Operator evolution from the similarity renormalization group

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Motivation

- Explosion of new NN interactions from chiral effective field theory (χ^{EFT}) in the last few years
- Previous SRG studies of operators were limited to phenomenological models or one χ^{EFT} interaction
- Revisit the question of how different potentials (regulator functions, cutoff, order, etc.) change under SRG transformations and how these transformations affect other operators

SRG formalism

- SRG transformations decouple low- and high-momenta in Hamiltonian

$$H(s) = U(s)H(0)U^{\dagger}(s)$$

where $s = 0 \rightarrow \infty$

- In practice, solve differential flow equation

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

with SRG generator $\eta(s) \equiv \frac{dU(s)}{ds}U^{\dagger}(s) = [G, H(s)]$

SRG formalism

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling

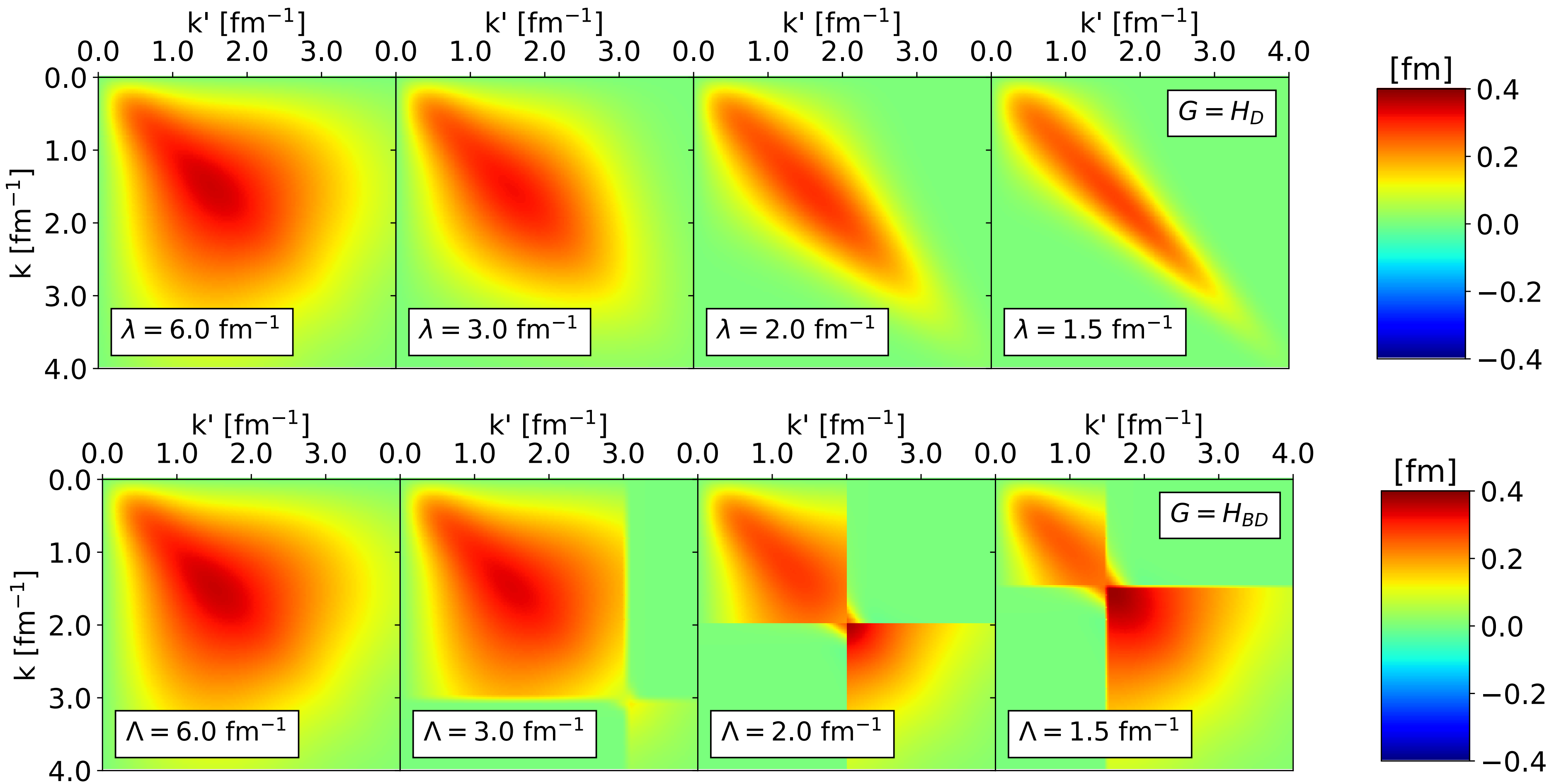
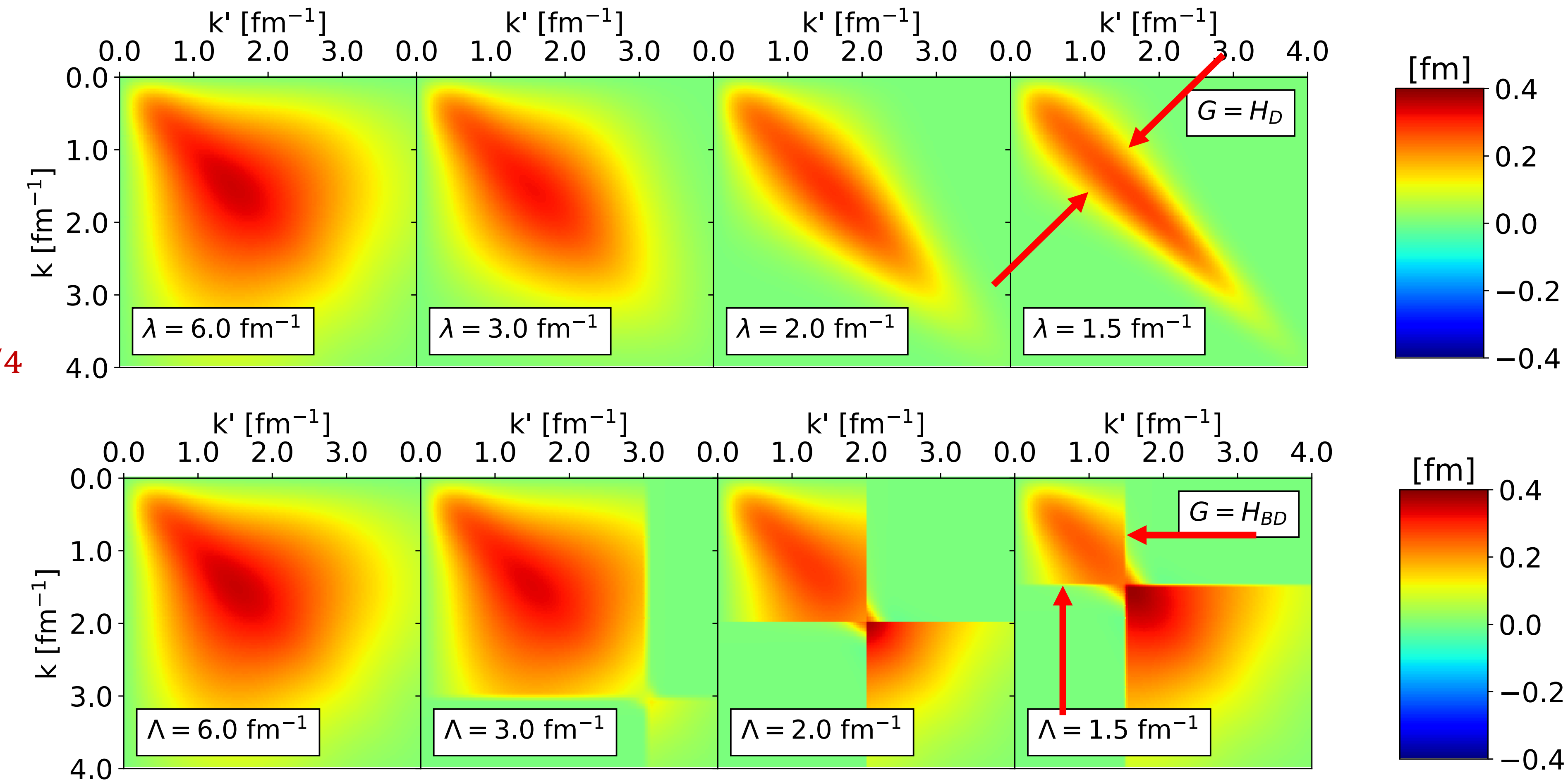


Fig. 1: SRG evolution of $V_\lambda(k, k')$ for several values of λ and Λ in the $^1P_1$ channel. Potentials from P. Reinert et al., Eur. Phys. J. A **54**, 86 (2018) which will be referred to as the RKE potentials.

SRG formalism

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling
- Parameters $\lambda = s^{-1/4}$ and Λ describe the decoupling of the evolved Hamiltonian



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SRG evolution of modern chiral potentials

- How do different potentials change under SRG transformations?

