Status of nuclear optical potentials and future prospects

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Introduction

- Nuclear reactions play a key role in answering questions such as the origin of heavy elements in the universe, fundamental symmetries, and the limits of nuclear stability
- Facilities seek to produce exotic isotopes and measure new data to better understand these areas

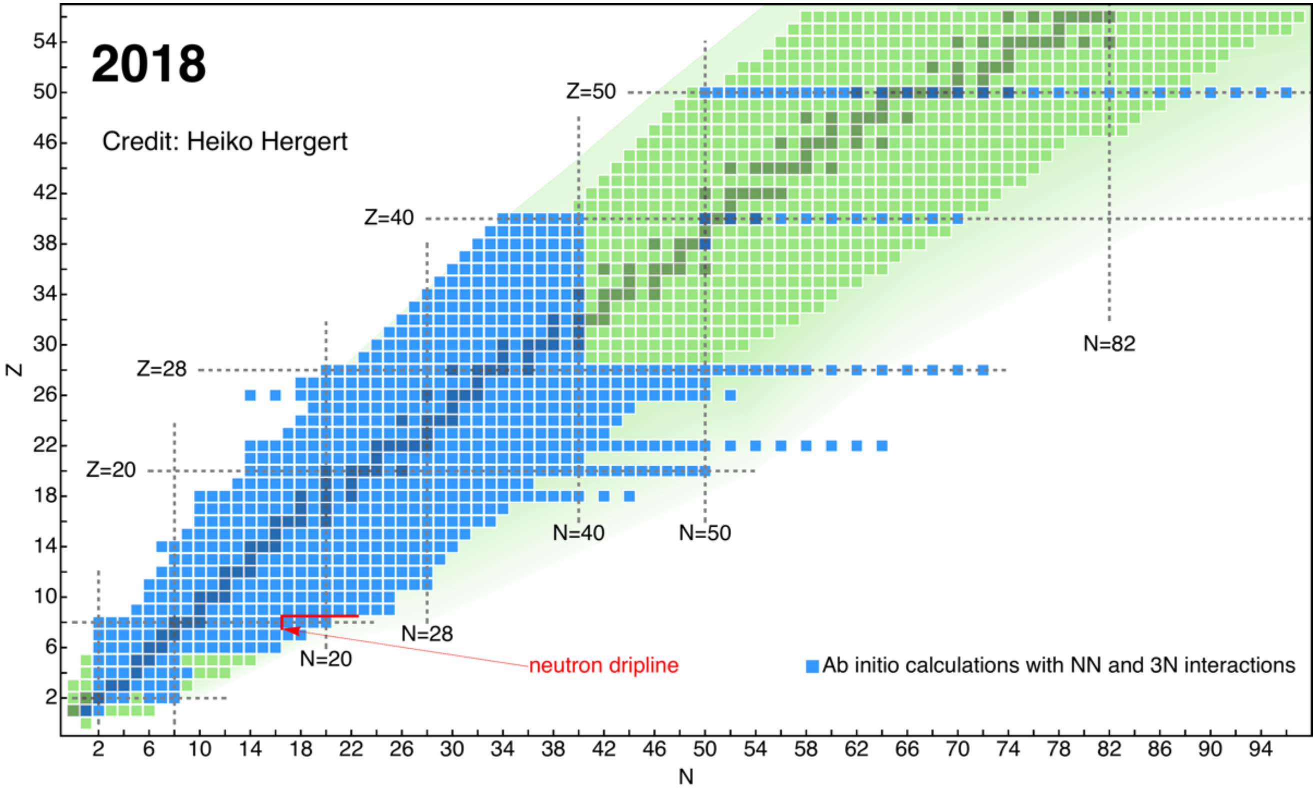
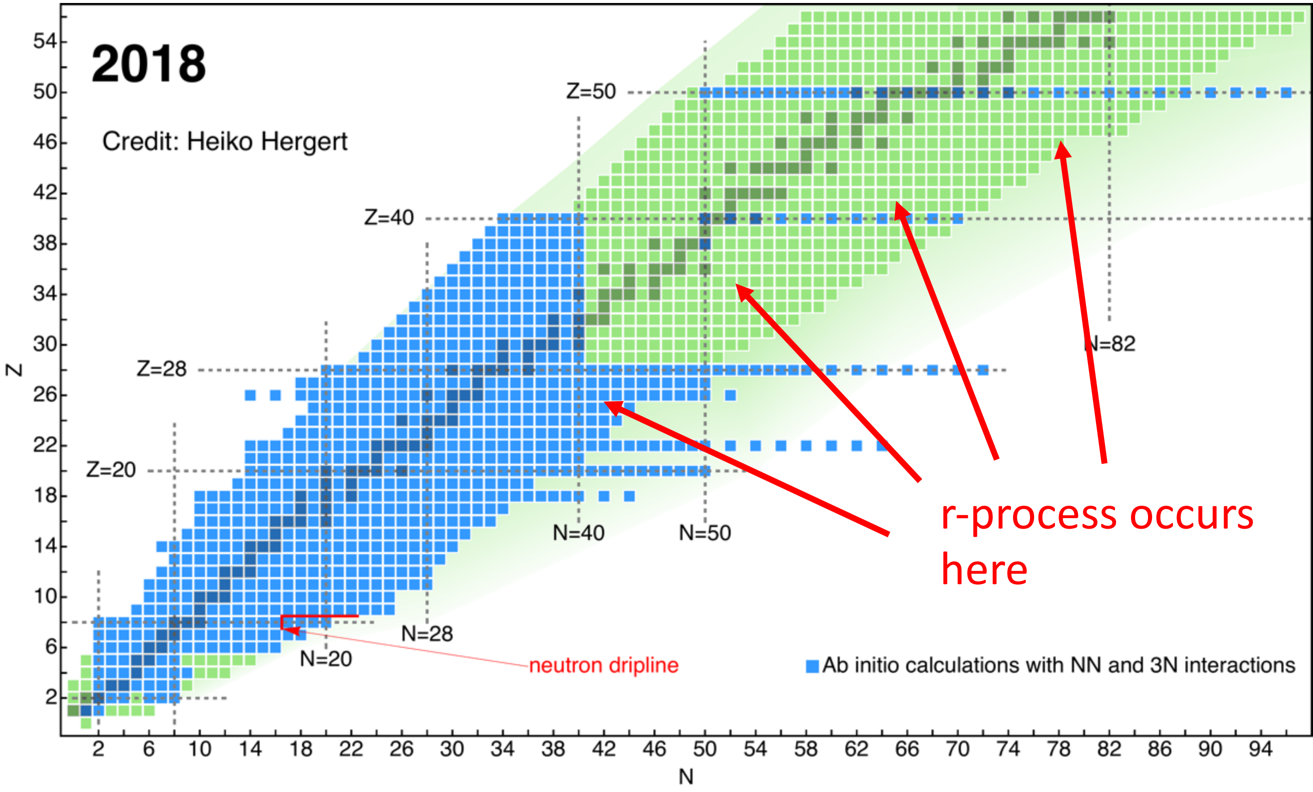


