880.05 PROBLEM SET #4 SOLUTIONS

1. "Skyrme-type model for nuclear matter."
- We take as a model of nuclear matter an energy per particle that includes: ($g=4$)
  - Kinetic energy $\frac{3}{5}k_F^2/2m$ with $k_F = \left(\frac{6\pi^2\rho}{4}\right)^{1/3} = \left(\frac{3\pi^2\rho}{2}\right)^{1/3}$
  - attractive two body energy $-\frac{|\lambda|}{2}\rho\left(1-\frac{1}{4}\right) = -\frac{3}{8}|\lambda|\rho$
  - repulsive 3-body energy $+\frac{|\beta|}{3!}\left(1-\frac{1}{4}\right)\left(1-\frac{2}{4}\right)\rho^2 = \frac{|\beta|}{16}\rho^2$

a) We'll use Mathematica to solve the problem for us. See the notebook on the next page

$$\Rightarrow \boxed{\lambda = -1024.58 \text{ MeV-fm}^3}, \quad \boxed{\beta = +14605.5 \text{ MeV-fm}^6}$$

b) See the plots and explanation in the notebook.

c) At equilibrium,

$$\boxed{\chi = k_F^2 \frac{\partial^2\epsilon}{\partial k_F^2} = 9\frac{\partial P}{\partial \rho} = 376 \text{ MeV}}$$

according to Mathematica. This is reasonably consistent with what one might expect. The noninteracting result is ~44 MeV.

These expectations usually come from more sophisticated mass formulas, fit to finite nuclei (energies, charge radii, spin-orbit splittings) and then extrapolated to nuclear matter (by just evaluating the energy functional).

To measure χ experimentally, the isoscalar monopole giant resonance energy is measured. This is a collective breathing mode $\Rightarrow$ energy scales with the compressibility (however, there is still ~~model~~ dependence!).

880.05: Problem 1, Problem set #4

Degeneracy factor g=4 for nuclear matter (2 spins times proton or neutron):

```
In[2]:= g = 4;
```

Define the Fermi momentum as a function of the density rho:

```
In[3]:= kf[rho_] := (6 Pi^2 rho / g) ^ (1 / 3)
```

The energy per particle is the sum of the kinetic energy, an attractive two–bodycontact energy, and a repulsive three–body contact energy. The coefficients of the latter two terms are from the conventions used in class; the units are MeV–fm^3and MeV–fm^6,respectively.

```
In[4]:= EoverA[rho_] := 3 / 5 hbarc^2 kf[rho]^2 / (2 m) + (3 / 8) λ rho + (1 / 16) β rho^2
```

Define the derivative of the energy with respect to density rho:

```
In[5]:= DerivEoverA[rho_] = D[EoverA[rho], rho]
```

Out[5]= $\frac{\left(\frac{3}{2}\right)^{2/3} \text{hbarc}^2 \pi^{4/3}}{5\, \text{m}\, \text{rho}^{1/3}} + \frac{\text{rho}\, \beta}{8} + \frac{3\,\lambda}{8}$

The nucleon mass is (on average) 939 MeV; we use hbarc = 197.33 Mev–fmto convert units.

```
In[14]:= m = 939; hbarc = 197.3;
```

Set up the equilibrium conditions:

```
rho0 = 0.16; EoverA0 = -16.;
```

Solve for λ and β by enforcing the equilibrium conditions. Note that the signs come out automagically.

```
In[10]:= ans = Solve[{DerivEoverA[rho0] == 0, EoverA[rho0] == EoverA0}, {λ, β}]

Out[10]= {{λ → -1024.58, β → 14605.5}}
```

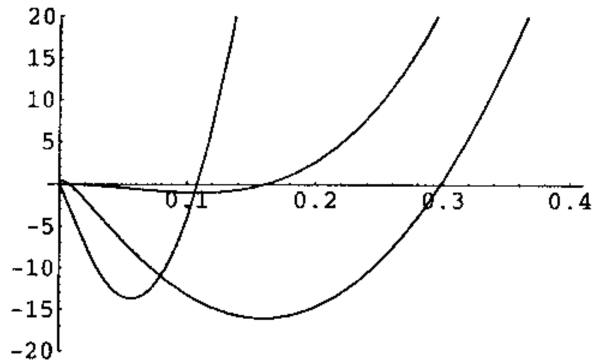
Evaluate the pressure and it's derivative (the latter identifies stable regions against long wavelength density fluctuations.

```
In[11]:= Pressure[rho_] = rho^2 DerivEoverA[rho];

In[12]:= DerivPressure[rho_] = D[Pressure[rho], rho];
```

