Model-independent determination of the axial mass parameter in quasielastic antineutrino-nucleon scattering

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Understanding the charged current quasielestic (CCQE) neutrino-nucleus interaction is important for precision studies of neutrino oscillations. The theoretical description of the interaction depends on the combination of a nuclear model with the knowledge of form factors. While the former has received considerable attention, the latter, in particular, the axial form factor, is implemented using the historical dipole model. Instead, we use a model-independent approach, presented in a previous study, to analyze the muon-antineutrino CCQE mineral oil data published by the MiniBooNE collaboration. We combine the cross section for scattering of antineutrinos off protons in carbon and hydrogen, using the same axial form factor for both. The extracted value of the axial mass parameter $m_A = 0.84^{+0.12}_{-0.04} \pm 0.11$ GeV is in very good agreement with the model-independent value extracted from MiniBooNE's neutrino data. Going beyond a one-parameter description of the axial form factor, we extract values of the axial form factor in the range of $Q^2 = 0.1\ldots1.0$ GeV^2, finding a very good agreement with the analogous extraction from the neutrino data. We discuss the implications of these results.

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I. INTRODUCTION

Future neutrino oscillation experiments plan to study the neutrino-mass ordering and search for CP violation in the lepton sector of the standard model. In order to do that, the charged current quasielestic (CCQE) neutrino-nucleus interaction must be known to high precision; see e.g. [1].

The neutrino interaction with quarks is determined by the standard model Lagrangian. For neutrino-nucleus scattering this interaction is folded twice. First, going from the quark level to the nucleon level form factors must be introduced. Second, going from the nucleon level to the nucleus level a nuclear model must be introduced. Thus neutrino-nucleus interaction is determined by the *combination* of form factors and a nuclear model. To understand neutrino-nucleus interaction it is important to get a handle on both.

The issue of whether the nuclear models used by the neutrino experiments are adequate was discussed by many authors. The question of the form factors has received a lot less attention [2]. In the isospin limit there are four form factors that contribute to the interaction: F_1, F_2, F_P, and F_A; see the appendix for details. Two of these, F_1 and F_2, can be related to the electric and magnetic form factors extracted in electron-proton scattering. F_P can be related to F_A in the $m_\pi \to 0$ limit, using partial conservation of axial-vector current (PCAC). Furthermore, its contribution to the cross section is suppressed by m_ℓ^2/m_N^2 where m_ℓ is the charged lepton mass and m_N is the nucleon mass; see the appendix. Even if the charged lepton is a muon, m_ℓ^2/m_N^2 is only about 0.01. Its contribution is further suppressed for an electron.

That leaves the axial form factor $F_A(q^2)$. Its value at $q^2 = 0$ can be determined from neutron decay. In particular, the latest value from the particle data group is $F_A(0) = -1.272$ [3]. A common model used in the literature to parametrize F_A is the dipole model,

$$F_A^{\mathrm{dipole}}(q^2) = \frac{F_A(0)}{[1 - q^2/(m_A^{\mathrm{dipole}})^2]^2}. \tag{1}$$

There are several problems with this model. First, it is motivated by similar older dipole models of the electromagnetic form factors. These are now known to be inadequate to describe electromagnetic form factor data. There is no reason to believe the dipole model is adequate to describe F_A. Second, this is a one-parameter model. One should not expect *any* one-parameter model to always adequately describe the form factor. As neutrino data become more accurate, the only improvement possible in this model is to reduce the error on m_A^{dipole}. Third, it is not clear what the physical meaning of m_A^{dipole} is. When different extractions of m_A^{dipole} disagree, is it a real discrepancy in the data or is it an artifact of the use of the dipole model? One would like to have a general improvable parametrization with its parameters having a model-independent interpretation.

The solution to this problem lies in using the so-called z expansion for F_A [2]. This method relies on the known analytic properties of the form factor to express it as a Taylor series in the variable $z(q^2)$, i.e. $F_A(q^2) = \sum_k a_k z^k(q^2)$ (see Sec. II below). The z expansion has by now become the standard tool in analyzing meson form factors; see e.g. [4]. It was first applied to baryons, in particular, the proton electric form factor in [5]. It has also been applied successfully to extract the nucleon axial mass in [2], the proton and the neutron magnetic radii in [6], the proton electric and magnetic radii in [7] (using also other methods), and the proton electric and magnetic radii in [8].[1] It has also been applied to analyze heavy-baryon form factors in [12–13] and the strange nucleon electromagnetic form factors in [14].

