Precision nucleon-nucleon potential at fifth order in the chiral expansion: Supplemental material

E. Epelbaum,[1] H. Krebs,[1] and U.-G. Meißner[2,3,4]

[1]*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[2]*Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*
[3]*Institut für Kernphysik, Institute for Advanced Simulation, and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[4]*JARA - High Performance Computing, Forschungszentrum Jülich, D-52425 Jülich, Germany*

(Dated: August 29, 2015)

PACS numbers: 13.75.Cs,21.30.-x

In addition to the uncertainty from the truncation of the chiral expansion of the nuclear force at N^{4}LO, the presented results are subject to additional sources of uncertainty, see the discussion in Ref. [1]. In particular, the impact of statistical uncertainties of the low-energy constants accompanying the contact interactions on NN phase shifts studied in Ref. [29] is still small at the N^{4}LO accuracy level. On the other hand, statistical (and systematic) uncertainties of the πN low-energy constants c_i, d_i and e_i used as input in our calculations as well as the uncertainties from using the Nijmegen PWA instead of experimental data and from adopting a particular fitting protocol might have significant impact on the results and need to be quantified in future studies.

Here we focus on yet another source of uncertainty associated with the implicit treatment of the three-pion (3π) exchange potential in our analysis. While the (nominally) leading 3π exchange potential at N^{3}LO is known to be weak compared to the two-pion exchange [30,31] and to have negligibly small effect on phase shifts, the subleading 3π exchange at N^{4}LO appears to be considerably stronger due to the large values of the πN low-energy constants c_i's [32]. The strongest subleading 3π exchange potentials are generated by the class-XIII diagrams in the notation of Ref. [32]. At the distance $r \sim M_\pi^{-1} \sim 1.4$ fm, the strongest subleading 3π exchange potential (class-XIII isovector central potential) is still about 20 times weaker than the strongest subleading 2π-exchange potential (isoscalar central one), in agreement with the power counting (suppression by Q^2) and its shorter range nature. Therefore, one may expect its effects on low-energy observables, at the level of accuracy of our calculations, to be well representable by contact interactions.

In order to validate such an implicit treatment of the 3π exchange potential, we have carried out a N^{4}LO fit for the intermediate value of the cutoff of $R = 1.0$ fm, in which we *explicitly* included the class-XIII 3π exchange potential $V_{3\pi}^{\rm XIII}$ from Ref. [32] using the same values of c_i as employed in our analysis and adopting exactly the same fitting protocol. We found no significant changes both in the quality of the description of the Nijmegen phase shifts and in the reproduction/predictions for observables. In particular, the results for the deuteron properties are summarized in Table I. Notice that the

TABLE I: Deuteron binding energy B_d (in MeV), asymptotic S state normalization A_S (in fm$^{-1/2}$) , asymptotic D/S state ratio η, radius r_d (in fm) and quadrupole moment Q (in fm^2) at N^{4}LO based on the implicit and explicit treatment of the 3π exchange for the cutoff $R = 1.0$ fm in comparison with empirical information. Also shown is the D-state probability P_D (in %). Notice that r_d and Q are calculated without taking into account meson-exchange current contributions and relativistic corrections. The star indicates an input quantity.

	implicit $V_{3\pi}$	explicit $V_{3\pi}^{\rm XIII}$	Empirical
B_d	2.2246*	2.2246*	2.224575(9)
A_S	0.8845	0.8844	0.8846(9)
η	0.0255	0.0255	0.0256(4)
r_d	1.974	1.974	1.97535(85)
Q	0.272	0.272	0.2859(3)
P_D	4.40	4.38	

difference for A_S is well within the theoretical uncertainty for this quantity which at N^{4}LO is expected to be ~ 0.0002 fm$^{-1/2}$, see [1] for more detail.

Next, we show in Table II the results for the total np cross section at the energies considered in our paper. The

TABLE II: Predictions for the np total cross section (in mb) at N^{4}LO in chiral EFT based on the implicit and explicit treatment of the 3π exchange for the cutoff $R = 1.0$ fm.

Energy	implicit $V_{3\pi}$	explicit $V_{3\pi}^{\rm XIII}$	NPWA
50 MeV	167.6 ± 0.2	167.6	167.6 . . . 168.0
96 MeV	78.2 ± 0.2	78.2	77.6 . . . 78.6
143 MeV	53.8 ± 0.7	53.6	53.2 . . . 54.1
200 MeV	41.9 ± 1.7	41.5	42.2 . . . 42.6

second column in this table gives the results shown by the filled red circles in Fig. 2 of our paper with the corresponding error bars for the cutoff choice of $R = 1.0$ fm.