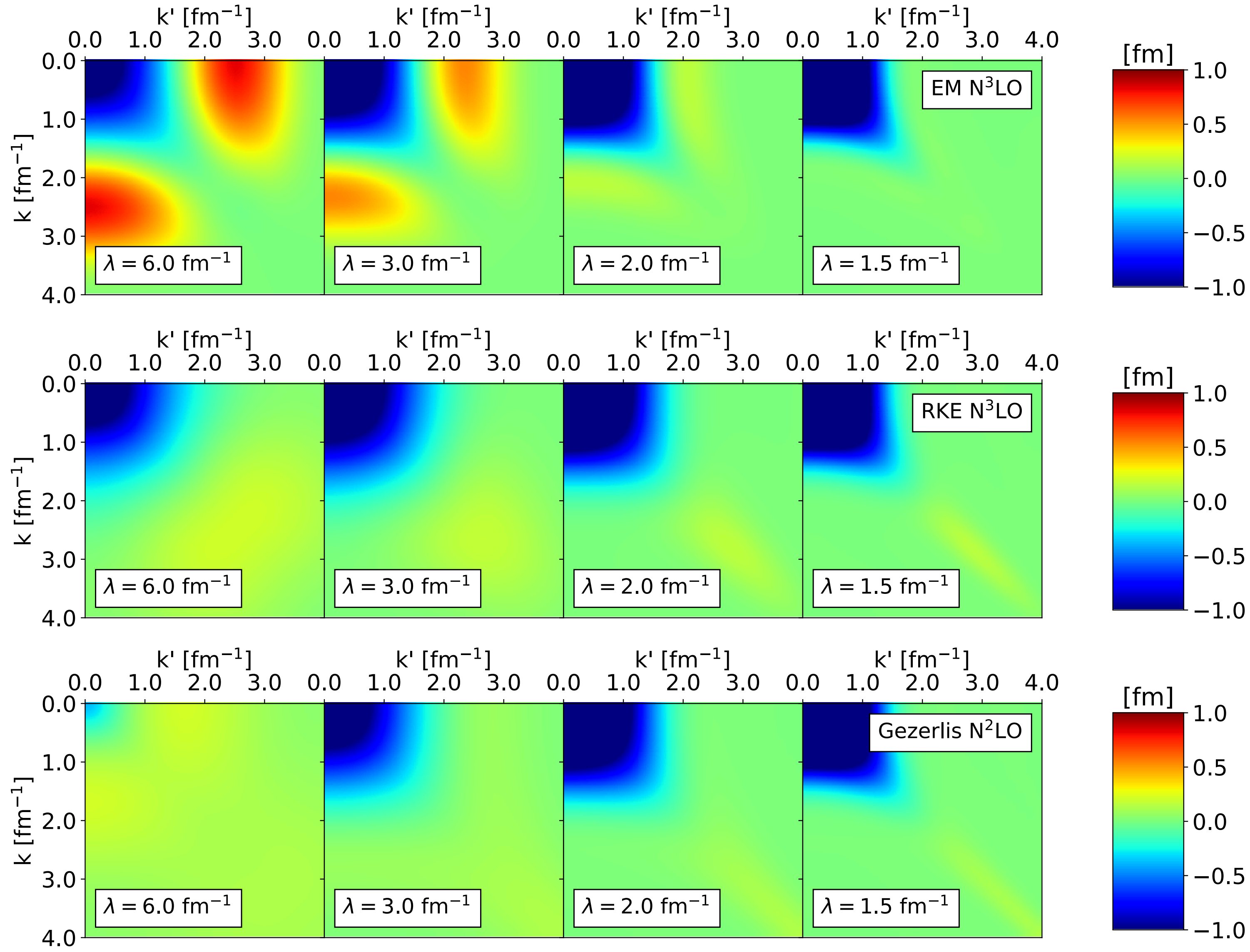


Fig. 2: SRG evolution of $V_\lambda(k,k')$ for several chiral potentials.

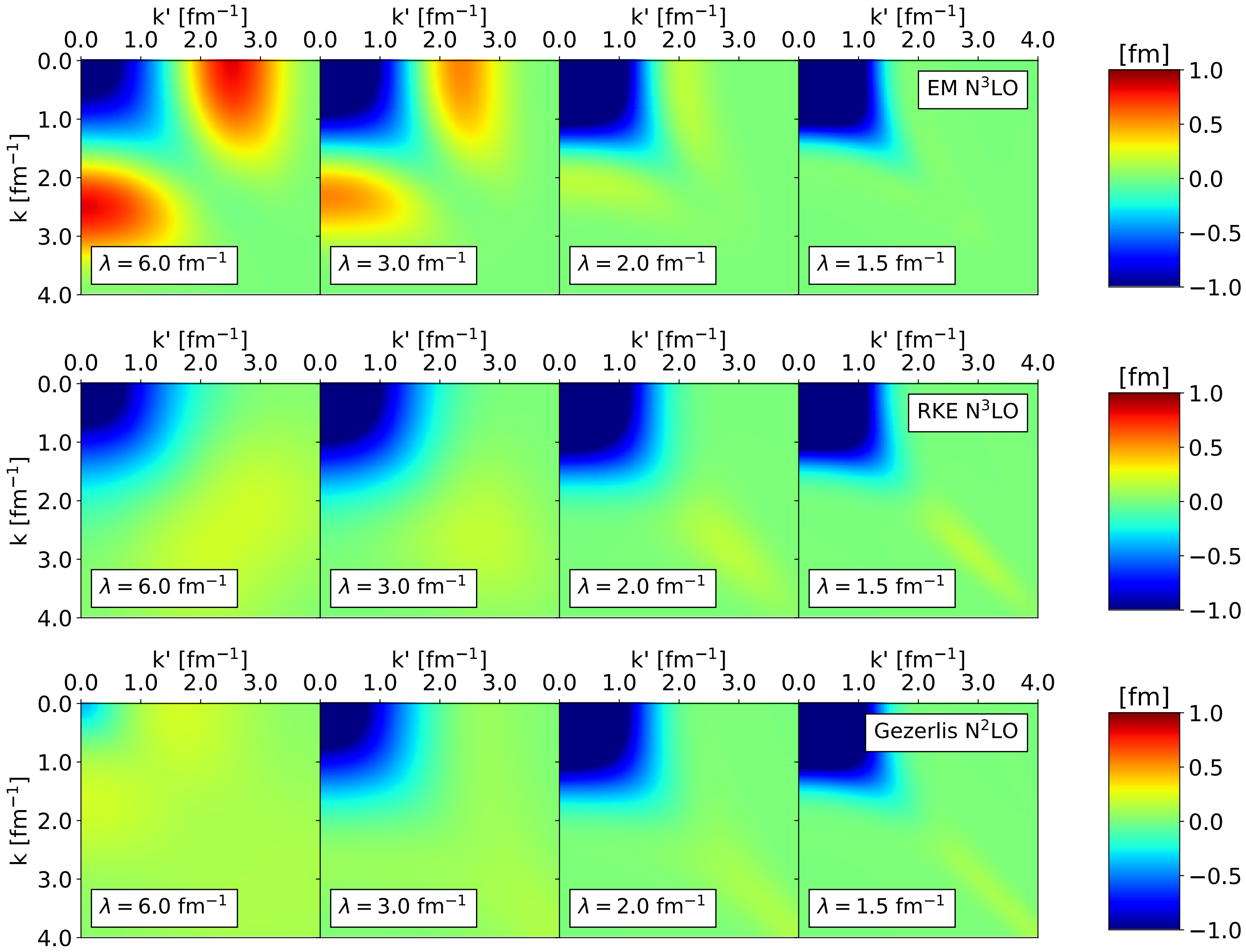
SRG evolution of modern chiral potentials

- How do different potentials change under SRG transformations?
- Use non-local Entem-Machleidt[1], semi-local RKE[2], and local Gezerlis et al.[3] potentials as examples

[1]D.R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003)

[2]P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A **54**, 86 (2018)

[3]A. Gezerlis, et al., Phys. Rev. C **90**, 054323 (2014)



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SRG evolution of modern chiral potentials

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- Use non-local Entem-Machleidt[1], semi-local RKE[2], and local Gezerlis et al.[3] potentials as examples
- Different potentials evolve to the same low-momentum matrix elements!

