Analyzing scale and scheme dependence in NN operators with the SRG

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Motivation

- Explosion of new NN interactions from chiral effective field theory (χ^{EFT}) in the last few years
  - Various schemes! (e.g., different regulators)
- Previous SRG studies of operators were limited to phenomenological models or one χ^{EFT} interaction

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  - Revisit this with new chiral interactions
- Goal: use SRG to analyze high-energy reactions at low resolution by consistently evolving wave function and corresponding operators

SRG formalism

- SRG transformations decouple low- and high-momenta in Hamiltonian

$$H(s) = U(s)H(0)U^{\dagger}(s)$$

where $s = 0 \rightarrow \infty$ and $U(s)$ is unitary

- In practice, solve differential flow equation

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

with SRG generator $\eta(s) \equiv \frac{dU(s)}{ds} U^{\dagger}(s) = [G, H(s)]$

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- G gives the scheme and s gives the scale

SRG formalism

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling scheme

RKE N[4]LO $\Lambda = 450$ MeV

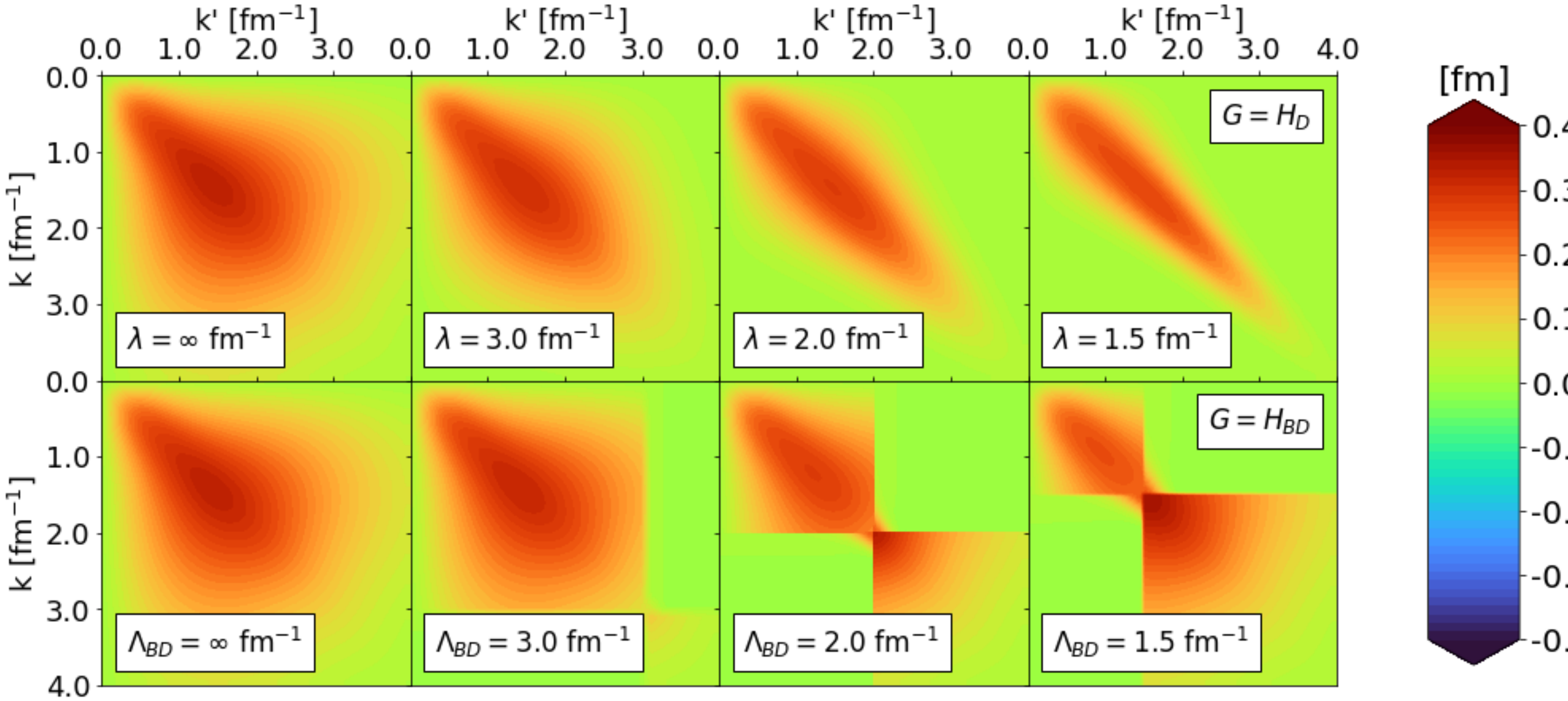
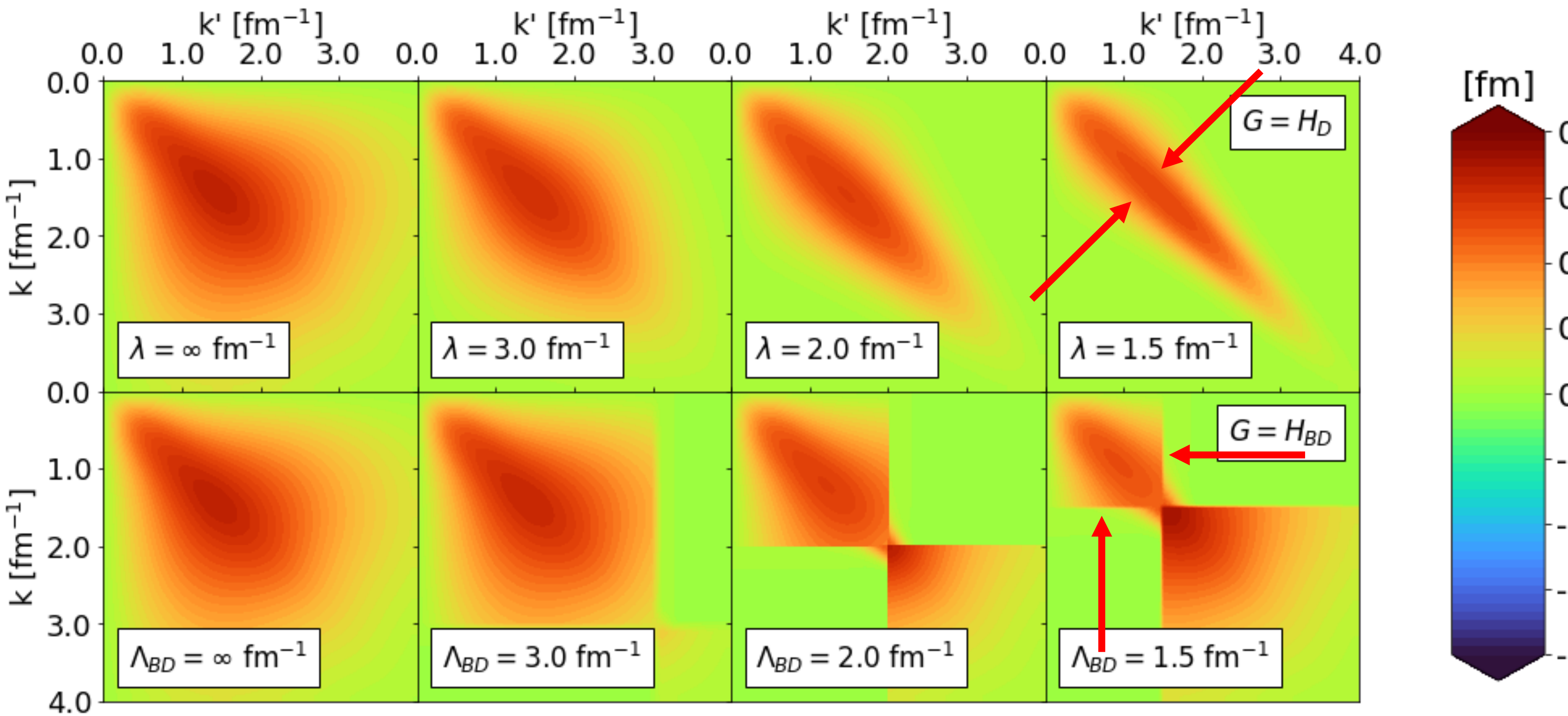


Fig. 1: SRG evolution of $V_\lambda(k, k')$ for several values of λ and Λ in the $^1P_1$ channel. Potentials from P. Reinert et al., Eur. Phys. J. A **54**, 86 (2018) which will be referred to as the RKE potentials.

