AJT notes (11/12/21)

$$
\langle \Psi_\alpha^{A-1} | \hat{U}_\lambda^\dagger \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger \hat{U}_\lambda | \Psi_0^A \rangle
$$

$$
= \langle \Phi | a_\alpha^\dagger \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger | \Phi \rangle \qquad (1)
$$

$$
\begin{aligned}
a_\alpha^\dagger \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger = {} & \underbrace{a_\alpha^\dagger a_{\vec{p}}}_{2} + \frac{1}{4} \sum_{1234} \delta\tilde{U}_{1234} \underbrace{a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}}}_{6} \\
& + \frac{1}{36} \sum_{5678910} \delta\tilde{U}_{5678910} \underbrace{a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10} a_9 a_8 a_{\vec{p}}}_{8} \\
& + \frac{1}{4} \sum_{abcd} \delta\tilde{U}^\dagger_{abcd} \underbrace{a_\alpha^\dagger a_{\vec{p}} a_a^\dagger a_b^\dagger a_d a_c}_{6} \\
& + \frac{1}{4} \frac{1}{4} \sum_{1234} \sum_{abcd} \delta\tilde{U}_{1234} \delta\tilde{U}^\dagger_{abcd} \underbrace{a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}} a_a^\dagger a_b^\dagger a_d a_c}_{10} \\
& + \frac{1}{36} \frac{1}{4} \sum_{5678910} \sum_{abcd} \delta\tilde{U}_{5678910} \delta\tilde{U}^\dagger_{abcd} \underbrace{a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10} a_9 a_8 a_{\vec{p}} a_a^\dagger a_b^\dagger a_d a_c}_{12} \\
& + \frac{1}{36} \sum_{mnopqr} \delta\tilde{U}^\dagger_{mnopqr} \underbrace{a_\alpha^\dagger a_{\vec{p}} a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p}_{8} \\
& + \frac{1}{4} \frac{1}{36} \sum_{1234} \delta\tilde{U}_{1234} \delta\tilde{U}^\dagger_{mnopqr} \underbrace{a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}} a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p}_{12} \\
& + \frac{1}{36} \frac{1}{36} \sum_{5678910} \sum_{mnopqr} \delta\tilde{U}_{5678910} \delta\tilde{U}^\dagger_{mnopqr} \underbrace{a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10} a_9 a_8 a_{\vec{p}} a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p}_{14} \qquad (2)
\end{aligned}
$$

* Split this up into terms organizing by total number of a's

(A) $a_\alpha^\dagger \hat{I} a_{\vec{p}} \hat{I}$ [2]

$a_\alpha^\dagger a_{\vec{p}}$ (3)

* Already normal-ordered.

(B) $a_\alpha^\dagger \delta\hat{U}_2 a_{\vec{p}} \hat{I}$ [6]

$a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}}$ (4)

* Already normal-ordered.

(C) $a_\alpha^\dagger \hat{I} a_{\vec{p}} \delta\hat{U}_2^\dagger$ [6]

$a_\alpha^\dagger a_{\vec{p}} a_a^\dagger a_b^\dagger a_d a_c = :a_\alpha^\dagger a_{\vec{p}} a_a^\dagger a_b^\dagger a_d a_c:$

$+ :a_\alpha^\dagger \overbrace{a_{\vec{p}} a_a^\dagger} a_b^\dagger a_d a_c: - :a_\alpha^\dagger \overbrace{a_{\vec{p}} a_a^\dagger a_b^\dagger} a_d a_c:$

$= a_\alpha^\dagger a_a^\dagger a_b^\dagger a_{\vec{p}} a_d a_c + \delta_{\vec{p}a} a_\alpha^\dagger a_b^\dagger a_d a_c - \delta_{\vec{p}b} a_\alpha^\dagger a_a^\dagger a_d a_c$

(5)

(D) $a_\alpha^\dagger \delta\hat{U}_3 a_{\vec{p}} \hat{I}$ [8]

4-body

$a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10} \cancel{a_9 a_8} a_{\vec{p}}$ * Already normal ordered (6)

E $a_\alpha^\dagger \, \hat{I} \, a_{\vec{\beta}} \, \delta \hat{U}_3^\dagger$ [8]

4-body

$$a_\alpha^\dagger a_{\vec{\beta}} \, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p = \cancel{:a_\alpha^\dagger a_{\vec{\beta}} \, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p:}$$

$$+ :a_\alpha^\dagger a_{\vec{\beta}} \, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p: - :a_\alpha^\dagger a_{\vec{\beta}} \, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p:$$

$$+ :a_\alpha^\dagger a_{\vec{\beta}} \, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p:$$

$$= \delta_{\vec{\beta} m} \, a_\alpha^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p - \delta_{\vec{\beta} n} \, a_\alpha^\dagger a_m^\dagger a_o^\dagger a_r a_q a_p$$

$$+ \delta_{\vec{\beta} o} \, a_\alpha^\dagger a_m^\dagger a_n^\dagger a_r a_q a_p \qquad (7)$$

F $a_\alpha^\dagger \, \delta \hat{U}_2 \, a_{\vec{\beta}} \, \delta \hat{U}_2^\dagger$ [10]

5-body

$$a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c = \cancel{:a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c:}$$

4-body

$$+ \cancel{\sum_{\text{singles}} (\ldots)} \qquad + :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c:$$

$$- :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c: + :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c:$$

$$- :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c: + :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c:$$

$$- :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 \, a_{\vec{\beta}} \, a_a^\dagger a_b^\dagger a_d a_c:$$

$$= \left(\delta_{4b}\delta_{3a} - \delta_{4a}\delta_{3b}\right) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_{\vec{p}}\, a_d a_c$$

$$+ \left(\delta_{3b}\delta_{\vec{p}a} - \delta_{3a}\delta_{\vec{p}b}\right) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_d a_c$$

$$+ \left(\delta_{4b}\delta_{\vec{p}a} - \delta_{4a}\delta_{\vec{p}b}\right) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_3 a_d a_c \qquad (8)$$