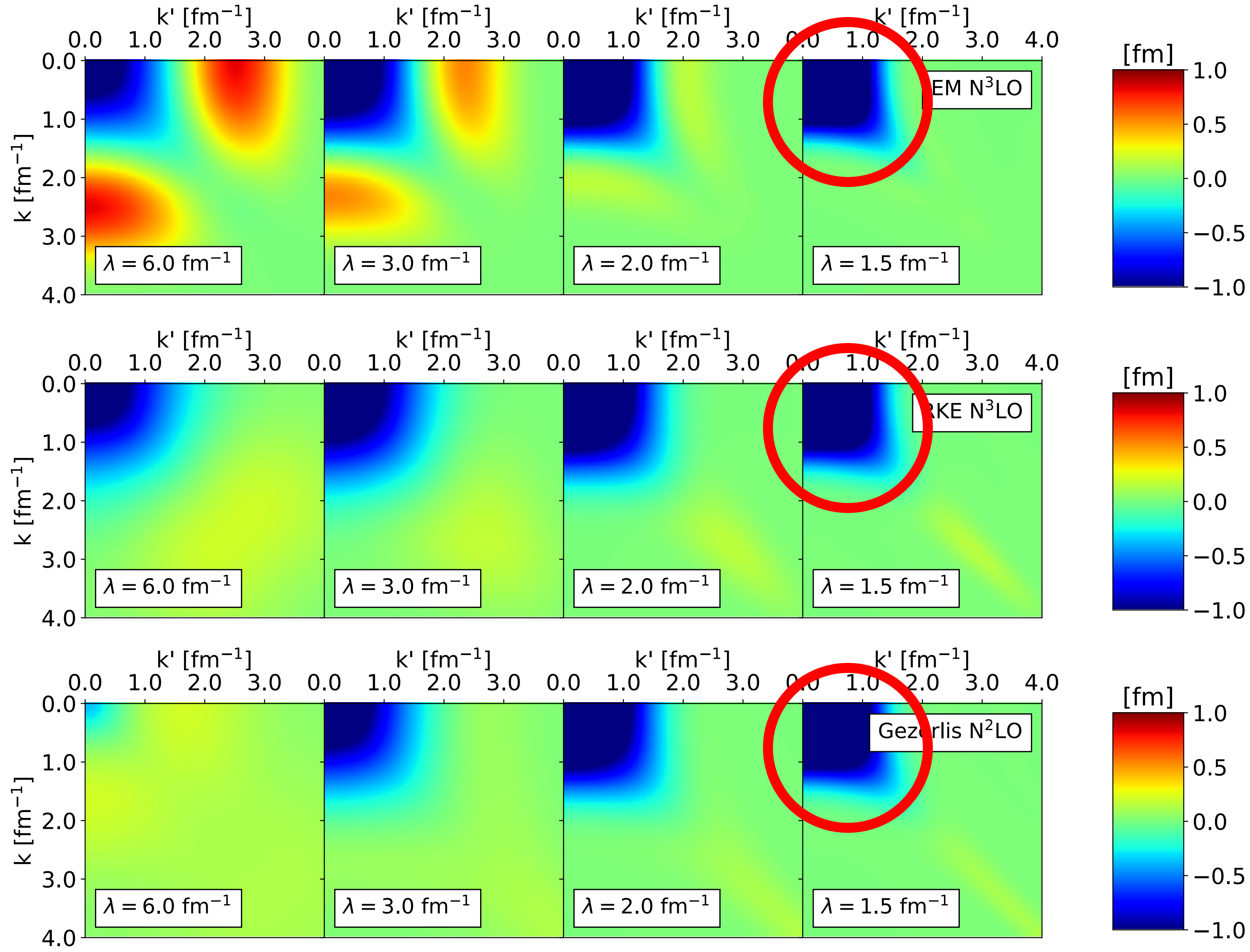


Fig. 2: SRG evolution of $V_\lambda(k,k')$ for several chiral potentials.

Universality in SRG-evolved potentials

- Evolved matrix elements collapse to the same lines

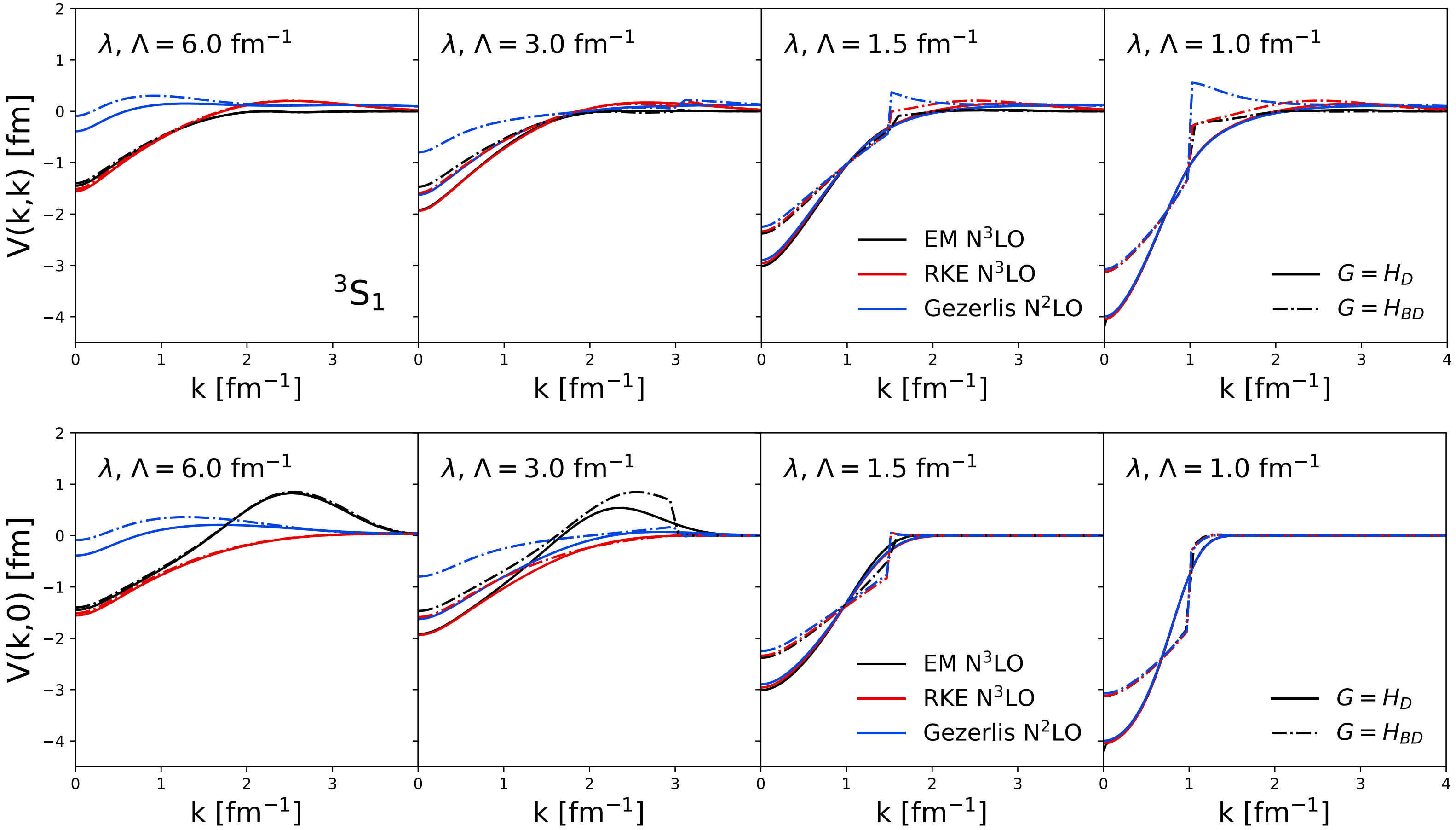
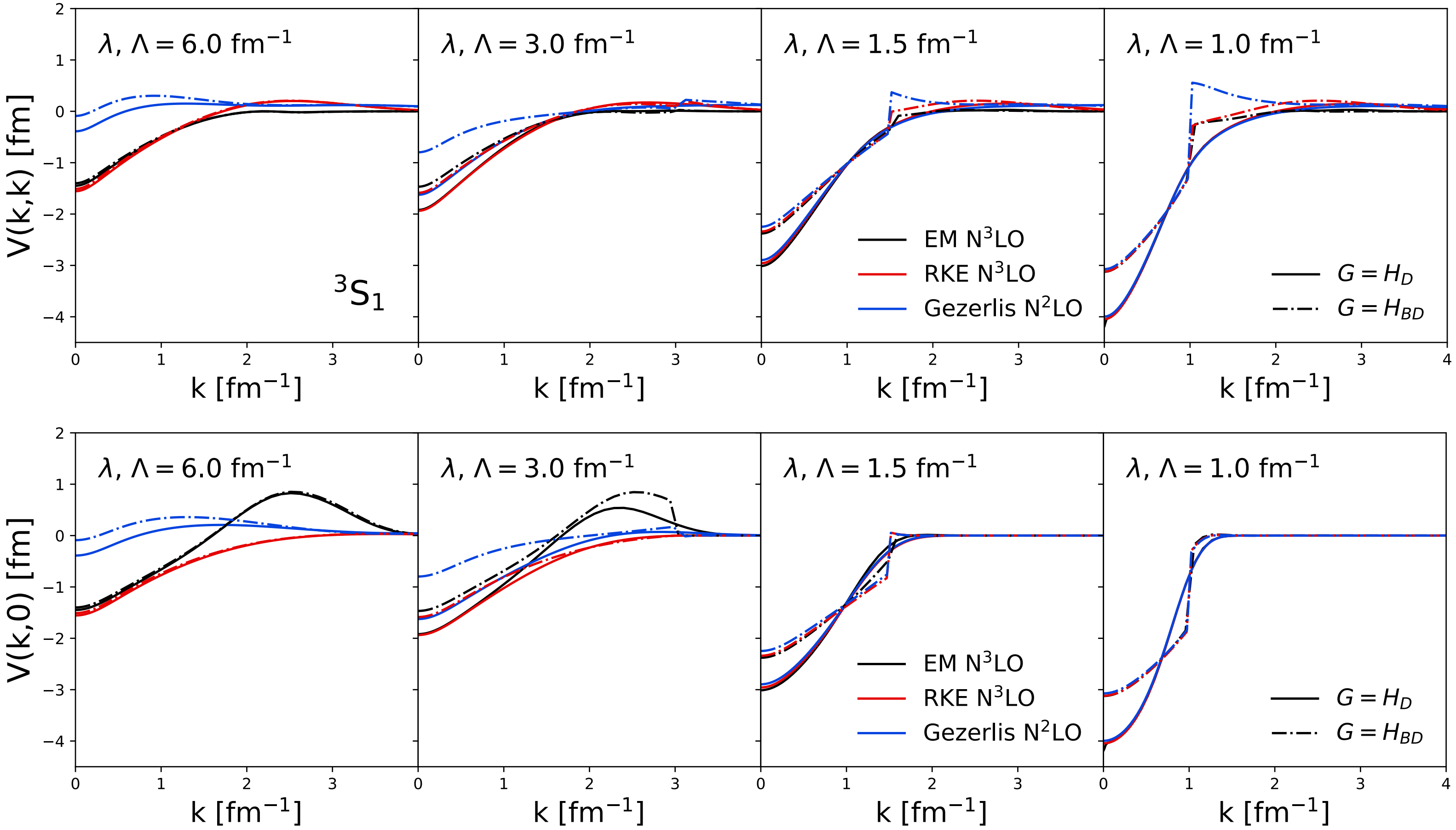
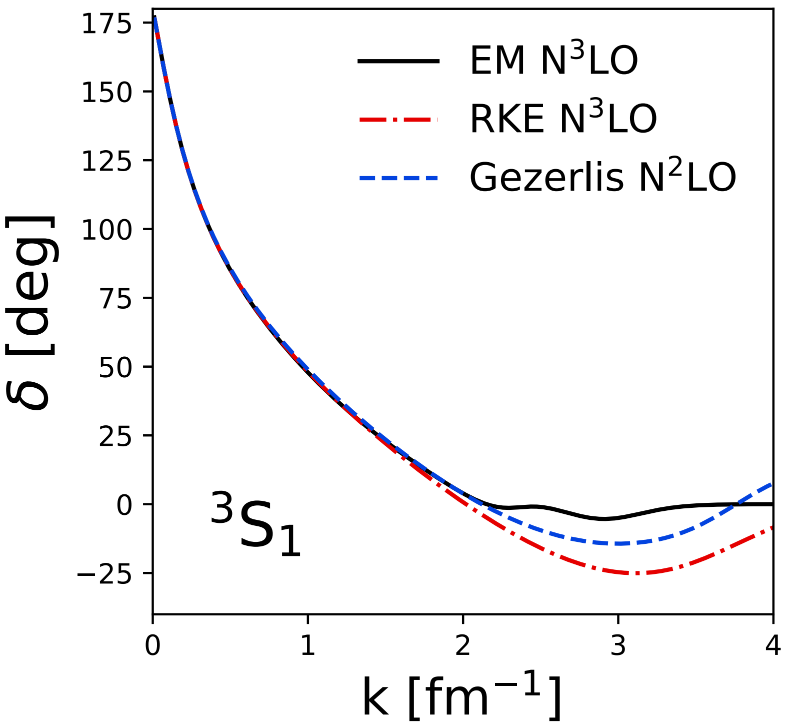


