

# Short-range correlation physics at low RG resolution

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*ajt, S.K. Bogner, and R.J. Furnstahl, arXiv:2006.11186*

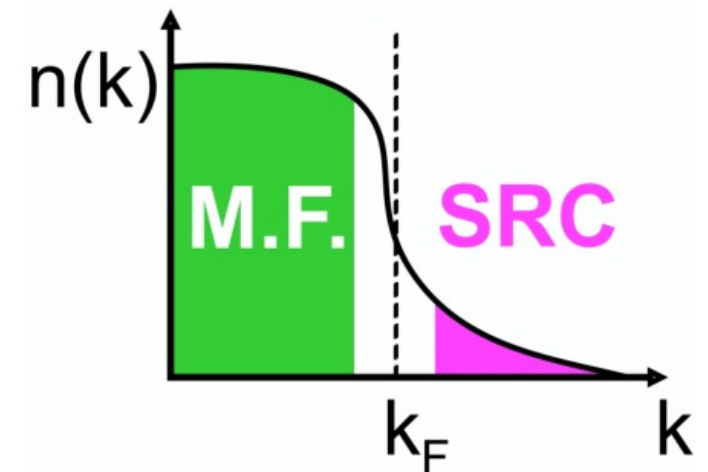
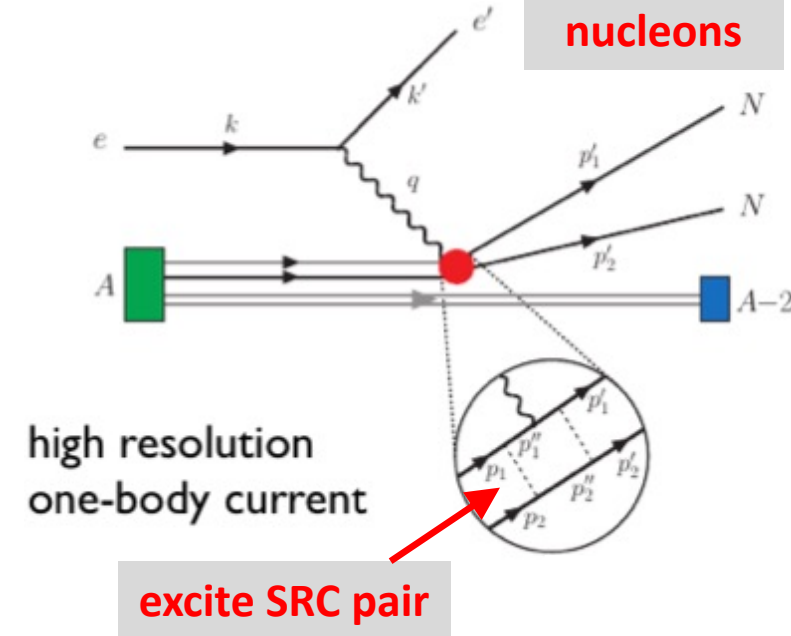
*Phys. Rev. C **102**, 034005 (2020)*

*ajt, S.K. Bogner, and R.J. Furnstahl, in progress*



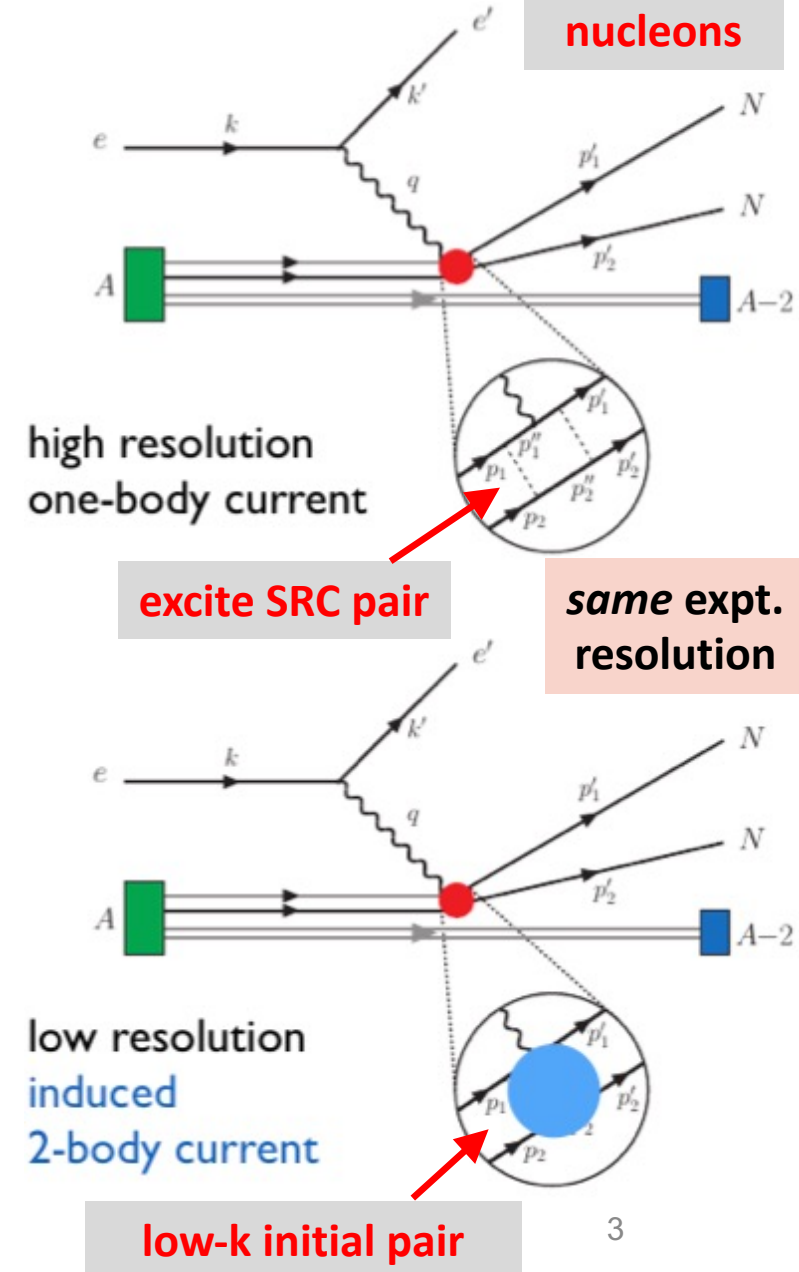
# Motivation

- Recent experiments have been able to isolate processes where short-range correlation (SRC) physics is dominant and well accounted for by SRC phenomenology
- SRC physics at **high RG resolution**
  - SRC pairs are components in the nuclear wave function with relative momenta above the Fermi momentum