The plots show where the pressure is positive (rho > 0.16/fm^3 or so) and where it is stable (rho>0.11/fm^3 or so).

```
In[13]:= Plot[{EoverA[rho] /. ans, Pressure[rho] /. ans, DerivPressure[rho] /. ans},
          {rho, 0, .4}, PlotRange → {-20, 20}]
```

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Out[13]= - Graphics -
```

Find the compressibility:

```
In[15]:= K = 9 DerivPressure[rho0] /. ans

Out[15]= {376.425}
```

2 "Lehmann representation for advanced and retarded functions."

- We'll pick the retarded function to do the Lehmann representation. The result for the advanced function follows trivially.

- We start with

$$\boxed{iG^R(\vec{x}t,\vec{x}'t') = \langle \Psi_0^N | \{\hat{\psi}_{H\alpha}(\vec{x}t), \hat{\psi}^\dagger_{H\beta}(\vec{x}'t')\} | \Psi_0^N \rangle \theta(t-t')}$$

and use

$$\boxed{\begin{aligned} \hat{\psi}_{H\alpha}(\vec{x}t) &= e^{i\hat{H}t}\hat{\psi}_\alpha(\vec{x})e^{-i\hat{H}t} \\ \hat{\psi}^\dagger_{H\beta}(\vec{x}'t') &= e^{i\hat{H}t'}\hat{\psi}^\dagger_\beta(\vec{x}')e^{-i\hat{H}t'} \end{aligned}}$$

with a complete set of intermediate states inserted between the field operators. In the $\psi\psi^\dagger$ ordering, only $|\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|$ survives while in the $\psi^\dagger\psi$ ordering, only $|\Psi_n^{N-1}\rangle\langle\Psi_n^{N-1}|$ survives:

$$\boxed{\begin{aligned} iG^R(\vec{x}t,\vec{x}'t') &= \sum_n \theta(t-t')\Big[\langle\Psi_0^N|e^{i\hat{H}t}\hat{\psi}_\alpha(\vec{x})e^{-i\hat{H}t}|\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|e^{i\hat{H}t'}\hat{\psi}^\dagger_\beta(\vec{x}')e^{-i\hat{H}t'}|\Psi_0^N\rangle \\ &\qquad + \langle\Psi_0^N|e^{i\hat{H}t'}\hat{\psi}^\dagger_\beta(\vec{x}')e^{-i\hat{H}t'}|\Psi_n^{N-1}\rangle\langle\Psi_n^{N-1}|e^{i\hat{H}t}\hat{\psi}_\alpha(\vec{x})e^{-i\hat{H}t}|\Psi_0^N\rangle\Big] \\ &= \sum_n \theta(t-t')\Big[e^{-i(E_n^{N+1}-E_0^{N})(t-t')}\langle\Psi_0^N|\hat{\psi}_\alpha(\vec{x})|\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|\hat{\psi}^\dagger_\beta(\vec{x}')|\Psi_0^N\rangle \\ &\qquad + e^{i(E_n^{N-1}-E_0^N)(t-t')}\langle\Psi_0^N|\hat{\psi}^\dagger_\beta(\vec{x}')|\Psi_n^{N-1}\rangle\langle\Psi_n^{N-1}|\hat{\psi}_\alpha(\vec{x})|\Psi_0^N\rangle\Big] \end{aligned}}$$

using $\hat{H}|\Psi_n^{N+1}\rangle = E_n^{N+1}|\Psi_n^{N+1}\rangle$ and so on.

- Now this is manifestly a function of $t-t'$

We'll use (from page 174)

$$\boxed{\int_{-\infty}^{\infty} d(t-t')\, \theta(t-t')\, e^{-i(E_n^{N+1}-E_0^N)(t-t')}\, e^{i\omega(t-t')} = \frac{i}{\omega-(E_n^{N+1}-E_0^N)+i\eta}}$$

and

$$\boxed{\int_{-\infty}^{\infty} d(t-t')\, \theta(t-t')\, e^{i(E_n^{N-1}-E_0^N)(t-t')}\, e^{i\omega(t-t')} = \frac{i}{\omega+(E_n^{N-1}-E_0^N)+i\eta}}$$

to find

