

# Short-range correlation physics at low RG resolution

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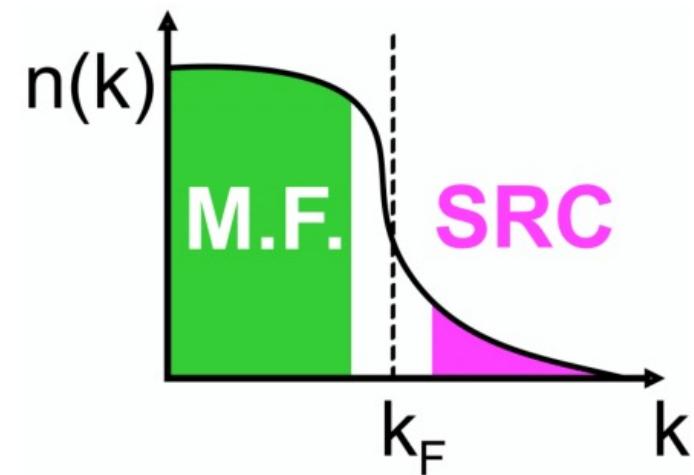
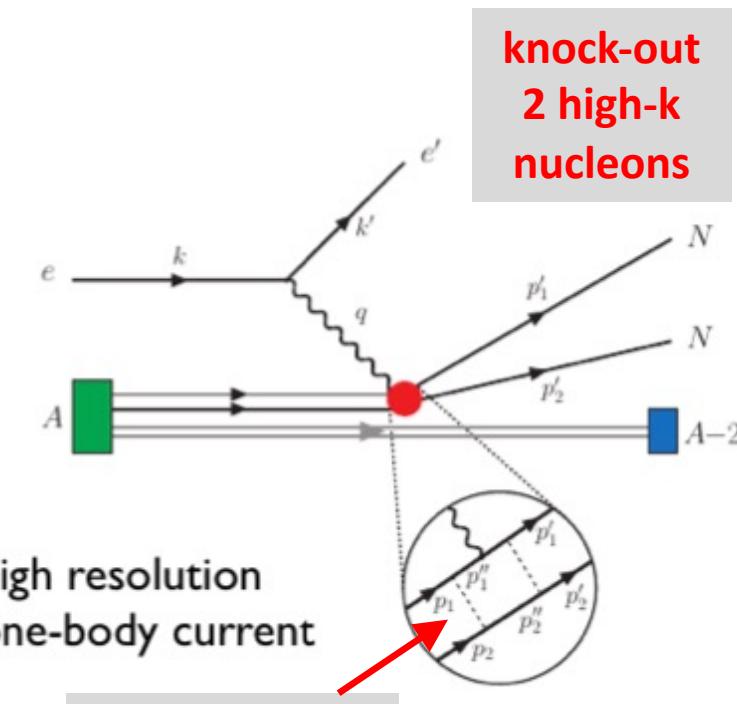
*ajt, S.K. Bogner, and R.J. Furnstahl, arXiv:2105.13936*

*Phys. Rev. C **104**, 034311 (2021)*



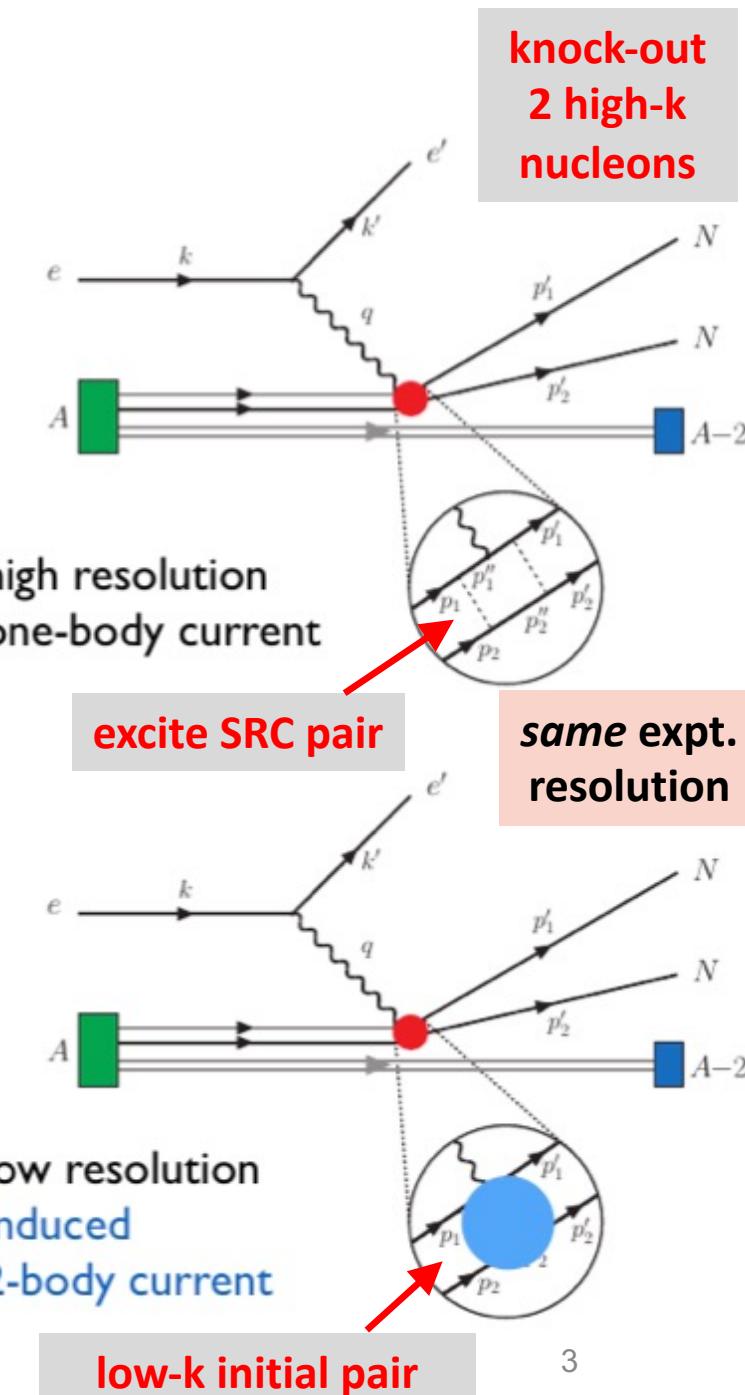
# Motivation

- Recent experiments have been able to isolate processes where short-range correlation (SRC) physics is dominant and well accounted for by SRC phenomenology
- SRC physics at **high RG resolution**
  - SRC pairs are components in the nuclear wave function with relative momenta above the Fermi momentum



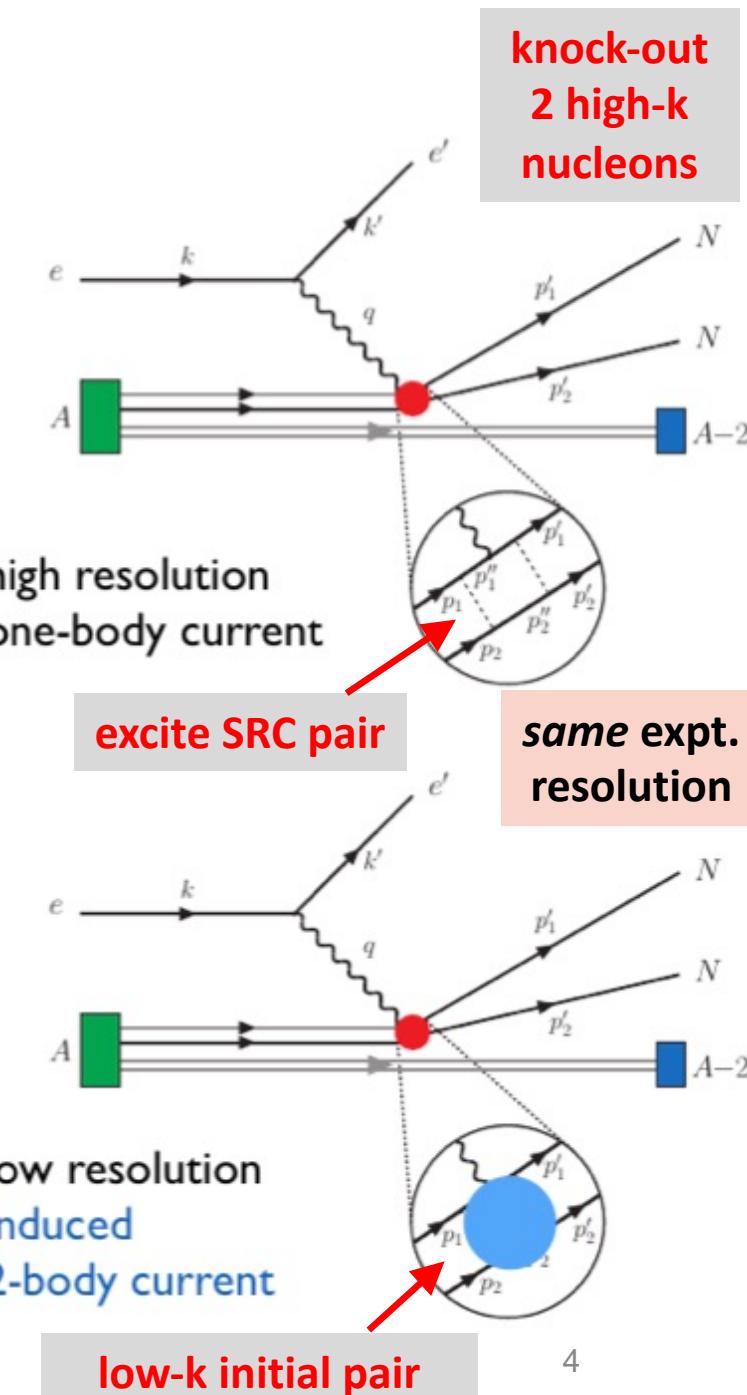
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  - The SRC *physics* is shifted into the reaction operators from the nuclear wave function (which becomes soft)
  - Operators do not become hard which simplifies calculations



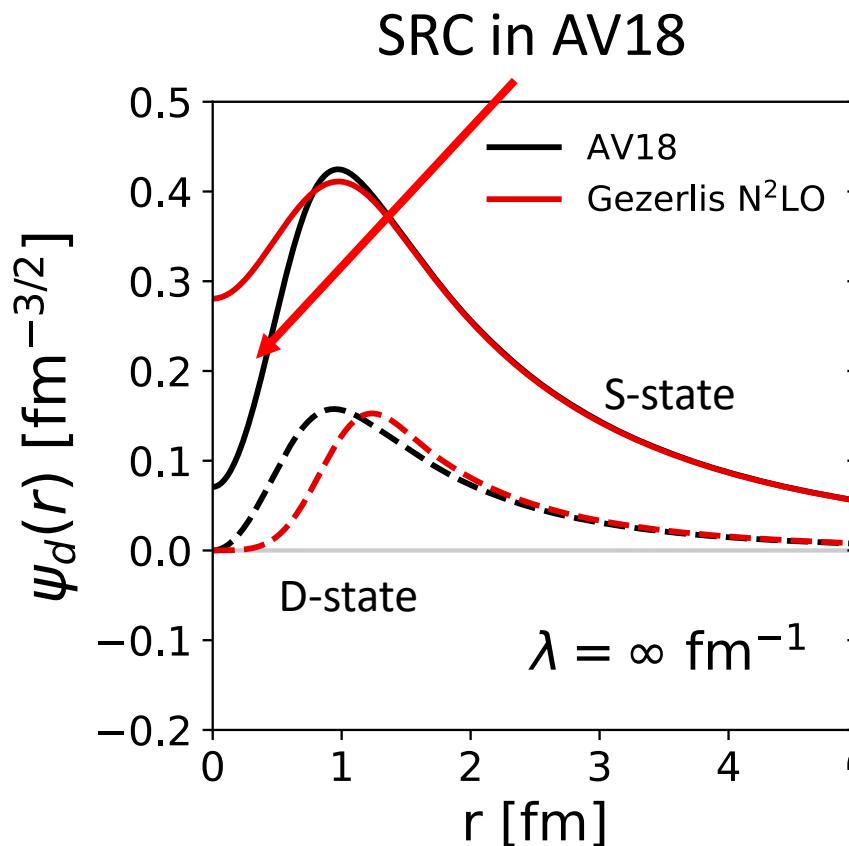
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  - Operators do not become hard which simplifies calculations
- Experimental resolution (set by momentum of probe) is the same in both pictures
- Same observables but different physical interpretation!



