SRG evolution of large-cutoff chiral effective field theory interactions

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The Ohio State University

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Motivation

- Revisit problem studied by Wendt et al.[1]
  - SRG evolve non-local chiral potentials at leading-order and high cutoffs
- Since then there has been an explosion of chiral potentials
- At leading-order chiral effective field theory is cutoff dependent
- Does the SRG evolve these potentials to a decoupled, cutoff independent form?

[1]K.A. Wendt, R.J. Furnstahl, R.J. Perry, Phys. Rev. C **83**, 034005 (2011)

Chiral effective field theory

- Regularization procedure is necessary in χ^{EFT} to tame divergences from high energy physics
- Several options used for regulator function (local, non-local; Gaussian, super-Gaussian; coordinate-space, momentum-space)
- Typical cutoffs: $\Lambda \sim 500$ MeV (~ 2.5 fm^{-1})

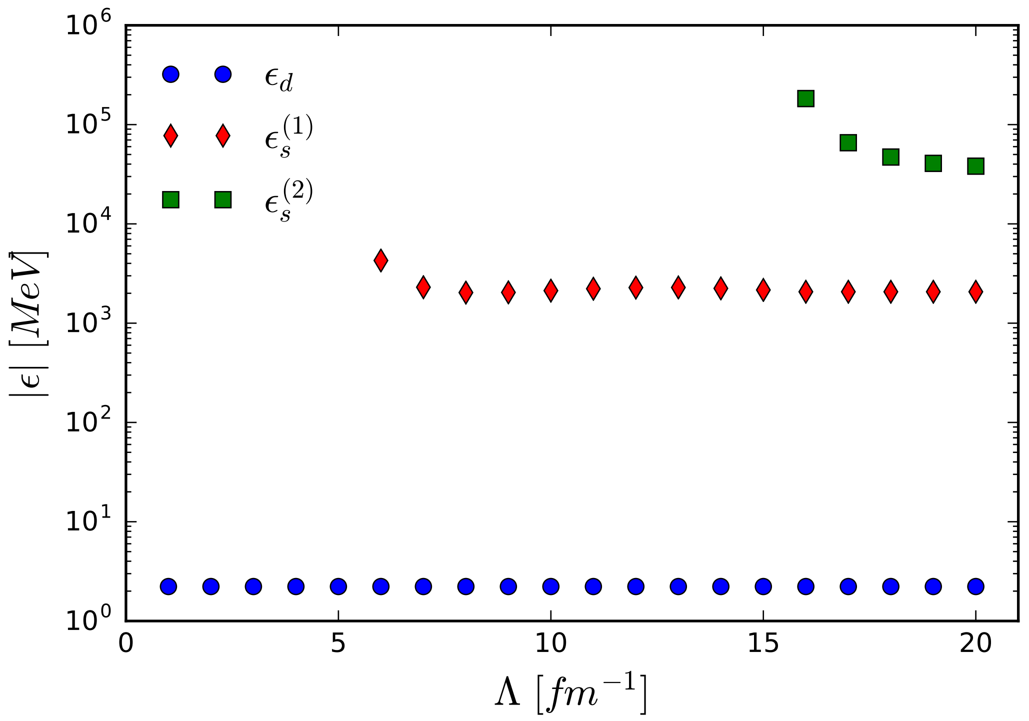
Chiral effective field theory

- Nogga et al.[2] took EFT cutoff Λ to much higher values to study cutoff dependence in non-local leading-order chiral nuclear interactions
- Tews et al.[3] studied regulator and cutoff dependence for local potentials at LO
- High cutoffs
  - At short distances (high momenta), the Hamiltonian is highly singular featuring spurious deeply bound states (which are non-physical) from one-pion exchange attractive tensor forces

[2]A. Nogga, R.G.E. Timmermans, and U. van Kolck, Phys. Rev. C **72**, 054006 (2005), [3]I. Tews, L. Huth, and A. Schwenk, Phys. Rev. C **98**, 024001 (2018)

Chiral effective field theory

- Spurious deeply bound states
  - From EFT perspective, deeply bound states are outside of range of EFT
  - In practice, do they corrupt low-momentum physics?
- Here we consider non-local and semi-local chiral potentials at LO in the $^3S_1$-$^3D_1$ channel



- Bound states energies of non-local potential as a function of Λ

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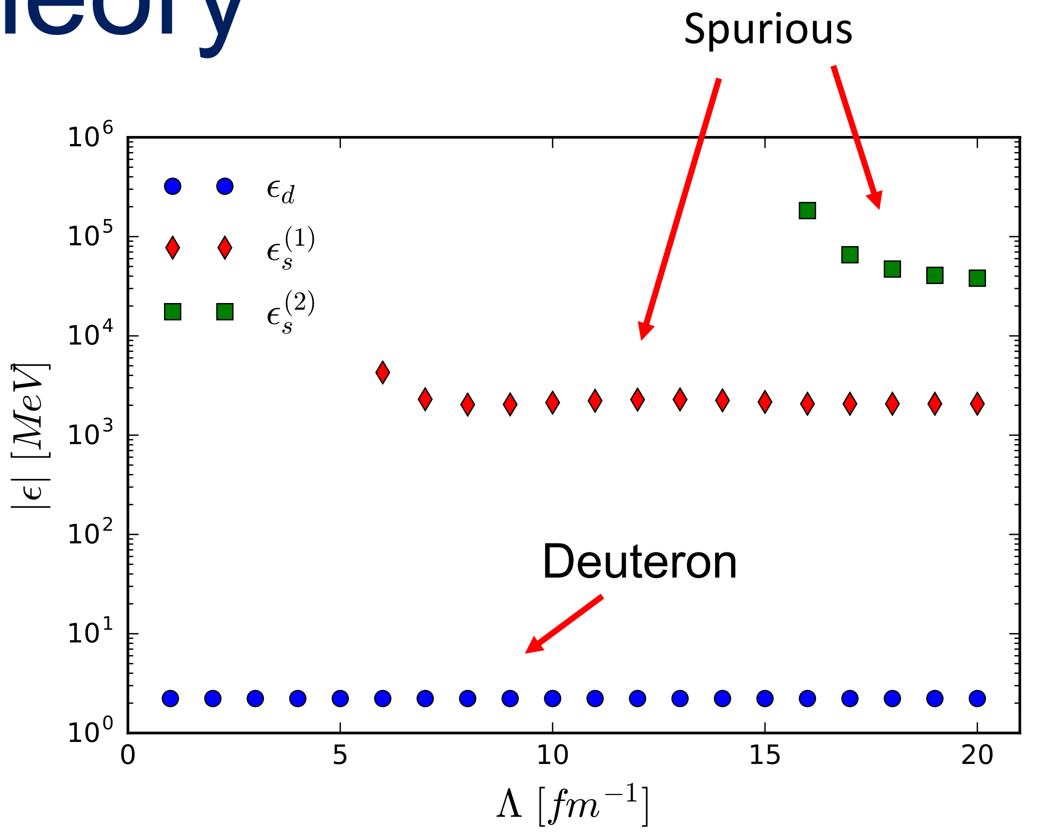

