Status of nuclear optical potentials and future prospects

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I. INTRODUCTION

Things to include:

– Why are nuclear reactions important? (Processes that help us understand nuclear structure amongst other things. Exotic nuclei are short-lived and must used reactions to study them.) List examples.

– r-process for motivation.

– How do optical potentials help us understand nuclear reactions?

– References: ...

II. FORMALISM

Things to include:

– Projectile strikes a nucleus: elastic scattering and more.

– Derivation of the general optical potential (equation (2.15) in Feshbach). Check.

– More general things from Thompson/Nunes: define optical potentials as complex potentials and consequences of this. Reaction cross section derivation?

– Drawbacks to this derivation. Transition to projection operator method.

– Relation to observables. Use Lippmann-Schwinger equation to bridge gap to observables: phase shifts and cross sections. Maybe this makes more sense in phenomenology section?

– Generalization: derivation with projection operators.

– References: [1], [2], [3].

Add Feshbach reference somewhere.

First, we present the optical potential for an $A+1$ particle system consisting of an incident nucleon and a target nucleus of mass number A. The system is described by the Schrödinger equation

$$\mathcal{H}\Psi = E\Psi, \tag{1}$$

with the Hamiltonian $\mathcal{H}$ given below:

$$\mathcal{H} = H_A(\mathbf{r}_1, \cdots, \mathbf{r}_A) + T_0 + V(\mathbf{r}_0, \cdots, \mathbf{r}_A). \tag{2}$$

The variables $\mathbf{r}_k$ correspond to position, spin, and isospin for the incident nucleon ($k = 0$) and each nucleon in the target nucleus ($k = 1 \cdots A$). T_0 is the kinetic energy of the incident nucleon and V is the potential energy of the $A+1$ system. H_A is the Hamiltonian for the target nucleus and satisfies the Schrödinger equation

$$H_A(\mathbf{r}_1, \cdots, \mathbf{r}_A)\psi_i(\mathbf{r}_1, \cdots, \mathbf{r}_A) = \epsilon_i \psi_i(\mathbf{r}_1, \cdots, \mathbf{r}_A), \tag{3}$$

for nuclear wave functions ψ_i and energies ϵ_i. Here, the index i corresponds to each state of the target nucleus with $i = 0$ being the ground state. The nuclear wave functions ψ_i form a complete, orthonormal set; thus, we expand the wave function Ψ as follows:

$$\Psi(\mathbf{r}_0, \cdots, \mathbf{r}_A) = \sum_i \psi_i(\mathbf{r}_1, \cdots, \mathbf{r}_A) u_i(\mathbf{r}_0). \tag{4}$$

Note, the factors u_i carry the $\mathbf{r}_0$ dependence.

Mention hard-core problem. In the following, we suppress the coordinate, spin, and isospin dependencies for brevity. We substitute 4 into the Schrödinger equation 1 and use the orthonormality of ψ_i to derive a system of equations for the amplitudes u_i:

$$(T_0 + V_{ii} + \epsilon_i - E)u_i = -\sum_{j \neq i} V_{ij} u_j, \tag{5}$$

where the potential matrix elements are

$$V_{ij}(\mathbf{r}_0) = \int d^3r_1 d^3r_2 \cdots d^3r_A \psi_i^* V \psi_j. \tag{6}$$

Next, we would like to derive an uncoupled equation for u_0 to describe elastic scattering in which the target nucleus is in its ground state with an incident nucleon of energy E. The other indices i describe an emergent nucleon in a different state (e.g. energy, spin, isospin, etc.) from the incident nucleon. It is convenient to define the vectors

$$\Phi \equiv \begin{pmatrix} u_1 \\ u_2 \\ \vdots \end{pmatrix}, \tag{7}$$

$$\mathbf{V}_0 = \begin{pmatrix} V_{01}, V_{02}, \cdots \end{pmatrix} \tag{8}$$

and the matrix operator $\mathbf{H}$

$$H_{ij} = T_0 \delta_{ij} + V_{ij} + \epsilon_i \delta_{ij}. \tag{9}$$

Then we can rewrite 5 as

$$(T_0 + V_{00} - E)u_0 = -\mathbf{V}_0\Phi, \tag{10a}$$

$$(\mathbf{H} - E)\Phi = -\mathbf{V}_0^\dagger u_0. \tag{10b}$$

We solve 10b for Φ

$$\Phi = \frac{1}{E - \mathbf{H} + i\eta}\mathbf{V}_0^\dagger u_0, \tag{11}$$

where $\eta \to 0^+$ to ensure only outgoing waves are present in exit channels for u_i with $i \geq 1$. Lastly, we substitute Φ into 10a to give

