

Short-range correlation physics at low RG resolution

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APS April Meeting – Virtual Meeting

April 19, 2021

ajt, S.K. Bogner, and R.J. Furnstahl, arXiv:2006.11186

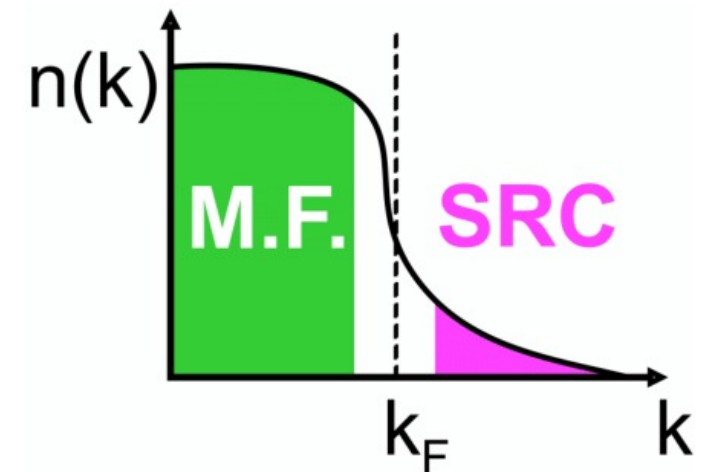
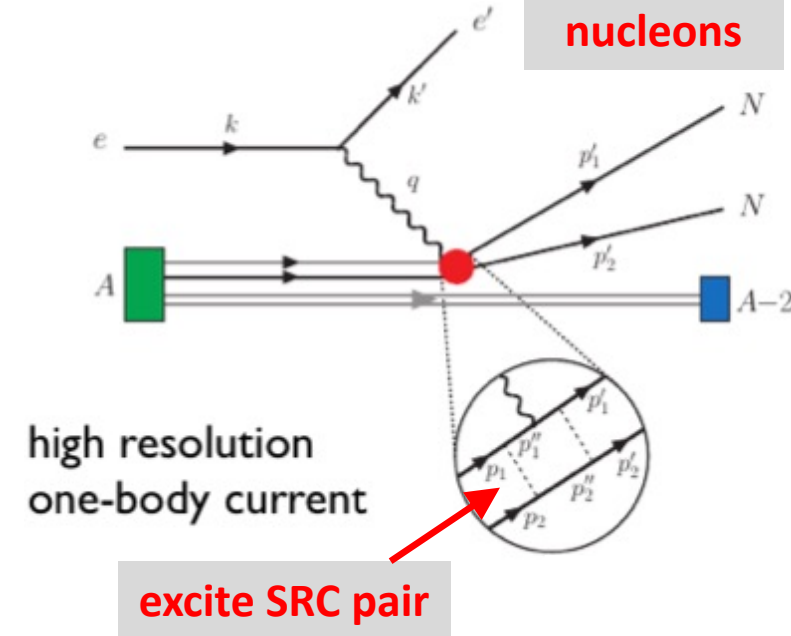
*Phys. Rev. C **102**, 034005 (2020)*

ajt, S.K. Bogner, and R.J. Furnstahl, in progress



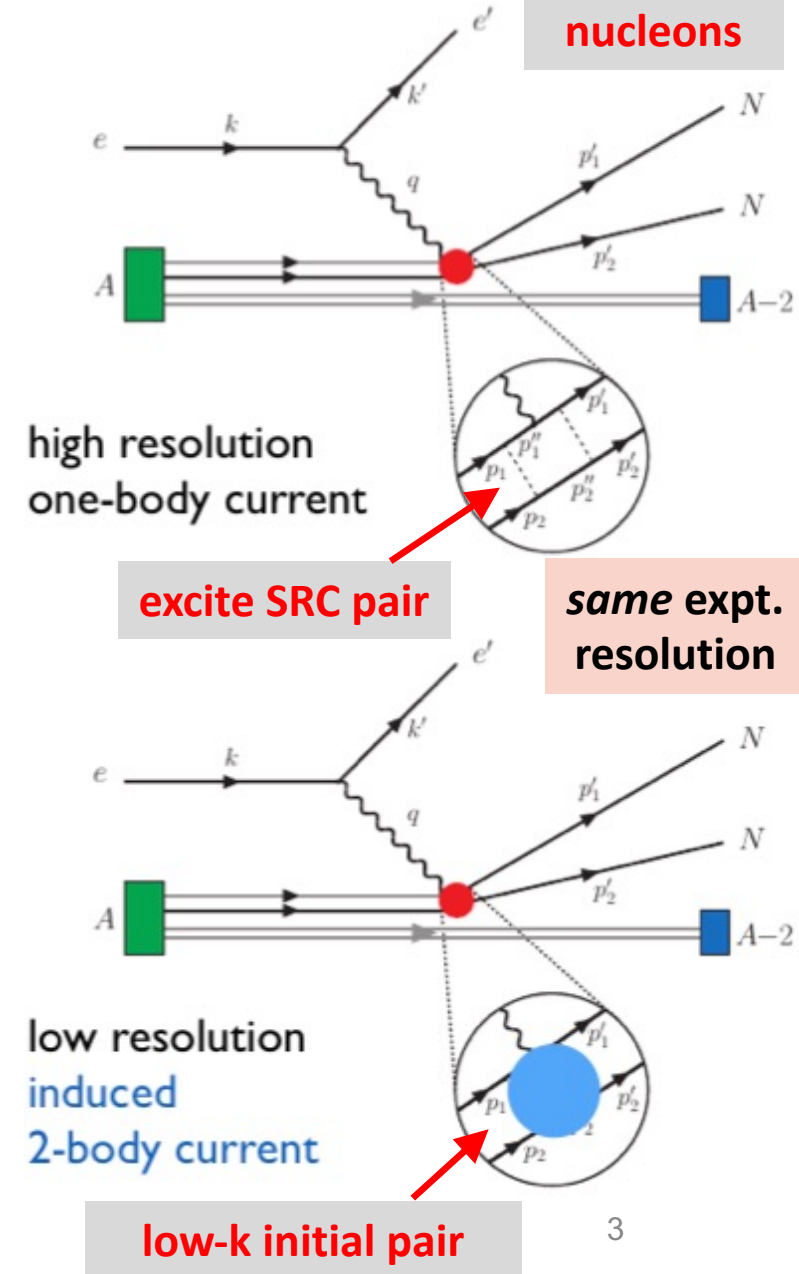
Motivation

- Recent experiments have been able to isolate processes where short-range correlation (SRC) physics is dominant and well accounted for by SRC phenomenology
- SRC physics at **high RG resolution**
 - SRC pairs are components in the nuclear wave function with relative momenta above the Fermi momentum



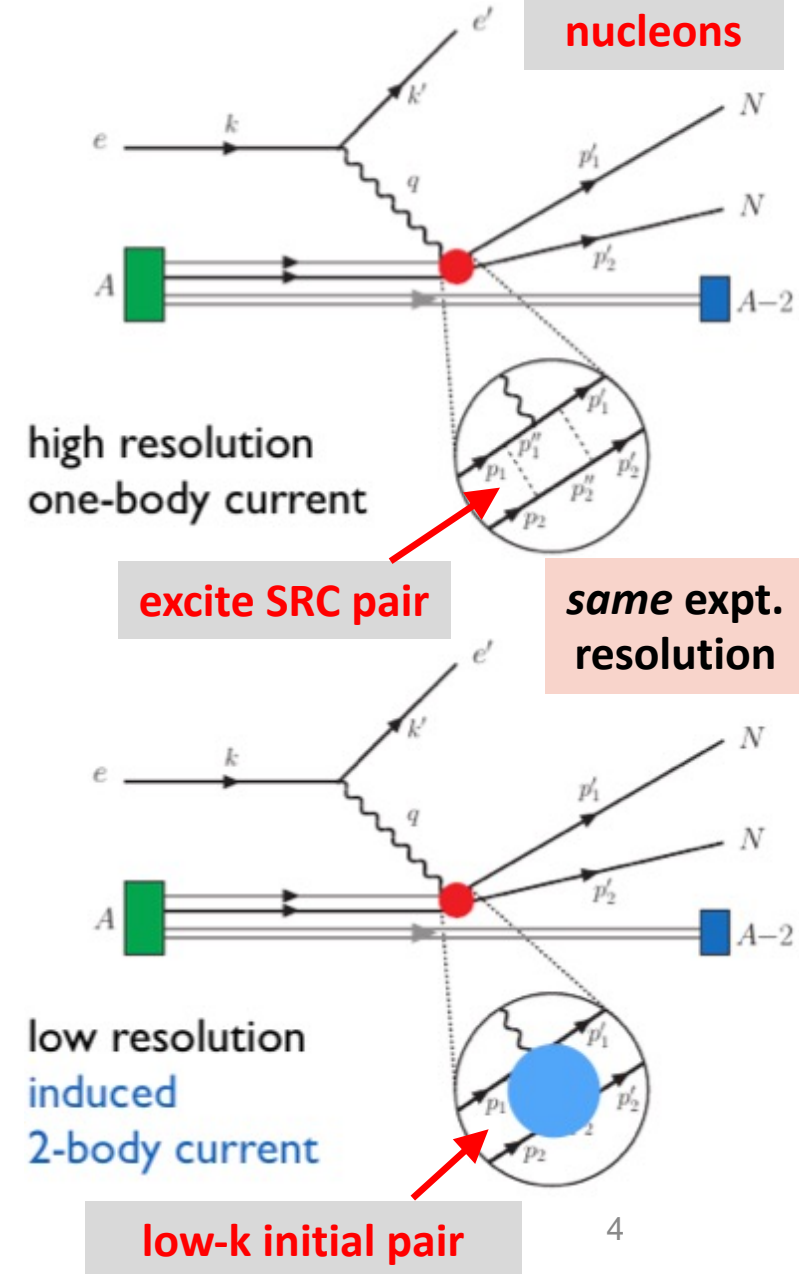
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 - The SRC *physics* is shifted into the reaction operators from the nuclear wave function (which becomes soft)
 - Operators do not become hard which simplifies calculations