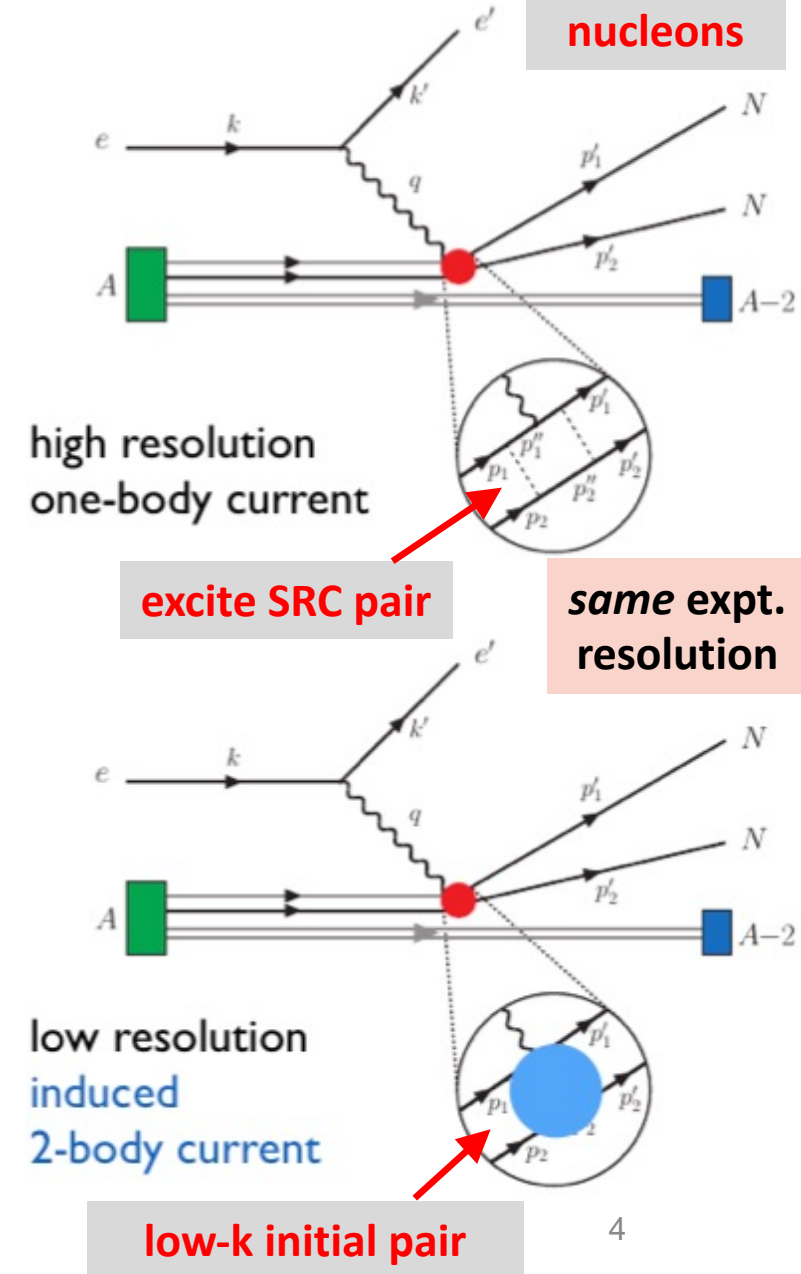
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- SRC physics at **low RG resolution**
  - The SRC *physics* is shifted into the reaction operators from the nuclear wave function (which becomes soft)
  - Operators do not become hard which simplifies calculations



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  - The SRC *physics* is shifted into the reaction operators from the nuclear wave function (which becomes soft)
  - Operators do not become hard which simplifies calculations
- Experimental resolution (set by momentum of probe) is the same in both pictures**
- Same observables but different physical interpretation!**

