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880.05 Class #1

Handouts:

- printouts of 880.05 homepage and info page (references only)
- "Recent developments in the nuclear many-body problem"
- phenomenological potentials

Course logistics:

- any conflict with meeting time? MW 10:30-12:18 Sm 2186
- step through info sheet
  - "Nuclear Many-Body Physics"
  - no text to buy but you may decide that one or more of the reserve texts are worth having (warning: many are expensive!)

    In place of a text, we will hand out exerpts when appropriate and I will post my notes in pdf format on the web page. I will try to post them promptly.

    password protected pages? → (Note: for copyright reasons, I can't post the exerpts.)

- "Nuclear" means nuclear examples but course material more general. Nuclear many-body problems have complications and we will always "do the easy problems first" (Also, the nuclear problems we're interested in are, at least in part, unsolved ⇒ learn tools to attack them.)

  Methods include path integrals and effective actions, effective field theory, renormalization group. Sounds scary but...

  your job to ask questions if I assume too much!

- Prerequisites are only the 1st year graduate courses. Field theory is not assumed. However, we'll use a lot of quantum and statistical mechanics, plus ideas from E&M (like Green's functions)

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- Instructors:
  Dick Furnstahl — my research specialization is nuclear many-body theory
  Achim Schwenk — postdoc in NTG, particular expertise in renormalization group methods applied to nuclear physics. PhD advisor at SUNY/Stony Brook was Gerry Brown, one of the many-body "greats".

- Schedule:
  - we'll take a break in the middle (use for extra questions)
  - problem sessions? (à la 2064 sessions?)

- Office hours:
  - dropping by Smith 4004 (Furnstahl) or 4080 (Schwenk) will work a lot of the time (just interrupt!), especially early (for Furnstahl)
  - schedule in class or via email
  - we'll figure out other times as we go

- Grading:
  - Only problem sets. — (schedule to be determined) "hands on" part of the course
  - You are strongly encouraged to collaborate except for the last problem set.

- Web page:
  - source of handouts (when possible), notes, problem sets, announcements
  - background (or supplementary) reading assignments will be from excerpts handed out ahead of time
  - Feedback — essential in class and out. PLEASE ask questions (and answer them!). Anonymous comments are fine.

- Philosophy: Downplay formal development when introducing new topics and build around the study of illustrative examples. In problem sets, extend, refine or treat analogous examples.

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References: (Course reserves are for 2-hour checkout; too short?)

- Fetter and Walecka is one of the great texts (ok, I'm biased: Walecka was my PhD advisor and I took the course from Fetter – also on my PhD committee). BUT 30 years old and although basic problems are still valid, new methods have developed (path integrals, EFT, etc.).
  Also expensive. But rigorous and correct and accessible.
- Negele and Orland: In many ways an updated FHW because uses path integrals. Philosophy is to do formal development in the main text and most examples and applications in the problems. Terrible pedagogy and not so great as a reference but thorough and correct and rigorous on path integral techniques.
- Nagaosa: Recent text with good coverage of path integral methods (including for superconductivity) and symmetry breaking.
- Stone: A book worth looking at for explanations clarifying tricky points. Combined particle physics and nonrelativistic many-body field theory. Not much depth because of wide range.
- Mattuck: "Guide to Feynman Diagrams..." is full of intuitive discussions of Feynman diagrams. Doesn't hit everything, but what it does cover is still relevant today. 2nd edition 1976.
- Mahan is a standard and encyclopedic reference for condensed matter applications.
- For nuclear background, Ring and Schuck and Siemens and Jensen are good for the many body problem, while Donoghue is good for low-energy QCD (which is nuclear physics!) · More references to come...

- We'll make review articles available to cover newer topics, such as effective field theory.