Fig. 1: Chart of nuclides with neutron number, N, counted horizontally and proton number, Z, counted vertically.
(Figure from H. Hergert.)

Introduction

- For example, the Facility for Rare Isotope Beams (FRIB) will target neutron-rich isotopes to study the rapid neutron-capture process (r-process)
- The r-process is responsible for the formation of roughly half the atomic nuclei past iron on the periodic table



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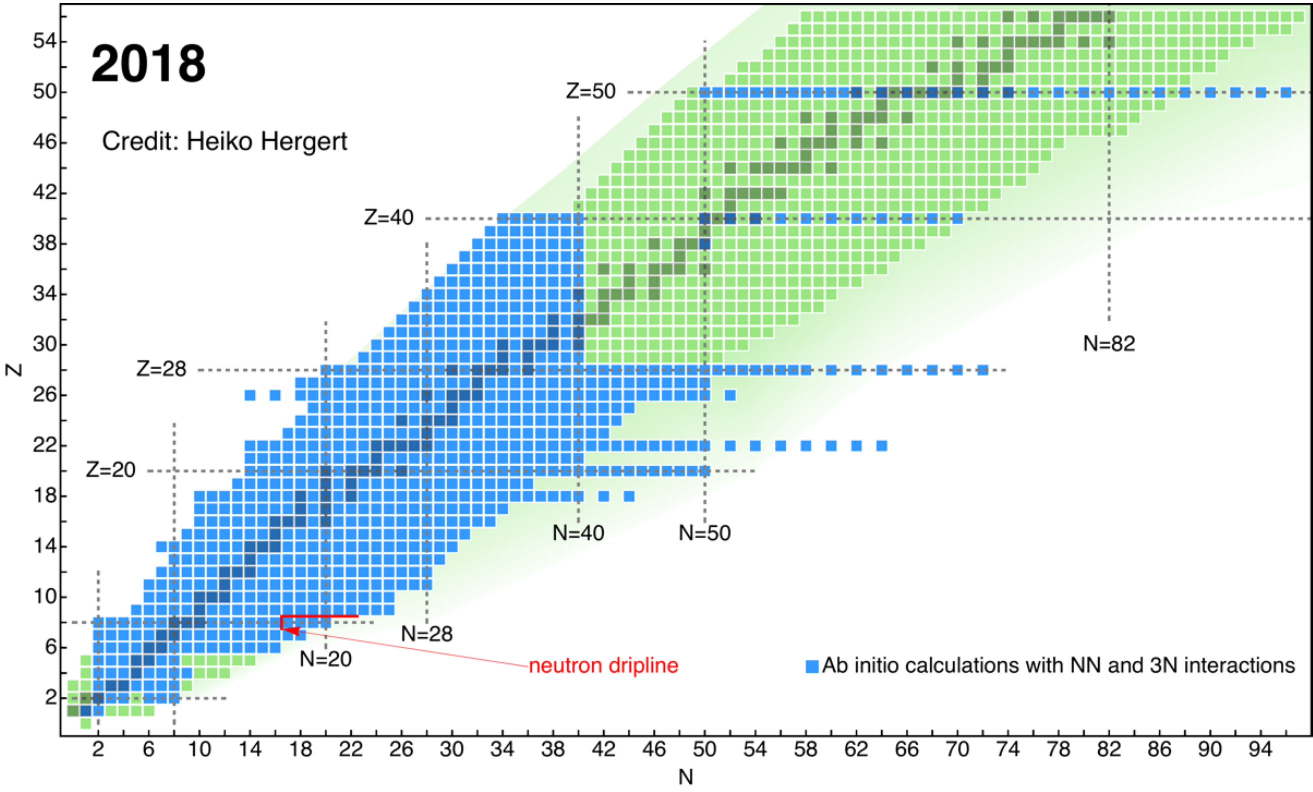
Introduction