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- Recent experiments have been able to isolate processes where short-range correlation (SRC) physics is dominant and well accounted for by SRC phenomenology
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 - The SRC *physics* is shifted into the reaction operators from the nuclear wave function (which becomes soft)
 - Operators do not become hard which simplifies calculations
- Experimental resolution (set by momentum of probe) is the same in both pictures**
- Same observables but different physical interpretation!**



Similarity Renormalization Group (SRG)

- Evolve operators to low RG resolution

$$O(s) = U(s)O(0)U^\dagger(s)$$

where $s = 0 \rightarrow \infty$ and
 $U(s)$ is unitary

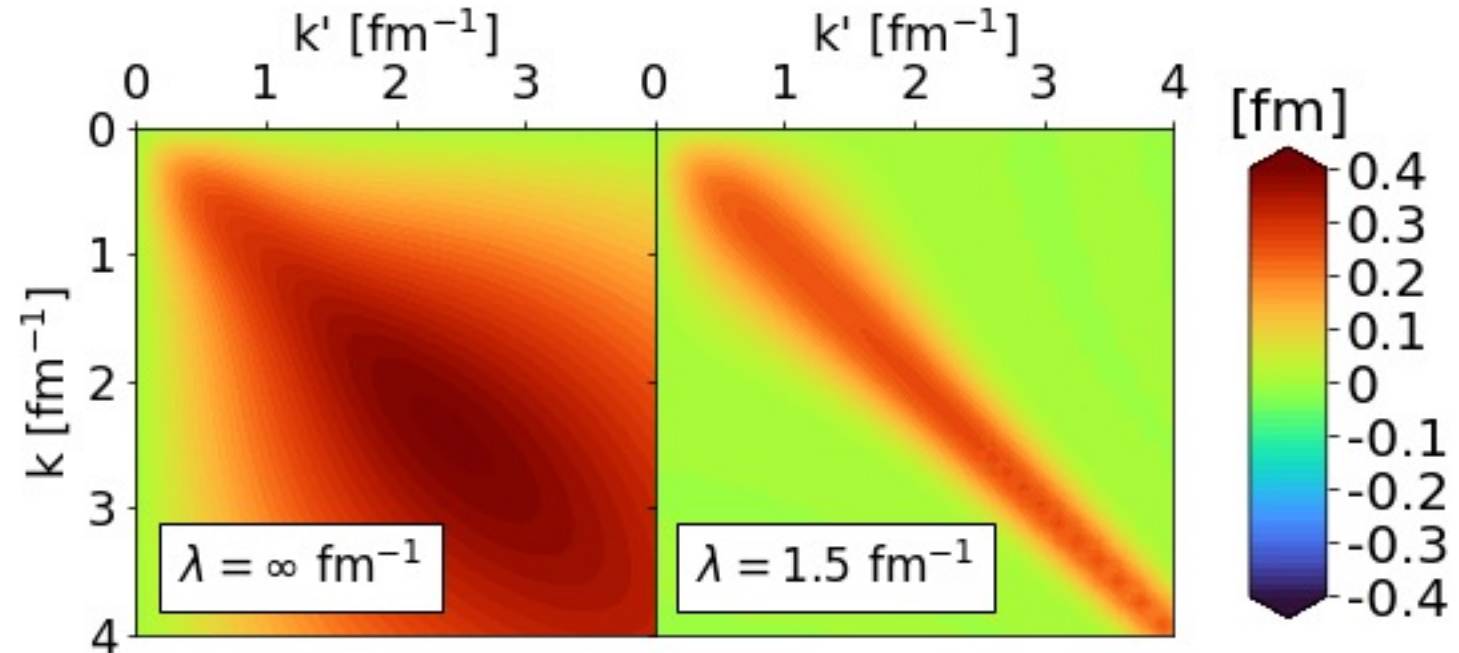


Fig. 1: Momentum space matrix elements of Argonne v18 (AV18) under SRG evolution in 1P_1 channel.

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- $\lambda = s^{-1/4}$ describes the decoupling scale of the RG evolved operator

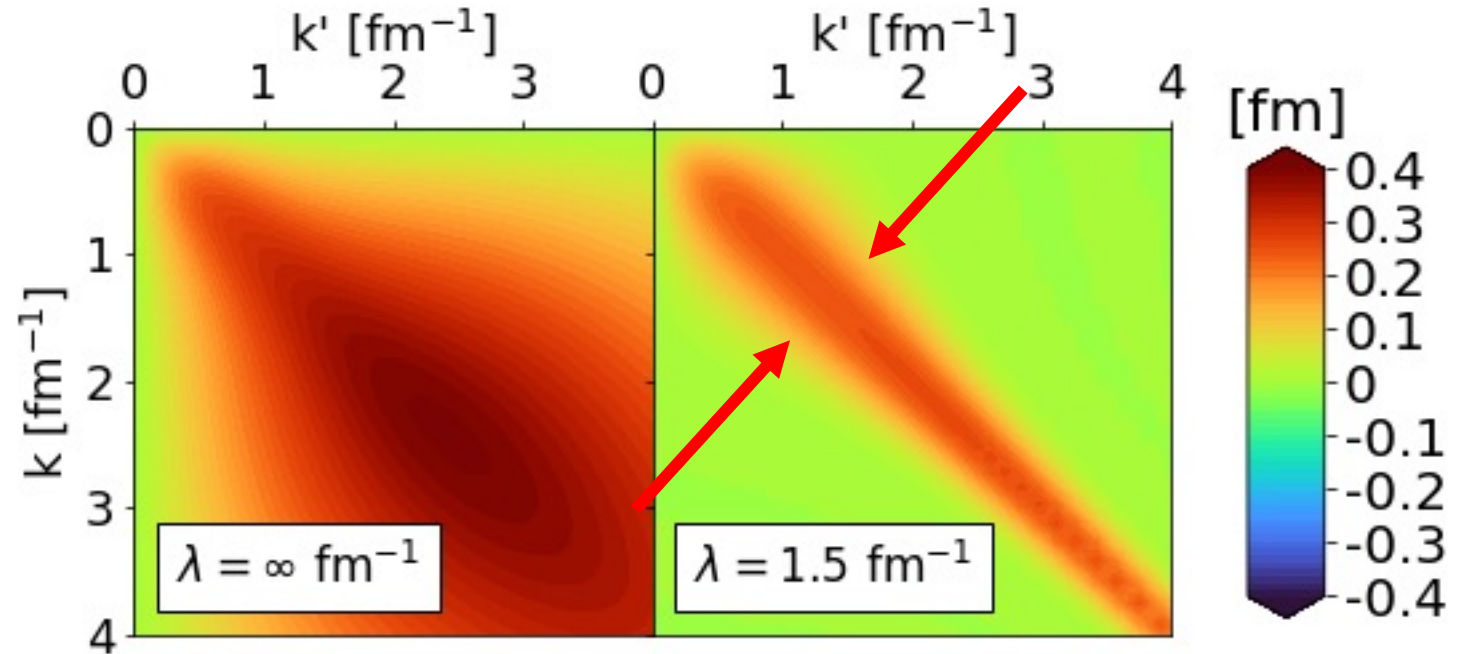


Fig. 1: Momentum space matrix elements of Argonne v18 (AV18) under SRG evolution in 1P_1 channel.

Deuteron wave function at low RG resolution

- AV18 wave function has significant SRC
- What happens to the wave function at low RG resolution?

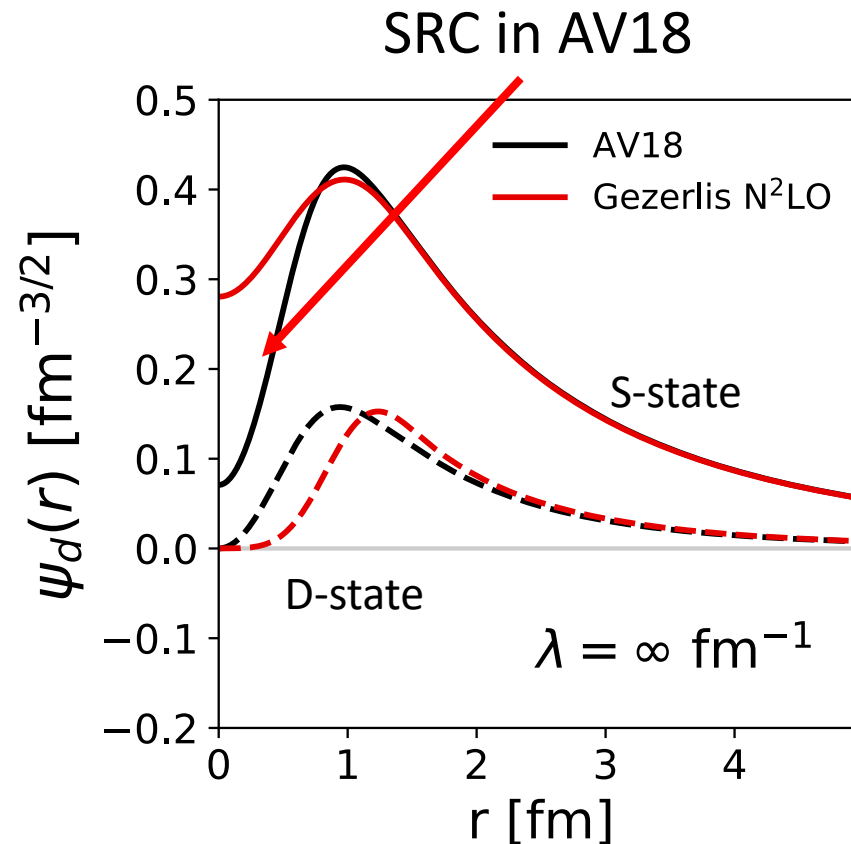


Fig. 2: SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis N2LO¹.

