 1 \,|\, +\tfrac{1}{2}\, +\tfrac{1}{2} \rangle \langle +\tfrac{1}{2}\, +\tfrac{1}{2} \,|\, 1\, 1 \rangle\, \delta\tilde{U}^{\dagger}_{^1S_0}\, \langle 1\, 1 \,|\, +\tfrac{1}{2}\, +\tfrac{1}{2} \rangle$$

$$+ \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 1\, 0 \rangle\, \delta\tilde{U}_{^1S_0}\, \langle 1\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 1\, 0 \rangle\, \delta\tilde{U}^{\dagger}_{^1S_0}\, \langle 1\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle$$

$$+ \langle -\tfrac{1}{2}\, +\tfrac{1}{2} \,|\, 1\, 0 \rangle\, \delta\tilde{U}_{^1S_0}\, \langle 1\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 1\, 0 \rangle\, \delta\tilde{U}^{\dagger}_{^1S_0}\, \langle 1\, 0 \,|\, -\tfrac{1}{2}\, +\tfrac{1}{2} \rangle$$

$$+ \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 0\, 0 \rangle\, \delta\tilde{U}_{^3S_1-^3S_1}\, \langle 0\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 0\, 0 \rangle\, \delta\tilde{U}^{\dagger}_{^3S_1-^3S_1}\, \langle 0\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle$$

$$+ \langle -\tfrac{1}{2}\, +\tfrac{1}{2} \,|\, 0\, 0 \rangle\, \delta\tilde{U}_{^3S_1-^3S_1}\, \langle 0\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 0\, 0 \rangle\, \delta\tilde{U}^{\dagger}_{^3S_1-^3S_1}\, \langle 0\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle$$

$$+ \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 0\, 0 \rangle\, \delta\tilde{U}_{^3S_1-^3D_1}\, \langle 0\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle \langle +\tfrac{1}{2}\, -\tfrac{1}{2} \,|\, 0\, 0 \rangle\, \delta\tilde{U}^{\dagger}_{^3D_1-^3S_1}\, \langle 0\, 0 \,|\, +\tfrac{1}{2}\, -\tfrac{1}{2} \rangle$$

$$+ \langle -\tfrac{1}{2}\, +\tfrac{1}{2} \,|\, 0\, 0 \rangle \quad \cdots \quad \langle 0\, 0 \,|\, -\tfrac{1}{2}\, +\tfrac{1}{2} \rangle$$

$$= \delta\tilde{U}_{^1S_0}\, \delta\tilde{U}^{\dagger}_{^1S_0} \Big|_{\substack{\tau'=p \\ \tau''=p}} + \frac{1}{4}\, \delta\tilde{U}_{^1S_0}\, \delta\tilde{U}^{\dagger}_{^1S_0} \Big|_{\substack{\tau'=p \\ \tau''=n}}$$

$$+ \frac{1}{4}\, \delta\tilde{U}_{^1S_0}\, \delta\tilde{U}^{\dagger}_{^1S_0} \Big|_{\substack{\tau'=n \\ \tau''=p}} + \frac{3}{4} \Big[\delta\tilde{U}_{^3S_1-^3S_1}\, \delta\tilde{U}^{\dagger}_{^3S_1-^3S_1} +$$

$$\delta\tilde{U}_{^3S_1-^3D_1}\, \delta\tilde{U}^{\dagger}_{^3D_1-^3S_1} \Big]_{\substack{\tau'=p \\ \tau''=n}} + \frac{3}{4} \Big[\delta\tilde{U}_{^3S_1-^3S_1}\, \delta\tilde{U}^{\dagger}_{^3S_1-^3S_1} +$$

$$\delta\tilde{U}_{^3S_1-^3D_1}\, \delta\tilde{U}^{\dagger}_{^3D_1-^3S_1} \Big]_{\substack{\tau'=n \\ \tau''=p}}$$

Then we finally have

$$= \frac{1}{2} \left(\frac{2}{\pi} \right)^2 \sum_{X = {^3S_1}, {^3D_1}} \int_0^{\infty} dk\, k^2 \int_0^{\infty} dK\, K^2 \int_{-1}^{1} \frac{d(\hat{K}\cdot\hat{k})}{2} \times$$

$$\Big[\delta U_{^1S_0}(k, |\vec{q} - \tfrac{1}{2}\vec{K}|)\, \delta U^{\dagger}_{^1S_0}(|\vec{q} - \tfrac{1}{2}\vec{K}|, k)\, \theta(k_F^p - |\tfrac{1}{2}\vec{K} + \vec{k}|) \times$$