# Similarity Renormalization Group (SRG)

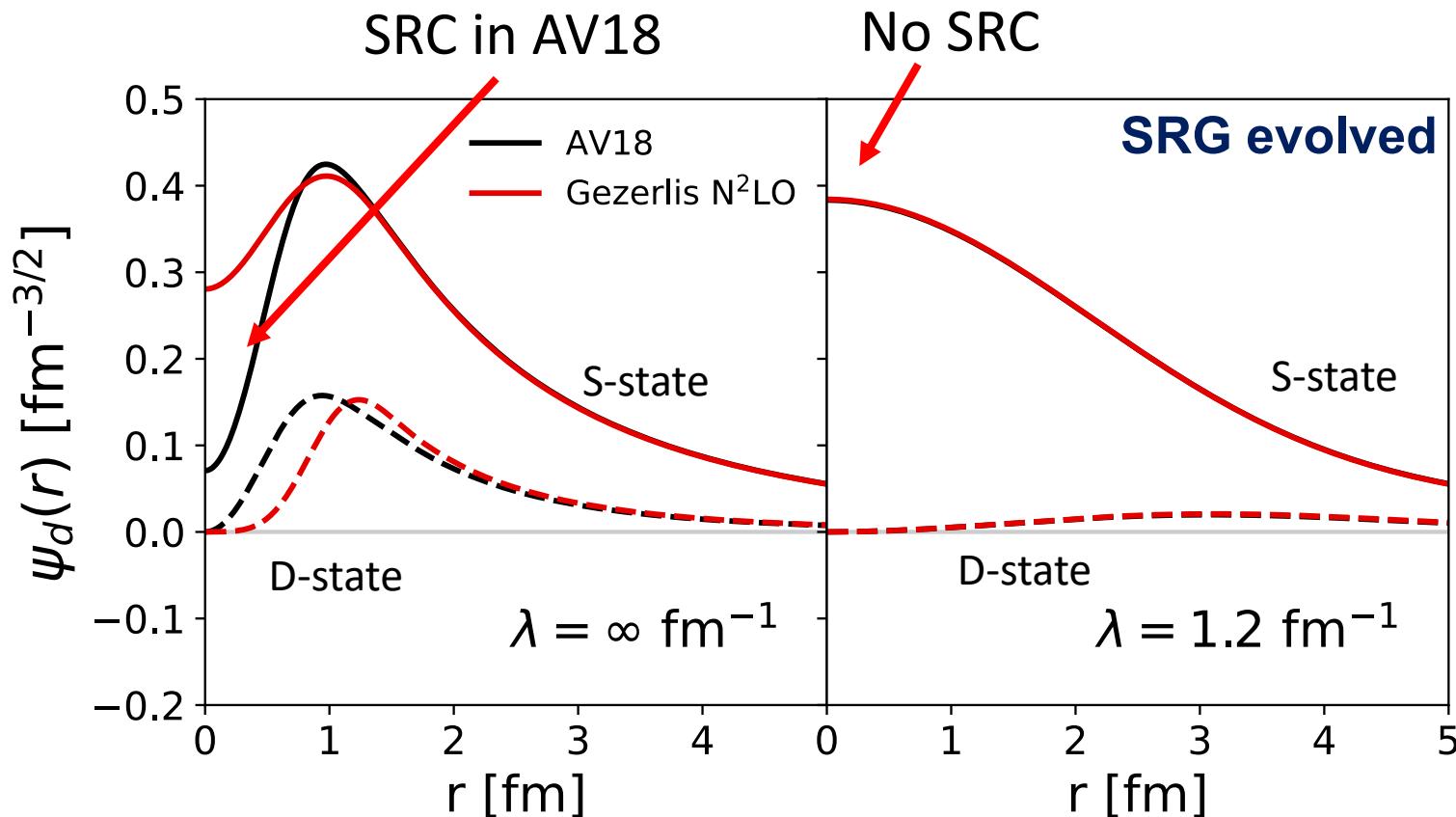
- AV18 wave function has significant SRC
- What happens to the wave function at low RG resolution?
- Use SRG to unitarily evolve to low RG resolution where  $\lambda$  gives the decoupling scale



**Fig. 1:** SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis N<sup>2</sup>LO<sup>1</sup>.

# Similarity Renormalization Group (SRG)

- SRC physics in AV18 is gone from wave function at low RG resolution
- Deuteron wave functions become soft and D-state probability goes down
- Observables such as asymptotic D-S ratio are the same



**Fig. 1:** SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis N<sup>2</sup>LO<sup>1</sup>.

# Momentum distributions at low RG resolution

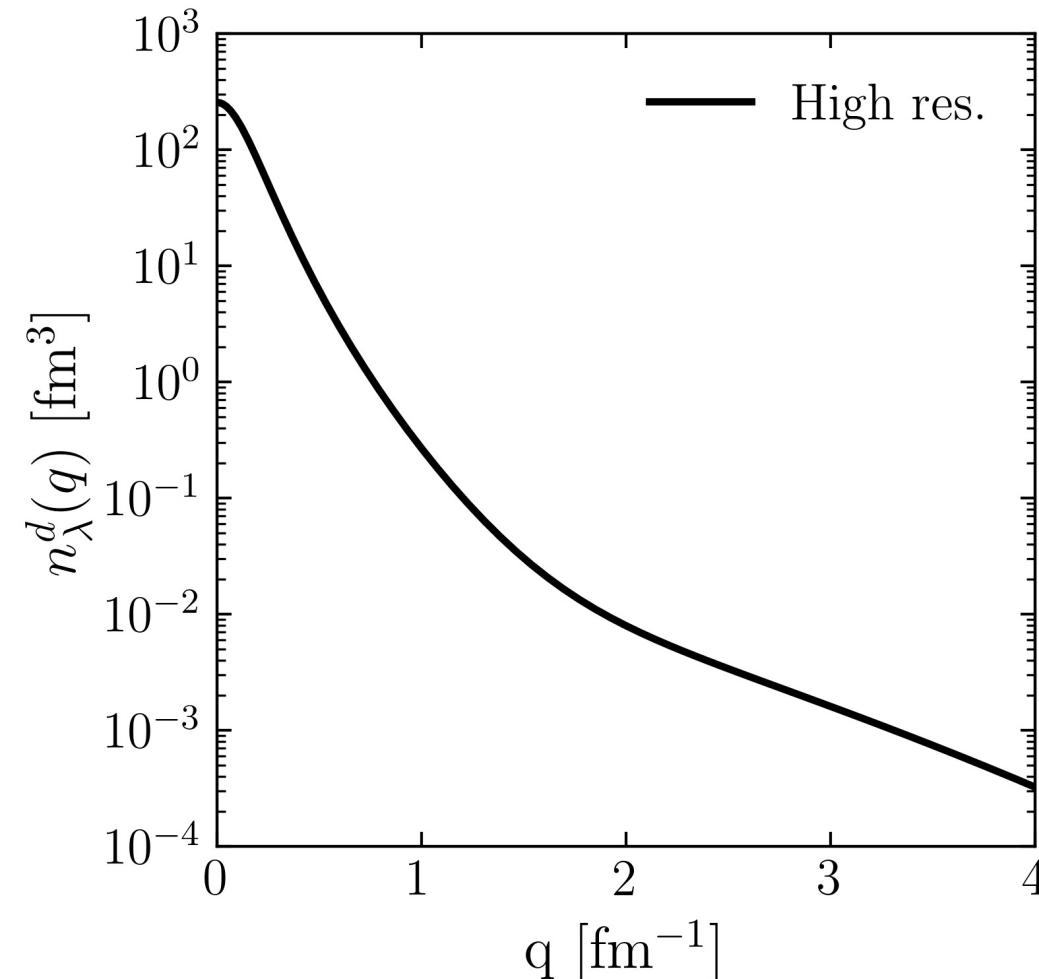
- Soft wave functions at low RG resolution
  - Where does the SRC physics go?

# Momentum distributions at low RG resolution

- Soft wave functions at low RG resolution
  - Where does the SRC physics go?
- SRC physics shifts to the operators  $\langle \psi_f^{hi} | U_\lambda^\dagger U_\lambda O^{hi} U_\lambda^\dagger U_\lambda | \psi_i^{hi} \rangle$
- Apply SRG transformations to momentum distribution operator

$$n^{hi}(\mathbf{q}) = a_{\mathbf{q}}^\dagger a_{\mathbf{q}}$$
$$U_\lambda = 1 + \frac{1}{4} \sum_{K, \mathbf{k}, \mathbf{k}'} \delta U_\lambda^{(2)}(\mathbf{k}, \mathbf{k}') a_{\frac{K}{2}+\mathbf{k}}^\dagger a_{\frac{K}{2}-\mathbf{k}}^\dagger a_{\frac{K}{2}-\mathbf{k}'} a_{\frac{K}{2}+\mathbf{k}'} + \dots$$

# Momentum distributions at low RG resolution



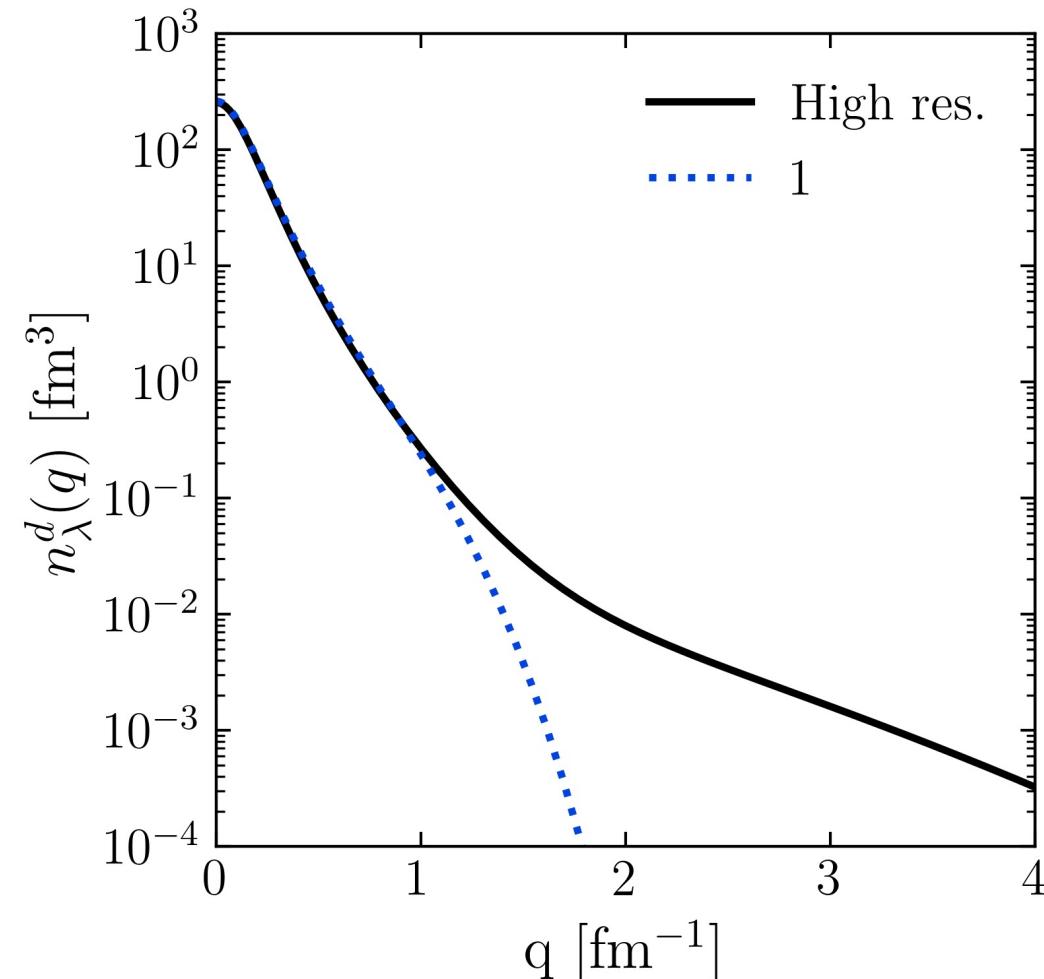
- Deuteron example

$$n^{lo}(\mathbf{q}) = (1 + \delta U) a_{\mathbf{q}}^\dagger a_{\mathbf{q}} (1 + \delta U^\dagger)$$

$$\langle \psi_d^{hi} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_d^{hi} \rangle$$