Fig. 3: Diagonal and far off-diagonal matrix elements of $V_\lambda(k, k')$ for several chiral potentials with band- and block-diagonal decoupling.

Universality in SRG-evolved potentials

- Universality in potential matrix elements is due to equivalent low-energy phase shifts[1]





[1]B. Dainton et al., Phys. Rev. C **89**, 014001 (2014)

SRG evolution for other operators

- SRG transformations will decouple the Hamiltonian but this behavior is not necessarily true for any operator

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SRG evolution for other operators

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- What are the characteristics of other evolved operators?
- Does universality hold for evolved operators?

Momentum projection operator

- We use the momentum projection operator $a_q^\dagger a_q$ as a test case

$$a_q^\dagger a_q \psi(k) = \psi(q)$$

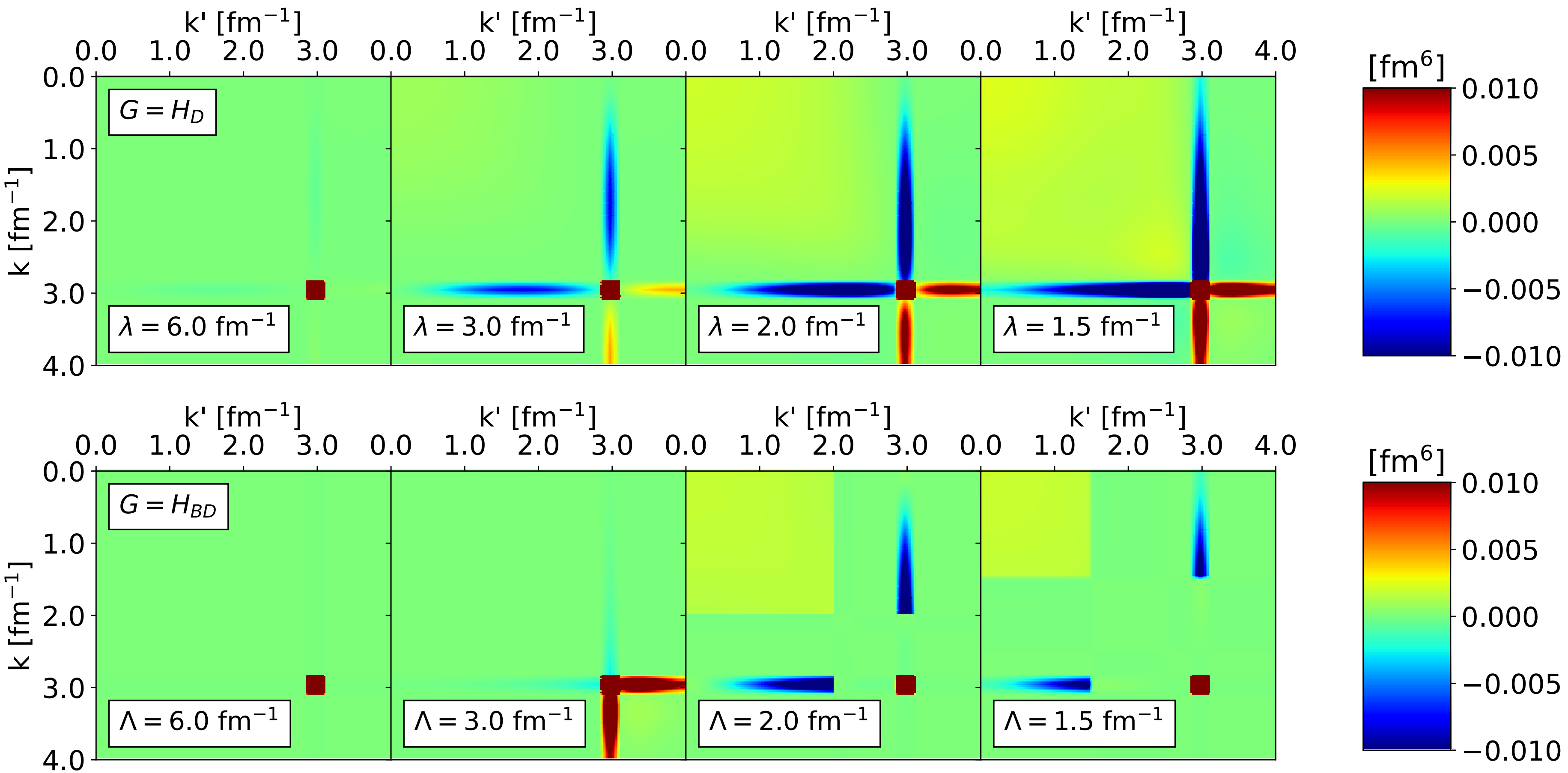

Fig. 4: SRG evolution of $a_q^\dagger a_q(k, k')$ for several values of λ and Λ where the transformations are done using the RKE N^{3}LO potential. Here $q = 3$ fm^{-1}.