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Overview of Course

e.g. atoms in a trap; nucleus

- We'll be considering nonrelativistic many-body problems
  - basic idea: given interactions between 2 (or few) particles, calculate observables for many particle system
  - nonrelativistic ⇒ momentum $p \ll$ particle mass M (so $v \ll c$)
    - aside: for nuclear case, relativistic covariance (time component of four-vector vs. scalar) is believed by some to be important
  - observables include ground state energy (and density distribution in finite system), equation of state, other thermodynamic functions, energies and lifetimes of excited states, linear response to external probes, ...

- This is Nuclear many-body physics but much more general for several reasons:
  - universal features for different physical systems will be emphasized
  - the problems of greatest interest are not solved, so we'll have to look at simpler ones as examples
  - interested primarily in tools for solving many-body problems

- See the review talk by Furnstahl on new developments in the nuclear many-body problem
  - plenary talk at conference of many-body experts
  - we'll learn the tools needed to carry out some of the program.
  - don't worry about understanding details — just a "teaser"
  - some aspects:
    - pairing phenomena (e.g. "color superconductivity")
    - effective field theory methods — applying to many-body problems
    - renormalization group methods and shell model

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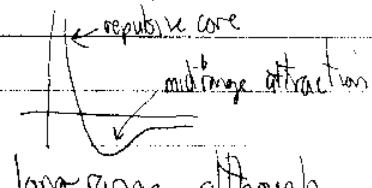
- Consider the handout with nucleon-nucleon and ^3He-^3He atomic potentials
  - these are (effective) potential between a neutron and a proton, or between two ^3He atoms as function of separation.
  - note that NN potential is for relative orbital angular momentum $L=0$ and total spin $S=0$ $\Rightarrow$ potential is different for different L,S, which complicates the nuclear problem!

- Note the similar qualitative features:



  - very strong short-range repulsion
  - attractive at intermediate range and long-range, although fall off much faster than Coulomb
  - universal figure number: 2.4 — we'll derive that later :)

- Where do these potentials come from?
  - One source is from a more fundamental underlying interaction
  - In ^3He, that interaction is quantum electrodynamics (QED), which reduces here primarily to the good old Coulomb interaction
  - Each ^3He is a few-body problem with Coulomb potential $V_c(\vec{r}_1-\vec{r}_2) \propto \frac{e^2}{|\vec{r}_1-\vec{r}_2|}$ between each of the two electrons

    and also a potential between the e^-'s and the (inert) nucleus.
  - Put two together and we can find the potential energy as a function of the separation of the nuclei
  - First done variationally by Slater and Kirkwood in 1931 [Exercise: look this up, in Physical Review 37 (1931) 682.]
  - Basic physics:
    - repulsion from Coulomb repulsion of overlapping electron clouds $\Rightarrow$ very steep function of separation r
    - attraction from induced electric dipoles: "mutual electric polarization" (2nd-order perturbation theory implies attractive if atoms are in their ground states) $\boxed{E^{(2)} = \sum_{j\neq 0} \frac{|\langle 0|H_1|j\rangle|^2}{E_0-E_j} < 0}$