SRG formalism

- $G = H_D(s)$ for band-diagonal decoupling and $G = H_{BD}(s)$ for block-diagonal decoupling scheme
- Parameters $\lambda = s^{-1/4}$ and Λ describe the decoupling scale of the evolved Hamiltonian

RKE N[4]LO $\Lambda = 450$ MeV



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SRG evolution of modern chiral potentials

- Variety of NN interactions with different schemes: non-local EMN[1] (500 MeV), semi-local RKE[2] (450 MeV), and local Gezerlis et al.[3] (1 fm) potentials as examples

[1]D.R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C **96**, 024004 (2017)
[2]P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A **54**, 86 (2018)
[3]A. Gezerlis, et al., Phys. Rev. C **90**, 054323 (2014)

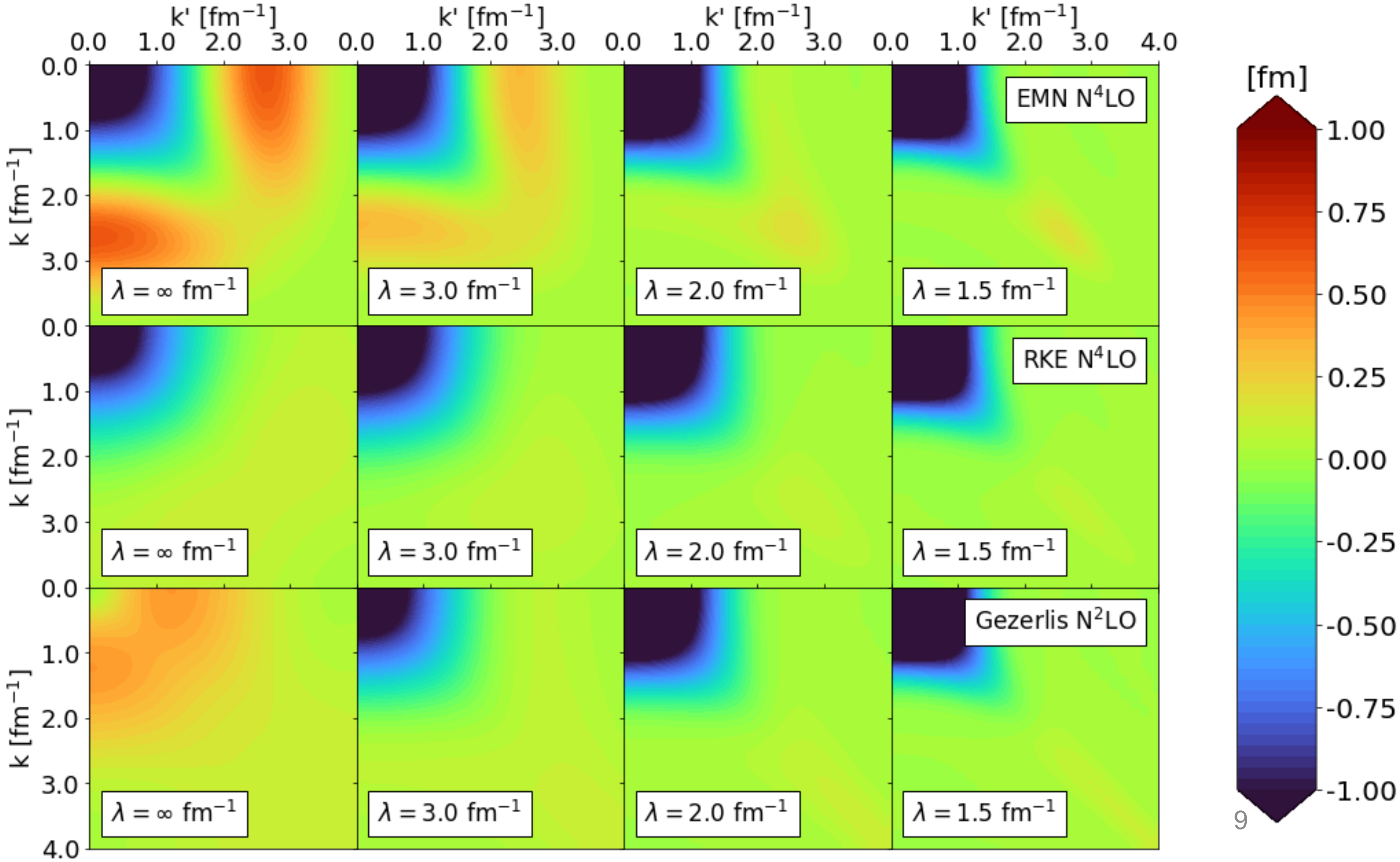
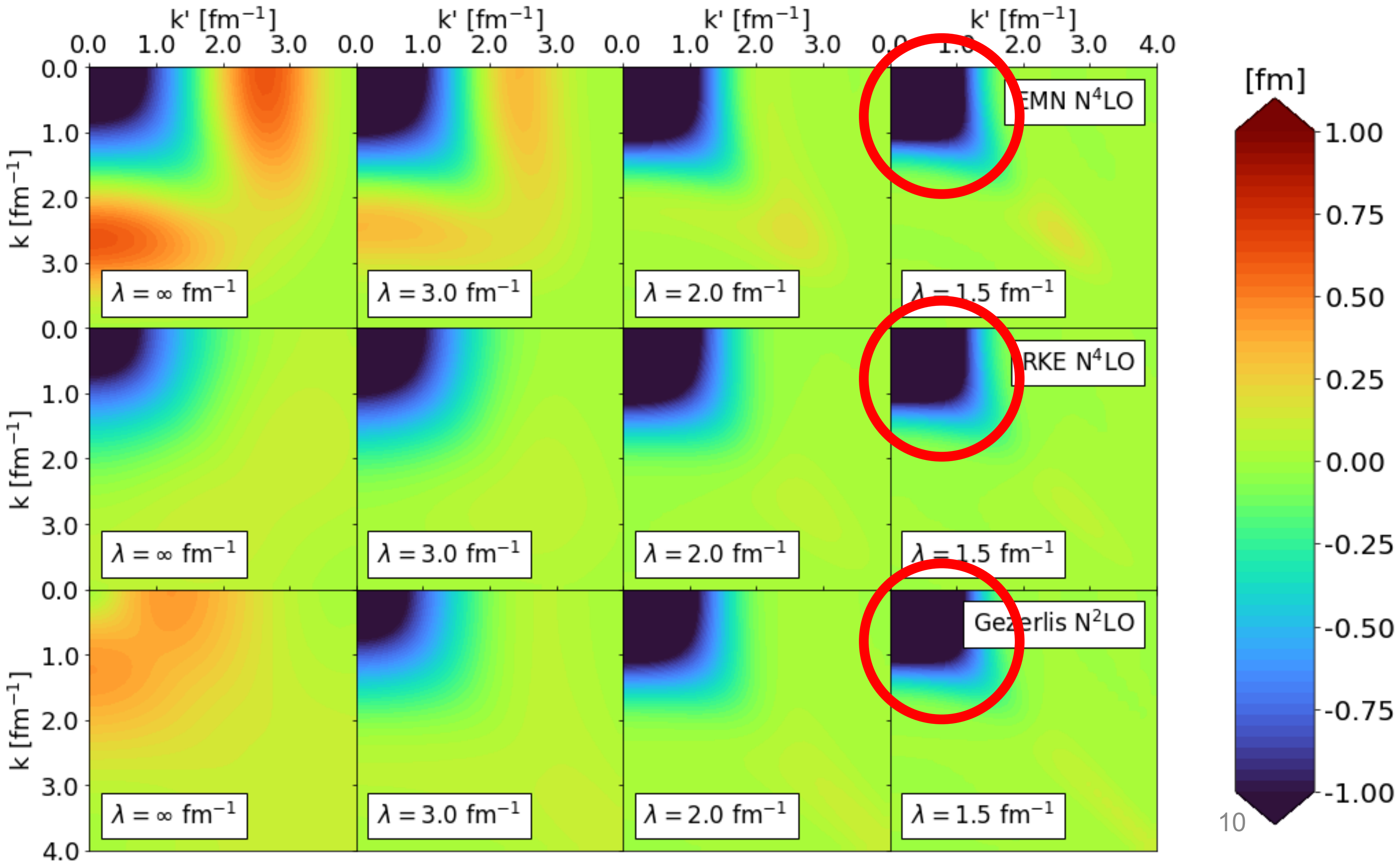


Fig. 2: SRG evolution of $V_\lambda(k,k')$ for several chiral potentials in the $^3S_1$ channel.

SRG evolution of modern chiral potentials

- Change the scale to lower resolution
- Different potentials are approximately the same at low resolution!



