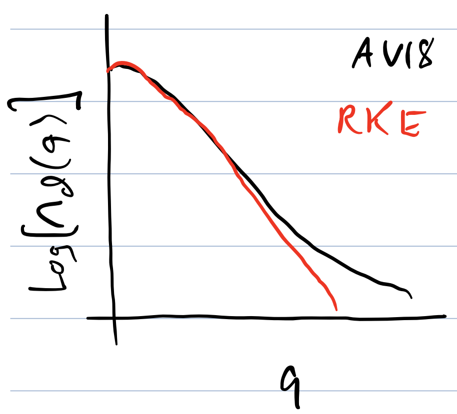
1. We can connect a hard potential (e.g., AV18) to a soft potential (e.g., RKE N4LO 450 MeV) through SRG evolution.

Here we will use the deuteron momentum distribution as an approximate tool to find the appropriate matching scale.

We expect a higher tail from the hard potential, that is,




Taking $|\psi_d^{\lambda'}\rangle = \hat{U}^{AV18}(\lambda')\,|\psi_d^{AV18}\rangle$ we can map to the deuteron momentum distribution of the soft potential $|\psi_d^{RKE}\rangle$

$$\Rightarrow \quad |\psi_d^{\lambda'}\rangle = \hat{U}^{AV18}(\lambda')\,|\psi_d^{AV18}\rangle \approx |\psi_d^{RKE}\rangle$$

for some matching $\lambda'$ ($\approx 4.5\ \mathrm{fm}^{-1}$).

2. Conversely, we can move backwards matching a soft potential to a hard one by applying the inverse SRG transformations:

$$\hat{H}^{AV18} \approx \left[\hat{U}^{AV18}(\lambda')\right]^{\dagger} \hat{H}^{RKE}\, \hat{U}^{AV18}(\lambda')$$

Following the procedure of evolving a one-body operator $a^{\dagger}_{\vec{q}} a_{\vec{q}}$ we start with constructing

$\hat{U}^{RKE}(\lambda)$ for a low RG resolution scale $\lambda \sim 1.35\ \mathrm{fm}^{-1}$ relying on

$$\hat{U}(\lambda) = \sum_{\alpha} |\Psi_\alpha(\lambda)\rangle\langle\Psi_\alpha(\infty)|$$

But if $\left[\hat{U}^{AV18}(\lambda')\right]^{\dagger} \hat{H}^{RKE}\, \hat{U}^{AV18}(\lambda')$ is our initial Hamiltonian, then the initial eigenstates are

$$|\Psi_\alpha(\infty)\rangle \rightarrow |\Psi_\alpha^{RKE}(\lambda')\rangle = \left[\hat{U}^{AV18}(\lambda')\right]^{\dagger} |\Psi_\alpha^{RKE}(\infty)\rangle$$

And the transformation we construct is then

$$\sum_\alpha |\psi_\alpha^{RKE}(\lambda)\rangle\langle\psi_\alpha^{RKE}(\lambda')|$$

$$= \sum_\alpha \hat{U}^{RKE}(\lambda)\,|\psi_\alpha^{RKE}(\infty)\rangle\langle\psi_\alpha^{RKE}(\infty)|\,\hat{U}^{AV18}(\lambda')$$

$$= \hat{U}^{RKE}(\lambda)\hat{U}^{AV18}(\lambda') \qquad \text{(using completeness)}$$

Then in evolving the 1-body operator $a^\dagger_{\vec{q}}\, a_{\vec{q}}$, we have

$$a^\dagger_{\vec{q}} a_{\vec{q}}(\lambda) = \hat{U}^{RKE}(\lambda)\,\hat{U}^{AV18}(\lambda')\, a^\dagger_{\vec{q}}\, a_{\vec{q}} \left[\hat{U}^{RKE}(\lambda)\,\hat{U}^{AV18}(\lambda')\right]^\dagger$$

$$= \hat{U}^{RKE}(\lambda)\left[\hat{U}^{AV18}(\lambda')\, a^\dagger_{\vec{q}} a_{\vec{q}}\, \hat{U}^{AV18\dagger}(\lambda')\right]\hat{U}^{RKE\dagger}(\lambda)$$

Therefore, the initial operator for the soft RKE potential can be approximated as

$$\hat{U}^{AV18}(\lambda')\, a^\dagger_{\vec{q}} a_{\vec{q}}\, \hat{U}^{AV18\dagger}(\lambda')$$

which is a 2-body operator (as opposed to the initial 1-body operator for AV18)

This should then move Levinger constant up to AV18 values