**Fig. 2:** Contributions to deuteron momentum distribution with AV18 and  $\lambda = 1.35$  fm<sup>-1</sup>.

# Momentum distributions at low RG resolution



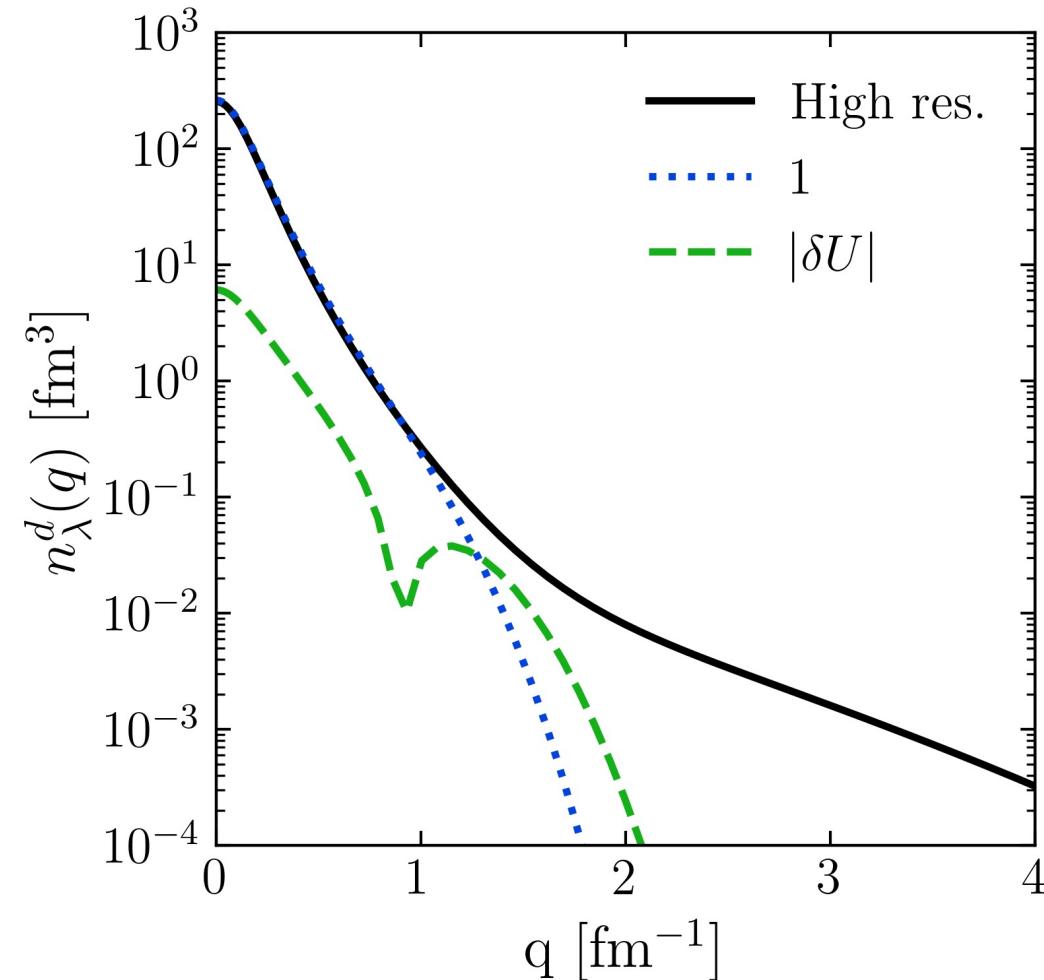
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$$\begin{aligned}\langle \psi_d^{hi} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_d^{hi} \rangle \\ \langle \psi_d^{lo} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_d^{lo} \rangle\end{aligned}$$

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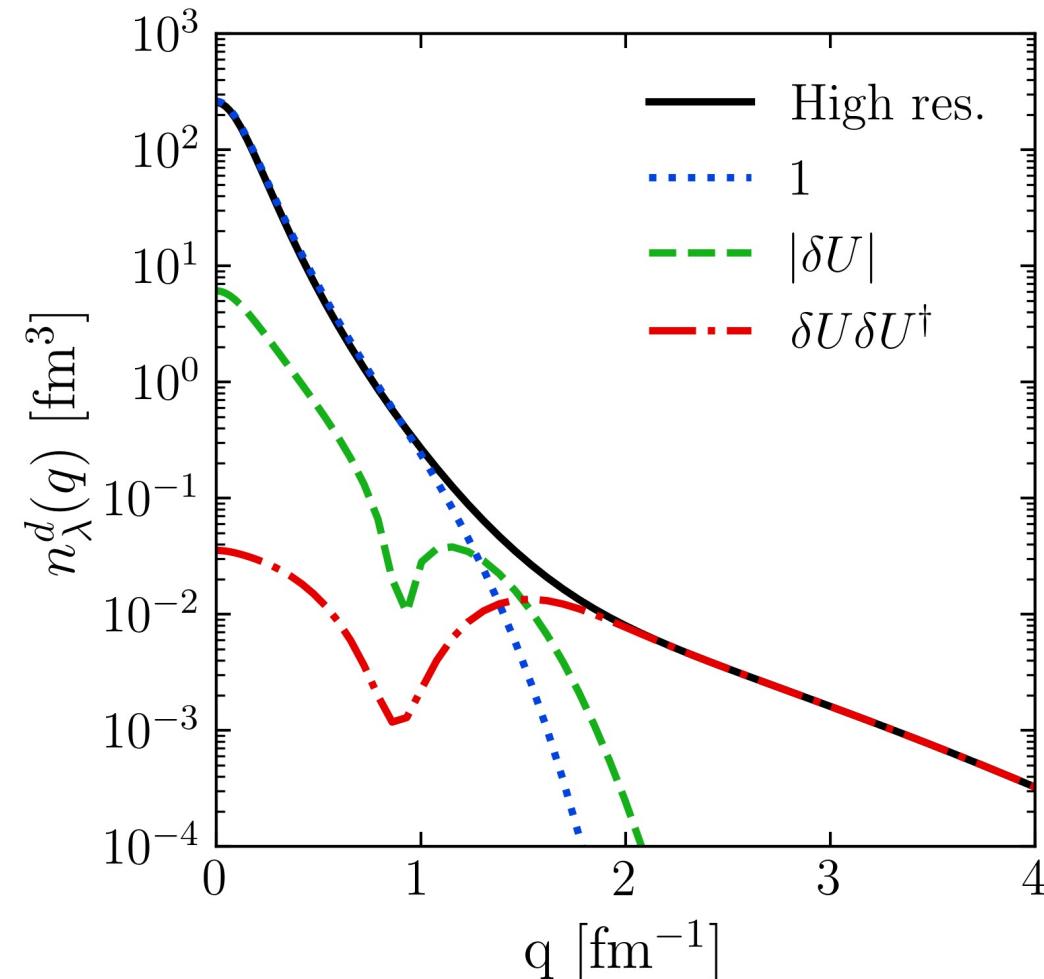
$$\langle \psi_d^{hi} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_d^{hi} \rangle$$

$$\langle \psi_d^{lo} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_d^{lo} \rangle$$

$$\langle \psi_d^{lo} | \delta U a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \delta U^\dagger | \psi_d^{lo} \rangle$$

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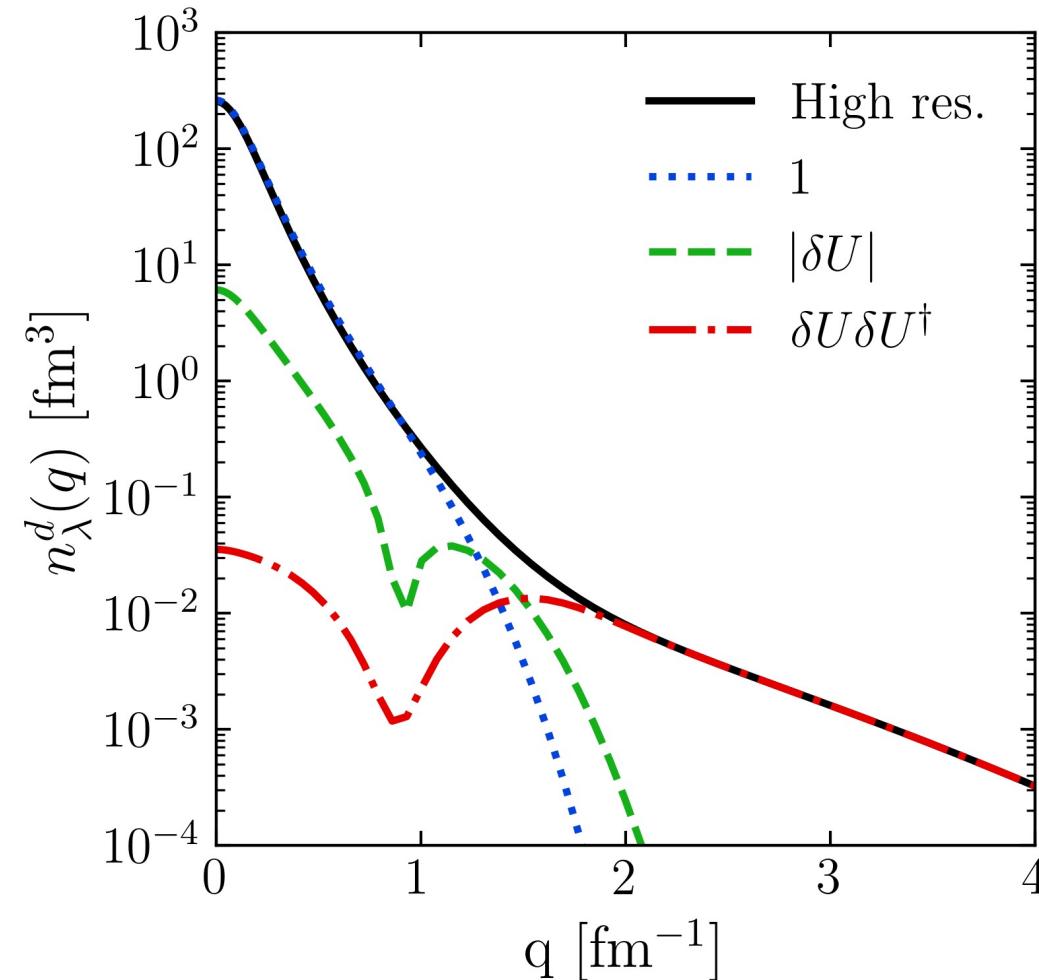
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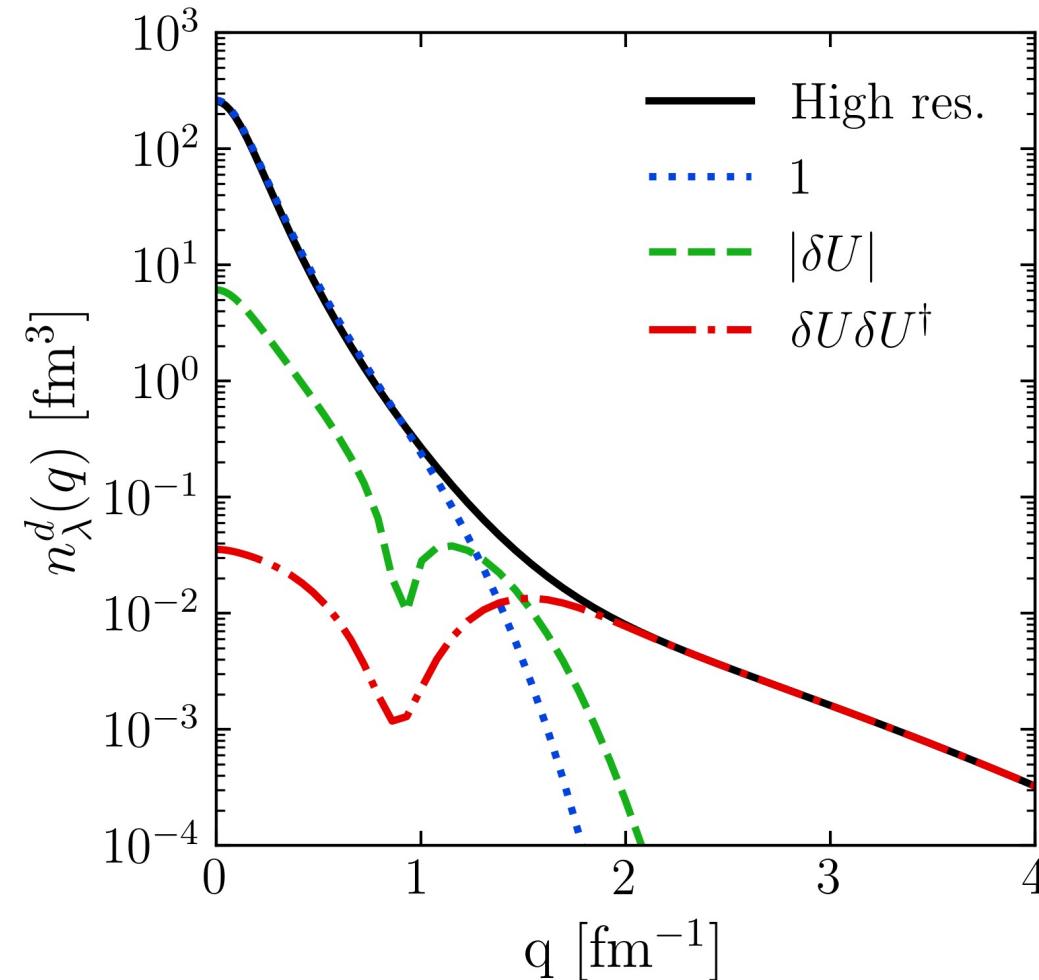