- Critical to understand nuclear reactions since facilities must use reactions to produce and study short-lived exotic nuclei



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Nuclear optical potential

- Can introduce a complex potential to model the effective projectile-nucleus interaction in scattering experiments
- These potentials are called optical potentials

$$U(\boldsymbol{r}) = V(\boldsymbol{r}) + iW(\boldsymbol{r})$$

- Properties of $U(\boldsymbol{r})$:

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- Analogous to a complex index of refraction for describing light absorption/refraction in absorbing materials

Nuclear scattering

- Projectile-nucleus scattering is a quantum many-body problem
  - An incident particle interacts with A nucleons (protons and neutrons) in a target nucleus
- Difficulties of nuclear many-body systems:
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  ii. Computational difficulty of the problem drastically increases with nuclear mass number A

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  2. Computational difficulty of the problem drastically increases with nuclear mass number A
- Optical potentials simplify the problem by giving an effective projectile-nucleus interaction that accounts for absorption of incident particles (inelastic scattering)

Formalism

- $A+1$ particle system consisting of incident nucleon and target nucleus of mass number A described by Schrödinger equation

$$\mathcal{H}\Psi = E\Psi$$

where the total Hamiltonian is

$$\mathcal{H}(\boldsymbol{r_0}, \ldots, \boldsymbol{r_A}) = H_A(\boldsymbol{r_1}, \ldots, \boldsymbol{r_A}) + T_0 + V(\boldsymbol{r_0}, \ldots, \boldsymbol{r_A})$$

Formalism

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- The nuclear Hamiltonian satisfies the Schrödinger equation

$$H_A \psi_i = \epsilon_i \psi_i$$

for wave functions ψ_i and energies ϵ_i where $i = 0$ is the ground state

Formalism

- The optical potential is given by

$$V_{opt}(\boldsymbol{r_0}) = V_{00} + \boldsymbol{V}_0 \frac{1}{E - \boldsymbol{H} + i\eta} \boldsymbol{V}_0^\dagger$$

where $V_{ij} = \langle \psi_i | V | \psi_j \rangle$, $H_{ij} = T_0 \delta_{ij} + V_{ij} + \epsilon_i \delta_{ij}$ for $i, j > 0$, and $\boldsymbol{V}_0 = (V_{01}, V_{02}, \dots)$

- $\eta \to 0^+$ to ensure only outgoing waves are present in exit channels

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Elastic scattering

Inelastic scattering

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$$V_{opt}(\boldsymbol{r_0}) = V_{00} + \boldsymbol{V}_0 \frac{1}{E - \boldsymbol{H} + i\eta} \boldsymbol{V}_0^{\dagger}$$

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Formalism

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- V_{opt} is complex, energy dependent, and non-local
- Cannot be evaluated for realistic systems

Phenomenology

- General form of phenomenological optical potential

$$V_{opt}(r,E) = V_C(r) - V_V(E)f(x_0) + (\frac{\hbar}{m_\pi c})^2 V_{SO}(E)\boldsymbol{\sigma}\cdot\boldsymbol{l}\frac{1}{r}\frac{d}{dr}f(x_{SO}) - i[W_V(E)f(x_W) - 4W_D(E)\frac{d}{dx_D}f(x_D)]$$

- Woods-Saxon form factors:

$$f(x_i) = \frac{1}{1+e^{x_i}}$$

where $x_i = (r - R_i)/a_i$ for nuclear radii and diffusivity parameters R_i and a_i

