

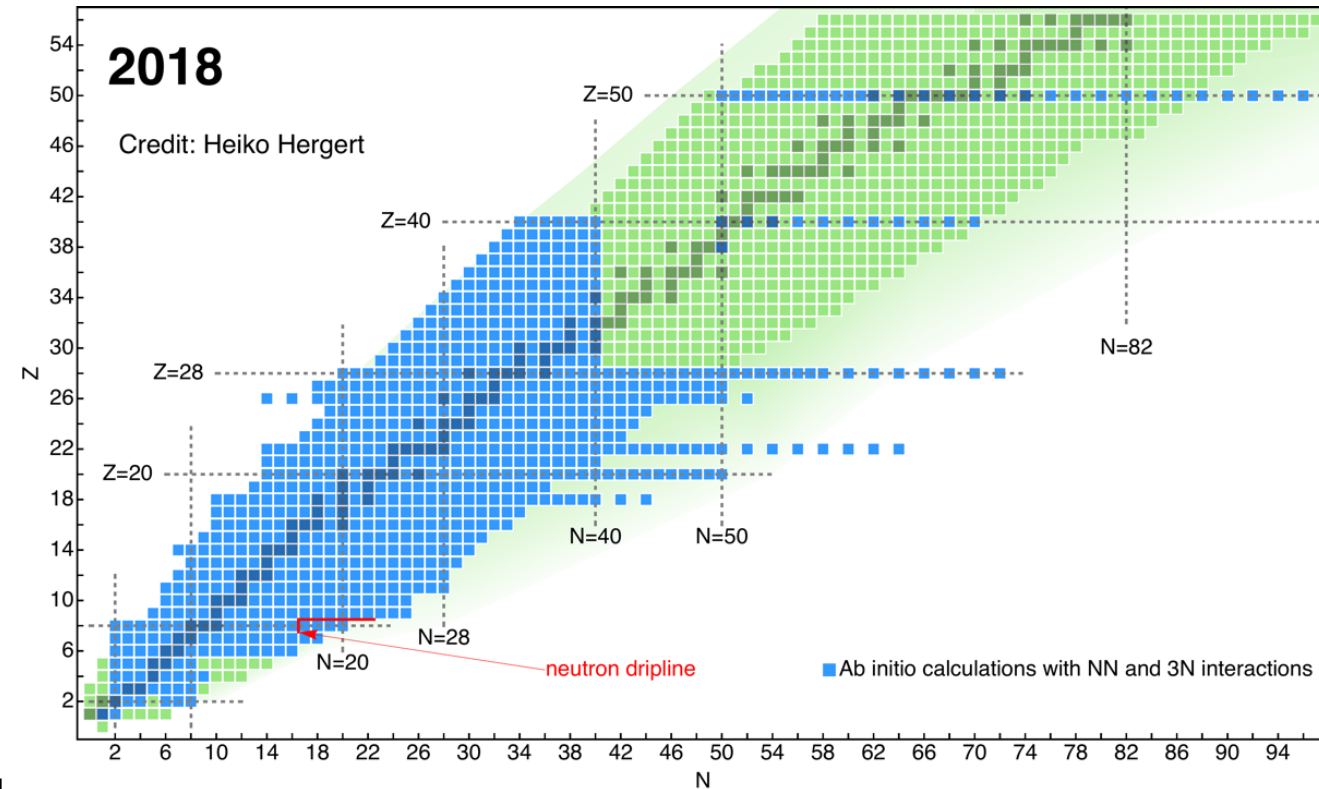
# Status of nuclear optical potentials and future prospects