Deuteron wave function at low RG resolution

- SRC physics in AV18 is gone from wave function at low RG resolution
- Deuteron wave functions become soft and D-state probability goes down
- Observables such as asymptotic D-S ratio are the same

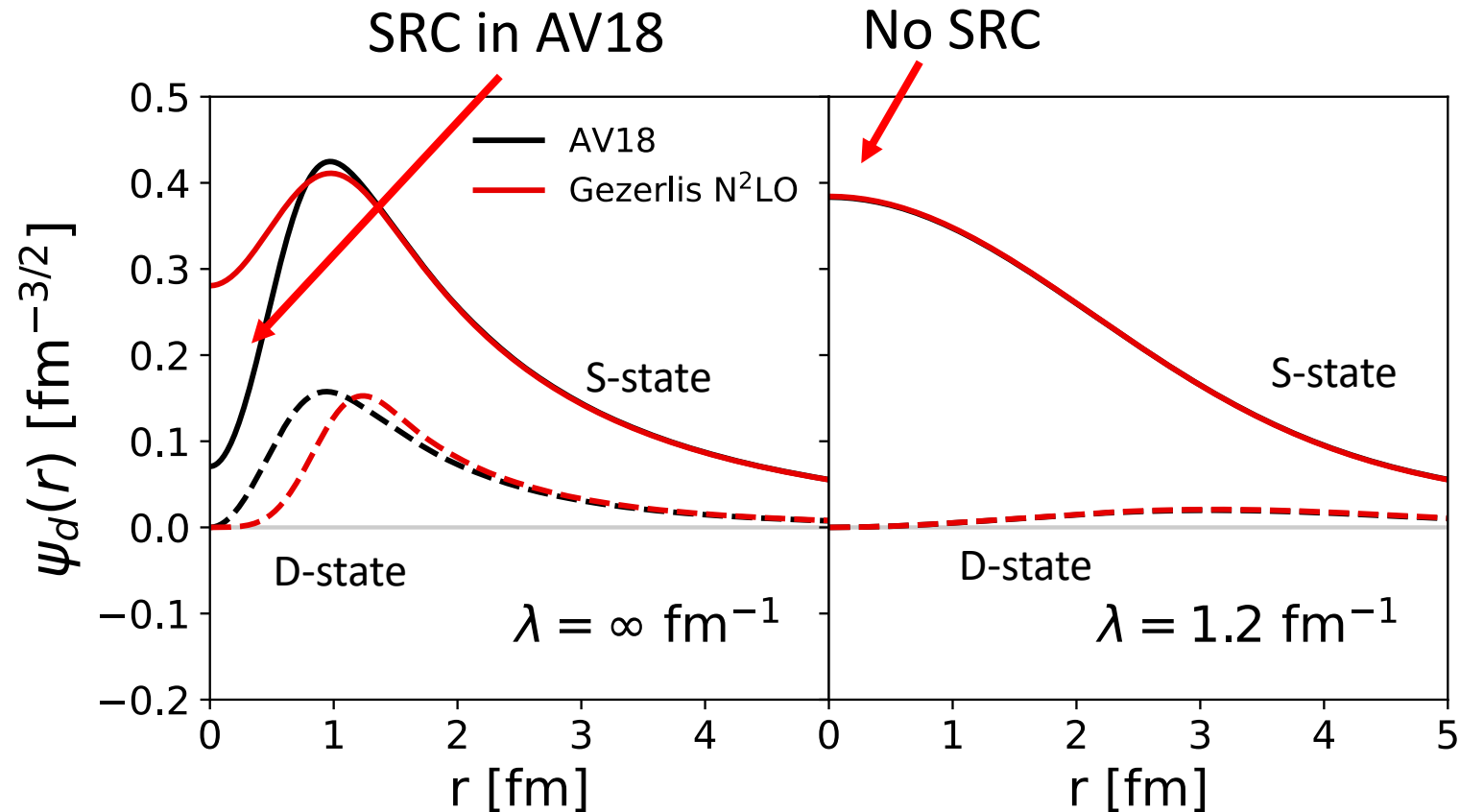


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Momentum distributions at low RG resolution

- Soft wave functions at low RG resolution
 - Where does the SRC physics go?

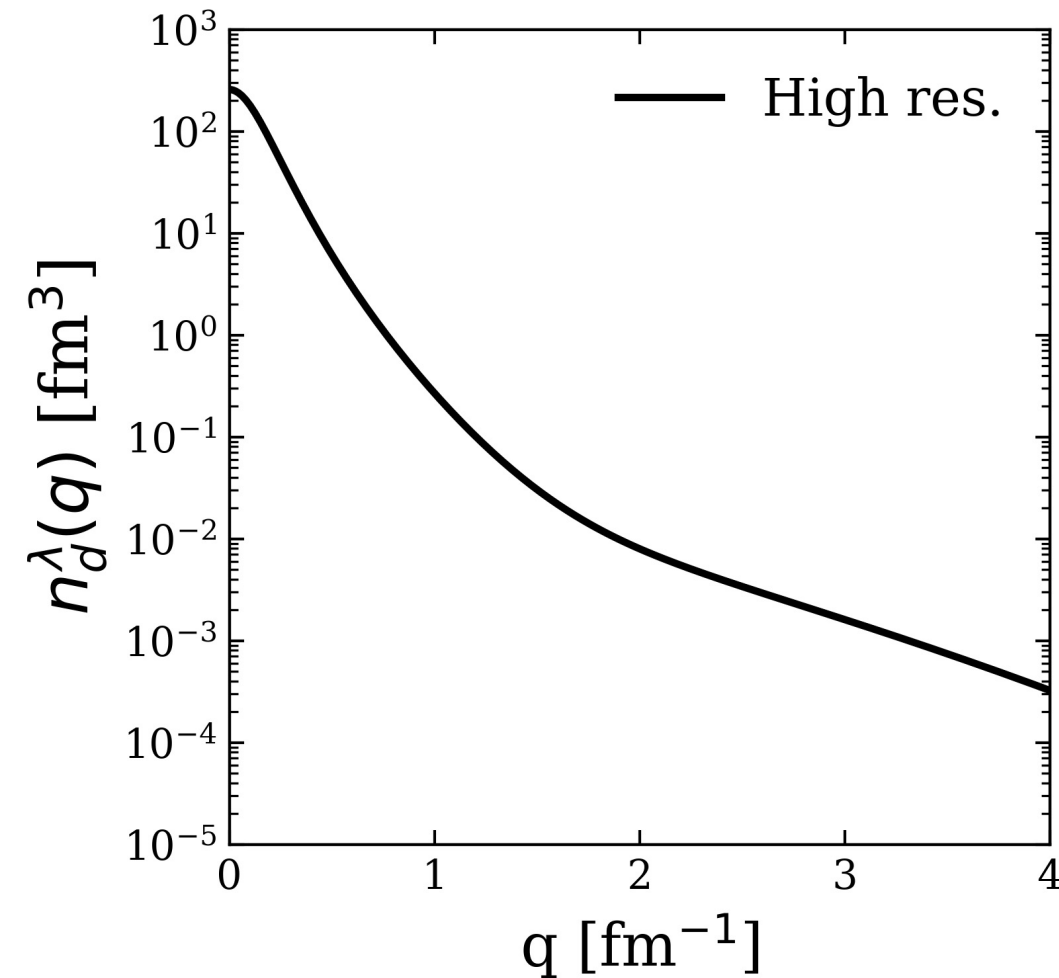
Momentum distributions at low RG resolution

- Soft wave functions at low RG resolution
 - Where does the SRC physics go?
- SRC physics shifts to the operators $\langle \psi_f^{hi} | U_\lambda^\dagger U_\lambda O^{hi} U_\lambda^\dagger U_\lambda | \psi_i^{hi} \rangle$
- Apply SRG transformations to momentum distribution operator

$$n^{hi}(\mathbf{q}) = a_q^\dagger a_q$$

$$U_\lambda = 1 + \frac{1}{4} \sum_{\mathbf{K}, \mathbf{k}, \mathbf{k}'} \delta U_\lambda^{(2)}(\mathbf{k}, \mathbf{k}') a_{\frac{\mathbf{K}}{2} + \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}'} a_{\frac{\mathbf{K}}{2} + \mathbf{k}'} + \dots$$

