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Parametrization of the coupled channels S matrix in the inelastic case: Relation to Arndt-Roper form

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A previously introduced parametrization of the S matrix for two coupled channels is extended to allow use of Blatt-Biedenharn type phase shifts. It is then shown how the K-matrix parametrization of Arndt and Roper is related to the other schemes.

I. INTRODUCTION

This paper is concerned with representations of the partial wave S matrix for a system of two coupled channels such as the $^3S_1$ and $^3D_1$ or the $^3P_2$ and $^3F_2$ states in NN scattering. Below the inelastic threshold, S is a symmetric unitary 2×2 matrix. Above threshold, this part of S, denoted S_e, is only a submatrix of a larger symmetric unitary S whose dimension is two plus the number of inelastic channels. Recent work has led to a number of proposals[1–7] for a convenient and physically meaningful choice of the six real parameters which characterize S_e and which fulfill the subunitarity constraint.[3] I have added to the list of these parametrizations and clarified the relations among them.[7] The proposal of Arndt and Roper[8] differs from the others in that they have parametrized the K_e matrix rather than S_e directly. Since the people who are most actively pursuing phase shift analysis of NN scattering data in fact use the Arndt-Roper (AR) parametrization in their published solutions,[9] it is useful to understand the relationship between the AR parameters and the others. This is the purpose of this paper.

In Sec. II the well-known Blatt-Biedenharn (BB) (Ref. 10) and Stapp ("bar") phase parameters[11] for the purely elastic case are reviewed and the transformation between them is clarified. In Sec. III, I show that the parametrizations of Refs. 1–7 can be amended to use a BB-type representation for the "elastic" parameters rather than the bar-type phases originally proposed. The transformation between these schemes involves a "hidden parameter" θ which was not evident in the purely elastic case. In Sec. IV I show how the AR parametrization is related to the others, drawing on the results of Secs. II and III. Section V discusses some specific cases from the published phase shifts of Ref. 9, and it is noted that the *a priori* choice $\phi=0$ in Ref. 9 restricts their analysis to the special case where there is only a single inelastic channel coupled to the two "elastic" channels. The conclusions form Sec. VI.

II. RELATIONS IN THE ELASTIC CASE

Any 2×2 matrix M may be written as

$$M=A+i\mathbf{B}\cdot\boldsymbol{\sigma} \tag{2.1}$$

in terms of four complex numbers. An overall phase may be factored out to make A real. The S matrix for two coupled channels is symmetric, which rules out a σ_y component. We then have

$$S=e^{2is}(a+ib\sigma_z+ic\sigma_x)\ . \tag{2.2}$$

Unitarity then requires b and c to be real, and

$$a^2+b^2+c^2=1\ . \tag{2.3}$$

It is then natural to take a,b,c as components of a unit vector in R_3. For example, the Blatt-Biedenharn parametrization

$$S=e^{2is}(e^{-i\epsilon\sigma_y}e^{2id\sigma_z}e^{i\epsilon\sigma_y}) \tag{2.4}$$

gives

$$a=\cos 2d\ ,\quad b=\sin 2d\cos 2\epsilon\ ,\quad c=\sin 2d\sin 2\epsilon\ , \tag{2.5}$$

as if $2d$ were the polar angle θ and 2ϵ the azimuthal angle ϕ. Here, the sum and difference of the BB eigenphases are

$$\begin{aligned}2s&=\delta_1+\delta_2\ ,\\ 2d&=\delta_1-\delta_2\ ,\end{aligned} \tag{2.6}$$

respectively. We can separate out s because the unit matrix commutes with everything.