Anthony Tropiano

August 6, 2019

# Introduction

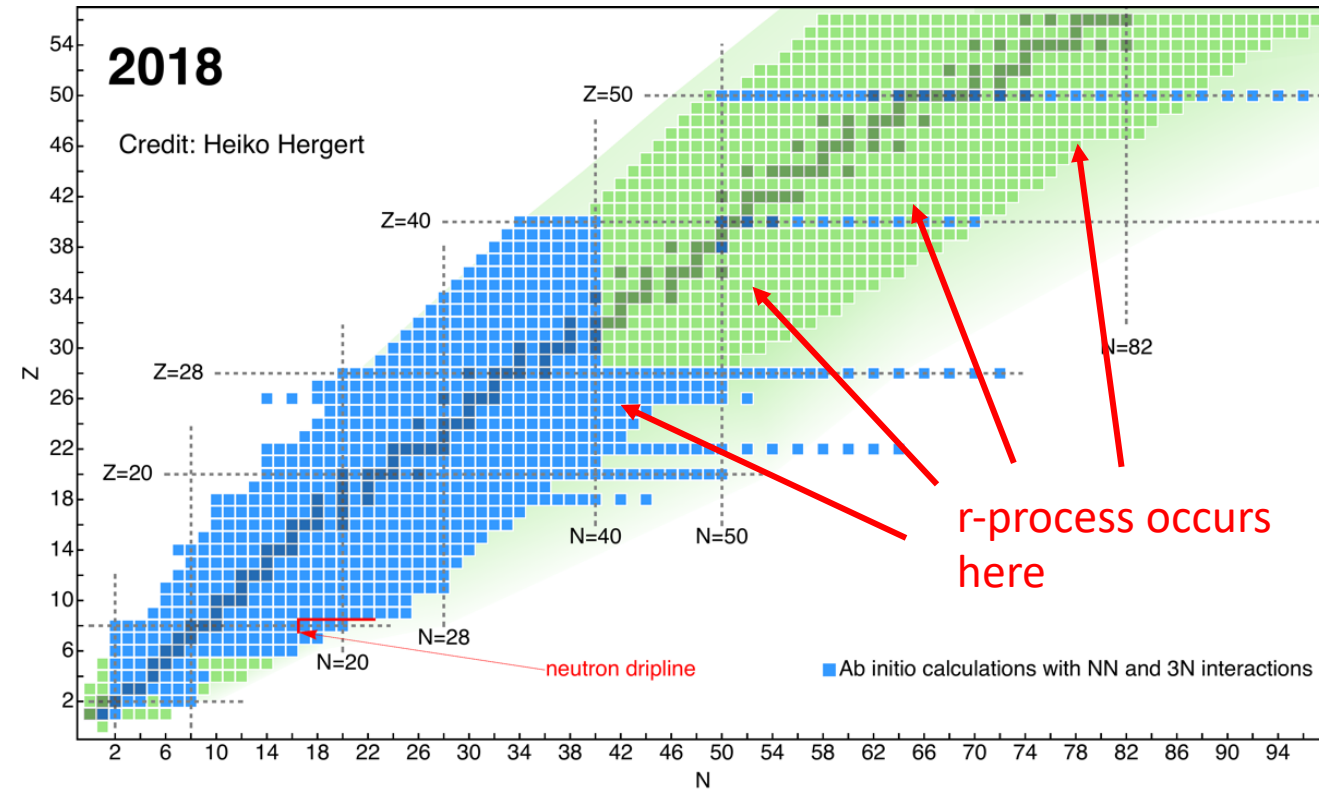
- Nuclear reactions play a key role in answering questions such as the origin of heavy elements in the universe, fundamental symmetries, and the limits of nuclear stability
- Facilities seek to produce exotic isotopes and measure new data to better understand these areas



**Fig. 1:** Chart of nuclides with neutron number,  $N$ , counted horizontally and proton number,  $Z$ , counted vertically.  
(Figure from H. Hergert.)

# Introduction

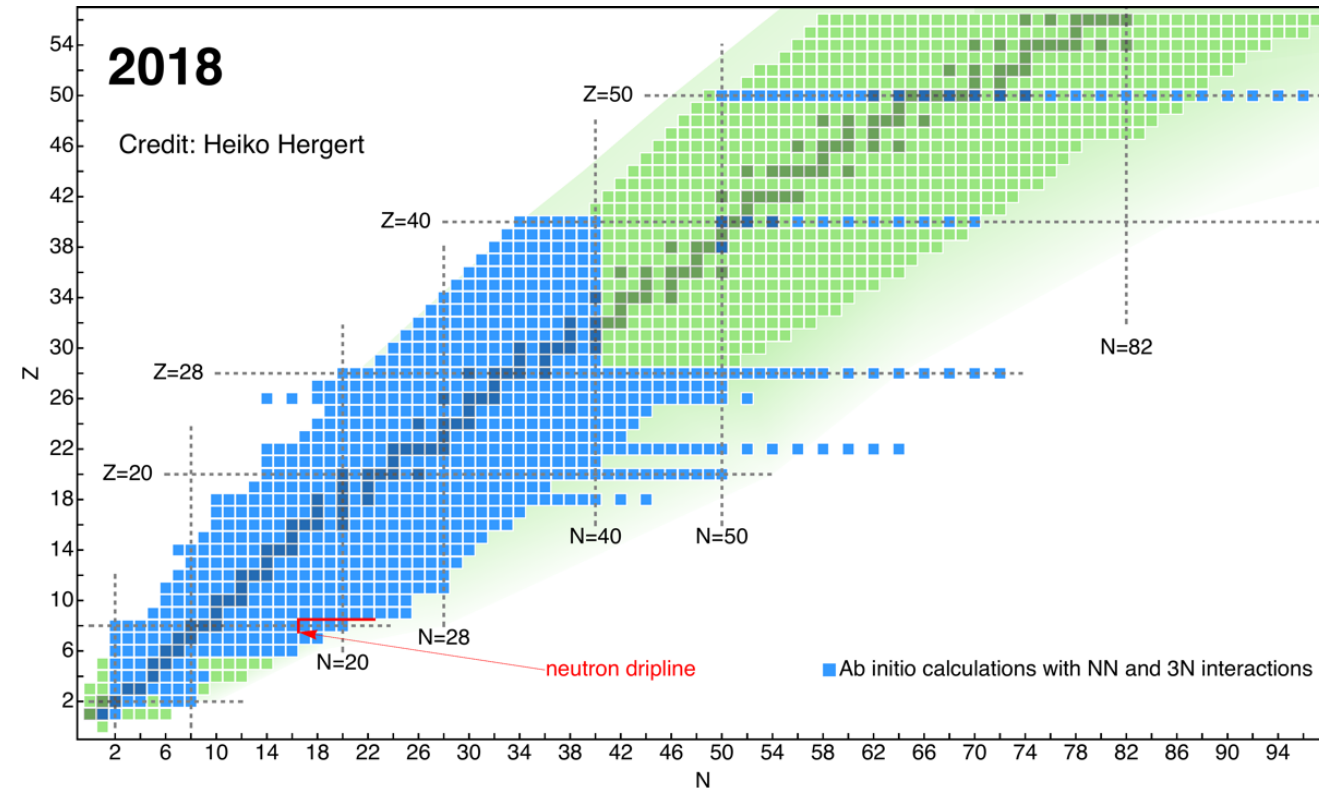
- For example, the Facility for Rare Isotope Beams (FRIB) will target neutron-rich isotopes to study the rapid neutron-capture process (r-process)
- The r-process is responsible for the formation of roughly half the atomic nuclei past iron on the periodic table