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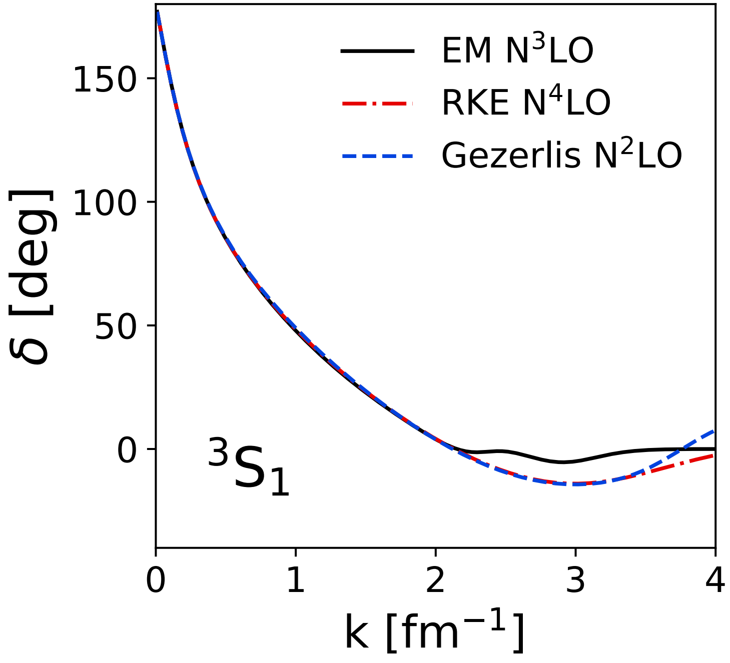
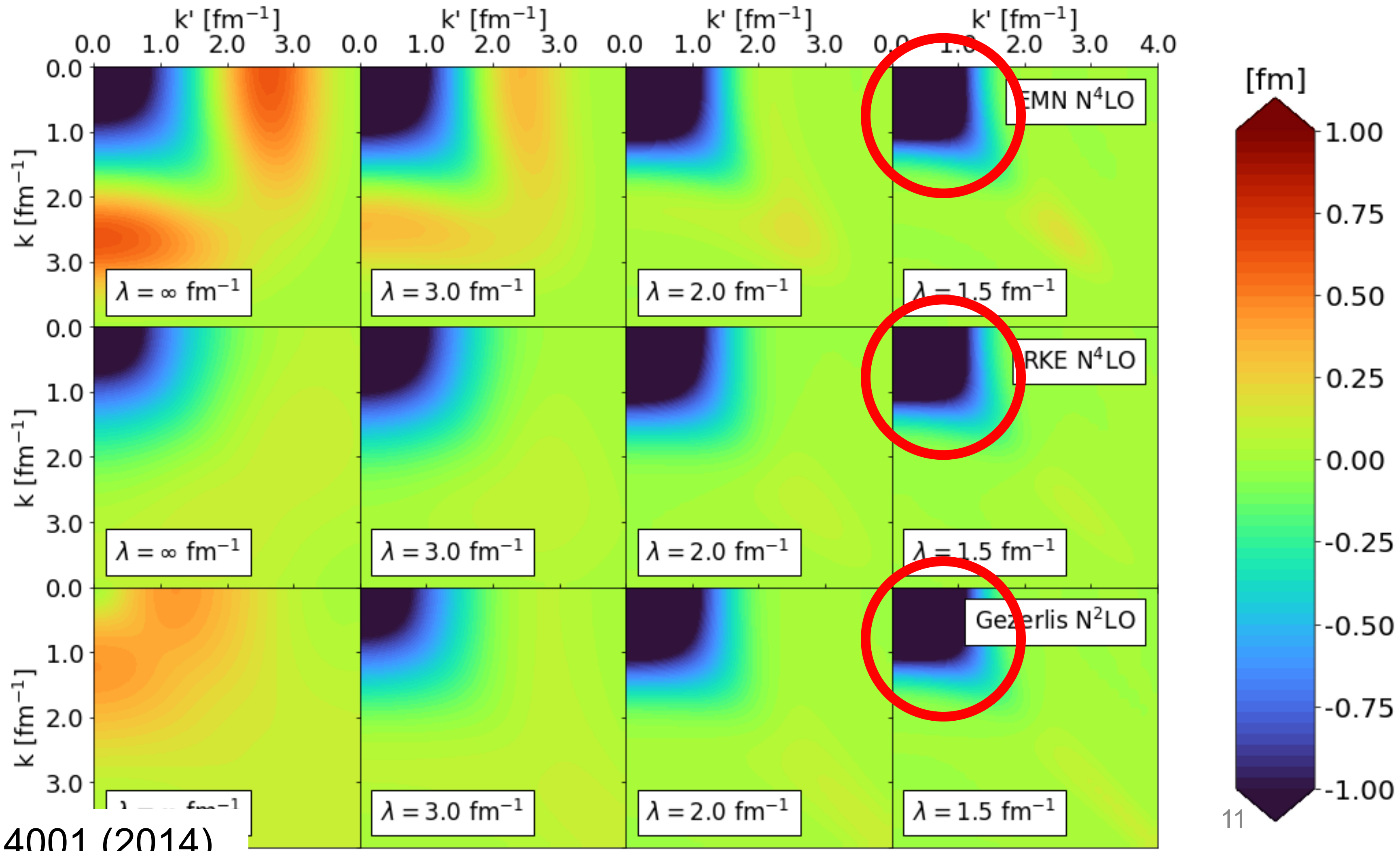
Universality: NN potentials

- Equivalent low-energy phase shifts ⟹ equivalent low-momentum matrix elements $V_\lambda(k,k')$[1]





[1]B. Dainton et al., Phys. Rev. C **89**, 014001 (2014)

Universality: Wave functions

- What happens to the wave functions from different NN interactions?
- Look at deuteron wave function in coordinate space as example

