Operator evolution notes

A. J. Tropiano[1]

[1] *Department of Physics, The Ohio State University, Columbus, OH 43210, USA*

(Dated: September 23, 2019)

A. Building SRG unitary transformations

Diagonalize initial and evolved Hamiltonians which we will call $H(0)$ and $H(s)$, respectively. This gives $\psi_\alpha(0)$ and $\psi_\alpha(s)$ for each eigenvalue indexed by α. Then the SRG unitary transformation can be computed by taking a sum over outer products of the evolved and initial wave functions:

$$U(s) = \sum_{\alpha=1}^{N} |\psi_\alpha(s)\rangle \langle\psi_\alpha(0)| \,, \tag{1}$$

where N is the dimension of the Hamiltonian matrix. Here the weights are factored into the wave functions, thus $U(s)$ is unitless.

To evolve operators, we simply apply $U(s)$:

$$O(s) = U(s)O(0)U^\dagger(s), \tag{2}$$

where $O(0)$ is the bare operator.

B. Momentum projection operator: $a^\dagger_q a_q(k,k')$

Applying $a^\dagger_q a_q(k,k')$ to a wave function $\psi(k)$ returns $\psi(q)$. For the discrete case, $\psi(k_i)$ is an $N \times 1$ vector and $a^\dagger_q a_q(k_i,k_j)$ is an $N \times N$ matrix where i, $j = 1 \cdots N$. Then $a^\dagger_q a_q(k,k')$ acting on $\psi(k)$ is a matrix multiplication, implying a continuous integration over $d^3k/(2\pi)^3 = 2/(\pi k^2 dk)$ in spherical coordinates. Therefore, we include a factor of $\pi/(2k_i k_j \sqrt{w_i w_j})$ in $a^\dagger_q a_q(k_i,k_j)$ where w represents the momentum weights. In matrix form,

$$a^\dagger_q a_q(k_i,k_j) = \frac{\pi \delta_{k_i q}\delta_{k_j q}}{2k_i k_j \sqrt{w_i w_j}}, \tag{3}$$

which has units fm^3. To evolve operators, we apply $U(s)$ at this point. For mesh-independent figures, we must divide by an additional factor of $k_i k_j \sqrt{w_i w_j}$. This operator is inherently mesh-dependent based off discretizing $\delta_{k_i q}\delta_{k_j q}$ above.

C. Momentum distribution function: $\phi^2(k)$

We diagonalize the Hamiltonian for eigenvectors ψ_α. In the $^3\text{S}_1$-$^3\text{D}_1$ coupled channel, the S-component is given by $\psi_\alpha[:N]$ and the D-component by $\psi_\alpha[N:]$ where N is the length of the momentum mesh. Then the momentum distribution of the state α is given by,

$$|\phi_\alpha(k)|^2 = |\psi_\alpha[:N]|^2 + |\psi_\alpha[N:]|^2. \tag{4}$$

This satisfies the normalization condition $\sum_{i=1}^{N} |\phi(k_i)|^2 = 1$, implying that the factor $k^2 dk$ (or in the discrete case, $k_i^2 w_i$) is factored into the wave function. For mesh-independent figures, divide by $k_i^2 w_i$.