6 $a_\alpha^\dagger\, \delta\hat{U}_3\, a_{\vec{p}}\, \delta\hat{U}_2^\dagger$ [12]

$$a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10} a_9 a_8 a_{\vec{p}}\, a_a^\dagger a_b^\dagger a_d a_c \qquad \text{(4-body at least)}$$

$$= \overset{\text{6-body}}{\cancel{:a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10} a_9 a_8 a_{\vec{p}}\, a_a^\dagger a_b^\dagger a_d a_c:}} + \overset{\text{5-body}}{\cancel{\sum_{\text{singles}} (\dots)}}$$

$$+ \overset{\text{4-body}}{\cancel{\sum_{\text{doubles}} (\dots)}} \qquad (9)$$

7 $a_\alpha^\dagger\, \delta\hat{U}_2\, a_{\vec{p}}\, \delta\hat{U}_3^\dagger$ [12]

$$a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p$$

$$= \overset{\text{6-body}}{\cancel{:a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p:}} + \overset{\text{5-body}}{\cancel{\sum_{\text{singles}} (\dots)}} + \overset{\text{4-body}}{\cancel{\sum_{\text{doubles}} (\dots)}}$$

$$+ :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p:$$

$$- :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p:$$

$$+ :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec p}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p :$$

$$- :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec p}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p :$$

$$+ :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec p}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p :$$

$$- :a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec p}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p :$$

$$= \Big[\delta_{4o}\left(\delta_{3n}\delta_{\vec p m} - \delta_{3m}\delta_{\vec p n}\right) + \delta_{4n}\left(\delta_{3m}\delta_{\vec p o} - \delta_{3o}\delta_{\vec p m}\right)$$

$$+ \delta_{4m}\left(\delta_{3n}\delta_{\vec p o} - \delta_{3o}\delta_{\vec p n}\right)\Big] a_\alpha^\dagger a_1^\dagger a_2^\dagger a_r a_q a_p \qquad (10)$$

(I) $a_\alpha^\dagger\, \delta\hat U_3\, a_{\vec p}\, \delta\hat U_3^\dagger$ [14]

$$a_\alpha^\dagger a_5^\dagger a_6^\dagger a_7^\dagger a_{10} a_9 a_8 a_{\vec p}\, a_m^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p \quad \text{(4-body at least)} \qquad (11)$$

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* Recombine terms

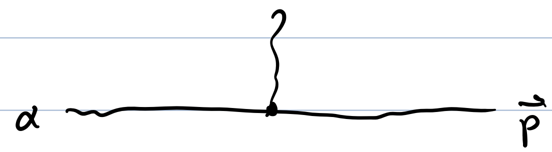
- Maybe organize diagrams?

$a_\alpha^\dagger \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger$ 3-body level

$$
\begin{aligned}
&\approx a_\alpha^\dagger a_{\vec{p}} + \frac{1}{4}\sum_{abcd} \delta\tilde{U}^\dagger_{abcd}\Big[\big(\delta_{\vec{p}a}\, a_\alpha^\dagger a_b^\dagger a_d a_c - \delta_{\vec{p}b}\, a_\alpha^\dagger a_a^\dagger a_d a_c\big) \\
&\quad + a_\alpha^\dagger a_a^\dagger a_b^\dagger a_{\vec{p}} a_d a_c\Big] + \frac{1}{4}\sum_{1234}\delta\tilde{U}_{1234}\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec{p}} \\
&\quad + \frac{1}{4}\frac{1}{4}\sum_{1234}\sum_{abcd}\delta\tilde{U}_{1234}\,\delta\tilde{U}^\dagger_{abcd}\Big[\big(\delta_{4b}\delta_{3a} - \delta_{4a}\delta_{3b}\big)\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_{\vec{p}} a_d a_c \\
&\qquad + \big(\delta_{3b}\delta_{\vec{p}a} - \delta_{3a}\delta_{\vec{p}b}\big)\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_d a_c \\
&\qquad + \big(\delta_{4b}\delta_{\vec{p}a} - \delta_{4a}\delta_{\vec{p}b}\big)\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_3 a_d a_c\Big] \\
&\quad + \frac{1}{36}\sum_{mnopqr}\delta\tilde{U}^\dagger_{mnopqr}\Big(\delta_{\vec{p}m}\, a_\alpha^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p \\
&\qquad - \delta_{\vec{p}n}\, a_\alpha^\dagger a_m^\dagger a_o^\dagger a_r a_q a_p + \delta_{\vec{p}o}\, a_\alpha^\dagger a_m^\dagger a_n^\dagger a_r a_q a_p\Big) \\
&\quad + \frac{1}{4}\frac{1}{36}\sum_{1234}\sum_{mnopqr}\delta\tilde{U}_{1234}\,\delta\tilde{U}^\dagger_{mnopqr}\Big[\delta_{4o}\big(\delta_{3n}\delta_{\vec{p}m} - \delta_{3m}\delta_{\vec{p}n}\big) \\
&\qquad + \delta_{4n}\big(\delta_{3n}\delta_{\vec{p}o} - \delta_{3o}\delta_{\vec{p}m}\big) + \delta_{4m}\big(\delta_{3n}\delta_{\vec{p}o} - \delta_{3o}\delta_{\vec{p}n}\big)\Big]\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_r a_q a_p
\end{aligned}
\tag{12}
$$

