- Look at spectroscopic factor but starting with Dickhoff scalar operator and keeping 2^{nd} term.

- Starting point: $|\Psi_i\rangle = |\Psi_0^A\rangle$

$$|\Psi_f\rangle = a^{\dagger}_{\vec{p}\sigma\tau}|\Psi_\alpha^{A-1}\rangle \qquad (1)$$

Scalar external probe

$$\hat{\rho}(\vec{q}) = \sum_{\vec{p}\sigma'\tau'}\sum_{\vec{p}'\sigma''\tau''}\langle\vec{p}\sigma'\tau'|e^{i\vec{q}\cdot\vec{r}}|\vec{p}'\sigma''\tau''\rangle a^{\dagger}_{\vec{p}\sigma'\tau'}a_{\vec{p}'\sigma''\tau''}$$

$$= \sum_{\vec{p}\sigma\tau} a^{\dagger}_{\vec{p}\sigma\tau}\, a_{\vec{p}-\hbar\vec{q}\,\sigma\tau'} \qquad (2)$$

$$\Rightarrow \langle\Psi_f|\hat{\rho}(\vec{q})|\Psi_i\rangle = \sum_{\vec{p}'\sigma'\tau'}\langle\Psi_\alpha^{A-1}|a_{\vec{p}\sigma\tau}\,a^{\dagger}_{\vec{p}'\sigma'\tau'}\,a_{\vec{p}'-\hbar\vec{q}\,\sigma'\tau'}|\Psi_0^A\rangle$$

$$= \sum_{\vec{p}'\sigma'\tau'}\langle\Psi_\alpha^{A-1}|\Big[\delta_{\vec{p}',\vec{p}}\,\delta_{\sigma'\sigma}\,\delta_{\tau'\tau}\,a_{\vec{p}'-\hbar\vec{q}\,\sigma'\tau'} + a^{\dagger}_{\vec{p}'\sigma'\tau'}\,a_{\vec{p}\sigma\tau}\,a_{\vec{p}'-\hbar\vec{q}\,\sigma'\tau'}\Big]|\Psi_0^A\rangle$$

$$= \langle\Psi_\alpha^{A-1}|a_{\vec{p}-\hbar\vec{q}\,\sigma\tau} + \sum_{\vec{p}'\sigma'\tau'} a^{\dagger}_{\vec{p}'\sigma'\tau'}\,a_{\vec{p}\sigma\tau}\,a_{\vec{p}'-\hbar\vec{q}\,\sigma'\tau'}|\Psi_0^A\rangle \qquad (3)$$

: Insert $\hat{U}_\lambda^\dagger \hat{U}_\lambda$ twice

$$\langle \Psi_\alpha^{A-1} | \hat{U}_\lambda^\dagger \hat{U}_\lambda \Big[a_{\vec{p}-\hbar\vec{q}\sigma\tau} + \sum_{\vec{p}'\sigma'\tau'} a^\dagger_{\vec{p}'\sigma'\tau'} a_{\vec{p}\sigma\tau} a_{\vec{p}'-\hbar\vec{q}\sigma'\tau'} \Big] \hat{U}_\lambda^\dagger U_\lambda | \Psi_0^A \rangle$$

$$= \langle \Psi_\alpha^{A-1}(\lambda) | \hat{U}_\lambda \Big[a_{\vec{p}-\hbar\vec{q}\sigma\tau} + \sum_{\vec{p}'\sigma'\tau'} a^\dagger_{\vec{p}'\sigma'\tau'} a_{\vec{p}\sigma\tau} a_{\vec{p}'-\hbar\vec{q}\sigma'\tau'} \Big] \hat{U}_\lambda^\dagger | \Psi_0^A(\lambda) \rangle$$

Assume $|\Psi_0^A(\lambda)\rangle \equiv |\Phi\rangle$ and $\langle \Psi_\alpha^{A-1}(\lambda)| = \langle \Phi | a_\alpha^\dagger$

$$= \langle \Phi | a_\alpha^\dagger \hat{U}_\lambda \Big[a_{\vec{p}-\hbar\vec{q}\sigma\tau} + \sum_{\vec{p}'\sigma'\tau'} a^\dagger_{\vec{p}'\sigma'\tau'} a_{\vec{p}\sigma\tau} a_{\vec{p}'-\hbar\vec{q}\sigma'\tau'} \Big] \hat{U}_\lambda^\dagger | \Phi \rangle \qquad (4)$$

- Evaluate w.r.t. single-particle (s.p.) states where

$$a_{\vec{k}\sigma\tau} = \sum_\beta \langle \vec{k}\sigma\tau | \beta \rangle a_\beta$$
$$\propto \phi_\beta(k) Y_\ell^{m_\ell}(\Omega_k) \qquad (5)$$

$$\hat{U}_\lambda = \hat{I} + \frac{1}{4} \sum_{\vec{K}\vec{k}\vec{k}'} \sum_{\sigma_i} \sum_{\tau_j} \langle \vec{k}\sigma_1\tau_1\sigma_2\tau_2 | \delta\tilde{U} | \vec{k}'\sigma_3\tau_3\sigma_4\tau_4 \rangle$$
$$\times a^\dagger_{\frac{\vec{K}}{2}+\vec{k}\sigma_1\tau_1} a^\dagger_{\frac{\vec{K}}{2}-\vec{k}\sigma_2\tau_2} a_{\frac{\vec{K}}{2}-\vec{k}'\sigma_4\tau_4} a_{\frac{\vec{K}}{2}+\vec{k}'\sigma_3\tau_3} + \ldots \qquad (5)$$