**Fig. 1:** Chart of nuclides with neutron number,  $N$ , counted horizontally and proton number,  $Z$ , counted vertically.  
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# Introduction

- Critical to understand nuclear reactions since facilities must use reactions to produce and study short-lived exotic nuclei



**Fig. 1:** Chart of nuclides with neutron number,  $N$ , counted horizontally and proton number,  $Z$ , counted vertically.  
(Figure from H. Hergert.)

# Nuclear scattering

- Projectile-nucleus scattering is a quantum many-body problem
  - An incident particle interacts with  $A$  nucleons (protons and neutrons) in a target nucleus
- Difficulties of nuclear many-body systems:
  - i. Often non-perturbative
  - ii. Computational difficulty of the problem drastically increases with nuclear mass number  $A$

# Nuclear optical potential

- Can introduce a complex potential to model the effective projectile-nucleus interaction in scattering experiments
- These potentials are called **optical potentials**

$$U(\mathbf{r}) = V(\mathbf{r}) + iW(\mathbf{r})$$

- Properties of  $U(\mathbf{r})$ :

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  - i. Non-Hermitian
  - ii. Gives non-unitary S-matrix
  - iii. For  $W < 0$ , gives an absorptive potential meaning a loss of flux

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- Optical potentials simplify nuclear scattering by giving an effective projectile-nucleus interaction that accounts for absorption of incident particles (inelastic scattering)

# Formalism

- $A + 1$  particle system consisting of incident nucleon and target nucleus of mass number  $A$  described by Schrödinger equation

$$\mathcal{H}\Psi = E\Psi$$

where the total Hamiltonian is

$$\mathcal{H}(\mathbf{r}_0, \dots, \mathbf{r}_A) = H_A(\mathbf{r}_1, \dots, \mathbf{r}_A) + T_0 + V(\mathbf{r}_0, \dots, \mathbf{r}_A)$$

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- The nuclear Hamiltonian satisfies the Schrödinger equation

$$H_A \psi_i = \epsilon_i \psi_i$$

for wave functions  $\psi_i$  and energies  $\epsilon_i$  where  $i = 0$  is the ground state

# Formalism

- The optical potential is given by

$$V_{opt}(\mathbf{r}_0) = V_{00} + \mathbf{V}_0 \frac{1}{E - \mathbf{H} + i\eta} \mathbf{V}_0^\dagger$$

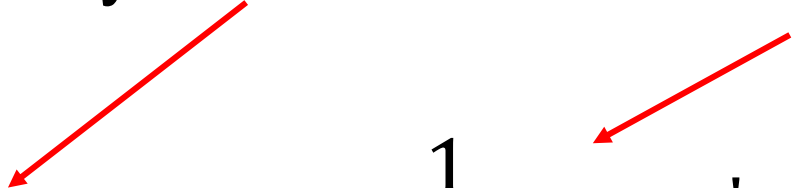
where  $V_{ij} = \langle \psi_i | V | \psi_j \rangle$ ,  $H_{ij} = T_0 \delta_{ij} + V_{ij} + \epsilon_i \delta_{ij}$  for  $i, j > 0$ , and  $\mathbf{V}_0 = (V_{01}, V_{02}, \dots)$

- $\eta \rightarrow 0^+$  to ensure only outgoing waves are present in exit channels

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Elastic scattering                      Inelastic scattering

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$$V_{opt}(\mathbf{r}_0) = V_{00} + \mathbf{V}_0 \frac{1}{E - \mathbf{H} + i\eta} \mathbf{V}_0^\dagger$$

- $V_{opt}$  is complex, energy dependent, and non-local



# Formalism

$$V_{opt}(\mathbf{r}_0) = V_{00} + \mathbf{V}_0 \frac{1}{E - \mathbf{H} + i\eta} \mathbf{V}_0^\dagger$$

- $V_{opt}$  is complex, energy dependent, and non-local
- Cannot be evaluated for realistic systems

# Phenomenology

- General form of phenomenological optical potential

$$V_{opt}(r, E) = V_C(r) - V_V(E)f(x_0) + \left(\frac{\hbar}{m_\pi c}\right)^2 V_{SO}(E) \boldsymbol{\sigma} \cdot \boldsymbol{l} \frac{1}{r} \frac{d}{dr} f(x_{SO}) - i[W_V(E)f(x_W) - 4W_D(E) \frac{d}{dx_D} f(x_D)]$$

- Woods-Saxon form factors:

$$f(x_i) = \frac{1}{1 + e^{x_i}}$$

where  $x_i = (r - R_i)/a_i$  for nuclear radii and diffusivity parameters  $R_i$  and  $a_i$