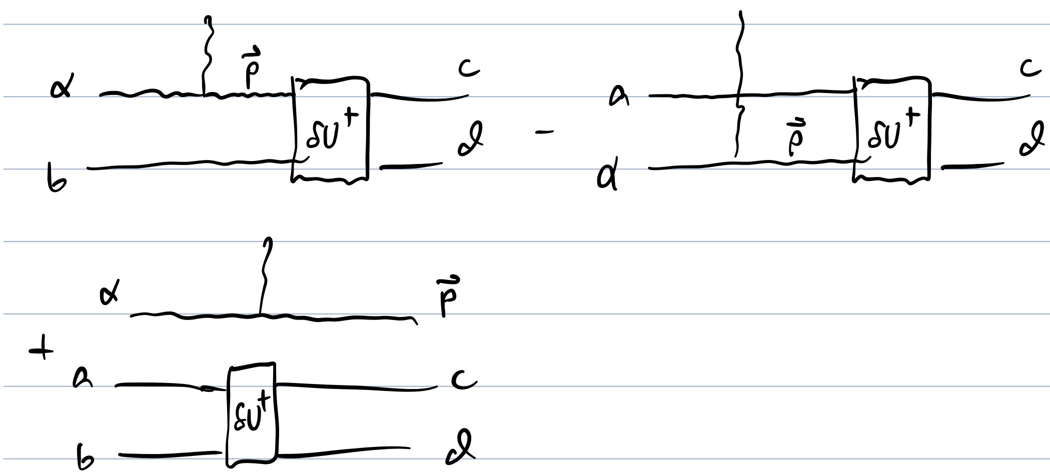
* Consider diagrams of contributions in evolved operator

I) $a_\alpha^\dagger \hat{I} a_{\vec p} \hat{I} \sim a_\alpha^\dagger a_{\vec p}$

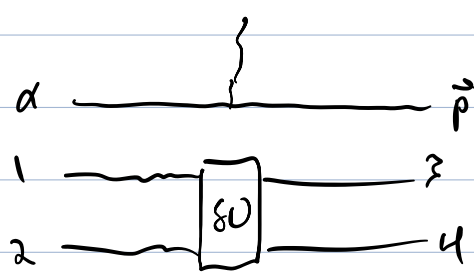


II) $a_\alpha^\dagger \hat{I} a_{\vec p} \delta\hat{U}^\dagger_{(2)}$

$$\sim \delta\tilde{U}^\dagger_{abcd}\left[\left(\delta_{\vec p a}\, a_\alpha^\dagger a_b^\dagger a_d a_c - \delta_{\vec p b}\, a_\alpha^\dagger a_a^\dagger a_d a_c\right) + a_\alpha^\dagger a_a^\dagger a_b^\dagger a_{\vec p} a_d a_c\right]$$



III) $a_\alpha^\dagger \delta\hat{U}_{(2)} a_{\vec p} \hat{I} \sim \delta\tilde{U}_{1234}\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_3 a_{\vec p}$

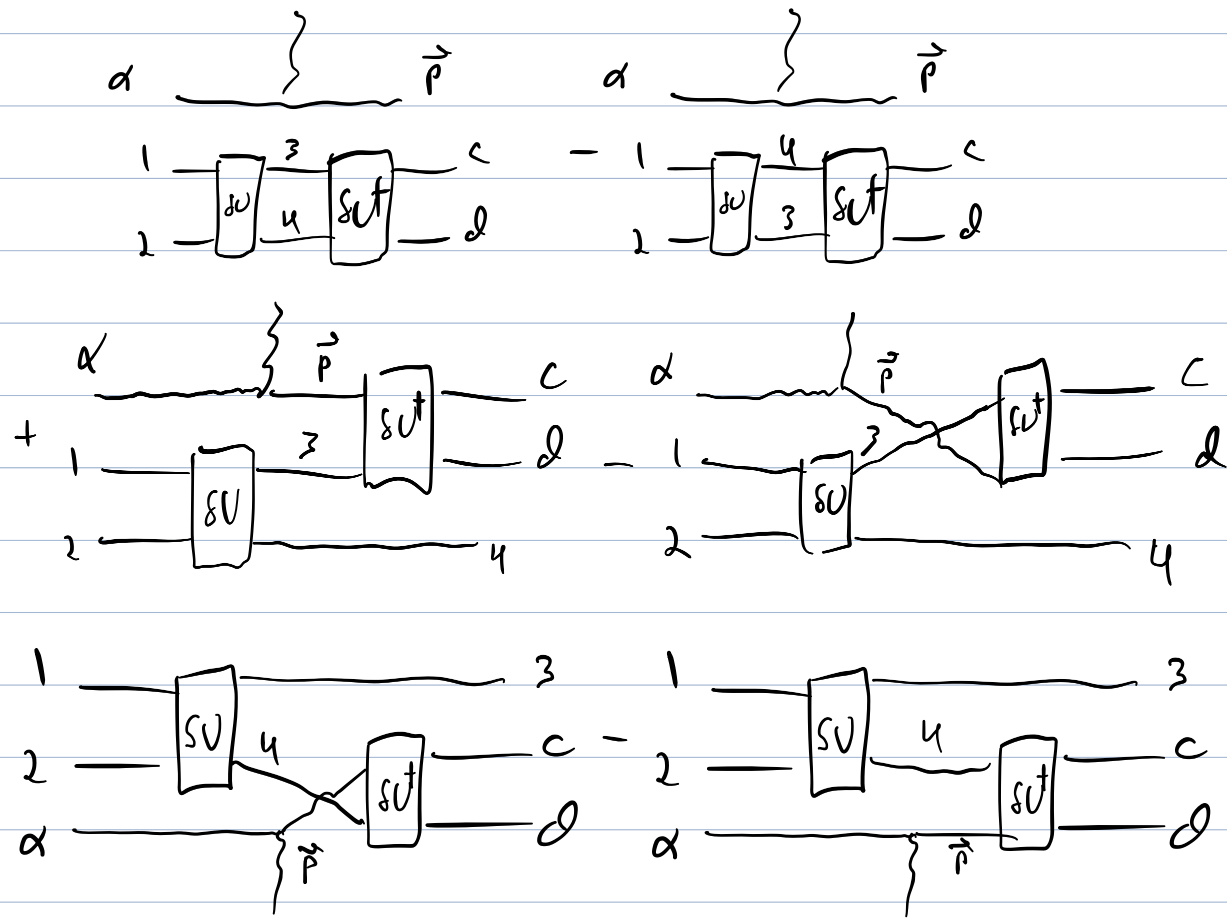


IV) $a_\alpha^\dagger\, \delta\hat U_{(2)}\, a_{\vec p}\, \delta\hat U^\dagger_{(2)}$

$$\sim \delta\tilde U_{1234}\, \delta\tilde U^\dagger_{abcd} \Big[(\delta_{4b}\delta_{3a} - \delta_{4a}\delta_{3b})\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_{\vec p}\, a_d a_c$$