Similarly, the bar phase convention of Stapp *et al.*

$$S=e^{2i\bar{s}}(e^{i\bar{d}\sigma_z}e^{2i\bar{\epsilon}\sigma_x}e^{i\bar{d}\sigma_z}) \tag{2.7}$$

gives

$$c=\sin 2\bar{\epsilon}\ ,\quad a=\cos 2\bar{\epsilon}\cos 2\bar{d}\ ,\quad b=\cos 2\bar{\epsilon}\sin 2\bar{d}\ , \tag{2.8}$$

in which $\pi/2-2\bar{\epsilon}$ is the polar angle and $2\bar{d}$ the azimuthal angle.

Comparing (2.4) and (2.7) shows that the sum phase s is the *same* in both representations. Equating (2.5) and (2.8) leads to the well-known relations between the BB and bar phases ϵ,d and $\bar{\epsilon},\bar{d}$ [see (2.13) below]. One can easily invent other parametrizations of S, interchanging the roles of a,b,c, but these appear to have no particular use.

Before going over to the inelastic case, it is convenient following Bryan[1] to "factorize" S in the form

$$S=\mathcal{O}\,\mathcal{O}^T\ , \tag{2.9}$$

where

$$\begin{aligned}\mathcal{O}&=e^{-i\epsilon\sigma_y}e^{id\sigma_z}\ (\mathrm{BB})\ ,\\ \bar{\mathcal{O}}&=e^{i\bar{d}\sigma_z}e^{i\bar{\epsilon}\sigma_x}\ (\mathrm{bar})\ .\end{aligned} \tag{2.10}$$

Because the overall phase is conserved we can omit it for the present. Clearly $\bar{\mathcal{O}}\neq\mathcal{O}$ even though

$$\bar{\mathcal{O}}\bar{\mathcal{O}}^T=S=\mathcal{O}\mathcal{O}^T\ .$$

Because both are products of unitary factors, we can write $\bar{\mathcal{O}}=\mathcal{O}U$ with $UU^T=1$, which requires $U=e^{i\theta\sigma_y}$ for some θ. Equating

$$\bar{\mathcal{O}}=\mathcal{O}e^{i\theta\sigma_y} \tag{2.11}$$

leads to four relations [not three as in (2.5) and (2.8), because σ_y does appear]

$$\begin{aligned}\cos\bar{d}\,\cos\bar{\epsilon}&=\cos d\,\cos(\epsilon-\theta)\ ,\\ \sin\bar{d}\,\sin\bar{\epsilon}&=\cos d\,\sin(\epsilon-\theta)\ ,\\ \sin\bar{d}\,\cos\bar{\epsilon}&=\sin d\,\cos(\epsilon+\theta)\ ,\\ \cos\bar{d}\,\sin\bar{\epsilon}&=\sin d\,\sin(\epsilon+\theta)\ ,\end{aligned} \tag{2.12}$$

from which we find

$$\tan 2\theta=\sin 2\bar{\epsilon}/\tan 2\bar{d}=\tan 2\epsilon\cos 2d\ , \tag{2.13}$$

$$\cos 2\theta=\cos 2\epsilon/\cos 2\bar{\epsilon}=\sin 2\bar{d}/\sin 2d\ . \tag{2.14}$$

These relations include the well-known $\epsilon,d\leftrightarrow\bar{\epsilon},\bar{d}$ transformations, but supplement them with the additional "hidden parameter" θ which does not appear in S in the elastic case. But even here this parameter is not completely superfluous. For example, when $\bar{\epsilon}=\pi/4$, (2.7) gives $e^{-2i\bar{s}}S=i\sigma_x$ independent of $\bar{d}$. Transforming to BB phases gives $\epsilon=\pi/4$, $d=\pi/4$. It would be impossible to transform back to the original $\bar{\epsilon},\bar{d}$ without additional information: θ [$=\pi/4-\bar{d}$ from (2.13)] supplies this. When $\bar{\epsilon}$ is infinitesimally close to $\pi/4$ the usual transformation is well defined, but including θ, one has a well-defined result even in the limit.

