

Longitudinal response functions for ^{40}Ca from quasi-elastic electron scattering

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Longitudinal response functions were extracted from quasi-elastic electron scattering data on ^{40}Ca at angles of 45° , 90° , and 140° with bombarding energies ranging from 130 to 840 MeV and for constant three-momentum transfers of 300, 330, 370, 410 and 450 MeV/c. Contrary to previously reported results, the present longitudinal response functions show no more than 20% missing strength when compared to the relativistic Fermi gas model. Calculations employing wavefunctions generated from a relativistic Hartree potential and using an effective momentum approximation yield results that are in much closer agreement with the experimental data and indicate essentially no longitudinal suppression

The basic assumption of quasi-elastic electron scattering from the nucleus is that the virtual photon is absorbed on one nucleon while the other nucleons act as spectators. This simple model predicts a broad peak in the cross section with its maximum located ap-

proximately at the energy loss corresponding to elastic scattering from the free nucleon. The width and shape of this peak are determined by the Fermi motion of the bound nucleons. In order for this quasi-elastic assumption to be valid the three-momentum transfer, q , to the nucleus must correspond to a reduced wavelength of the virtual photon less than the internucleon separation, or $q > 250$ MeV/c.

During the past decade experiments with energy and momentum transfers that meet the quasi-elastic conditions have been carried out for ^{12}C [1], ^{40}Ca [2,3], ^{48}Ca [2,3], ^{56}Fe [3,4], and ^{238}U [5]. Unpublished data for ^{208}Pb [6] are also available. Early calculations using the relativistic Fermi gas model were

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reasonably successful in reproducing approximately the positions and widths of the quasi-elastic peaks observed in these measurements. However, there were apparent discrepancies between the theoretical predictions and the experimentally separated longitudinal and transverse structure functions. Large suppressions were reported in the total longitudinal strength as compared to the Fermi gas calculations (on the order of 50% for ^{56}Fe [4] and 45% for ^{40}Ca [3]). This original finding in ^{56}Fe led to many papers invoking varied mechanisms to explain the longitudinal suppression while not disturbing the observed agreement with the transverse structure function [7–9]. But discrepancies also exist between the different experimental data sets. The ^{238}U data from Bates [5] showed essentially the full expected longitudinal strength at $q = 500 \text{ MeV}/c$, while the ^{208}Pb data from Saclay showed approximately 40% of the expected longitudinal strength at the same momentum transfer.

The present experiment was performed at the MIT Bates Linear Accelerator Center in order to study carefully the question of longitudinal strength suppression in ^{40}Ca . The Bates recirculated linac can deliver electron beam currents up to $40 \mu\text{A}$ at energies approaching 1 GeV at 1% duty factor. The scattered electrons were momentum analyzed by the ELSSY dispersion matched magnetic spectrometer [10]. The useful momentum acceptance of this spectrometer is about 6%, so a number of different magnet settings was required to map out the entire available inelastic response.

This experiment on ^{40}Ca was designed to take advantage of the higher energy capability of the Bates recirculated linac in order to obtain momentum transfers up to $600 \text{ MeV}/c$ at a scattering angle of 45.5° . Quasi-elastic scattering data were obtained at this angle for energies from 348 to 841 MeV on ^{40}Ca [11]. These data were combined with those from a previous Bates experiment [2] at scattering angles of 90° and 140° to obtain Rosenbluth separations of the quasi-elastic peak for three-momentum transfers up to $450 \text{ MeV}/c$.