**Fig. 2:** Contributions to deuteron momentum distribution with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ .

- For high- $q$ , the  $\delta U_\lambda \delta U_\lambda^\dagger$  2-body term dominates

$$\approx \sum_{K,k,k'} \delta U_\lambda(\mathbf{k}, \mathbf{q}) \delta U_\lambda^\dagger(\mathbf{q}, \mathbf{k}') a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

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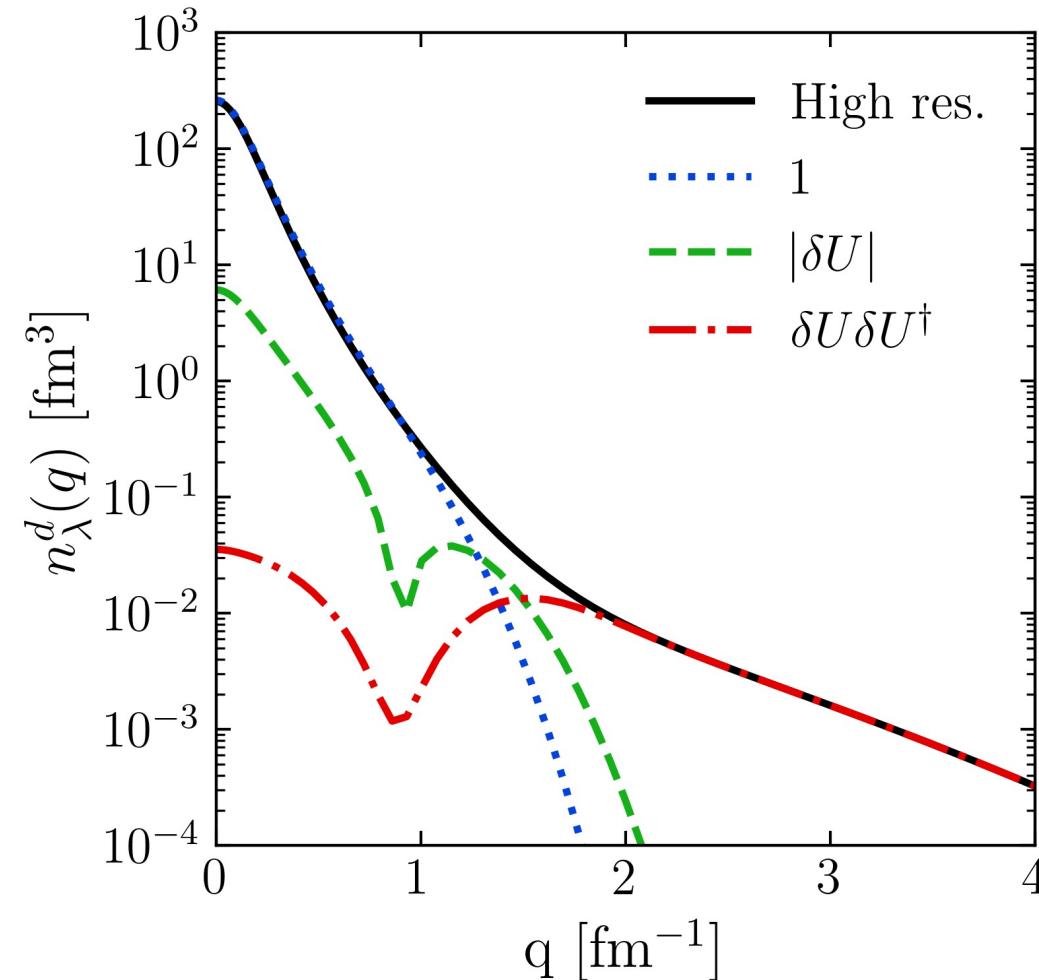
**Factorization:**  $\delta U_\lambda(\mathbf{k}, \mathbf{q}) \approx F_\lambda^{lo}(\mathbf{k}) F_\lambda^{hi}(\mathbf{q})$

$$\approx |F_\lambda^{hi}(\mathbf{q})|^2 \sum_{\mathbf{K}, \mathbf{k}, \mathbf{k}'}^{\lambda} F_\lambda^{lo}(\mathbf{k}) F_\lambda^{lo}(\mathbf{k}') a_{\frac{\mathbf{K}}{2} + \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}'} a_{\frac{\mathbf{K}}{2} + \mathbf{k}'}$$



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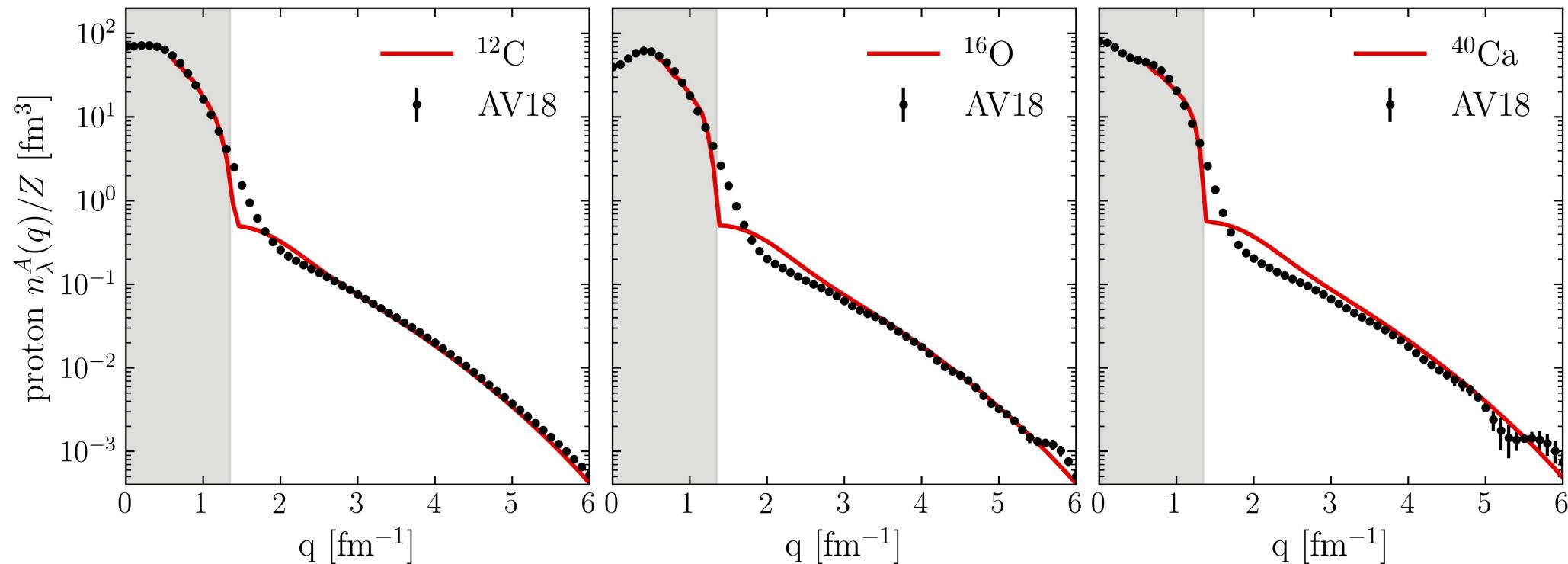
$$\approx \sum \delta U_\lambda(\mathbf{k}, q) \delta U_\lambda^\dagger(\mathbf{q}, \mathbf{k}') a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}^\dagger$$

Apply this strategy to nuclear momentum distributions using local density approximation (LDA)!

$$\approx |F_\lambda^{lo}(q)|^2 \sum_{\mathbf{K}, \mathbf{k}, \mathbf{k}'} F_\lambda^{lo}(\mathbf{k}) F_\lambda^{lo}(\mathbf{k}') a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