III. RELATIONS IN THE INELASTIC CASE

When two elastic channels are coupled to other inelastic channels, S_e is symmetric, but is now only a submatrix of a larger unitary matrix. Bryan has shown that S_e may be written in the form

$$S_e=e^{i\bar{\Delta}}e^{i\bar{\epsilon}\sigma_x}\bar{N}e^{i\bar{\epsilon}\sigma_x}e^{i\bar{\Delta}}\ , \tag{3.1}$$

with $\bar{N}$ real and symmetric. (Bryan and previous authors[1-8] did not write a bar over their phase parameters. This is done here to distinguish them from the BB-type parameters to be discussed.) Various representations for $\bar{N}$ have been proposed, among which I advocate[7]

$$\bar{N}=\begin{bmatrix}\cos 2\bar{\alpha} & -\sin 2\bar{\alpha}\sin\bar{\mu}\\ -\sin 2\bar{\alpha}\sin\bar{\mu} & \cos 2\bar{\eta}\cos^2\bar{\mu}-\cos 2\bar{\alpha}\sin^2\bar{\mu}\end{bmatrix}\ , \tag{3.2}$$

while Klarsfeld has proposed[3]

$$\bar{N}=e^{-i\bar{\omega}\sigma_y}\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}e^{i\bar{\omega}\sigma_y}\ , \tag{3.3}$$

where the eigenvalues are called eigeninelasticities for obvious reasons. In view of (2.11) we can write

$$\begin{aligned}S_e=\bar{\mathcal{O}}\bar{N}\bar{\mathcal{O}}^T&=\mathcal{O}e^{i\theta\sigma_y}\bar{N}e^{-i\theta\sigma_y}\mathcal{O}^T\\ &=\mathcal{O}N\mathcal{O}^T\\ &=e^{-i\epsilon\sigma_y}e^{i\Delta}Ne^{i\Delta}e^{i\epsilon\sigma_y}\ ,\end{aligned} \tag{3.4}$$

where N is simply a rotated $\bar{N}$. If one adopts Klarsfeld's representation (3.3), the eigeninelasticities are unchanged, and

$$N=e^{-i\omega\sigma_y}\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}e^{i\omega\sigma_y}\ , \tag{3.5}$$

with $\omega=\bar{\omega}-\theta$. Or one could use (3.2) and obtain an altered set of values α,μ,η for the inelastic parameters. Whichever choice is made for N, one can have either a bar phase representation (3.1) or a BB-type representation as in (3.4). The "hidden" parameter θ is required to transform between them. The observables are the same.

If S_e is given in numerical form (as would occur in a phase shift analysis of experimental data), we can solve for the BB phase parameters δ_1, δ_2, ϵ, and N by a procedure which turns out to be almost identical to that given by Bryan[4] for the bar-type phase case. This procedure is outlined in the Appendix.

IV. RELATIONSHIP TO THE PARAMETRIZATION OF ARNDT AND ROPER

S_e, which we will now denote simply as S, can be written

$$S=(1+iK)(1-iK)^{-1}\ . \tag{4.1}$$

In the purely elastic case, $K=K_r$ is real and symmetric, and this makes S unitary and symmetric. Arndt and Roper[8] actually express K_r in terms of $\bar{\delta}_1\bar{\delta}_2\bar{\epsilon}$, but there is some advantage to using the BB representation because then

$$K_r = e^{-i\epsilon\sigma_y}\tan\Delta e^{i\epsilon\sigma_y}$$
$$\equiv VTV^\dagger \tag{4.2}$$

is diagonalized by the *same* rotation as is S. T is the matrix with $\tan\delta_i$ on the diagonal.