Momentum distributions at low RG resolution



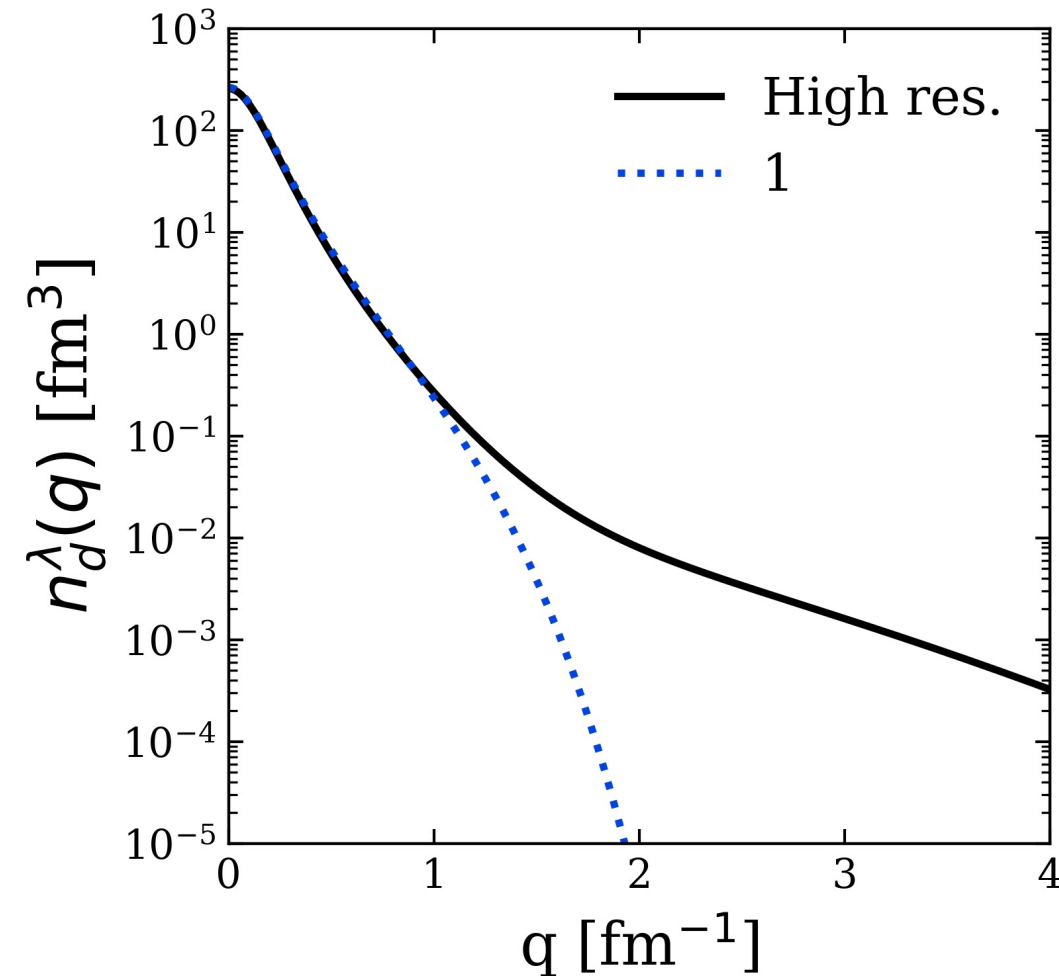
- Deuteron example

$$n^{lo}(\mathbf{q}) = (1 + \delta U) a_{\mathbf{q}}^\dagger a_{\mathbf{q}} (1 + \delta U^\dagger)$$

$$\langle \psi_d^{hi} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_d^{hi} \rangle$$

Fig. 3: Contributions to deuteron momentum distribution with AV18 and $\lambda = 1.35$ fm⁻¹.

Momentum distributions at low RG resolution



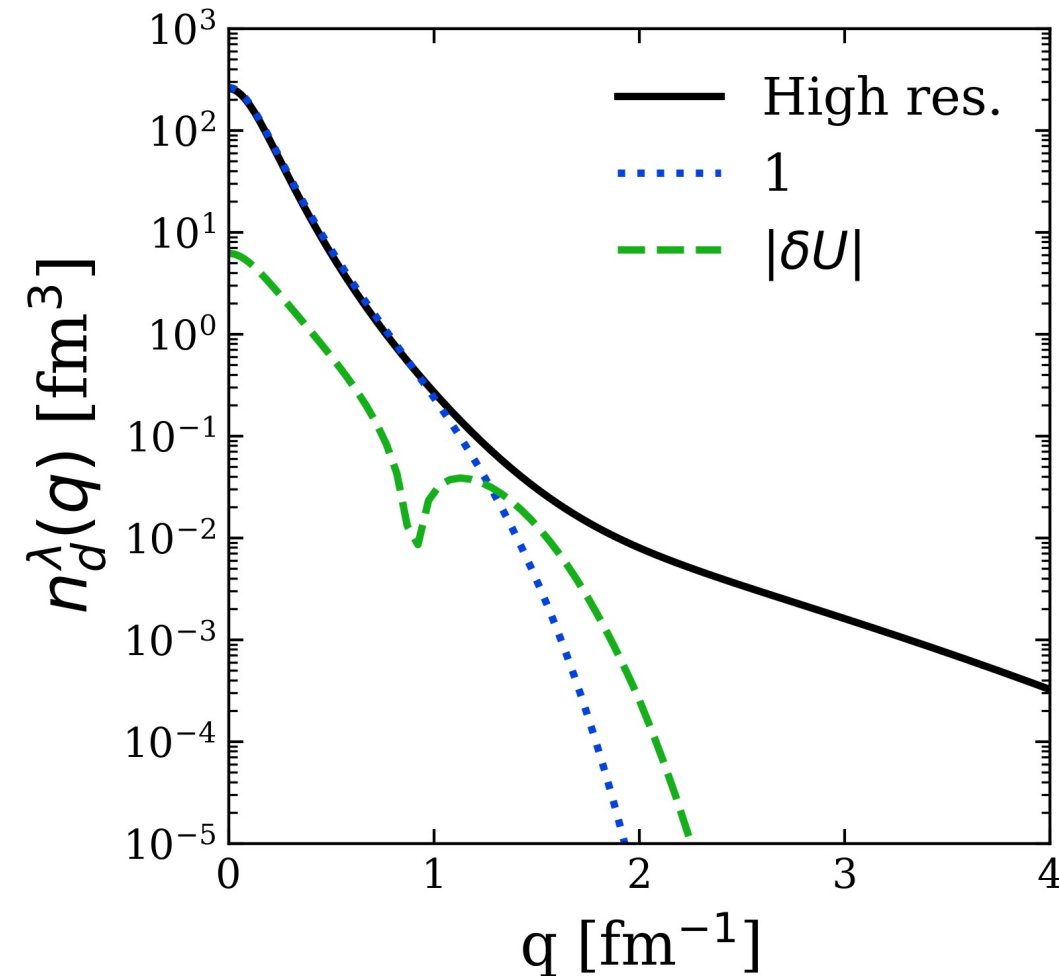
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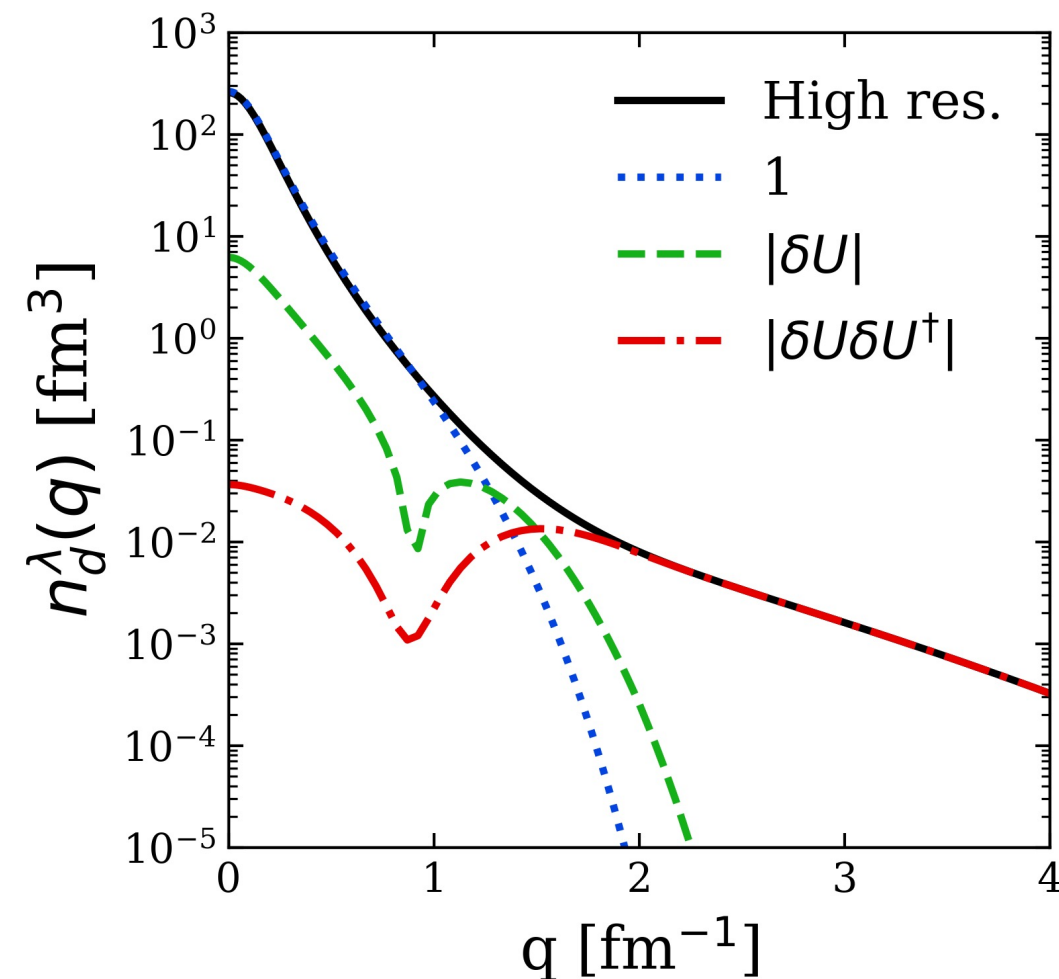
$$\langle \psi_d^{hi} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | \psi_d^{hi} \rangle$$

$$\langle \psi_d^{lo} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | \psi_d^{lo} \rangle$$

$$\langle \psi_d^{lo} | \delta U a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \delta U^{\dagger} | \psi_d^{lo} \rangle$$

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Momentum distributions at low RG resolution