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- We use the momentum projection operator $a_q^\dagger a_q$ as a test case

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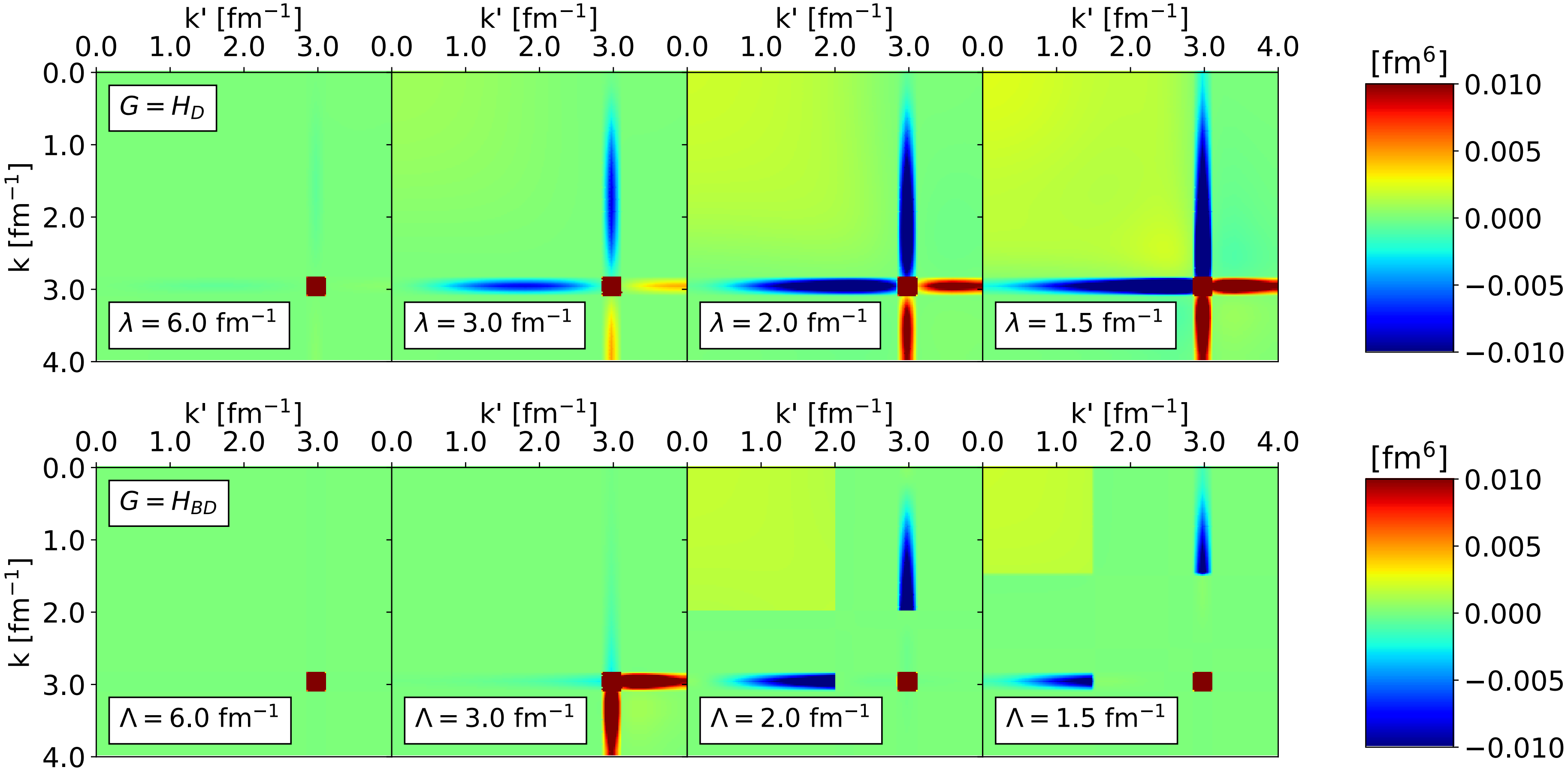
- Initially starts out as a δ-function
- SRG transformations induce low-momentum contributions



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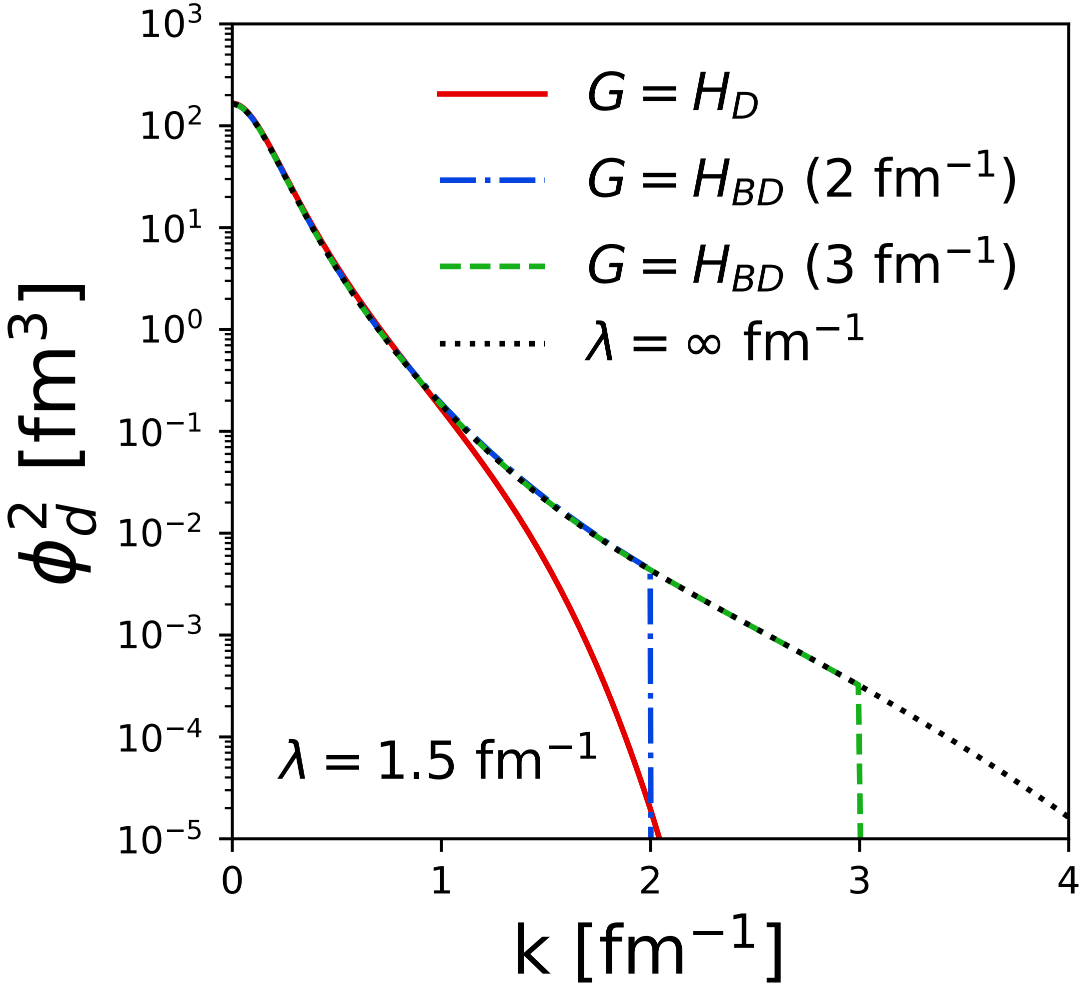