To include inelasticity, an imaginary part

$$K_i = \begin{bmatrix} \tan^2\rho_1 & \tan\rho_1\tan\rho_2\cos\phi \\ \tan\rho_1\tan\rho_2\cos\phi & \tan^2\rho_2 \end{bmatrix} \tag{4.3}$$

is introduced into (4.1), bringing in three new parameters. Writing

$$Q = V^\dagger K_i V ,$$

we have

$$S = V(1+iT-Q)(1-iT+Q)^{-1}V^\dagger , \tag{4.4}$$
$$= Ve^{i\Delta}(1-e^{-i\Delta}Q\cos\Delta)(1+e^{i\Delta}Q\cos\Delta)^{-1}e^{i\Delta}V^\dagger , \tag{4.5}$$
$$= \mathscr{O}M\mathscr{O}^T , \tag{4.6}$$

where

$$M = (1-e^{-i\Delta}Q\cos\Delta)(1+e^{i\Delta}Q\cos\Delta)^{-1} \tag{4.7}$$

contains both the effects of inelasticity and some residual phase shifts. In actual applications at energies below 1 GeV, the inelastic effects are small. To leading order in Q we have

$$M \simeq 1-2\cos\Delta Q\cos\Delta \equiv 1-2P , \tag{4.8}$$

which is purely real and gives directly N of Eq. (3.4). To second order in P we have

$$M = 1-2P+2P^2+2iPTP$$
$$\approx e^{iPTP}(1-2P+2P^2 \ldots)e^{iPTP} . \tag{4.9}$$

Since K_i, Q, and P are symmetric, so is PTP, which we split into diagonal and off-diagonal parts

$$PTP = \Delta'' + \epsilon''\sigma_x . \tag{4.10}$$

If we wish to transform (4.6) back into bar phase parameters, as preferred by Arndt *et al.*, we must combine $\Delta+\Delta''$ and ϵ as BB phases, and transform using (2.13) and (2.14) into equivalent bar phases $\overline{\Delta},\overline{\epsilon}'$. We can then add $\overline{\epsilon}'+\epsilon''=\overline{\epsilon}$ to give the final bar mixing parameter. We then have

$$S \approx \overline{\mathscr{O}}e^{-i\theta\sigma_y}(1-2P+2P^2 \ldots)e^{i\theta\sigma_y}\overline{\mathscr{O}}^T , \tag{4.11}$$
$$\equiv \overline{\mathscr{O}}\,\overline{N}_{AR}\overline{\mathscr{O}}^T , \tag{4.12}$$

with θ determined by the above transformation. It should be noted that the $\overline{\delta}_1\overline{\delta}_2\overline{\epsilon}$ in Eq. (4.12) are those of Eq. (3.1) and *not* the values given by Arndt and Roper in their parametrization of K_r. When we add $\Delta+\Delta''$, the average phase shift s is changed, so it is clear that we will not return to the original value of $\overline{\Delta}$.

V. APPLICATION

Arndt *et al.*[9] have published phase parameters for NN scattering at energies up to 1 GeV. Inelastic effects are included for the coupled $^3P_2$-$^3F_2$ (Ref. 9, Table IV, pp. 106–7) and $^3S_1$-$^3D_1$ systems (Ref. 9, p. 108) at $E>300$ MeV. From their published phase parameters the exact S may be calculated according to Eq. (4.1) and the Bryan-Sprung parameters extracted either exactly from Eq. (3.1) or approximately from Eqs. (4.9)–(4.14). Some examples are shown in Table I, taken from the $^3P_2$-$^3F_2$ coupled waves because the inelastic effects are largest. The parameters are in degrees except for the λ_i, which are real numbers. They are grouped in blocks of six: $\delta_1\delta_2\epsilon$ above and the inelastic parameters $\rho_1\rho_2\phi$ or $\alpha\mu\eta$ below. The λ_i and ω are equivalent to these as they also parametrize N.

There are several interesting points to be noted. The most striking is that the inelastic parameter η is always zero, or equivalently one of the λ_i is equal to unity. In general, $\det(1-N^2)\geq 0$ and equality holds in the case of a single inelastic channel.[1,3] According to Ref. 8, the choice $\phi=0$ in the phase shift analysis of Arndt *et al.*[4] also corresponds to a single inelastic channel, so these are consistent. When $\phi=0$, K_i has a zero eigenvalue, as will Q and P. In Eq. (4.7) there is always a $Q\cos\Delta$ acting to the right, so the exact N will have a unit eigenvalue, which is sufficient to make $\det(1-N^2)=(1-\lambda_1^2)(1-\lambda_2^2)$ vanish.