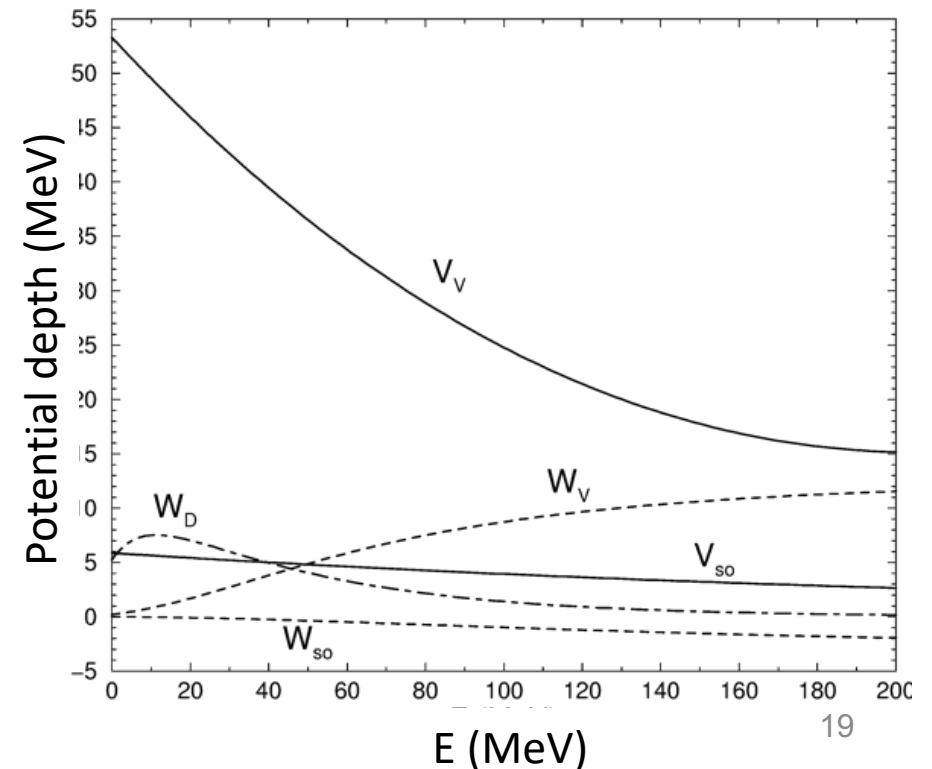
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- Obtained by  $\chi^2$  minimization fitting scattering observables with radii and diffusivity parameters

**Fig. 2:** Potential well depths as a function of laboratory energy  $E$  for each of the terms above including an imaginary spin-orbit term,  $W_{SO}$ . (A. J. Koning and J. P. Delaroche, Nucl. Phys. A **713**, 213 (2003).)



# Phenomenology

- Ambiguity in fitting
  - Several sets of parameters can give a good fit
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- Ambiguity in fitting
  - Several sets of parameters can give a good fit
  - Heavily dependent on data sets used
- Cannot extend to exotic nuclei where no data are present
- No reliable way to quantify uncertainty in phenomenological optical potentials