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SRG for LO chiral EFT with high cutoffs

- Wendt et al.[1] studied band-diagonal SRG evolution for non-local interaction at large-cutoffs
- SRG decouples low- and high-momenta by 'rotating' Hamiltonian:

$$\widetilde{H}(s) = U(s) H U^{\dagger}(s)$$

where $s = 0 \rightarrow \infty$

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where $s = 0 \rightarrow \infty$

- In practice, solve differential flow equation

$$\frac{d\tilde{H}}{ds} = \left[\eta(s), \tilde{H}(s)\right]$$

with SRG generator $\eta(s) \equiv \frac{dU}{ds} U^{\dagger}(s) = [G, \tilde{H}(s)]$

- $G = T_{rel}$ or $\tilde{H}_d(s)$ (Wegner) for band-diagonal and $G = \tilde{H}_{BD}(s)$ for block-diagonal decoupling

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SRG for LO chiral EFT with high cutoffs

- For typical cutoffs, same $\widetilde{H}(s)$ regardless of $\mathrm{G} = T_{rel}$ or $\widetilde{H}_d(s)$!
- For high cutoff $\Lambda = 9\ \mathrm{fm}^{-1}$
  - Wegner: keeps spurious bound state safely outside low-momentum block
  - T_{rel} : spurious bound state distorts the low-momentum block of $\widetilde{H}(s)$
  - Choice in G matters at high cutoffs!

$\Lambda = 4\ \mathrm{fm}^{-1}$

$G = T_{rel}$

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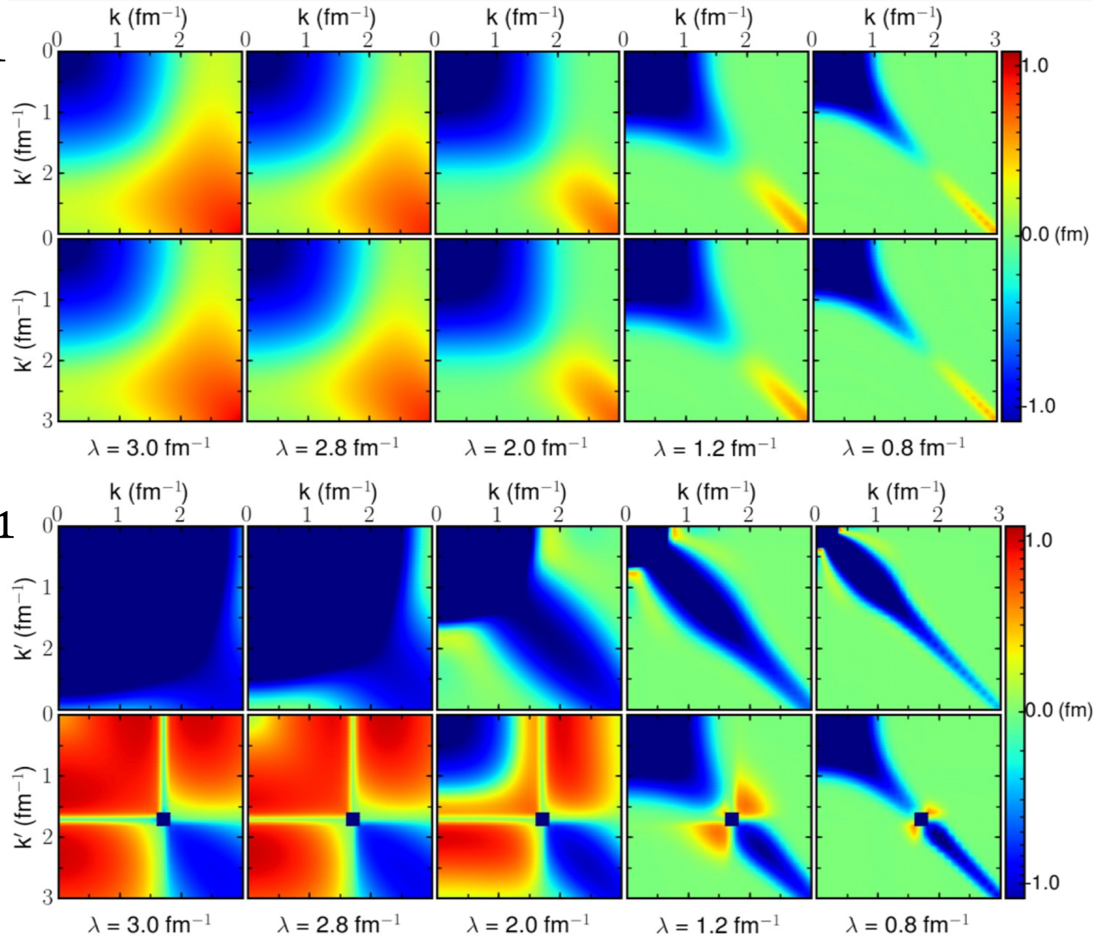

SRG evolution of $V_\lambda(k, k')$ for several values of $\lambda = s^{-1/4}$

Figure from K.A. Wendt, R.J. Furnstahl, R.J. Perry, Phys. Rev. C **83**, 034005 (2011)