The second interesting point is that the Arndt-Roper and Bryan-Sprung parameters are very close in value, while the perturbative estimate [Eqs. (4.9)–(4.12)] gives a good account of their (small) difference. This justifies the expansion in powers of P as an aid to understanding the relation between the two representations. Since the corrections, Δ'' and ϵ'', are of second order in P, it is not surprising that the three "elastic" parameters should be close. To understand why ρ_1 and ρ_2 should be approximately equal to α and μ, respectively, we can expand 4.8 to second order in the ρ_i, neglecting the δ's and ϵ. This gives

$$N \simeq \begin{bmatrix} 1-2\rho_1^2 & -2\rho_1\rho_2 \\ -2\rho_1\rho_2 & 1-2\rho_2^2 \end{bmatrix} = \begin{bmatrix} \cos2\alpha & -\sin2\alpha\sin\mu \\ -\sin2\alpha\sin\mu & \cos2\eta\cos^2\mu-\cos2\alpha\sin^2\mu \end{bmatrix}$$
$$\simeq \begin{bmatrix} 1-2\alpha^2 & -2\alpha\mu \\ -2\alpha\mu & \cos2\eta(1-\mu^2)-\mu^2 \end{bmatrix} , \tag{5.1}$$

from which we see $\alpha=\rho_1$, $\mu=\rho_2$, and $\cos2\eta=1$. Had we not set $\phi=0$, the identity of ρ_1,ρ_2 to α,μ would not be expected. The last row in Table I, "Test case" illustrates this.

Since according to Ref. 7 α and μ represent the coupling parameters from channels 1 and 2 to the inelastic channel 3, it is not surprising that ρ_1 and ρ_2 from Eq. (4.3) should have the same significance when there is no other

TABLE I. Comparison of AR phase parameters to those of Sprung (Ref. 7) and Klarsfeld (Ref. 3).

		AR			DWLS			Klarsfeld			
		δ_1	δ_2	ϵ	δ_1	δ_2	ϵ				
E (MeV)		ρ_1	ρ_2	ϕ	α	μ	η	λ_1	λ_2	ϖ	θ
600	exact	17.990	−2.840	−0.800	17.999	−2.837	−0.795				
		9.100	4.100	0.000	8.636	4.133	0.000	1.000	0.94474	25.390	
	approximate				17.999	−2.837	−0.795				
					8.618	4.148	0.000	1.000	0.94485	25.514	−0.898
700	exact	16.030	−5.180	−0.560	16.119	−5.169	−0.529				
		16.000	5.300	0.000	15.336	5.321	0.000	1.000	0.84409	18.683	
	approximate				16.119	−5.168	−0.530				
					15.271	5.325	0.000	1.000	0.84521	18.775	−0.611
800	exact	12.520	−7.730	−0.660	12.778	−7.711	−0.590				
		21.400	5.900	0.000	20.815	5.900	0.000	1.000	0.72896	15.129	
	approximate				12.773	−7.709	−0.593				
					20.542	5.862	0.000	1.000	0.73545	15.246	−0.767
900	exact	7.720	−10.350	−0.950	8.069	−10.331	−0.868				
		25.400	6.300	0.000	25.040	6.257	0.000	1.000	0.62221	13.133	
	approximate				8.056	−10.327	−0.876				
					24.297	6.129	0.000	1.000	0.64243	13.306	−1.289
1000	exact	2.050	−12.970	−1.350	2.139	−12.966	−1.331				
		28.500	6.500	0.000	28.302	6.360	0.000	1.000	0.53136	11.625	
	approximate				2.152	−12.963	−1.331				
					26.817	6.099	0.000	1.000	0.57494	11.869	−2.323
Test case		45.0	15.0	30.0	46.072	18.161	32.148				
		35.264	39.231	54.735	20.188	−1.262	29.283				

inelastic channel.