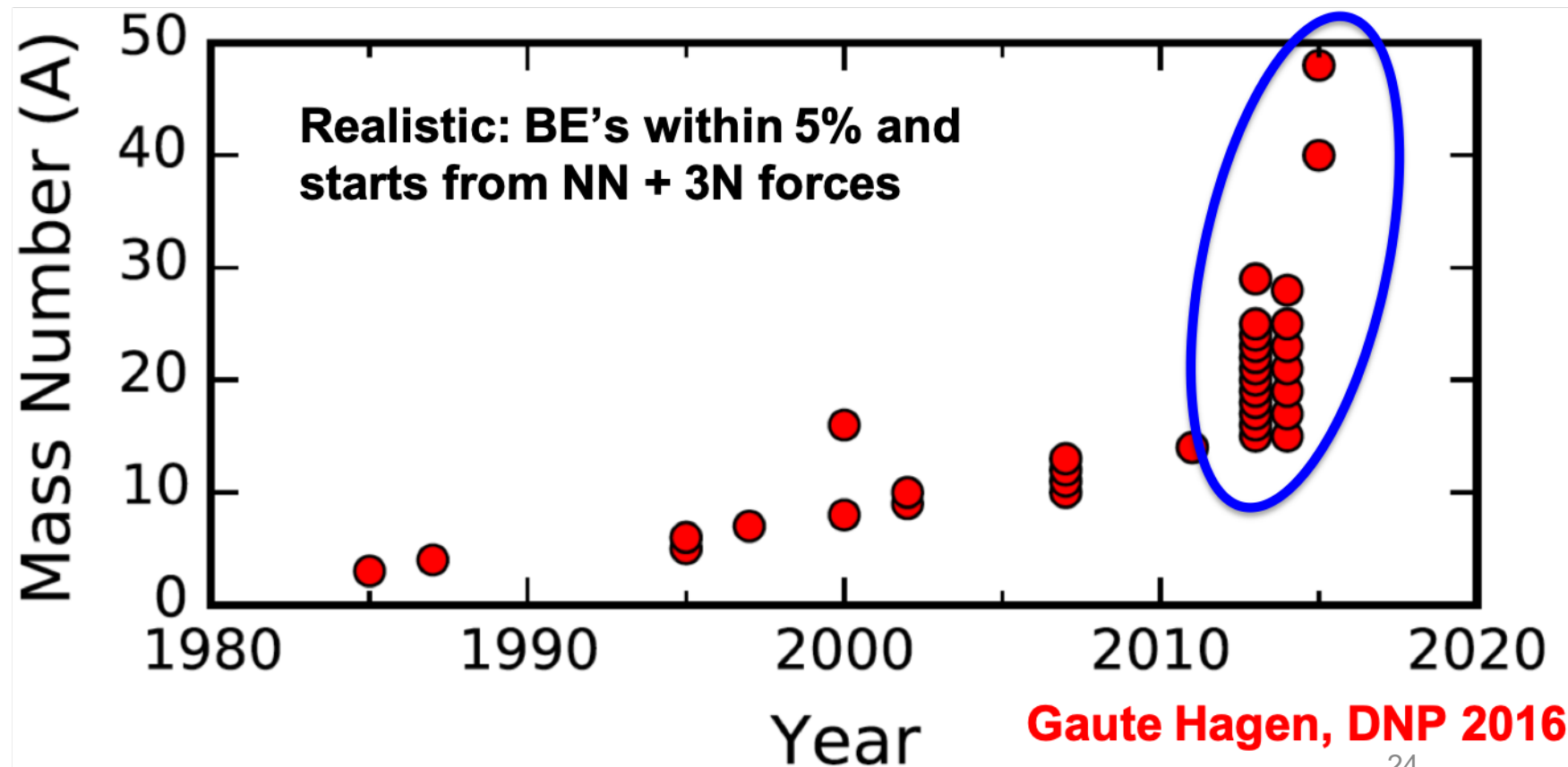
# Microscopic approaches

- Microscopic optical potentials are models based off realistic nuclear structure inputs
- Can overcome shortcomings of phenomenological models (e.g. predictive power, uncertainty quantification)

# Microscopic approaches

- Microscopic nuclear structure has made enormous progress in the past decade

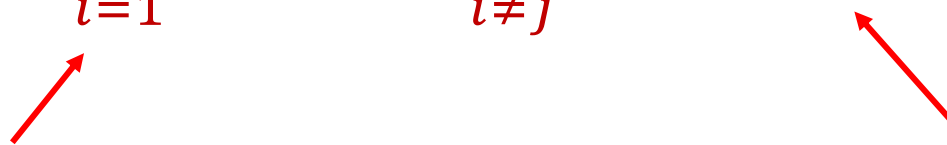
**Fig. 3:** Binding energies for A-body nuclei within 5% of the experimental value calculated from *ab initio* methods. (Figure from G. Hagen.)





# Multiple-scattering approach

- Basic idea is to express the optical potential  $U$  in terms of the NN  $T$ -matrix and nuclear density
- Define projection operators  $P$  and  $Q$  which project onto elastic and inelastic channels, respectively
- Apply the spectator expansion to the optical potential

$$U = \sum_{i=1}^A \tau(0, i) + \sum_{i \neq j}^A \tau(0, i) Q G_0(E) \tau(0, j) + \dots$$


Projectile interacts with one nucleon

Projectile interacts with two nucleons

# Multiple-scattering approach

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where

$$G_0(E) = \frac{1}{E - \mathcal{H}_0 + i\eta}, \mathcal{H}_0 \text{ non-interacting projectile-nucleus Hamiltonian}$$

$$\tau(0, i) = V(0, i) + V(0, i) G_0(E) Q \tau(0, i) = \hat{\tau}(0, i) - \hat{\tau}(0, i) G_0(E) P \tau(0, i)$$

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$$U \approx \sum_{i=1}^A \tau(0, i)$$

- Ordered by projectile-target interactions
- Take first term in spectator expansion
- Make impulse approximation
  - Assume  $\hat{t}(0, i)$  is the free NN  $T$ -matrix
- Valid when the energy of the incident projectile is much larger than the binding energy of the struck nucleon ( $E > 100$  MeV)

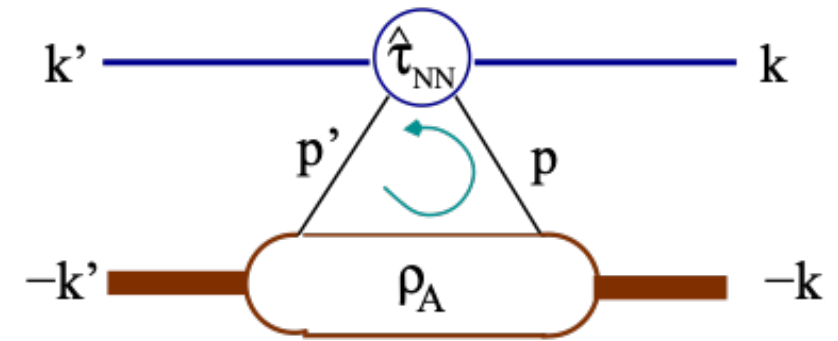
# Multiple-scattering approach

- Write optical potential in momentum-space in terms of the NN  $T$ -matrix and nuclear density

$$U(\mathbf{q}, \mathbf{K}; E) = \sum_{\alpha=p,n} \int d^3P \, \eta(\mathbf{P}, \mathbf{q}, \mathbf{K}) \hat{t}_{\alpha}(\mathbf{k}, \mathbf{k}') \rho_{\alpha}(\mathbf{P} - \frac{(A-1)\mathbf{q}}{2A}, \mathbf{P} + \frac{(A-1)\mathbf{q}}{2A})$$

