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INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

NCSM effective potentials

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Lecture 3

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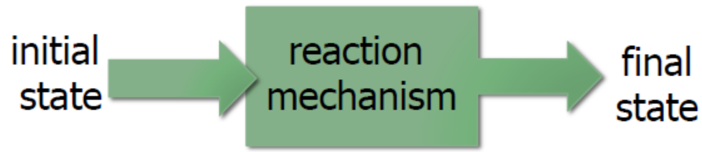
Exotic Nuclei are usually short lived:

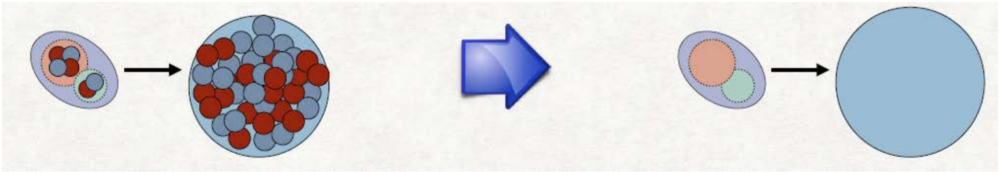
Have to be studied with reactions in inverse kinematics

e.g. direct reaction:



Challenge:

- **In the continuum, theory can solve the few-body problem exactly.**



Many-body problem

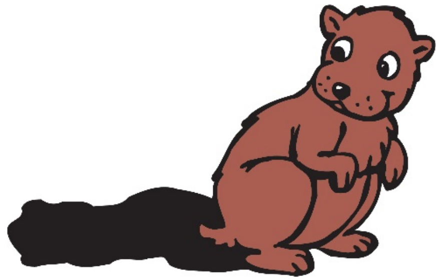
Few-body problem

Example (d,p) Reactions:

Reduce Many-Body to Few-Body Problem

Solve few-body problem

“Shadow” ?



Challenges & Opportunities

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

- **Nucleon-nucleon interaction believed to be well known:**
  today: chiral interactions

- **Effective proton (neutron) interactions:**
  - purely phenomenological optical potentials fitted to data
  - optical potentials with theoretical guidance
  - microscopic optical potentials

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Isolate relevant degrees of freedom

Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on P space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly
(Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

Isolate relevant degrees of freedom

Formally: separate Hilbert space into P and Q space, and calculate in P space

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Effective Interactions: non-local and energy dependent

History: **Phenomenological optical potentials**

Either fitted to a large global data set OR to a restricted data set

Most general form of optical potential

- Σ_i [$V_{A,Z,N,E}(r)$ + i $W_{A,Z,N,E}(r)$] $\text{Operator}_{(i)}$
- **Functions are of Woods-Saxon type**

Have central and spin orbit term

Fit cross sections, angular distributions polarizations, for a set of nuclei (lightest usually ^{12}C).

No connection to microscopic theory

Today's Goal: effective interaction from *ab initio* methods

Start from many-body Hamiltonian with 2 (and 3) body forces

Theoretical foundations laid by Feshbach and Watson in the 1950s

Feshbach:

→ **effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)** energy ~ 10 MeV

Rotureau, Danielewicz, Hagen, Jansen, Nunes
PRC 95, 024315 (2017)

Idini, Barbieri, Navratil
J.Phys.Conf. 981. 012005 (2018)
Acta Phys. Polon. B48, 273 (2017)

Watson:

→ **Multiple scattering expansion, e.g. spectator expansion (current truncation to 2 active particles)**

Spectator Expansion:

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Expansion in:
- particles active in the reaction
- antisymmetrized in active particles

"fast reaction", i.e. ≥ 100 MeV

Elastic Scattering (Watson approach)

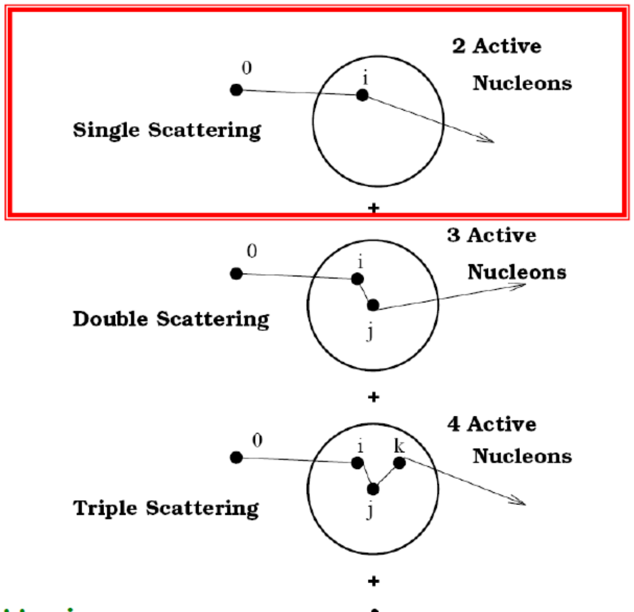
- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $\mathbf{P} = |\Phi_0\rangle\langle\Phi_0|$
  - With $\mathbf{1=P+Q}$ and $\mathbf{[P,G_0]=0}$
- For elastic scattering one needs: $\mathbf{P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P}$

$$\boxed{\begin{aligned} \mathbf{T} &= \mathbf{U + U\,G_0(E)\,P\,T} \\ \mathbf{U} &= \mathbf{V + V\,G_0(E)\,Q\,U} \end{aligned}}$$

$\Leftarrow$ **effective (optical) potential**

Up to here exact

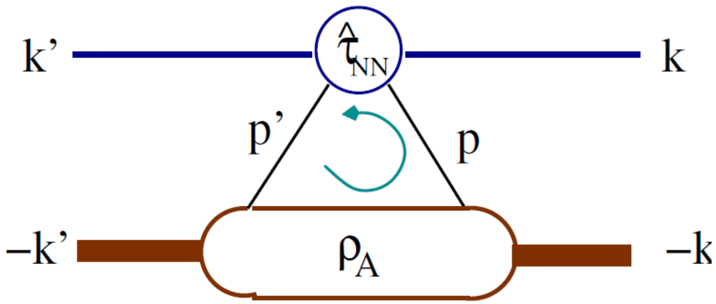
Spectator Expansion of U :

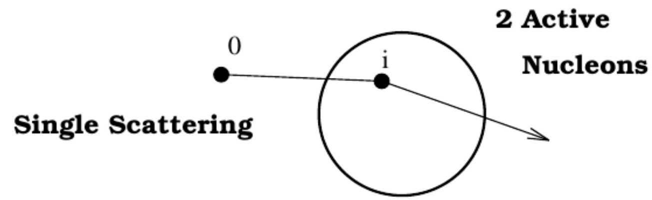


1st order: single scattering: $\mathbf{U^{(1)} \approx \Sigma^{A}_{i=0}\,\tau_{0i}}$

Chinn, Elster, Thaler, PRC 47, 2242 (1993)

Computing the first order folding potential $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$





NN scattering amplitudes

Nuclear one-body density

$$U_{el}(\mathbf{q},\mathbf{K}) = \sum_{i=n,p} \int d\mathbf{P}\; \eta(\mathbf{q},\mathbf{K},\mathbf{P})\; \hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}(\frac{A+1}{A}\mathbf{K} - \mathbf{P}), \mathcal{E}\right) \rho_i(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q})$$

$$\mathbf{K} \equiv \frac{\mathbf{k}+\mathbf{k}'}{2} \qquad \mathbf{P} \equiv \frac{\mathbf{k}_i+\mathbf{k}_i'}{2} + \frac{\mathbf{K}}{A}$$

$$\mathrm{q} \equiv \mathrm{k}' - \mathrm{k}$$

Same NN Interaction can now be used for NN t-matrix and one-body density matrix

Effective Potential is non-local and energy dependent

Details of implementation designed for energies ≥ 100 MeV

No-Core-Shell Model One-Body Density Matrices

Local Density: Derivation in

C. Cockrell, J. P. Vary, and P. Maris, Phys. Rev. **C86**, 034325 (2012), arXiv:1201.0724.

s.f. ≡ space fixed

$$\rho_{s.f.}(\vec{r}) = \sum_K \frac{\langle JMK0|JM\rangle}{\sqrt{2J+1}} Y_K^{*0}(\hat{r})\rho_{s.f.}^{(K)}(r)$$

with

$$\int \rho_{s.f.}(\vec{r}) d^3r = A,$$

Normalization in coordinate space

Multipoles:

$$\rho_{s.f.}^{(K)}(r) = \sum_{n_1,l_1,n_2,l_2,j_1,j_2} R_{n_1l_1}(r)R_{n_2l_2}(r)\frac{-1}{\sqrt{2K+1}}\left\langle l_1\frac{1}{2}j_1\,||Y_K||\,l_2\frac{1}{2}j_2\right\rangle\left\langle AJ\lambda\left|\left|(a^\dagger_{n_1l_1j_1}\tilde{a}_{n_2l_2j_2})^{(K)}\right|\right|AJ\lambda\right\rangle$$

↑ Coefficients from NCSM calculation, e.g. Maris, ~~Vary~~

HO wave functions

$$R_{nl}(r) = \left[\frac{2(2\nu)^{l+3/2}\Gamma(n+1)}{\Gamma(n+l+\frac{3}{2})}\right]^{\frac{1}{2}} r^l e^{-\nu r^2} L_n^{l+\frac{1}{2}}(2\nu r^2)$$

$$\nu = \frac{mc^2\hbar\Omega}{2\hbar^2c^2}$$

characterizes calc.

