AJT Notes (3/19/21)

- Start with single-particle momentum distribution in partial waves then use $|\psi_\delta^A(r)|^2$ to define $\rho_N^A(r)$.

$$\hat{n}_A^\tau(q) = V\Big[2\,\theta(k_F^\tau - q) + \frac{2}{\pi}\int_0^\infty dk\,k^2 \int_{-1}^{1} d(\hat{q}\cdot\hat{k}) \times$$

$$\Big\{\delta U_{^1S_0}(k,k)\big[\theta(k_F^p - |\vec{q}-2\vec{k}|) + \tfrac{1}{2}\theta(k_F^n - |\vec{q}-2\vec{k}|)\big] +$$

$$\frac{2J+1}{2}\delta U_{^3S_1-^3S_1}(k,k)\,\theta(k_F^n - |\vec{q}-2\vec{k}|)\,\theta(k_F^p - q)\Big\} +$$

$$\frac{1}{4}\left(\frac{2}{\pi}\right)^2 \sum_{x={}^3S_1,{}^3D_1} \int_0^\infty dk\,k^2 \int_0^\infty dK\,K^2 \int_{-1}^{1} d(\hat{K}\cdot\hat{k})\Big\{\delta U_{^1S_0}(k,|\vec{q}-\tfrac{1}{2}\vec{K}|)\times$$

$$\delta U_{^1S_0}^\dagger(|\vec{q}-\tfrac{1}{2}\vec{K}|,k)\,\theta(k_F^p - |\tfrac{1}{2}\vec{K}+\vec{k}|)\,\theta(k_F^p - |\tfrac{1}{2}\vec{K}-\vec{k}|) +$$

$$\frac{1}{4}\delta U_{^1S_0}(k,|\vec{q}-\tfrac{1}{2}\vec{K}|)\,\delta U_{^1S_0}^\dagger(|\vec{q}-\tfrac{1}{2}\vec{K}|,k)\big[\theta(k_F^p - \ldots)\,\theta(k_F^n - \ldots) +$$

$$\theta(k_F^n - \ldots)\,\theta(k_F^p - \ldots)\big] + \frac{2J+1}{4}\delta U_{^3S_1-x}(k,|\vec{q}-\tfrac{1}{2}\vec{K}|)\times$$

$$\delta U_{x-^3S_1}^\dagger(|\vec{q}-\tfrac{1}{2}\vec{K}|,k)\big[\theta(k_F^p - |\tfrac{1}{2}\vec{K}+\vec{k}|)\,\theta(k_F^n - |\tfrac{1}{2}\vec{K}-\vec{k}|) +$$

$$\theta(k_F^n - |\tfrac{1}{2}\vec{K}+\vec{k}|)\,\theta(k_F^p - |\tfrac{1}{2}\vec{K}-\vec{k}|)\big]\Big] \qquad (1)$$

So for Deuteron : $T = 0,\ S = 1$

$$\hat{n}^p_d(q) = V\Big[\, 2\,\Theta(k_F^p - q) + \frac{2}{\pi}\frac{(2J+1)}{2}\int_0^\infty dk\, k^2 \int_{-1}^{1} d(\hat{q}\cdot\hat{k}) \times$$

$$\delta U_{^3S_1-^3S_1}(k,k)\,\Theta(k_F^n - |\vec{q} - 2\vec{k}|)\,\Theta(k_F^p - q) \;+$$

$$\frac{1}{4}\left(\frac{2}{\pi}\right)^2 \frac{(2J+1)}{2} \sum_{x={}^3S_1,{}^3D_1} \int_0^\infty dk\, k^2 \int_0^\infty dK\, K^2 \int_{-1}^{1} d(\hat{K}\cdot\hat{k}) \times$$

$$\delta U_{^3S_1-x}(k, |\vec{q} - \tfrac{1}{2}\vec{K}|)\, \delta U^\dagger_{x-^3S_1}(|\vec{q} - \tfrac{1}{2}\vec{K}|, k)\, \Theta(k_F - |\tfrac{1}{2}\vec{K} + \vec{k}|)$$

$$\Theta(k_F - |\tfrac{1}{2}\vec{K} - \vec{k}|)\,\Big] \qquad (2)$$

$$k_F(r) \approx \left(3\pi^2 |\psi_d^d(r)|^2\right)^{1/3} \qquad (3)$$

$$\hat{n}_d(q) = V\Big[\, 2\,\Theta(k_F - q) + \frac{2}{\pi}\frac{\overset{?}{(2J+1)}}{2}\int_0^\infty dk\, k^2 \times$$

$$\int_{-1}^{1} d(\hat{q}\cdot\hat{k})\, \delta U_{^3S_1-^3S_1}(k,k)\, \Theta(k_F - |\vec{q} - \tfrac{1}{2}\vec{k}|)\, \Theta(k_F - q) \;+$$

$$\frac{1}{8}\left(\frac{2}{\pi}\right)^2 (2J+1) \sum_{x={}^3S_1,{}^3D_1} \int_0^\infty dk\, k^2 \int_0^\infty dK\, K^2 \int_{-1}^{1} d(\hat{K}\cdot\hat{k}) \times$$

$$\delta U_{^3S_1-x}(k, |\vec{q} - \tfrac{1}{2}\vec{K}|)\, \delta U^\dagger_{x-^3S_1}(|\vec{q} - \tfrac{1}{2}\vec{K}|, k)\, \Theta(k_F - |\tfrac{1}{2}\vec{K} + \vec{k}|) \times$$

$$\Theta(k_F - |\tfrac{1}{2}\vec{K} - \vec{k}|)\,\Big] \qquad (4)$$

Then $\langle n_d(q) \rangle_d = 4\pi \int_0^\infty dr\, r^2 \, \frac{n_d(q)}{V}$ (5)

$$\rho^d(q) = q^2 \langle n_d(q) \rangle_d / A \quad (6)$$

Check $\int dq\, \rho^d(q) = 1$. Also check

$\int \rho^A(q)\, dq = 1$. When both are correctly normalized then do

$$a_2^A = \frac{\int_2^\infty dq\, \rho^A(q)}{\int_2^\infty dq\, \rho^d(q)} \quad (7)$$

* Questions :

- Factor of $2J+1$ for deuteron ?
- Sensitivity to K_array ?
- Sensitivity to X_array ?
- Check equations in single_particle_momentum_dist.py
- $\int dq\, \rho^A(q) = Z/A$ for asymmetric nuclei?