Nucleon-Nucleon and ^3He-^3He Potentials

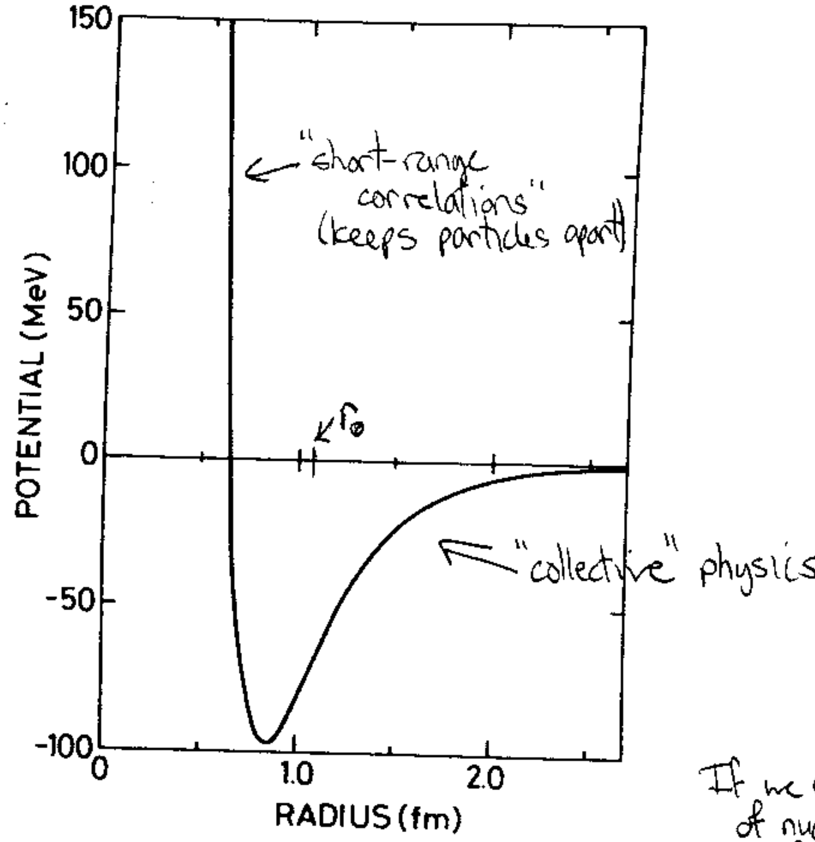


Figure 2.4b Phenomenology of nucleon-nucleon scattering for L = S = 0.
Reid's phenomenological potential (MeV) as a function of the nucleons' separation (fm). [*From R. V. Reid*]

From Siemens and Jensen, "Elements of Nuclei"

If we characterize density of nuclei or liquid helium as $R \sim r_0 A^{1/3}$ then $r_0 \sim 1.1$ fm for nuclei and 2.44 Å for ^3He.
One might expect $r_0 \sim c$, the core radius. (close packed spheres predicts $r_0 \sim 0.55c$), but considerably larger in nuclear case. Implication is that nuclear matter is "delicately" bound

which implies sensitive cancellations and that many-body forces will be important. Hard problem.

From Vollhardt and Wölfle, "The Superfluid Phases of Helium 3"

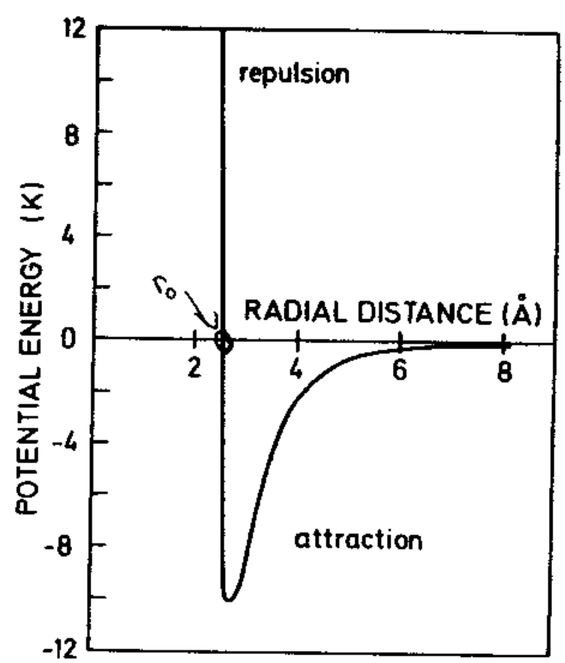


Figure 2.4 Interaction potential of two ^{3}He atoms as a function of separation.