where  $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ ,  $\mathbf{K} = \frac{1}{2}(\mathbf{k} + \mathbf{k}')$ , and

$$\mathbf{P} = \frac{1}{2}(\mathbf{p} + \mathbf{p}')$$



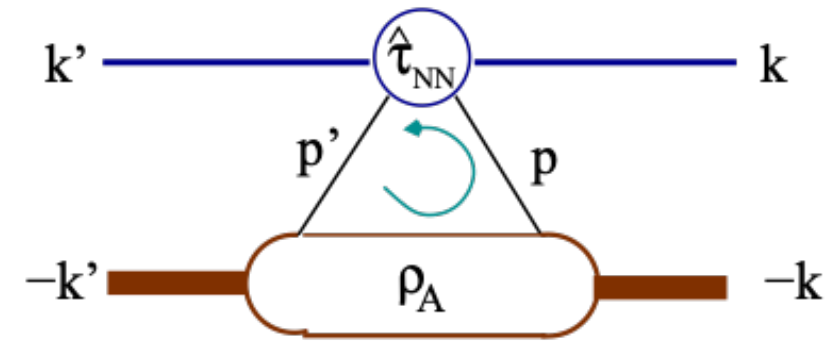
**Fig. 4:** Diagram of the single scattering term in the spectator expansion. (M. Burrows, et al., Phys. Rev. C **99**, 044603 (2019).)

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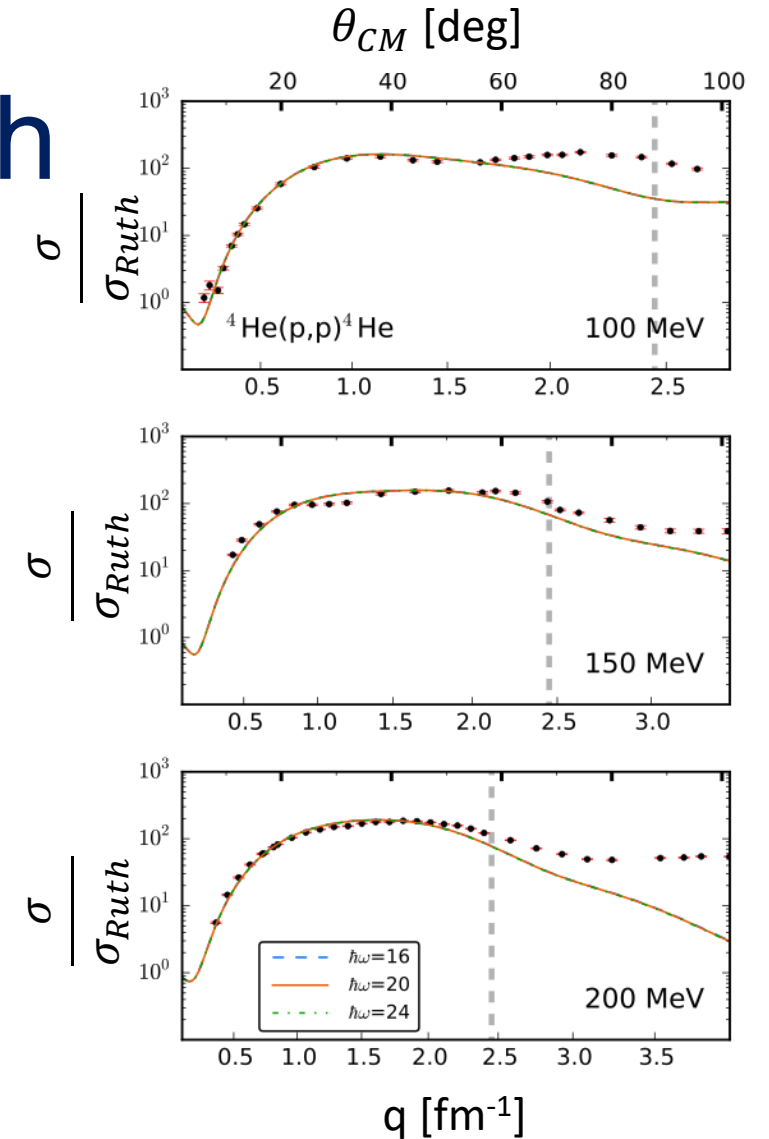
- $\eta$  relates the NN zero-momentum frame to the NA zero-momentum frame
- $\rho_{\alpha}$  represents the one-body density matrix



**Fig. 4:** Diagram of the single scattering term in the spectator expansion. (M. Burrows, et al., Phys. Rev. C **99**, 044603 (2019).)

# Multiple-scattering approach

- Multiple-scattering approach at first order describes experiments well for  $100 < E < 200 \text{ MeV}$  up to 60 degrees in center-of-mass frame
- At larger angles, three-nucleon forces (3NF's) become important
- Difficult to implement 3NF's



**Fig. 5:** Cross section for elastic proton scattering from  $^4\text{He}$  using the multiple-scattering approach. (M. Burrows, et al., Phys. Rev. C **99**, 044603 (2019).)