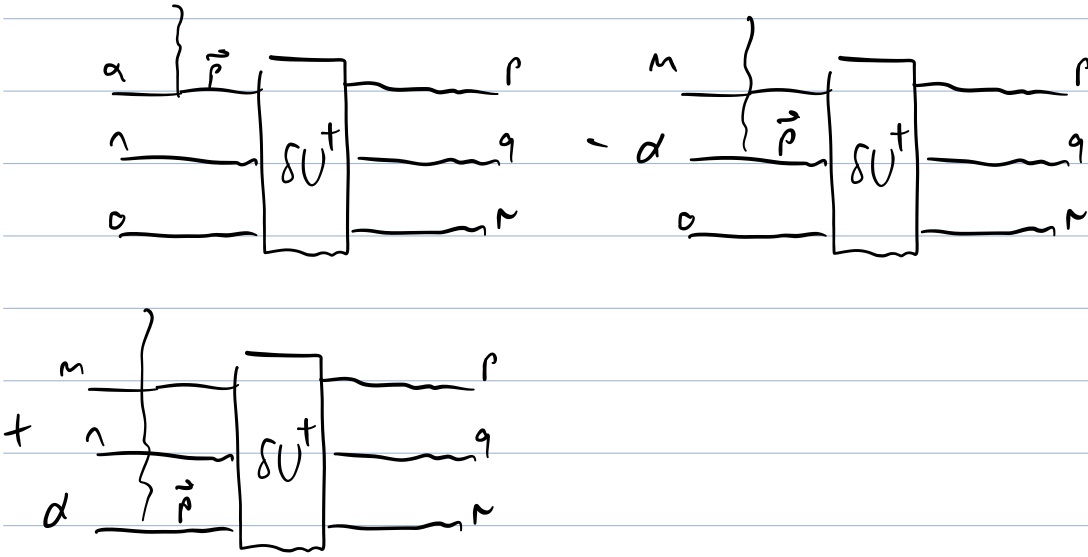
$$+ (\delta_{3b}\delta_{\vec p a} - \delta_{3a}\delta_{\vec p b})\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_d a_c$$

$$+ (\delta_{4b}\delta_{\vec p a} - \delta_{4a}\delta_{\vec p b})\, a_\alpha^\dagger a_1^\dagger a_2^\dagger a_3 a_d a_c \Big]$$



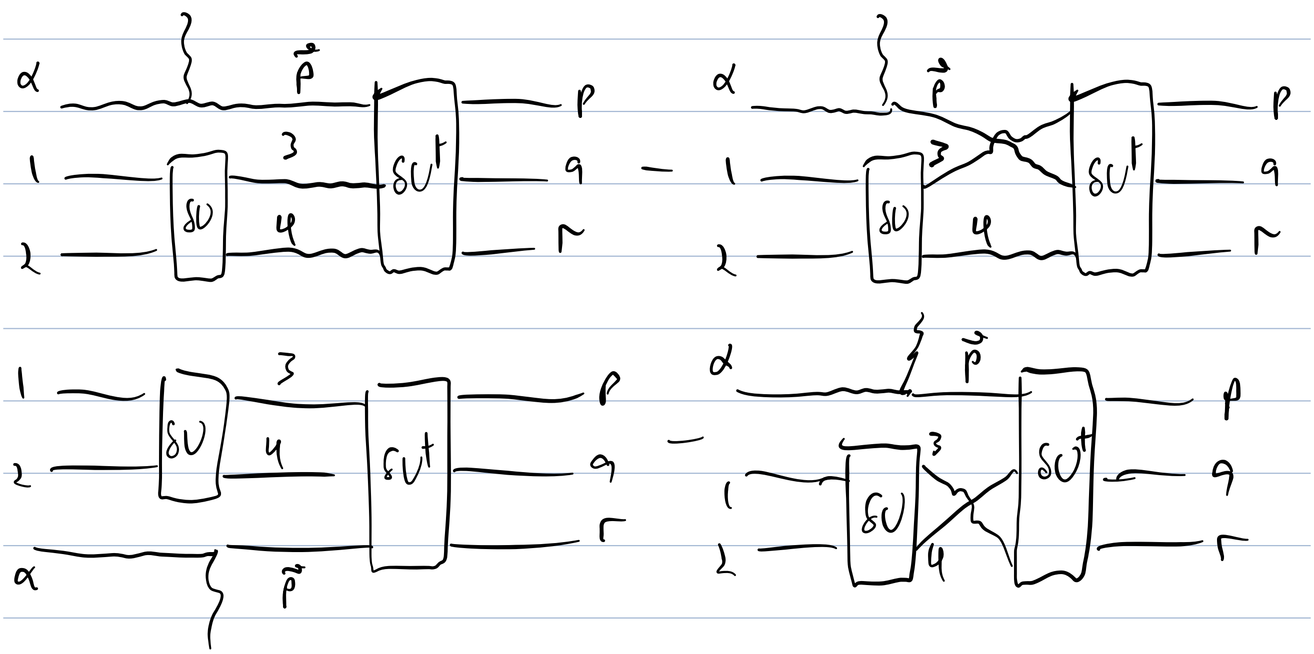
V) $a_\alpha^\dagger\, \hat I\, a_{\vec p}\, \delta\hat U^\dagger_{(3)}$ $\sim \delta\tilde U^\dagger_{mnopqr} \Big(\delta_{\vec p m}\, a_\alpha^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p$