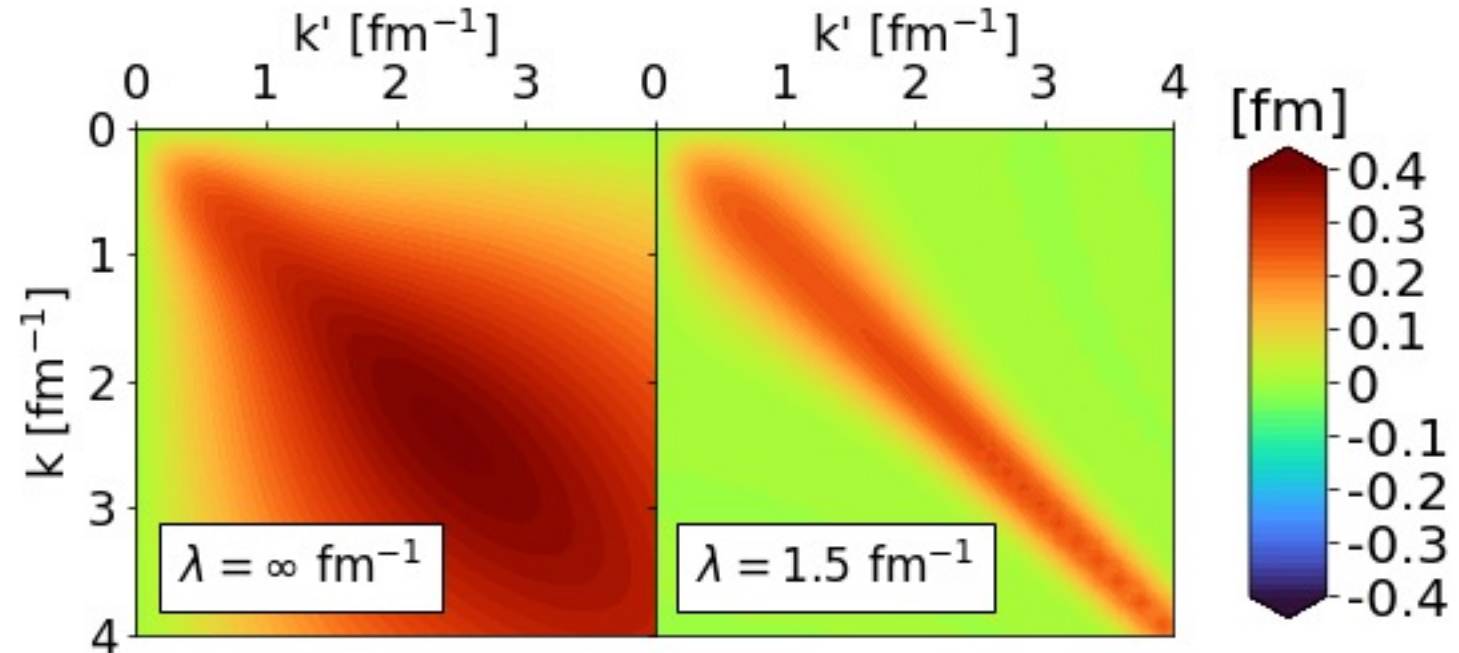


# Similarity Renormalization Group (SRG)

- Evolve operators to low RG resolution

$$O(s) = U(s)O(0)U^\dagger(s)$$

where  $s = 0 \rightarrow \infty$  and  
 $U(s)$  is unitary



**Fig. 1:** Momentum space matrix elements of Argonne v18 (AV18) under SRG evolution in  $^1P_1$  channel.

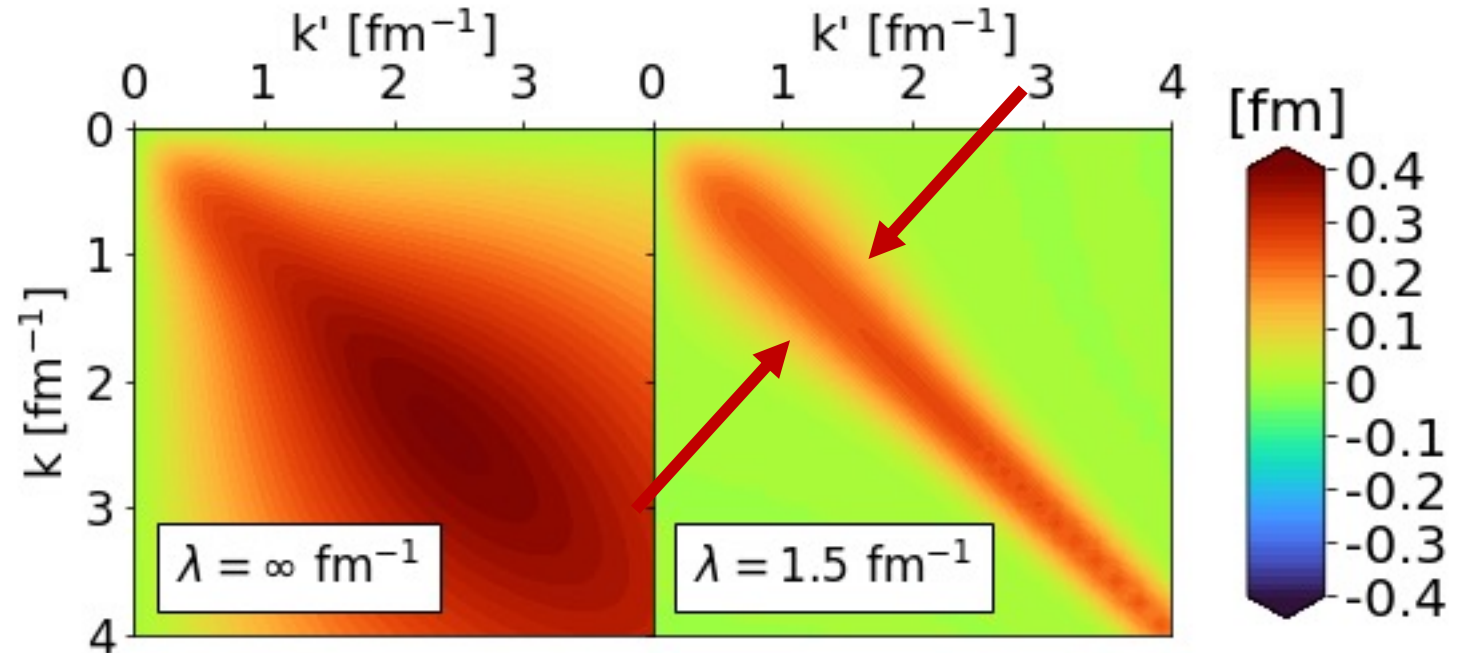
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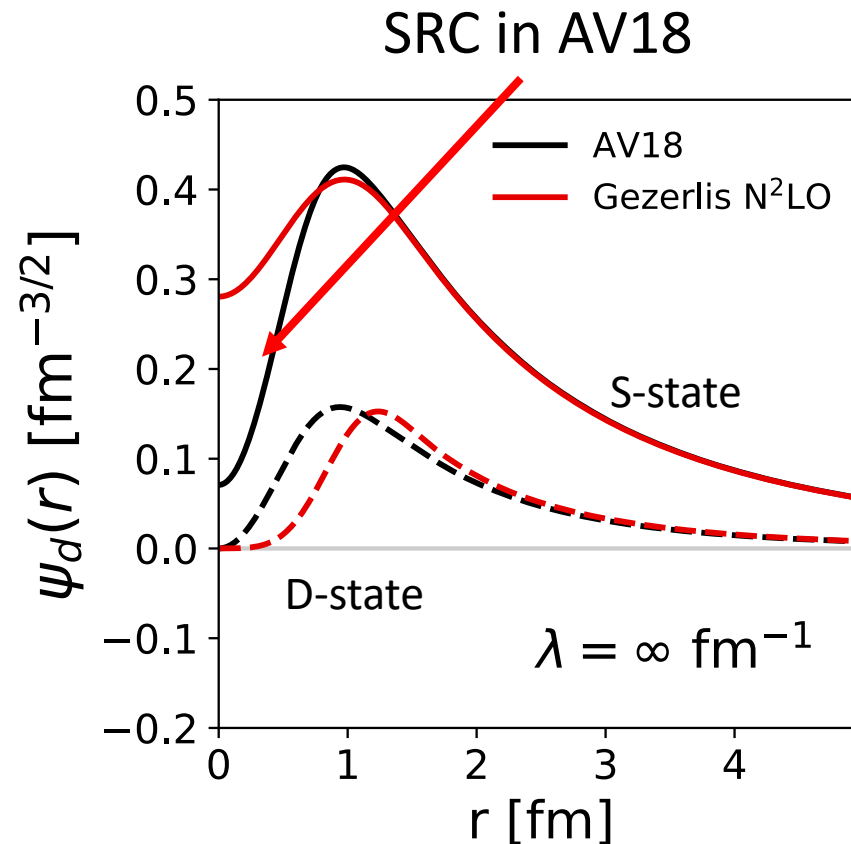
- $\lambda = s^{-1/4}$  describes the decoupling scale of the RG evolved operator



**Fig. 1:** Momentum space matrix elements of Argonne v18 (AV18) under SRG evolution in  $^1P_1$  channel.

# Deuteron wave function at low RG resolution

- AV18 wave function has significant SRC
- What happens to the wave function at low RG resolution?