$$\boxed{G^R_{\alpha\beta}(\vec{x},\vec{x}';\omega) = \sum_n \frac{\langle \Psi_0^N|\hat{\psi}_\alpha(\vec{x})|\Psi_n^{N+1}\rangle\langle \Psi_n^{N+1}|\hat{\psi}^\dagger_\beta(\vec{x}')|\Psi_0^N\rangle}{\omega-(E_n^{N+1}-E_0^N)+i\eta} + \sum_n \frac{\langle \Psi_0^N|\hat{\psi}^\dagger_\beta(\vec{x}')|\Psi_n^{N-1}\rangle\langle \Psi_n^{N-1}|\hat{\psi}_\alpha(\vec{x})|\Psi_0^N\rangle}{\omega+(E_n^{N-1}-E_0^N)+i\eta}}$$

The advanced function has $-i\eta$'s.

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3. "Spin-dependent force."

- If we now have a spin-dependent force

$$\boxed{V_s(\vec{x}_1,\vec{x}_2)_{\alpha\beta,\lambda\mu} = \lambda_s \vec{\sigma}(1)_{\alpha\beta}\cdot\vec{\sigma}(2)_{\lambda\mu}\,\delta^3(\vec{x}_1-\vec{x}_2)}$$

in three-dimensions, the major difference is the replacement $\delta^{(1)}_{\alpha\beta} \to \vec{\sigma}^{(1)}_{\alpha\beta}$ and $\delta_{\lambda\mu}{}^{(2)} \to \vec{\sigma}(2)_{\lambda\mu}$.

a) The interaction term in the Lagrangian with potential $V(\vec{x}_1,\vec{x}_2)_{\alpha\beta,\lambda\mu}$ is ($\delta(t_1-t_2)$ is implicit)

$$\boxed{-\tfrac{1}{2}\psi^\dagger_\alpha(x_1)\psi^\dagger_\lambda(x_2)\,V(\vec{x}_1,\vec{x}_2)_{\alpha\beta,\lambda\mu}\,\psi_\mu(x_2)\psi_\beta(x_1)}$$

With $V(\vec{x}_1,\vec{x}_2)_{\alpha\beta,\lambda\mu} = \lambda\,\delta_{\alpha\beta}\delta_{\lambda\mu}\,\delta^3(\vec{x}_1-\vec{x}_2)$ we get our previous term:

$$\boxed{-\tfrac{\lambda}{2}\psi^\dagger_\alpha(x)\psi^\dagger_\lambda(x)\psi_\lambda(x)\psi_\alpha(x)}$$

so now we get

$$\boxed{-\tfrac{\lambda_s}{2}\psi^\dagger_\alpha(x)\psi^\dagger_\lambda(x)\psi_\mu(x)\psi_\beta(x)\,\vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\lambda\mu}}$$

b) The Feynman rule for 

was $(\delta_{\alpha\beta}\delta_{\lambda\mu}+\delta_{\alpha\mu}\delta_{\lambda\beta})(-i\lambda)$

for the spin-independent case, and then $-g$ for each $\delta_{\alpha\alpha}$. Now the rule replaces the δ's with σ's:

$$\Rightarrow \boxed{(\vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\lambda\mu}+\vec{\sigma}_{\alpha\mu}\vec{\sigma}_{\lambda\beta})(-i\lambda_s)}$$

and $-g$ for each $\delta_{\alpha\alpha}$ still, if there are any.

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c) The labeled bow-tie diagram is

$$iG^0_{\alpha\beta}(x_1,x_1^+)\, iG^0_{\lambda\mu}(x_1,x_1^+) \times (-i\lambda) \times \left(\vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\lambda\mu} + \vec{\sigma}_{\alpha\mu}\cdot\vec{\sigma}_{\lambda\beta}\right)$$

- From the G^0's, we get $\delta_{\alpha\beta}\delta_{\lambda\mu}$

$$\Rightarrow \vec{\sigma}_{\alpha\alpha}\cdot\vec{\sigma}_{\lambda\lambda} + \vec{\sigma}_{\alpha\mu}\cdot\vec{\sigma}_{\mu\alpha} = 3\delta_{\alpha\alpha} \Rightarrow -3g$$

as a spin factor.

- $\int d^3x_1 \int_{-\infty}^{\infty} dt_1 \Rightarrow VT \Rightarrow$ divide this out to get the energy density
- symmetry factor and over $i \Rightarrow \frac{i}{2}$
- $iG^0(x_1,x_1^+) = -\int \frac{d^3k}{(2\pi)^3}\,\theta(k_F-|\vec{k}|)$