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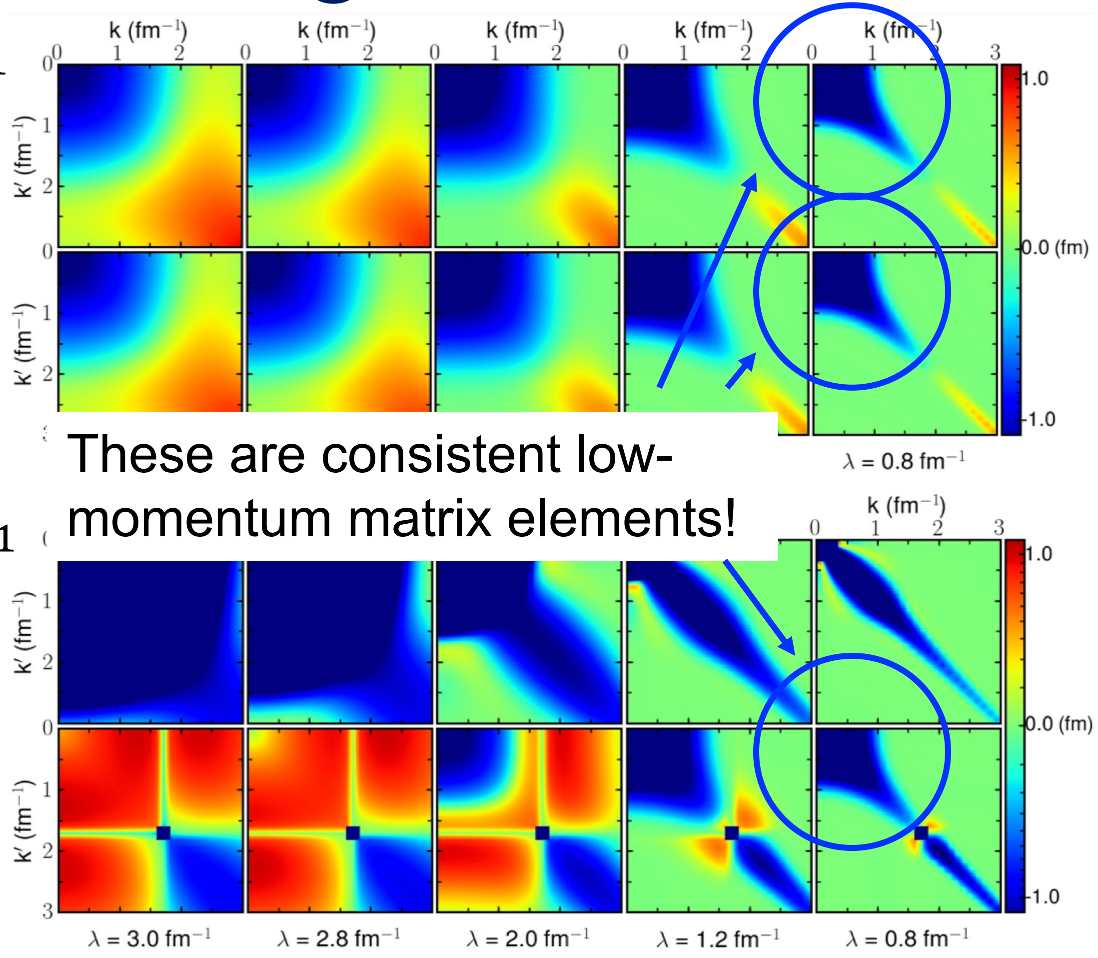

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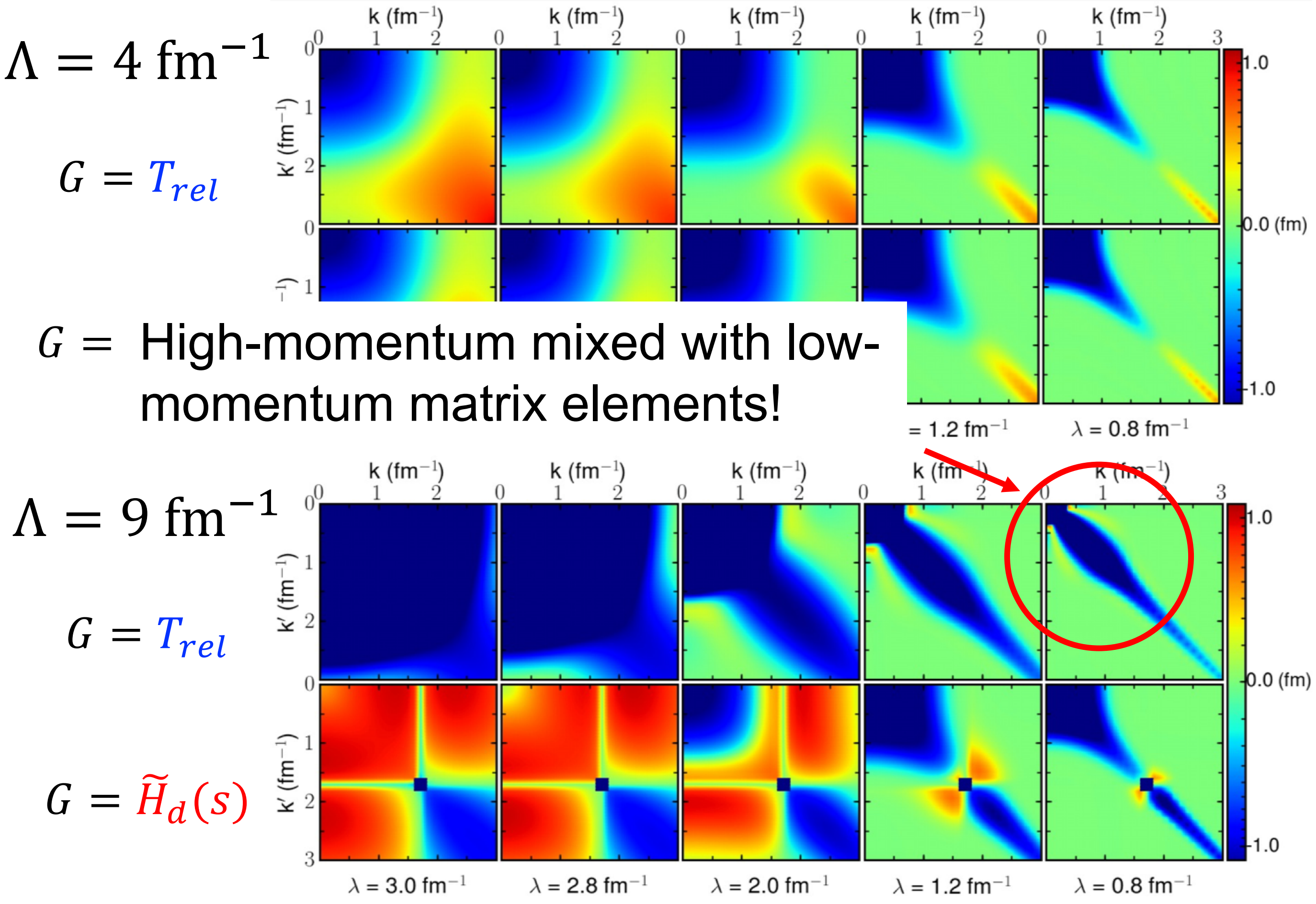