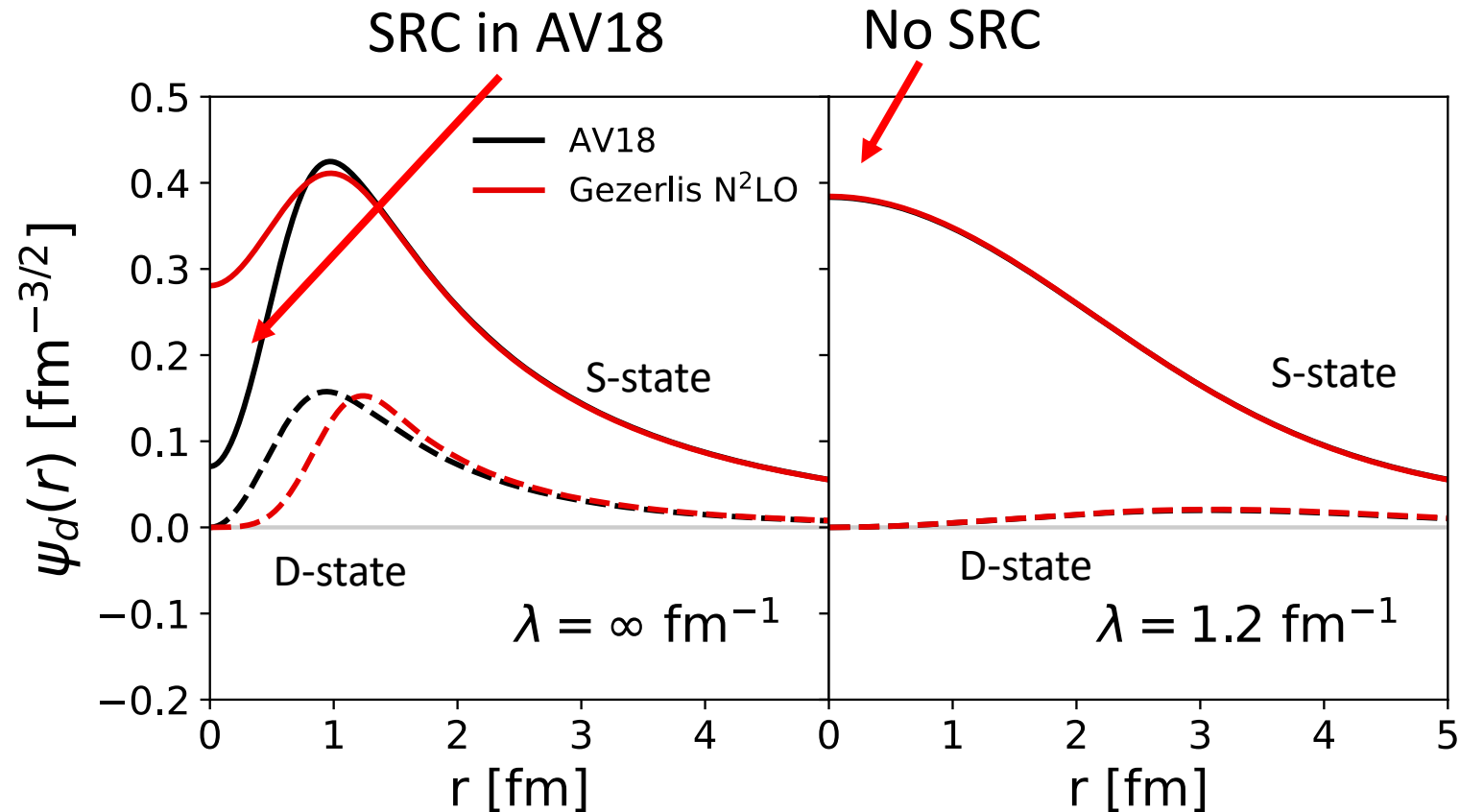
**Fig. 2:** SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis  $N^2\text{LO}$ <sup>1</sup>.

<sup>1</sup>A. Gezerlis et al., Phys. Rev. C **90**, 054323 (2014)



# Deuteron wave function at low RG resolution

- SRC physics in AV18 is gone from wave function at low RG resolution
- Deuteron wave functions become soft and D-state probability goes down
- Observables such as asymptotic D-S ratio are the same



**Fig. 2:** SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis N2LO<sup>1</sup>.



# Momentum distributions at low RG resolution

- Soft wave functions at low RG resolution
  - Where does the SRC physics go?

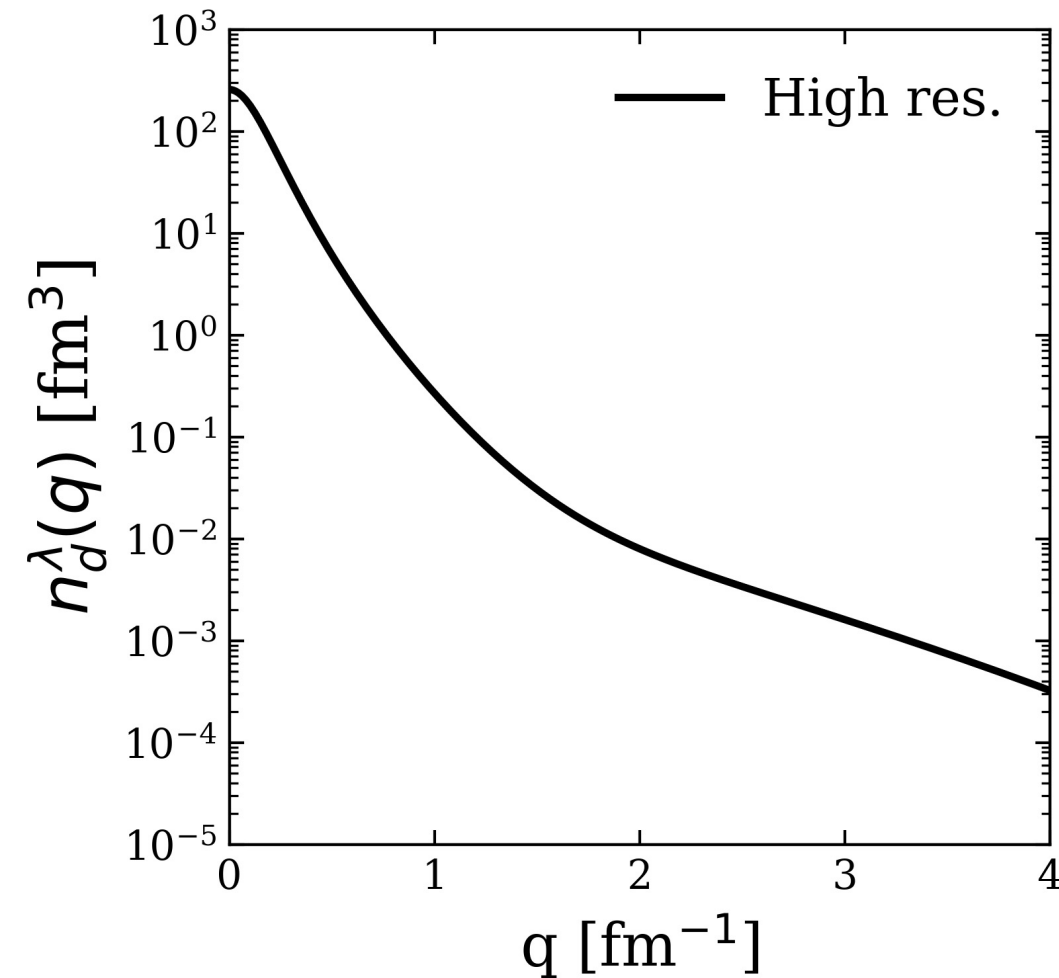
# Momentum distributions at low RG resolution

- Soft wave functions at low RG resolution
  - Where does the SRC physics go?
- SRC physics shifts to the operators  $\langle \psi_f^{hi} | U_\lambda^\dagger U_\lambda O^{hi} U_\lambda^\dagger U_\lambda | \psi_i^{hi} \rangle$
- Apply SRG transformations to momentum distribution operator

$$n^{hi}(\mathbf{q}) = a_q^\dagger a_q$$

$$U_\lambda = 1 + \frac{1}{4} \sum_{\mathbf{K}, \mathbf{k}, \mathbf{k}'} \delta U_\lambda^{(2)}(\mathbf{k}, \mathbf{k}') a_{\frac{\mathbf{K}}{2} + \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}'} a_{\frac{\mathbf{K}}{2} + \mathbf{k}'} + \dots$$