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- For nucleon-nucleon case, we also know the underlying interaction for each ⇒ quantum chromodynamics (or QCD).
- Each nucleon is also a composite system of quarks interacting via gluon exchange (cf. electrons and photon exchange). But we don't have a simple analog to Coulomb ⇒ can't generate N-N potential from first principles (at present)

- Long-range is understood as originating from one-pion exchange (pion is a spin-zero meson with charge $-1, +1, 0$ and rest energy of about 140 MeV/c^2) as (essentially) rigorous consequence of QCD (effective field theory!)
- Shorter-range is more phenomenological
  - 2 pion exchange attraction (heavier, so shorter range)
  - vector (spin 1) meson ρ, ω exchange ⇒ short-range repulsion (cf. spin-1 photon exchange between same charges)

- One of the major tasks in the nuclear many-body problem has been to find this potential
  - How do you know it is correct? Calculate experimental observables and compare to data.
  - One two-body bound state (deuteron)
  - lots of two-nucleon scattering data (phase shifts)
  - (later) you will explore this data and address the question of how features of the phase shift imply this qualitative form.

  - This task is to a large degree complete: several potentials reproduce data at energies up to inelastic processes occur with χ^2/dof of close to 1.

- "Several"? You mean the potential is not unique? More below...

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• What might we expect *qualitatively* from a many-body system with such a potential?

$\Rightarrow$ compare to ideal gas $PV = nRT$ (n is # of moles)

① hard core repulsion means particles can't use entire volume (eg. "excluded volume") so

$$\boxed{V \to (V - nb)}$$ with b constant

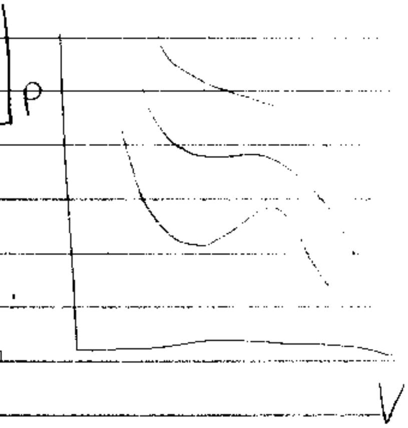
② attraction means the pressure (on the container for example) is reduced (since momentum is less or consider limit of N-body bound state), so

$$\boxed{P = \frac{nRT}{V-nb} \to P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}}$$

(more effective if closer $\Rightarrow$ V smaller, see Huang's book for a more detailed rationale for the form)

Combined $\Rightarrow$ $$\boxed{\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT}$$

$\Rightarrow$ Van der Waals equation of state



Recall that there is a liquid-gas phase transition.

And, in fact, there is one in the nuclear case!

• Probed in low-energy heavy-ion collisions.

• Liquid helium has very interesting physics $\to$ it is a superfluid
  • a consequence of the attractive interaction $\Rightarrow$ pairing as in a superconductor (much more later!)
  • liquid ^3He very different than ^4He $\Rightarrow$ fermions vs. bosons is important!
  • nuclei also exhibit pairing (we'll explore this later)

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· Is the potential unique?

· No. For example, it is local as given here. One can construct non-local potentials that reproduce the same scattering and bound-state data.

Usual S-eqn (1-D) (eigenvalue eqn)
$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)}$$

· Kinetic energy term is nonlocal slightly:

discrete approximation:
$$\boxed{\frac{d^2\psi(x)}{dx^2} \xrightarrow{x \to x_i} \frac{\psi(x_{i+1}) - 2\psi(x_i) + \psi(x_{i-1})}{2\Delta x}}$$

involves $\psi(x)$ at nearby points in a simple way.

Do the same for $V(x)$

$$\boxed{V(x_i)\psi(x_i) \to \sum_j V(x_i, x_j)\psi(x_j) \to \int dx'\, V(x,x')\psi(x')}$$ "nonlocal potential"

· It is also energy independent (same potential no matter what relative energy of the particles).

· The potential is designed to fit scattering over a wide range of momenta. What if we only care about low momenta?

· Low momentum ⇒ long wavelength

· cf. multipole expansion: at large wavelength, complicated charge or current distribution behaves like leading multipole (point charge or point dipole or ...)

⇒ replace complicated potential by simpler version that reproduces data in limited range.