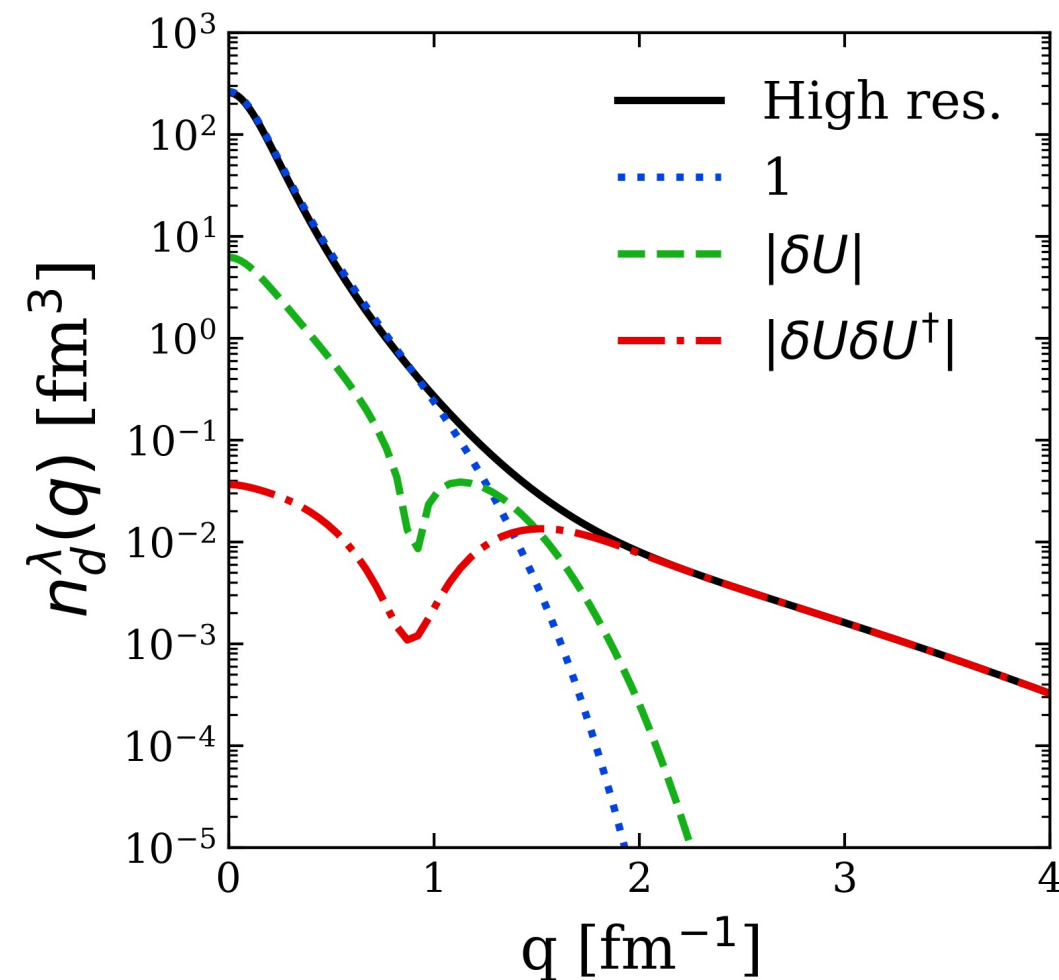
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Momentum distributions at low RG resolution



- For high- q , the $\delta U_\lambda \delta U_\lambda^\dagger$ 2-body term dominates

$$\approx \sum_{K,k,k'} \delta U_\lambda(\mathbf{k}, \mathbf{q}) \delta U_\lambda^\dagger(\mathbf{q}, \mathbf{k}') a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

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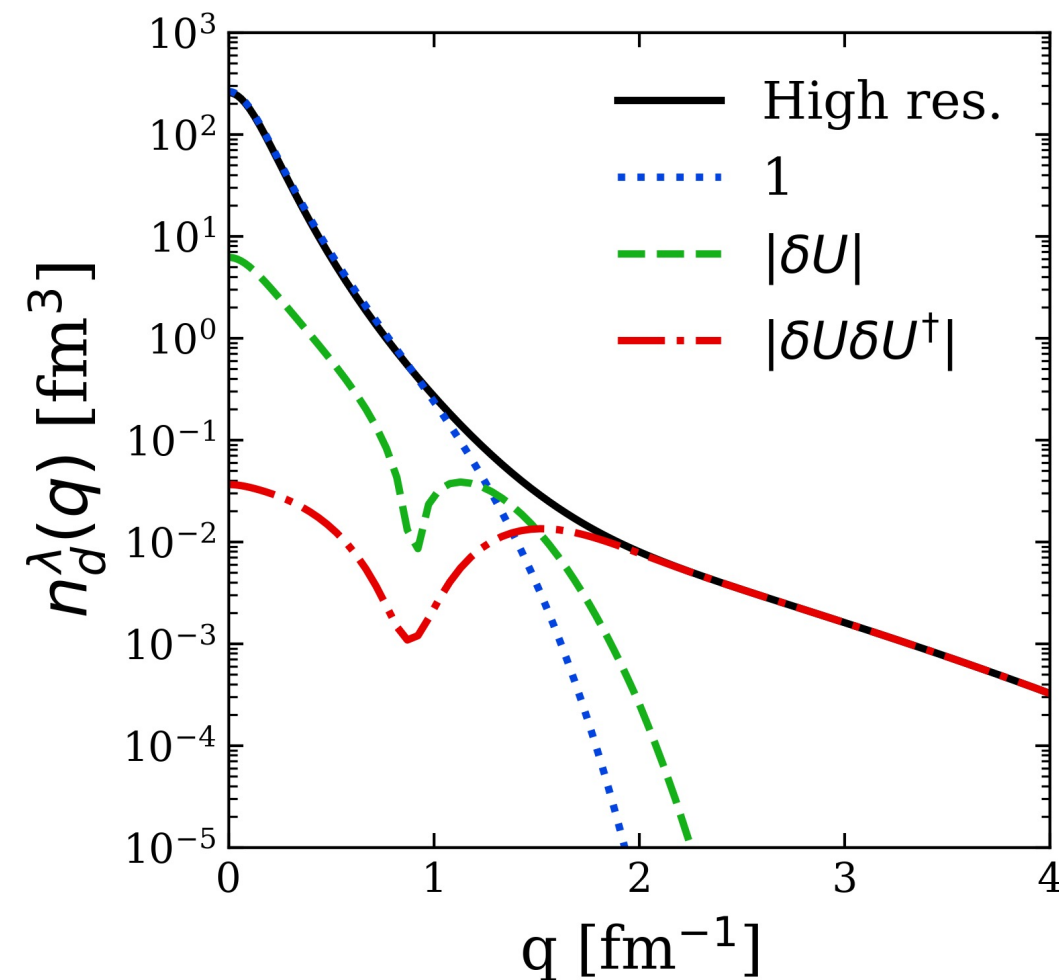


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↓

Factorization: $\delta U_\lambda(\mathbf{k}, \mathbf{q}) \approx F_\lambda^{lo}(\mathbf{k}) F_\lambda^{hi}(\mathbf{q})$

↓

$$\approx |F_\lambda^{hi}(\mathbf{q})|^2 \sum_{K,k,k'} F_\lambda^{lo}(\mathbf{k}) F_\lambda^{lo}(\mathbf{k}') a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

Momentum distributions at low RG resolution

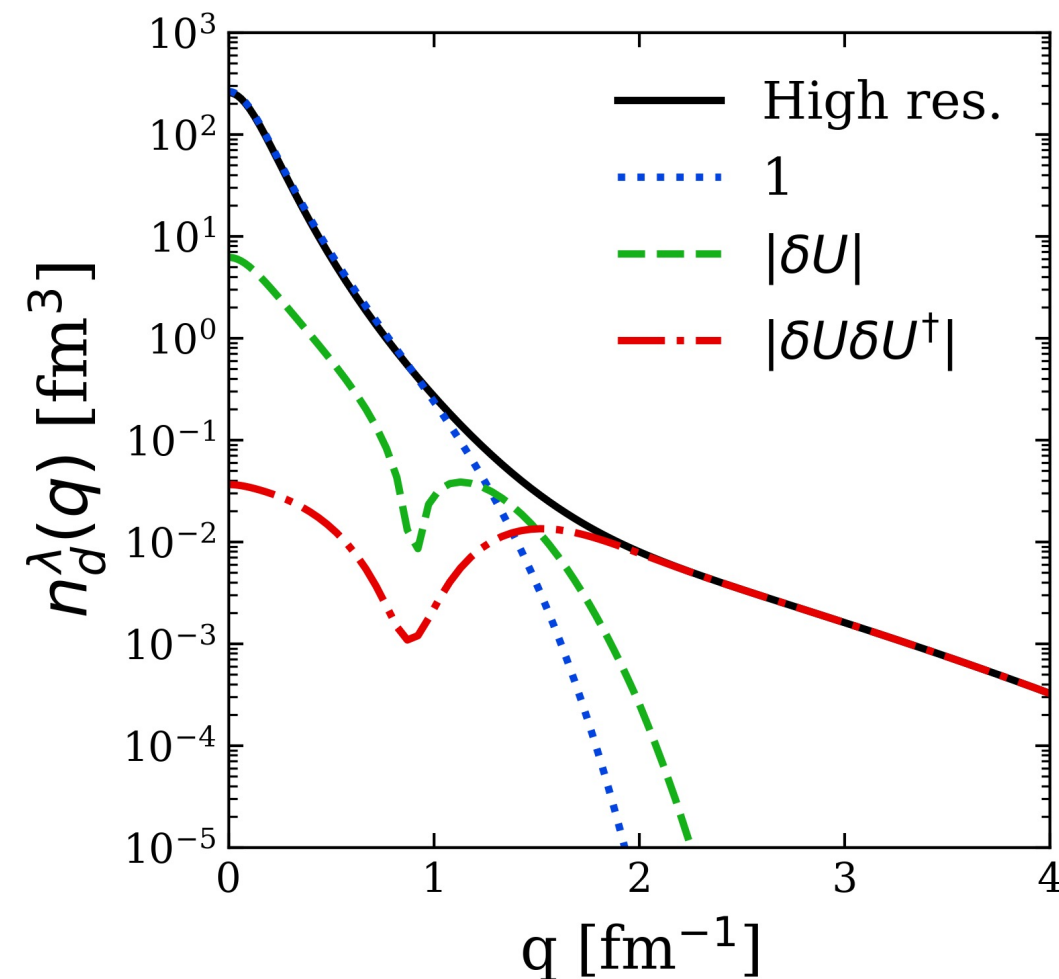


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Apply this strategy to nuclear momentum distributions using local density approximation (LDA)!