# Momentum distributions at low RG resolution



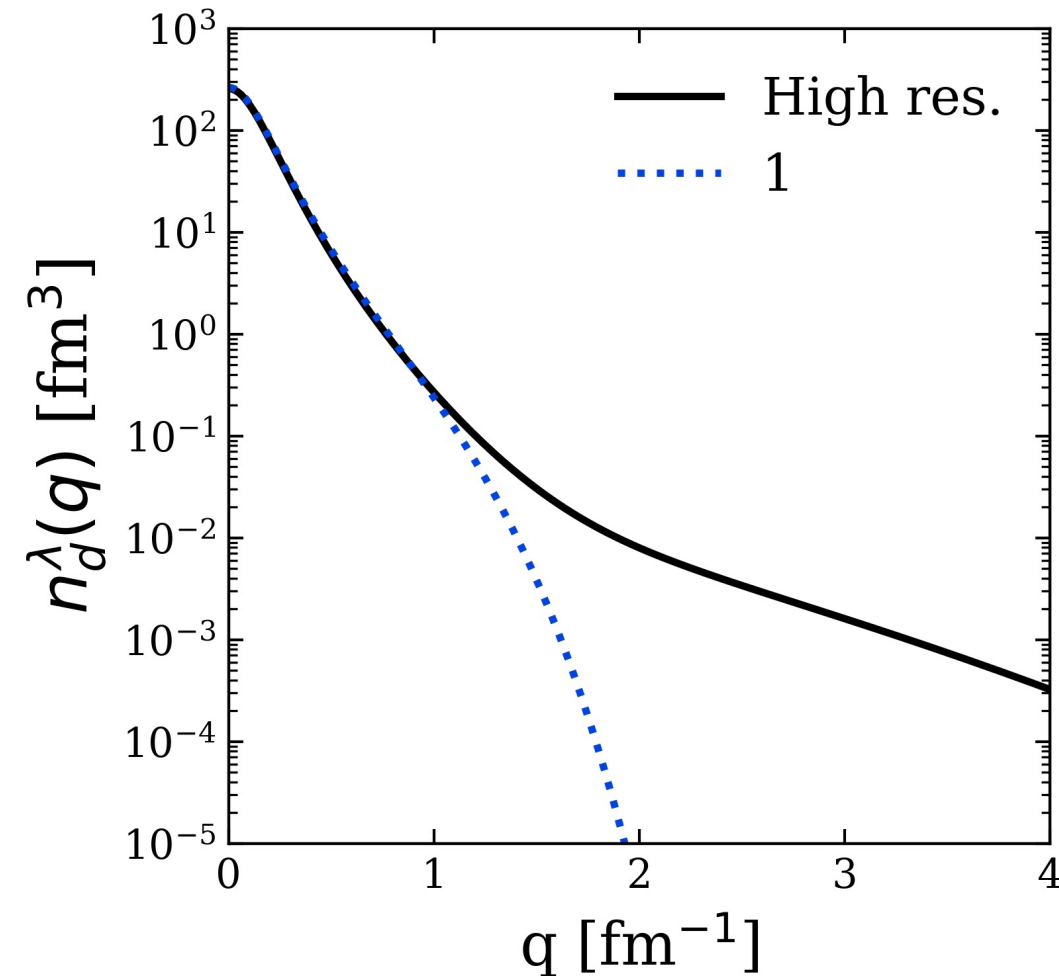
- Deuteron example

$$n^{lo}(\mathbf{q}) = (1 + \delta U) a_{\mathbf{q}}^\dagger a_{\mathbf{q}} (1 + \delta U^\dagger)$$

$$\langle \psi_d^{hi} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_d^{hi} \rangle$$

**Fig. 3:** Contributions to deuteron momentum distribution with AV18 and  $\lambda = 1.35$  fm<sup>-1</sup>.

# Momentum distributions at low RG resolution



- Deuteron example

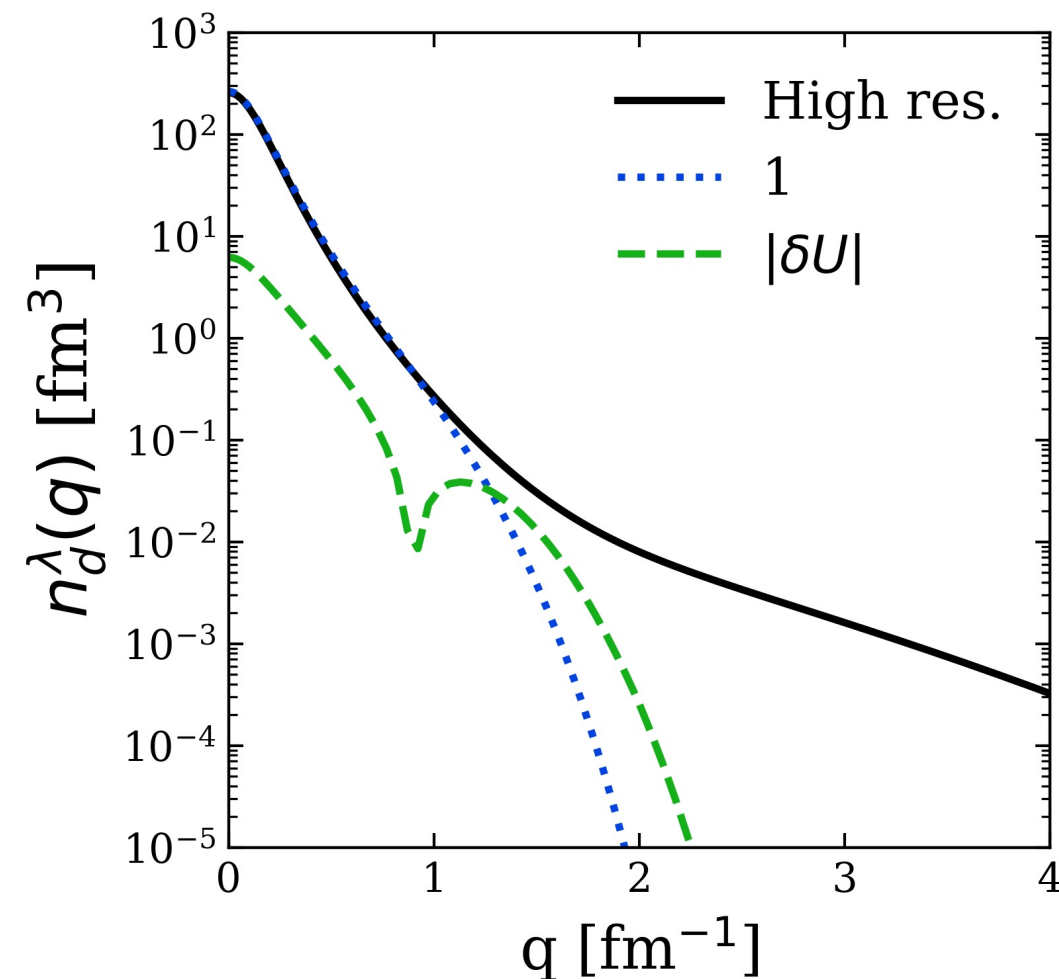
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$$\langle \psi_d^{lo} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | \psi_d^{lo} \rangle$$