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Motivation

- High cutoff potentials were used as a test case for the Magnus expansion (analogous problem with IMSRG intruder states)
  - Magnus agreed with SRG result but struggled to evolve very high cutoffs ($\Lambda \sim 20$ fm^{-1})
- In the process, this led to several questions

Motivation

- How do different regulator functions (non-local, semi-local, local) affect SRG evolution?
- Do matrix elements of potential approach universal form at lower λ?
- How does the spurious bound state decouple for Wegner generators?
- What happens to spurious bound state(s) with block-diagonal SRG evolution?
- Can we gain insight into renormalizability for chiral EFT formulations?

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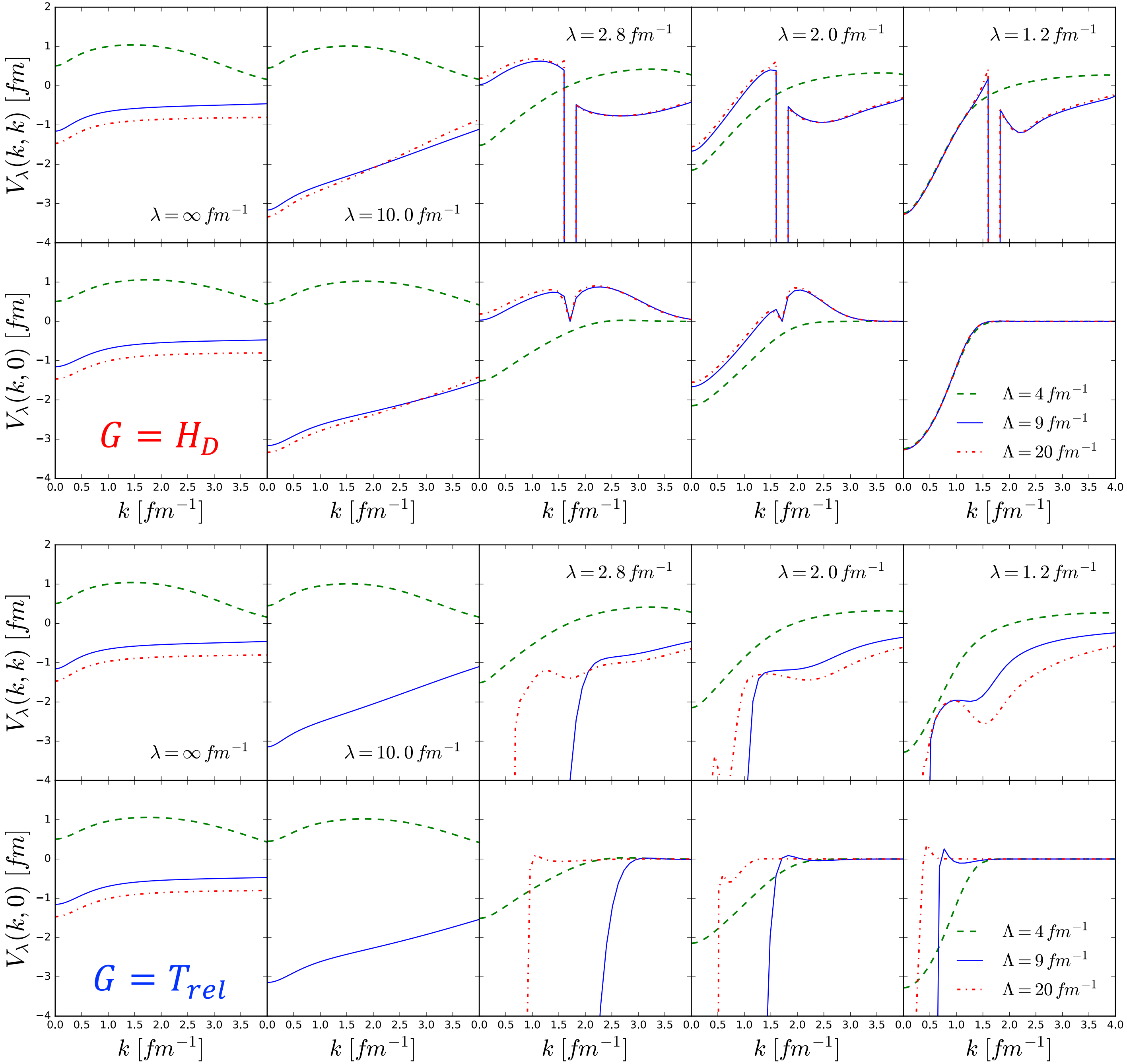
Band-diagonal evolution

- Non-local regulator
  - Matrix elements of $V_\lambda(k,k')$ are independent of Λ at low-momentum for Wegner evolution
  - Wegner generator decouples the spurious bound state at $k\sim1.83$ fm^{-1}

  But is mesh dependent!