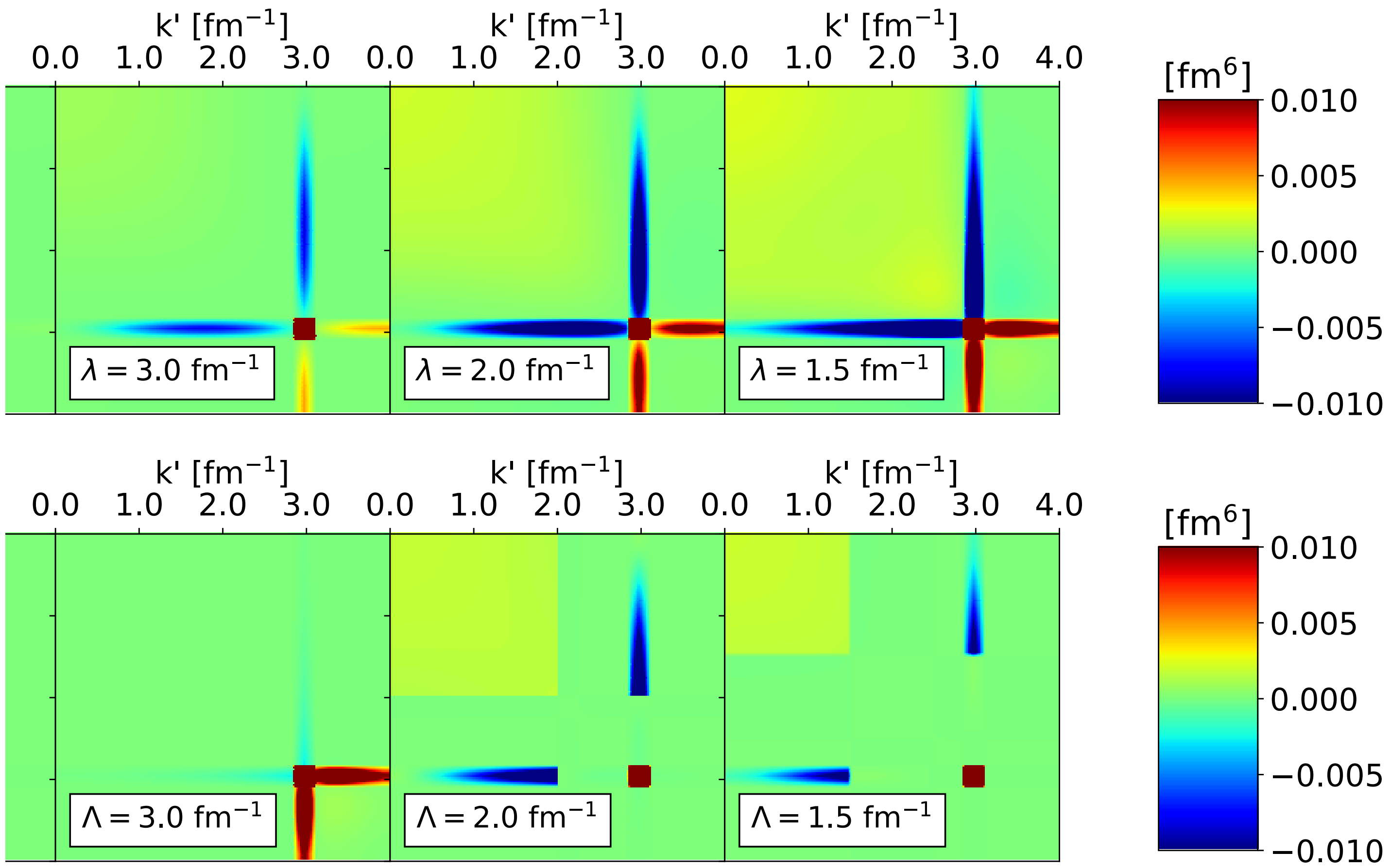

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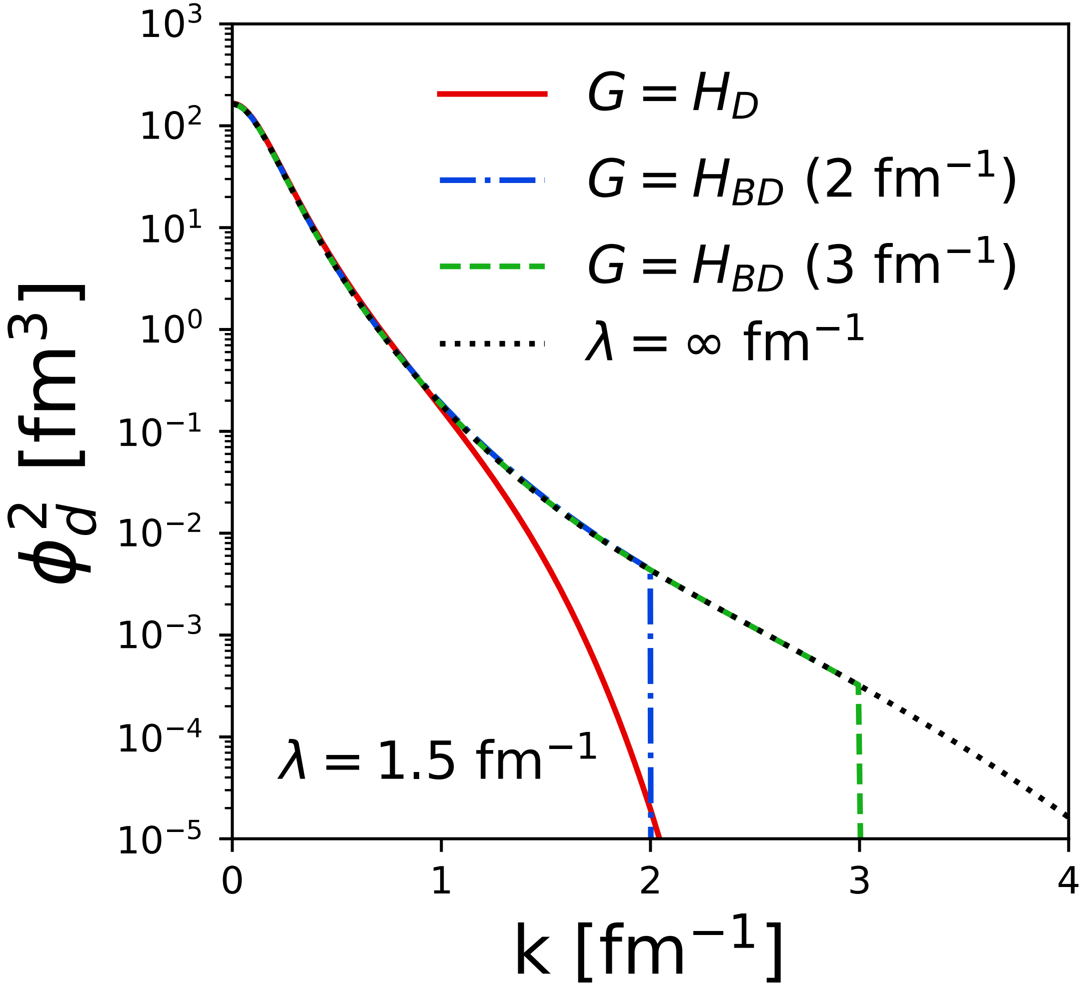
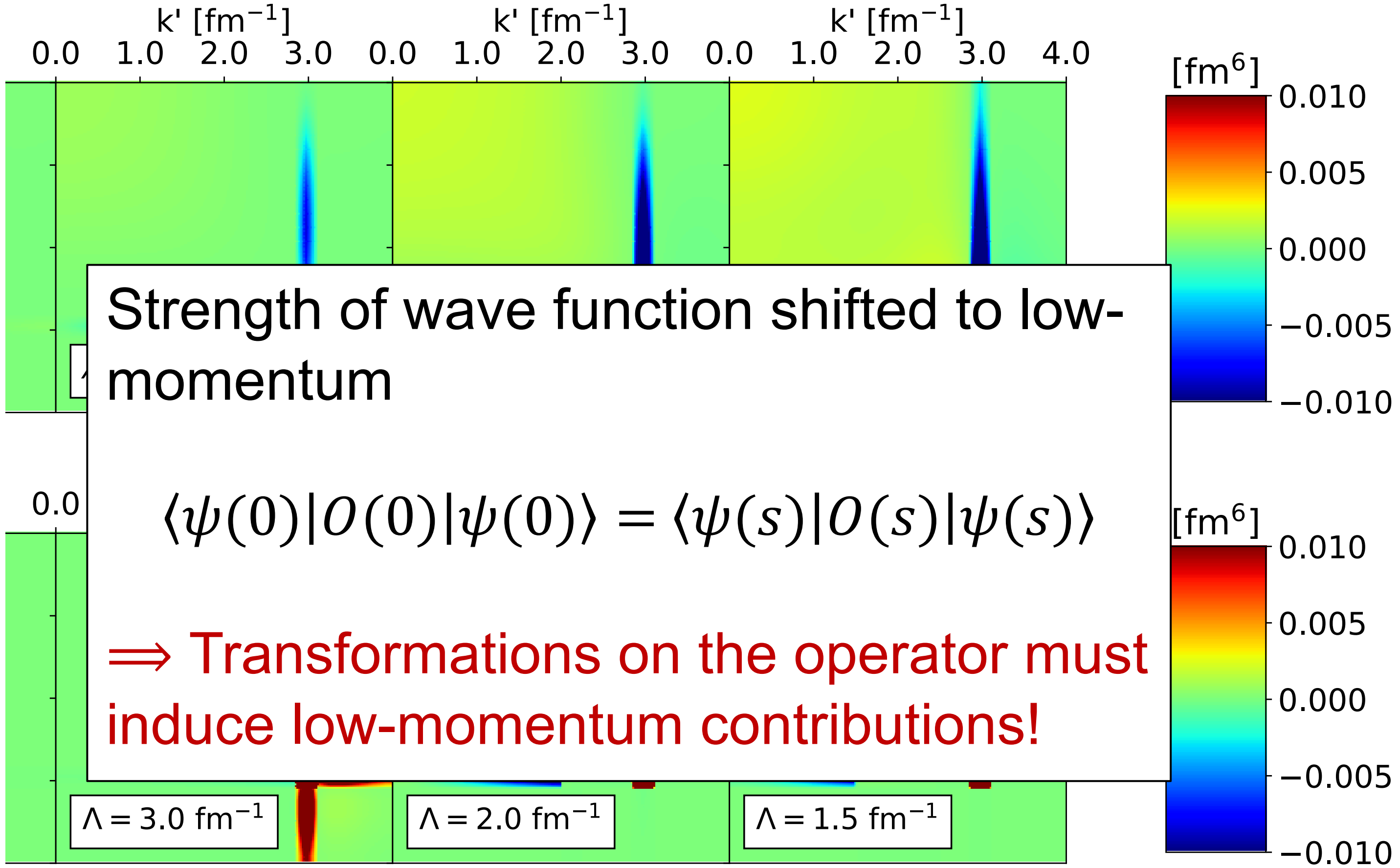