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- Obtained by χ^2 minimization fitting scattering observables with radii and diffusivity parameters

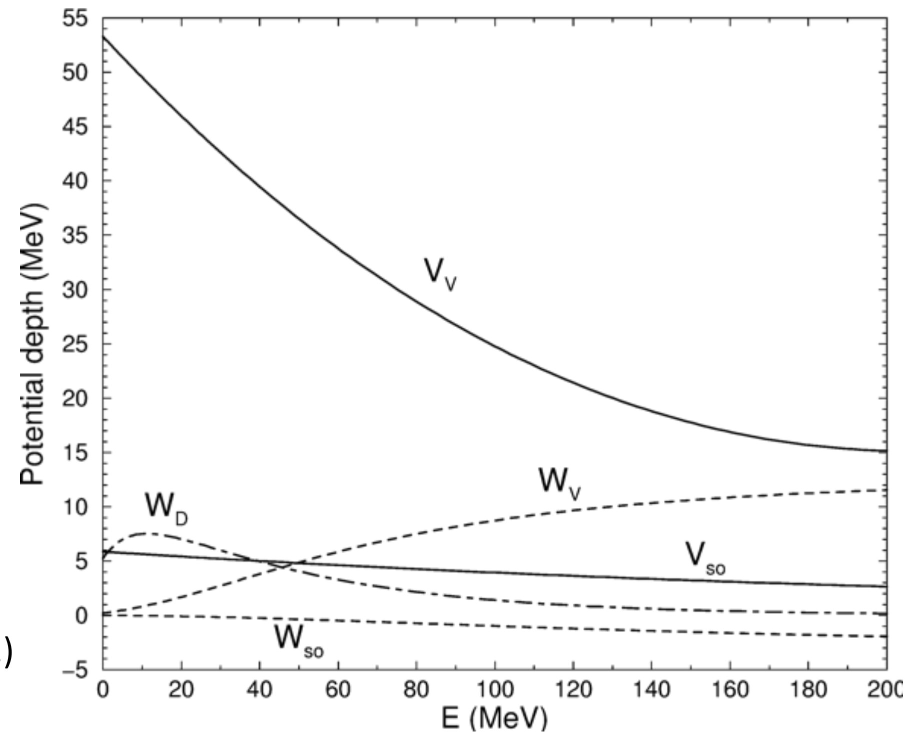

Fig. 2: Potential well depths as a function of laboratory energy E for each of the terms above including an imaginary spin-orbit term, W_{SO}. (A. J. Koning and J. P. Delaroche, Nucl. Phys. A **713**, 213 (2003).)

Phenomenology

- Ambiguity in fitting
  - Several sets of parameters can give a good fit
  - Heavily dependent on data sets used

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  - Several sets of parameters can give a good fit
  - Heavily dependent on data sets used
- Cannot extend to exotic nuclei where no data are present
- No reliable way to quantify uncertainty in phenomenological optical potentials