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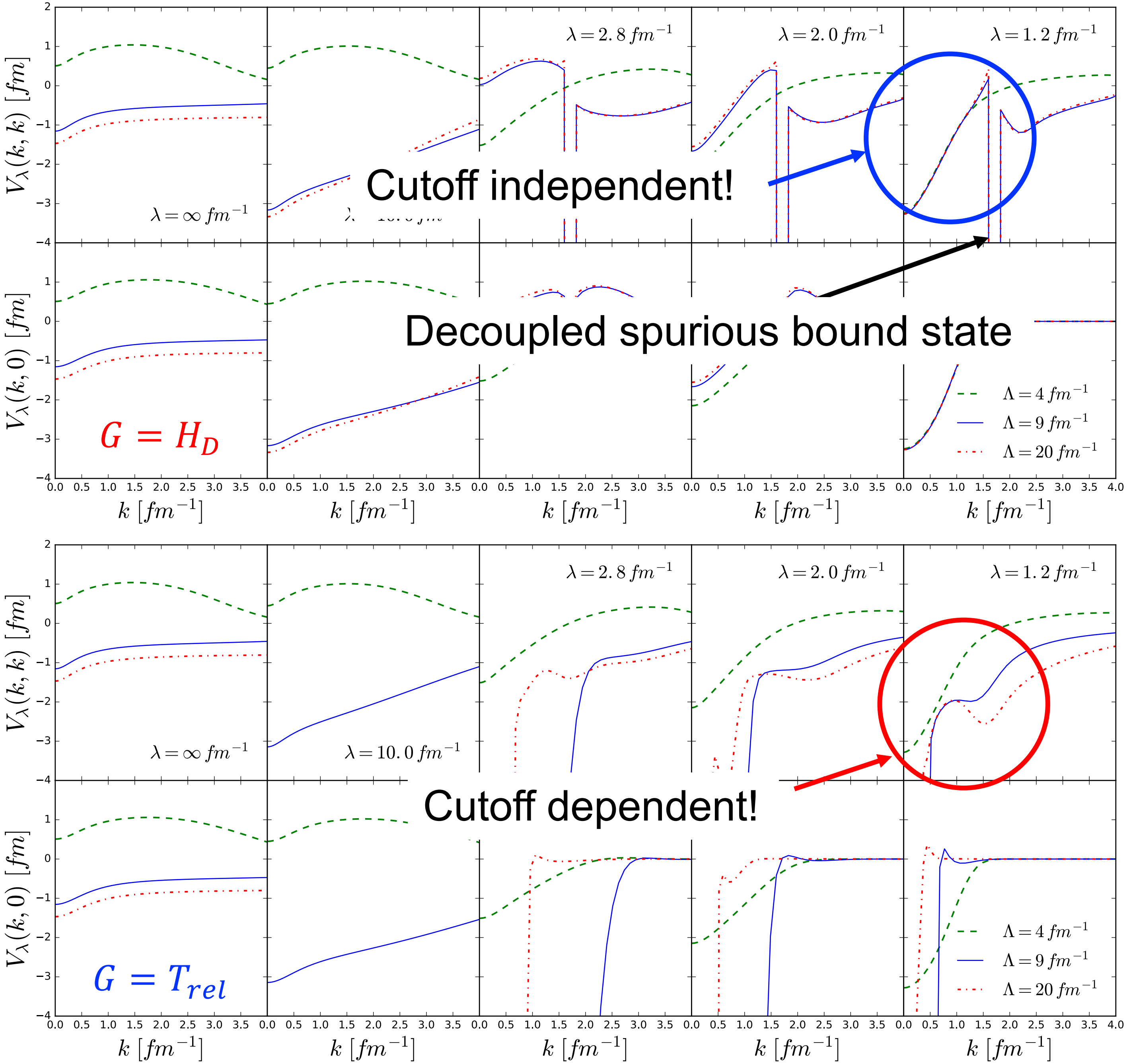
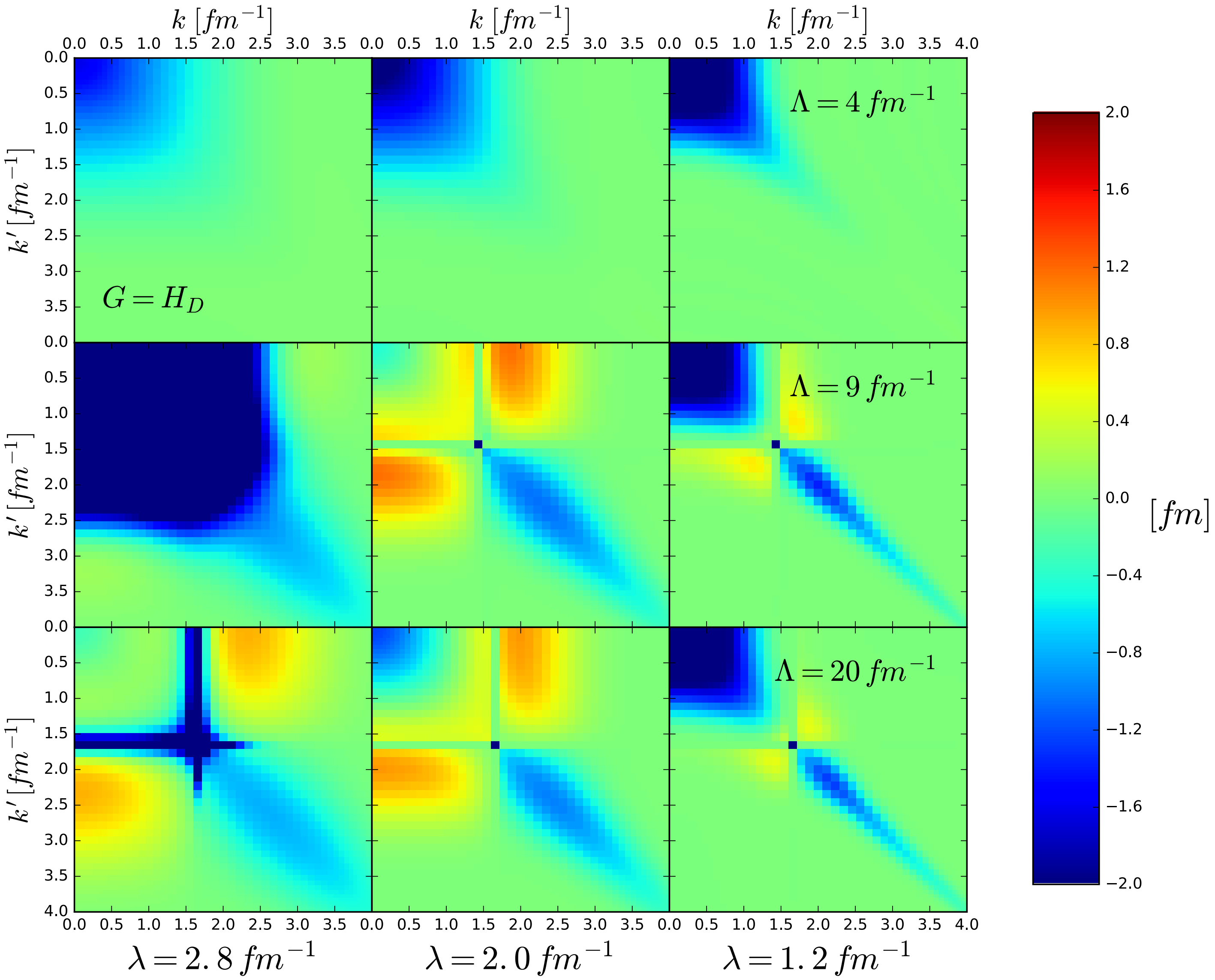


