AJT notes (11/5/21)

Goal: calculate SRG evolved spectroscopic factor where

$$S_\alpha = \int d^3p \, |\langle \Psi_\alpha^{A-1} | a_{\vec{p}} | \Psi_0^A \rangle|^2 \tag{1}$$

- Here $\alpha = \{n, j, l, s, t\}$.

Strategy: start with inserting $\hat{U}_\lambda^\dagger \hat{U}_\lambda$

$$\langle \Psi_\alpha^{A-1} | a_{\vec{p}} | \Psi_0^A \rangle = \langle \Psi_\alpha^{A-1} | \hat{U}_\lambda^\dagger \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger \hat{U}_\lambda | \Psi_0^A \rangle$$

$$= \langle \Psi_{\alpha,\lambda}^{A-1} | \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger | \Psi_{0,\lambda}^A \rangle$$

Let the SRG-evolved wave function be an uncorrelated state

$$|\Psi_\lambda\rangle \approx \prod_{\beta \text{ occ.}} a_\beta^\dagger |0\rangle \tag{2}$$

Then we can write

$$\langle \Psi_{\alpha,\lambda}^{A-1} | = \langle \Psi_{0,\lambda}^A | a_\alpha^\dagger \tag{3}$$

To simplify notation: $|\Psi_{0,\lambda}^A\rangle \equiv |\Phi\rangle$

$$\Rightarrow \langle \Phi | a_\alpha^\dagger \hat{U}_\lambda a_{\vec{p}} \hat{U}_\lambda^\dagger | \Phi \rangle \tag{4}$$

Strategy:

- Expand $\hat{U}_\lambda$ and $\hat{U}_\lambda^\dagger$ to 2-body
- Apply Wick's Theorem on operator and truncate 3-body +

$$\hat{U}_\lambda = I + \frac{1}{4}\int d^3K \int d^3k \int d^3k' \, \langle \delta\tilde{U}(\vec{k},\vec{k}')\rangle$$

$$\times \; a^\dagger_{\frac{\vec{K}}{2}+\vec{k}} \, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}} \, a_{\frac{\vec{K}}{2}-\vec{k}'} \, a_{\frac{\vec{K}}{2}+\vec{k}'} \tag{5}$$

$$\hat{U}_\lambda^\dagger = I + \frac{1}{4}\int d^3K' \int d^3k'' \int d^3k''' \, \langle \delta\tilde{U}^\dagger(\vec{k}'',\vec{k}''')\rangle$$

$$\times \; a^\dagger_{\frac{\vec{K}'}{2}+\vec{k}''} \, a^\dagger_{\frac{\vec{K}'}{2}-\vec{k}''} \, a_{\frac{\vec{K}'}{2}-\vec{k}'''} \, a_{\frac{\vec{K}'}{2}+\vec{k}'''} . \tag{6}$$

Then

$$a^\dagger_{\vec\alpha} \, \hat{U}_\lambda \, a_{\vec\beta} \, \hat{U}_\lambda^\dagger$$

$$= a^\dagger_{\vec\alpha} a_{\vec\beta} + \frac{1}{4}\int d^3K \int d^3k \int d^3k' \, \delta\tilde{U}(\vec{k},\vec{k}') \, a^\dagger_{\vec\alpha} \, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}} \, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}} \, a_{\frac{\vec{K}}{2}-\vec{k}'} \, a_{\frac{\vec{K}}{2}+\vec{k}'} \, a_{\vec\beta}$$

$$+ \frac{1}{4}\int d^3K' \int d^3k'' \int d^3k''' \, \delta\tilde{U}^\dagger(\vec{k}'',\vec{k}''') \, a^\dagger_{\vec\alpha} \, a_{\vec\beta} \, a^\dagger_{\frac{\vec{K}'}{2}+\vec{k}''} \, a^\dagger_{\frac{\vec{K}'}{2}-\vec{k}''} \, a_{\frac{\vec{K}'}{2}-\vec{k}'''} \, a_{\frac{\vec{K}'}{2}+\vec{k}'''}$$

$$\Big[+ \frac{1}{16}\int d^3K \int d^3K' \int d^3k \int d^3k' \int d^3k'' \int d^3k''' \, \delta\tilde{U}(\vec{k},\vec{k}') \, \delta\tilde{U}^\dagger(\vec{k}'',\vec{k}''')\rangle$$

$$\times \; a^\dagger_{\vec\alpha} \, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}} \, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}} \, a_{\frac{\vec{K}}{2}-\vec{k}'} \, a_{\frac{\vec{K}}{2}+\vec{k}'} \, a_{\vec\beta} \, a^\dagger_{\frac{\vec{K}'}{2}+\vec{k}''} \, a^\dagger_{\frac{\vec{K}'}{2}-\vec{k}''} \, a_{\frac{\vec{K}'}{2}-\vec{k}'''} \, a_{\frac{\vec{K}'}{2}+\vec{k}'''} \Big] \tag{7}$$