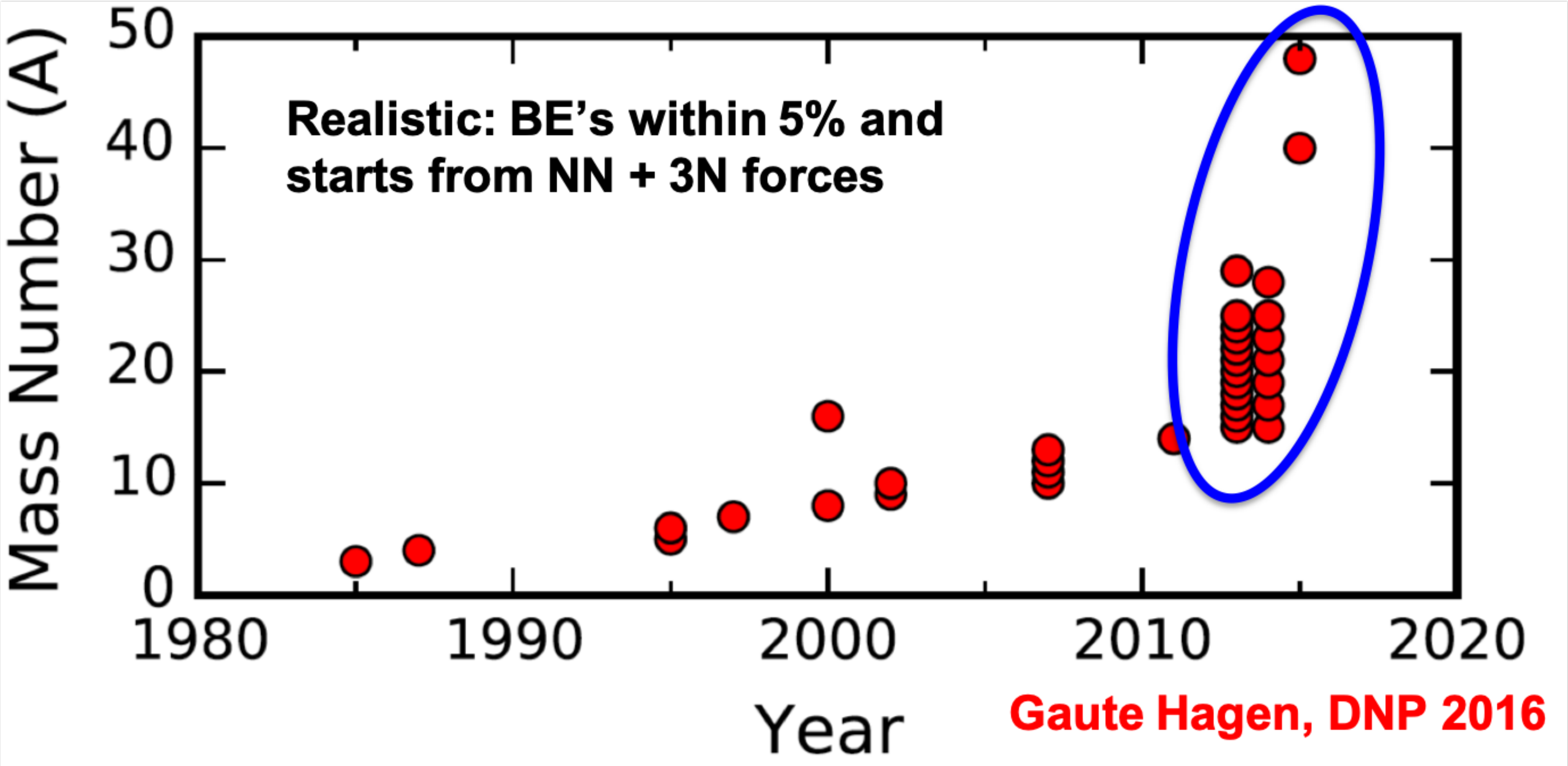
Microscopic approaches

- Microscopic optical potentials are models based off realistic nuclear structure inputs
- Can overcome shortcomings of phenomenological models (e.g. predictive power, uncertainty quantification)

Microscopic approaches

- Microscopic nuclear structure has made enormous progress in the past decade

Fig. 3: Binding energies for A-body nuclei within 5% of the experimental value calculated from *ab initio* methods. (Figure from G. Hagen.)



Multiple-scattering approach

- Basic idea is to express the optical potential U as a convolution of the NN T-matrix and nuclear density
- Define projection operators P and Q which project onto elastic and inelastic channels, respectively
- Apply the spectator expansion to the optical potential

$$U = \sum_{i=1}^{A} \tau(0,i) + \sum_{i \neq j}^{A} \tau(0,i) Q G_0(E) \tau(0,j) + \cdots$$

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where

$G_0(E) = \frac{1}{E - \mathcal{H}_0 + i\eta}$, $\mathcal{H}_0$ non-interacting projectile-nucleus Hamiltonian

$$\tau(0,i) = \hat{\tau}(0,i) - \hat{\tau}(0,i) G_0(E) P \tau(0,i)$$

$$\hat{\tau}(0,i) = V(0,i) + V(0,i) G_0(E) \hat{\tau}(0,i)$$

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- Ordered by projectile-target interactions

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Projectile interacts with one nucleon

Projectile interacts with two nucleons

Multiple-scattering approach

$$U \approx \sum_{i=1}^{A} \tau(0, i)$$

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$$U \approx \sum_{i=1}^{A} \tau(0,i)$$

- Ordered by projectile-target interactions
- Take first term in spectator expansion
- Make impulse approximation: assume $\hat{\tau}(0,i)$ is the free NN T-matrix
- Valid when the energy of the incident projectile is much larger than the binding energy of the struck nucleon ($E > 100$ MeV)

Multiple-scattering approach

- Write optical potential as convolution of the effective interaction with the target's ground state

$$U(\boldsymbol{q},\boldsymbol{K};E) = \sum_{\alpha=p,n} \int d^3P\, \eta(\boldsymbol{P},\boldsymbol{q},\boldsymbol{K})\hat{\tau}_\alpha(\boldsymbol{k},\boldsymbol{k}')\rho_\alpha\left(\boldsymbol{P} - \frac{(A-1)\boldsymbol{q}}{2A}, \boldsymbol{P} + \frac{(A-1)\boldsymbol{q}}{2A}\right)$$

where $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$, $\boldsymbol{K} = \frac{1}{2}(\boldsymbol{k} + \boldsymbol{k}')$, and

