Recap of what works:

- For unevolved case we have

$$\hat{n}_{\sigma\tau}(\vec{k}) \sim \langle F | a^\dagger_{\vec{k},\sigma,\tau} a_{\vec{k}\sigma\tau} | F \rangle$$

$$\sim V \Theta(k_F - k) \quad [fm^3]$$

Then for nuclear averaging:

$$\langle \hat{n}_{\sigma\tau}(k) \rangle = \int d^3r \, \underbrace{\hat{n}_{\sigma\tau}(k; k_F(r))}_{V}$$

$$= 4\pi \int_0^\infty r^2 dr \, \Theta(k_F(r) - k)$$

In the code:

$$\rho^A_\tau(k) = 4\pi \frac{1}{(2\pi)^3} \underbrace{\sum_\sigma}_{=2 \text{ (two spins)}} k^2 \frac{\langle \hat{n}_{\sigma\tau}(k) \rangle}{A}$$

$$= \left[4\pi \frac{1}{(2\pi)^3} 2 \right] k^2 \frac{\langle \hat{n}_\tau(k) \rangle}{A}$$

Then $\int_0^\infty dk \, \rho^A(k)$ as expected

We can see the overall factor by expecting

$$\frac{1}{A}\sum_{\vec{k},\sigma,\tau}\langle \hat{n}_{\sigma\tau}(k)\rangle = 1$$

$$\downarrow$$

$$\frac{1}{A}\sum_{\sigma,\tau}\int\frac{d^3k}{(2\pi)^3}\langle \hat{n}_{\sigma\tau}(k)\rangle = \frac{2\times 4\pi}{A}\frac{1}{(2\pi)^3}\sum_{\tau}\int_0^\infty dk\, k^2\langle \hat{n}_{\sigma\tau}(k)\rangle = 1$$

Now let's evolve $\hat{n}_{\sigma\tau}(k)$ with $\hat{U}$ and $\hat{U}^\dagger$

$$\hat{U} = 1 + \frac{1}{4}\sum_{\tau_i}\sum_{\sigma_j}\sum_{\vec{K}\vec{k}\vec{k}'}\langle \vec{k}\sigma\tau\sigma'\tau'|\delta\tilde{U}|\vec{k}'\sigma''\tau''\sigma'''\tau'''\rangle \times$$

$$a^\dagger_{\frac{\vec{K}}{2}+\vec{k}\sigma\tau}\, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}\sigma'\tau'}\, a_{\frac{\vec{K}}{2}-\vec{k}'\sigma'''\tau'''}\, a_{\frac{\vec{K}}{2}+\vec{k}'\sigma''\tau''}$$

Split $\hat{U}\,\hat{n}_{\sigma\tau}(k)\,\hat{U}^\dagger$ into 4 pieces:

Term 1: $1\times\hat{n}_{\sigma\tau}(k)\times 1$

$$\boxed{=\ \theta(k_F - k)}\quad \text{as before}$$

Term 2: $$\frac{1}{4}\sum_{\tau_i}\sum_{\sigma_j}\sum_{\vec{K}\vec{k}'\vec{k}''}\langle \vec{k}'\sigma'\tau'\sigma''\tau''|\delta\tilde{U}|\vec{k}''\sigma'''\tau'''\sigma^{(4)}\tau^{(4)}\rangle\times$$

$$a^\dagger_{\frac{\vec{K}}{2}+\vec{k}'\sigma'\tau'}\, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}'\sigma''\tau''}\, a_{\frac{\vec{K}}{2}-\vec{k}''\sigma^{(4)}\tau^{(4)}}\, a_{\frac{\vec{K}}{2}+\vec{k}''\sigma'''\tau'''}\times$$

$$a^\dagger_{\vec{k}\sigma\tau}\, a_{\vec{k}\sigma\tau}$$

Only nonzero contractions w.r.t. $|0\rangle$:

$$a^\dagger a^\dagger \underbrace{a\, a}\, \overbrace{a\, a^\dagger}\, a \to 2\, \delta_{\frac{\vec{K}}{2}+\vec{k}'', \vec{k}}\, \delta_{\sigma'''\sigma}\, \delta_{\tau'''\tau}$$

(antisymmetrized $\to$ factor of 2)

Then do final 4 w.r.t. $|F\rangle$

$$\langle F |\, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}'}\, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}'}\, a_{\frac{\vec{K}}{2}-\vec{k}\,\sigma^{(4)}\tau^{(4)}}\, a_{\vec{k}\sigma\tau}\, | F\rangle$$

$$\Rightarrow 2\, \delta_{\frac{\vec{K}}{2}+\vec{k}', \vec{k}}\, \delta_{\sigma'\tau', \sigma\tau}\, \delta_{\vec{k}, \vec{k}'}\, \delta_{\sigma''\tau'', \sigma^{(4)}\tau^{(4)}}\, \Theta(k_F - k)$$