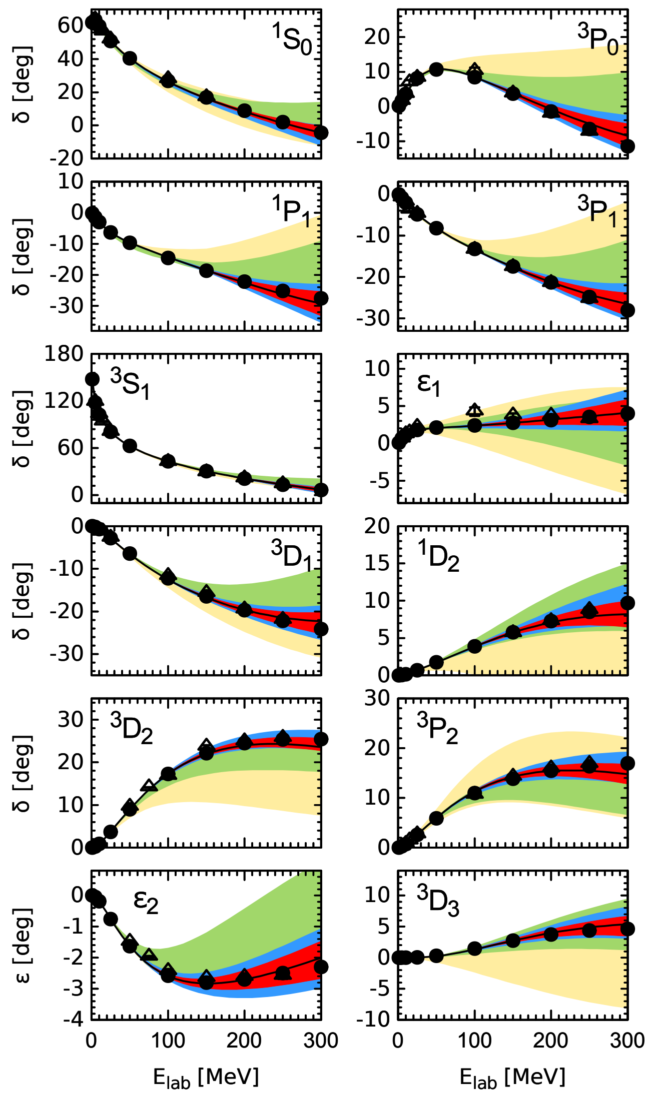

FIG. 1: Results for the np S-, P- and D-waves and the mixing angles ϵ_1, ϵ_2 up to N^{4}LO based on the cutoff of $R = 1.0$ fm. The notation is the same as in Fig. 3 of our paper. The red bands show estimated theoretical uncertainty at N^{4}LO without the explicit inclusion of the 3π exchange calculated using Eq. (2) in our paper. Black solid lines depict the results of the fit at N^{4}LO where the subleading class-XIII 3π exchange potential is included explicitly.

Similarly to the deuteron properties, the effects of the explicit inclusion of the 3π exchange in the total cross section are well within the theoretical accuracy estimated using Eq. (2) of our paper and based on the implicit treatment of the 3π exchange. The same conclusion holds true for phase shifts as visualized in Fig. 1. Finally, Fig. 2 shows how the results for the observables at the highest considered energy of $E_{\mathrm{lab}} = 200$ MeV, where effects due to the explicit treatment of the 3π exchange are expected to be most pronounced, change if the class-XIII 3π exchange potential is treated explicitly. We do not show experimental data to avoid a too cluttered plot. Again, we observe that the changes in our results due to the explicit treatment of the 3π exchange appear to be well within the estimated theoretical accuracy.

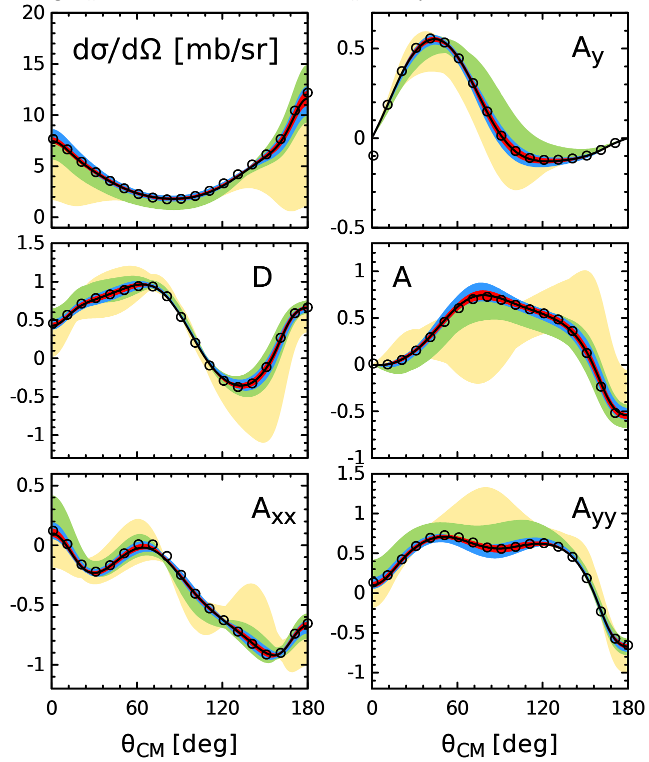

FIG. 2: Predictions for selected np scattering observables at $E_{\mathrm{lab}} = 200$ MeV calculated up to N^{4}LO based on the cutoff of $R = 1.0$ fm. The notation is the same as in Fig. 4 of our paper. The red bands show estimated theoretical uncertainty at N^{4}LO without the explicit inclusion of the 3PE calculated using Eq. (2) in the paper. Black solid lines depict the results of the fit at N^{4}LO where the subleading class-XIII 3PE is included explicitly.

In summary, we have verified that the explicit treatment of the 3π exchange does not appreciably affect any of our results for the intermediate cutoff value of $R = 1.0$ fm.