$$\boldsymbol{P} = \frac{1}{2}(\boldsymbol{p} + \boldsymbol{p}')$$

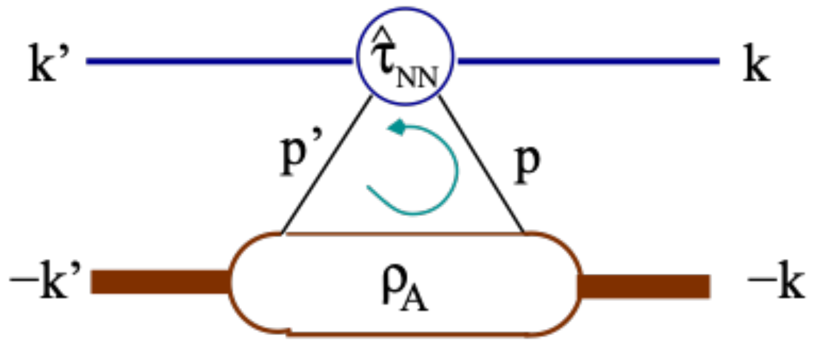

Fig. 4: Diagram of the single scattering term in the spectator expansion. (M. Burrows, et al., Phys. Rev. C **99**, 044603 (2019).)

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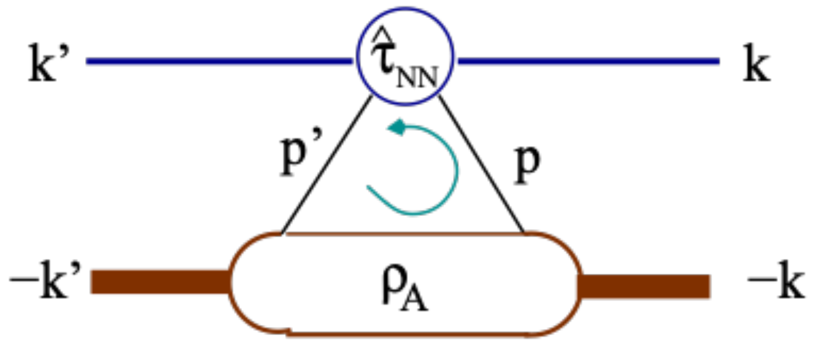
- η relates the NN zero-momentum frame to the NA zero-momentum frame
- ρ_α represents the one-body density matrix



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Multiple-scattering approach

- Multiple-scattering approach at first order describes experiments well for $100 < E < 200$ MeV up to 60 degrees in center-of-mass frame
- At larger angles, three-nucleon forces (3NF's) become important
- Difficult to implement 3NF's