**Fig. 3:** Contributions to deuteron momentum distribution with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ .

# Momentum distributions at low RG resolution



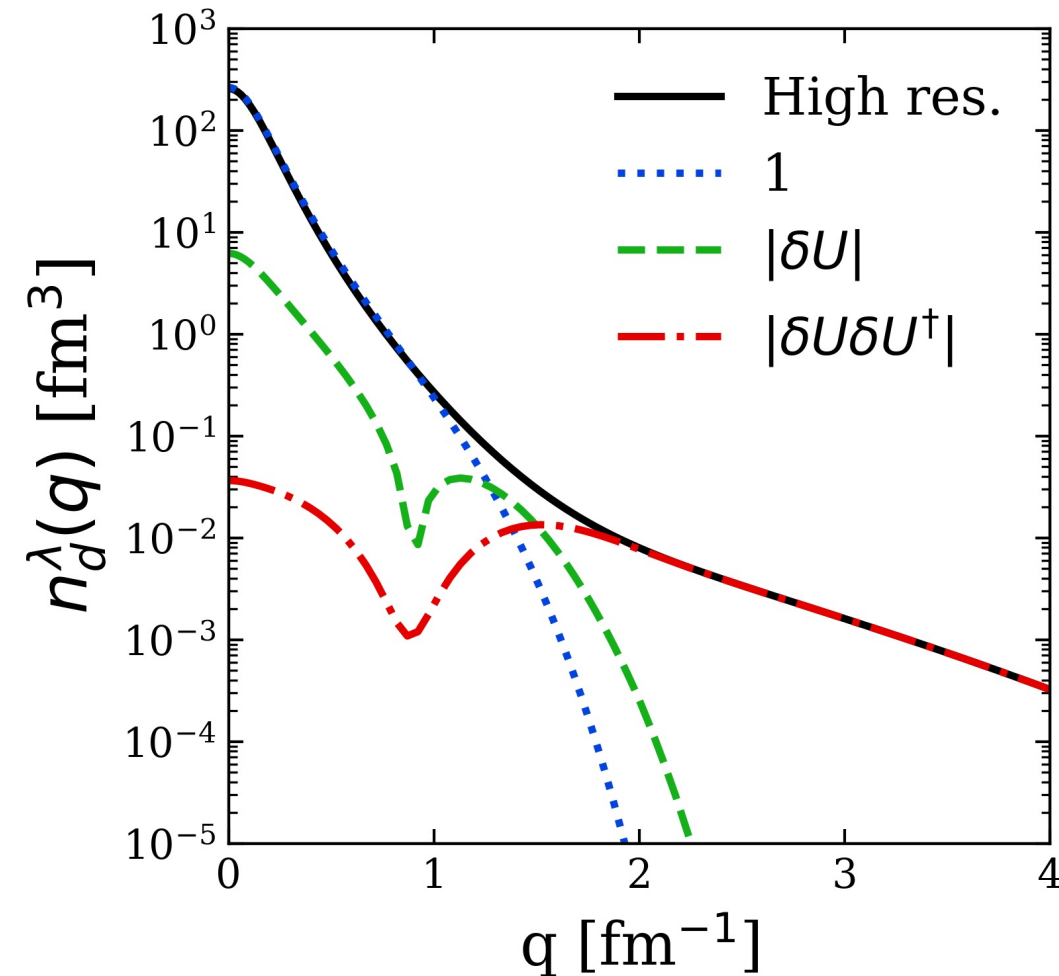
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**Fig. 3:** Contributions to deuteron momentum distribution with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ .

# Momentum distributions at low RG resolution



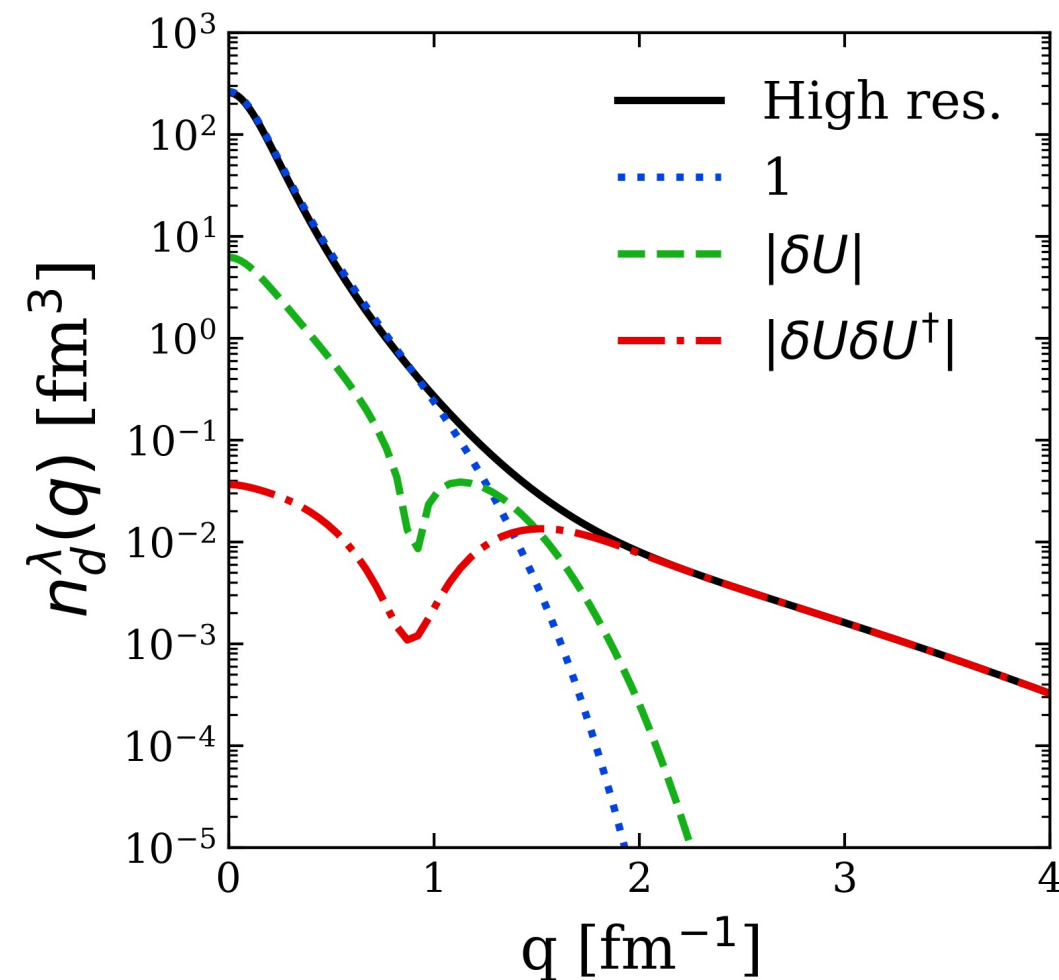
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# Momentum distributions at low RG resolution