# Proton momentum distributions

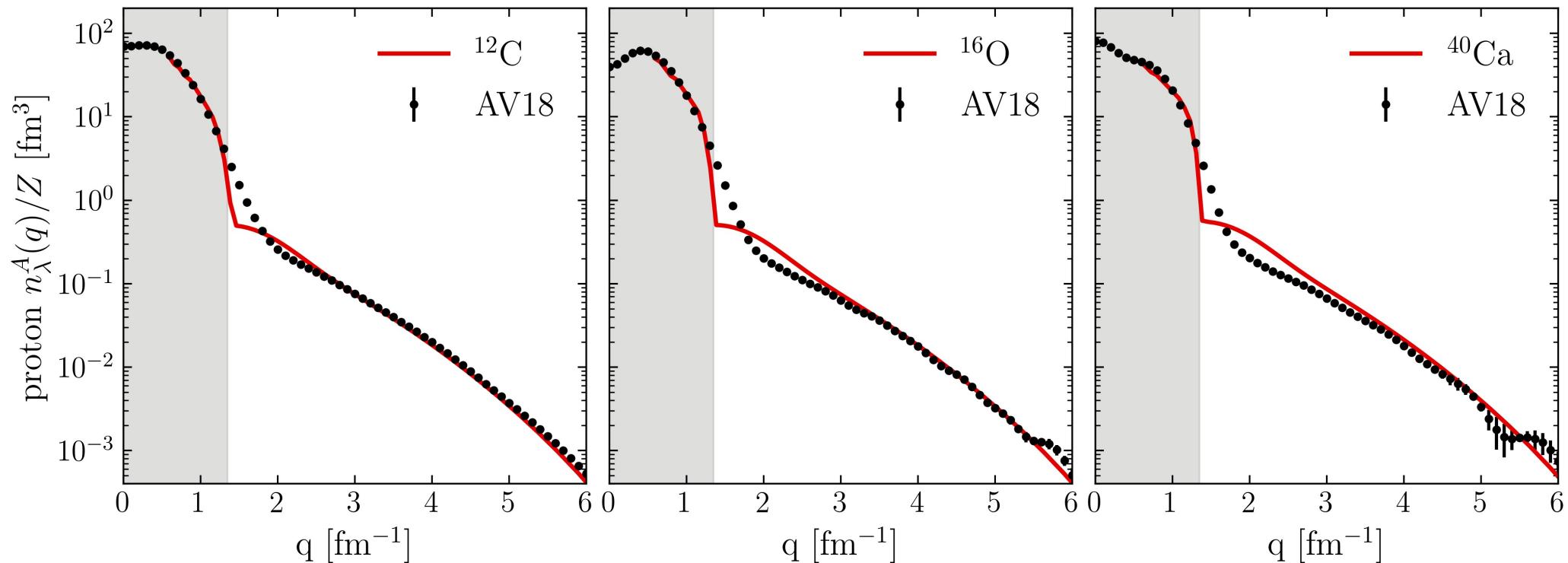
- Low RG resolution calculations reproduce momentum distributions of AV18 data<sup>1</sup> (high RG resolution calculation)



**Fig. 3:** Proton momentum distributions for <sup>12</sup>C, <sup>16</sup>O, and <sup>40</sup>Ca under LDA with AV18,  $\lambda = 1.35$  fm<sup>-1</sup>, and densities from Skyrme EDF SLy4 using the HFBRAD code<sup>2</sup>.

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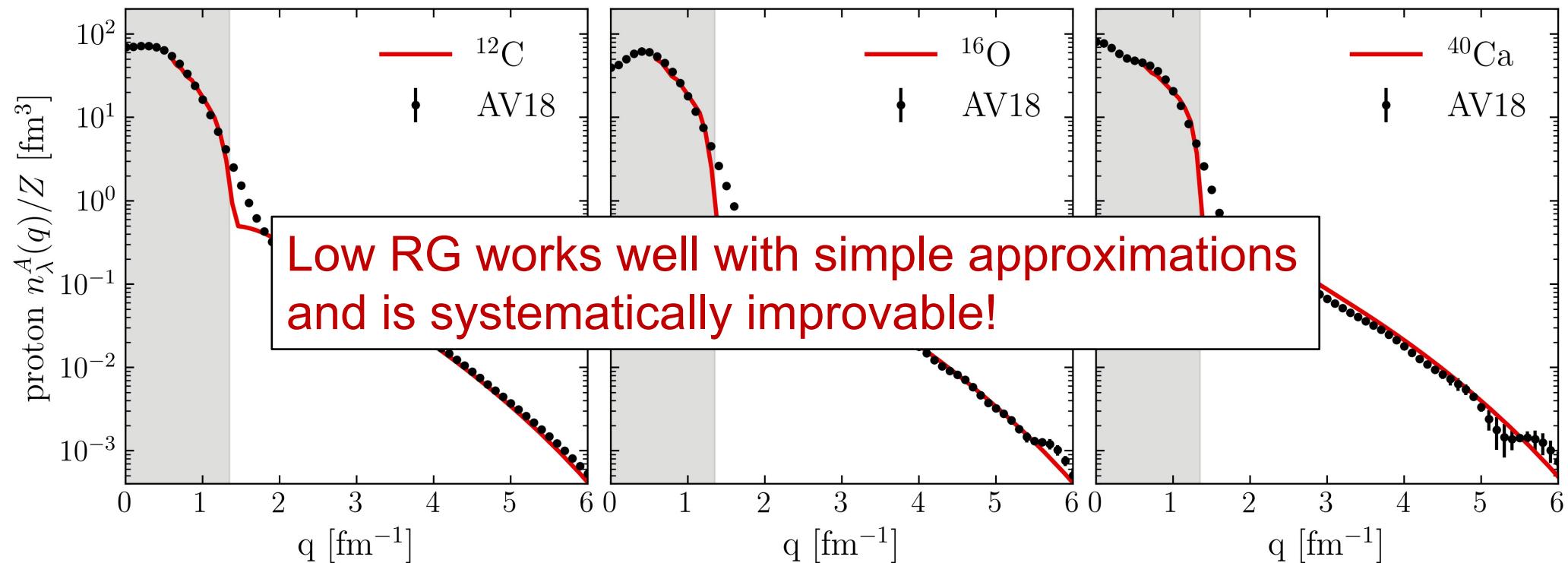
- **Universality:** High- $q$  dependence from universal function  $\approx |F_{\lambda}^{hi}(q)|^2$  fixed by 2-body and insensitive to nucleus



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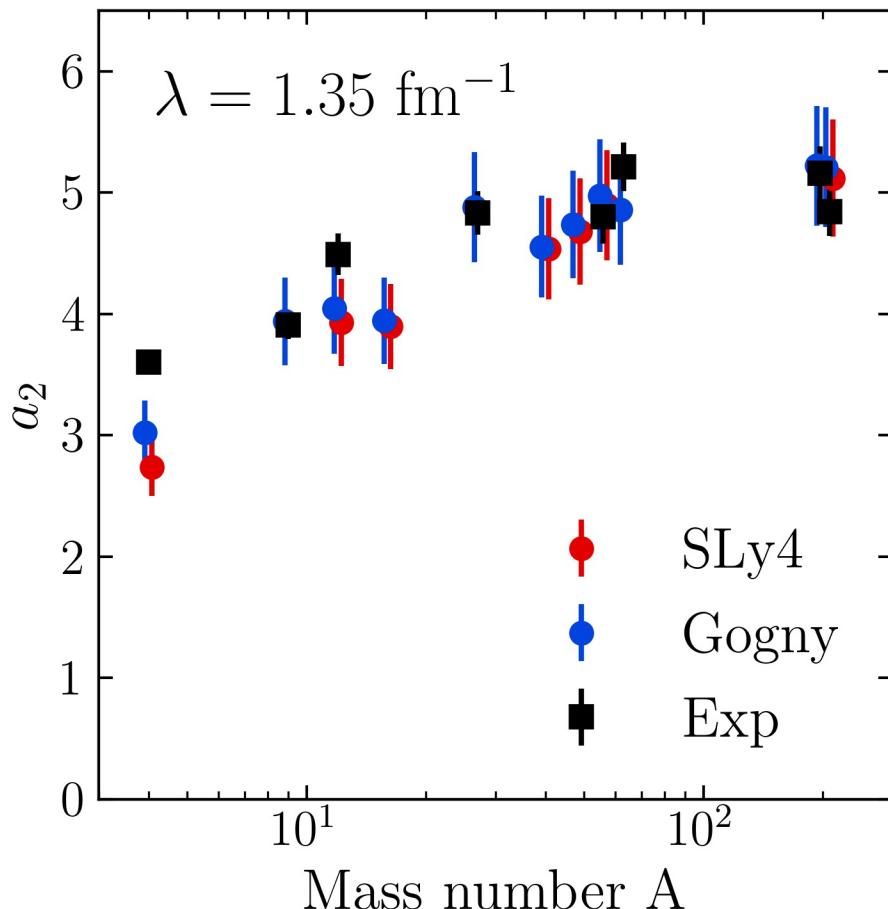
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# SRC scaling factors



**Fig. 4:**  $a_2$  scale factors using single-nucleon momentum distributions under LDA (SLy4 in red<sup>1</sup>, Gogny<sup>2</sup> in blue) with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$  compared to experimental values<sup>3</sup>.

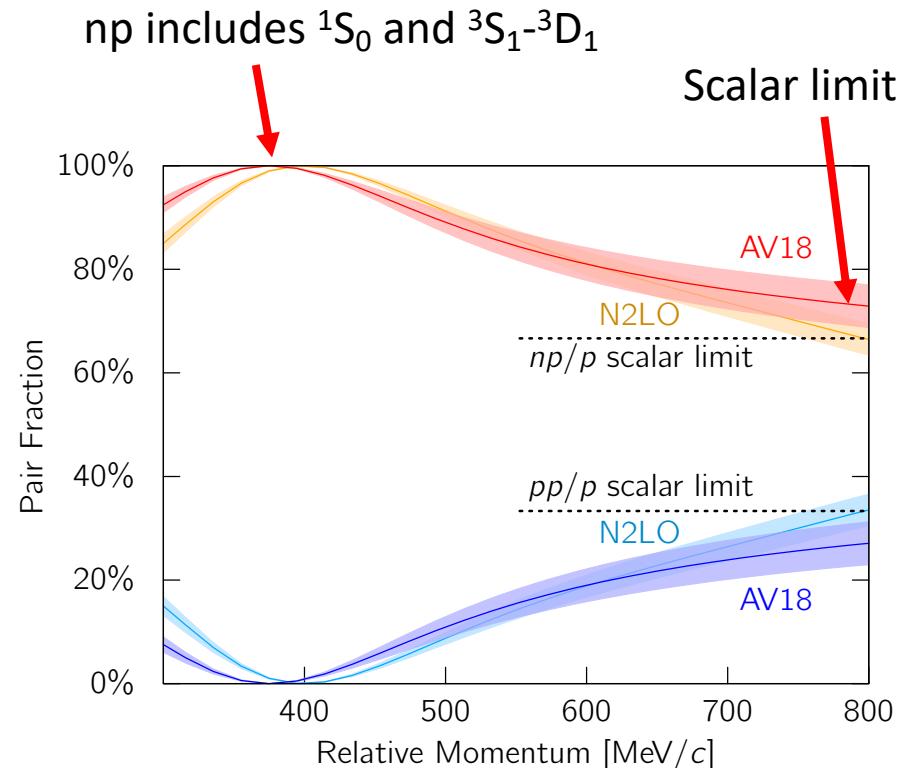
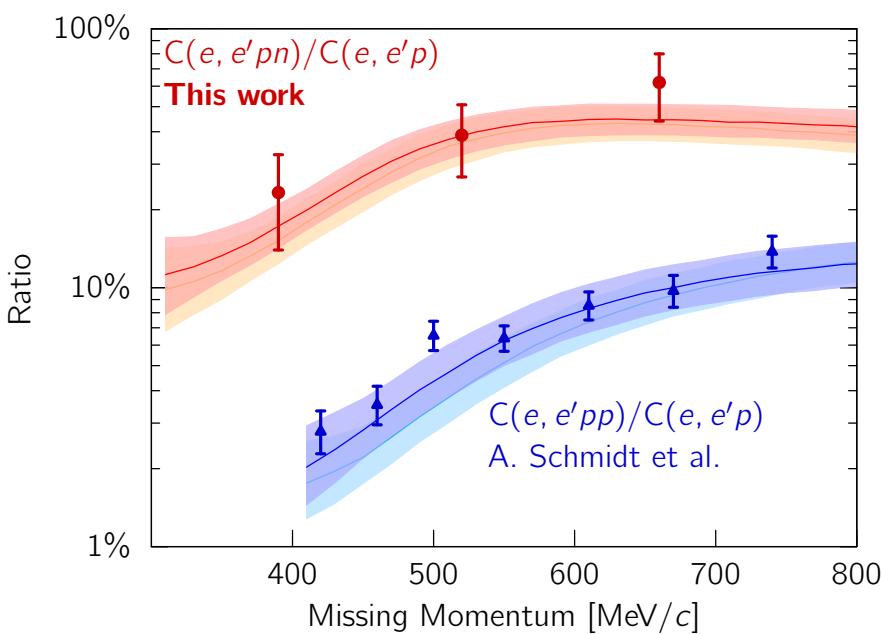
- SRC scaling factors

$$a_2 = \lim_{q \rightarrow \infty} \frac{P^A(q)}{P^d(q)} \approx \frac{\int_{\Delta p^{high}} dq P^A(q)}{\int_{\Delta p^{high}} dq P^d(q)}$$

where  $P^A(q)$  is the single-nucleon probability distribution in nucleus A with error bars from varying  $\Delta p^{high}$