In the plane wave Born approximation the inclusive quasi-elastic cross section can be written as

$$\frac{d^2\sigma}{d\Omega_e dE_f} = \sigma_M \left[\frac{q_\mu^4}{q^4} S_L + \left(\tan^2 \frac{\theta}{2} - \frac{q_\mu^2}{2q^2} \right) S_T \right], \quad (1)$$

where $q_\mu^2 = \omega^2 - q^2$ is the four-momentum squared, ω is the energy transferred to the nucleus, Ω_e is the solid angle, E_f is the final electron energy and σ_M is the Mott cross section. At large scattering angles the $\tan^2 \frac{1}{2}\theta$ term causes the total response to be dominated by S_T , the transverse response function. In previous measurements where $\theta \geq 90^\circ$ the longitudinal response function, S_L , contributed only (10–20)% of the total. Therefore, small uncertainties in the cross section measurements would lead to relatively large uncertainties in the extracted value of S_L . In the experiment reported here the longitudinal response at 45.5° contributes (30–60)% of the total cross section.

Another source of uncertainty in the Rosenbluth separation is the Coulomb distortion of the electron. Eq. (1) is only valid for plane waves, and it can be applied to experimental results only after Coulomb distortion effects are removed from the data. These Coulomb distortion effects were recently investigated for the $(e, e'p)$ reaction in the quasi-elastic region [13], and it was found that an effective momentum approximation (EMA) is quite accurate in correcting these distortion effects for nuclei as light as calcium. In this approximation the Coulomb field changes both the incoming and outgoing momenta, $p_{i,f}$, to an effective momentum $p_{i,f}^{\text{eff}} = p_{i,f} + 1.5\alpha Z/R_C$, where R_C is the mean uniform radius for nuclear charge. The effective three-momentum transfer is then given by $q^{\text{eff}} = p_i^{\text{eff}} - p_f^{\text{eff}}$. The subscripts i and f refer to the initial and final states, respectively, and the labels eff and pw refer, respectively, to the effective values and the plane wave values. In addition, due to the focusing effect of the nuclear Coulomb field, the incoming electron flux must be renormalized by a factor $(p_i^{\text{eff}}/p_i)^2$. Thus,

$$\left(\frac{d^2\sigma}{d\Omega_e dk_o'} \right)_{\text{eff}} = \left(\frac{p_i^{\text{eff}}}{p_i} \right)^2 \left(\frac{d^2\sigma(p_i^{\text{eff}}, p_f^{\text{eff}})}{d\Omega_e dk_o'} \right)_{\text{pw}}, \quad (2)$$

Eq. (2) can be used to compare theoretical calculations to the cross section data or to convert the experimental cross sections to equivalent plane wave cross sections.

The present data were compared to two different theoretical models. The first of these was the relativistic Fermi gas model which is described by Alberico et

al [12] and was the baseline model for comparison in previous measurements. The only free parameters in this model are the nuclear Fermi momentum which was set to 240 MeV/ c and the effective nuclear binding energy which was set to zero. The second model was a single particle calculation by Jin et al [13,14], using nuclear wave functions derived from a relativistic Hartree model. In this calculation, in the spirit of the quasi-elastic assumption, the cross section was the incoherent sum of single-nucleon promotions from occupied states. The promoted nucleon does not necessarily leave the nucleus without further interaction. However, since these subsequent interactions do not affect the (e, e') cross sections, the potential used in comparisons with the inclusive quasi-elastic data does not contain an absorptive (imaginary) part. The free Dirac current operator, including the anomalous magnetic moments and nucleon form factors, was used in these calculations and the bound and continuum nucleon wave functions used were solutions to the Dirac equation with scalar and vector potentials calculated in the relativistic Hartree model.

It has been emphasized previously [14] that within the framework of a given model, it is necessary that the total response functions at constant bombarding energy and angle be understood before trying to understand the results of a Rosenbluth separation. The data at constant bombarding energy used in the present separations at scattering angles of 45°, 90° and 140° were compared to the Hartree model, and good agreement was found between theory and experiment. Representative examples of this comparison at 45° are shown in fig. 1. Comparisons with our data at 90° and 140° appear in ref. [14]. In general there is good agreement between theory and experiment except at the highest bombarding energies where contributions due to excitation of the Δ resonance become important.