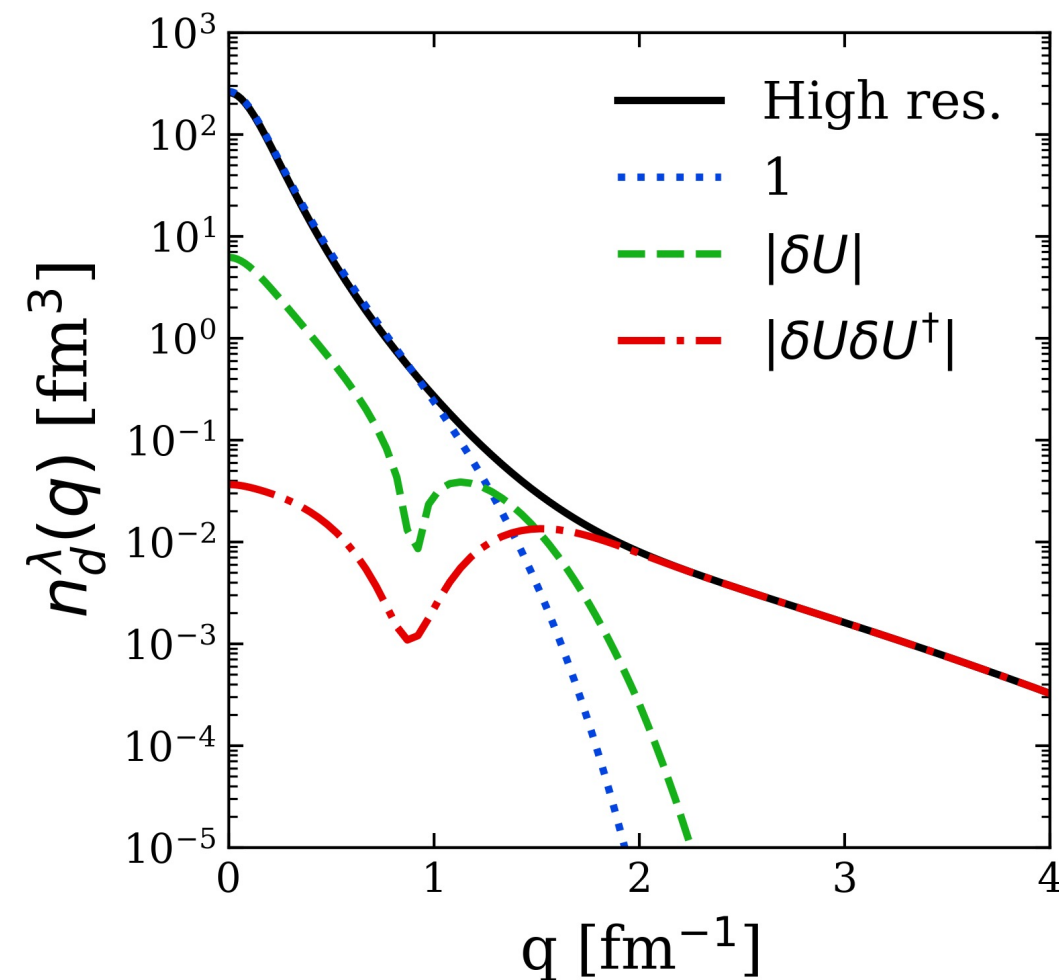
- For high- $q$ , the  $\delta U_\lambda \delta U_\lambda^\dagger$  2-body term dominates

$$\approx \sum_{K,k,k'} \delta U_\lambda(\mathbf{k}, \mathbf{q}) \delta U_\lambda^\dagger(\mathbf{q}, \mathbf{k}') a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

**Fig. 3:** Contributions to deuteron momentum distribution with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ .



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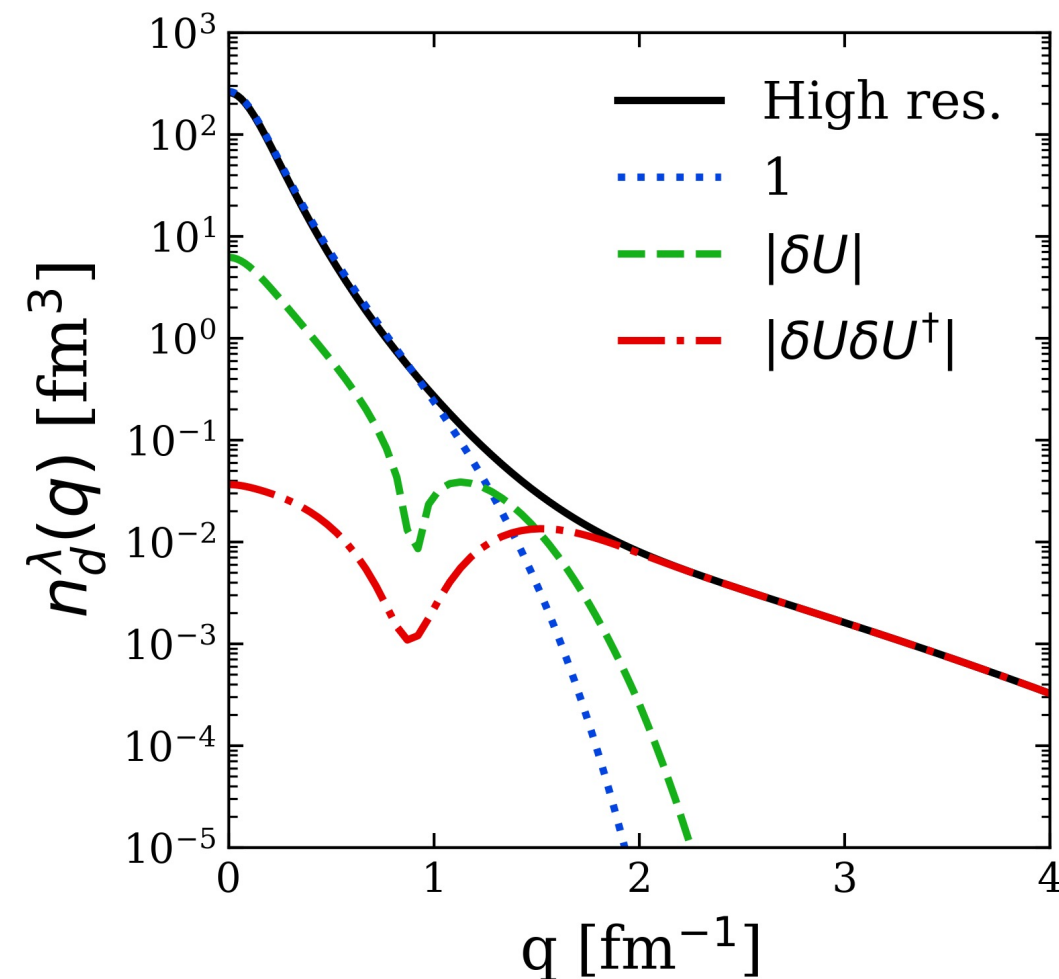
↓

**Factorization:**  $\delta U_\lambda(\mathbf{k}, \mathbf{q}) \approx F_\lambda^{lo}(\mathbf{k}) F_\lambda^{hi}(\mathbf{q})$

↓

$$\approx |F_\lambda^{hi}(\mathbf{q})|^2 \sum_{K,k,k'} F_\lambda^{lo}(\mathbf{k}) F_\lambda^{lo}(\mathbf{k}') a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

# Momentum distributions at low RG resolution



**Fig. 3:** Contributions to deuteron momentum distribution with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ .

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Apply this strategy to nuclear momentum distributions using local density approximation (LDA)!  $F_\lambda^{hi}(\mathbf{q})$