Also shown in Table I are exact and approximate values of Klarsfeld's parameters $\lambda_1\lambda_2$ and ϖ. These agree well. The hidden parameter θ is important in achieving this agreement.

VI. CONCLUSION

I have shown how the phase parameters of Arndt and Roper[8] are related to the representations proposed by other authors[1-7] in recent years. A perturbative expansion was used to explain why, in practical cases, the numerical values of the two sets of parameters are close.

In the inelastic case we can use either a BB-type or a bar-type representation for the three "elastic" parameters. The transformation between these two schemes involves the "hidden parameter" θ which does not appear in the purely elastic S matrix.

The published phase shifts of Arndt *et al.*[9] correspond to the case where there is only a single inelastic channel, due to their *a priori* choice $\phi=0$. This means that in the geometrical model of the parameter space of Kermode and Cooper,[6] only the surface and none of the interior region has been explored. An alternative choice of $\phi=\pi/2$ would have explored the $z=0$ plane. It is true that at the inelastic threshold the system point starts from a vertex of the allowed region (see Fig. 1 of Ref. 7), but it would be surprising if the system remained forever on the surface. It would be interesting to explore some other fixed value of ϕ, even if the data are insufficient to allow a complete search. The analysis of Dubois *et al.*,[12] for example, is restricted to the interior of the allowed region.

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APPENDIX

We assume that S is given in the form

$$S=\begin{bmatrix} R_{11}e^{2i\delta_{11}} & iR_{12}e^{2i\delta_{12}} \\ iR_{12}e^{2i\delta_{12}} & R_{22}e^{2i\delta_{22}} \end{bmatrix} . \tag{A1}$$

We wish to parametrize this S in the form (3.4), which we can write

$$e^{-2is}S=e^{-i\epsilon\sigma_y}e^{id\sigma_z}Ne^{id\sigma_z}e^{i\epsilon\sigma_y}\equiv\hat{S} , \tag{A2}$$

where $2s=\delta_1+\delta_2$, $2d=\delta_1=\delta_1-\delta_2$, and we have used the fact that the unit operator commutes with everything. Defining

$$\begin{gathered}\theta_a=\delta_{11}-s , \quad \theta_b=\delta_{22}-s , \quad \phi=\delta_{11}+\delta_{22}-2\delta_{12} , \\ N_+=(N_{11}+N_{22})/2 , \quad N_-=(N_{11}-N_{22})/2 ,\end{gathered} \tag{A3}$$

we have

$$\begin{aligned} R_{11}e^{2i\theta_a}&=\hat{S}_{11} \\ &=N_+(\cos 2d+i\sin 2d\cos 2\epsilon) \\ &\quad +N_-(\cos 2d\cos 2\epsilon+i\sin 2d)-N_{12}\sin 2\epsilon , \end{aligned} \tag{A4}$$

$$\begin{aligned} R_{22}e^{2i\theta_b}&=\hat{S}_{22} \\ &=N_+(\cos 2d-i\sin 2d\cos 2\epsilon) \\ &\quad -N_-(\cos 2d\cos 2\epsilon-i\sin 2d)+N_{12}\sin 2\epsilon , \end{aligned} \tag{A5}$$

$$\begin{aligned} iR_{12}e^{2i(\theta_a+\theta_b-\phi)}&=\hat{S}_{12} \\ &=N_{12}\cos 2\epsilon+N_-\cos 2d\sin 2\epsilon \\ &\quad +iN_+\sin 2d\sin 2\epsilon . \end{aligned} \tag{A6}$$