$$\left\langle l_1\frac{1}{2}j_1||Y_K||l_2\frac{1}{2}j_2\right\rangle = \frac{1}{\sqrt{4\pi}}\hat{j}_1\hat{j}_2\hat{l}_1\hat{l}_2(-1)^{j_2+\frac{1}{2}}\langle l_10l_20|K0\rangle\begin{Bmatrix} j_1 & j_2 & K \\ l_2 & l_1 & \frac{1}{2}\end{Bmatrix}$$

e.g. $\boxed{K=0}$ multi-pole of the diagonal one-body density

$$\rho^{(0)}_{s.f.} = \sum_{n_1,l,n_2,j}\left[\frac{2(2\nu)^{l+3/2}\Gamma(n_1+1)}{\Gamma(n_1+l+\frac{3}{2})}\right]^{\frac{1}{2}}\left[\frac{2(2\nu)^{l+3/2}\Gamma(n_2+1)}{\Gamma(n_2+l+\frac{3}{2})}\right]^{\frac{1}{2}} r^{2l}e^{-2\nu r^2}L^{l+\frac{1}{2}}_{n_1}(2\nu r^2)L^{l+\frac{1}{2}}_{n_2}(2\nu r^2)$$
$$\times\frac{\sqrt{2j+1}}{\sqrt{4\pi}}\Delta_{jl\frac{1}{2}}(-1)\left\langle AJ\lambda||(a^{\dagger}_{n_1lj}\tilde{a}_{nlj})^{(0)}||AJ\lambda\right\rangle \qquad (1.1.13)$$

Fourier transform is

$$\rho_{s.f.}(\vec{q}) = \int \rho(\vec{r})e^{-i\vec{q}\cdot\vec{r}}d^3r$$

multipole expansion of plane waves,

$$e^{-i\vec{q}\cdot\vec{r}} = 4\pi\sum_{lm}Y^{*m}_l(\hat{r})Y^m_l(\hat{q})(-i)^l j_l(qr)$$

Final result:

$$\boxed{\rho_{s.f.}(\vec{q})} = 4\pi\sum_K C_K(-i)^K Y^0_K(\hat{q})\int dr\, r^2\rho^{(K)}_{s.f.}(r)j_K(qr) \qquad C_K = \frac{\langle JMK0|JM\rangle}{\sqrt{2J+1}}$$

Work out that:

$$\boxed{\rho^{(0)}_{s.f.}(\vec{q}\to 0) \quad = \quad A}$$

momentum space normalization

For all reaction calculations one needs translationally invariant quantities

Center-of-Mass Removal

[Galilei invariance for nonrelativistic formulation]

General Hamiltonion

with two-body forces

$$H = \frac{1}{2m}\sum_{i=1}^{A} \vec{p_i}^{\,2} + \sum_{i<j}^{A} V\left(|\vec{r_i} - \vec{r_j}|\right)$$

Add HO potential

$$H = \frac{1}{2m}\sum_{i=1}^{A} |\vec{p_i}|^{\,2} + \frac{1}{2}m\omega^2\sum_{i}^{A} |\vec{r_i}|^2 + \sum_{i<j}^{A} V\left(|\vec{r_i} - \vec{r_j}|\right)$$

aside: for relativistic calculations: fulfill Poincaré invariance

Define relative and c.m. coordinates

$$\vec{R} = \frac{1}{A}\sum_{i}^{A}\vec{r_i} \qquad \vec{z_i} = \vec{r_i} - \vec{R}$$

$$\vec{P} = \sum_{i}^{A}\vec{p_i} \qquad \vec{\zeta_i} = \vec{p_i} - \frac{\vec{P}}{A},$$

$$H = \frac{1}{2mA}|\vec{P}|^{\,2} + \frac{1}{2m}\sum_{i=1}^{A}|\zeta_i|^{\,2} + \frac{1}{2}m\omega^2\sum_{i}^{A}|r_i|^2 + \sum_{i<j}^{A} V\left(|\vec{r_i} - \vec{r_j}|\right)$$

Goal: Write Hamiltonian in such a way that c.m. pieces and intrinsic pieces are separated

The Hamiltonian then can be split into a center-of-mass (c.m.) piece,

$$H_{c.m.} = \frac{1}{2mA}P^2 + \frac{1}{2}Am\omega^2 R^2$$

and an intrinsic piece,

$$H_{int} = \frac{1}{2m}\sum_{i=1}^{A}\zeta_i^2 + \frac{m\omega^2}{2}\sum_{i=1}^{A} z^2 + \sum_{i<j}^{A} V(|\vec{r_i} - \vec{r_j}|) \ .$$

The intrinsic piece of the Hamiltonian can be rewritten in terms of single particle states,

$$H_{int} = \frac{1}{2mA}\sum_{i<j}^{A}|\vec{p_i} - \vec{p_j}|^2 + \frac{m\omega^2}{2A}\sum_{i<j}^{A}|\vec{r_i} - \vec{r_j}|^2 + \sum_{i<j}^{A} V(|\vec{r_i} - \vec{r_j}|)$$

Addive terms in a Hamiltonian $\longrightarrow$ product ansatz in wave function

$$\boxed{|\Psi_i JM\rangle = |\Psi_{int_i} JM\rangle \otimes |\phi_{cm} 0s\rangle}$$

we want this piece !

Compute c.m. piece:

$$\left\langle \vec{R}|\phi_{cm}, 0s\right\rangle = \phi_{0s}(\vec{R}) = R_{00}(R)Y_0^0(\theta,\phi) = \frac{2}{(b_{cm}^2)^{3/4}}\frac{1}{\pi^{1/4}}e^{\frac{-R^2}{2b_{cm}^2}} \times \frac{1}{\sqrt{4\pi}}$$

$$b_{cm}^2 = \frac{b^2}{A}$$

How to compute ?

The momentum space representation of the density matrix elements in the space-fixed frame is defined as

$$\langle \Psi_i J_f M_f \,|\hat{\rho}(\vec{q})|\, \Psi_i J_i M_i \rangle \equiv \left\langle \Psi_i J_f M_f \left| \sum_n e^{-i\vec{q}\cdot\hat{\vec{r}_n}} \right| \Psi_i J_i M_i \right\rangle . \tag{1.7.16}$$

Using relative coordinates $\hat{\vec{z}}_n = \hat{\vec{r}}_n - \hat{\vec{R}}$, this becomes

$$\langle \Psi_i J_f M_f \,|\hat{\rho}(\vec{q})|\, \Psi_i J_i M_i \rangle = \left\langle \Psi_i J_f M_f \left| \sum_n e^{-i\vec{q}\cdot\hat{\vec{z}_n}} e^{-i\vec{q}\cdot\vec{R}} \right| \Psi_i J_i M_i \right\rangle . \tag{1.7.17}$$

With $|\Psi_i JM\rangle = |\Psi_{int_i} JM\rangle \otimes |\phi_{cm} 0s\rangle$ one obtains

$$\langle \Psi_i J_f M_f \,|\hat{\rho}(\vec{q})|\, \Psi_i J_i M_i \rangle = \left\langle \phi_{cm_i} 0s | e^{-i\vec{q}\cdot\vec{R}} | \phi_{cm_i} 0s \right\rangle \underbrace{\left\langle \Psi_{int_i} J_f M_f | \sum_n e^{-i\vec{q}\cdot\hat{\vec{z}_n}} | \Psi_{int_i} J_i M_i \right\rangle}_{\text{in relative coordinates}} .$$

$$\left\langle \phi_{cm_i} 0s | e^{-i\vec{q}\cdot\vec{R}} | \phi_{cm_i} 0s \right\rangle = e^{-\frac{q^2 b_{cm}^2}{4}}$$

analytic !

$$\begin{aligned} \underbrace{\langle \Psi_i J_f M_f \,|\hat{\rho}_{t.i}(\vec{q})|\, \Psi_i J_i M_i \rangle}_{\text{translationally invariant (t.i) local one-body density}} &= e^{\frac{q^2 b_{cm}^2}{4}} \langle \Psi_i J_f M_f \,|\hat{\rho}(\vec{q})|\, \Psi_i J_i M_i \rangle \\ &= e^{\frac{q^2 b^2}{4A}} \underbrace{\langle \Psi_i J_f M_f \,|\hat{\rho}(\vec{q})|\, \Psi_i J_i M_i \rangle}_{\text{calculated}} . \end{aligned}$$

Space-Fixed Non-Local One-Body Density

$$\rho(\vec{r},\vec{r'}) = \left\langle \phi' \left| \sum_{i=1}^{A} \delta^3(r_i - r)\delta^3(r_i' - r') \right| \phi \right\rangle$$

$$\rho(\vec{r},\vec{r'}) = \left\langle A\lambda' J' M' \left| \sum_{i=1}^{A} \frac{\delta(r_i - r)}{r^2} \frac{\delta(r_i' - r')}{r'^2} \sum_{\mu\nu} \sum_{\mu'\nu'} Y_\mu^\nu(\hat{r}_i) Y_\mu^{*\nu}(\hat{r}) Y_{\mu'}^{*\nu'}(\hat{r'}) Y_{\mu'}^{\nu'}(\hat{r_i'}) \right| A\lambda J M \right\rangle$$