The longitudinal response functions obtained from Rosenbluth separations performed on the experimental data are shown in fig. 2 for six values of q . Also shown in fig. 2 are calculations based on the relativistic Fermi gas model [12] (dotted lines) and on the relativistic Hartree shell model [14] (solid lines). The relativistic Fermi gas model predictions show rather poor agreement with the data, which define a peak that is noticeably broader and lower than that pre-

dicted by this theory. The relativistic Hartree shell model calculations, on the other hand, are generally in good quantitative agreement with the data except at the two lowest momentum transfers which are on the margin of the quasielastic region.

As mentioned above, a point of controversy has been the total experimental longitudinal strength, taking the predictions of the relativistic Fermi gas model as a benchmark. In order to address this question, our longitudinal response functions were integrated over the same range of ω as in ref. [3] and divided by the integrated strength predicted by the relativistic Fermi gas model. The results are shown as solid squares in fig. 3. Also shown in fig. 3 as solid circles are the same ratios reported by Meziani et al [3]. There is a clear disagreement between these two experimental results. The present measurements indicate a reduction of the longitudinal strength of less than 20% from the relativistic Fermi gas value, whereas the results of Meziani et al, exhibit a reduction of up to 45% relative to the Fermi gas prediction.

In order to be certain that the observed discrepancy was not attributable to differences in data analysis, the radiatively corrected differential cross sections from Meziani et al [3] were analyzed using the same codes as were used to analyze the present data. This analysis reproduced the results of Meziani et al within the quoted errors. It therefore appears that the source of the discrepancy between these two sets of measurements does not lie in the analysis of the radiatively corrected data.

In conclusion we note that the present results for the total longitudinal strength for quasi-elastic electron scattering from ^{40}Ca are no more than 20% below the predictions of the relativistic Fermi gas model. Calculations using our relativistic Hartree model produce results that are in agreement with the cross sections in the range of electron energy 470–840 MeV and with extracted longitudinal response functions for $q > 330$ MeV/ c . These results are in strong disagreement with previous experiments which reported reductions of as much as 45% in the measured total longitudinal strength when compared to the predictions of the relativistic Fermi gas model. The cause of the discrepancy is not understood at this time, and its resolution will probably require further experimental investigation.