In s.p. basis, we write

$$\hat{U}_\lambda = \hat{I} + \frac{1}{4} \sum_{1234} \sum_{\rho\sigma\mu\nu} \delta\tilde{U}_{1234} \langle \rho\sigma | 12 \rangle \langle 34 | \mu\nu \rangle a_\rho^\dagger a_\sigma^\dagger a_\nu a_\mu$$
$$+ \ldots \qquad (6)$$

where $\sum_{1234} \equiv \sum_{\vec{q}\vec{k}\vec{k}'} \sum_{\sigma_i} \sum_{\tau_j}$, $\delta\tilde{U}_{1234} \equiv \langle \vec{k}\sigma_1\tau_1\sigma_2\tau_2 | \delta\tilde{U} | \vec{k}'\sigma_3\tau_3\sigma_4\tau_4 \rangle$

- Evaluate operator at 2-body level:

$$\sum_{\beta} \langle \vec{p}-\hbar\vec{q}\,\sigma\tau | \beta \rangle\, a^\dagger_\alpha a_\beta$$

$$+ \frac{1}{4} \sum_{\beta} \sum_{1234} \sum_{\rho\sigma\mu\nu} \langle \vec{p}-\hbar\vec{q}\,\sigma\tau | \beta \rangle\, \delta\tilde{U}_{1234} \langle \rho\sigma | 12 \rangle \langle 34 | \mu\nu \rangle\, a^\dagger_\alpha a_\beta a^\dagger_\rho a^\dagger_\sigma a_\nu a_\mu$$

$$+ \sum_{\beta\gamma\delta} \sum_{\vec{p}'\sigma'\tau'} \langle \beta | \vec{p}'\sigma'\tau' \rangle \langle \vec{p}\sigma\tau | \delta \rangle \langle \vec{p}'-\hbar\vec{q}\,\sigma'\tau' | \gamma \rangle\, a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma$$

$$+ \frac{1}{4} \sum_{\beta\gamma\delta} \sum_{\vec{p}'\sigma'\tau'} \sum_{1234} \sum_{\rho\sigma\mu\nu} \langle \beta | \vec{p}'\sigma'\tau' \rangle \langle \vec{p}\sigma\tau | \delta \rangle \langle \vec{p}'-\hbar\vec{q}\,\sigma'\tau' | \gamma \rangle$$

$$\times\, \delta\tilde{U}_{1234} \langle \rho\sigma | 12 \rangle \langle 34 | \mu\nu \rangle\, a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma\, a^\dagger_\rho a^\dagger_\sigma a_\nu a_\mu + \cdots \qquad (7)$$

$$a^\dagger_\alpha a_\beta a^\dagger_\rho a^\dagger_\sigma a_\nu a_\mu = \{ a^\dagger_\alpha a_\beta \overset{\text{3-body}}{\cancel{a^\dagger_\rho}} a^\dagger_\sigma a_\nu a_\mu \}$$

$$+ \{ a^\dagger_\alpha \overbrace{a_\beta a^\dagger_\rho} a^\dagger_\sigma a_\nu a_\mu \}$$

$$- \{ a^\dagger_\alpha \overbrace{a_\beta a^\dagger_\rho a^\dagger_\sigma} a_\nu a_\mu \}$$

$$= \delta_{\beta\rho}\, a^\dagger_\alpha a^\dagger_\sigma a_\nu a_\mu - \delta_{\beta\sigma}\, a^\dagger_\alpha a^\dagger_\rho a_\nu a_\mu \qquad (8)$$

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$$a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma a^\dagger_\rho a^\dagger_\sigma a_\nu a_\mu = \{a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma a^\dagger_\rho a^\dagger_\sigma a_\nu a_\mu\} \quad \text{4-body}$$

$$+ \{a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma a^\dagger_\rho a^\dagger_\sigma a_\nu a_\mu\}$$

$$- \{a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma a^\dagger_\rho a^\dagger_\sigma a_\nu a_\mu\}$$

$$= \delta_{\delta\sigma}\delta_{\gamma\rho}\, a^\dagger_\alpha a^\dagger_\beta a_\nu a_\mu - \delta_{\delta\rho}\delta_{\gamma\sigma}\, a^\dagger_\alpha a^\dagger_\beta a_\nu a_\mu \qquad (9)$$