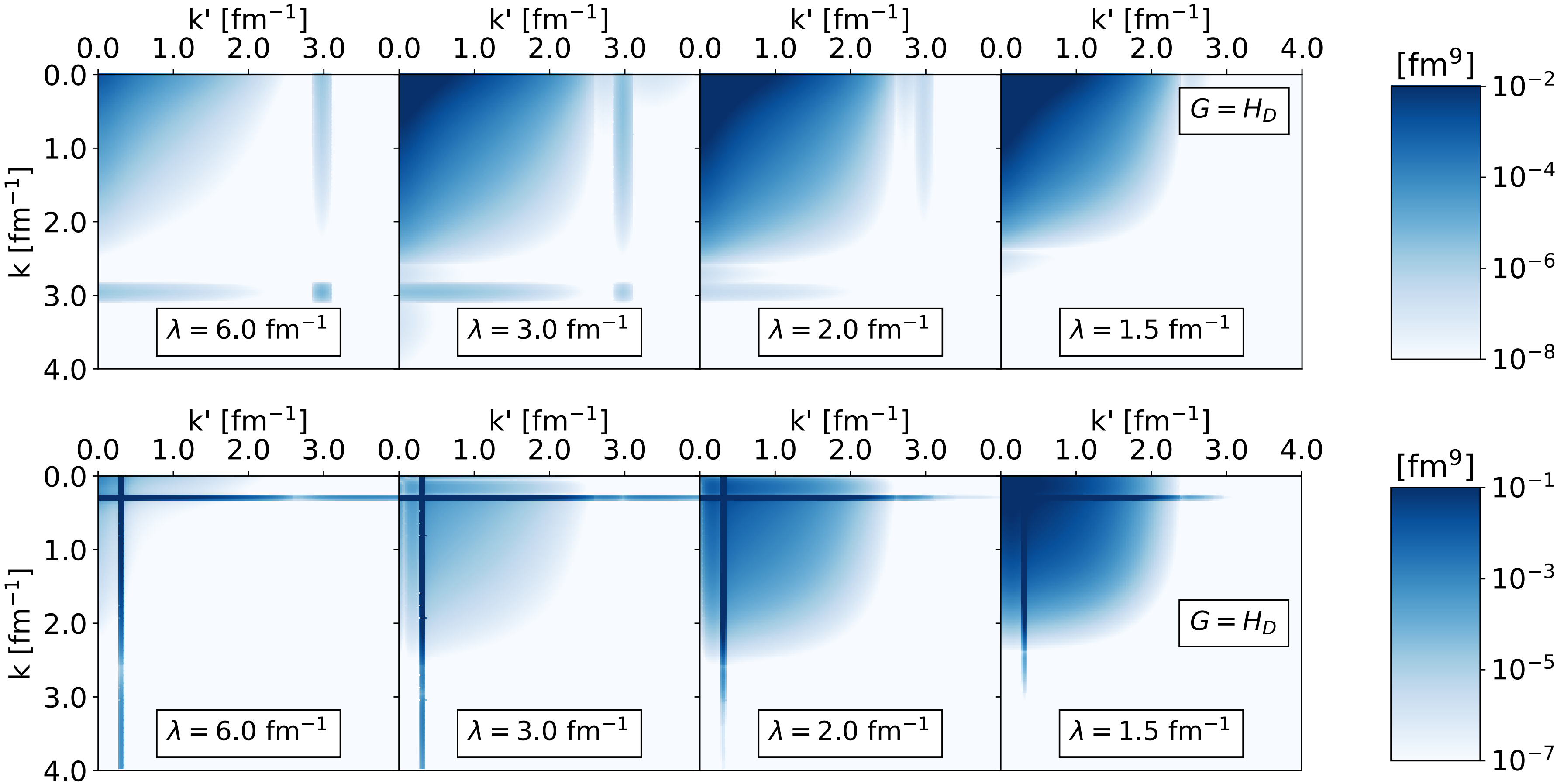
Strength of wave function shifted to low-momentum

$$\langle \psi(0) | O(0) | \psi(0) \rangle = \langle \psi(s) | O(s) | \psi(s) \rangle$$

$\Rightarrow$ Transformations on the operator must induce low-momentum contributions!

Fig. 4: SRG evolution of $a_q^\dagger a_q(k,k')$ for several values of λ and Λ where the transformations are done using the RKE N^{3}LO potential. Here $q = 3$ fm^{-1}.

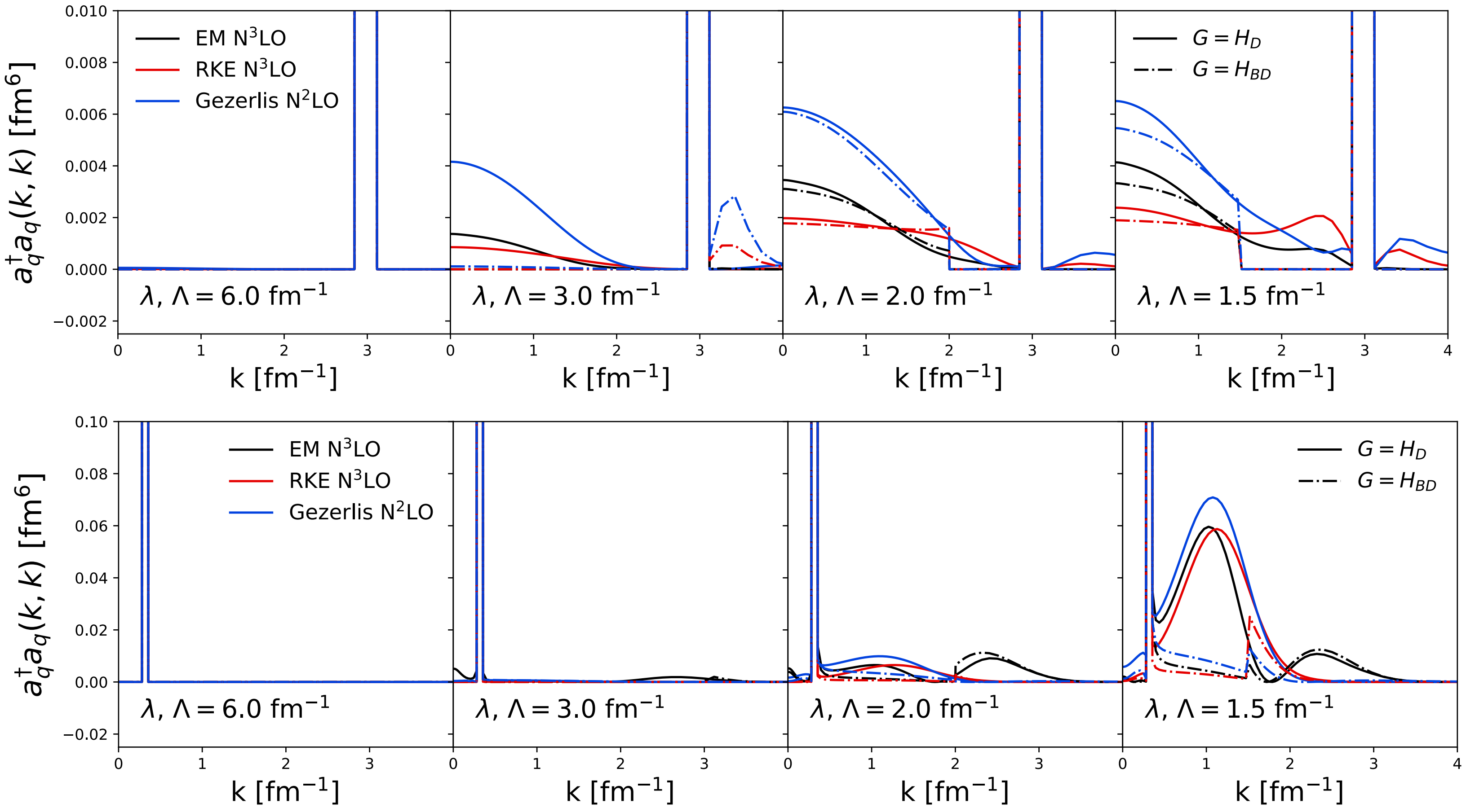
Momentum projection operator



- Evolution of high-momentum operator ($q = 3$ fm^{-1}) shifts strength to low-momentum matrix elements
- Low-momentum operator ($q = 0.3$ fm^{-1}) retains the same momentum scale

Fig. 6: Integrand of $\langle\psi_d|a_q^\dagger a_q|\psi_d\rangle$ in momentum-space for $q = 3$ (top) and 0.3 (bottom) fm^{-1} with SRG transformations for several values of λ where the transformations are done using the RKE N^{3}LO potential.

Momentum projection operator



- Matrix elements do not collapse for different cases

Fig. 5: Diagonal matrix elements of $a_q^\dagger a_q(k,k')$ for $q = 3$ (top) and 0.3 (bottom) fm⁻¹ with SRG transformations from several chiral potentials with band- and block-diagonal decoupling.