Expand in multipoles

$$\begin{aligned} \rho(\vec{r},\vec{r'}) &= \sum_{\mu\mu'} \sum_{K=|\mu-\mu'|}^{\mu+\mu'} (-1)^{J'-M'} \begin{pmatrix} J' & K & J \\ -M' & 0 & M \end{pmatrix} \mathcal{Y}_{K0}^{*\mu\mu'}(\hat{r},\hat{r'}) \times \\ &\quad \frac{1}{\hat{K}} \sum_{\alpha\beta} \left\langle \alpha \left\| \frac{\delta(r_i - r)}{r^2} \frac{\delta(r_i' - r')}{r'^2} \mathcal{Y}_K^{\mu\mu'}(\hat{r}_i,\hat{r_i'}) \right\| \beta \right\rangle \langle A\lambda' J' \| (a_\alpha^\dagger \tilde{a}_\beta)^{(K)} \| A\lambda J \rangle \end{aligned}$$

After evaluating reduced matrix elements

$$\rho(\vec{r},\vec{r'}) = \sum_{Kll'} (-1)^{J'-M'} \begin{pmatrix} J' & K & J \\ -M' & 0 & M \end{pmatrix} \mathcal{Y}_{K0}^{*l'l}(\hat{r},\hat{r'}) \tilde{\rho}_{Kll'}(r,r')$$

$$\tilde{\rho}_{K\mu\mu'}(r,r') = \sum_{njn'j'} \hat{j}\hat{j'}(-1)^{\mu'+\mu+j+\frac{1}{2}+K} \begin{Bmatrix} \mu' & \mu & K \\ j & j' & \frac{1}{2} \end{Bmatrix} R_{n'\mu'j'}(r') R_{n\mu j}(r) \langle A\lambda' J' \| (a_\alpha^\dagger \tilde{a}_\beta)^{(K)} \| A\lambda J \rangle$$

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Derivation of Center of Mass Contribution

Variables:

$$
\begin{aligned}
\vec{q} &= \vec{p}' - \vec{p} \\
\vec{\mathcal{K}} &= \frac{1}{2}(\vec{p} + \vec{p}') \\
\vec{\zeta} &= \frac{1}{2}(\vec{r} + \vec{r}') \\
\vec{Z} &= \vec{r}' - \vec{r} \,,
\end{aligned}
$$

momentum transfer

displacement

Change of variables $\vec{r}, \vec{r}' \rightarrow \vec{Z}, \vec{\zeta}$ with help of Talmi-Moshinsky brackets

separate ζ and Z into relative and c.m. components,

$$
\begin{aligned}
\zeta &= \zeta_{rel} + \zeta_{c.m.} \\
Z &= Z_{rel} \,.
\end{aligned}
$$

Momentum space representation

$$\left\langle \Psi' J' M' \middle| \hat{\rho}(\vec{q}, \vec{\mathcal{K}}) \middle| \Psi J M \right\rangle \equiv \left\langle \Psi' J' M' \middle| e^{-i\vec{q}\cdot\vec{\zeta}} e^{-i\vec{\mathcal{K}}\cdot\vec{Z}} \middle| \Psi J M \right\rangle$$

Evaluate and obtain translationally invariant $\hat{\rho}(\vec{q}, \vec{\mathcal{K}})_{t.i.}$

$$= \sum_{n' n l' j j} \sum_{K} \sum_{n_q l_q n_\mathcal{K} l_\mathcal{K}} \text{clebsches} \cdot \text{brackets} * \text{angular functions} * R_{n_q l_q}(q)\, R_{n_\mathcal{K} l_\mathcal{K}}(\mathcal{K}) * \langle A \lambda' J' \| (a^\dagger_\alpha a_\beta)^K \| A \lambda J \rangle$$

new angular variables

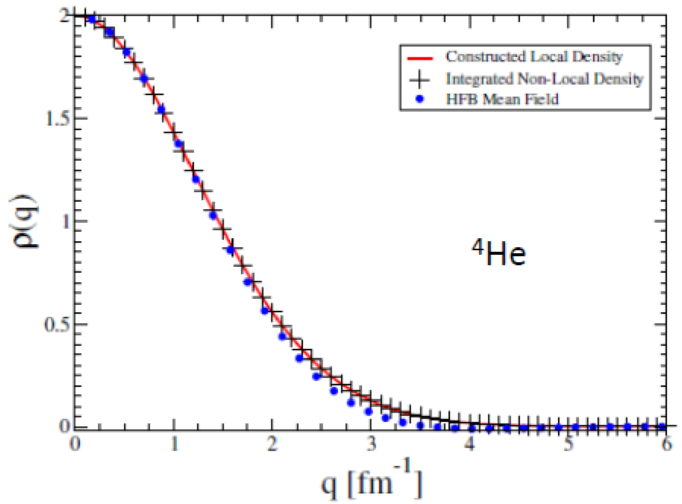
Reality Check:

local density $\rho(\vec{q}) = \int d^3 K \, \rho(\vec{q}, \vec{K})$

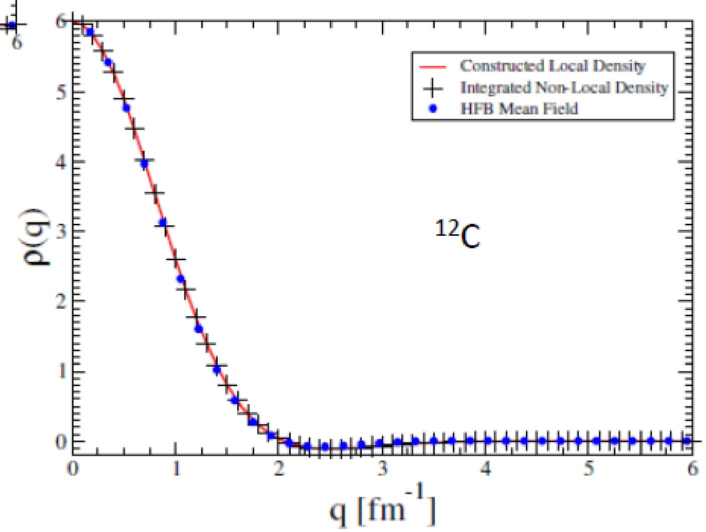
compared with directly constructed $\rho(q)$

Here ground state, i.e. $\rho^{(0)}(q)$

$K = 0$



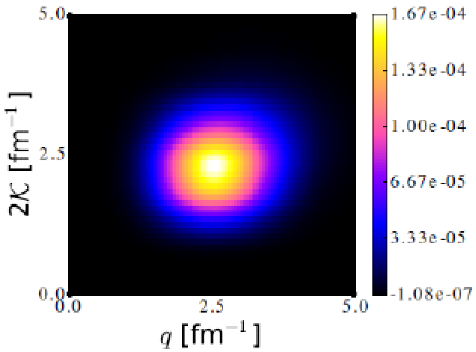
Compared to Hartree-Fock-Bogoliubov calculation using the Gogny DS1 potential

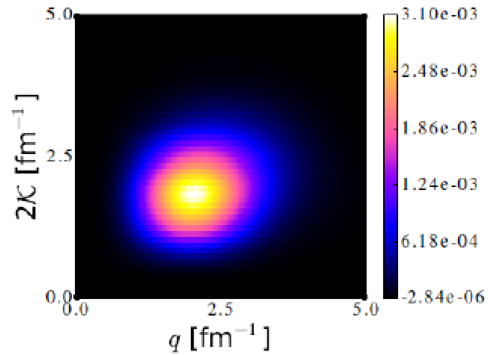


Successively sum contributions over l_q and $l_{\mathcal{K}}$.

Non-local one-body density matrix for ^{4}He

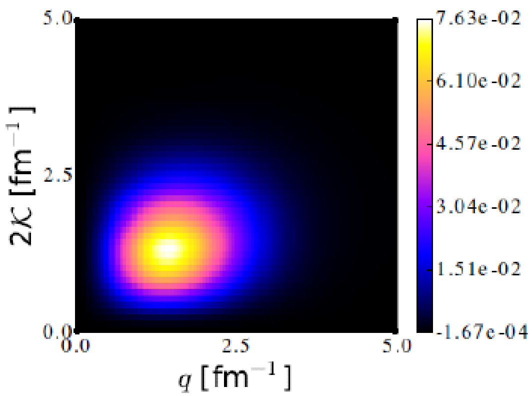
$\sum_{l_q=5}^{14} \rho_{K=0}(q,\mathcal{K})$