$$F_\lambda^{hi}(\mathbf{q})$$

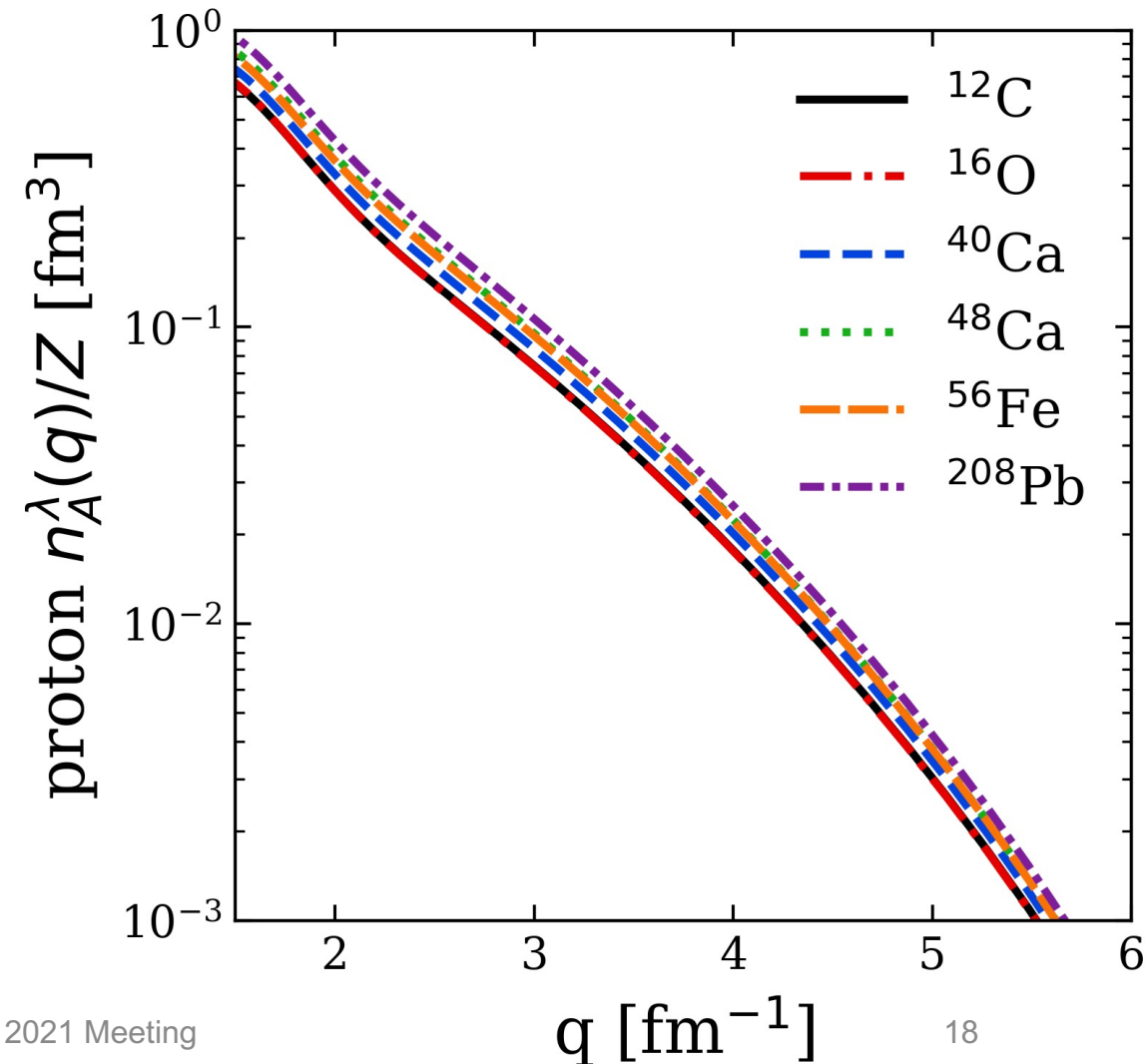
$$\approx |F_\lambda^{lo}(\mathbf{q})|^2 \sum_{\mathbf{K}, \mathbf{k}, \mathbf{k}'} F_\lambda^{lo}(\mathbf{k}) F_\lambda^{lo}(\mathbf{k}') a_{\frac{\mathbf{K}}{2} + \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}'} a_{\frac{\mathbf{K}}{2} + \mathbf{k}'}$$

Preliminary LDA results

- **Universality**

- High- q tail collapses to universal function $\approx |F_\lambda^{hi}(\mathbf{q})|^2$ fixed by 2-body

Fig. 4: Proton momentum distribution under LDA with AV18, $\lambda = 1.35 \text{ fm}^{-1}$, and densities from Skyrme potential SLy4 using the HFBRAD code¹.



Preliminary LDA results

- **Low RG resolution** calculations reproduce momentum distributions of AV18 data¹ (high RG resolution calculation)
 - **Low RG works well with simple approximations and is systematically improvable**
 - *Absolute normalization still a work in progress (scaled up by one overall factor)*

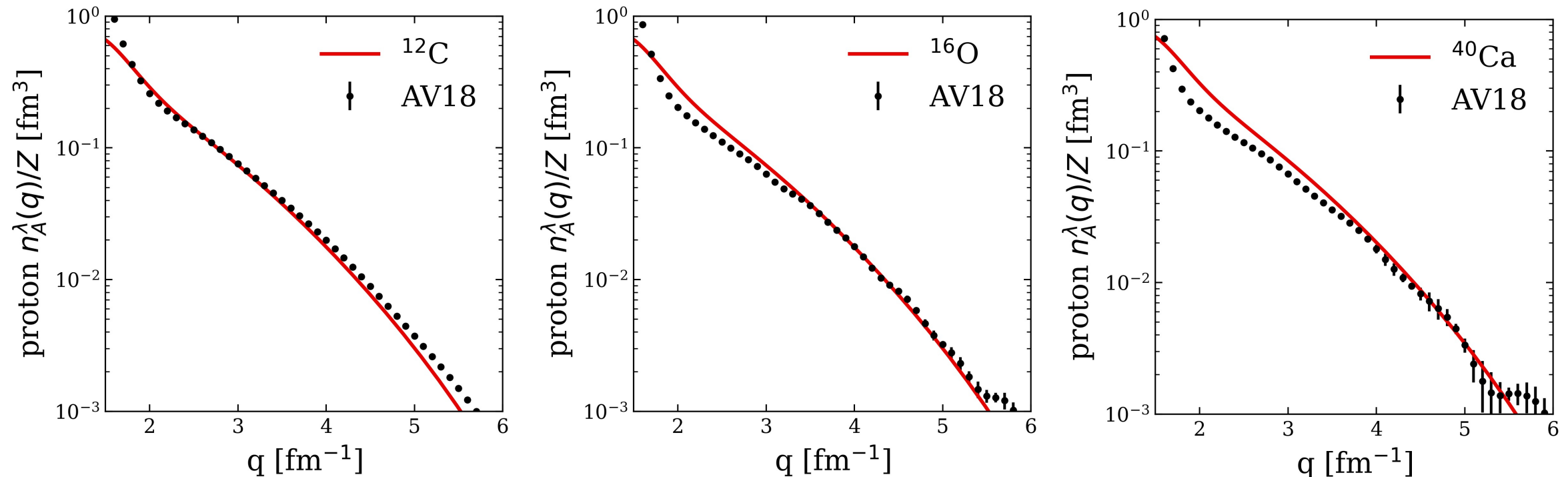


Fig. 5: Proton momentum distributions for ¹²C, ¹⁶O, and ⁴⁰Ca under LDA with AV18 and $\lambda = 1.35$ fm⁻¹.