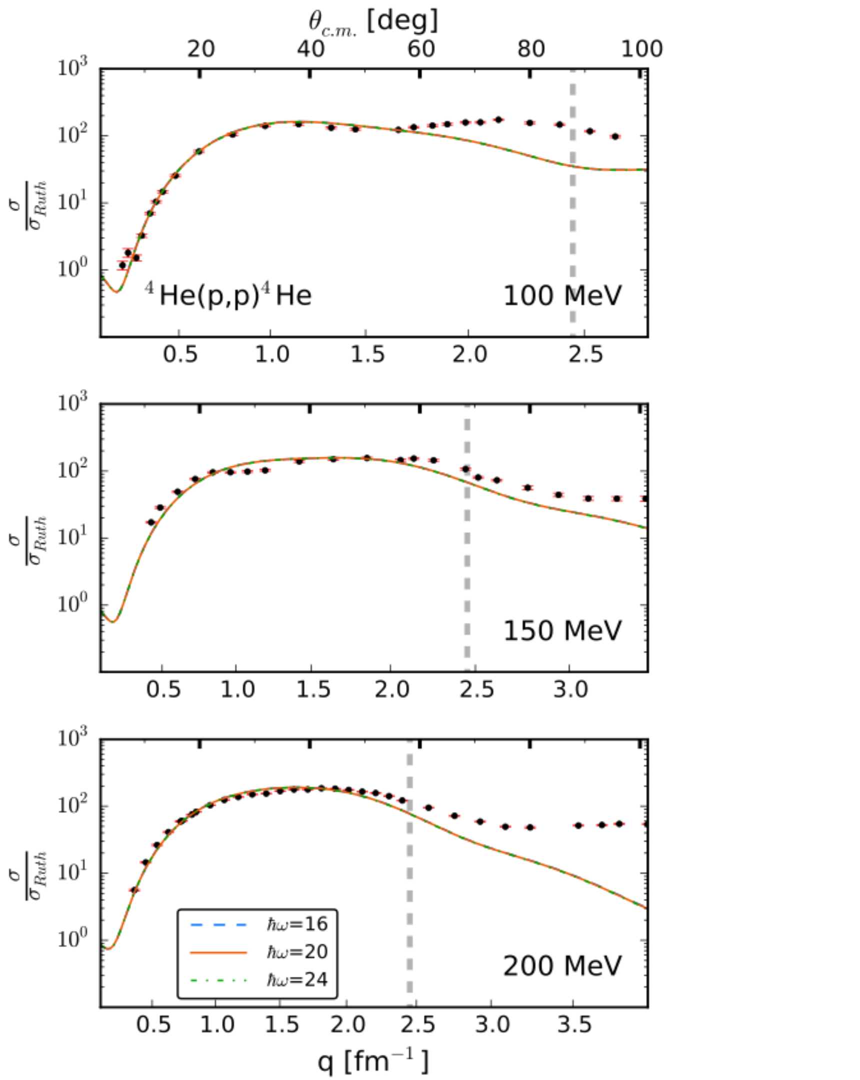

Fig. 5: Cross section for elastic proton scattering from ^{4}He using the multiple-scattering approach. (M. Burrows, et al., Phys. Rev. C **99**, 044603 (2019).)

Nucleon self-energy with chiral interactions

- The optical potential for scattering states is identified with single particle self-energy
- This approach calculates the nucleon self-energy in nuclear matter using interactions derived from chiral effective field theory (χ^{EFT})

Nucleon self-energy with chiral interactions

- χ^{EFT} gives a low-energy description of the nuclear force involving proton, neutron, and pion degrees of freedom
- Nucleons interact via pion exchanges (long-range) and contact forces (short-range)
- Requires a regularization procedure to separate the high- and low-energy physics via a momentum-space cutoff

Nucleon self-energy with chiral interactions

- Compute first- and second-order contributions to the nucleon self-energy Σ with effective potentials V_{2N}^{eff} derived from χ^{EFT}
- V_{2N}^{eff} consist of an NN potential with an effective, medium-dependent NN interaction (depends on 3NF)

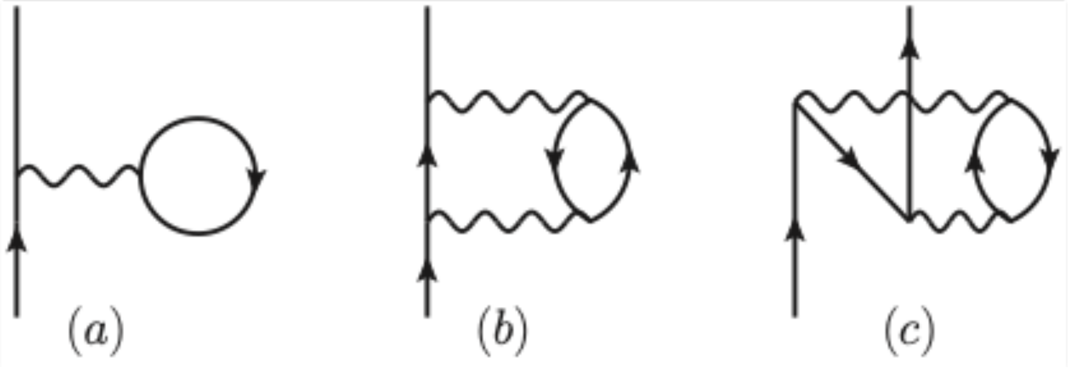

Fig. 6: First- and second-order contributions to the nucleon self-energy where the solid lines indicate nucleon propagators and the wavy lines indicate the in-medium, anti-symmetrized NN interaction. (T. R. Whitehead, et al., Phys. Rev. C **100**, 014601 (2019).)

Nucleon self-energy with chiral interactions

- The optical potential is given by

$$U_N(E;k_f^p,k_f^n) = V_N(E;k_f^p,k_f^n) + \mathrm{i}W\left(E;k_f^p,\ k_f^n\right)$$

$$V_N(E;k_f^p,k_f^n) = Re\Sigma_N(q,E(q);k_f^p,\ k_f^n)$$

$$W_N(E;k_f^p,k_f^n) = \frac{M_N^{k*}}{M} Im\Sigma_N(q,E(q);k_f^p,\ k_f^n)$$

where $N = \mathrm{p},\mathrm{n}$ and $\frac{M_N^{k*}}{M} = [1 + \frac{M}{k}\frac{\partial}{\partial k}V_N\left(k,E(k)\right)]$ defines the effective k-mass