Term 1 : $\sim a_{\alpha}^{\dagger} a_{\vec{p}}$

$:a_{\alpha}^{\dagger} a_{\vec{p}}: = a_{\alpha}^{\dagger} a_{\vec{p}}$

Term 2 : $\sim a^{+}a^{+}a^{+}aaa$

$:a^{+}a^{+}a^{+}aaa: = \cancel{a^{+}a^{+}a^{+}aaa}$ 3-body

Term 4 : $\sim a^{+}a^{+}a^{+}aaaa^{+}a^{+}aa$

- Every term of $a^{+}a^{+}a^{+}aaaa^{+}a^{+}aa$ is 3-body or higher. Truncate.

Term 3 : $\sim a^{+}aa^{+}a^{+}aa$

* Because we are not using $\vec{K}, \vec{k}, \vec{k}'$ anymore relabel third term $\vec{k}' \to \vec{k}$, etc.

$$a_\alpha^\dagger a_{\vec p}\, a^\dagger_{\frac{\vec K}{2}+\vec k}\, a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}$$

$$\simeq \overset{\text{3-body}}{\cancel{:a_\alpha^\dagger a_{\vec p}\, a^\dagger_{\frac{\vec K}{2}+\vec k}\, a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}:}}$$

$$+ \sum_{\substack{\text{single}\\ \text{contractions}}} :a_\alpha^\dagger a_{\vec p}\, a^\dagger_{\frac{\vec K}{2}+\vec k}\, a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}:$$

$$+ \sum_{\substack{\text{double}\\ \text{cont.}}} \overset{=0}{\cancel{:a_\alpha^\dagger a_{\vec p}\, a^\dagger_{\frac{\vec K}{2}+\vec k}\, a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}:}}$$

$$= :a_\alpha^\dagger \overbrace{a_{\vec p}\, a^\dagger_{\frac{\vec K}{2}+\vec k}}\, a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}:$$

$$- :a_\alpha^\dagger \overbrace{a_{\vec p}\, a^\dagger_{\frac{\vec K}{2}+\vec k}\, a^\dagger_{\frac{\vec K}{2}-\vec k}}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}:$$

$$= \delta^3\!\left(\vec p - \frac{\vec K}{2} - \vec k\right) a_\alpha^\dagger a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'}$$

$$- \delta^3\!\left(\vec p - \frac{\vec K}{2} + \vec k\right) a_\alpha^\dagger a^\dagger_{\frac{\vec K}{2}+\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'} \qquad (8)$$

* with antisymmetrized $\delta\tilde{V}^+$, these two contributions end up being identical. Multiply by 2.

$$= a_\alpha^\dagger a_{\vec p} + \frac{1}{2}\int d^3K \int d^3k \int d^3k'\, \delta\tilde{V}^+(\vec k, \vec k')\, \delta^3\!\left(\vec p - \frac{\vec K}{2} - \vec k\right)$$

$$\times\, a_\alpha^\dagger a^\dagger_{\frac{\vec K}{2}-\vec k}\, a_{\frac{\vec K}{2}-\vec k'}\, a_{\frac{\vec K}{2}+\vec k'} \qquad (9)$$

5

- Now we change from field operators $a_{\vec{p}}$ (really should be $\psi(\vec{p})$) to s.p. SM state creation and annihilation operator

In general:

$$a_{\vec{k}} = \sum_{\beta} \phi_\beta(\vec{k})\, a_\beta \qquad (10)$$

where $\phi_\beta(\vec{k})$ is the s.p. momentum distribution

$$1 = \frac{2}{\pi}\int_0^\infty dk\, k^2\, |\phi_\beta(k)|^2 \quad \text{(from code test)}$$

$$[\phi_\beta(\vec{k})] = fm^{3/2}$$

$$= \sum_\beta \phi_\beta(\vec{p})\, a_\alpha^\dagger a_\beta + \frac{1}{2}\sum_\beta \int d^3K \int d^3k \int d^3k'\, \delta U^\dagger(\vec{k},\vec{k}')$$

$$\times\, \delta^3\!\left(\vec{p}-\tfrac{\vec{K}}{2}-\vec{k}\right)\phi_\beta\!\left(\tfrac{\vec{K}}{2}+\vec{k}'\right) a_\alpha^\dagger a^\dagger_{\frac{\vec{K}}{2}-\vec{k}}\, a_{\frac{\vec{K}}{2}-\vec{k}'}\, a_\beta \qquad (11)$$