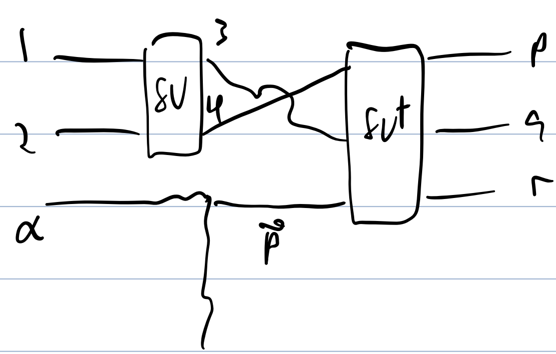
$$- \delta_{\vec p n}\, a_\alpha^\dagger a_m^\dagger a_o^\dagger a_r a_q a_p + \delta_{\vec p o}\, a_\alpha^\dagger a_m^\dagger a_n^\dagger a_r a_q a_p \Big)$$



VI) $a^\dagger_\alpha \, \delta\hat{U}_{(2)} \, a_{\vec{p}} \, \delta\hat{U}^\dagger_{(3)} \sim \delta\tilde{U}_{1234} \, \delta\tilde{U}^{\dagger}_{mnopqr} \Big[\delta_{4o}(\delta_{3n}\delta_{\vec{p}m} - \delta_{3m}\delta_{\vec{p}n})$

$+ \delta_{4n}(\delta_{3n}\delta_{\vec{p}o} - \delta_{3o}\delta_{\vec{p}m}) + \delta_{4m}(\delta_{3n}\delta_{\vec{p}o} - \delta_{3o}\delta_{\vec{p}n}) \Big] a^\dagger_\alpha a^\dagger_1 a^\dagger_2 a_r a_q a_p$





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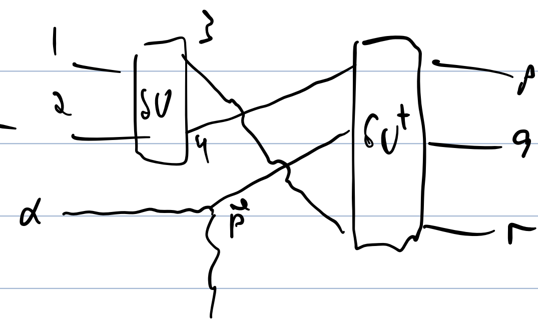
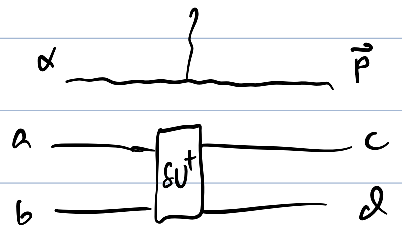


(12/8/21) Update on disconnected diagrams

* Consider the disconnected diagrams:



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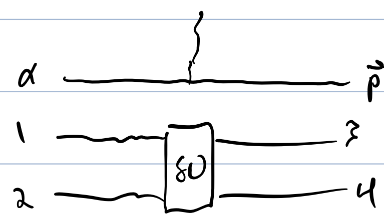

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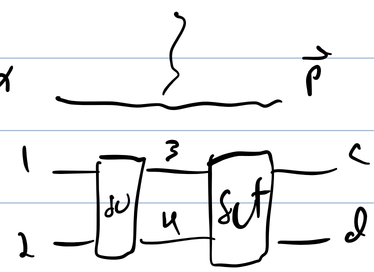

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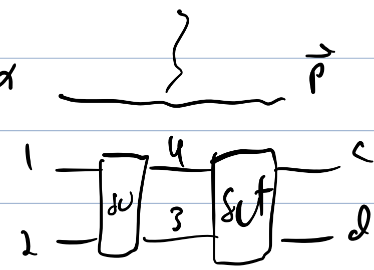


Recall $\hat{U}_\lambda^\dagger \hat{U}_\lambda = \hat{I}$

$$\parallel$$

$$\left[\hat{I} + \frac{1}{4}\sum \delta U_\lambda^\dagger \ldots\right]\left[\hat{I} + \frac{1}{4}\sum \delta U_\lambda \ldots\right]$$

$$= \hat{I} + \frac{1}{4}\sum_{\vec{K}\vec{k}\vec{k}'}\left[\delta U_\lambda^\dagger(\vec{k},\vec{k}') + \delta U_\lambda(\vec{k},\vec{k}') + \frac{1}{2}\sum_{\vec{k}''}\delta U_\lambda^\dagger(\vec{k},\vec{k}'')\,\delta U_\lambda(\vec{k}'',\vec{k}')\right]$$

$$\times\, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}}\, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}}\, a_{\frac{\vec{K}}{2}-\vec{k}'}\, a_{\frac{\vec{K}}{2}+\vec{k}'} \qquad (13)$$

$$\Rightarrow \delta U_1^+(\vec{k},\vec{k}') + \delta U_1(\vec{k},\vec{k}') + \frac{1}{2}\sum_{\vec{k}''} \delta U_1^+(\vec{k},\vec{k}'')\,\delta U_1(\vec{k}'',\vec{k}') = 0 \qquad (14)$$