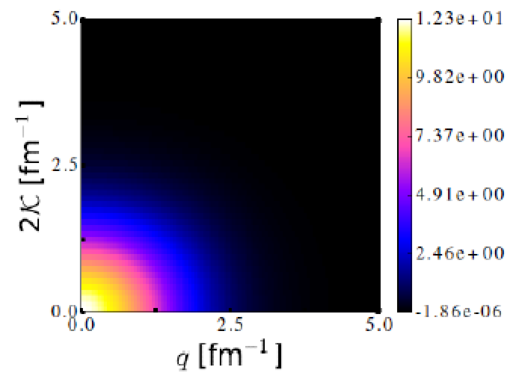




$\sum_{l_q=4}^{14}$

$\sum_{l_q=2}^{14}$





$\sum_{l_q=0}^{14}$

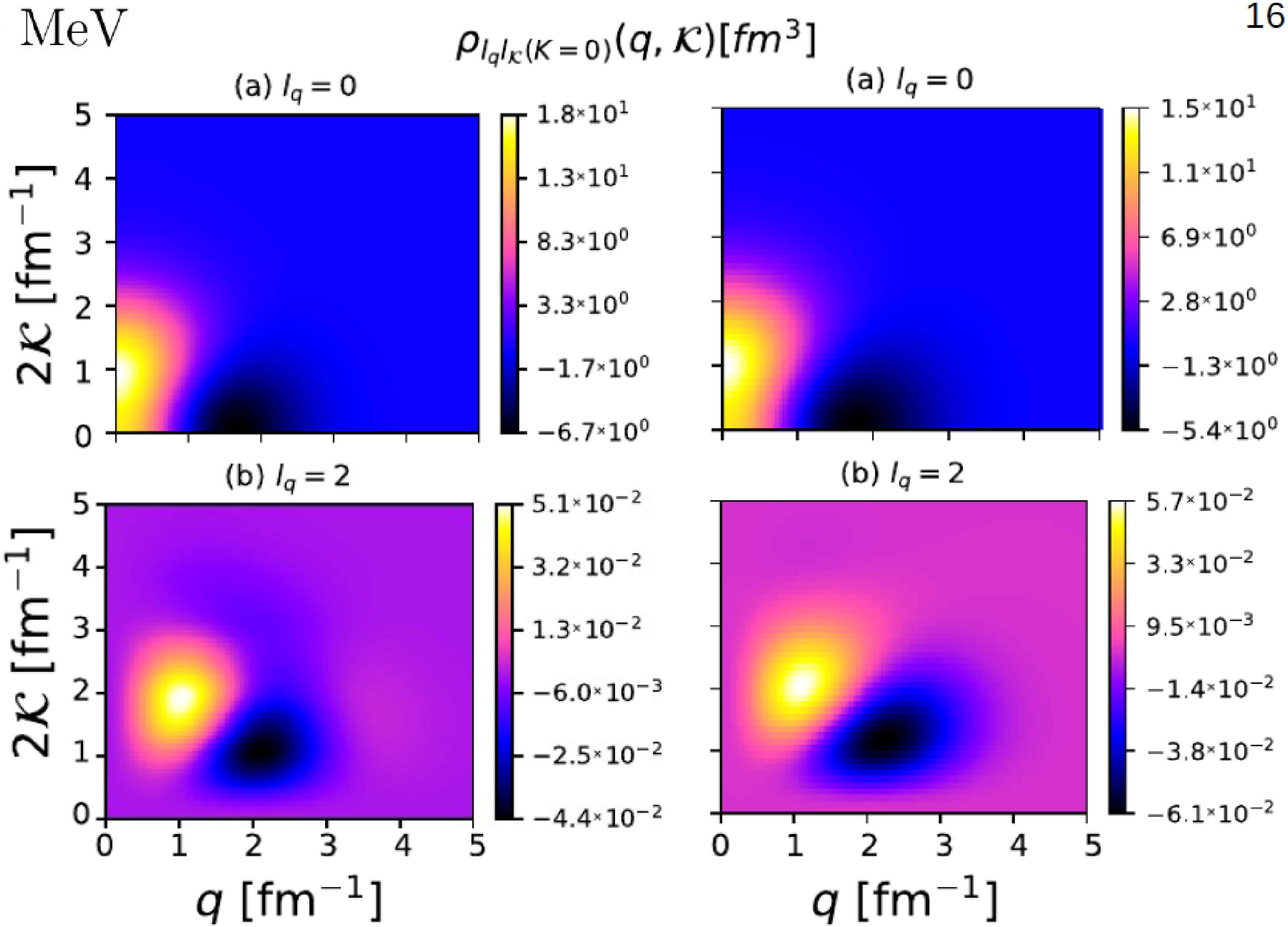
Reduced matrix elements provided by P. Maris, $N_{max} = 14$, JISP16, $\hbar\Omega = 20$

N_{max}=8

$\hbar\omega$=20 MeV

Proton distribution

^{16}O



NNLO$_{opt}$

JISP16

Observables in p+A elastic scattering

- p == proton or neutron
- A == spin-zero nucleus (closed shell)
- Independent vectors: $\vec{k}, \vec{k}', \ \vec{k} \times \vec{k}'$ or $\vec{k} \pm \vec{k}, \vec{k} \times \vec{k}'$
- Elastic scattering: |k| = |k'|
- Most general form of scattering amplitude
- spin-1/2 $\rightarrow$ spin-0: $A \cdot 1 + \vec{\sigma} \cdot \vec{C}$
- Assuming rotational invariance and parity conservation

$$M = A \cdot 1 + C\, \vec{\sigma} \cdot \left(\hat{k} \times \hat{k}'\right)$$
$$= A(k,\theta) + \underbrace{C(k,\theta)}_{\text{Spin-flip amplitude}} \vec{\sigma} \cdot \hat{N}$$

Explicitly:

- k and k' span scattering plane (x-z - plane)
- y-plane $\vec{\sigma} \cdot \hat{\mathrm{N}} = \sigma_{\mathrm{y}}$
- With standard Pauli spinors:

$$\frac{d\sigma}{d\Omega}(\theta, +\hat{y} \rightarrow +\hat{y}) = \left|\chi_{+y}\left(A + C\sigma_y\right)\chi_{+y}\right| = \left|A + C\right|^2$$

$$\frac{d\sigma}{d\Omega}(\theta, +\hat{y} \rightarrow -\hat{y}) = 0$$

- **Unpolarized cross section:**
  - **Average of initial states and sum of all final states**

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{1}{2}\left[\frac{d\sigma}{d\Omega}(\theta, i \rightarrow +\hat{y}) + \frac{d\sigma}{d\Omega}(\theta, i \rightarrow -\hat{y})\right]$$

$$= \left|A(\theta)\right|^2 + \left|C(\theta)\right|^2$$

Analyzing Power A_y

- Spin of the outgoing projectile is measured
- Incident beam is unpolarized

$$A_y = \frac{\frac{d\sigma}{d\Omega}(\theta, i \to +\hat{y}) - \frac{d\sigma}{d\Omega}(\theta, i \to -\hat{y})}{\frac{d\sigma}{d\Omega}(\theta, i \to +\hat{y}) + \frac{d\sigma}{d\Omega}(\theta, i \to -\hat{y})} = \frac{2\,\Re e\left(A^*(\theta)\,C(\theta)\right)}{|A(\theta)|^2 + |C(\theta)|^2}$$

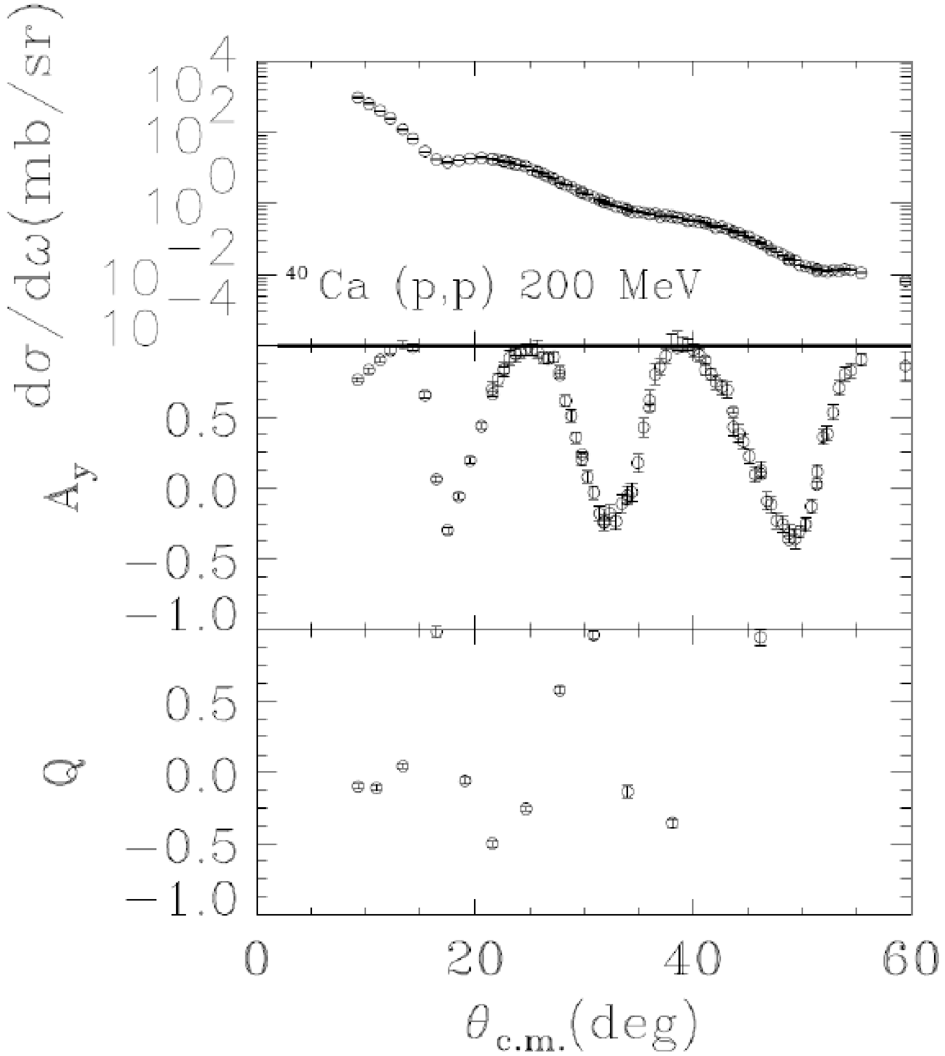
$\Rightarrow$ Spin dependence out of the scattering plane