In particular, [2] has used the z expansion to extract axial mass in a model-independent way. The axial mass is defined in terms of the form factor slope at $q^2 = 0$: $m_A = [F'_A(0)/2F_A(0)]^{-1/2}$ [2]. Assuming the dipole model this definition coincides with m_A^{dipole}. But in general the two are not equal. The extraction is model independent since the value of m_A is independent of the number of parameters used in the fit. From the MiniBooNE muon-neutrino data [2,15] found $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV. This value is consistent with fits to an illustrative data set for pion electroproduction, $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV. Using the dipole model, [2] has found $m_A^{\text{dipole}} = 1.29 \pm 0.05$ GeV (neutrino scattering) and $m_A^{\text{dipole}} = 1.00 \pm 0.02$ GeV (electroproduction). One could conclude in this case that the discrepancy is an artifact of the use of the dipole model.

The results of [2] assumed the relativistic Fermi gas (RFG) nuclear model of Smith and Moniz [16]. Still, it was possible to extract one of the RFG parameters ϵ_b from the MiniBooNE data without an assumption on m_A. In particular, [2] has found $\epsilon_b = 0.028 \pm 0.03$ GeV in agreement with the value $\epsilon_b = 0.025$ GeV, as extracted from electron scattering data on nuclei in [17], but less consistent with the value used by MiniBooNE [15], $\epsilon_b = 0.034 \pm 0.09$ GeV.

The agreement between the model-independent extraction of the axial mass from neutrino and pion electroproduction data is very encouraging. It is important to confirm these results by applying the same method to other neutrino data sets. For example, MiniBooNE has released data on muon-antineutrino-nucleus scattering [1]. An important difference from the neutrino-nucleus case is that the antineutrino scatters off protons in carbon and hydrogen, as opposed to only neutrons in carbon for the neutrino. As a result one has to combine scattering off "free" protons in hydrogen with scattering off "bound" protons in carbon. In particular, [1] has used *different* values of the axial mass for protons in carbon ($m_A^{\text{dipole}} = 1.35$ GeV) and protons in hydrogen ($m_A^{\text{dipole}} = 1.02$ GeV). Since the axial mass is a fundamental property of the nucleon, such a treatment is problematic. The main goal of this paper is the model-independent extraction of the axial mass of the nucleon from the antineutrino-nucleus scattering data, using the *same* axial form factor for protons in carbon and hydrogen.

The paper is structured as follows. In Sec. II we briefly review the theoretical framework behind the model-independent extraction. In Sec. III we present the results of model-independent extraction of the axial mass and the axial form factor from the data. We present our conclusions in Sec. IV. For completeness, we have collected in the appendix formulas for the differential cross section for the free and bound nucleon, including the effects of flux averaging.

II. THEORETICAL FRAMEWORK

Most of the theoretical framework concerning the use of the z expansion was discussed in detail in [2]. Here we briefly review it.

The nucleon matrix elements depend on four form factors, F_1, F_2, F_p, and F_A; see the appendix. Two of them, F_1 and F_2, can be related using isospin symmetry to the electromagnetic form factors measured in electron-proton scattering. In extracting m_A and F_A from the MiniBooNE data we generally try to follow their choices for the input functions and parameters. Thus we use the BBA2003 parametrization [18] for F_1 and F_2, used in [1]. For F_p we use the pion pole approximation[2] $F_P(q^2) \approx 2m_N^2 F_A(q^2)/(m_\pi^2 - q^2)$. This leaves F_A, which is the focus of our analysis.

The axial form factor is analytic in the cut $t = q^2$ plane outside a cut that starts at the three-pion threshold, $t \geq t_{\text{cut}} = 9m_\pi^2$. The domain of analyticity can be mapped onto the unit circle via the transformation

$$z(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}}, \qquad (2)$$

where t_0 is a free parameter determined by $z(t_0, t_{\text{cut}}, t_0) = 0$. In our analysis we take $t_0 = 0$, but the results do not depend on this choice [2]. We express the form factor as a power series in $z(q^2) = z(q^2, t_{\text{cut}}, 0)$,

$$F_A(q^2) = \sum_{k=0}^{\infty} a_k z(q^2)^k. \qquad (3)$$

[1]Some other studies do not bound the coefficients of the z expansion [9,10] or modify it [11].

[2]The values we use for the various input parameters are listed in Table I. In the following we assume that the errors due the variation of these parameters are small compared to the uncertainty on F_A. Also, due to the suppression of the F_P contribution we do not consider uncertainties associated with this approximation.

TABLE I. Numerical values for input parameters.