- The diagrams correspond to

$$= \frac{1}{4}\sum_{1234}\left(\delta\tilde{U}^+_{1234}\, a_\alpha^+ a_1^+ a_2^+ a_{\vec{p}}\, a_4 a_3 + \delta\tilde{U}_{1234}\, a_\alpha^+ a_1^+ a_2^+ a_4 a_3 a_{\vec{p}}\right)$$

$$+ \frac{1}{4}\frac{1}{2}\sum_{1234}\sum_{ab} \delta\tilde{U}_{12ab}\,\delta\tilde{U}^+_{ab34}\; a_\alpha^+ a_1^+ a_2^+ a_{\vec{p}}\, a_4 a_3$$

$$\{a_{\vec{p}}, a_i\} = 0 \Rightarrow a_\alpha^+ a_1^+ a_2^+ a_4 a_3 a_{\vec{p}} = a_\alpha^+ a_1^+ a_2^+ a_{\vec{p}}\, a_4 a_3$$

$$= \frac{1}{4}\sum_{1234}\Big[\underbrace{\delta\tilde{U}^+_{1234} + \delta\tilde{U}_{1234} + \frac{1}{2}\sum_{ab}\delta\tilde{U}_{12ab}\,\delta\tilde{U}^+_{ab34}}_{=0}\Big]\, a_\alpha^+ a_1^+ a_2^+ a_{\vec{p}}\, a_4 a_3$$

$\Rightarrow$ These disconnected diagrams cancel

Update formula for $a_\alpha^+ \hat{U}_1\, a_{\vec{p}}\, \hat{U}_1^+$ and diagrams

$a_\alpha^\dagger \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger$ 3-body level

$$\approx a_\alpha^\dagger a_{\vec{p}} + \frac{1}{4}\sum_{abcd} \delta\tilde{U}^\dagger_{abcd}\left(\delta_{\vec{p}a}\, a_\alpha^\dagger a_b^\dagger a_d a_c - \delta_{\vec{p}b}\, a_\alpha^\dagger a_a^\dagger a_d a_c\right)$$

$$+ \frac{1}{4}\frac{1}{4}\sum_{1234}\sum_{abcd} \delta\tilde{U}_{1234}\, \delta\tilde{U}^\dagger_{abcd}\Big[\left(\delta_{3b}\delta_{\vec{p}a} - \delta_{3a}\delta_{\vec{p}b}\right) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_4 a_d a_c$$

$$+ \left(\delta_{4b}\delta_{\vec{p}a} - \delta_{4a}\delta_{\vec{p}b}\right) a_\alpha^\dagger a_1^\dagger a_2^\dagger a_3 a_d a_c\Big]$$

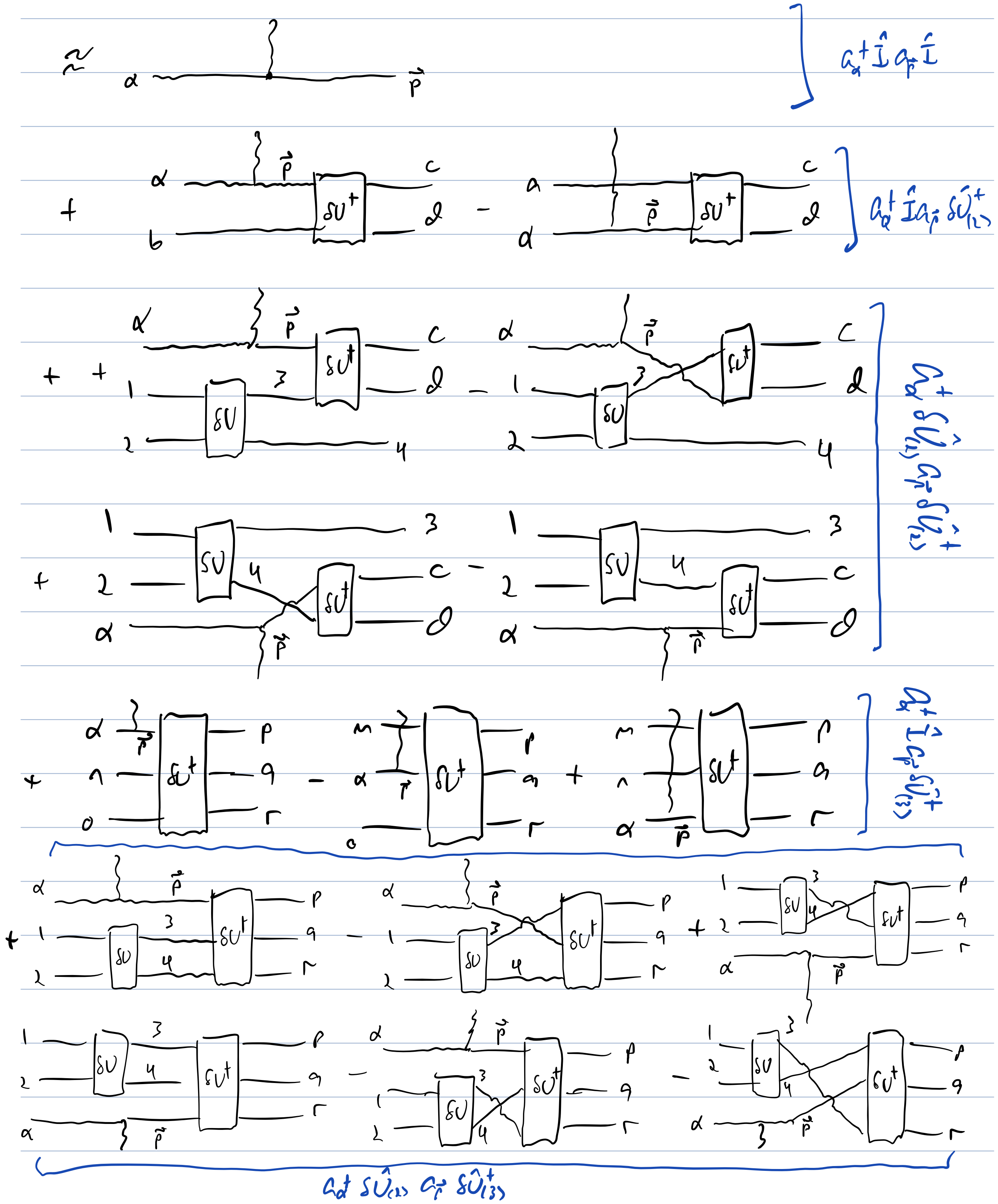
$$+ \frac{1}{36}\sum_{mnopqr} \delta\tilde{U}^\dagger_{mnopqr}\Big(\delta_{\vec{p}m}\, a_\alpha^\dagger a_n^\dagger a_o^\dagger a_r a_q a_p$$