Spin rotation parameter Q

- Measures the rotation of the spin vector in the scattering plane
- $+x \rightarrow \pm z$

$$Q = \frac{\frac{d\sigma}{d\Omega}(\theta, \hat{x} \rightarrow +\hat{z}) - \frac{d\sigma}{d\Omega}(\theta, \hat{x} \rightarrow -\hat{z})}{\frac{d\sigma}{d\Omega}(\theta, \hat{x} \rightarrow +\hat{z}) + \frac{d\sigma}{d\Omega}(\theta, \hat{x} \rightarrow -\hat{z})} = \frac{2\,\Im\mathrm{m}\left(A(\theta)C^*(\theta)\right)}{|A(\theta)|^2 + |C(\theta)|^2}$$

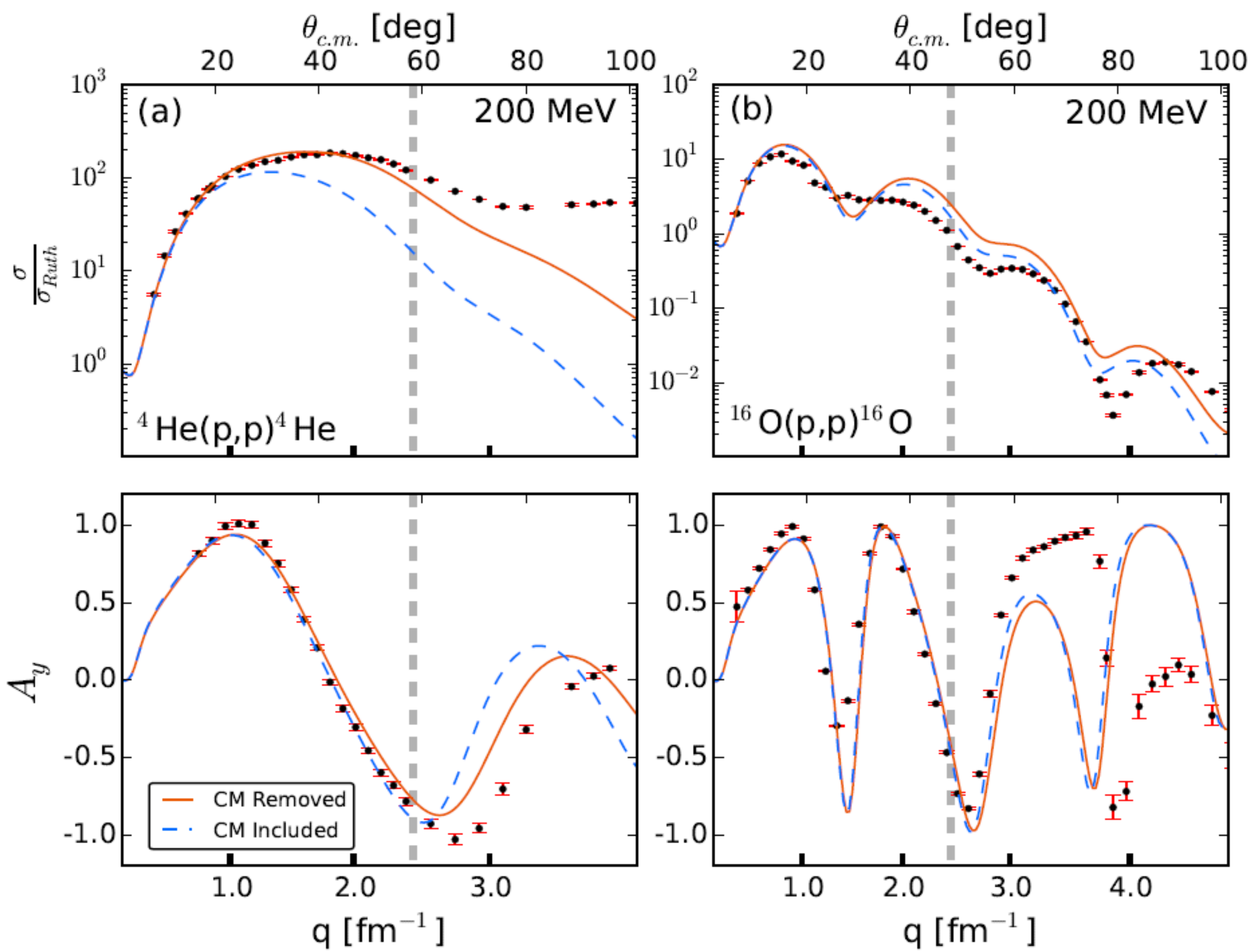
$\Rightarrow$ Spin dependence within the scattering plane



In addition one has

- **total cross section**
- **reaction cross section**

Effect of center-of-mass



NN amplitude: $f_{NN}(k'k;E) = C \langle k' | t_{NN}(E) | k \rangle$

Variables (E,k',k,φ) ⇒ (E, q, K, θ)

with $q = k' - k$

$K = ½ (k' + k)$

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis

Struck nucleon "*i*" : target basis

$$\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i\cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)}\otimes\mathbf{1}+\mathbf{1}\otimes\boldsymbol{\sigma}^{(i)})\cdot\hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{n}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) - H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) + H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN})$$

- -

$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})\right]$$ Off-shell

Most general form

NN amplitude: $f_{NN}(k'k;E) = C\,\langle k' | t_{NN}(E) | k \rangle$

Variables (E,k',k,φ) ⇒ (E, q, K, θ)

with $q = k' - k$

$K = \frac{1}{2}(k' + k)$

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis

Struck nucleon "*i*" : target basis

Usual assumption:
Spin saturated
ground state

$$
\begin{aligned}
\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = {} & A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i\cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)}\otimes\mathbf{1} + \mathbf{1}\otimes\boldsymbol{\sigma}^{(i)})\cdot\hat{\mathrm{n}}_{NN} \\
& + M(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{n}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{n}}_{NN}) \\
& + (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) - H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}}) \\
& + (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) + H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN})
\end{aligned}
$$

$$
+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})\right]
$$

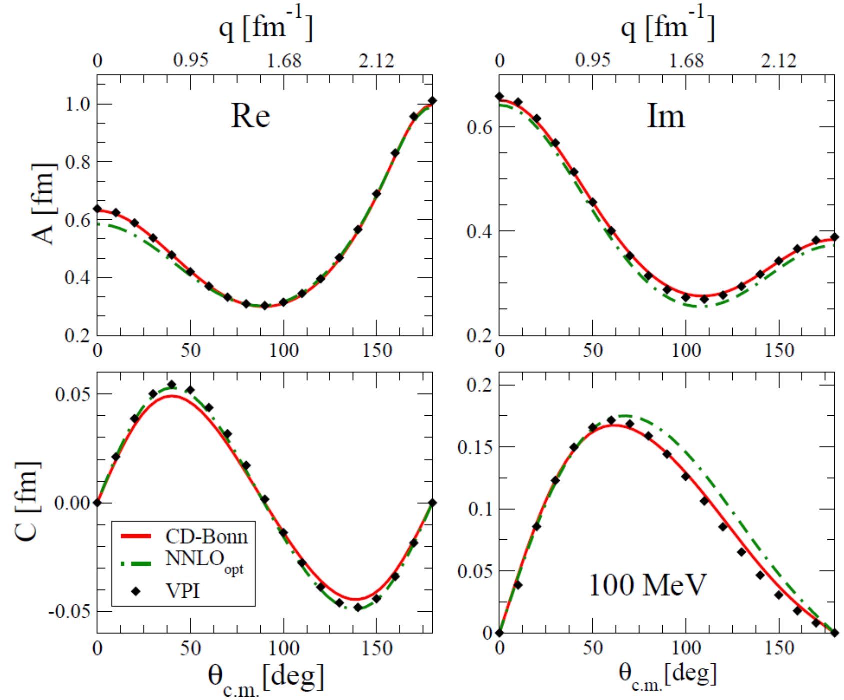
Off-shell

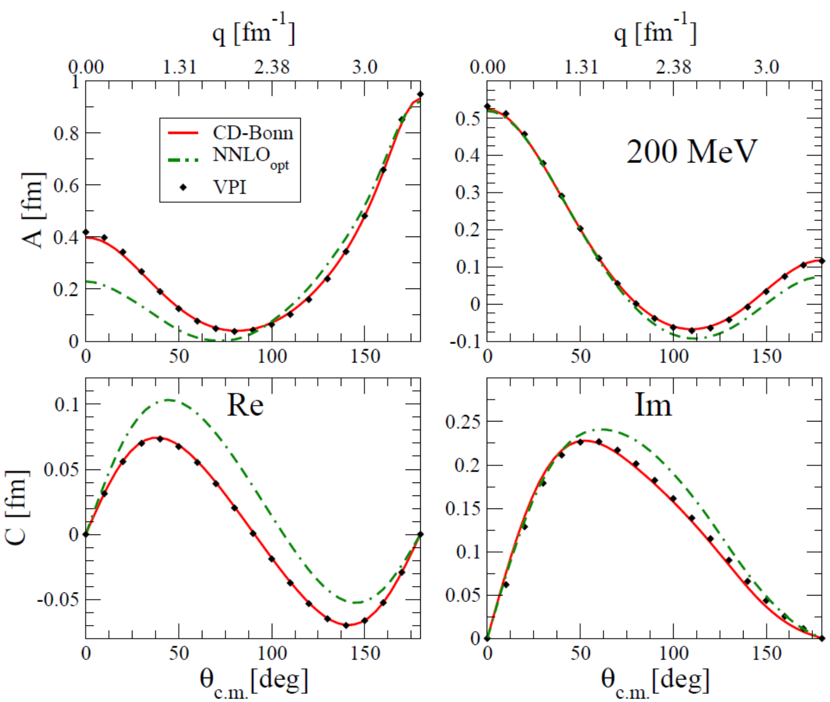
Remark: Spin dependence is not explicit in usual definition of one-body density matrix