Parameter	Value	Reference
$\lvert V_{ud}\rvert$	0.9742	[3]
μ_p	2.793	[3]
μ_n	−1.913	[3]
m_μ	0.1057 GeV	[3]
G_F	1.166×10^{-5} GeV^{-2}	[3]
m_N	0.9389 GeV	[3]
m_π	0.140 GeV	[3]
$F_A(0)$	−1.272	[3]
ϵ_b	0.025 GeV	[17]
p_F	0.220 GeV	[15]

For $t_0 = 0$, a_0 is equal to $F_A(0) = -1.272$. The axial mass is determined from the slope of F_A at $q^2 = 0$, i.e. $m_A = [F'_A(0)/2F_A(0)]^{-1/2}$. For the choice of $t_0 = 0$, m_A will depend only on a_1. To ensure that m_A does not depend on the number of parameters, the coefficients must be bounded. As discussed in detail in [2] we use in our fits the uniform bounds[3] of $|a_k| \le 5$ and $|a_k| \le 10$. In practice we fit only a finite number of parameters $0 < k \le k_{\max}$. For definiteness our default is $k_{\max} = 7$, but we have checked that fitting a larger number of parameters, i.e. $k_{\max} = 8$, 9, and 10, does not change the results.

As in the MiniBooNE analysis we use the RFG nuclear model, but similar to the analysis of [2], we use the binding energy of $\epsilon_b = 0.025$ GeV from [17].

The antineutrino can scatter off protons in carbon and in hydrogen. MiniBooNE reports the double differential cross section per nucleon for the mineral oil used in the experiment.[4] The mineral oil is composed of C_nH_{2n+2}, $n \sim 20$; see Sec. III A of [1]. This implies that on average there are 2.1 hydrogen nuclei for every carbon nucleus. Considering the large distance between the carbon and the hydrogen nuclei compared to the typical nuclear size, we can add the cross sections directly, ignoring interference, and divide by the number of protons, in this case 8.1. We thus have

$$\frac{d\sigma_{\text{mineral oil,per nucleon,avg}}}{dE_\ell d\cos\theta_\ell} = \frac{1}{8.1}\left(6\frac{d\sigma_{\text{carbon,per nucleon,avg}}}{dE_\ell d\cos\theta_\ell} + 2.1\frac{d\sigma_{\text{hydrogen,avg}}}{dE_\ell d\cos\theta_\ell}\right), \quad (4)$$

where avg. denotes flux averaging. The expressions for $d\sigma_{\text{hydrogen,avg}}$ and $d\sigma_{\text{carbon,per nucleon,avg}}$. are given in Eqs. (A15) and (A17). In particular, we use the *same* axial form factor for carbon and hydrogen cross sections.

The expression in (4) can be compared to MiniBooNE's reported per-nucleon mineral oil differential cross section to extract m_A. In order to do that we form the error matrix

$$E_{ij} = (\delta\sigma_i)^2\delta_{ij} + (\delta N)^2\sigma_i\sigma_j, \quad (5)$$

where $\sigma_i = (d\sigma/dE_\mu d\cos\theta_\mu)\Delta E_\mu \Delta\cos\theta_\mu$ denotes a partial cross section, $\delta\sigma_i$ denotes the shape uncertainty from Table XIV of [1], and $\delta N = 0.13$ is the normalization error from [1]. We form the chi-squared function

$$\chi^2 = \sum_{ij}(\sigma_i^{\text{expt}} - \sigma_i^{\text{theory}})E_{ij}^{-1}(\sigma_j^{\text{expt}} - \sigma_j^{\text{theory}}), \quad (6)$$

and minimize χ^2 to find best fit values for m_A. The error on m_A is determined from the $\Delta\chi^2 = 1$ intervals.

In order to study the m_A sensitivity to $Q^2 = -q^2$, we consider subsets of the MiniBooNE data with a cut on Q^2. For a free proton at rest, the Q^2 can be determined from the observed muon energy and scattering angle assuming a quasielastic scattering. Due to nuclear effects Q^2 cannot be determined unambiguously and as a proxy we use the "reconstructed" Q^2 of [2],

$$Q^2_{\text{rec}} = 2E_\nu^{\text{rec}}E_\mu - 2E_\nu^{\text{rec}}\sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu - m_\mu^2, \quad (7)$$