$$\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right) + \frac{1}{4}\,\delta U_{^1S_0}\left(k, \left|\vec{q} - \tfrac{1}{2}\vec{K}\right|\right) \delta U_{^1S_0}\left(\left|\vec{q} - \tfrac{1}{2}\vec{K}\right|\right) \times$$

$$\Big(\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} + \vec{k}\right|\right)\Theta\left(k_F^n - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right) + \Theta\left(k_F^n - \left|\tfrac{1}{2}\vec{K} + \vec{k}\right|\right) \times$$

$$\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right)\Big)$$

$$+ \frac{3}{4}\,\delta U_{^3S_1 - X}\left(k, \left|\vec{q} - \tfrac{1}{2}\vec{K}\right|\right) \delta U^{\dagger}_{X - ^3S_1}\left(\left|\vec{q} - \tfrac{1}{2}\vec{K}\right|, k\right) \times$$

$$\Big(\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} + \vec{k}\right|\right)\Theta\left(k_F^n - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right) + \Theta\left(k_F^n - \left|\tfrac{1}{2}\vec{K} + \vec{k}\right|\right)\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right)\Big)\Big]$$

(7)

Taking only S-wave contributions:

$$\Lambda_\lambda^p(q) = V\Big[2\Theta(k_F^p - q) + \frac{2}{\pi}\int_0^\infty \underbrace{dk\,k^2}_{\mathrm{fm}^{-3}} \int_{-1}^{1} d(\hat{q}\cdot\hat{k}) \Big\{ \underbrace{\delta U_{^1S_0}(k,k)}_{\mathrm{fm}^{3}} \Big[\Theta\left(k_F^p - |\vec{q} - 2\vec{k}|\right) +$$

$$\frac{1}{2}\Theta\left(k_F^n - |\vec{q} - 2\vec{k}|\right)\Big] + \frac{3}{2}\,\delta U_{^3S_1 - ^3S_1}(k,k)\,\Theta\left(k_F^n - |\vec{q} - 2\vec{k}|\right)\Big]\Theta(k_F^p - q) +$$

$$\frac{1}{4}\left(\frac{2}{\pi}\right)^2 \sum_{X = ^3S_1, ^3D_1} \int_0^\infty \underbrace{dk\,k^2}_{\mathrm{fm}^{-3}} \int_0^\infty \underbrace{dK\,K^2}_{\mathrm{fm}^{-3}} \int_{-1}^{1} d(\hat{K}\cdot\hat{k}) \Big\{ \underbrace{\delta U_{^1S_0}\left(k, \left|\vec{q} - \tfrac{1}{2}\vec{K}\right|\right)}_{\mathrm{fm}^{3}} \times$$

$$\underbrace{\delta U^{\dagger}_{^1S_0}}_{\mathrm{fm}^{3}}\left(\left|\vec{q} - \tfrac{1}{2}\vec{K}\right|, k\right) \Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} + \vec{k}\right|\right)\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right) +$$

$$+ \frac{1}{4}\,\delta U_{^1S_0}\left(\vec{k}, \left|\vec{q} - \tfrac{1}{2}\vec{K}\right|\right) \delta U^{\dagger}_{^1S_0}\left(\left|\vec{q} - \tfrac{1}{2}\vec{K}\right|, k\right) \Big(\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} + \vec{k}\right|\right)\Theta\left(k_F^n - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right)$$

$$+ \Theta\left(k_F^n - \left|\tfrac{1}{2}\vec{K} + \vec{k}\right|\right)\Theta\left(k_F^p - \left|\tfrac{1}{2}\vec{K} - \vec{k}\right|\right)\Big) +$$

$$\frac{3}{4}\,\delta U_{{}^3S_1-x}\left(k, |\vec{q}-\tfrac{1}{2}\vec{K}|\right)\delta U^{\dagger}_{x-{}^3S_1}\left(|\vec{q}-\tfrac{1}{2}\vec{K}|, k\right) \times$$

$$\Big(\Theta(k_F^p - |\tfrac{1}{2}\vec{K}+\vec{k}|)\,\Theta(k_F^n - |\tfrac{1}{2}\vec{K}-\vec{k}|) + \Theta(k_F^n - |\tfrac{1}{2}\vec{K}+\vec{k}|) \times$$

$$\Theta(k_F^p - |\tfrac{1}{2}\vec{K}-\vec{k}|)\Big)\Big\}\Big] \qquad (8)$$

* For $\hat{n}_\lambda^{\hat{}}(q)$ swap every instance of p with n and vice versa.