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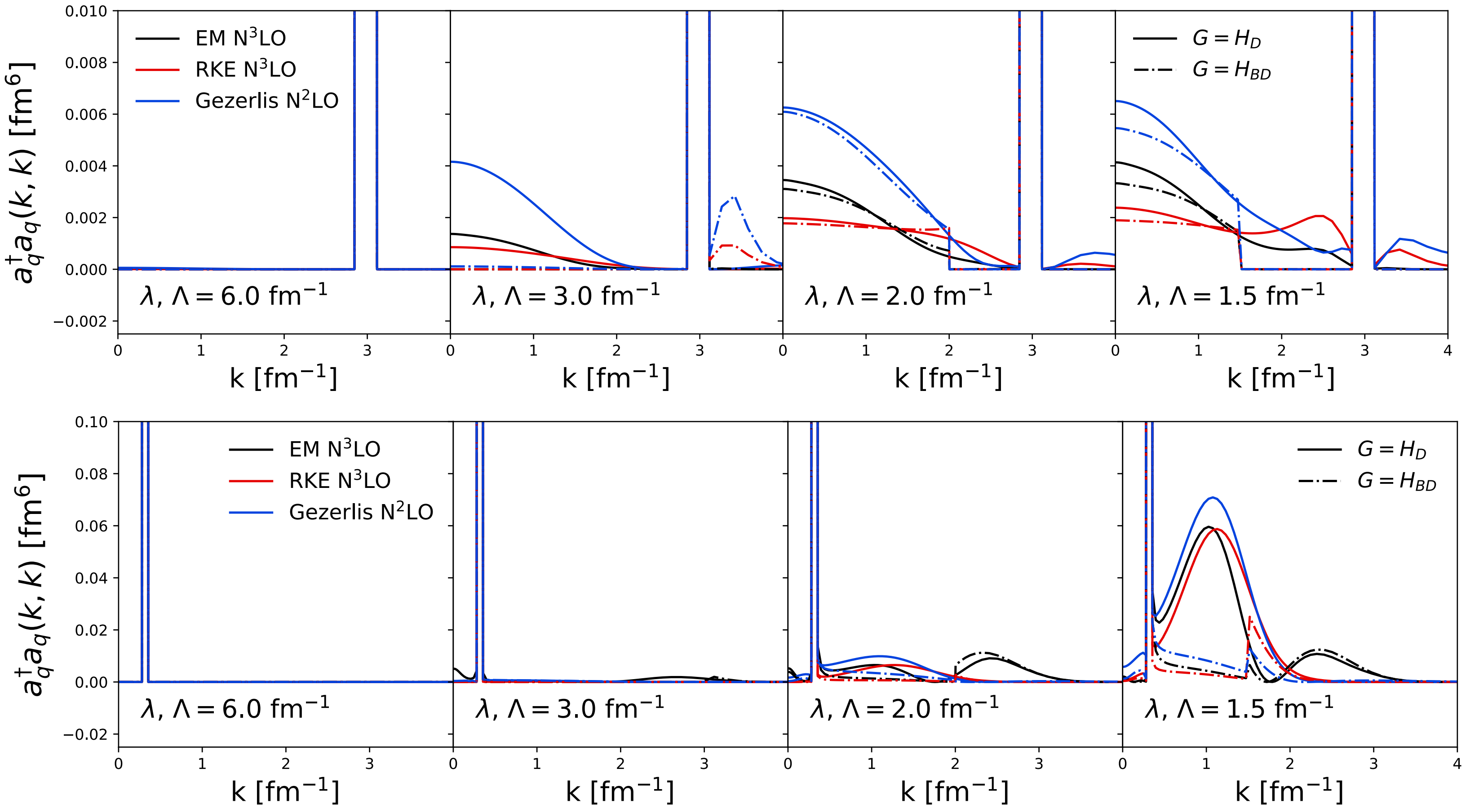

- Matrix elements do not collapse for different cases
- Initial $|\phi_d(q)|^2$ is not the same for various potentials

⇒ No universality in SRG-evolved $a_q^\dagger a_q(k, k')$ operator

Fig. 5: Diagonal matrix elements of $a_q^\dagger a_q(k, k')$ for $q = 3$ (top) and 0.3 (bottom) fm^{-1} with SRG transformations from several chiral potentials with band- and block-diagonal decoupling.

Summary

- Different SRG softened interactions collapse to universal form at low-energy if corresponding phase shifts are the same
- Universality depends on the pattern of SRG decoupling – the SRG generator (band- or block-diagonal)
- Other operators do not necessarily decouple like evolved potentials but reflect changes in the evolved wave function

Outlook

- Further test SRG-evolution of other operators ($\hat{r}^2$, $\hat{Q}$, etc.)

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- Recent interest in high cutoff potentials and spurious, deeply bound states[1]
  - Spurious bound states can affect decoupling and universality (choice in $G = H_D$ or T_{rel} is important)[2]
  - How do other operators change with these potentials?

[1]I. Tews et al., Phys. Rev. C **98**, 024001 (2018), [2]K.A. Wendt et al., Phys. Rev. C **83**, 034005 (2011)

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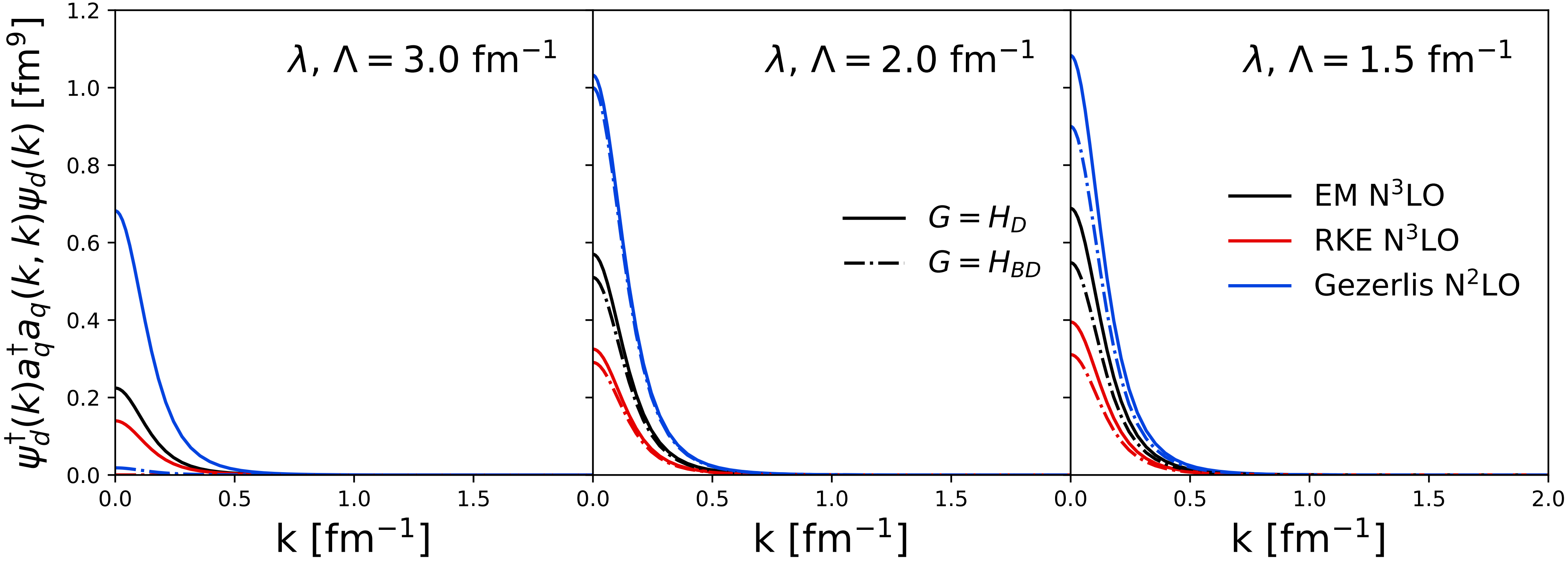
- Recent interest in high cutoff potentials and spurious, deeply bound states[1]
  - Spurious bound states can affect decoupling and universality (choice in $G = H_D$ or T_{rel} is important)[2]
  - How do other operators change with these potentials?
- The Magnus expansion provides an improved approach to the SRG[3]
  - Do the same characteristics of operator evolution, universality, and generator dependence hold in this approach?
  - What does this imply for IMSRG calculations?

[1]I. Tews et al., Phys. Rev. C **98**, 024001 (2018), [2]K.A. Wendt et al., Phys. Rev. C **83**, 034005 (2011), [3]T.D. Morris et al., Phys. Rev. C **92**, 034331 (2015)

Extras


$\psi_d^\dagger(k) a_q^\dagger a_q(k,k) \psi_d(k)$ [fm^9]
$\lambda, \Lambda = 3.0$ fm^{-1}
$\lambda, \Lambda = 2.0$ fm^{-1}
$\lambda, \Lambda = 1.5$ fm^{-1}
$G = H_D$
$G = H_{BD}$
EM N^{3}LO
RKE N^{3}LO
Gezerlis N^{2}LO
k [fm^{-1}]
k [fm^{-1}]
k [fm^{-1}]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
2.0