$$\Rightarrow \boxed{\mathcal{E}_{1s} = (-i)(i)(-1)^2\left(+\frac{\lambda_s}{2}\right)(-3g)\left(\int\frac{d^3k}{(2\pi)^3}\theta(k_F-|\vec{k}|)\right)^2 = -\frac{3\lambda_s}{2}\frac{1}{g}\rho^2}$$

So this is just 3 times the Fock term from the spin-independent interaction. The averaging over spin makes the Hartree term vanish.

d) The anomalous graph vanishes because of the integrals, not the spin sum, which is the only difference. So it still vanishes. For the beachball diagram, we only have to redo the spin sum. We can do this using the expressions on (125) or (130):

$$\Rightarrow \delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}\delta_{\alpha_3\beta_3}\delta_{\alpha_4\beta_4}\left(\vec{\sigma}_{\alpha_1\beta_3}\cdot\vec{\sigma}_{\alpha_2\beta_4}+\vec{\sigma}_{\alpha_1\beta_4}\cdot\vec{\sigma}_{\alpha_2\beta_3}\right)\left(\vec{\sigma}_{\alpha_3\beta_1}\cdot\vec{\sigma}_{\alpha_4\beta_2}+\vec{\sigma}_{\alpha_3\beta_2}\cdot\vec{\sigma}_{\alpha_4\beta_1}\right)$$

$$= 2(+3g^2+3g) = 6g(g+1)$$ from Mathematica. [using $\delta_{\alpha\alpha}\to -g$]

4. "Fermi liquid theory in one spatial dimension."
We'll follow through the discussion in Negele and Orland Chapter 6, converting from 3 to 1 dimension as we go.

· We have a uniform gas of spin-1/2 fermions (so $g=2$), which is N particles in "volume" L.
· The Fermi momentum and density are related by

$$\boxed{\rho = \frac{g k_F}{\pi} = \frac{2k_F}{\pi}}$$

· The variation of the energy is

$$\boxed{\delta E = \sum_{k,\sigma} \epsilon_k^0 \,\delta n(k,\sigma) + \frac{1}{2L} \sum_{k\sigma, k'\sigma'} f(k\sigma, k'\sigma')\,\delta n(k,\sigma)\,\delta n(k',\sigma')}$$

with $\boxed{\epsilon_k^0 \equiv k^2/2m}$

· The quasiparticle energy is

$$\boxed{\epsilon_{k\sigma} = \frac{\delta E}{\delta n(k,\sigma)} = \epsilon_k^0 + \frac{1}{L}\sum_{k'\sigma'} f(k\sigma, k'\sigma')\,\delta n(k'\sigma')}$$

and the quasiparticle interaction is

$$\boxed{\frac{\delta^2 E}{\delta n(k\sigma)\,\delta n(k'\sigma')} = \frac{1}{L} f(k\sigma, k'\sigma') = \frac{1}{L} f(k'\sigma', k\sigma)}$$

a) To find the effective mass in terms of $f(k'\sigma', k\sigma)$, we consider the momentum per length:

$$\boxed{P = \frac{1}{L}\sum_{k,\sigma} k\, n(k,\sigma)}$$

which is also the sum of quasiparticle velocities $\frac{\partial \epsilon_k}{\partial k}$ times their mass:

$$\boxed{P = \frac{1}{L}\sum_{k,\sigma} m \frac{\partial \epsilon_k}{\partial k}\, n(k,\sigma)}$$

We can take the functional derivative wrt $n(k,\sigma)$ after setting these equal

$$\boxed{\frac{\delta}{\delta n(k,\sigma)}\left[\sum_{k'\sigma'} k' n(k',\sigma') = \sum_{k',\sigma'} m \frac{\partial \epsilon_{k'}}{\partial k'} n(k',\sigma')\right]}$$

$$\Rightarrow \boxed{k = m\frac{\partial \epsilon_k}{\partial k} + m\sum_{\sigma'}\int \frac{dk'}{2\pi}\frac{\partial}{\partial k'}\left(\frac{\delta \epsilon_{k'}}{\delta n(k,\sigma)}\right) n(k',\sigma')}$$