Wolfenstein Amplitudes A and C

NNLO$_{opt}$ fitted to E_{lab}=125 MeV

→ max. momentum transfer ≈ 2.45 fm^{-1}



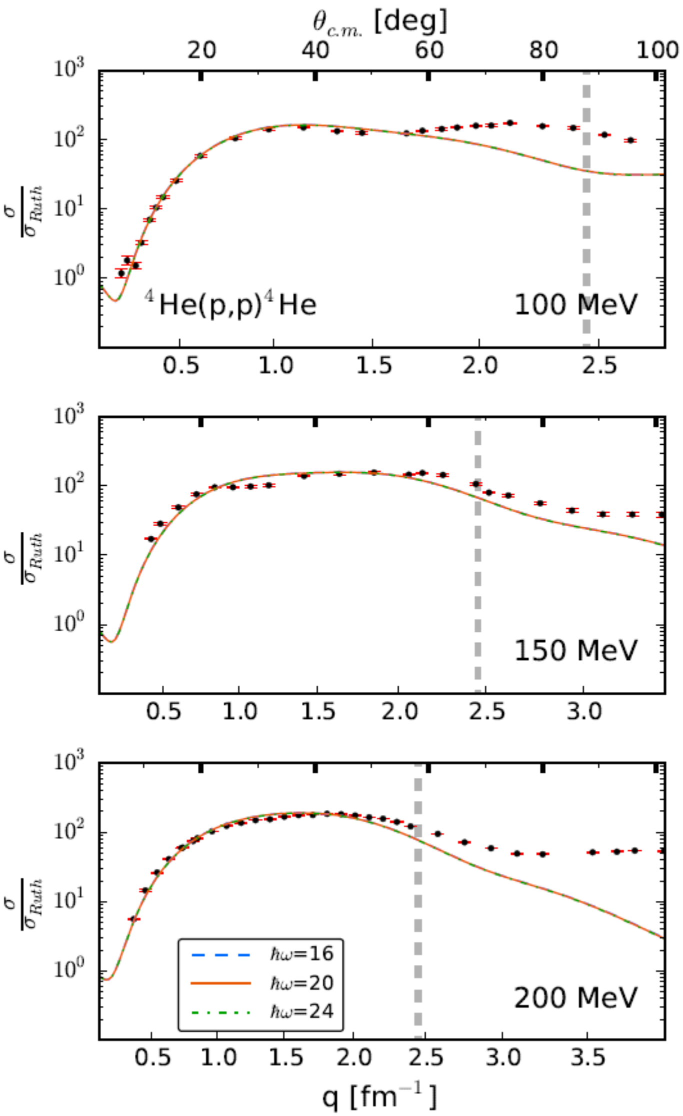


$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

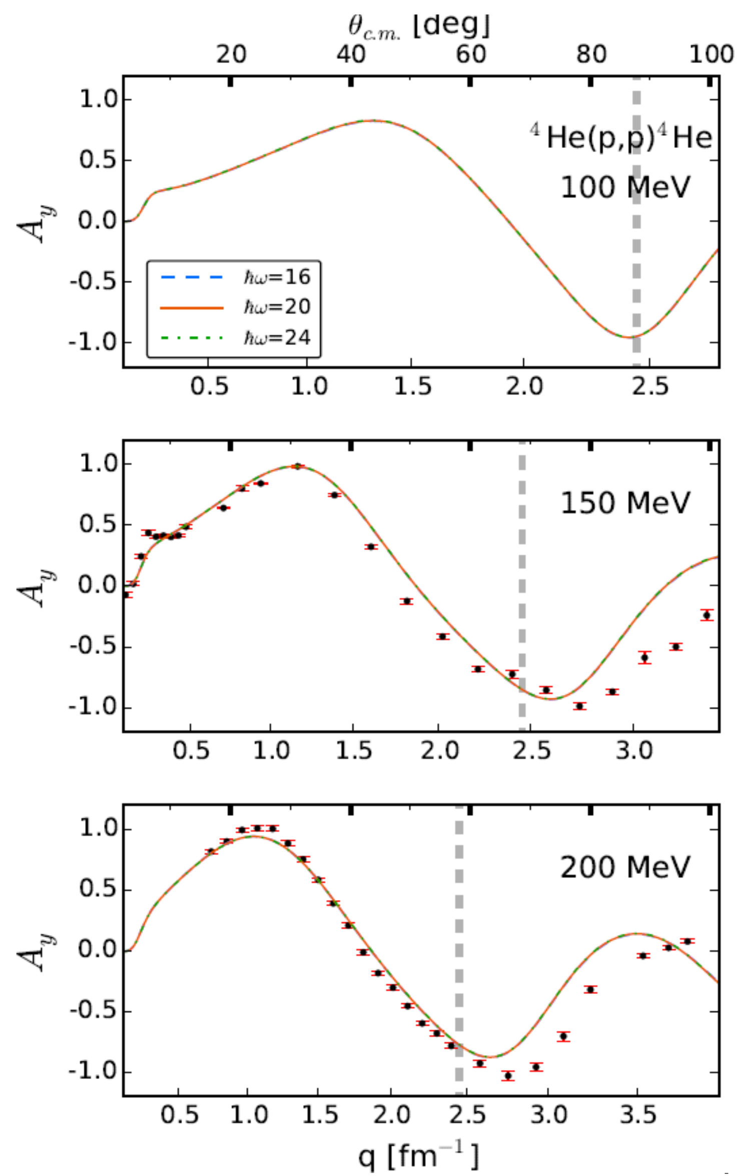
$$q \approx 480 \text{ MeV} = 2.45 \text{ fm}^{-1}$$

