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Wednesday 880.05

· On left board (for reference): "anharmonic oscillator"

$$\boxed{\hat{H}(\hat{p},\hat{x}) = :\hat{H}(\hat{p},\hat{x}): = \frac{\hat{p}^2}{2m} + V(\hat{x}) \overset{\text{example}}{=} \frac{\hat{p}^2}{2m} + \frac{1}{2}a\hat{x}^2 + \frac{1}{4}\lambda\hat{x}^4}$$

(note: this will look like ϕ function field theory examples, but very different since long-distance confining forces)

$$Z = \mathrm{Tr}\, e^{-\beta\hat{H}} = \int dx \langle x | e^{-\beta\hat{H}} | x\rangle$$

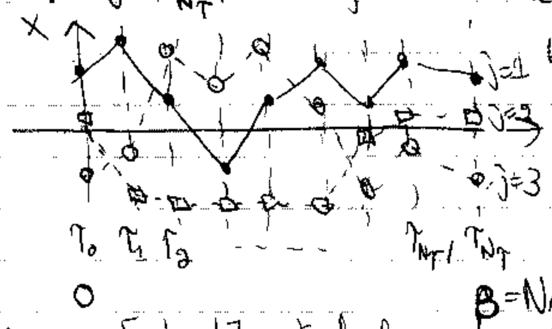
$Z(\beta)$

$$= \int dx \int_{x(0)=x}^{x(\beta)=x} \mathcal{D}(x(\tau))\, e^{-\frac{1}{\hbar}\int_0^\beta d\tau \left[\frac{m}{2}\left(\frac{dx(\tau)}{d\tau}\right)^2 + V(x(\tau))\right]}$$

$$= \int_{x(\beta)=x(0)} \mathcal{D}[x(\tau)]\, e^{-\frac{1}{\hbar}\int_0^\beta d\tau \left[\frac{m}{2}\left(\frac{dx(\tau)}{d\tau}\right)^2 + V(x(\tau))\right]} = \int_{x(\beta)=x(0)} \mathcal{D}[x(\tau)]\, e^{-S_E[x(\tau)]/\hbar}$$

$$\left[\int d^3x_1 \int d^3x_2 \int d^3x_3 \cdots \int d^3x_{N_\tau}\right] = \int_{-\infty}^{\infty} \prod_{k=1}^{N_\tau} dx_k \left(\frac{m}{2\pi\epsilon\hbar}\right)^{\frac{3N_\tau}{2}} e^{-\frac{\epsilon}{\hbar}\sum_{i=1}^{N_\tau}\left[\frac{m}{2}\left(\frac{x_i - x_{i-1}}{\epsilon}\right)^2 + V(x_{i-1})\right]}$$

where $\beta = N_\tau \epsilon$ (Monday N_τ was M but notation is too confusing) and $x_0 = x_{N_\tau}$. The trajectories $x(\tau) \to \{x(\tau_i)\}_j$ begin and end at same x



· Recall that the $\frac{\hat{p}^2}{2m}$ went away after

$$\int \frac{d^3p_n}{(2\pi)^3} e^{-\epsilon\frac{p_n^2}{2m} + ip_n(x_n - x_{n-1})} = \left(\frac{m}{2\pi\epsilon}\right)^{3/2} e^{-\epsilon\frac{m}{2}\left(\frac{x_n - x_{n-1}}{\epsilon}\right)^2}$$

can be absorbed in defining Z or hidden in $\mathcal{D}$.

Note: Using $\tau \in [-\beta/2, \beta/2]$ instead of $\tau \in [0,\beta]$ is often simpler.

· Plan for today:

① Quick comments on MATLAB PS#1

② Recap of intro to stochastic evaluation of path integrals

③ Brief follow-ups to one-particle path integral
- spectral decomposition, perturbation theory, correlation functions

④ Generalization to many-particle states — (anti) symmetrization
- issues for numerical evaluation

Monday → ⑤ Intro to alternative based on coherent states and field operators

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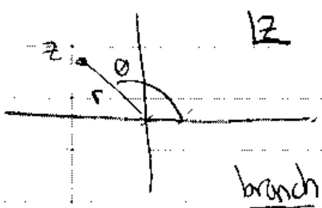
MATLAB PS#2 Comments [any questions on diagrams?]

- Well executed solutions – we'll build on that when possible. (Eg. MATLAB code to illustrate SVM for PS#2.)

- Most (if not all) of you encountered an error evaluating logm(A) when A had real negative eigenvalue.
  "Principal matrix logarithm not defined for A with nonpositive real eigenvalues. A non-principal matrix log is returned."
  - let's remember what is going on in complex plane

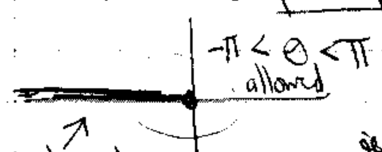


$\mathbb{Z}$ polar form $z = re^{i\theta}$

$\Rightarrow \ln z = \ln r + \ln e^{i\theta} = \ln r + i\theta$

branch point at $z=0$, multivalued as $\theta \to 0 \to 2\pi \to 4\pi \to$ ... keeps changing.

So define principal logarithm ($\Rightarrow$ single valued) by excluding negative real axis:



- Not defined here for $\theta = \pi$ or $\theta = -\pi$ (although typically $-\pi < \theta \le \pi$ is used)
  ie is logm(-1) = $\cancel{\ln 1}^{0} + i\pi$ or $-i\pi$
  1x1 matrix (chooses $+i\pi$)

For a matrix, runs into this if eigenvalues lie on branch cut.

- MATLAB for loops
  M = randn(5) + i*randn(5)

one way: eye(5) + M + M*M/2 + M*M*M/6 + ... (or M^3/factorial(6))

another way:

```
sum = 0
for i = 0:10        % or whatever
  sum = sum + M^i/factorial(i)
end
disp(sum);
```

print ("display") final result

exclude ; to print each intermediate answer

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- Recap & intro to stochastic evaluation of path integrals

See Joaquin Drut notes.