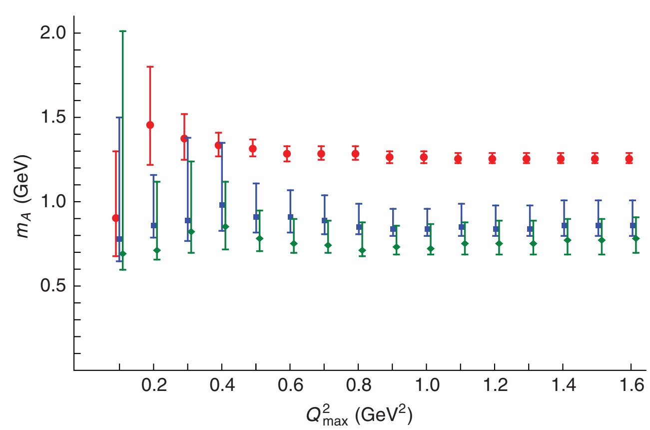

FIG. 1 (color). Extracted value of m_A versus $Q^2_{\max}$. Dipole model results for m_A^{dipole} are shown by the red circles; z expansion results with $|a_k| \le 5$ are shown by the blue squares; z expansion results with $|a_k| \le 10$ are shown by the green diamonds.

[3]It should be noted that the uniform bounds of 5, 10 also imply that $m_A > 0.599$, 0.424 GeV[6]. In such cases one can perform fits with bounds on all a_k apart from a_1. In practice, this is not a problem as the extracted values we find are larger than these lower bounds.

[4]MiniBooNE also reported "hydrogen subtracted" data [1]. We do not use them for two reasons. First, the subtraction relies on the event generator, since the scattering off carbon and hydrogen cannot be distinguished in the data. Second, they use a different axial mass for protons in hydrogen and protons in carbon.

where E_ν^{rec} approximates the neutrino energy in the nucleon rest frame,

$$E_\nu^{\mathrm{rec}} = \frac{m_N E_\mu - m_\mu^2/2}{m_N - E_\mu + \sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu}. \tag{8}$$

III. RESULTS

A. Axial mass extraction

As described in the previous section, $F_A(q^2)$ is the only undetermined part of (4). We extract m_A^{dipole} using the dipole model from (1) and m_A using the z expansion from (3). We present results for data with $Q^2_{\mathrm{rec}} \le Q^2_{\mathrm{max}}$, where $Q^2_{\mathrm{max}} = 0.1, 0.2, \ldots, 1.6\ \mathrm{GeV}^2$. The cut $Q^2_{\mathrm{max}} = 1.6\ \mathrm{GeV}^2$ includes the entire MiniBooNE data set. We apply the bounds $|a_k| \le 5$ and $|a_k| \le 10$ and use $k_{\mathrm{max}} = 7$. Our results are presented in Fig. 1. The z expansion results lie systematically below values extracted using the dipole model. The same effect was found in [2] analyzing MiniBooNE's neutrino data. Taking $Q^2_{\mathrm{max}} = 1.0\ \mathrm{GeV}^2$ for definiteness, we find

$$m_A = 0.84^{+0.12}_{-0.04} \pm 0.11\ \mathrm{GeV} \quad \text{(antineutrino scattering)}, \tag{9}$$

where the first error is experimental, using the fit with $|a_k| \le 5$, and the second error represents residual form factor shape uncertainty, taken as the maximum change of the 1σ interval when the bound is increased to $|a_k| \le 10$. This result is in very good agreement with the value extracted in [2] from neutrino data

$$m_A = 0.85^{+0.22}_{-0.07} \pm 0.09\ \mathrm{GeV} \quad \text{(neutrino scattering)}. \tag{10}$$

For comparison, a fit that uses the dipole model and the same Q^2_{max} gives $m_A^{\mathrm{dipole}} = 1.27^{+0.03}_{-0.04}$ GeV. This value is in good agreement with the value found in [2] $m_A^{\mathrm{dipole}} = 1.29 \pm 0.05$ GeV.

For both the neutrino and antineutrino data sets the errors on the model-independent axial mass are not symmetric. In particular, the magnitude of the upper error bar is larger than the magnitude of the lower error bar. It was suggested [19] that it might be due to the fact that m_A^2 is inversely proportional to the slope F_A. We can check this hypothesis by extracting the "axial radius" [2] $r_A = [6F_A'(0)/F_A(0)]^{1/2}$ from the data. Using $Q^2_{\mathrm{max}} = 1.0\ \mathrm{GeV}^2$ we find

$$r_A = 0.81^{+0.05}_{-0.10} \pm 0.14\ \mathrm{fm} \quad \text{(antineutrino scattering)}, \tag{11}$$

which is consistent with (9). We conclude that the asymmetry in the errors is not due to the inverse relation between m_A^2 and $F_A'(0)$.