- Good agreement with  $a_2$  values from experiment<sup>3</sup> and LCA calculations<sup>4</sup>

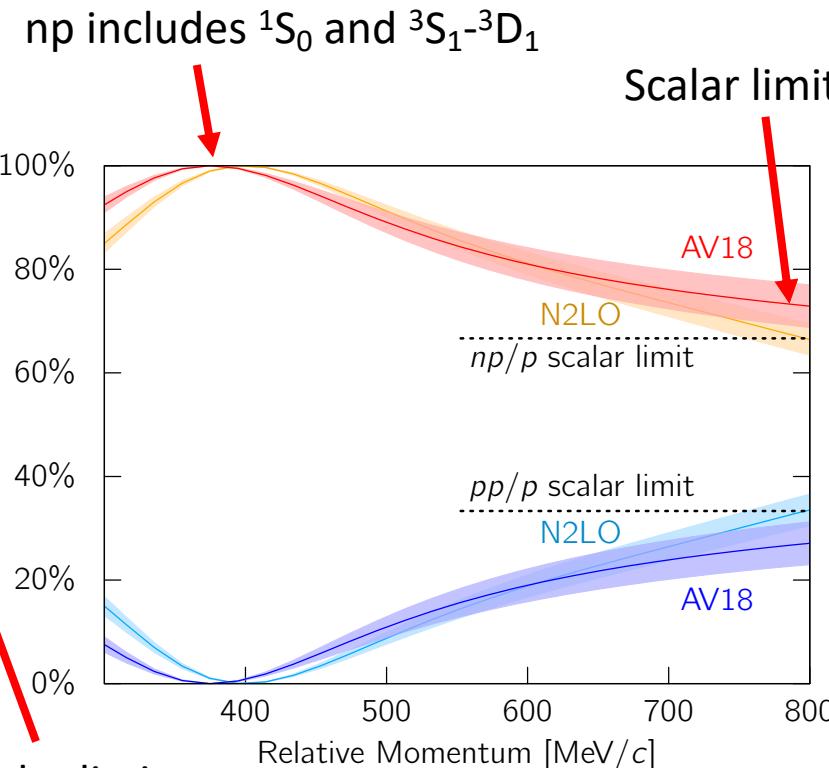
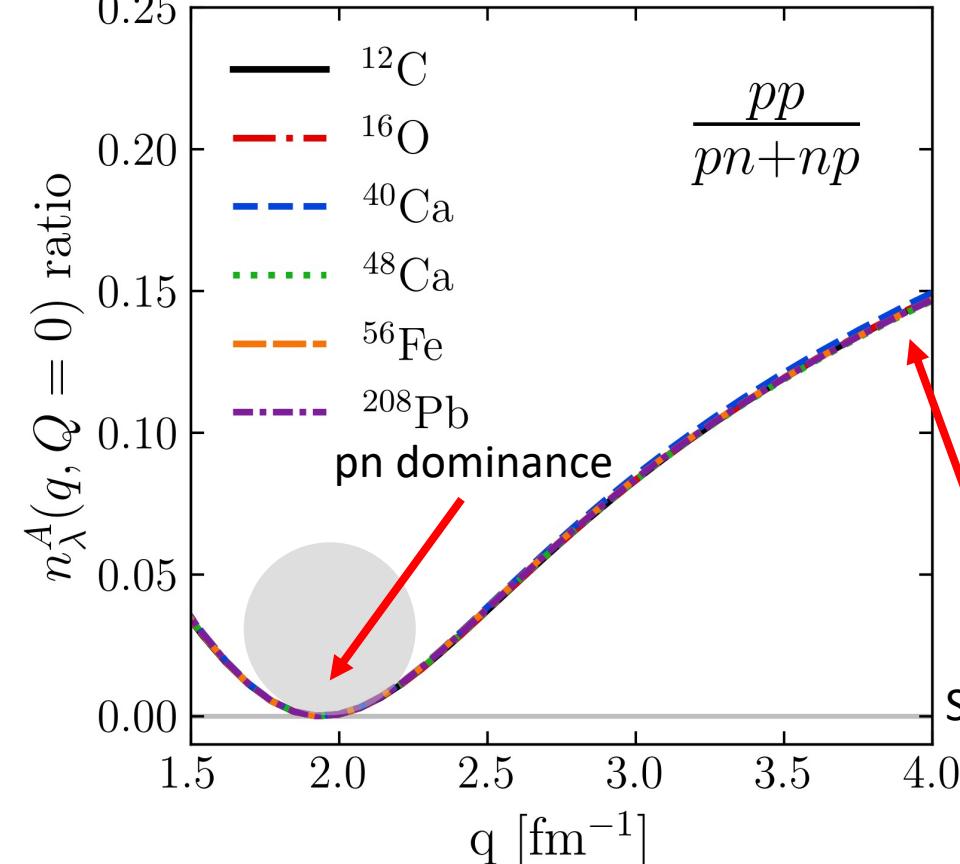
# SRC phenomenology



**Fig. 5:** (a) Ratio of two-nucleon to single-nucleon electron-scattering cross sections for carbon as a function of missing momentum. (b) Fraction of np to p and pp to p pairs versus the relative momentum. Figure from CLAS collaboration publication<sup>1</sup>.

- At **high RG resolution**, the tensor force and the repulsive core of the NN interaction kicks nucleon pairs into SRCs
- np dominates because the tensor force requires spin triplet pairs (pp are spin singlets)
- **Do we describe this physics at low RG resolution?**

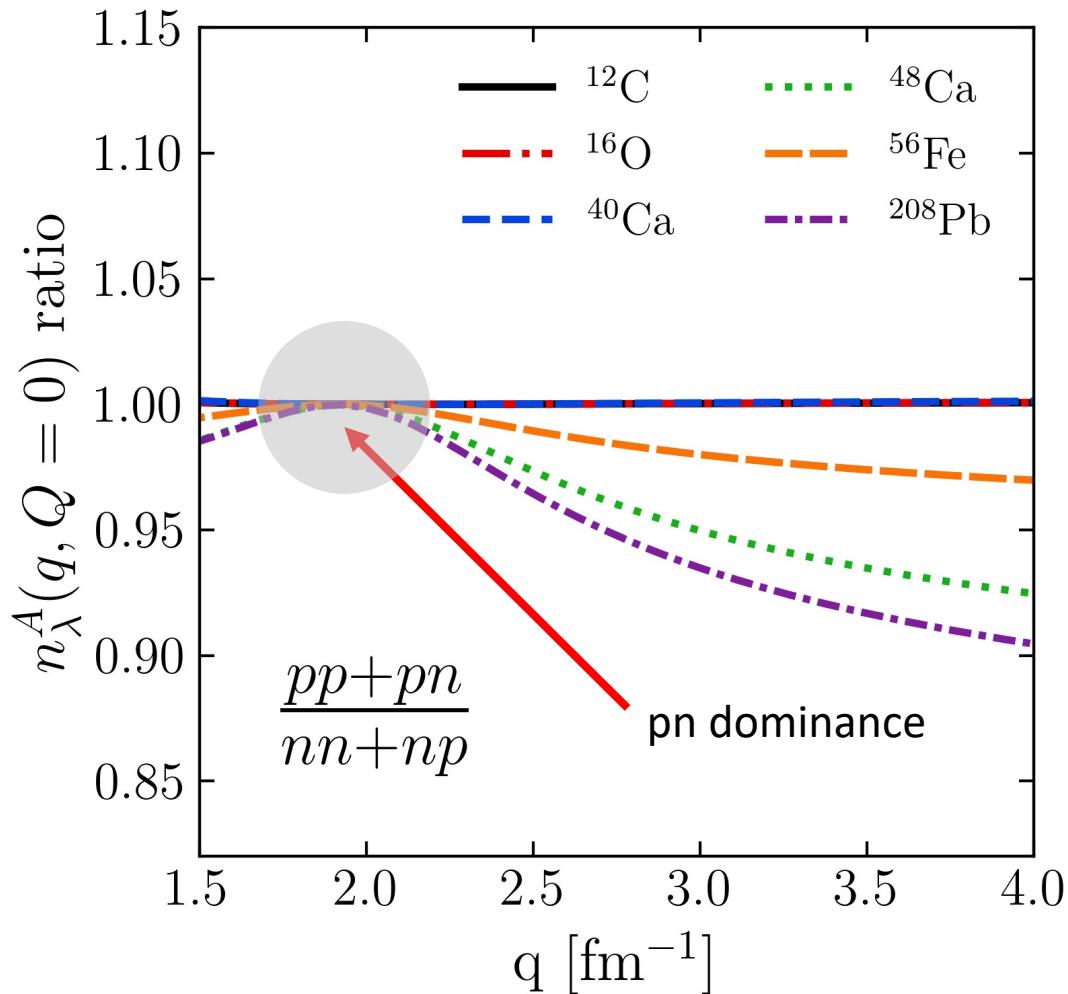
# SRC phenomenology



- At low RG resolution, SRCs are suppressed in the wave function and shifted into the operator
- Physics is established in the 2-body system – **can apply to any nucleus!**
- **Low RG resolution** picture reproduces the characteristics of cross section ratios using simple approximations

**Fig. 6:** pp/pn ratio of pair momentum distributions under LDA with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ .

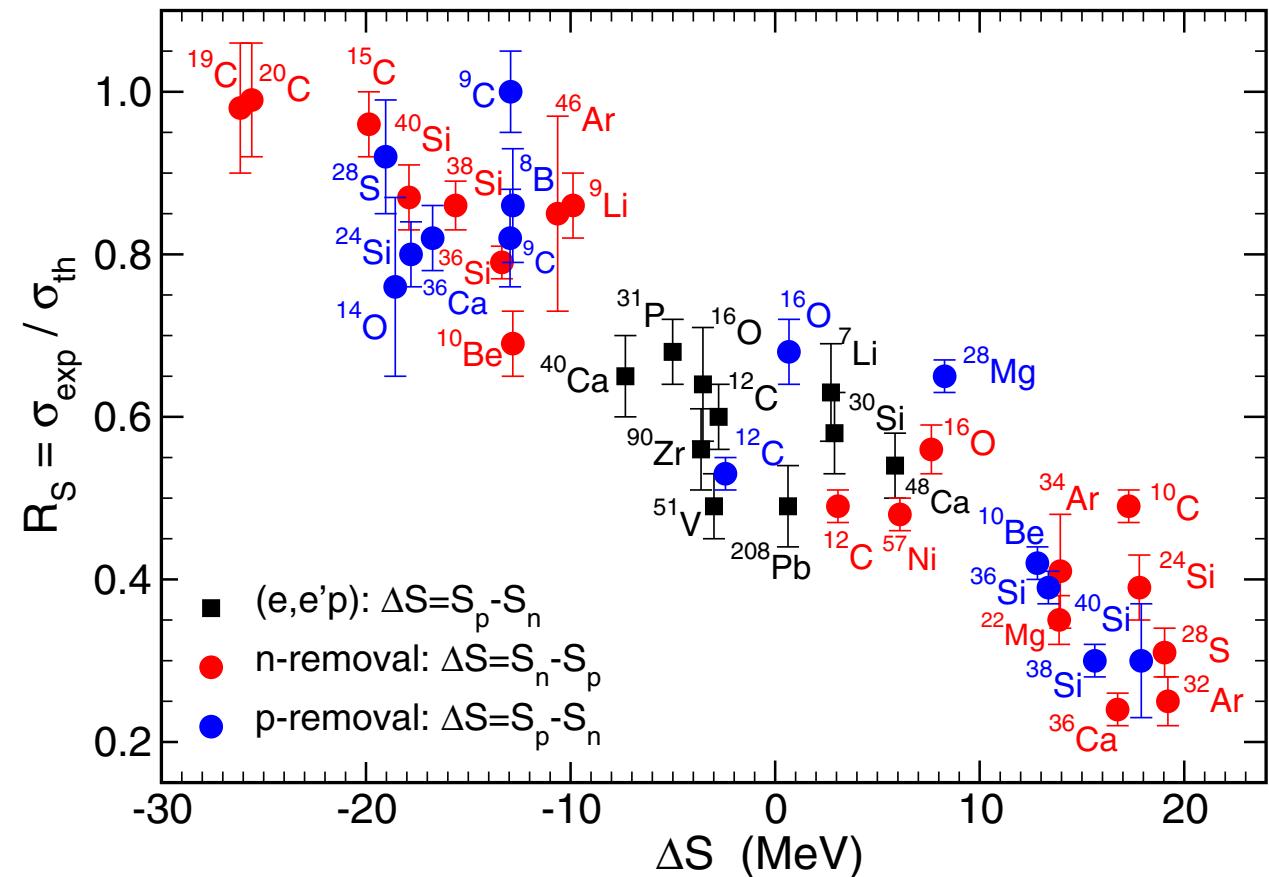
# SRC phenomenology



- Ratio  $\sim 1$  independent of N/Z in pn dominant region
- Ratio  $< 1$  for nuclei where  $N > Z$  and outside pn dominant region

**Fig. 7:**  $(pp+pn)/(nn+np)$  ratio of pair momentum distributions under LDA with AV18 and  $\lambda = 1.35$  fm $^{-1}$ . Anthony Tropiano, APS DNP 2021 Meeting

# Other exclusive knockout reactions



**Fig. 8:**  $R$  as a function of  $\Delta S$ . Red (blue) points correspond to neutron-removal (proton-removal) cases. Solid black squares correspond to electron-induced proton knockout data. Figure from J. A. Tostevin and A. Gade, Phys. Rev. C **90**, 057602 (2014).

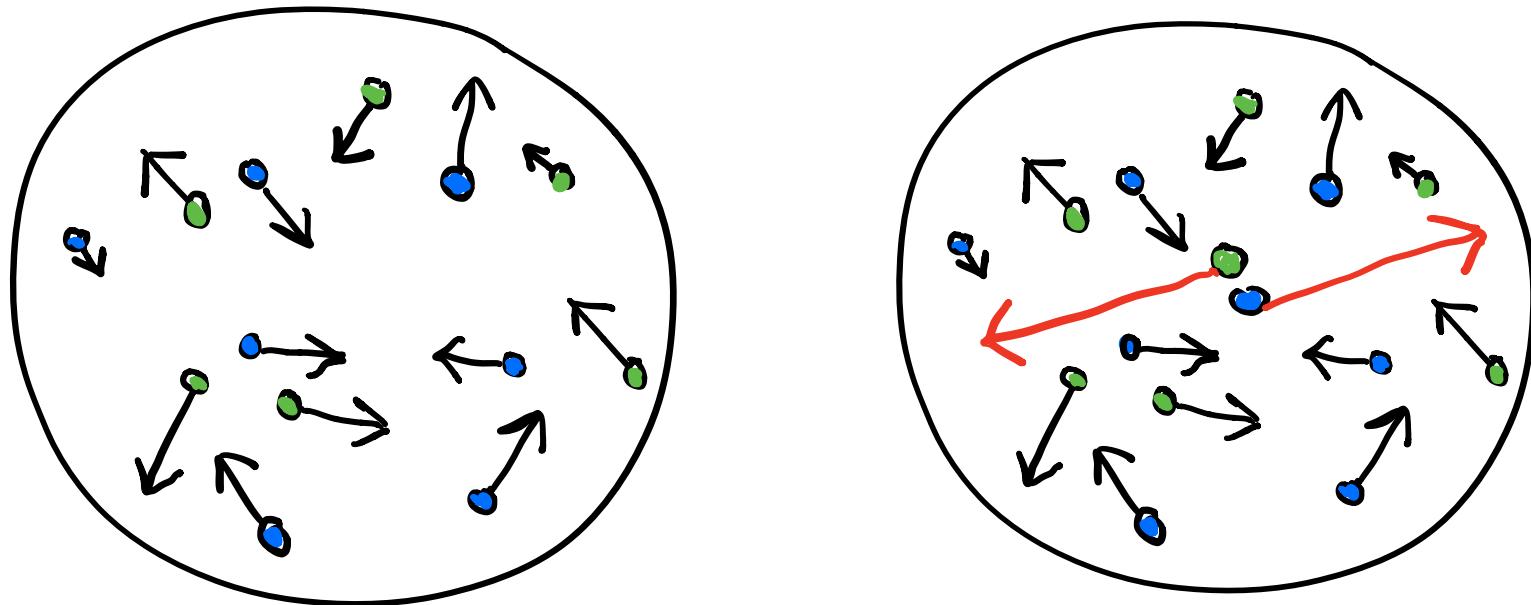
- Origin of  $R = \frac{\sigma_{\text{exp}}}{\sigma_{\text{theory}}} < 1$  is not understood
- Mismatch of scale between one-body (**high RG**) operator and shell model structure (**low RG**) gives  $\sigma_{\text{theory}} > \sigma_{\text{exp}}$
- In progress: apply SRG to reaction operator consistently

# Summary and outlook

- Simple approximations work and are systematically improvable at low RG resolution
- Results suggest that we can analyze high-energy nuclear reactions using low RG resolution structure (e.g., shell model) and consistently evolved operators
  - Matching resolution scale between structure and reactions is crucial!
- Ongoing work:
  - Extend to cross sections and test scale/scheme dependence of extracted properties
  - Investigate impact of various corrections: 3-body terms, final state interactions, etc.
  - Apply to more complicated knock-out reactions (SRG with optical potentials)
  - Implement uncertainty quantification in low RG resolution calculations

# Extras

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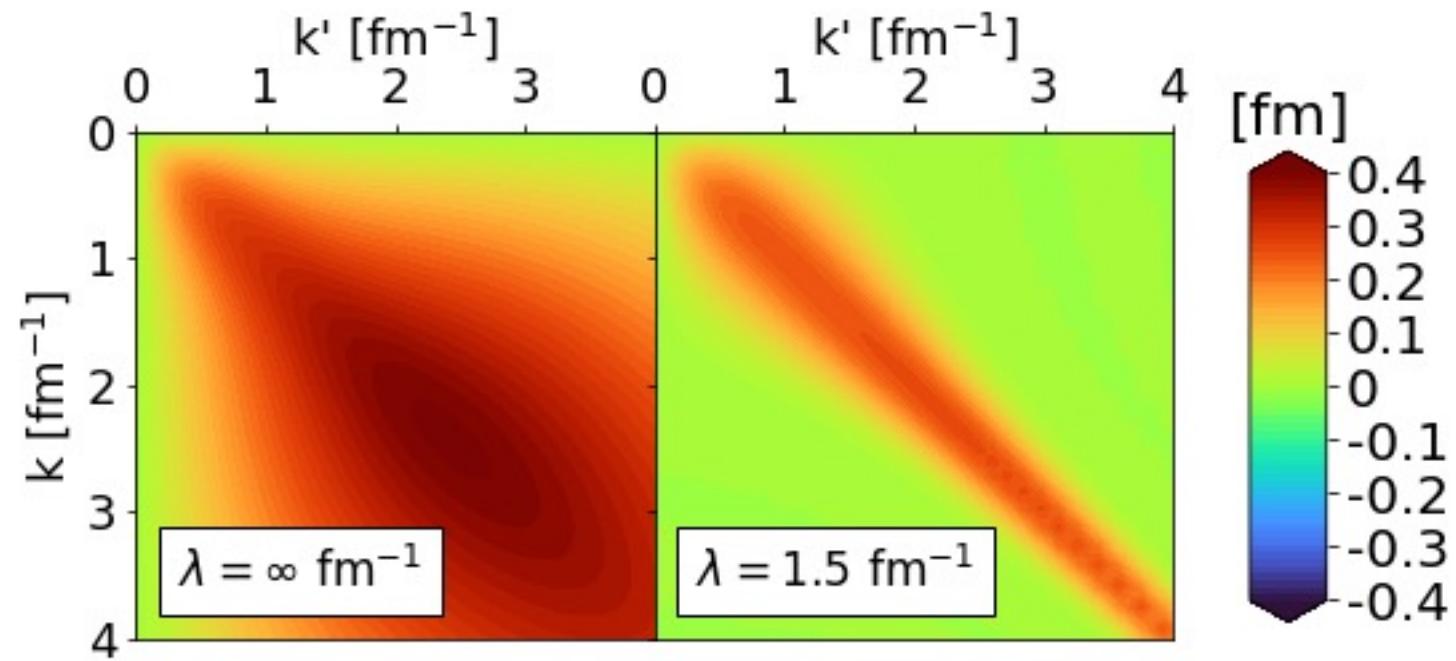
**Fig. 9:** Cartoon snapshots of a nucleus at (left) low-RG and (right) high-RG resolutions. The back-to-back nucleons at high-RG resolution are an SRC pair with small center-of-mass momentum.