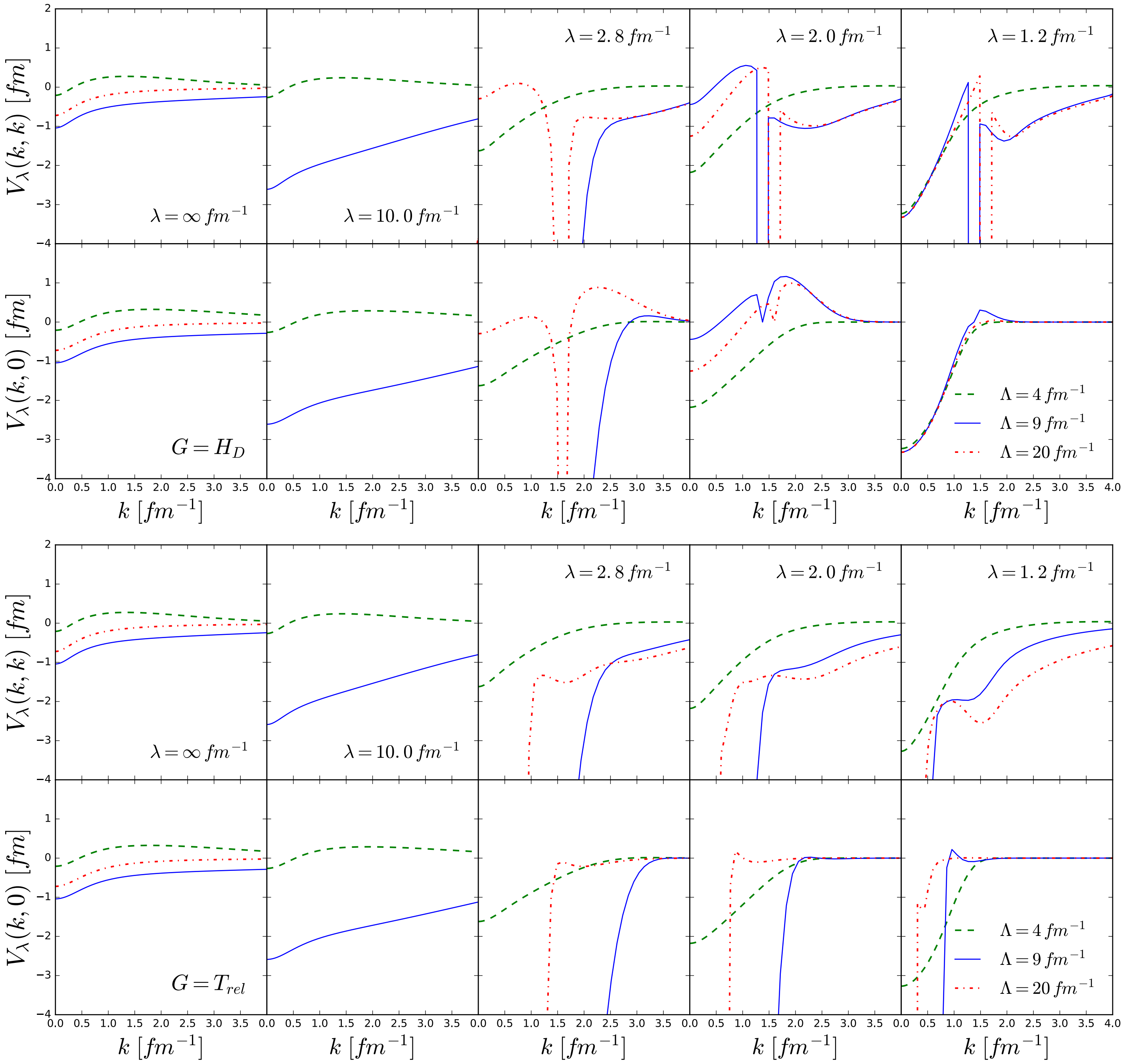
Band-diagonal evolution

- Semi-local regulator
  - Local one-pion exchange and non-local contact interaction
  - Similar to non-local case: Wegner generator decouples the spurious bound state at $k{\sim}1.49$ and $1.71\ \mathrm{fm}^{-1}$ for $\Lambda = 9$ and $20\ \mathrm{fm}^{-1}$ respectively