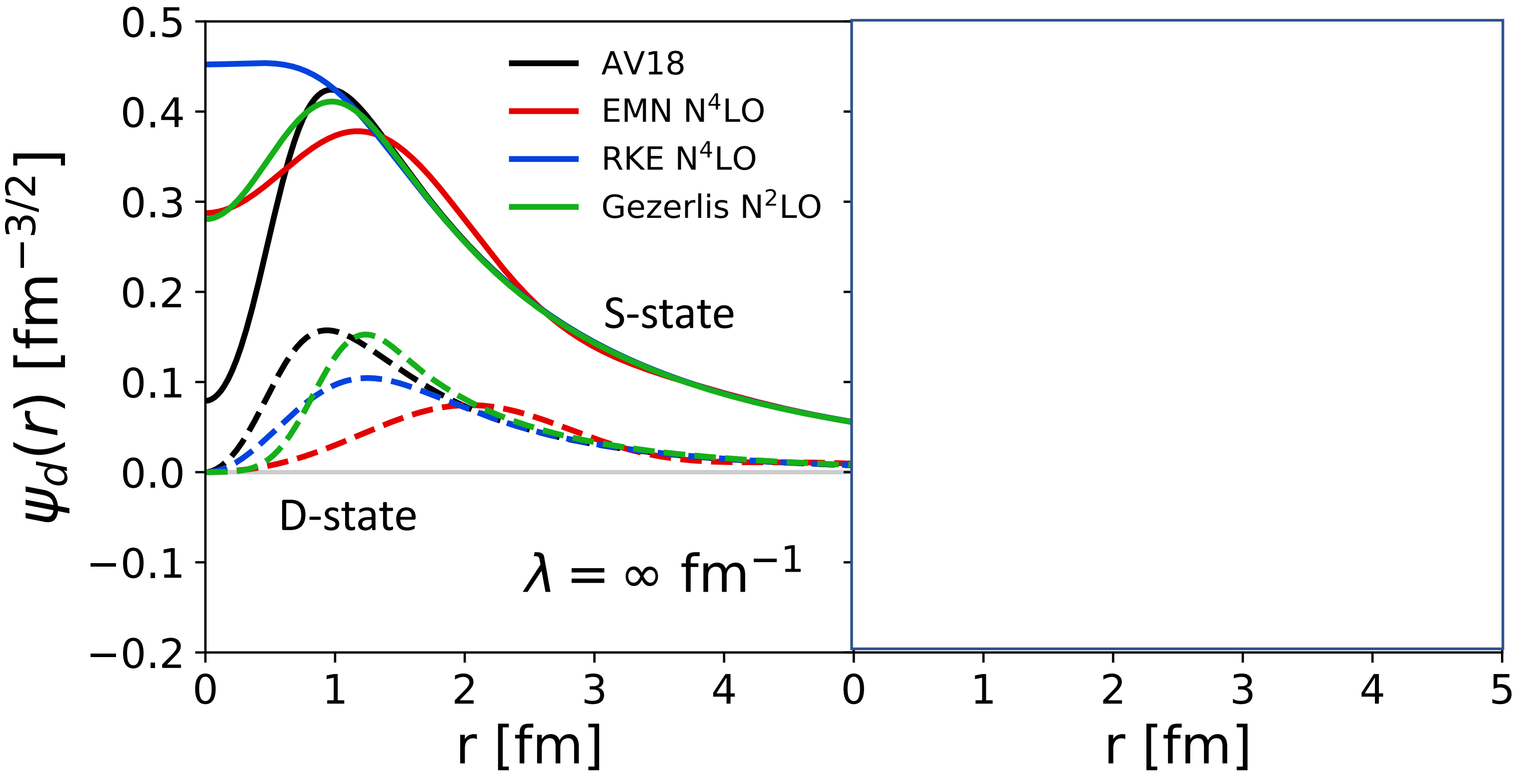
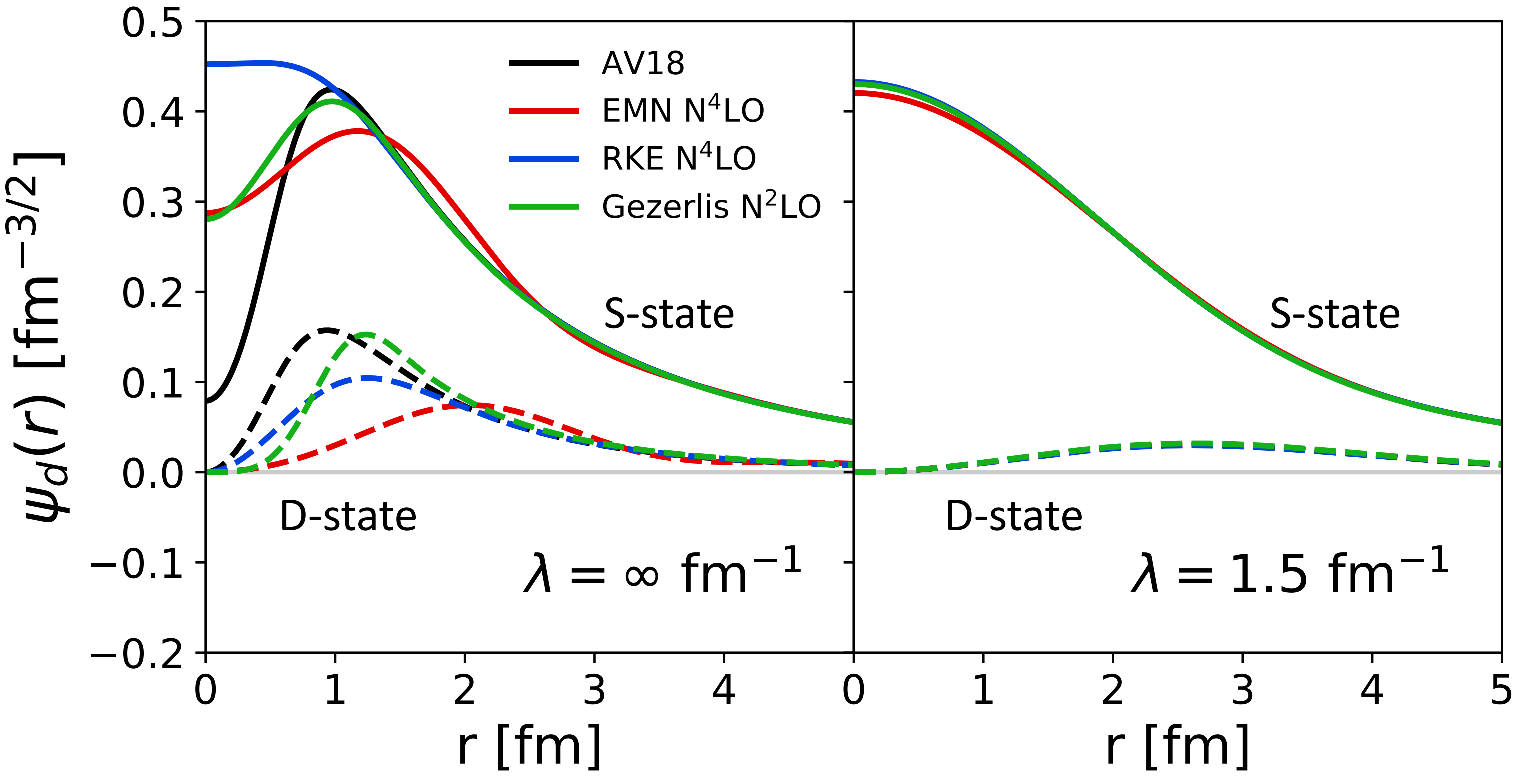


Fig. 3: SRG evolution of deuteron wave function in coordinate space for several interactions.

Universality: Wave functions

- Natural consequence: the low-energy states between drastically different potentials also exhibit universality



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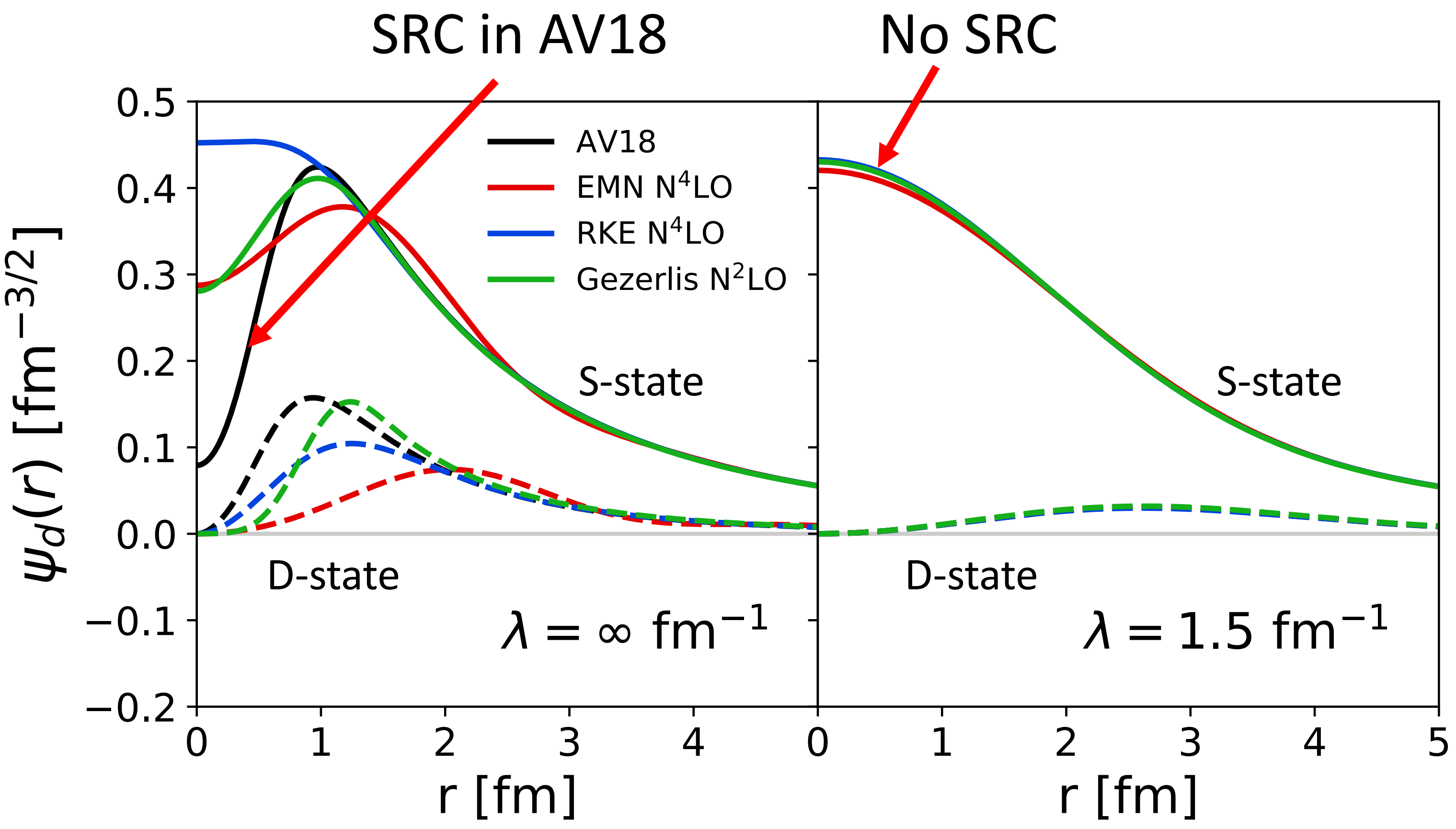
- SRC physics in AV18 (scheme dependent) is gone from wave function at low resolution