The fact that the errors are not symmetric implies that larger values of m_A are acceptable from the fit. Since larger m_A tends to increase the cross section, one can take the asymmetry to be an indication that the nuclear cross section is too small. To check this option qualitatively, we have multiplied the carbon cross section by a constant factor and repeated the fits. We find that if the factor is larger than 1, the errors indeed become more symmetric. If it is smaller than 1, the asymmetry grows. There is very little change in the value of m_A^{dipole}. We find similar results if we follow the same procedure for the MiniBooNE neutrino data analyzed in [2]. Therefore there are hints that the carbon cross section is too small, which might imply an issue with the nuclear model. It would be interesting to explore in more detail the combined effect of the z expansion and a change in the nuclear model. This is left to a future study.

B. Extraction of $F_A(q^2)$

The model-independent approach allows us to extract values of $F_A(q^2)$ from data and not just one parameter such as the axial mass. We fit $F_A(q^2)$ to the entirety of MiniBooNE's muon-antineutrino CCQE data, using the z expansion with $k_{\mathrm{max}} = 7$ and $|a_k| \le 10$. Figure 2 shows the extracted values for $Q^2 = 0.1, 0.2, \ldots, 1.0\ \mathrm{GeV}^2$. We compare these results to a dipole fit that assumes $m_A^{\mathrm{dipole}} = 1.27^{+0.03}_{-0.04}$ GeV.

These results can be compared to the extraction of $F_A(q^2)$ in [2] from MiniBooNE's muon-neutrino CCQE data [15]. Figure 3 compares the two extractions. The very good agreement between the two extractions is clear.

The shape of the form factor hints at a nonzero curvature. The curvature can be extracted in a model-independent way using the z expansion. In order to do that we extract a_2 of Eq. (3). We use a cut of $Q^2_{\mathrm{max}} = 1.0\ \mathrm{GeV}^2$, $k_{\mathrm{max}} = 7$ and enforce $|a_k| \le 10$ for $k \ge 3$, i.e. leaving a_1 and a_2

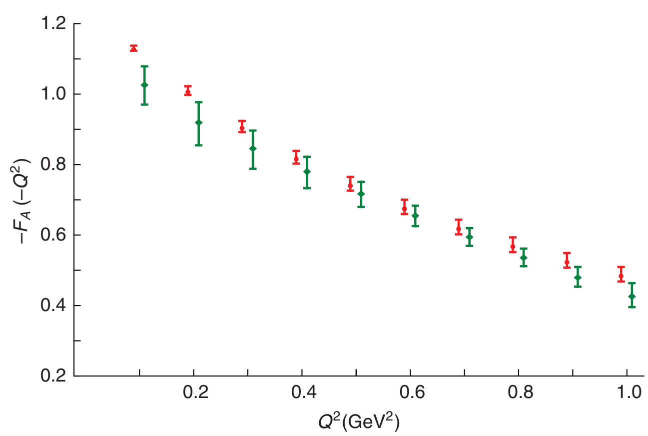

FIG. 2 (color). Comparison of the axial-vector form factor F_A as extracted using the z expansion (green diamonds) and dipole ansatz (red circles).

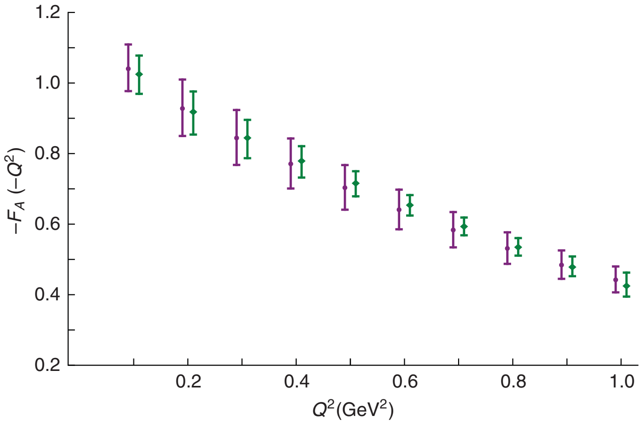

FIG. 3 (color). Comparison of extraction of the axial-vector form factor F_A using the z expansion from neutrino data (purple circles) and from antineutrino data (green diamonds).

unbounded [recall that $a_0 = F_A(0) = -1.272$]. We find $a_2 = -9.6^{+4.7}_{-2.6}$, which can be compared to $a_2 = -8^{+6}_{-3}$ found in [2]. In both cases we see an indication for a nonzero curvature, but it is poorly constrained. A similar extraction of a_1, equivalent to the extraction of the axial mass, gives $a_1 = 3.4^{+0.9}_{-1.0}$ which is in good agreement with $a_1 = 2.9^{+1.1}_{-1.0}$ of [2].