$$(T_0 + V_{00} - \mathbf{V}_0\frac{1}{E - \mathbf{H} + i\eta}\mathbf{V}_0^\dagger - E)u_0 = 0, \tag{12}$$

and define the optical potential as

$$V_{opt} = V_{00} - \mathbf{V}_0\frac{1}{E - \mathbf{H} + i\eta}\mathbf{V}_0^\dagger. \tag{13}$$

Drawbacks of this derivation.

We can see from Eq. 13 that the optical potential is complex and energy dependent. The factor of $i\eta$ leads to V^{opt} being complex, and thus, non-hermitian. The factor of $i\eta$ accounts for incident particles leaving the entrance channel u_0 to an exit channel u_i where $i \geq 1$. This only occurs if reactions are possible, that is, $E > \epsilon_1$. Because V^{opt} is not hermitian, the S matrix is not unitary giving rise to complex scattering phase shifts.

Furthermore, the potential is non-local... Use reference [4].

– Generalization by using projection operators.

– What does P and Q do?

– Examples? (Feshbach.)

The projection operators satisfy the following relations: $P + Q = 1$, $P^2 = P$, and $Q^2 = Q$. We act on Eq. 1 with P and Q and use the projection operator relations to obtain two equations:

$$(E - \mathcal{H}_{PP})P\Psi = \mathcal{H}_{PQ}Q\Psi, \tag{14a}$$

$$(E - \mathcal{H}_{QQ})Q\Psi = \mathcal{H}_{QP}P\Psi. \tag{14b}$$

Solving 14b for $Q\Psi$ yields

$$Q\Psi = \frac{1}{E - \mathcal{H}_{QQ}}\mathcal{H}_{QP}P\Psi. \tag{15}$$

Note, if P does not include all open channels, then a factor of $i\eta$ where $\eta \rightarrow 0^+$ must be inserted in the denominator as before to account for... Finish this note. Substituting $Q\Psi$ into Eq. 14a and rearranging gives

$$(E - \mathcal{H}_{PP} - \mathcal{H}_{PQ}\frac{1}{E - \mathcal{H}_{QQ}}\mathcal{H}_{QP})P\Psi = 0, \tag{16}$$

where the effective Hamiltonian is

$$H_{eff} = \mathcal{H}_{PP} + \mathcal{H}_{PQ}\frac{1}{E - \mathcal{H}_{QQ}}\mathcal{H}_{QP}. \tag{17}$$

Advantages of the projection operator formulation. Wider applicability with generalization of P.

III. PHENOMENOLOGY

Things to include:

– Form of the potential: Woods-Saxon shape, coulomb component, spin-orbit force. (Basic example in Thompson/Nunes.). Can start with discussion similar to Thompson/Nunes.

– Fit strength, radii, and diffuseness of complex potential.

– Surface and volume component from Dickhoff?

– Issue: fitting ambiguities, extractions to exotic regions of the nuclear chart.

– Make sure to touch on phenomenology of optical potentials in modern experimental analyses (key word is modern!)

– References: [5] section 3, [6].

IV. MICROSCOPIC OPTICAL POTENTIALS

Things to include:

– Successes and limitations.

– Motivation: predictions for exotic region of the nuclear chart.

– Major methods: Multiple scattering (see references in Dickhoff paper) and Green's function based methods (coupled cluster). SRG evolution.

– Coupled cluster Green's function [7].

– References: [5] section 4, [8] G-matrix interaction, [9] self-consistent Green's function, [7].

V. THEORETICAL ISSUES

Things to include:

– Fitting ambiguities for phenomenological potential.

– Uncertainty quantification.

– Add this to outlook in conclusion?

– References: [10].

VI. CONCLUSION

Summary and outlook.

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