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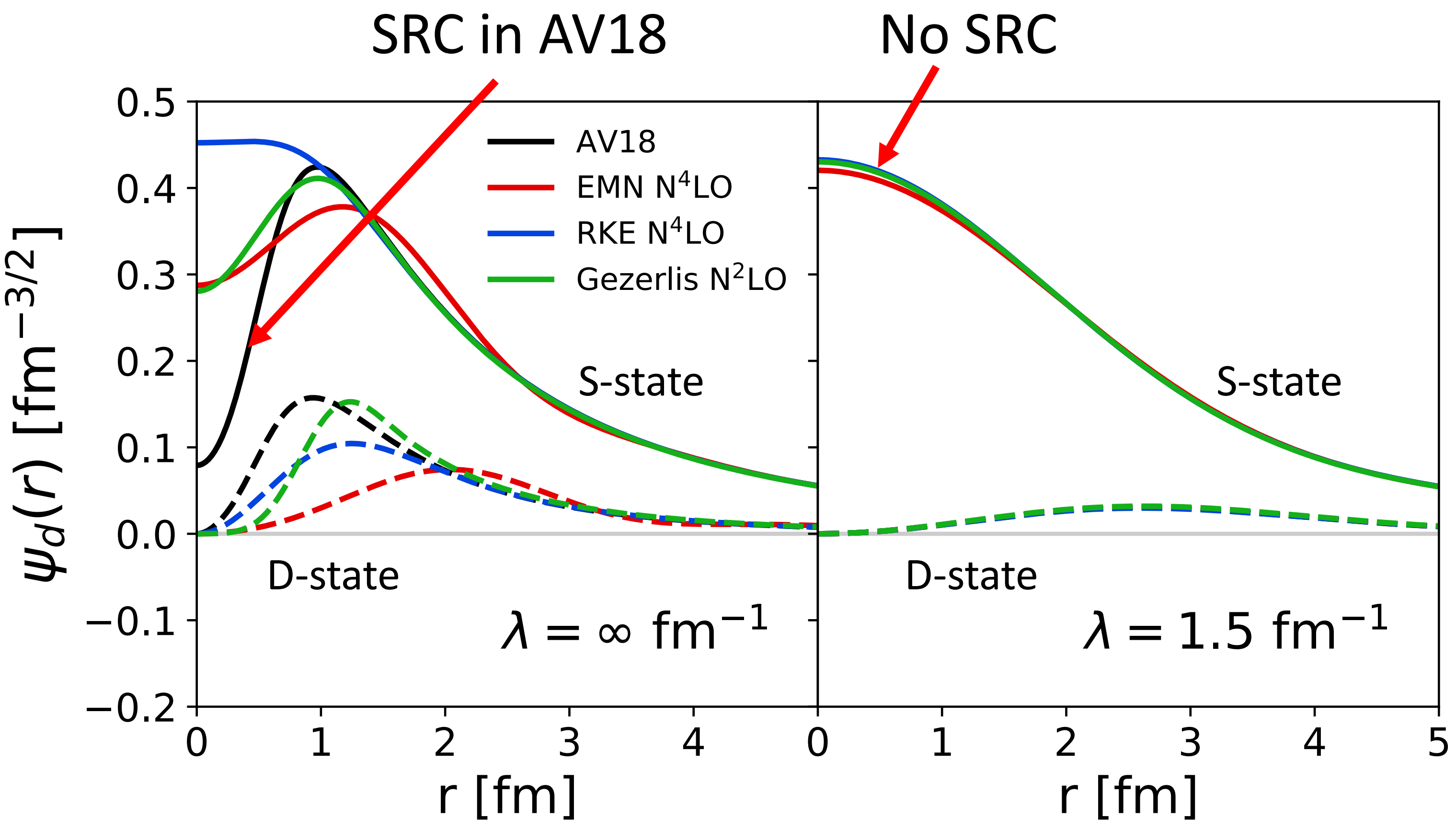
Universality: Wave functions

- Natural consequence: the low-energy states between drastically different potentials also exhibit universality
- SRC physics in AV18 (scheme dependent) is gone from wave function at low resolution
- All deuteron wave functions become soft and D-state probability goes down
- Observables such as asymptotic D-S ratio the same



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Connection to experiments

- In analyzing scattering observables, there is scale and scheme dependence in factorization of structure and reaction
- General problem for any matrix element $\langle \psi_f | O | \psi_i \rangle$

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Connection to experiments

- In analyzing scattering observables, there is scale and scheme dependence in factorization of structure and reaction
- General problem for any matrix element $\langle\psi_f|O|\psi_i\rangle$
- Tune the scale (e.g. λ) with SRG transformations making a potential with SRC physics like AV18 much softer like a high-order chiral potential
- Can use low-resolution wave function to calculate high-energy reactions by consistently evolving the operator

$$\langle\psi_f(0)|O(0)|\psi_i(0)\rangle = \langle\psi_f(s)|O(s)|\psi_i(s)\rangle$$

- Mismatch of scales leads to incorrect observable (e.g., theory knock-out cross section compared to experiment)

Where does the short-distance physics go?

- Use simple operator $a_q^\dagger a_q$ where q is the relative momentum
  $a_q^\dagger a_q \sim \delta(k-q)\delta(k'-q)$

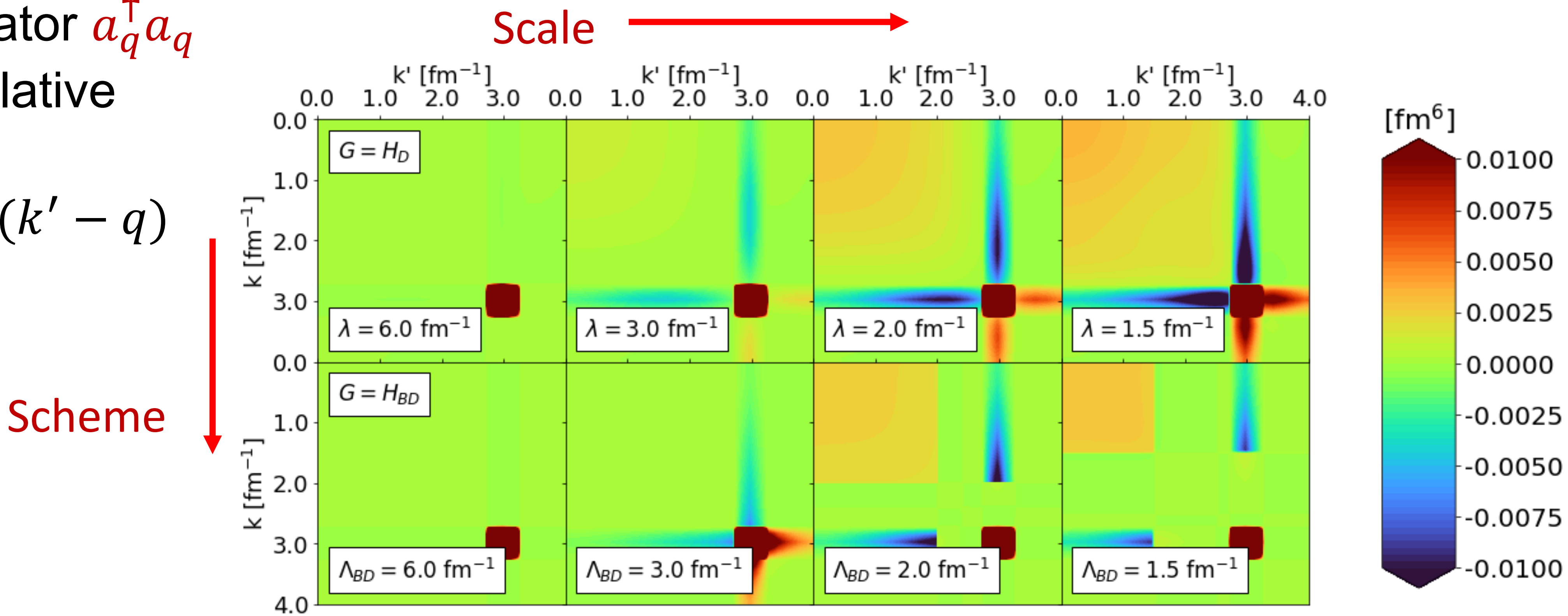
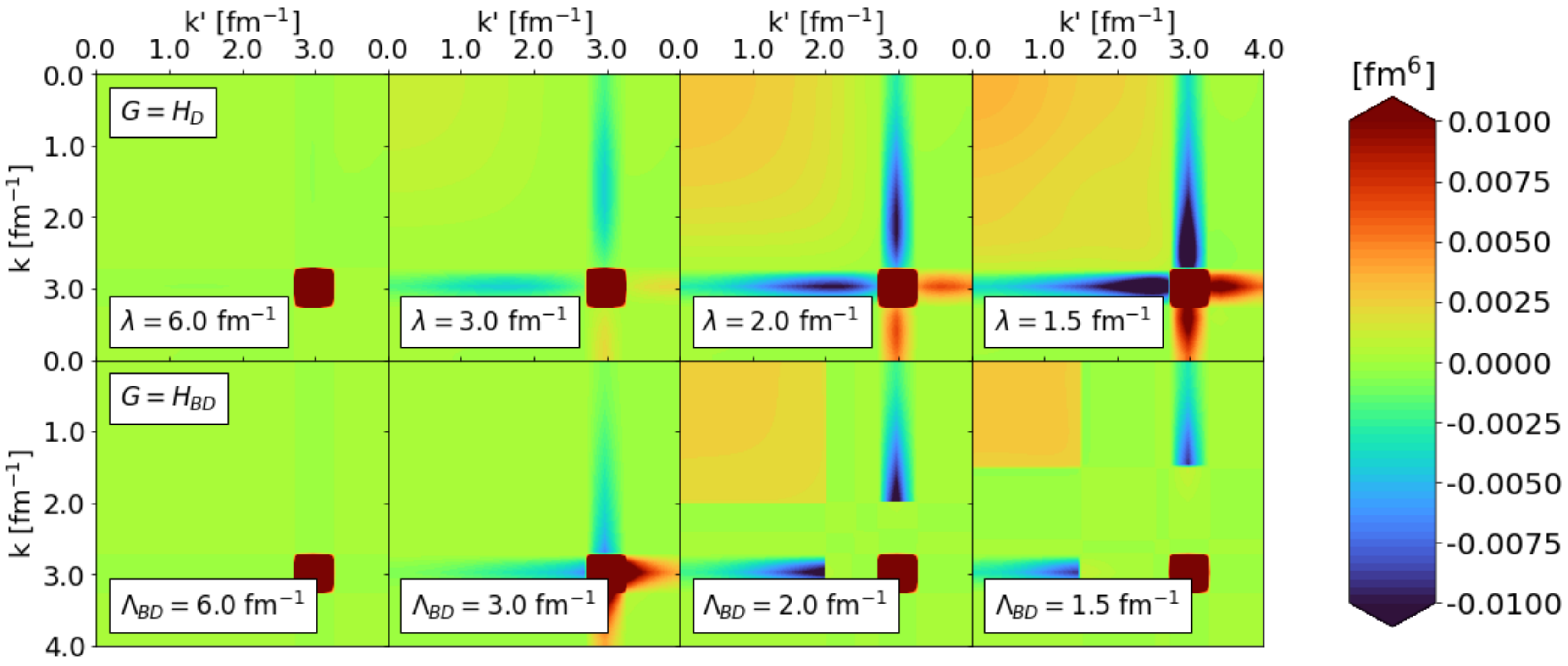