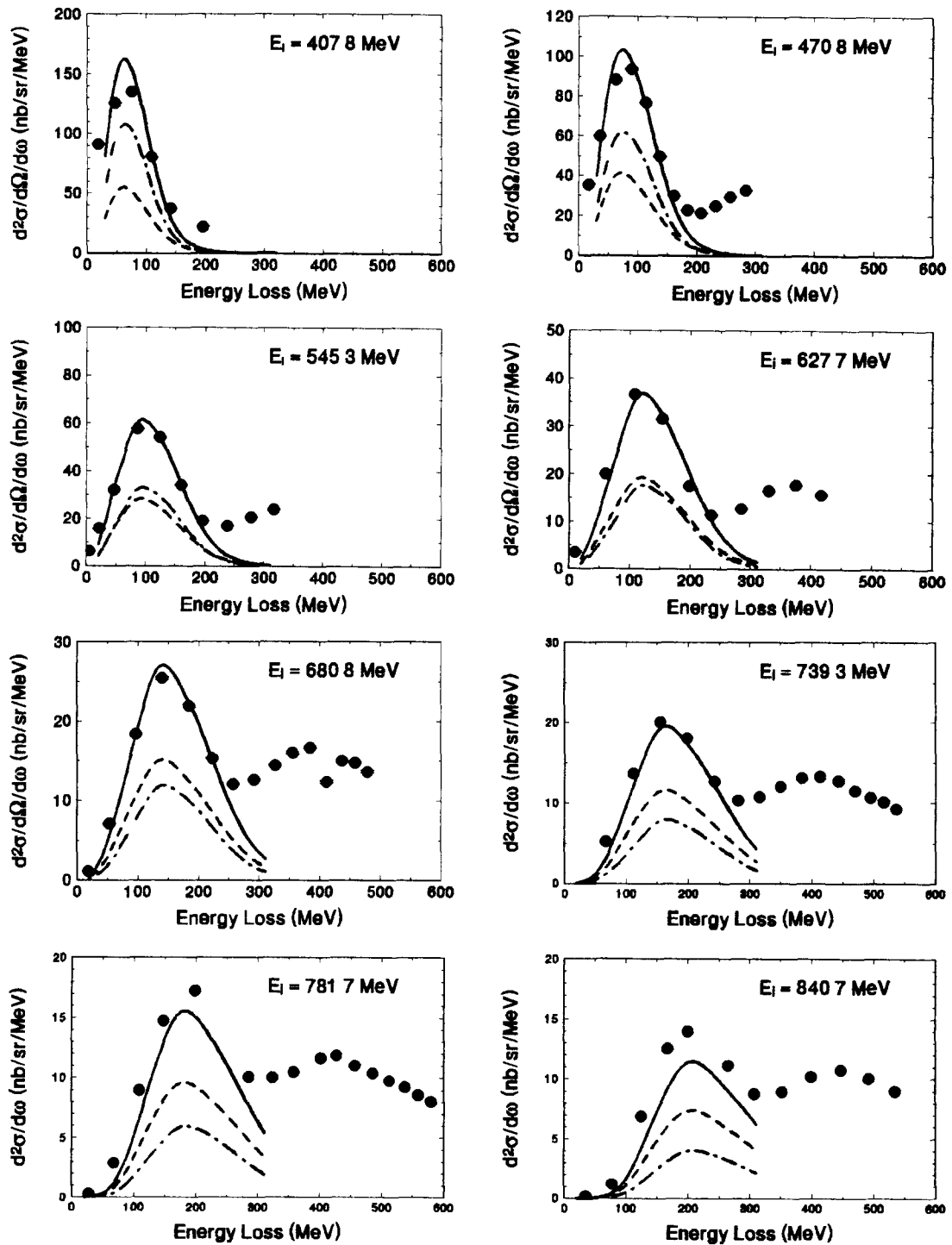


Fig 1 Comparison of theoretical [14] and experimental differential cross sections at 45.5° for several bombarding energies. The solid curves are the total theoretical differential cross sections, the dot-dashed curves are the longitudinal contributions and the dashed curves are transverse contributions to the cross sections. The data from this experiment are shown as filled circles.