$$- \delta_{\vec{p}n}\, a_\alpha^\dagger a_m^\dagger a_o^\dagger a_r a_q a_p + \delta_{\vec{p}o}\, a_\alpha^\dagger a_m^\dagger a_n^\dagger a_r a_q a_p\Big)$$

$$+ \frac{1}{4}\frac{1}{36}\sum_{1234}\sum_{mnopqr} \delta\tilde{U}_{1234}\, \delta\tilde{U}^\dagger_{mnopqr}\Big[\delta_{4o}\left(\delta_{3n}\delta_{\vec{p}m} - \delta_{3m}\delta_{\vec{p}n}\right)$$

$$+ \delta_{4n}\left(\delta_{3m}\delta_{\vec{p}o} - \delta_{3o}\delta_{\vec{p}m}\right) + \delta_{4m}\left(\delta_{3n}\delta_{\vec{p}o} - \delta_{3o}\delta_{\vec{p}n}\right)\Big] a_\alpha^\dagger a_1^\dagger a_2^\dagger a_r a_q a_p \tag{15}$$

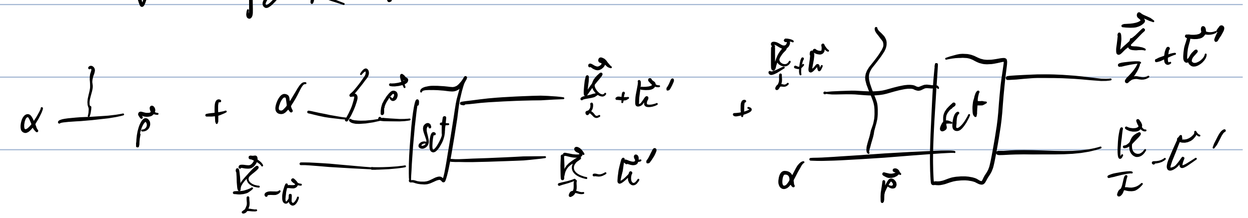
$a_\alpha^\dagger \hat{U}_t a_{\vec{p}} \hat{U}_t^\dagger$



- Evaluate first 3 diagrams assuming Fermi gas $|\Psi^A_{0\alpha}\rangle \equiv |F\rangle$ where

$$\langle F|\, a^\dagger_{\vec k} a_{\vec k} \,|F\rangle = \begin{cases} 1 & k<k_F \\ 0 & k>k_F \end{cases} = \Theta(k_F - k) \qquad (16)$$

If you average over $k_F(R)$ then $\frac{1}{V}\int d^3R\, \Theta(k_F(R)-k)$ where $V = \int d^3R$?



Change $a^\dagger_\alpha$ to momentum: $a^\dagger_\alpha = \int d^3p'\, \phi_\alpha(\vec p')\, a^\dagger_{\vec p'}$ (17)

$$\langle F| \Big[\int d^3\vec p'\, \phi_\alpha(\vec p') \Big(a^\dagger_{\vec p'} a_{\vec p} + \frac{1}{4}\int d^3K \int d^3k \int d^3k'$$

$$\times \Big(\delta U^\dagger(\vec k, \vec k')\, \delta^3\!\left(\vec p - \tfrac{\vec K}{2} - \vec k\right) a^\dagger_{\vec p'} a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}$$

$$- \delta U^\dagger(\vec k, \vec k')\, \delta^3\!\left(\vec p - \tfrac{\vec K}{2} + \vec k\right) a^\dagger_{\vec p'} a^\dagger_{\frac{\vec K}{2}+\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'} \Big)\Big) |F\rangle \qquad (18)$$

$$= \int d^3\vec p'\, \phi_\alpha(\vec p') \langle F|\, a^\dagger_{\vec p'} a_{\vec p} \,|F\rangle$$

$$+ \frac{1}{4} \int d^3\vec p' \int d^3K \int d^3k \int d^3k'\, \phi_\alpha(\vec p') \Big[\delta U^\dagger(\vec k, \vec k')\, \delta^3\!\left(\vec p - \tfrac{\vec K}{2} - \vec k\right)$$

$$\times \langle F | a^\dagger_{\vec p'} a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'} | F \rangle - \delta U^\dagger(\vec k, \vec k')\, \delta^3\!\left(\vec p - \frac{\vec K}{2} + \vec k\right) a$$

$$\times \langle F | a^\dagger_{\vec p'} a^\dagger_{\frac{\vec K}{2}+\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'} | F \rangle \Big] \qquad (19)$$

$$\langle F | a^\dagger_{\vec p'} a_{\vec p} | F \rangle = \delta^3(\vec p' - \vec p)\, \Theta(k_F - p)$$

$$\langle F | a^\dagger_{\vec p'} a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'} | F \rangle = \delta^3\!\left(\vec p' - \frac{\vec K}{2} - \vec k'\right) \delta^3(\vec k - \vec k')$$
$$\times\, \Theta\!\left(k_F - \left|\frac{\vec K}{2}+\vec k\right|\right) \Theta\!\left(k_F - \left|\frac{\vec K}{2}-\vec k\right|\right) - \delta^3\!\left(\vec p' - \frac{\vec K}{2} + \vec k'\right) \delta^3(\vec k + \vec k')\, \Theta\!\left(k_F - \left|\frac{\vec K}{2}+\vec k\right|\right) \Theta\!\left(k_F - \left|\frac{\vec K}{2}-\vec k\right|\right) \qquad (20)$$