Nucleon self-energy with chiral interactions

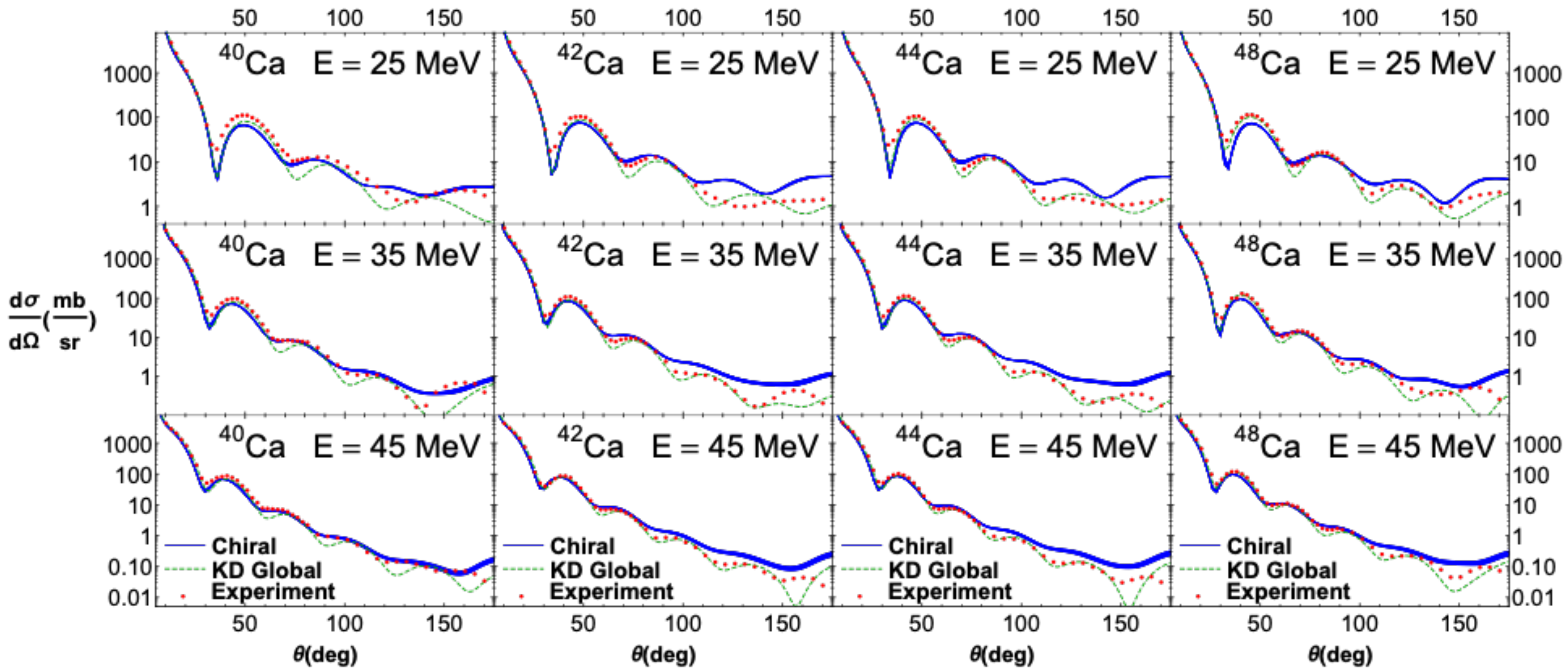

Fig. 7: Cross section for elastic scattering of protons from calcium isotopes at several lab energies. Blue lines correspond to microscopic cross sections and green lines correspond to a phenomenological model. (T. R. Whitehead, et al., Phys. Rev. C **100**, 014601 (2019).)

Nucleon self-energy with chiral interactions

- Well-suited to describe low-energy scattering
- Momentum-space cutoff of the EFT limits the capability of this approach $E < 200\ \mathrm{MeV}$

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- Phenomenological models are constrained by scattering data and work well where data are available
    - Ambiguity in fitting
    - Lack predictive power
    - No means for uncertainty quantification
- Microscopic methods use NN interactions from nuclear structure as inputs in computing optical potentials
    - Extends to reactions involving rare isotopes
    - Offers a means to quantify theoretical uncertainty estimates

Outlook

- Currently microscopic approaches struggle in precision across kinematic ranges or nuclei
  - Can be used to guide phenomenological models
  - Necessary to understand what components are key in computing microscopic models

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  - Necessary to understand what components are key in computing microscopic models
- Need to further understand uncertainty quantification in optical potentials to reliably compare different models
- Can use renormalization group (RG) methods to investigate scheme dependence in factorization of nuclear structure from the scattering probe

Extras

Nucleon self-energy with chiral interactions

- Compute first- and second-order contributions to the nucleon self-energy

$$\Sigma_{2N}^{(1)}(q;k_f) = \sum_i \langle \boldsymbol{q}\boldsymbol{h}_i | V_{2N}^{eff} | \boldsymbol{q}\boldsymbol{h}_i \rangle n_i$$

$$\Sigma_{2N}^{(2a)}(q,\omega;k_f) = \frac{1}{2}\sum_{ijk} \frac{\left| \langle \boldsymbol{p}_i \boldsymbol{p}_k | V_{2N}^{eff} | \boldsymbol{q}\boldsymbol{h}_j \rangle \right|^2}{\omega + \epsilon_j - \epsilon_i - \epsilon_k + i\eta} \bar{n}_i n_j \bar{n}_k$$