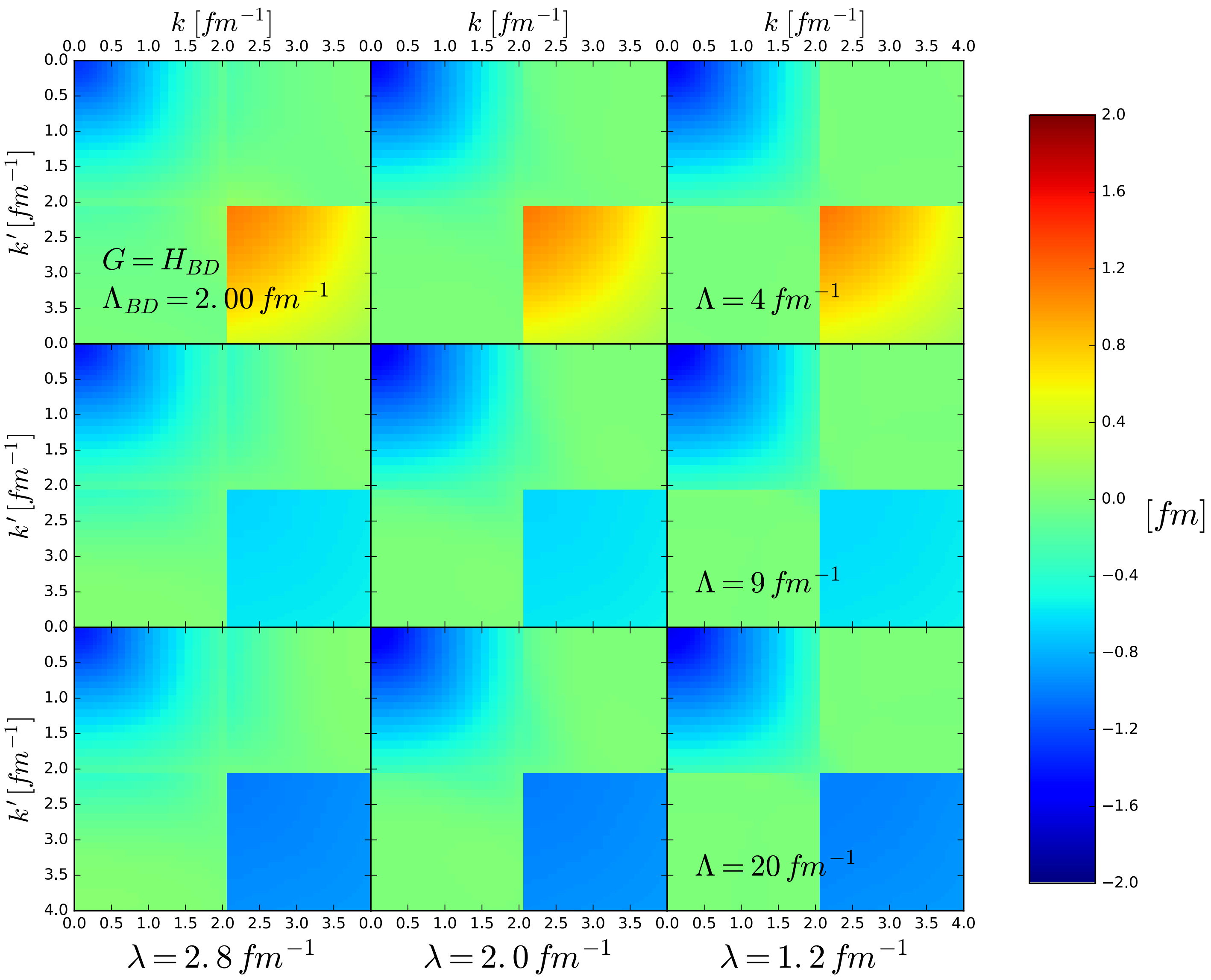
Band-diagonal evolution

- Semi-local regulator
  - Worse matrix element equivalence than non-local case (ε_s decoupled at lower k) for Wegner
  - T_{rel} behavior is the same as before
  - For band-diagonal generators, decoupling of ε_s depends on momentum mesh and regulator
  - Is decoupling cleaner for block-diagonal evolution?



Block-diagonal evolution

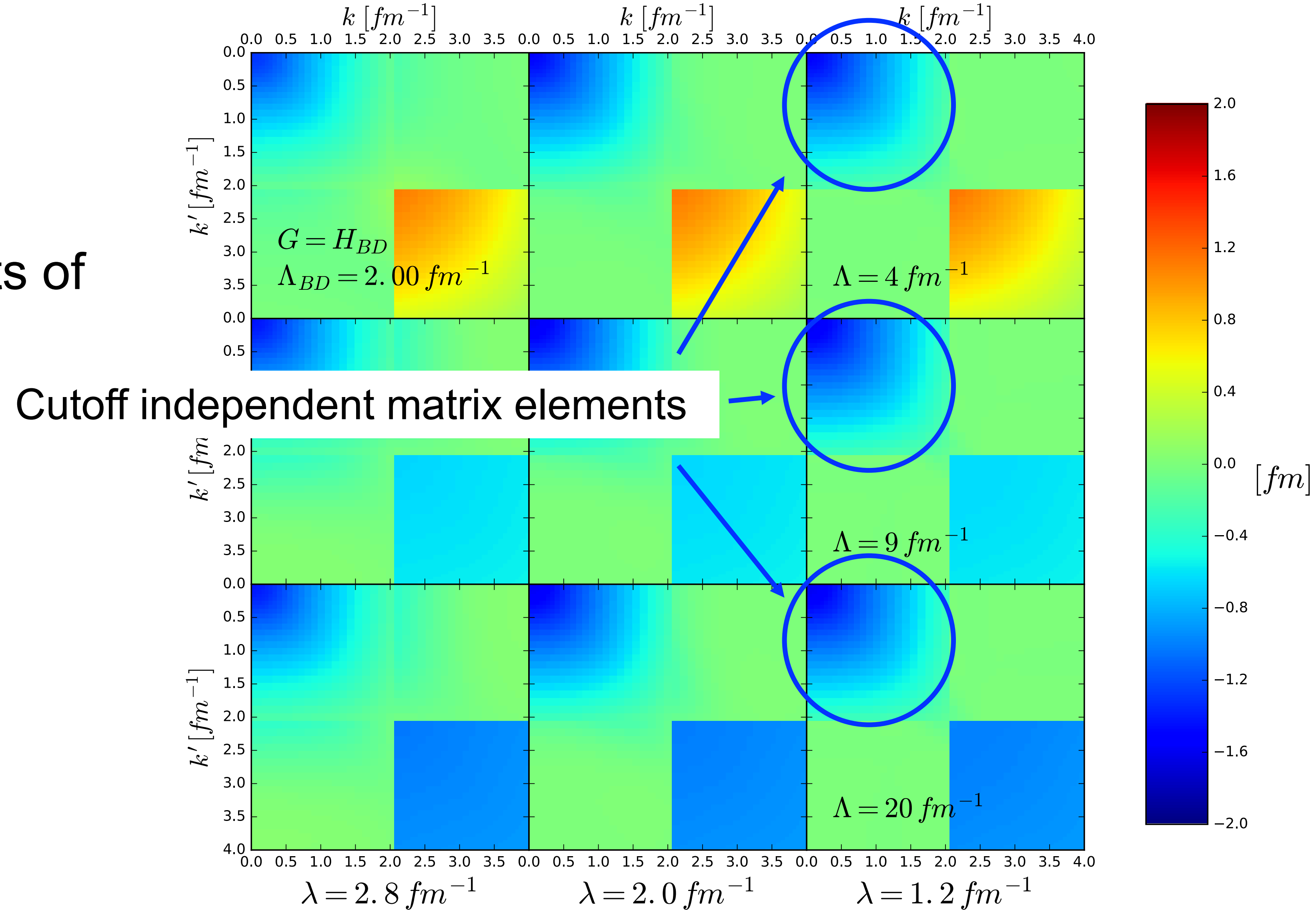
- $G = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$ with projection operator P and $Q = 1 - P$
- Decouples Hamiltonian into sub-blocks at momentum Λ_{BD}
- Diagonalize sub-blocks to see where ε_s decouples
  - ε_s decouples at $k \sim 4.47$ fm^{-1} much higher than $G = \widetilde{H}_d$ where $k \sim 1.83$ fm^{-1}