$$\langle F | a^\dagger_{\vec p'} a^\dagger_{\frac{\vec K}{2}+\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'} | F \rangle = \delta^3\!\left(\vec p' - \frac{\vec K}{2} - \vec k'\right) \delta^3(\vec k + \vec k')$$
$$\times\, \Theta\!\left(k_F - \left|\frac{\vec K}{2}-\vec k\right|\right) \Theta\!\left(k_F - \left|\frac{\vec K}{2}+\vec k\right|\right) - \delta^3\!\left(\vec p' - \frac{\vec K}{2} + \vec k'\right) \delta^3(\vec k - \vec k')\, \Theta\!\left(k_F - \left|\frac{\vec K}{2}-\vec k\right|\right) \Theta\!\left(k_F - \left|\frac{\vec K}{2}+\vec k\right|\right) \qquad (21)$$

Solve system of equations:

I) $\vec p = \frac{\vec K}{2} + \vec k \qquad \frac{\vec K}{2} = \vec p - \vec k \qquad \Theta(k_F - p)\,\Theta(k_F - |\vec p - 2\vec k|)$

$\vec p' = \frac{\vec K}{2} + \vec k' \quad \Rightarrow \quad \vec p' = \vec p$

$\vec k = \vec k'$

II) $\vec p = \frac{\vec K}{2} + \vec k \qquad \frac{\vec K}{2} = \vec p - \vec k \qquad \Theta(k_F - p)\,\Theta(k_F - |\vec p - 2\vec k|)$

$\vec p' = \frac{\vec K}{2} - \vec k' \quad \Rightarrow \quad \vec p' = \vec p$

$\vec k = -\vec k'$

III) $\vec p = \frac{\vec K}{2} - \vec k \qquad \frac{\vec K}{2} = \vec p + \vec k \qquad \Theta(k_F - p)\,\Theta(k_F - |\vec p + 2\vec k|)$

$\vec p' = \frac{\vec K}{2} + \vec k' \quad \Rightarrow \quad \vec p' = \vec p$

$\vec k = -\vec k'$

IV) $\vec p = \frac{\vec K}{2} - \vec k \qquad \vec k = \vec k' \quad \rightarrow \quad \vec p' = \vec p \qquad \Theta(k_F - p)\,\Theta(k_F - |\vec p + 2\vec k|)$

$\vec p' = \frac{\vec K}{2} - \vec k' \qquad \frac{\vec K}{2} = \vec p + \vec k$

$$= \phi_\alpha(\vec{p})\, \Theta(k_F - p) \Big[1 + \frac{1}{4} \times 8 \int d^3k \, \Big(\delta U^\dagger(\vec{k},\vec{k})\, \Theta(k_F - |\vec{p}-2\vec{k}|)$$

$$- \delta U^\dagger(\vec{k},-\vec{k})\, \Theta(k_F - |\vec{p}-2\vec{k}|) - \delta U^\dagger(\vec{k},-\vec{k})\, \Theta(k_F - |\vec{p}+2\vec{k}|)$$

$$+ \delta U^\dagger(\vec{k},\vec{k})\, \Theta(k_F - |\vec{p}+2\vec{k}|) \Big) \Big]$$

$$= \phi_\alpha(\vec{p})\, \Theta(k_F - p) \Big[1 + 4 \int d^3k \, \delta U^\dagger(\vec{k},\vec{k}) \Big(\Theta(k_F - |\vec{p}-2\vec{k}|)$$

$$+ \Theta(k_F - |\vec{p}+2\vec{k}|) \Big] \qquad (22)$$

$$S_\alpha = \int d^3p \, \Big| \phi_\alpha(\vec{p})\, \Theta(k_F - p) \Big[1 + 4 \int d^3k \, \delta U^\dagger(\vec{k},\vec{k}) \Big(\Theta(k_F - |\vec{p}-2\vec{k}|)$$

$$+ \Theta(k_F - |\vec{p}+2\vec{k}|) \Big) \Big] \Big|^2 \qquad (23)$$

Before we integrated over R to get $k_F^\tau(R) = [3\pi^2 \rho^\tau(R)]^{1/3}$

This would give incorrect units here. How to choose k_F?

We can connect $\delta U^\dagger$ term to depletion from SRC. From unitarity

$$\Big[\delta\tilde{U}_\lambda(\vec{k},\vec{k}') + \delta\tilde{U}_\lambda^\dagger(\vec{k},\vec{k}') + \frac{1}{2} \int d^3q \, \delta\tilde{U}_\lambda(\vec{k},\vec{q})\, \delta\tilde{U}_\lambda^\dagger(\vec{q},\vec{k}') \Big] = 0 \qquad (24)$$

Let $\vec{k}' = \vec{k}$. Then $\delta \tilde{U}_\lambda(\vec{k},\vec{k}) = \delta \tilde{U}_\lambda^\dagger(\vec{k},\vec{k})$

and we have

$$\delta \tilde{U}_\lambda^\dagger(\vec{k},\vec{k}) = -\frac{1}{4}\int d^3q \; \delta \tilde{U}_\lambda(\vec{k},\vec{q})\,\delta \tilde{U}_\lambda^\dagger(\vec{q},\vec{k}) \tag{25}$$

Substitute (25) into (23):

$$S_\alpha = \int d^3p \,\Big| \phi_\alpha(\vec{p})\,\Theta(k_F - p)\Big[1 - \int d^3k \int d^3q \; \delta U_\lambda(\vec{k},\vec{q})\,\delta U_\lambda^\dagger(\vec{q},\vec{k})$$

$$\times \Big(\Theta(k_F - |\vec{p}-2\vec{k}|) + \Theta(k_F - |\vec{p}+2\vec{k}|)\Big)\Big]\Big|^2 \tag{26}$$

SRC depletion from 2nd term