Since $\hat{S}$ differs from N by unitary factors, $\operatorname{Im}\det\hat{S}=\operatorname{Im}\det N=0$, giving

$$R_{11}R_{22}\sin 2(\theta_a+\theta_b)+R_{12}^2\sin 2(\theta_a+\theta_b-\phi)=0 \tag{A7}$$

or

$$\tan 2(\theta_a+\theta_b)=\frac{R_{12}^2\sin 2\phi}{R_{11}R_{22}+R_{12}^2\cos 2\phi} , \tag{A8}$$

which is formally identical to Eq. (36) of Ref. 4, although the symbols have different meanings. Since $\theta_a-\theta_b=\delta_{11}-\delta_{22}$ is known, we can solve immediately for θ_a,θ_b and thereby determine the overall phase $2s=\delta_1+\delta_2$. Recall that ϕ is a known quantity. From (A6),

$$\operatorname{Im}\hat{S}_{12}=N_+\sin 2d\sin 2\epsilon=R_{12}\cos 2(\theta_a+\theta_b-\phi) ,$$

while (A4) and (A5) give

$$\begin{aligned}\operatorname{Im}(\hat{S}_{11}-\hat{S}_{22})&=2N_+\sin 2d\cos 2\epsilon \\ &=R_{11}\sin 2\theta_a+R_{22}\sin 2\theta_b .\end{aligned}$$

Accordingly,

$$\tan 2\epsilon=\frac{2R_{12}\cos 2(\theta_a+\theta_b-\phi)}{R_{11}\sin 2\theta_a+R_{22}\sin 2\theta_b} , \tag{A9}$$

which is, again, formally the same as Eq. (38) of Ref. 4. Similarly,

$$\begin{aligned}\frac{\operatorname{Im}(\hat{S}_{11}-\hat{S}_{22})}{\operatorname{Re}(\hat{S}_{11}+\hat{S}_{22})}&=\tan 2d\cos 2\epsilon \\ &=\frac{R_{11}\sin 2\theta_a-R_{22}\sin 2\theta_b}{R_{11}\cos 2\theta_a+R_{22}\cos 2\theta_b} .\end{aligned} \tag{A10}$$

We now know d and ϵ as well as s, so the elastic parameters have been deduced. The elements of the matrix N then follow from

$$\begin{aligned}\operatorname{Re}(\hat{S}_{11}+\hat{S}_{22})&=2N_+\cos 2d \\ &=R_{11}\cos 2\theta_a+R_{22}\cos 2\theta_b ,\end{aligned} \tag{A11}$$

$$\mathrm{Im}(\hat{S}_{11}+\hat{S}_{22})=2N_{-}\sin 2d$$
$$=R_{11}\sin 2\theta_a+R_{22}\sin 2\theta_b\ , \tag{A12}$$

$$N_{12}=\mathrm{Re}\hat{S}_{12}\cos 2\epsilon-\tfrac{1}{2}\mathrm{Re}(\hat{S}_{11}-\hat{S}_{22})\sin 2\epsilon$$
$$=-R_{12}\sin(\theta_a+\theta_b-\phi)\cos 2\epsilon$$
$$-\tfrac{1}{2}(R_{11}\cos 2\theta_a-R_{22}\cos 2\theta_b)\sin 2\epsilon\ . \tag{A13}$$

In case $2d$ is small, we may use

$$N_{-}=\mathrm{Re}\hat{S}_{12}\sin 2\epsilon+\tfrac{1}{2}\mathrm{Re}(\hat{S}_{11}-\hat{S}_{22})\cos 2\epsilon\ . \tag{A14}$$

Once N is known we may use (3.2) or (3.3) to deduce the corresponding "inelastic" parameters.

It is interesting that (A11) is similar to Eq. (39) of Ref. 4, except that one divides by $\cos 2d$ rather than $\cos 2\epsilon$ to deduce N_{+}. In any case, it is no more difficult to deduce the parameters in the BB case than in the "bar" case.

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