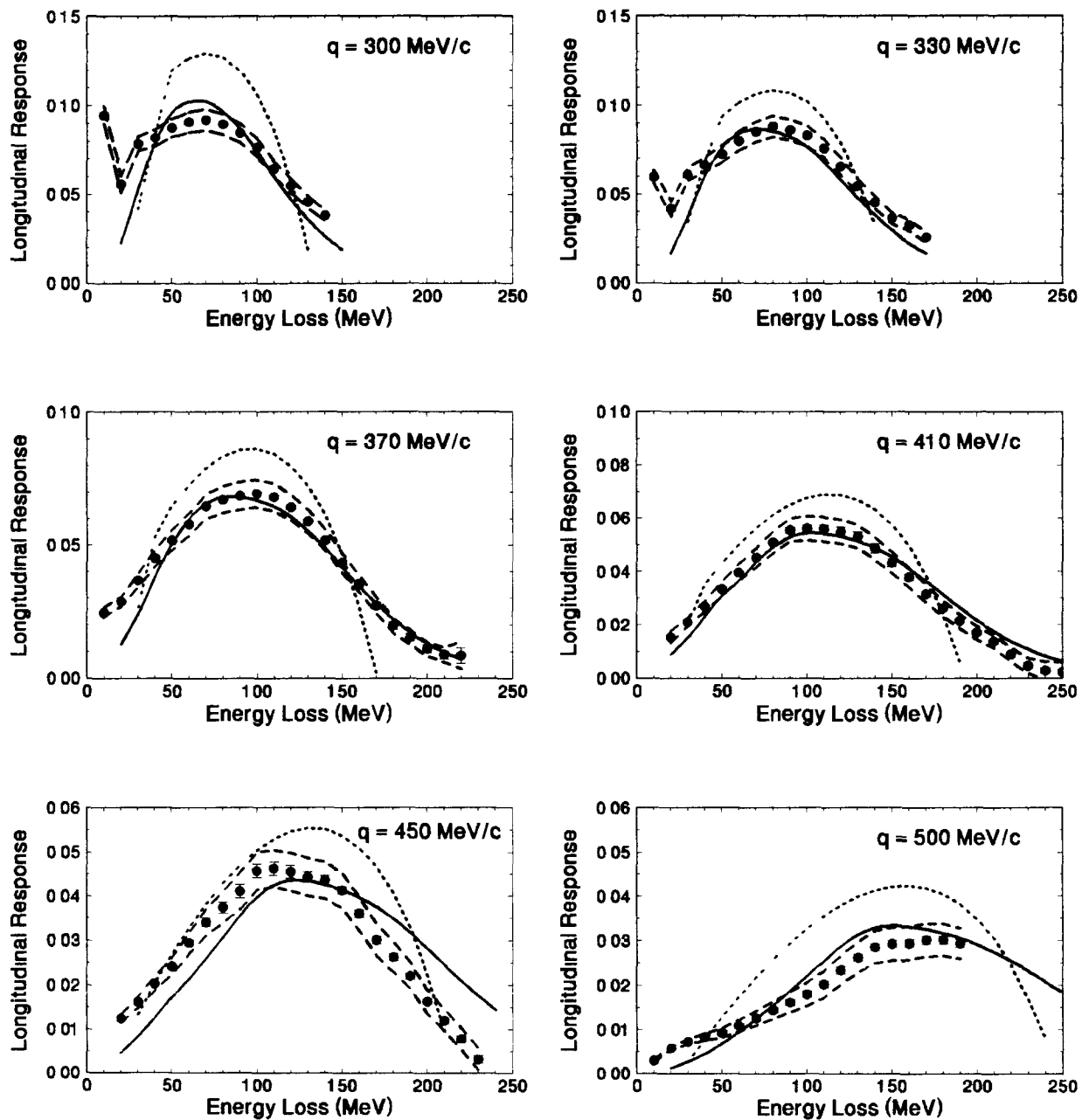


Fig. 2 Longitudinal response functions for ^{40}Ca . Filled circles: values measured in the present experiment, dashed lines: limits of estimated systematic errors for the present experiment, dotted line: prediction of the relativistic Fermi gas model [12], solid line: prediction of the relativistic Hartree model [13]. The error bars on the data points represent statistical errors only. The curves for the relativistic Fermi gas model were arbitrarily shifted 20 MeV toward larger ω for approximate alignment of the experimental and theoretical quasi-elastic peaks.

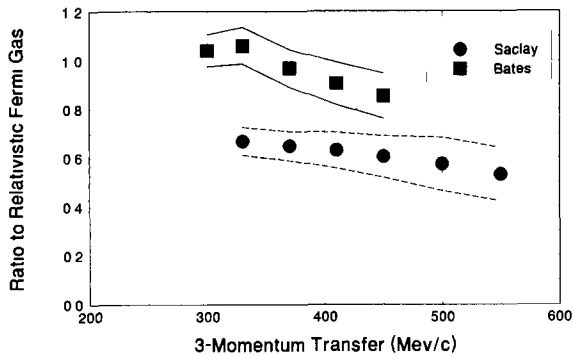


Fig. 3 Ratio of experimental total longitudinal strength for ^{40}Ca to predictions of the relativistic Fermi gas model as reported from Bates (present experiment) and from Saclay [3]. The solid lines are the systematic error limits on the Bates data, and the dashed lines are the systematic error limits on the Saclay data. Statistical errors are always smaller than the plotted symbols.

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