Evaluate w.r.t. to $|\Phi\rangle$

$$= \sum_\beta \Big[\phi_\beta(\vec{p})\, \langle \Phi | a_\alpha^\dagger a_\beta | \Phi \rangle$$

$$+ \frac{1}{2}\int d^3K \int d^3k \int d^3k'\, \delta U^\dagger(\vec{k},\vec{k}')\, \delta^3\!\left(\vec{p}-\tfrac{\vec{K}}{2}-\vec{k}\right) \qquad (12)$$

$$\times \left[\phi_\beta\left(\frac{\vec{K}}{2}+\vec{k}'\right) \langle \Phi | a_\alpha^\dagger a_\beta | \Phi \rangle \langle a^\dagger_{\frac{\vec{K}}{2}-\vec{k}}\, a_{\frac{\vec{K}}{2}-\vec{k}'} \rangle \right.$$

$$\left. - \phi_\beta\left(\frac{\vec{K}}{2}-\vec{k}'\right) \langle \Phi | a_\alpha^\dagger a_\beta | \Phi \rangle \langle a^\dagger_{\frac{\vec{K}}{2}-\vec{k}}\, a_{\frac{\vec{K}}{2}+\vec{k}'} \rangle \right]$$

* Unsure about this step

Argue $\langle a^\dagger_{\frac{\vec{K}}{2}-\vec{k}}\, a_{\frac{\vec{K}}{2}-\vec{k}'} \rangle = \delta^3\left(\frac{\vec{K}}{2}-\vec{k}-\frac{\vec{K}}{2}+\vec{k}'\right)$

$$= \delta^3(\vec{k}'-\vec{k})$$

$$= \sum_\beta \left[\phi_\beta(\vec{p})\,\delta_{\alpha\beta} + \frac{1}{2}\int d^3K \int d^3k \int d^3k' \right.$$

$$\times\ \delta\tilde{U}^\dagger(\vec{k},\vec{k}')\,\delta^3\left(\vec{p}-\frac{\vec{K}}{2}-\vec{k}\right)\delta_{\alpha\beta}\left[\phi_\beta\left(\frac{\vec{K}}{2}+\vec{k}'\right)\right.$$

$$\left.\left.\times\ \delta^3(\vec{k}'-\vec{k}) - \phi_\beta\left(\frac{\vec{K}}{2}-\vec{k}'\right)\delta^3(\vec{k}'+\vec{k})\right]\right] \qquad (13)$$

$\delta\tilde{U}^\dagger$ antisymmetrized $\rightarrow$ identical contributions since $-\delta\tilde{U}^\dagger(\vec{k},-\vec{k}) = \delta\tilde{U}^\dagger(\vec{k},\vec{k})$

$$= \phi_\alpha(\vec{p}) + \int d^3K \int d^3k \int d^3k'\ \delta\tilde{U}^\dagger(\vec{k},\vec{k}')$$

$$\times\ \delta^3\left(\vec{p}-\frac{\vec{K}}{2}-\vec{k}\right)\delta^3(\vec{k}'-\vec{k})\,\phi_\beta\left(\frac{\vec{K}}{2}+\vec{k}'\right) \qquad (14)$$

$\vec{K}_L = \vec{p} - \vec{k}$ (factor of 8)

$\vec{k}' = \vec{k} \Rightarrow \vec{K}_L + \vec{k}' = \vec{p}$

$$= \phi_\alpha(\vec{p}) + 8 \int d^3k \, \delta \tilde{U}^\dagger(\vec{k}, \vec{k}) \phi_\beta(\vec{p})$$

$$= \phi_\alpha(\vec{p}) \left[1 + 8 \int d^3k \, \delta \tilde{U}^\dagger(\vec{k}, \vec{k}) \right] \quad (15)$$

$\downarrow$ Partial waves like last time

$$= \phi_\alpha(\vec{p}) \Big[1 + 16 \sum_{L,S,T}{}' \sum_J (2J+1) \frac{2}{\pi} \int_0^\infty dk \, k^2$$

$$\times (k(LS)JT | \delta U^\dagger | k(LS)JT) \sum_{\tau'} |\langle \tau \tau' | T \, \tau+\tau' \rangle|^2 \Big] \quad (16)$$

Then the SF is given by

$$S_\alpha(\lambda) = \int d^3p \, |\phi_\alpha(\vec{p})|^2$$

$$\times \left[1 + 16 \sum_{L,S,T}{}' \sum_J (2J+1) \frac{2}{\pi} \int_0^\infty dk \, k^2 (k(LS)JT | \delta U^\dagger | k(LS)JT) \sum_{\tau'} |\langle \tau\tau' | T \, \tau+\tau' \rangle| \right]^2 \quad (17)$$