# Nucleon self-energy with chiral interactions

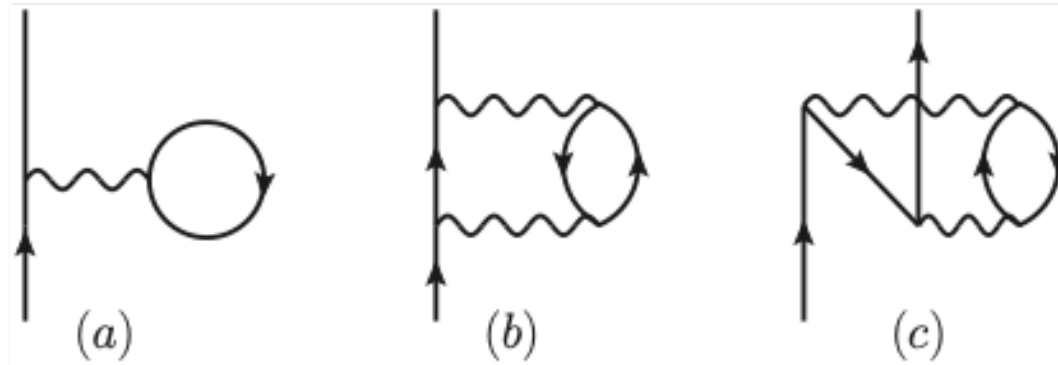
- The optical potential for scattering states is identified with single particle self-energy
- This approach calculates the nucleon self-energy in nuclear matter using interactions derived from chiral effective field theory ( $\chi^{EFT}$ )

# Nucleon self-energy with chiral interactions

- $\chi^{EFT}$  gives a low-energy description of the nuclear force involving proton, neutron, and pion degrees of freedom
- Nucleons interact via pion exchanges (long-range) and contact forces (short-range)
- Requires a regularization procedure to separate the high- and low-energy physics via a momentum-space cutoff

# Nucleon self-energy with chiral interactions

- Compute first- and second-order contributions to the nucleon self-energy  $\Sigma$  with effective potentials  $V_{2N}^{eff}$  derived from  $\chi^{EFT}$
- $V_{2N}^{eff}$  consist of an NN potential with an effective, medium-dependent NN interaction (depends on 3NF)

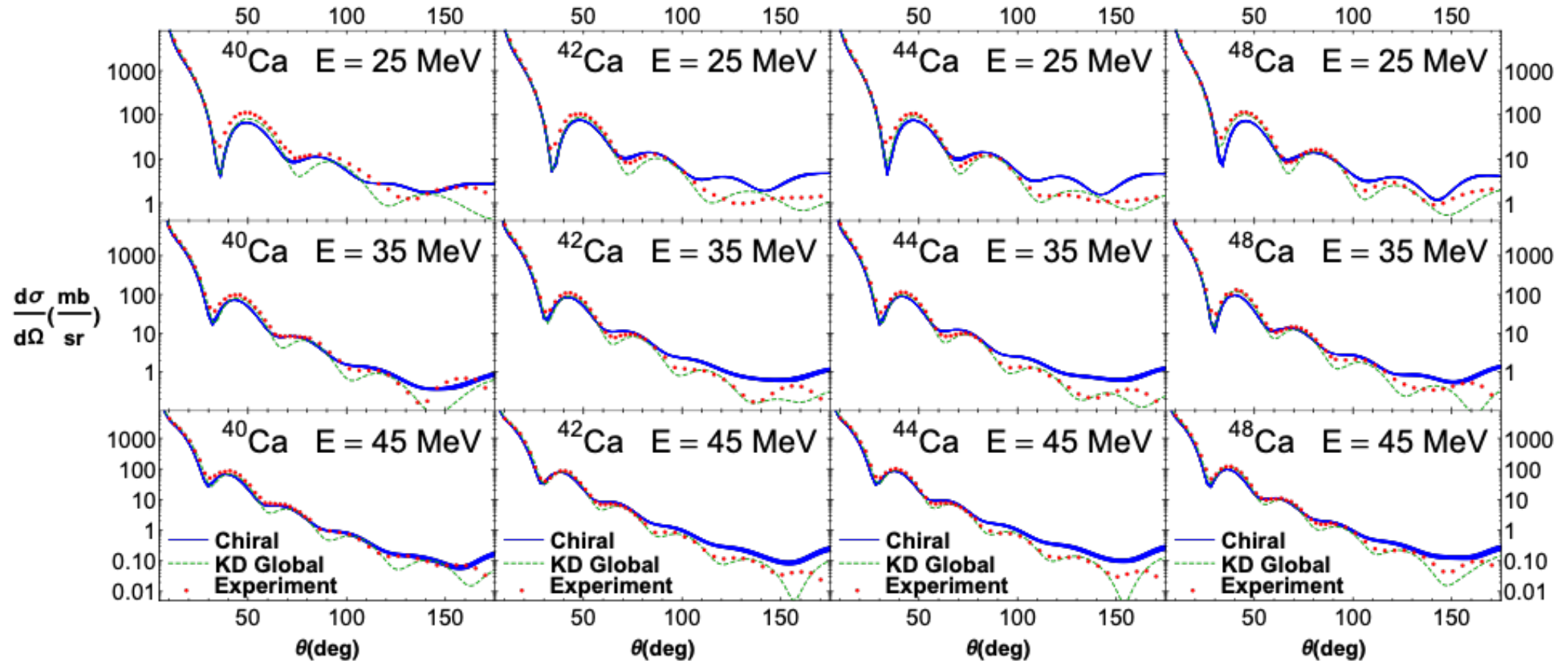


**Fig. 6:** First- and second-order contributions to the nucleon self-energy where the solid lines indicate nucleon propagators and the wavy lines indicate the in-medium, anti-symmetrized NN interaction. (T. R. Whitehead, et al., Phys. Rev. C **100**, 014601 (2019).)