One could use the model-independent approach to extract values of $F_A(q^2)$ from other neutrino data sets. These extractions can be used to tabulate values of $F_A(q^2)$ similar to the electromagnetic form factors; see e.g. [20]. Such tables would be a much better approach than trying to reconcile various data sets using just one degree of freedom.

IV. SUMMARY

The improved precision of experiments requires us to move from historical *ad hoc* models of form factors to a model-independent approach. The field of flavor physics has gone through such a change and currently the use of the z expansion is the standard method in analyzing exclusive decays; see e.g. [4]. Motivated by the "proton radius puzzle," studies of the proton and neutron electromagnetic form factors have started to implement this model-independent method. A similar shift is needed for the use of the axial form factor in neutrino experiments. In particular, one would like to separate effects coming from the nuclear models and effects from the form factors. The first application of the z expansion to the extraction of the axial mass was performed in [2].

In this paper we have applied the method of [2] to analyze the muon antineutrino CCQE cross section data published by the MiniBooNE experiment [1]. An important difference from the neutrino data analyzed in [2] is that the antineutrino interacts both with protons in carbon and in hydrogen, while the neutrino interacts only with the neutrons in carbon. As a result, in extracting the axial mass, or more generally, the axial form factor, one has to combine cross sections for protons in hydrogen and protons in carbon. While the former is described using form factors alone, the latter requires also the use of a nuclear model. It is important to use the same axial form factor for both.

We have extracted the axial mass using a model-independent approach from MiniBooNE's muon-antineutrino CCQE data [1]. We find $m_A = 0.84^{+0.12}_{-0.04} \pm 0.11$ GeV. This result is in very good agreement with the analogues model-independent extraction of m_A from MiniBooNE's muon-neutrino CCQE data [15], $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV [2]. For comparison, a fit using the dipole model gives $m_A^{\text{dipole}} = 1.27^{+0.03}_{-0.04}$ GeV, consistent with the value of $m_A^{\text{dipole}} = 1.29 \pm 0.05$ reported in [2].

Our extraction relies on a specific nuclear model, the RFG model, used also by the MiniBooNE experiment. Our study does not address whether this model is adequate or should be modified. Since the upper error bar on the model-independent m_A is about three times larger than the lower error bar, it is possible that some modification of the nuclear model is also needed. It would be interesting to combine the z expansion with other nuclear models.

Going beyond a one-parameter comparison, we have extracted values of the axial form factor as a function of Q^2. The results are shown in Fig. 2. These results can be compared to the similar extraction of the axial mass from MiniBooNE's muon neutrino CCQE data [15]; see Fig. 3. We find very good agreement between the two data sets. It would be beneficial to extract values of F_A from other neutrino experiments. Ideally one would like to have a world data set for values of the axial form factor, similar to that of electromagnetic form factors; see e.g. [20].

In summary, we find very good agreement between MiniBooNE's neutrino and antineutrino data sets when using model-independent extraction of the axial form factor in general and the axial mass in particular. These model-independent methods should be applied to other data sets and in combination with other nuclear models.

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APPENDIX: QUASIELASTIC (ANTI)NEUTRINO NUCLEON SCATTERING CROSS SECTION

The relevant part of the weak-interaction Lagrangian is

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{ud} \bar{\ell}\gamma^\alpha(1-\gamma_5)\nu \bar{u}\gamma_\alpha(1-\gamma_5)d + \text{H.c.}. \tag{A1}$$

The cross section for $\nu(k) + n(p) \to \ell^-(k') + p(p')$ on a free neutron is

$$\sigma_{\text{free}} = \frac{1}{4|k\cdot p|}\int \frac{d^3k'}{(2\pi)^3 2E_{\boldsymbol{k}'}} \times \int \frac{d^3p'}{(2\pi)^3 2E_{\boldsymbol{p}'}} \overline{|\mathcal{M}^2|}(2\pi)^4\delta^4(k+p-k'-p'), \tag{A2}$$

where the spin-averaged, squared amplitude is

$$\overline{|\mathcal{M}^2|} = \frac{G_F^2|V_{ud}|^2}{4} L^{\mu\nu}\sum_{\text{spins}} \langle p(p')|\bar{u}\gamma_\mu(1-\gamma_5)d|n(p)\rangle \times \langle p(p')|\bar{u}\gamma_\nu(1-\gamma_5)d|n(p)\rangle^*. \tag{A3}$$

The leptonic tensor neglecting the neutrino mass is ($\epsilon^{0123} = -1$)

$$L^{\mu\nu} = 8(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k' - i\epsilon^{\mu\nu\rho\sigma}k_\rho k'_\sigma). \tag{A4}$$