Nmax=18

^{4}He



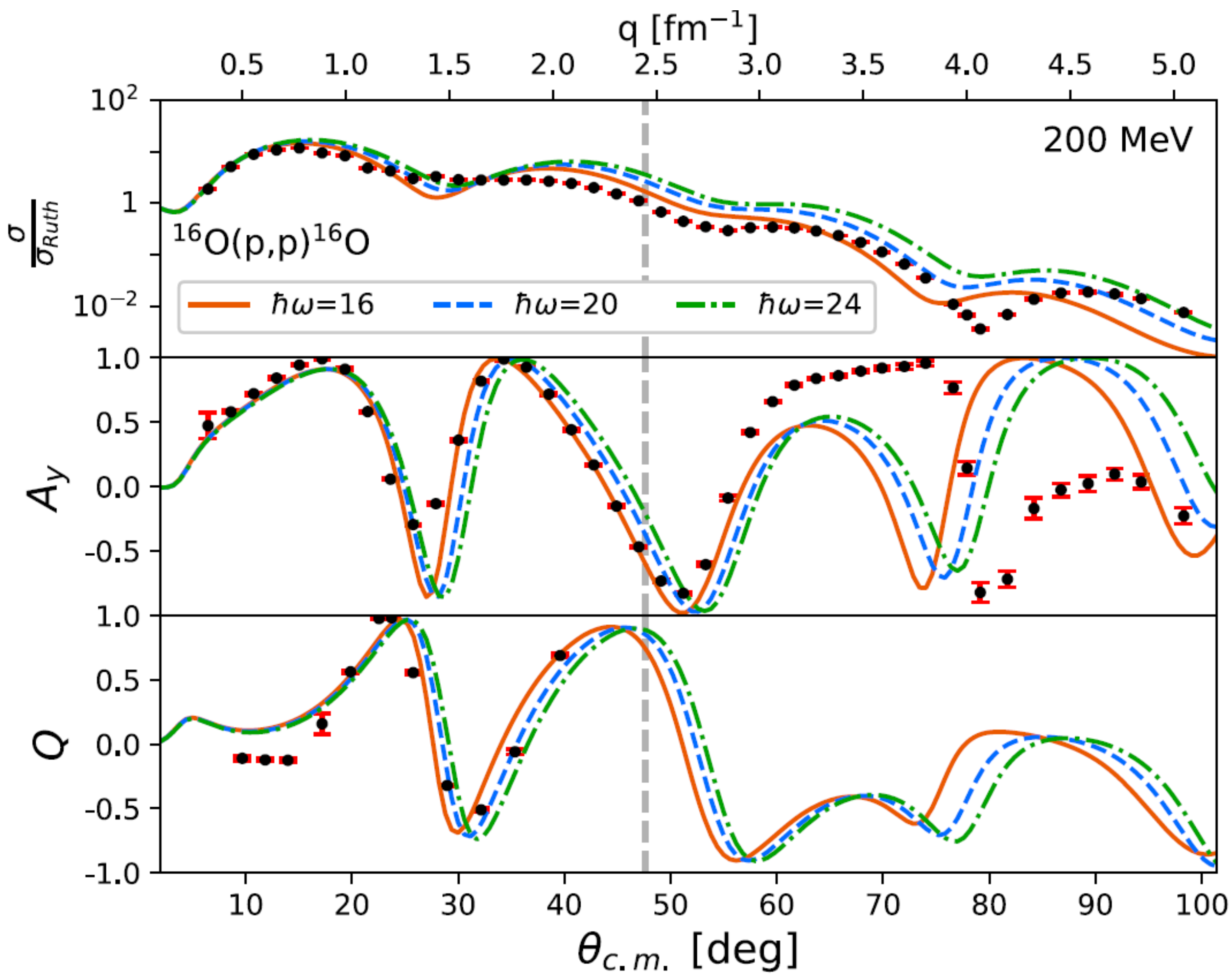
NNLO$_{opt}$ fitted up to Elab=125 MeV



Burrows, Elster, Weppner, Launey, Maris, Nogga, Popa
arXiv:1810.06442

^{16}O

N_{max}=10



$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

$$q \approx 480 \text{ MeV} = 2.45 \text{ fm}^{-1}$$

$NNLO_{opt}$ fitted up to Elab=125 MeV

Burrows, Elster, Weppner, Launey, Maris, Nogga, Popa
arXiv:1810.06442

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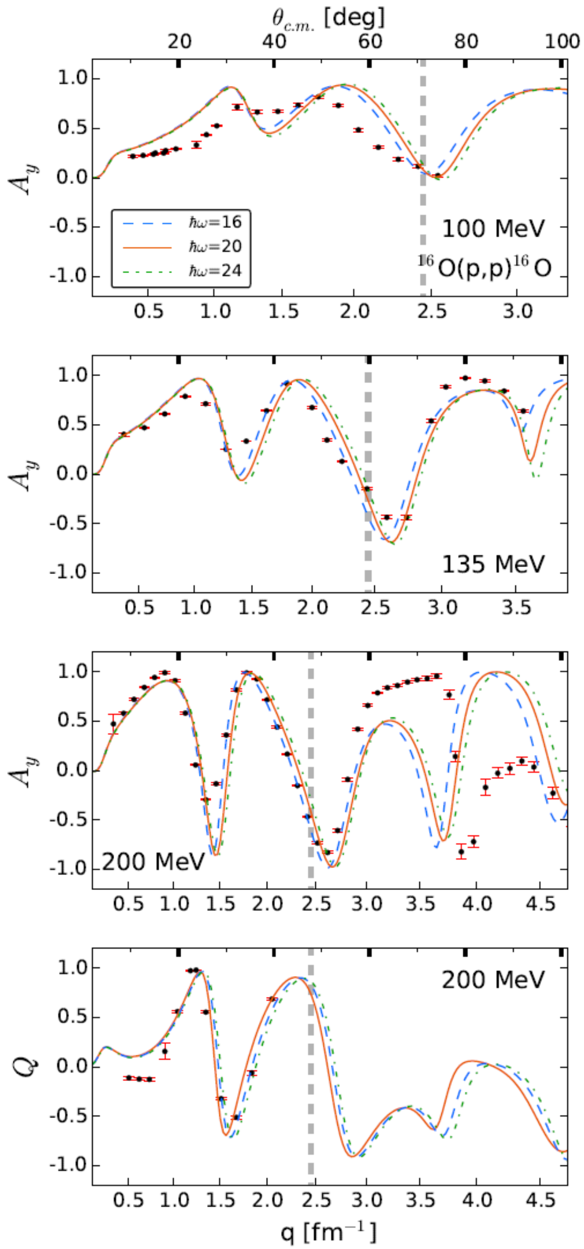


$\theta_{c.m.}$ [deg]
20
40
60
80
100
1.0
0.5
0.0
-0.5
-1.0
A_y
$\hbar\omega$=16
$\hbar\omega$=20
$\hbar\omega$=24
100 MeV
^{16}O(p,p)^{16}O
0.5
1.0
1.5
2.0
2.5
3.0
135 MeV
3.5
200 MeV
4.0
4.5
Q
q [fm^{-1}]

Previous calculations

Weppner, Elster, Hüber, PRC 57, 1378 (1998)

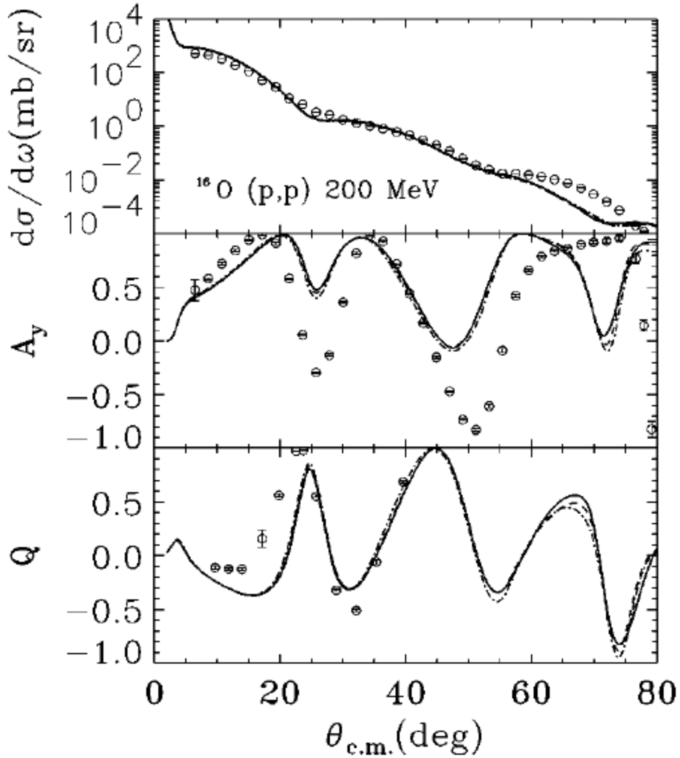


FIG. 1. The angular distribution of the differential cross section ($d\sigma/d\Omega$), analyzing power (A_y), and spin rotation function (Q) are shown for elastic proton scattering from ^{16}O at 200 MeV laboratory energy. The solid line represents the calculation performed with a first-order full-folding optical potential based on the DH density [14] and the CD-Bonn model [2]. The dashed line uses the NijmI model instead, the dash-dotted line the NijmII model [1]. The data are taken from Ref. [19].

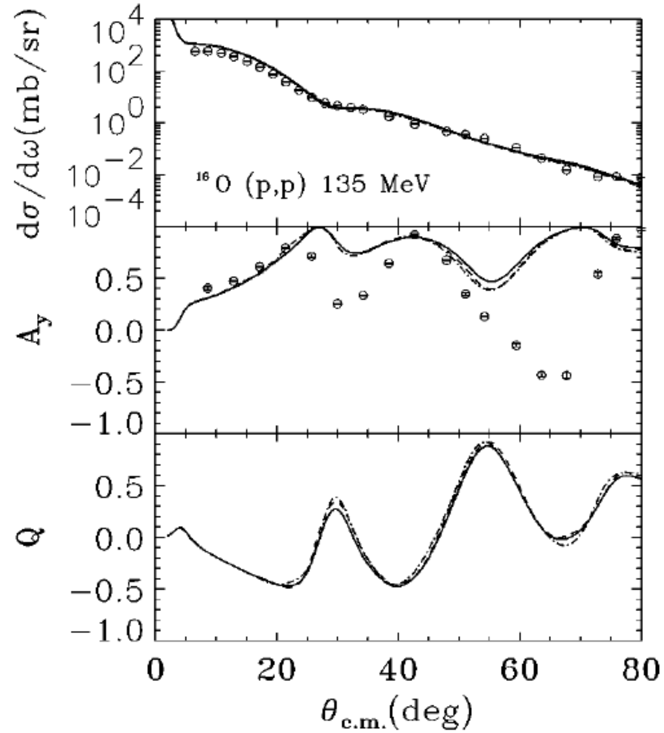
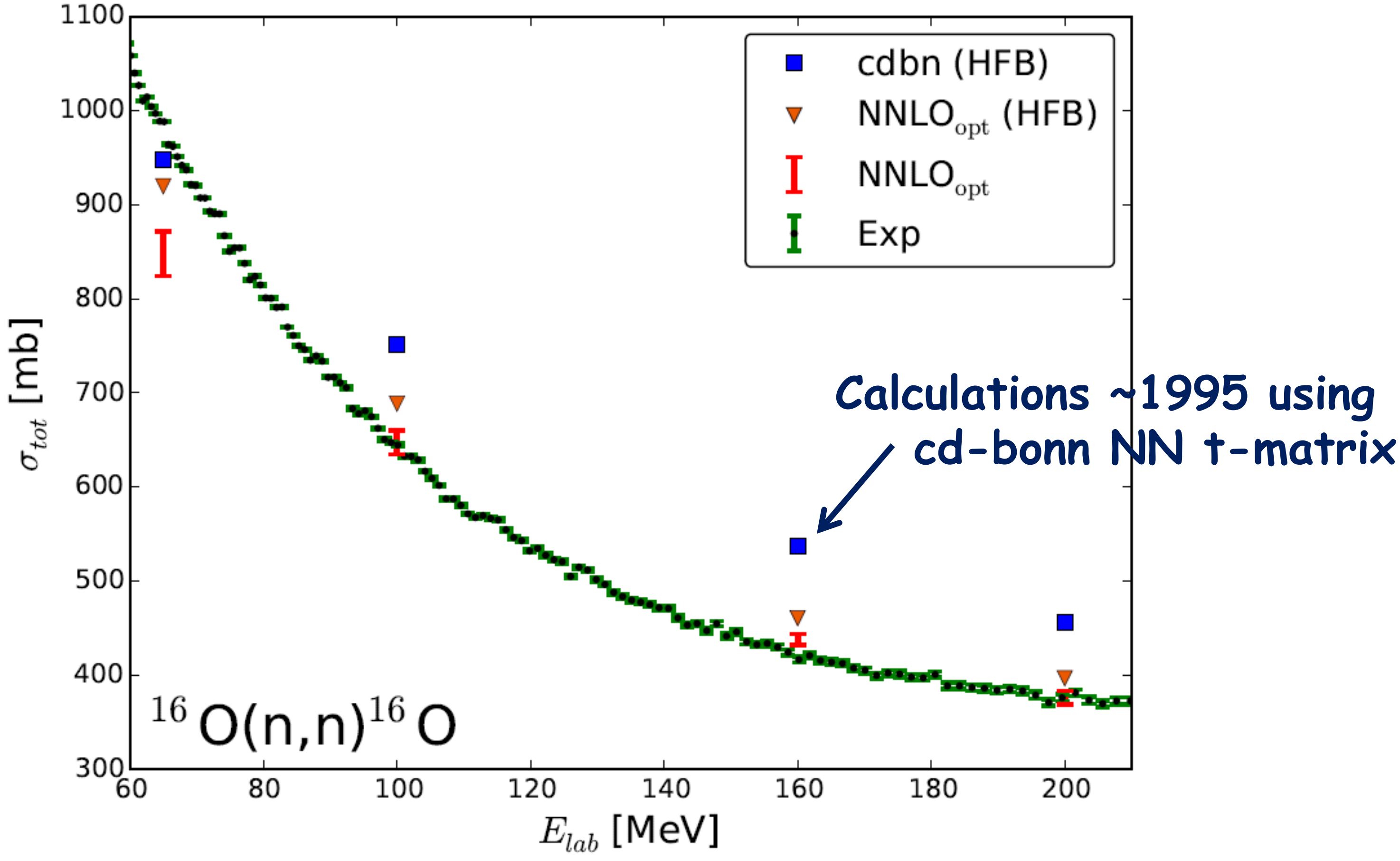
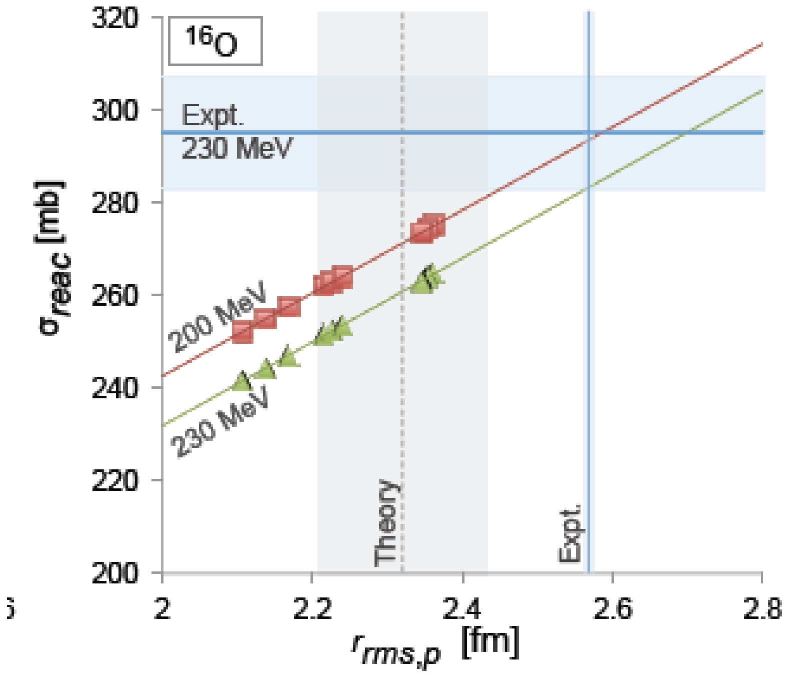