Fig. 4: SRG evolution of $a_q^\dagger a_q$ for $q = 3$ fm^{-1}.
Transformations done with RKE N^{4}LO 450 MeV.

Where does the short-distance physics go?

- Use simple operator $a_q^\dagger a_q$ where q is the relative momentum
  $a_q^\dagger a_q \sim \delta(k-q)\delta(k'-q)$
- Smooth induced contributions at low momentum reproduce UV physics of the original NN potential



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Scheme dependence in evolved $a_q^\dagger a_q$

- SRG induced terms in $a_q^\dagger a_q$ reflects difference in UV physics (scheme dependence from NN interaction)

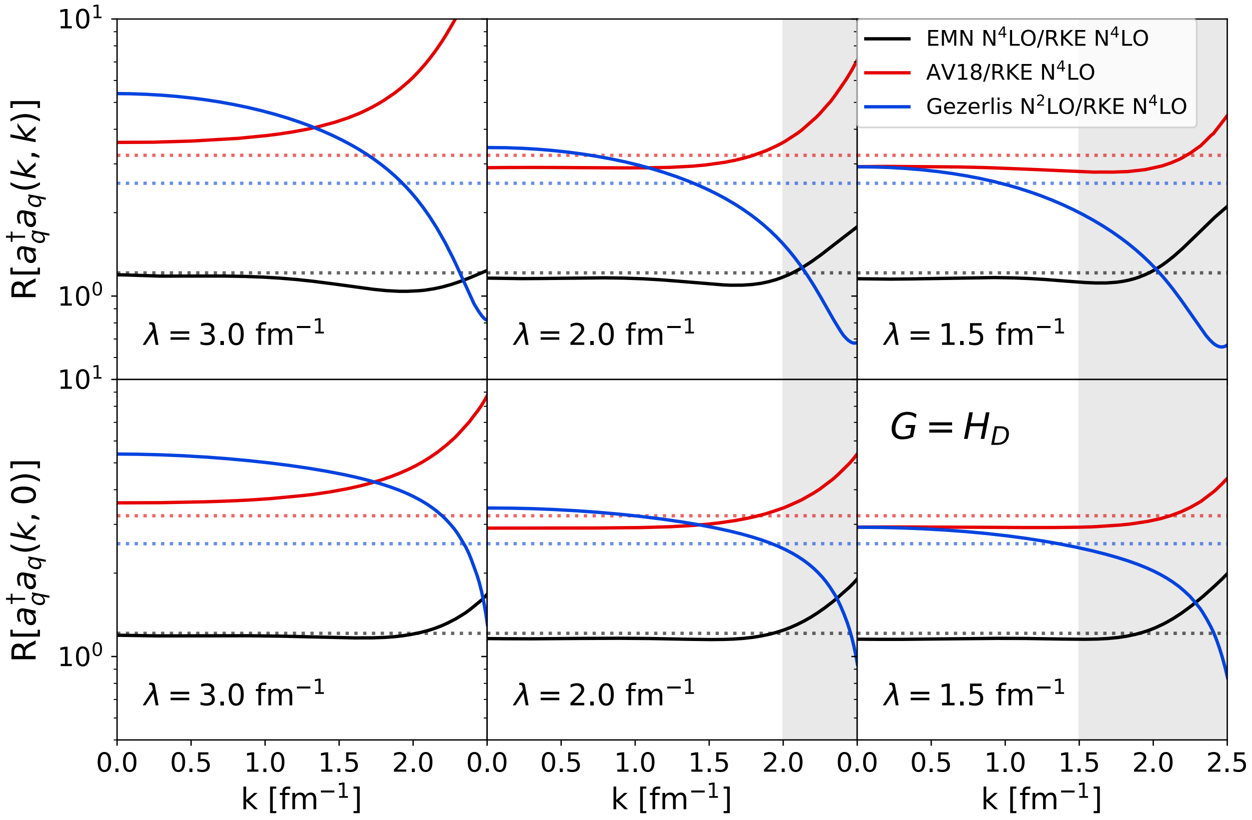


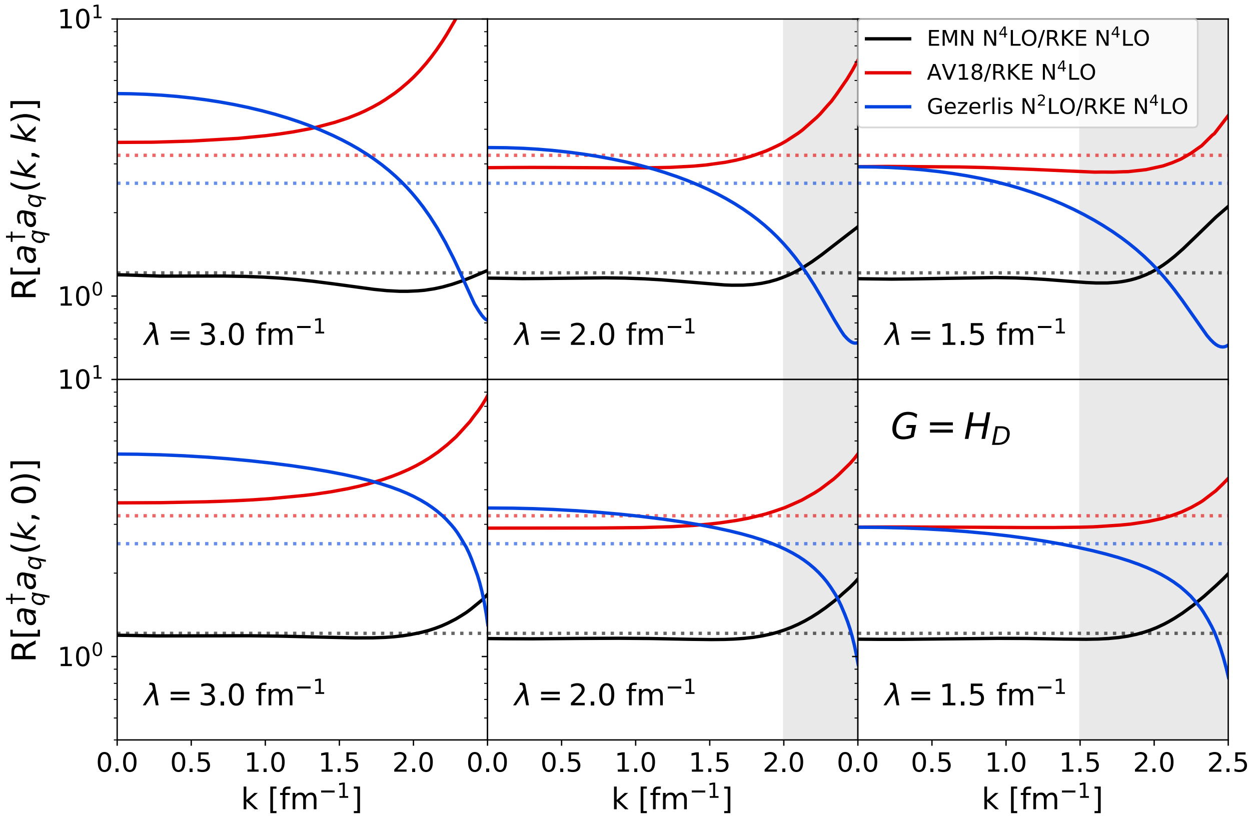
Fig. 5: Ratios of $a_q^\dagger a_q(k,k')$ isolating the diagonal and far off-diagonal matrix elements. Dotted lines indicate the ratio of wave functions $|\psi(q)|^2$.