The hadronic matrix element appearing in (A3) is parametrized by

$$\langle p(p')|\bar{u}\gamma_\mu(1-\gamma_5)d|n(p)\rangle = \bar{u}^{(p)}(p')\Gamma_\mu(q)u^{(n)}(p), \tag{A5}$$

where $q = k - k' = p' - p$ and we have defined the vertex function

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2) + \frac{q_\mu}{m_N}F_S(q^2) + \gamma_\mu\gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{m_N}\gamma_5 F_T(q^2) + \frac{q_\mu}{m_N}\gamma_5 F_P(q^2). \tag{A6}$$

Notice that Eqs. (A5)–(A6) define the relative phases between the form factors. In particular, they determine the sign of the ratio of $F_A(0)$ to $F_1(0)$ measured in neutron decay [3].

We may write the cross section of (A2) as

$$\sigma_{\text{free}} = \frac{G_F^2|V_{ud}|^2}{16|k\cdot p|}\int \frac{d^3k'}{(2\pi)^3 2E_{\boldsymbol{k}'}} L^{\mu\nu}\hat{W}_{\mu\nu}, \tag{A7}$$

where the nucleon structure function is

$$\hat{W}_{\mu\nu} = \int \frac{d^3p'}{(2\pi)^3 2E_{\boldsymbol{p}'}}(2\pi)^4\delta^4(p-p'+q)H_{\mu\nu}. \tag{A8}$$

The hadronic tensor is

$$H_{\mu\nu} = \text{Tr}[(\not{p}' + m_p)\Gamma_\mu(q)(\not{p} + m_n)\bar{\Gamma}_\nu(q)], \tag{A9}$$

where, as usual, $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$. We may similarly analyze antineutrino scattering, $\bar{\nu}(k) + p(p) \to \ell^+(k') + n(p')$, using (A7), taking $L^{\mu\nu} \to L^{\nu\mu}$, and making the replacements $m_n \leftrightarrow m_p$, $\Gamma_\mu(q) \to \bar{\Gamma}_\mu(-q)$ in $H_{\mu\nu}$.

Imposing time-reversal invariance shows that $F_i(q^2)$ are real. We assume isospin symmetry in the following, in which case F_S and F_T vanish, $m_n = m_p = m_N$, and $\bar{\Gamma}_\mu(-q) = \Gamma_\mu(q)$. The hadronic tensor has the time-reversal invariant decomposition

$$H_{\mu\nu} = -g_{\mu\nu}H_1 + \frac{p_\mu p_\nu}{m_N^2}H_2 - i\frac{\epsilon_{\mu\nu\rho\sigma}}{2m_N^2}p^\rho q^\sigma H_3 + \frac{q_\mu q_\nu}{m_N^2}H_4 + \frac{(p_\mu q_\nu + q_\mu p_\nu)}{2m_N^2}H_5. \tag{A10}$$

The H_is are expressed in terms of the form factors F_i as

$$\begin{aligned} H_1 &= 8m_N^2F_A^2 - 2q^2[(F_1+F_2)^2 + F_A^2], \\ H_2 &= H_5 = 8m_N^2(F_1^2 + F_A^2) - 2q^2F_2^2, \\ H_3 &= -16m_N^2F_A(F_1+F_2), \\ H_4 &= -\frac{q^2}{2}(F_2^2 + 4F_P^2) - 2m_N^2F_2^2 - 4m_N^2(F_1F_2 + 2F_AF_P). \end{aligned} \tag{A11}$$

In the rest frame of the nucleon, let E_ℓ and $|\vec{P}_\ell| = \sqrt{E_\ell^2 - m_\ell^2}$ be the energy and 3-momentum of the charged lepton, and let θ_ℓ be the angle between the 3-momenta of the leptons. Also in that frame $k\cdot p = E_\nu m_N$. Using

$$\int \frac{d^3p'}{(2\pi)^3 2E_{\boldsymbol{p}'}}\delta^4(p-p'+q) = \delta(2p\cdot q + q^2), \qquad \int \frac{d^3k'}{(2\pi)^3 2E_{\boldsymbol{k}'}} = \pi\int dE_\ell d\cos\theta|\vec{P}_\ell|, \tag{A12}$$

we have

$$\frac{d\sigma_{\mathrm{free}}}{dE_\ell d\cos\theta_\ell} = \frac{G_F^2|V_{ud}|^2}{8\pi m_N}\delta(2p\cdot q+q^2)|\vec{P}_\ell|\Big\{2(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)H_1 + (E_\ell + |\vec{P}_\ell|\cos\theta_\ell)H_2 \pm \frac{1}{m_N}[(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)(E_\nu + E_\ell) - m_\ell^2]H_3 + \frac{m_\ell^2}{m_N^2}(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)H_4 - \frac{m_\ell^2}{m_N}H_5\Big\}, \tag{A13}$$

where $p\cdot q = m_N(E_\nu - E_\ell)$ and $q^2 = m_\ell^2 - 2E_\nu E_\ell + 2E_\nu|\vec{P}_\ell|\cos\theta_\ell$, and where the upper (lower) sign is for neutrino (antineutrino) scattering.