Preliminary LDA results

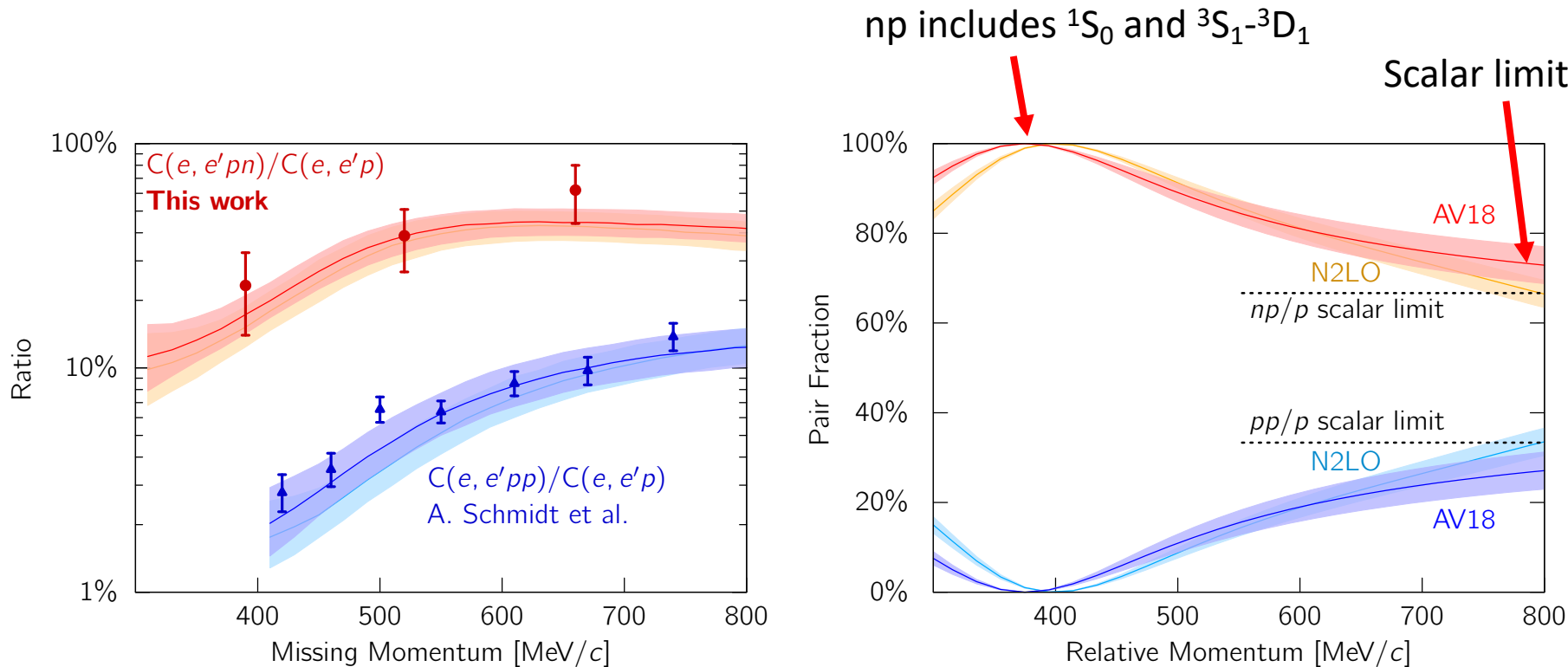
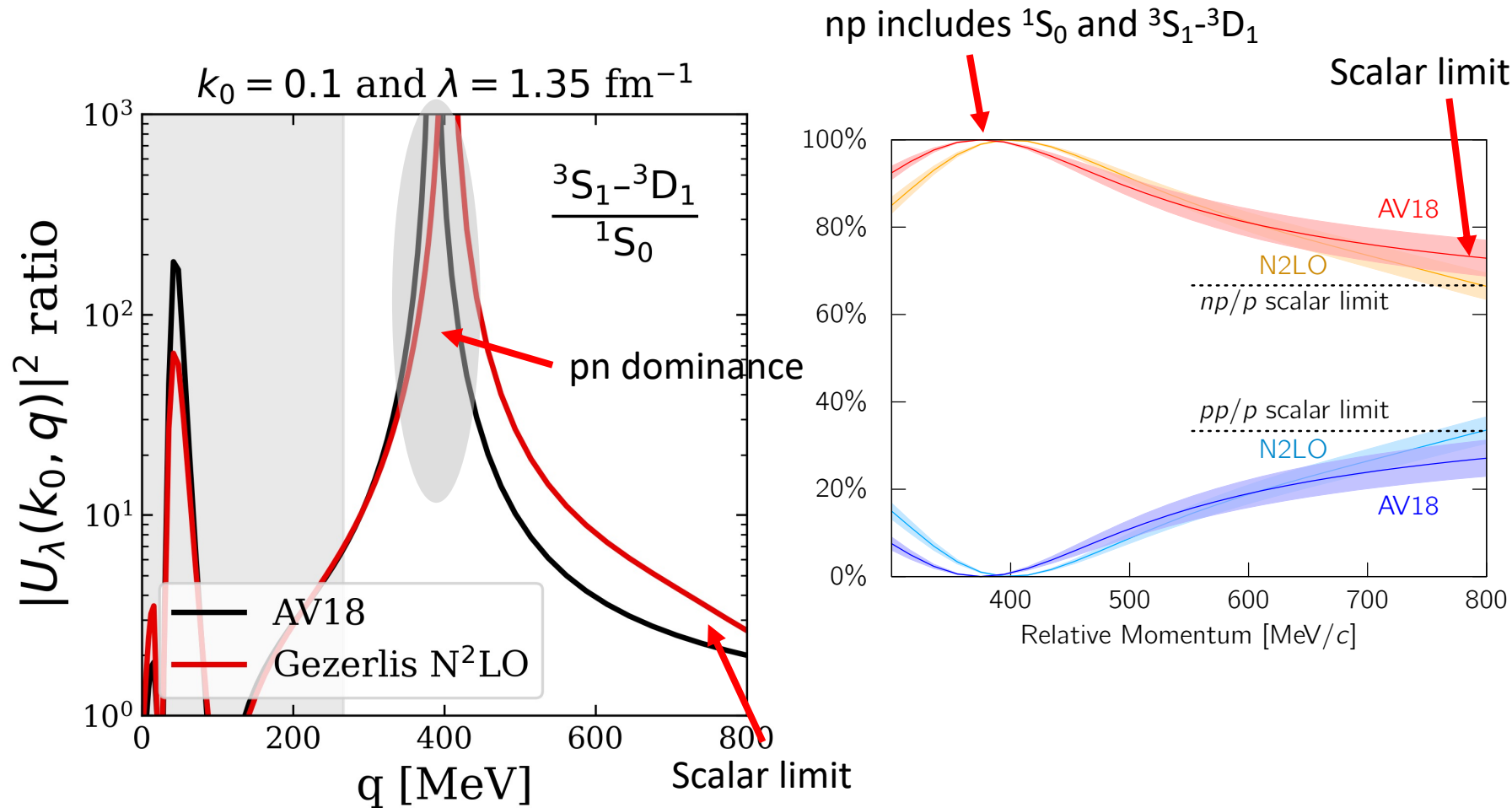


Fig. 6: (a) Ratio of two-nucleon to single-nucleon electron-scattering cross sections for carbon as a function of missing momentum. (b) Fraction of np to p and pp to p pairs versus the relative momentum. Figure from CLAS collaboration publication¹.

- At **high RG resolution**, the tensor force and the repulsive core of the NN interaction kicks nucleon pairs into SRCs
- np dominates because the tensor force requires spin triplet pairs (pp are spin singlets)
- **Do we describe this physics at low RG resolution?**

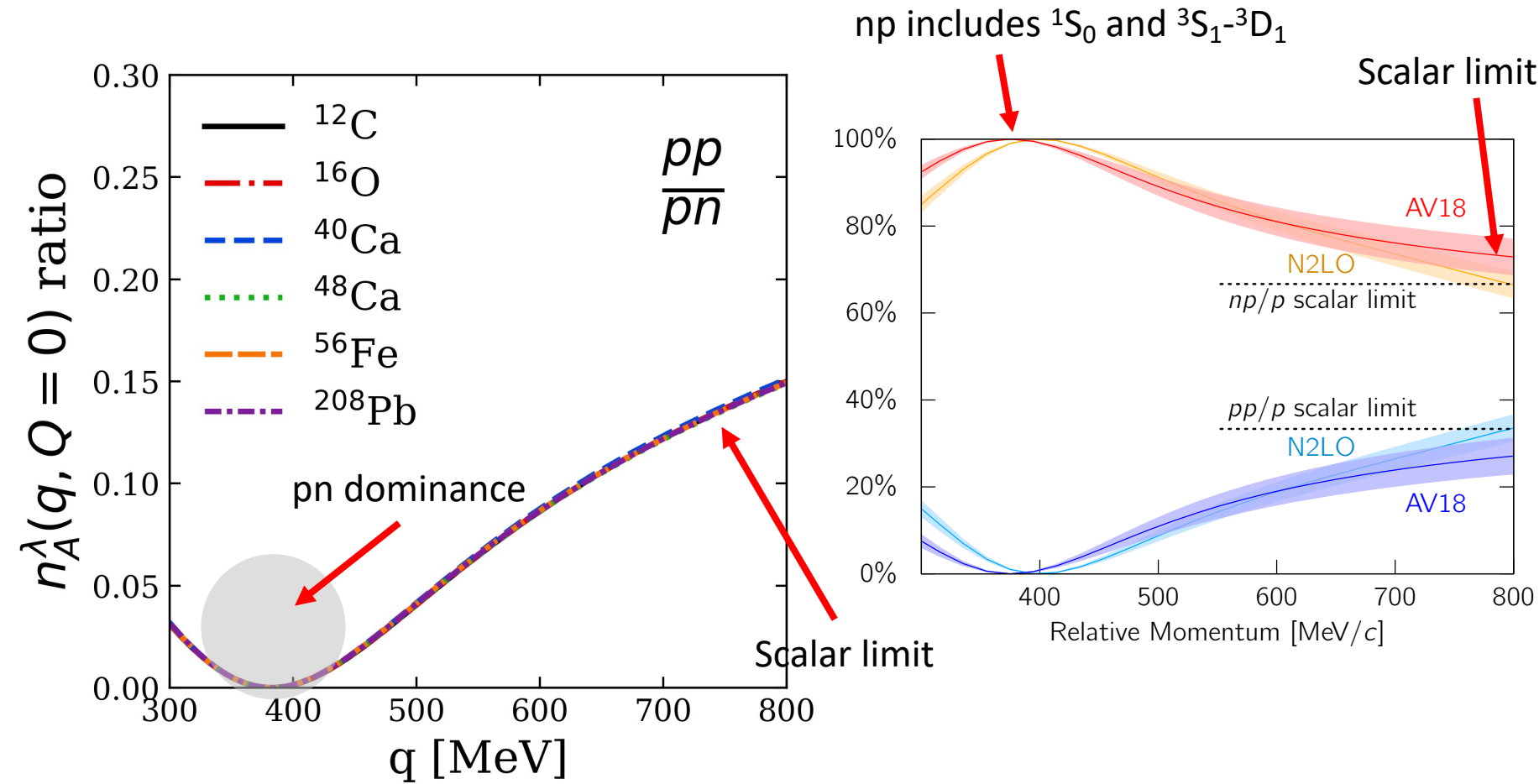
Preliminary LDA results



- At **low RG resolution**, SRCs are suppressed in the wave function
- Consider the ratio of 3S_1 - 3D_1 to 1S_0 evolved momentum projection operator $a_q^\dagger a_q$
- **This physics is established in the 2-body system!**
- **Can apply to any nucleus!**

Fig. 7: 3S_1 - 3D_1 to 1S_0 ratio of SRG-evolved momentum projection operators $a_q^\dagger a_q$.