Scheme dependence in evolved $a_q^\dagger a_q$

- SRG induced terms in $a_q^\dagger a_q$ reflects difference in UV physics (scheme dependence from NN interaction)
- At low-k ratio of $a_q^\dagger a_q$ approximately match the ratio of wave functions at high-momentum q:

$$|\psi\ (q)|^2/|\psi'\ (q)|^2$$

- Flatness at low-k indicates factorization of low- and high-resolution physics



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Where does the short-distance physics go?

Consistently evolve the wave functions!

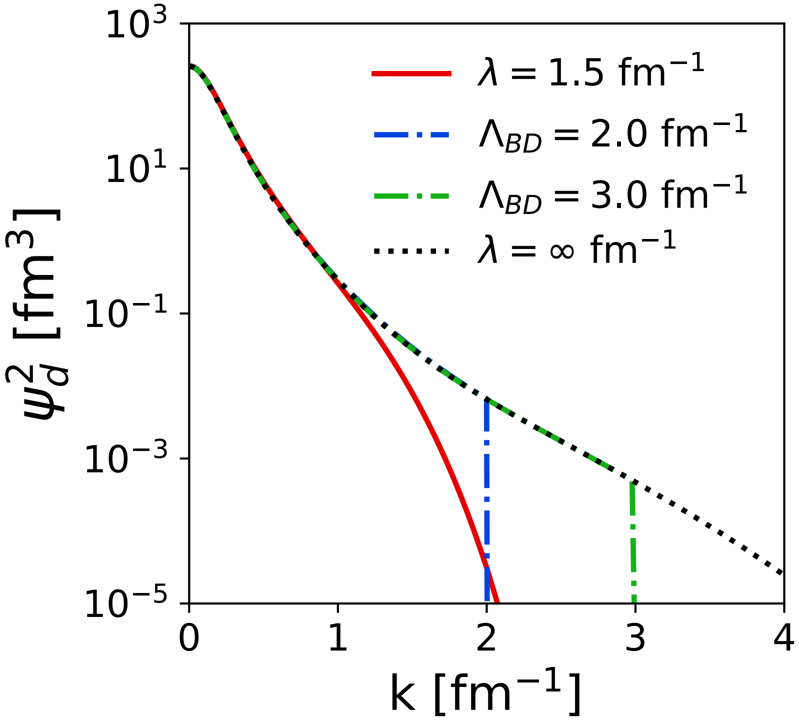
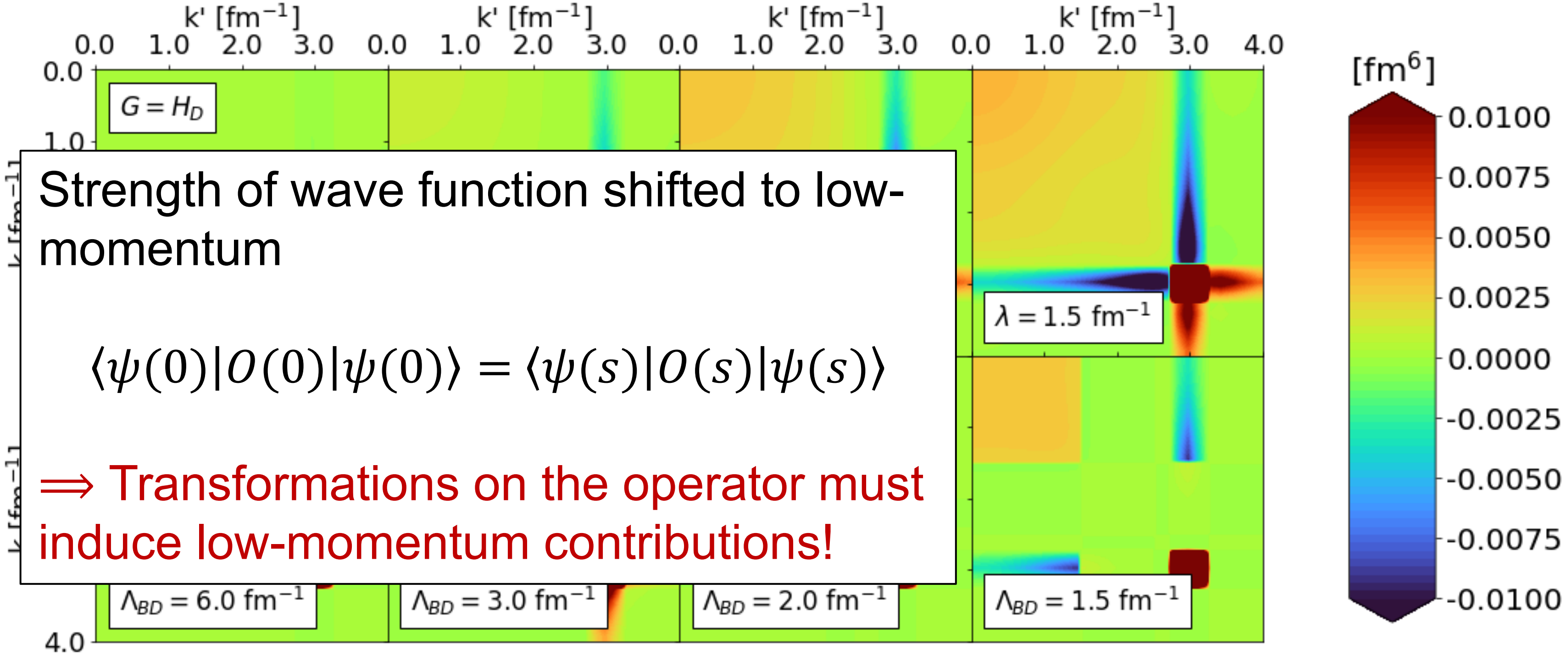


Fig. 6: SRG evolution of $\psi_d^2(k)$.



Strength of wave function shifted to low-momentum

$$\langle \psi(0)|O(0)|\psi(0)\rangle = \langle \psi(s)|O(s)|\psi(s)\rangle$$

$\Rightarrow$ Transformations on the operator must induce low-momentum contributions!

Fig. 4: SRG evolution of $a_q^\dagger a_q$ for $q = 3$ fm^{-1}.
Transformations done with RKE N^{4}LO 450 MeV.

High-momentum operator at low resolution

- Expectation value $\langle \psi_d | a_q^\dagger a_q | \psi_d \rangle$ is driven to low-momentum
- Note, each panel gives the correct result from unitarity of transformation!