We now consider the case of antineutrino-proton scattering. To find the flux-averaged cross section we use the reported antineutrino flux [1] to create a function $f(E_{\bar\nu})$. This function is normalized to one, i.e. $\int dE_{\bar\nu} f(E_{\bar\nu}) = 1$. To obtain the flux-averaged cross section, we write the delta function in (A13) as

$$\delta(2p\cdot q+q^2) = \frac{\delta(E_{\bar\nu} - E_0)}{2(m_N - E_\ell + |\vec{P}_\ell|\cos\theta)}, \quad \text{with} \quad E_0 = \frac{2E_\ell m_N - m_\ell^2}{2(m_N - E_\ell + |\vec{P}_\ell|\cos\theta)}, \tag{A14}$$

multiply $d\sigma_{\mathrm{free}}$ by $f(E_{\bar\nu})$ and integrate over $E_{\bar\nu}$. The flux-averaged hydrogen cross section is

$$\frac{d\sigma_{\mathrm{hydrogen,avg.}}}{dE_\ell d\cos\theta_\ell} = \int dE_{\bar\nu} f(E_{\bar\nu})\frac{d\sigma_{\mathrm{free}}}{dE_\ell d\cos\theta_\ell} = \frac{f(E_0)G_F^2|V_{ud}|^2|\vec{P}_\ell|}{16\pi m_N(m_N - E_\ell + |\vec{P}_\ell|\cos\theta)} \times\Big\{2(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)H_1 + (E_\ell + |\vec{P}_\ell|\cos\theta_\ell)H_2 \pm \frac{1}{m_N}[(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)(E_\nu + E_\ell) - m_\ell^2]H_3 + \frac{m_\ell^2}{m_N^2}(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)H_4 - \frac{m_\ell^2}{m_N}H_5\Big\}. \tag{A15}$$

The MiniBooNE differential cross section data are given in bins of $\cos\theta$ and $T_\mu = E_\ell - m_\ell$ with sizes of 0.1 and 0.1 GeV, respectively. The flux data are given in bins of $E_{\bar\nu}$ of size 0.05 GeV. We use the center of the bin for the cross section data and Eq. (A14) to find E_0 and then round it to the closest value of $E_{\bar\nu}$ from the center of the bin. One can also use the exact value of E_0 which would change the value of Q^2 and Q^2_{rec} in Eq. (7) but not $f(E_0)$. We have checked that this different choice has a very small effect on the value of the axial mass, well within our reported error bars.

The carbon scattering cross section per nucleon is given in [2] as

$$\frac{d\sigma_{\mathrm{carbon,per\ nucleon}}}{dE_\ell d\cos\theta_\ell} = \frac{1}{6}\frac{G_F^2|V_{ud}|^2|\vec{P}_\ell|}{16\pi^2 m_T}\Big\{2(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)W_1 + (E_\ell + |\vec{P}_\ell|\cos\theta_\ell)W_2 \pm \frac{1}{m_T}[(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)(E_\nu + E_\ell) - m_\ell^2]W_3 + \frac{m_\ell^2}{m_T^2}(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)W_4 - \frac{m_\ell^2}{m_T}W_5\Big\}, \tag{A16}$$

where W_i are given in Eq. (46) of [2], and the upper (lower) sign is for neutrino (antineutrino) scattering. We have divided by six the number of neutrons (protons) in carbon to obtain the cross section per nucleon. To obtain the flux-averaged cross section we multiply by $f(E_{\bar\nu})$ and integrate over $E_{\bar\nu}$,

$$\frac{d\sigma_{\mathrm{carbon,per\ nucleon,avg.}}}{dE_\ell d\cos\theta_\ell} = \int dE_{\bar\nu} f(E_{\bar\nu})\frac{d\sigma_{\mathrm{carbon,per\ nucleon}}}{dE_\ell d\cos\theta_\ell}. \tag{A17}$$

In practice, since $f(E_{\bar\nu})$ is a discrete function, for each bin we use the central value and sum over all of the bins.

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