Relabel σ, τ

$$\boxed{\text{Term 2: } \sum_{\tau'} \sum_{\sigma'} \langle \vec{k}\sigma\tau\sigma'\tau' |\, \delta\tilde{U}\, | \vec{k}\sigma\tau\sigma'\tau' \rangle \times \Theta(k_F - k)}$$

$$\boxed{\text{Similarly Term 3: } \sum_{\tau'} \sum_{\sigma'} \langle \vec{k}\sigma\tau\sigma'\tau' |\, \delta\tilde{U}^\dagger\, | \vec{k}\sigma\tau\sigma'\tau' \rangle\, \Theta(k_F - k)}$$

Term 4:

$$\frac{1}{16} \sum_{\tau_i \tau_i} \sum_{\sigma_i \sigma_i} \sum_{\vec{K}\vec{k}'\vec{k}''} \sum_{\vec{K}\vec{k}'\vec{k}''} \langle \vec{k}'\sigma\tau'\sigma''\tau'' |\, \delta\tilde{U}\, | \vec{k}''\sigma'''\tau'''\sigma^{(4)}\tau^{(4)} \rangle \times$$

$$\langle \vec{k}''\sigma'''\tau'''\sigma^{(4)}\tau^{(4)} |\, \delta\tilde{U}^\dagger\, | \vec{k}'\sigma'\tau'\sigma''\tau'' \rangle\, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}'\sigma\tau'}\, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}'\sigma''\tau''} \times$$

$$a_{\frac{\vec{K}}{2}-\vec{k}'' \sigma^{(4)}\tau^{(4)}} a_{\frac{\vec{K}}{2}+\vec{k}'' \sigma'''\tau'''} a^{+}_{\vec{k}\sigma\tau} a_{\vec{k}\sigma\tau} a^{+}_{\frac{\vec{K}}{2}+\vec{k}'' \sigma'''\tau'''} a^{+}_{\frac{\vec{K}}{2}-\vec{k}'' \sigma^{(4)}\tau^{(4)}} \times$$

$$a_{\frac{\vec{K}}{2}-\vec{k}' \sigma''\tau''} a_{\frac{\vec{K}}{2}+\vec{k}' \sigma'\tau'}$$

$$a_1^+ a_2^+ a_4 a_3 a_5^+ a_5 a_8^+ a_4^+ a_7 a_6 \quad = \delta_{\frac{\vec{K}}{2}+\vec{k}''\sigma'''\tau''', \frac{\vec{K}}{2}+\vec{k}'', \sigma'''\tau'''} \quad \sum_{\vec{k}'', \sigma''', \tau'''}$$

$$a^+ a^+ a a a^+ a a^+ a^+ a a \qquad \delta_{\frac{\vec{K}}{2}-\vec{k}''\sigma^{(4)}\tau^{(4)}, \vec{k}\sigma\tau} \quad \sum_{\vec{K}} \sigma^{(4)}\tau^{(4)}$$

$$\delta_{\frac{\vec{K}}{2}-\vec{k}''\sigma^{(4)}\tau^{(4)}, \vec{k}\sigma\tau} \quad \sum_{\vec{k}''} \sigma^{(4)}\tau^{(4)}$$

$$a^+ a^+ a a a^+ a a^+ a^+ a a \qquad \to \langle \vec{k}'\sigma'\tau'\sigma''\tau'' | \delta\tilde{U} | \vec{k}\sigma\tau\sigma'''\tau''' \rangle$$

(factor of 4)

$$a^+ a^+ a a a^+ a a^+ a^+ a a$$

$$\langle F | a^{+}_{\frac{\vec{K}}{2}+\vec{k}'\sigma'\tau'} a^{+}_{\frac{\vec{K}}{2}-\vec{k}'\sigma''\tau''} a_{\frac{\vec{K}}{2}-\vec{k}'\sigma''\tau''} a_{\frac{\vec{K}}{2}+\vec{k}'\sigma'\tau'} | F \rangle$$

$$2\, \delta_{\frac{\vec{K}}{2}+\vec{k}', \frac{\vec{K}}{2}+\vec{k}'} \, \delta_{\frac{\vec{K}}{2}-\vec{k}', \frac{\vec{K}}{2}-\vec{k}'} \qquad \sum_{\vec{K}\vec{k}} \sum_{\sigma'\tau'\sigma''\tau''}$$