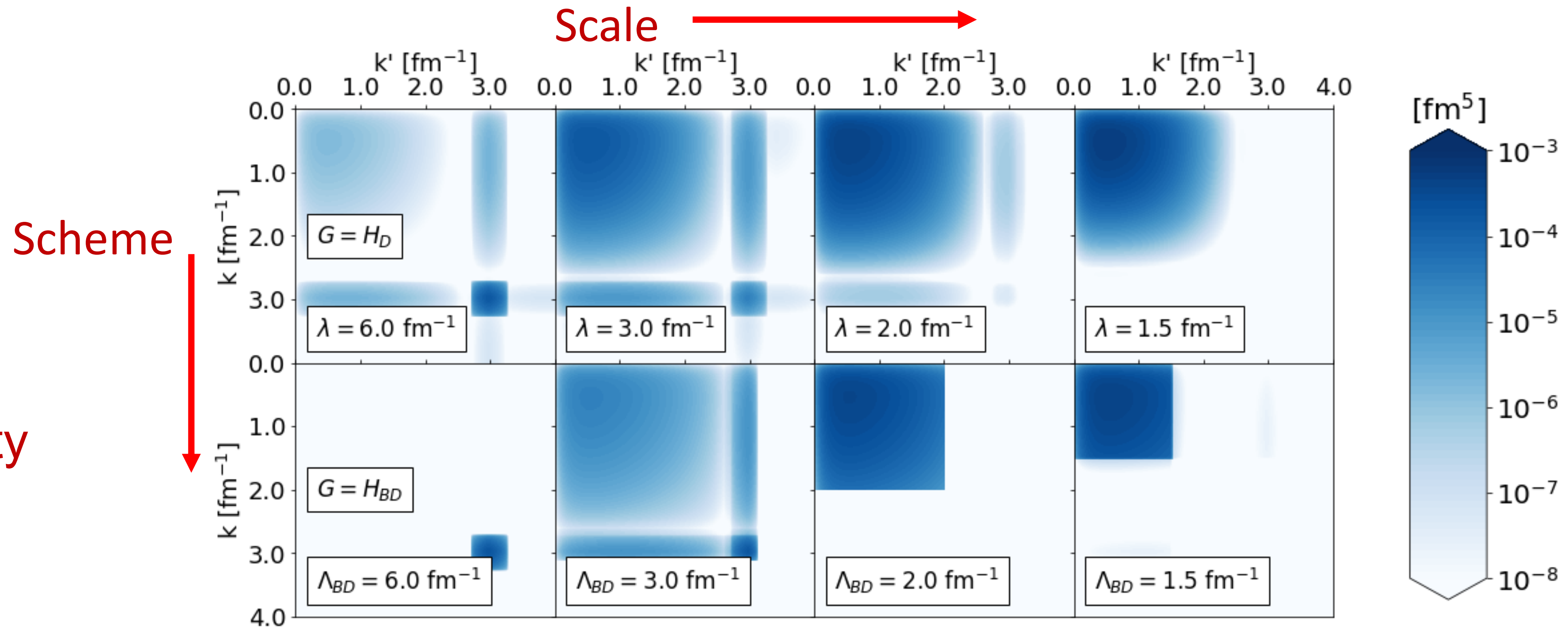


Fig. 7: SRG-evolved matrix elements of $\langle \psi | a_q^\dagger a_q | \psi \rangle$ with RKE N^{4}LO .

Summary and outlook

- Universality holds in drastically different chiral potentials
  - At low resolution, different interactions are the same
- Universality shows in low-energy states
- Evolved (non-Hamiltonian) operators reflect scheme dependence from different potentials
- Results suggest one can analyze high-energy nuclear reactions with low-resolution structure (e.g., shell model) if evolved operator used (and correct initial operator)

Back up slides

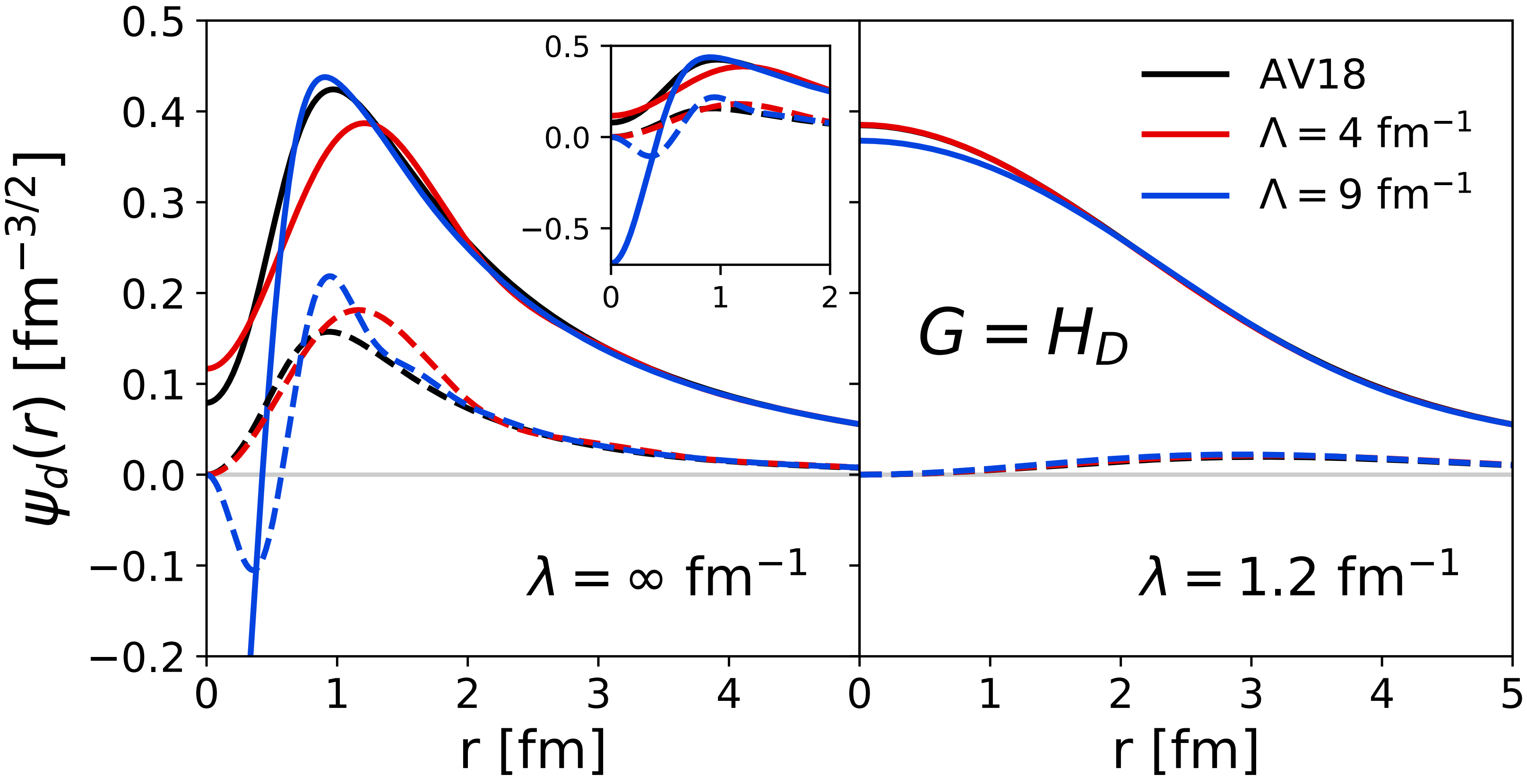


Fig. 8: SRG evolution of deuteron wave function in coordinate space for AV18 and two LO chiral models at high momentum-space cutoffs Λ.

Back up slides

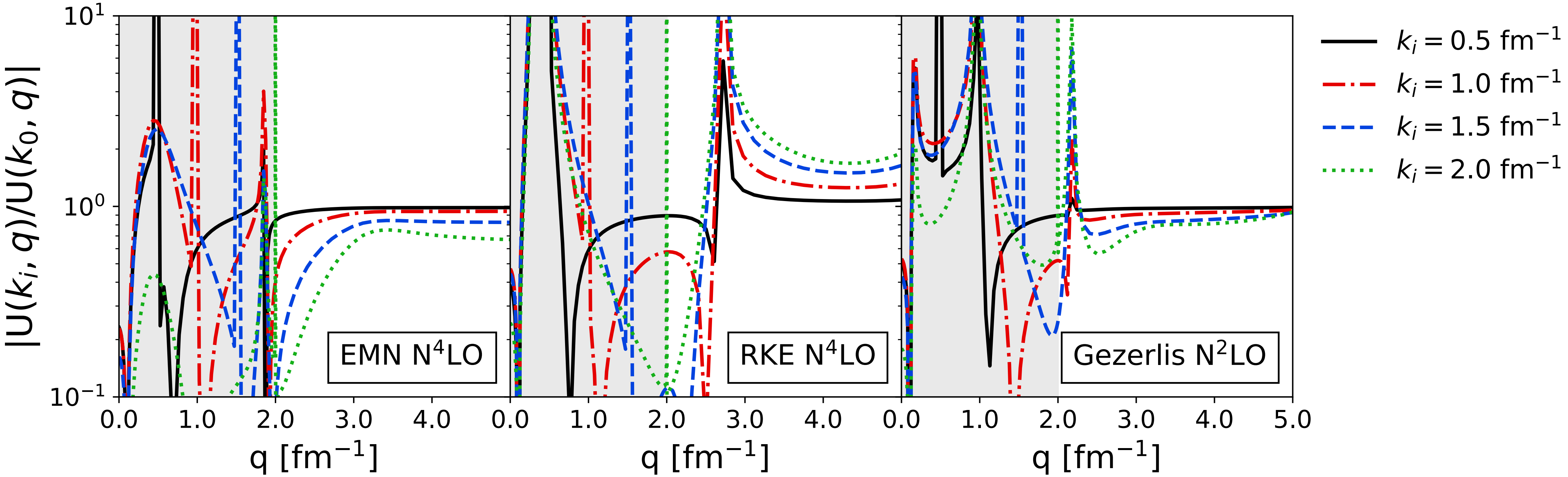


Fig. 9: Ratio of SRG transformations $U(k,q)$ at low- and high-momentum values with respect to high-momentum q, and fixing the low-momentum of the denominator k_0 and varying the low-momentum of the numerator k_i.