# Nucleon self-energy with chiral interactions

- Optical potential given in terms of  $\Sigma$
- Well-suited to describe low-energy scattering
- Momentum-space cutoff of the EFT limits the capability of this approach  $E < 200 \text{ MeV}$

# Nucleon self-energy with chiral interactions



**Fig. 7:** Cross section for elastic scattering of protons from calcium isotopes at several lab energies. Blue lines correspond to microscopic cross sections and green lines correspond to a phenomenological model. (T. R. Whitehead, et al., Phys. Rev. C **100**, 014601 (2019).)

# Summary

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- Phenomenological models are constrained by scattering data and work well where data are available
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- Nuclear optical potentials play an important role in modeling nuclear reactions
- Phenomenological models are constrained by scattering data and work well where data are available
  - Ambiguity in fitting
  - Lack predictive power
  - No means for uncertainty quantification
- Microscopic methods use NN interactions from nuclear structure as inputs in computing optical potentials
  - Extends to reactions involving rare isotopes
  - Offers a means to quantify theoretical uncertainty estimates



# Outlook

- Currently microscopic approaches struggle in precision across kinematic ranges or nuclei
  - Can be used to guide phenomenological models
  - Necessary to understand what components are key in computing microscopic models

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# Outlook

- Factorization of nuclear structure from the scattering probe is important for extracting information on process independent quantities
- The Similarity Renormalization Group (SRG) is a tool that can be used to analyze scale and scheme dependence of optical potentials

# Extras

# Nuclear scattering

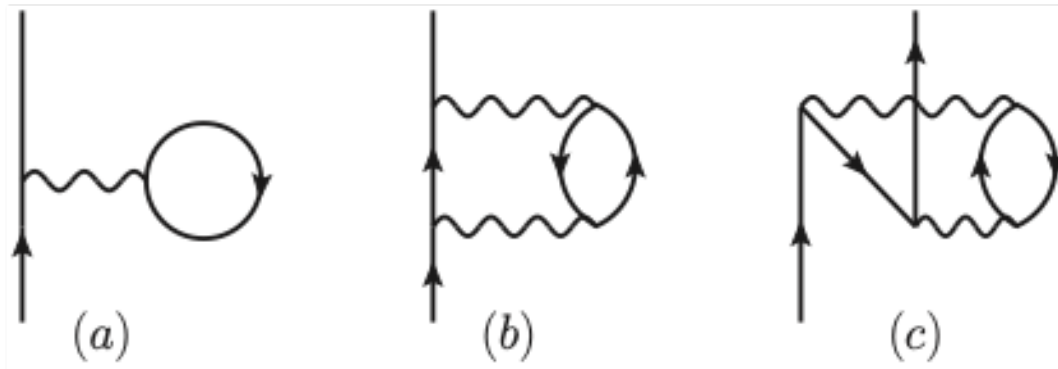
- Projectile-nucleus scattering is a quantum many-body problem
  - An incident particle interacts with  $A$  nucleons (protons and neutrons) in a target nucleus
- Difficulties of nuclear many-body systems:
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- Optical potentials simplify the problem by giving an effective projectile-nucleus interaction that accounts for absorption of incident particles (inelastic scattering)

# Nucleon self-energy with chiral interactions

- Compute first- and second-order contributions to the nucleon self-energy

$$\Sigma_{2N}^{(1)}(q; k_f) = \sum_i \langle \mathbf{q} \mathbf{h}_i | V_{2N}^{eff} | \mathbf{q} \mathbf{h}_i \rangle n_i$$

$$\Sigma_{2N}^{(2a)}(q, \omega; k_f) = \frac{1}{2} \sum_{ijk} \frac{|\langle \mathbf{p}_i \mathbf{p}_k | V_{2N}^{eff} | \mathbf{q} \mathbf{h}_j \rangle|^2}{\omega + \epsilon_j - \epsilon_i - \epsilon_k + i\eta} \bar{n}_i n_j \bar{n}_k$$



# Nucleon self-energy with chiral interactions

- The optical potential is given by

$$U_N(E; k_f^p, k_f^n) = V_N(E; k_f^p, k_f^n) + iW_N(E; k_f^p, k_f^n)$$

$$V_N(E; k_f^p, k_f^n) = \text{Re}\Sigma_N(q, E(q); k_f^p, k_f^n)$$

$$W_N(E; k_f^p, k_f^n) = \frac{M_N^{k*}}{M} \text{Im}\Sigma_N(q, E(q); k_f^p, k_f^n)$$

where  $N = p, n$  and  $\frac{M_N^{k*}}{M} = [1 + \frac{M}{k} \frac{\partial}{\partial k} V_N(k, E(k))]^{-1}$  defines the effective k-mass

# Outlook

- Currently microscopic approaches struggle in precision across kinematic ranges or nuclei
  - Can be used to guide phenomenological models
  - Necessary to understand what components are key in computing microscopic models
- Need to further understand uncertainty quantification in optical potentials to reliably compare different models
- Can use renormalization group (RG) methods to investigate scheme dependence in factorization of nuclear structure from the scattering probe