Non-local interaction

Block-diagonal evolution

- For Λ_{BD} = 2 fm^{-1} low-momentum matrix elements of evolved potential are independent of EFT Λ

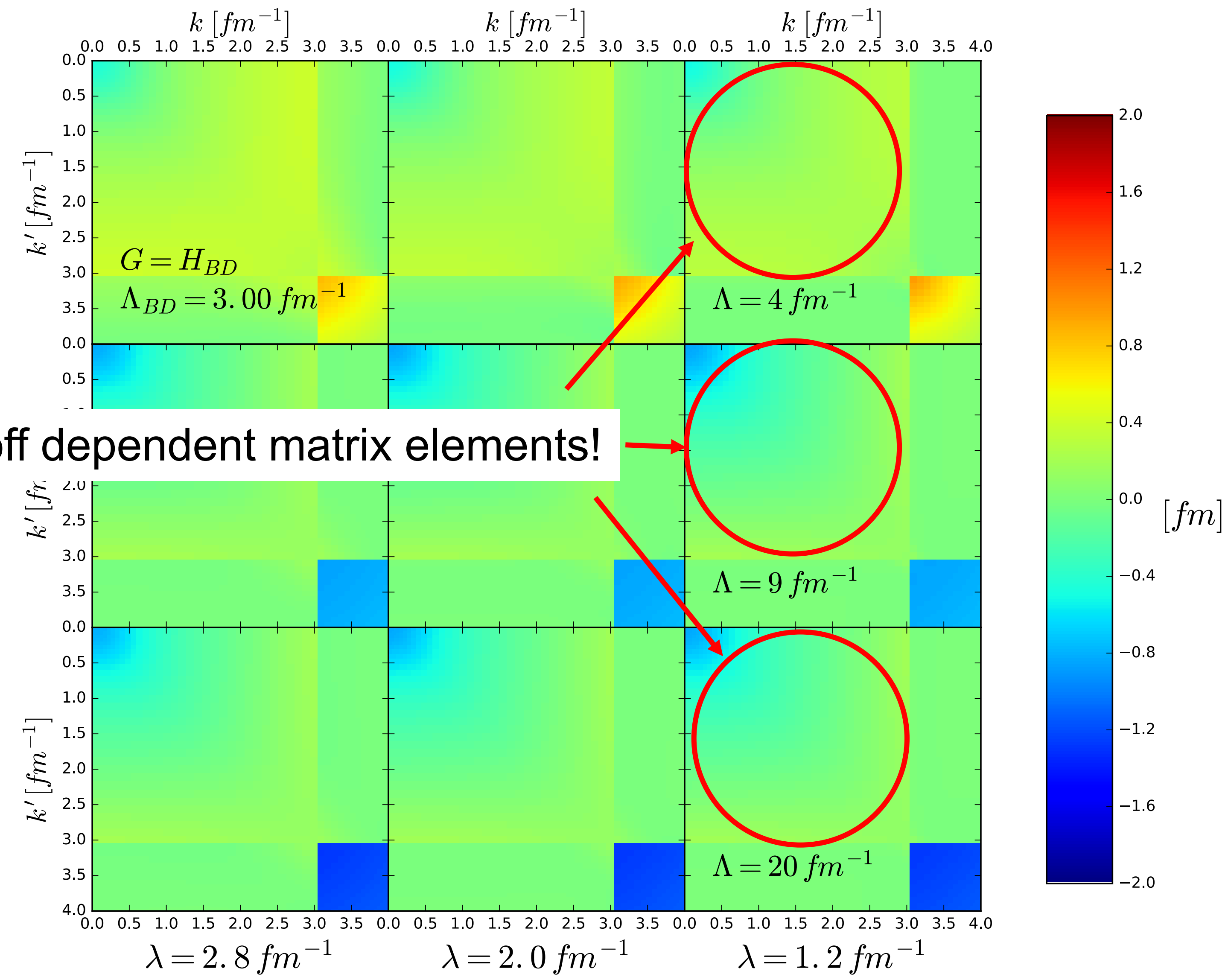


Non-local interaction

Block-diagonal evolution

- For $\Lambda_{BD} = 3\ \text{fm}^{-1}$ low-momentum matrix elements are not equivalent
  - Low-momenta couples to modes affected by spurious bound state!

Cutoff dependent matrix elements!



Non-local interaction

Conclusion

- Summary
  - SRG-evolved potentials flow to universal low-momentum matrix elements when ε_s decouples from low-momenta (band-diagonal Wegner or block-diagonal with low Λ_{BD})
  - Non-local and semi-local potentials have similar SRG evolution but ε_s decouples differently
  - Block-diagonal generators decouples ε_s at higher momentum than band-diagonal generators

Conclusion

- Future work
  - SRG evolve local chiral potential and other channels
  - Determine what controls where spurious bound state decouples
  - Use smooth cutoff for block-diagonal generator (cf. sharp cutoff)
  - Explore issues of renormalizability in χ^{EFT}