FIG. 5. Same as Fig. 1, except that the projectile energy is 135 MeV. The data are taken from Ref. [22].

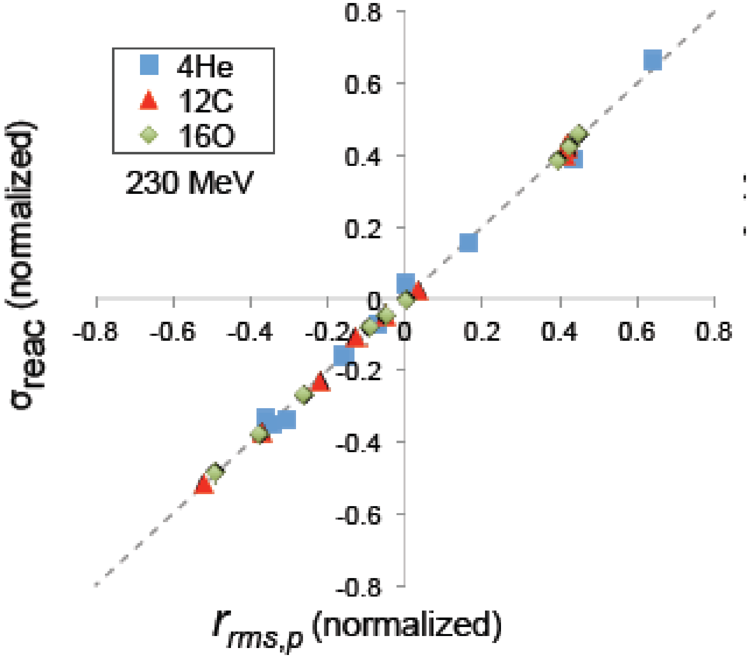
Total cross section for neutron scattering



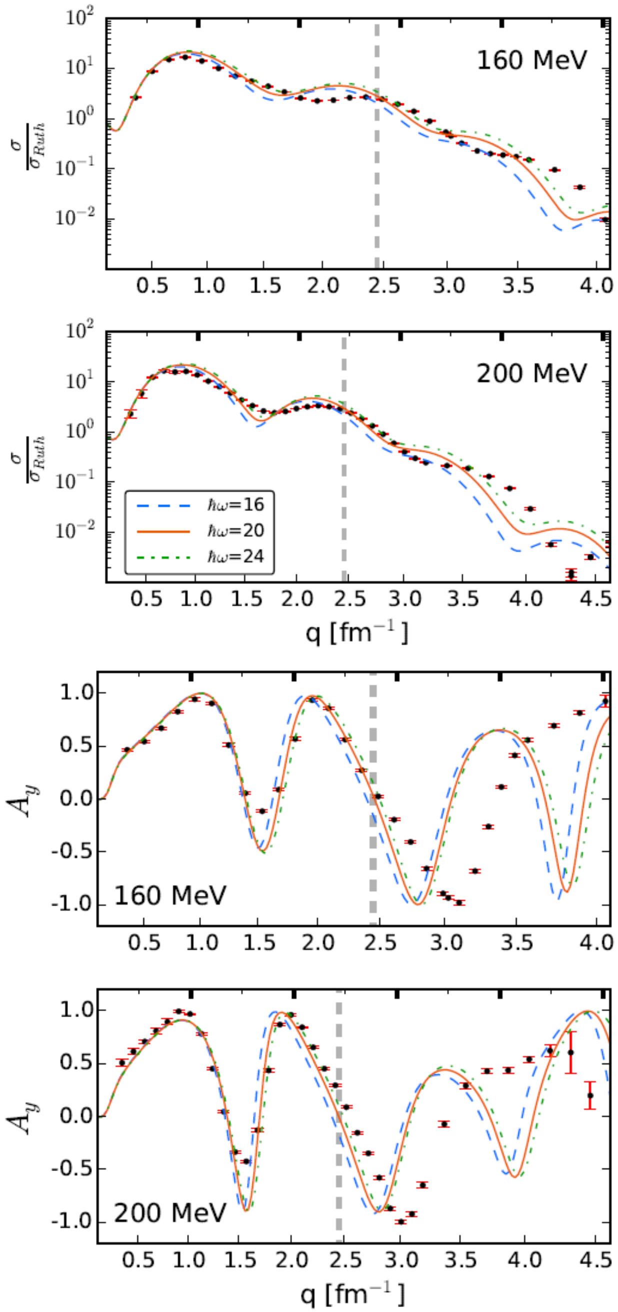
Reaction cross section and point proton radius



Reaction cross section and point proton radius



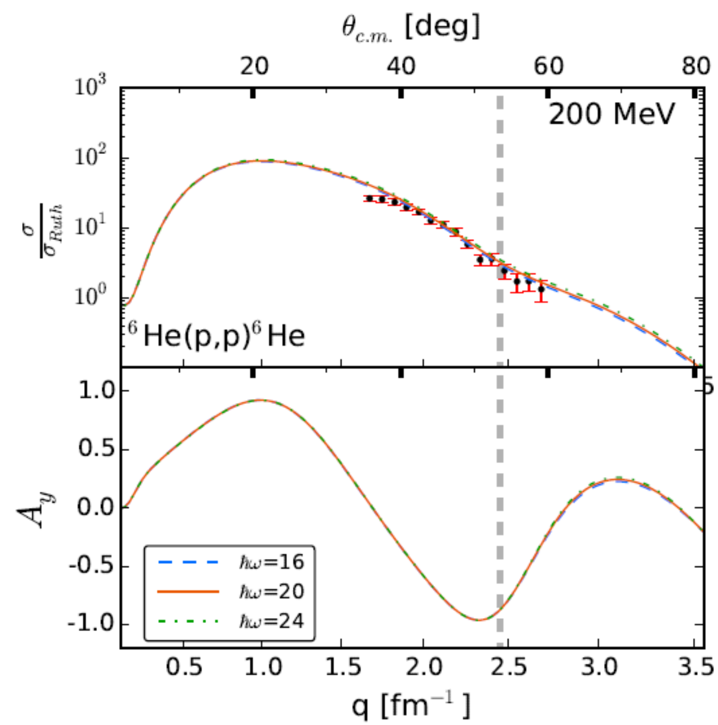
$^{12}C(p,p)^{12}C$



Note:

Implementation of first order term
(past, present, all groups)

only exact for spin saturated ground states
(≡ spin-flip of struck target nucleon neglected)



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