where we've converted to an integral (note that the L's have dropped out) and switched the order of differentiation.

From the expression for $\epsilon_{k,\sigma}$,

$$\Rightarrow \frac{k}{m} = \frac{\partial \epsilon_k}{\partial k} + \sum_{\sigma'}\int \frac{dk'}{2\pi}\frac{\partial f(k\sigma, k'\sigma')}{\partial k'} n(k',\sigma')$$

integrating by parts

$$= \frac{\partial \epsilon_k}{\partial k} - \sum_{\sigma'}\int \frac{dk'}{2\pi} f(k\sigma, k'\sigma')\frac{\partial n(k',\sigma')}{\partial k'}$$

(we can integrate by parts since n vanishes for large $|k|$).

· At zero temperature, $\boxed{n(k,\sigma) = \Theta(k_f - |k|)}$

$$\Rightarrow \boxed{\frac{\partial n(k',\sigma')}{\partial k'} = -\hat{k}\,\delta(k_f - |k|)} \qquad \text{where } \hat{k} \equiv \text{sign}(k) = \begin{cases} +1 & k>0 \\ -1 & k<0\end{cases}$$

The effective mass is defined at the Fermi surface as

$$\boxed{\left.\frac{d\epsilon_k}{dk}\right|_{k_F} \equiv \frac{k_F}{m^*} = v_f}$$

so evaluating the $\frac{k}{m}$ expression for $|k| = k_f$! (which means $k = +k_f$ or $-k_f$!)

$$\boxed{\begin{aligned}\frac{k}{m} &= \frac{k}{m^*} + \sum_{\sigma'}\int \frac{dk'}{2\pi} f(k\sigma, k'\sigma')\,\hat{k}\,\delta(k_f - |k|) \\ &= \frac{k}{m^*} + \sum_{\sigma'}\frac{1}{2\pi}\left(f(k\sigma, k_f\sigma') - f(k\sigma, -k_f\sigma')\right)\end{aligned}}$$

With $f(k\sigma, k'\sigma') \equiv f(k,k') + 4\sigma\cdot\sigma' \phi(k,k')$
and evaluating at $k = k_f$,

$$\boxed{\frac{k_f}{m} = \frac{k_f}{m^*} + \frac{g}{2\pi}\left[f(k_f,k_f) - f(k_f,-k_f)\right] = \frac{k_f}{m^*} + \frac{g}{\pi}\frac{F_1}{N(0)}}$$

(the ϕ term drops out since $\sum_{\sigma'} \sigma' = 0$).

We've used the definition

$$\boxed{\begin{aligned} F_{0,1} &= N(0)\tfrac{1}{2}\left(f(k_f,k_f) \pm f(k_f,-k_f)\right) \\ Z_{0,1} &= N(0)\tfrac{1}{2}\left(\phi(k_f,k_f) \pm \phi(k_f,-k_f)\right) \end{aligned}}$$

with

$$\boxed{N(0) = \frac{g}{\pi}\frac{m^*}{k_f}}$$

substituting the last,

$$\frac{k_f}{m} = \frac{k_f}{m^*} + \frac{k_f}{m^*}F_1 \Rightarrow \boxed{\frac{m^*}{m} = 1 + F_1}$$ as desired.

b) The specific heat at constant "volume" is defined as

$$\boxed{c_v = \frac{1}{L}\frac{\partial E}{\partial T}\bigg|_L = \frac{1}{L}\sum_{k\sigma}\frac{\delta E}{\delta n(k\sigma)}\frac{\delta n(k\sigma)}{\delta T}}$$

Since the temperature change leads to a change in occupation numbers.

Now $$\boxed{\frac{\delta E}{\delta n(k\sigma)} = \epsilon_{k\sigma} = \epsilon_k^0 + \frac{1}{L}\sum_{k'\sigma'} f(k\sigma, k'\sigma')\,\delta n(k'\sigma') = \epsilon_k^0 + O(T^2)}$$

at low T. We take the other derivative using

$$\frac{\delta}{\delta T}\frac{1}{e^{(\epsilon_{k\sigma}-\mu)/T}+1} = \frac{-1}{\left(e^{(\epsilon_{k\sigma}-\mu)/T}+1\right)^2} e^{(\epsilon_{k\sigma}-\mu)/T}\left[-\frac{(\epsilon_k-\mu)}{T^2} + \frac{1}{T}\frac{\partial(\epsilon_{k\sigma}-\mu)}{\partial T}\right]$$

Now $\frac{\delta n(k\sigma)}{\delta \epsilon_{k\sigma}} = -\frac{1}{\left(e^{(\epsilon_{k\sigma}-\mu)/T}+1\right)^2} e^{(\epsilon_{k\sigma}-\mu)/T} \frac{1}{T}$