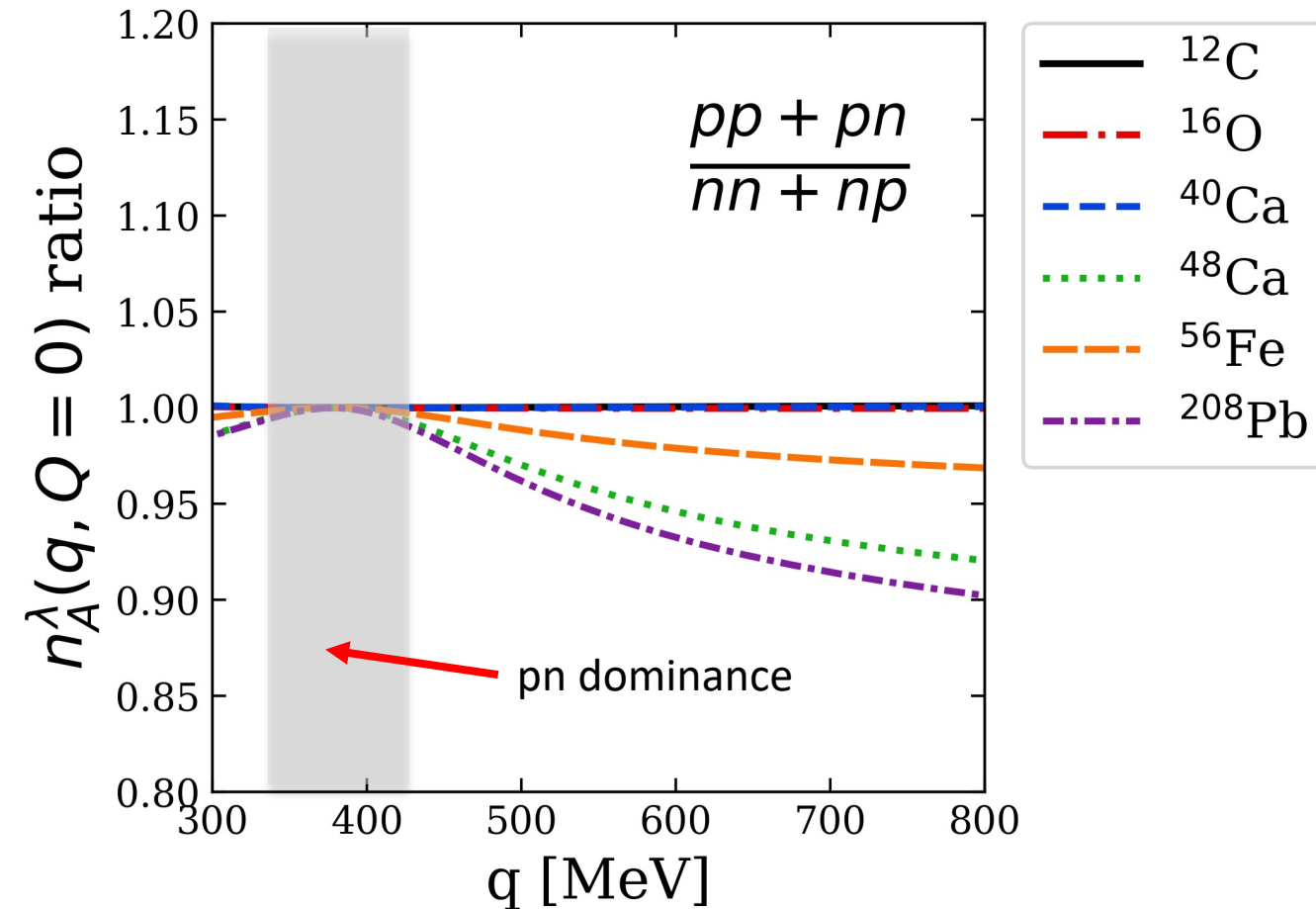
Preliminary LDA results



- Reproduces the characteristics of cross section ratios using **low RG resolution** operator with simple approximations

Fig. 8: pp/pn ratio of pair momentum distributions under LDA with AV18 and $\lambda = 1.35 \text{ fm}^{-1}$.

Preliminary LDA results



- Ratio ~ 1 independent of N/Z in pn dominant region
- Ratio < 1 for nuclei where $N > Z$ and outside pn dominant region

Fig. 9: $(pp+pn)/(nn+np)$ ratio of pair momentum distributions under LDA with AV18 and $\lambda = 1.35 \text{ fm}^{-1}$. Anthony Tropiano, APS April 2021 Meeting

Preliminary LDA results

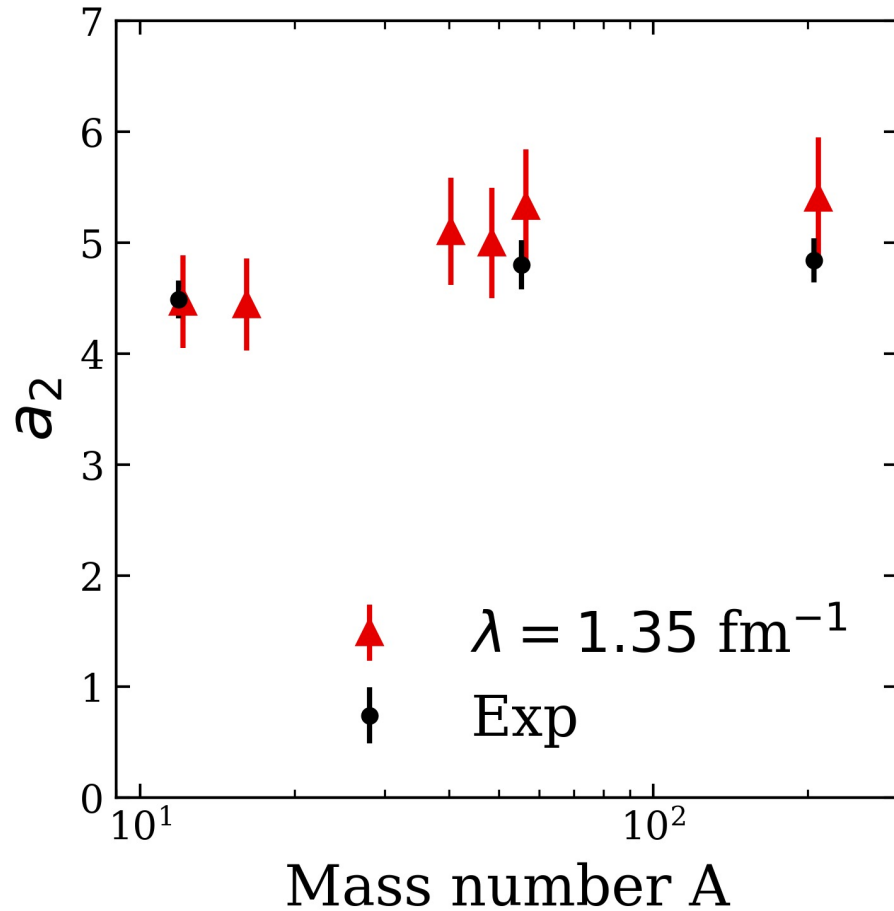


Fig. 10: a_2 scale factors using single-nucleon momentum distributions under LDA with AV18 and $\lambda = 1.35 \text{ fm}^{-1}$ compared to experimental values¹.

- SRC scale factors

$$a_2 = \lim_{q \rightarrow \infty} \frac{P^A(q)}{P^d(q)} \approx \frac{\int_{\Delta p^{high}} dq P^A(q)}{\int_{\Delta p^{high}} dq P^d(q)}$$

where $P^A(q)$ is the single-nucleon probability distribution in nucleus A with error bars from varying Δp^{high}

- Decent agreement with experiment¹ and LCA calculations² but need to further test systematics

¹B. Schmookler et al. (CLAS), Nature **566**, 354 (2019)
²J. Ryckebusch et al., Phys. Rev. C **100**, 054620 (2019)

Summary and outlook

- Simple approximations work and are systematically improvable at low RG resolution
- Results suggest that we can analyze high-energy nuclear reactions using low RG resolution structure (e.g., shell model) and consistently evolved operators
 - Matching resolution scale between structure and reactions is crucial!
- Ongoing work:
 - Extend to cross sections and test scale/scheme dependence of extracted properties
 - Further investigate how final state interactions and physical interpretations depend on the RG scale
 - Apply to more complicated knock-out reactions (SRG with optical potentials) – see Mostofa Hisham's talk (X13.00004)