# SRG decoupling

- Evolve operators to low RG resolution

$$O(s) = U(s)O(0)U^\dagger(s)$$

where  $s = 0 \rightarrow \infty$  and  
 $U(s)$  is unitary



**Fig. 10:** Momentum space matrix elements of Argonne v18 (AV18) under SRG evolution in  ${}^1\text{P}_1$  channel.

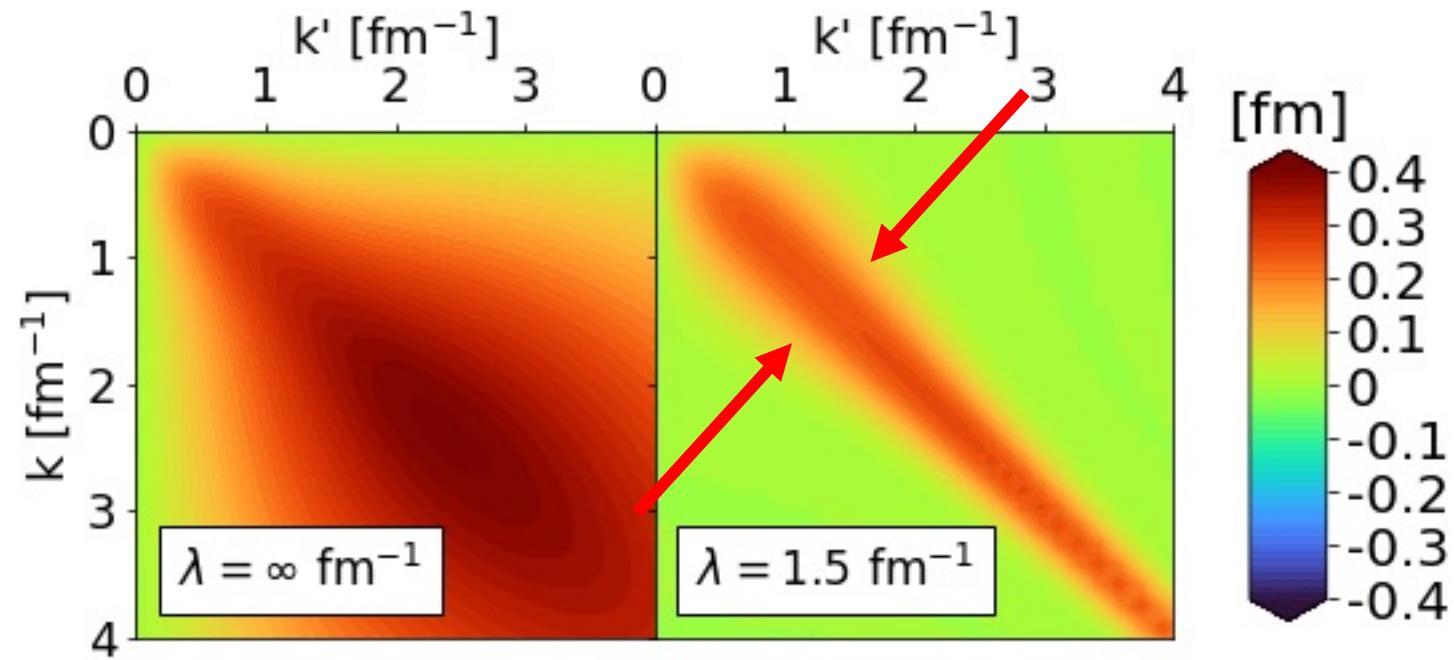
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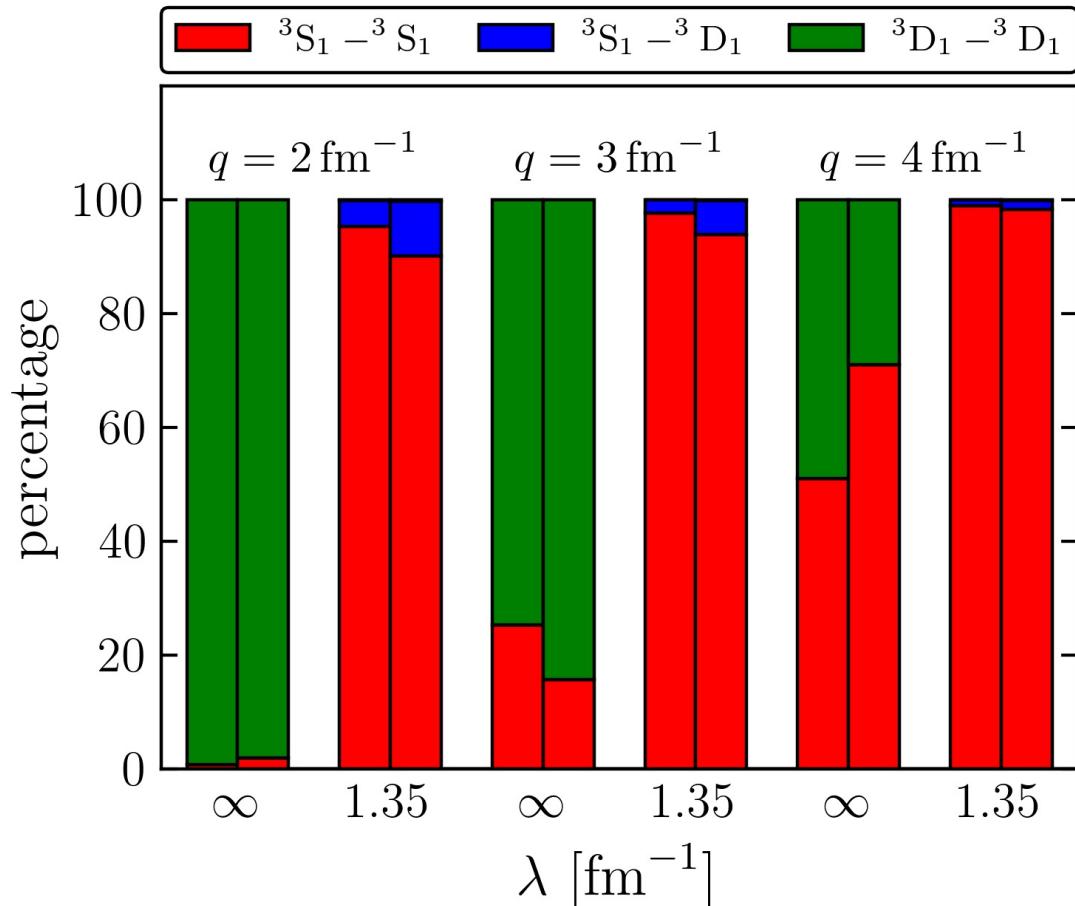
where  $s = 0 \rightarrow \infty$  and  $U(s)$  is unitary

- $\lambda = s^{-1/4}$  describes the decoupling scale of the RG evolved operator



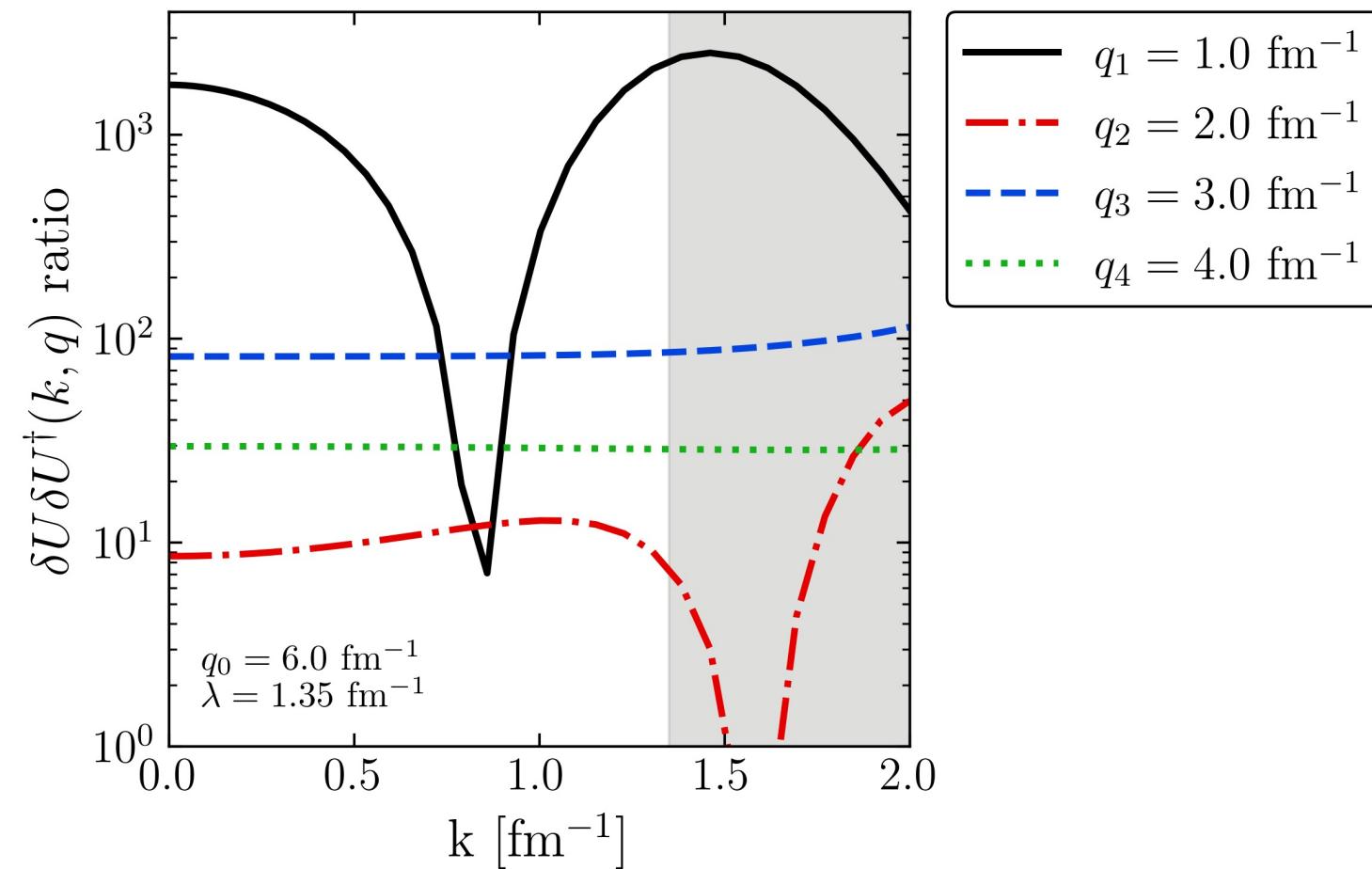
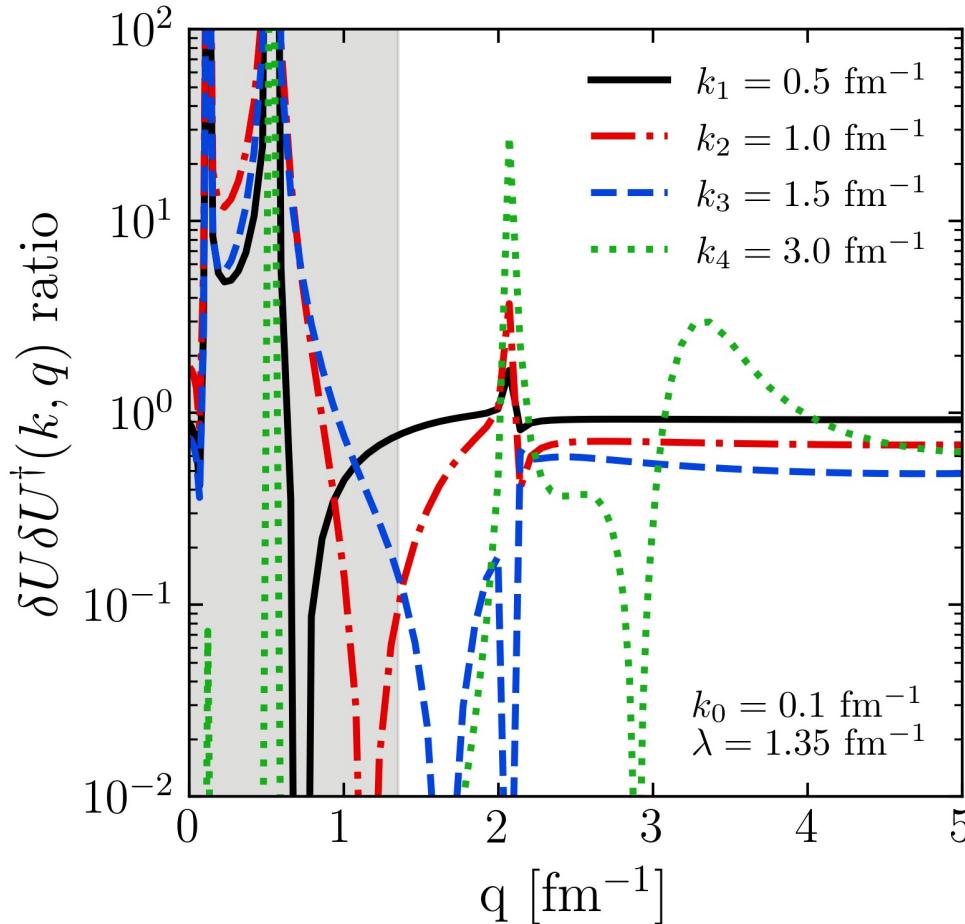
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# S-state and D-state contributions



**Fig. 11:** Percentage contributions from each channel to the matrix element of the momentum distribution in the deuteron at several relative momentum values  $q$  for AV18 (left bar at each  $\lambda$  value) and Gezerlis N<sup>2</sup>LO (right bar) potentials. We compare unevolved and SRG-evolved (both wave function and operator, so the net matrix element is unchanged) results where  $\lambda = 1.35 \text{ fm}^{-1}$

# Factorization



# SRG $\lambda$ dependence in momentum distributions