⇒ basic idea of effective field theory and renormalization group methods we'll discuss is to do this systematically

· Here, replace potential by delta function: $\boxed{V(\vec{x}_1 - \vec{x}_2) = \lambda\, \delta^3(\vec{x}_1 - \vec{x}_2)}$.

Reproduces very low-energy scattering. Actually excellent for atoms in traps!

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So, given a potential, how do we solve a many-body problem?

The many-particle Hamiltonian (N particles) is

$$\boxed{\begin{aligned} H &= \sum_{i=1}^{N} T(x_i) + \frac{1}{2}\sum_{i\neq j=1}^{N} V(x_i, x_j) \\ &= \sum_{i=1}^{N} T(x_i) + \sum_{i<j} V(x_i, x_j) \end{aligned}}$$

} avoid double counting

where T is the kinetic energy and V the potential energy.
(including any external one-body potential)

Here $x_i \equiv \{\vec{x}_i, s_z, t_z, \ldots\}$ denotes all coordinates of i^{th} particle.

- e.g. $T(x_i) = -\frac{\hbar^2\nabla_i^2}{2m}$ usually

- Then time-dependent S-eqn is

$$\boxed{i\hbar\frac{\partial}{\partial t}\Psi(x_1, \ldots, x_N; t) = H\Psi(x_1, \ldots, x_N; t)}$$

- If we find Ψ (or the energy eigenstates), we can calculate all we want!

- How might we solve this? Enumerate N=1 methods:
  - solve differential equation in coordinate space (integrodiff. eq in general)
    $\Rightarrow$ very hard when multidimensional
  - solve integral equation in momentum space
  - write $H = H_0 + H_1$ with H_0 solvable and do perturbation theory in H_1 (one choice is $H_0 = T$, $H_1 = V$)
  - diagonalize H in a complete basis
  - use the variational principle: minimize $\frac{\langle\psi_{trial}|H|\psi_{trial}\rangle}{\langle\psi_{trial}|\psi_{trial}\rangle}$ with respect to parameters in trial wave function $|\psi_{trial}\rangle$
  - project out the ground state starting from a wave function $|\varphi_0\rangle$ with non-zero overlap by applying $e^{-H\tau}|\varphi_0\rangle$
    - e.g. in one-d with one particle,
      $\varphi_0(x) = a_0\psi_0(x) + a_1\psi_1(x) + \ldots$ where $H\psi_i(x) = E_i\psi_i(x)$ are exact eigenstates

Then $e^{-H\tau}\varphi_0(x) = a_0 e^{-E_0\tau}\psi_0(x) + a_1 e^{-E_1\tau}\psi_1(x) + \ldots \xrightarrow[\tau\to\infty]{} a_0 e^{-E_0\tau}\psi_0$

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- This might remind you of two things:
  - a Boltzmann factor $e^{\beta H} \Rightarrow T \leftrightarrow \frac{1}{\beta}$
  - formal solution to time evolution:

$$\boxed{i\hbar \frac{\partial}{\partial t} \Psi(x,t) = H\Psi(x,t) \Rightarrow \Psi(x,t) = e^{iHt/\hbar}\Psi(x,0)}$$

$\Rightarrow$ evolution in imaginary time.

- We'll see both connections later!

- At the start of FtW (1971), they basically say that solving for $\Psi(x_1, \ldots, x_N)$ is impractical and that motivates alternative (2nd quant.)
  - But not really true for nuclei! "Green's function monte carlo"
  - Variational and a form of projection (GFMC) are now possible up to $A \sim 10$ (A is total of protons + neutrons)
    - multidimensional integrals solved by Monte Carlo methods
  - See Furnstahl talk for pictures.
  - Important $\Rightarrow$ show that 3-body forces are essential for quantitative description
  - Also shell model — diagonalizing, but solve problems of too large basis space by converting to equivalent answer in small space $\Rightarrow$ new $H_{\text{effective}}$

- Nevertheless, to deal with larger systems, we'll consider 2nd quantization.