* Take 1 term of last one as example

$$\frac{1}{4}\sum_{\beta\gamma\delta}\sum_{\vec{p}'\sigma'\tau'}\sum_{1234}\sum_{\rho\sigma\mu\nu}\langle\beta|\vec{p}'\sigma'\tau'\rangle\langle\vec{p}\sigma\tau|\delta\rangle\langle\vec{p}'-\hbar\vec{q}\,\sigma'\tau'|\gamma\rangle$$

$$\times\, \delta\tilde{U}_{1234}\langle\rho\sigma|12\rangle\langle34|\mu\nu\rangle\,\delta_{\delta\sigma}\delta_{\gamma\rho}\, a^\dagger_\alpha a^\dagger_\beta a_\nu a_\mu$$

$$\langle\Phi|a^\dagger_\alpha a^\dagger_\beta a_\nu a_\mu|\Phi\rangle = \delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}$$

$$= \frac{1}{4}\sum_{\beta\gamma\delta}\sum_{\vec{p}'\sigma'\tau'}\sum_{1234}\sum_{\rho\sigma\mu\nu}\langle\beta|\vec{p}'\sigma'\tau'\rangle\langle\vec{p}\sigma\tau|\delta\rangle\langle\vec{p}'-\hbar\vec{q}\,\sigma'\tau'|\gamma\rangle$$

$$\times\, \delta\tilde{U}_{1234}\langle\rho\sigma|12\rangle\langle34|\mu\nu\rangle\,\delta_{\delta\sigma}\delta_{\gamma\rho}\left(\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}\right)$$

$$\sum_{\vec{p}'} a^\dagger_\alpha a^\dagger_{\vec{p}'-\vec{q}}\, a_{\vec{p}'}\, a_{\vec{p}}$$

$$= \frac{1}{4}\sum_{\beta\gamma\delta}\sum_{\vec{p}'\sigma'\tau'}\sum_{1234}\langle\beta|\vec{p}'\sigma'\tau'\rangle\underbrace{\langle\vec{p}\sigma\tau|\delta\rangle}\langle\vec{p}'-\hbar\vec{q}\,\sigma'\tau'|\gamma\rangle$$

$$\Phi_\beta\, Y_\ell^m(\;)$$

$$\times\, \delta\tilde{U}_{1234}\langle\gamma\delta|12\rangle\Big(\langle34|\alpha\beta\rangle - \langle34|\beta\alpha\rangle\Big) \qquad (10)$$

$$\propto \sum_{\beta\gamma\delta} \phi_\beta^*(\vec{p}\,') \, \phi_\delta(\vec{p}) \, \phi_\gamma(\vec{p}\,'-\hbar\vec{q}) \, \phi_\gamma^*\left(\frac{\vec{K}}{2}+\vec{k}\right)$$

$$\times \phi_\delta^*\left(\frac{\vec{K}}{2}-\vec{k}\right) \left[\phi_\alpha\left(\frac{\vec{K}}{2}+\vec{k}\,'\right)\phi_\beta\left(\frac{\vec{K}}{2}-\vec{k}\,'\right) - \phi_\alpha\left(\frac{\vec{K}}{2}-\vec{k}\,'\right)\phi_\beta\left(\frac{\vec{K}}{2}+\vec{k}\,'\right)\right]$$

$$\propto |\phi_\beta|^2 |\phi_\delta|^2 |\phi_\gamma|^2 \phi_A \, \delta V^2 \sum_{\vec{K}\vec{k}\vec{k}'} \sum_{\vec{p}\,'}$$

$$fm^3 \quad fm^3 \quad fm^3 \quad fm^{3/2} \quad fm^3 \quad fm^{-9} \quad fm^{-3}$$

$$= fm^{3/2} \quad \checkmark$$

- Dickhoff assumption is that momentum of ejectile ($\vec{p}$) is much larger than typical momenta for particles in bound state:

$$a_{\vec{p}} |\Psi_0^N\rangle \ll a_{\vec{p}-\hbar\vec{q}} |\Psi_0^N\rangle$$

For us, $a_{\vec{p}}$ will be contracted with $\delta U^\dagger a^\dagger a^\dagger a a$ and a's on the far left are at soft momenta. But we see this term ends up being proportional to $\phi_\delta(\vec{p})$ which should weight it much lower.

Note, this is NOT $\phi_\delta(\vec{p}-\hbar\vec{q}) \equiv \phi_\delta(\vec{p}_m)$ where we've defined $\vec{p}_m$.

Also $|\Psi_f\rangle = a_{\vec{p}}^{\dagger} |\Psi_\alpha^{A-1}\rangle$ is a good approximation for high $\vec{p}$

For us, $|\Psi_f\rangle = a_{\vec{p}}^{\dagger} \hat{U}_\lambda^{\dagger} \hat{U}_\lambda |\Psi_\alpha^{A-1}\rangle$

$$= a_{\vec{p}}^{\dagger} \hat{U}_\lambda^{\dagger} |\Psi_\alpha^{A-1}(\lambda)\rangle$$

$$= a_{\vec{p}}^{\dagger} \hat{U}_\lambda^{\dagger} a_\alpha |\Psi_0^A(\lambda)\rangle$$