$$\delta_{\sigma'\sigma'} \, \delta_{\sigma''\sigma''}$$

$$\boxed{\begin{aligned} &= \frac{1}{2} \sum_{\sigma\sigma'\sigma''\sigma'''} \sum_{\tau'\tau''\tau'''} \sum_{\vec{K}\vec{k}'} \langle \vec{k}'\sigma'\tau'\sigma''\tau'' | \delta\tilde{U} | \vec{k}-\tfrac{\vec{K}}{2}\, \sigma\tau\sigma'''\tau''' \rangle \times \\ &\langle \vec{k}-\tfrac{\vec{K}}{2}\, \sigma\tau\sigma'''\tau''' | \delta\tilde{U}^{+} | \vec{k}'\sigma'\tau'\sigma''\tau'' \rangle\, \Theta\left(k_F - \left|\tfrac{\vec{K}}{2}+\vec{k}'\right|\right) \times \\ &\Theta\left(k_F - \left|\tfrac{\vec{K}}{2}-\vec{k}'\right|\right) \end{aligned}}$$

So $\hat{\Lambda}^{\lambda}_{\sigma\tau}(k) = \Big[1 + \sum_{\sigma'\tau'}\Big(\langle \vec{k}\sigma\tau\sigma'\tau' | \delta\tilde{U} | \vec{k}\sigma\tau\sigma'\tau'\rangle$

$+ \langle \vec{k}\sigma\tau\sigma'\tau' | \delta\tilde{U}^{+} | \vec{k}\sigma\tau\sigma'\tau'\rangle\Big)\Big]\Theta(k_F - k)$

$+ \frac{1}{2}\sum_{\sigma'\sigma''\sigma'''}\sum_{\tau'\tau''\tau'''}\sum_{\vec{K}\vec{k}'}\langle \vec{k}'\sigma'\tau'\sigma''\tau'' | \delta\tilde{U} | \vec{k}-\frac{\vec{K}}{2}\sigma\tau\sigma'''\tau'''\rangle \times$

$\langle \vec{k}-\frac{\vec{K}}{2}\sigma\tau\sigma'''\tau''' | \delta\tilde{U}^{+} | \vec{k}'\sigma'\tau'\sigma''\tau''\rangle \times$

$\Theta\left(k_F^{\tau'} - \left|\frac{\vec{K}}{2} + \vec{k}'\right|\right)\Theta\left(k_F^{\tau''} - \left|\frac{\vec{K}}{2} - \vec{k}'\right|\right)$

$$\sum_{\vec{K}\vec{k}'} \to \frac{1}{(2\pi)^6}\int d^3K \int d^3k'$$

$$\langle \hat{\Lambda}^{\infty}_{N\sigma}(k)\rangle = \int d^3r\, \Theta(k_F - k)$$

$k_F(r)$

$$= 4\pi \int_0^{\infty} dr\, r^2\, \Theta(k_F - k)$$

$$\frac{1}{(2\pi)^3}\sum_{\sigma}\int d^3k \langle \hat{\Lambda}^{\infty}_{N\sigma}(k)\rangle = N$$

$$\frac{1}{(2\pi)^3}\sum 4\pi \int^{\infty} dk\, k^2 \langle \hat{\Lambda}^{\infty}_{N\sigma}(k)\rangle = N$$

$(2\pi)^3$ σ ν_a

$$\langle \hat{n}^{\lambda}_{N\sigma}(k) \rangle$$

$$[UHU^{+}] = [H]$$

$$\int d^3k \, d^3k' \, \langle p | U | k \rangle \langle k | H | k' \rangle \langle k' | U^{+} | p' \rangle$$

$$U_{ij} = \frac{1}{\sqrt{\frac{2}{\pi}} p_i \sqrt{w_i}} \tilde{U}_{ij} \frac{1}{\sqrt{\frac{2}{\pi}} k_j \sqrt{w_j}}$$

$$\hat{U} \hat{n}_{N\sigma}(k) \hat{U}^{+}$$

$$\sim \frac{1}{2} \sum_{\sigma\sigma''\sigma'''} \sum_{\tau\tau''\tau'''} \sum_{\vec{K}\vec{k}'} \langle \vec{k}'\sigma\tau'\sigma''\tau'' | \tilde{U} | \vec{k} - \tfrac{\vec{K}}{2} \sigma\tau\sigma'''\tau''' \rangle$$

$$\times \langle \vec{k} - \tfrac{\vec{K}}{2} \sigma\tau\sigma'''\tau''' | \tilde{U}^{+} | \vec{k}'\sigma'\tau'\sigma''\tau'' \rangle \times$$

$$\Theta\left(k_F^{\tau'}(r) - \left|\tfrac{\vec{K}}{2} + \vec{k}'\right|\right) \Theta\left(k_F^{\tau''}(r) - \left|\tfrac{\vec{K}}{2} - \vec{k}'\right|\right)$$