$$\Rightarrow \boxed{\frac{\delta}{\delta T} n(k\sigma) = \frac{\delta n(k\sigma)}{\delta \epsilon_{k\sigma}} \left(-\frac{(\epsilon_{k\sigma}-\mu)}{T} + \frac{\partial}{\partial T}(\epsilon_{k\sigma}-\mu) \right)}$$

So
$$\boxed{C_V = \frac{1}{L} \sum_{k\sigma} \epsilon_{k\sigma} \frac{\delta n(k\sigma)}{\delta \epsilon_{k\sigma}} \left(-\frac{(\epsilon_{k\sigma}-\mu)}{T} + \frac{\partial}{\partial T}(\epsilon_k - \mu) \right)}$$

Now we use

$$\boxed{\frac{\partial}{\partial \epsilon} n(k\sigma) \xrightarrow[\beta \to \infty]{} -\delta(\epsilon-\mu) - \frac{\pi^2}{6\beta^2} \frac{\partial^2}{\partial \epsilon^2} \delta(\epsilon-\mu) + \mathcal{O}(\beta^{-4})}$$

and use $\epsilon_k \to \epsilon_k^0$ and convert the integral over k to one over $\epsilon_k^0 \equiv \epsilon$ using

$$\epsilon_k = \mu + (k-k_f)\frac{\partial \epsilon}{\partial k}\Big|_{k_f} \equiv \mu + (k-k_f) v_f = \mu + (k-k_f)\frac{k_f}{m^*}$$

$\Rightarrow dk = \frac{m^*}{k_f} d\epsilon$ at the Fermi surface

$$\Rightarrow C_V = \frac{g\, m^*}{\pi k_f} \int d\epsilon \, \epsilon \left(-\frac{\pi^2 T^2}{6} \frac{\partial^2}{\partial \epsilon^2} \delta(\epsilon-\mu) \right) \left(-\frac{(\epsilon-\mu)}{T} \right) + \mathcal{O}(T^2)$$

integrating by parts twice $= g \frac{m^*}{k_f} \frac{\pi^2 T}{6} \cdot 2 \int_0^\infty d\epsilon \, \delta(\epsilon-\mu)$

or $\boxed{C_V = \frac{g \pi T m^*}{3 k_f}}$

c) The sound velocity follows from

$$\boxed{c^2 = \frac{1}{m} \frac{\partial P}{\partial \rho} = \frac{1}{m} \rho \frac{\partial \mu}{\partial \rho}}$$

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The latter follows from $F(T,L,N) \equiv L\,f(T,\frac{N}{L})$

$$\Rightarrow P = -\frac{\partial F}{\partial L} = -f + \rho\frac{\partial f}{\partial \rho} = -f + \rho\mu \Rightarrow \frac{\partial P}{\partial \rho} = \rho\frac{\partial \mu}{\partial \rho}$$

We can find $\frac{\partial \mu}{\partial \rho}$ using $\mu = \epsilon(k_f, n(k\sigma))$

$$\Rightarrow \frac{\partial \mu}{\partial \rho} = \frac{\partial \epsilon^0}{\partial k_f}\frac{\partial k_f}{\partial \rho} + \sum_{\sigma'}\int \frac{dk}{2\pi} f(k_f\sigma, k\sigma')\frac{\delta n}{\delta k_f}\frac{\partial k_f}{\partial \rho}$$

Now $\rho = \frac{g k_f}{\pi} \Rightarrow \frac{\partial k_f}{\partial \rho} = \frac{\pi}{g}$; $\frac{\partial \epsilon^0}{\partial k_f} = \frac{k_f}{m^*}$; $\frac{\delta n}{\delta k_f} = \delta(|k|-k_f)$

$$\Rightarrow \rho\frac{\partial \mu}{\partial \rho} = \rho\frac{k_f}{m^*}\frac{\pi}{g} + \rho\sum_{\sigma'}\frac{1}{2\pi}\frac{\pi}{g}\times\left[\underbrace{f(k_f\sigma, k_f\sigma') + f(k_f\sigma, -k_f\sigma')}_{\frac{2F_0}{N(0)} = \frac{2\pi k_f F_0}{g m^*}}\right]$$

$\frac{2F_0}{N(0)}$ and Z_0 term averages out

$$= \frac{k_f^2}{m^*} + \frac{k_f^2 F_0}{m^*}$$

$$\Rightarrow \boxed{c^2 = \frac{k_f^2}{m m^*}(1+F_0)}$$