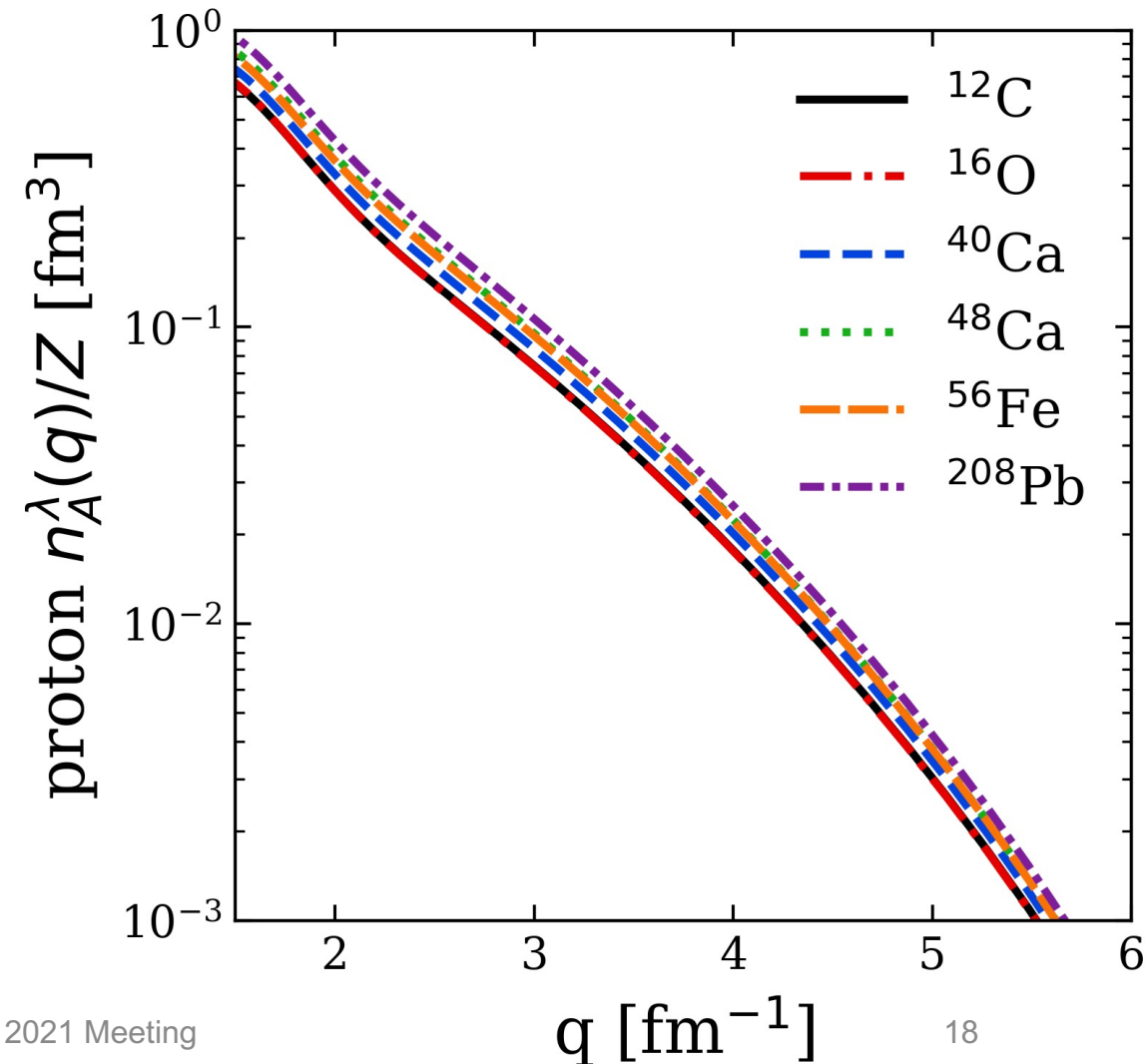
$$\approx |F_\lambda^{lo}(\mathbf{q})|^2 \sum_{\mathbf{K}, \mathbf{k}, \mathbf{k}'} F_\lambda^{lo}(\mathbf{k}) F_\lambda^{lo}(\mathbf{k}') a_{\frac{\mathbf{K}}{2} + \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}}^\dagger a_{\frac{\mathbf{K}}{2} - \mathbf{k}'} a_{\frac{\mathbf{K}}{2} + \mathbf{k}'}$$

# Preliminary LDA results

- **Universality**

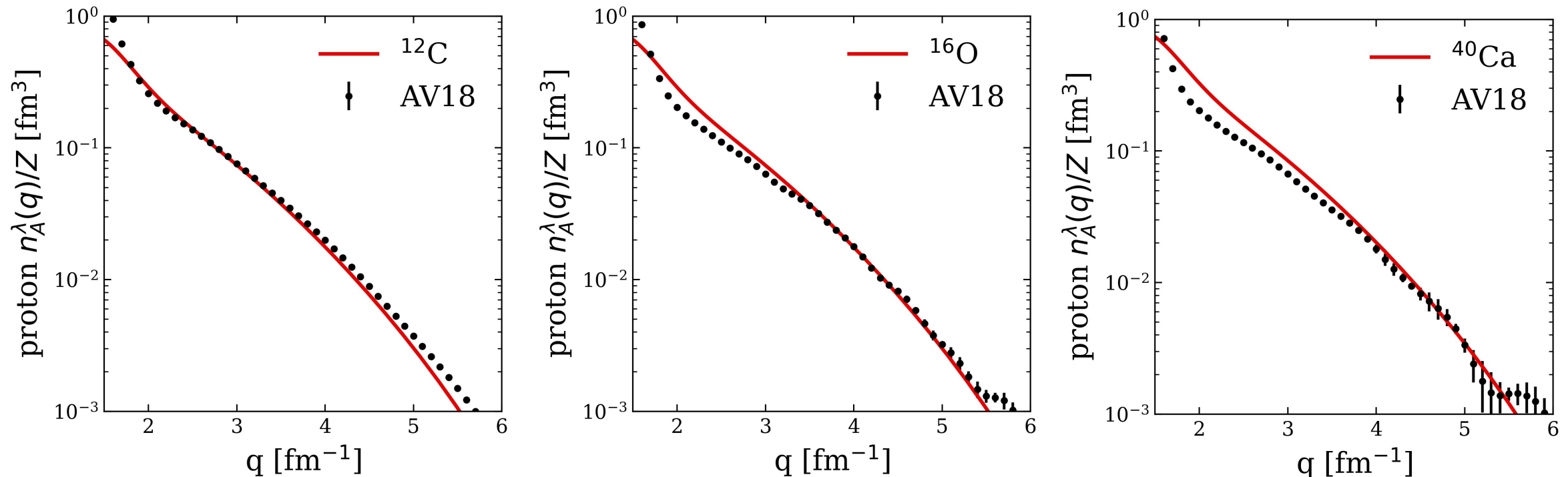
- High- $q$  tail collapses to universal function  $\approx |F_\lambda^{hi}(\mathbf{q})|^2$  fixed by 2-body

**Fig. 4:** Proton momentum distribution under LDA with AV18,  $\lambda = 1.35 \text{ fm}^{-1}$ , and densities from Skyrme potential SLy4 using the HFBRAD code<sup>1</sup>.



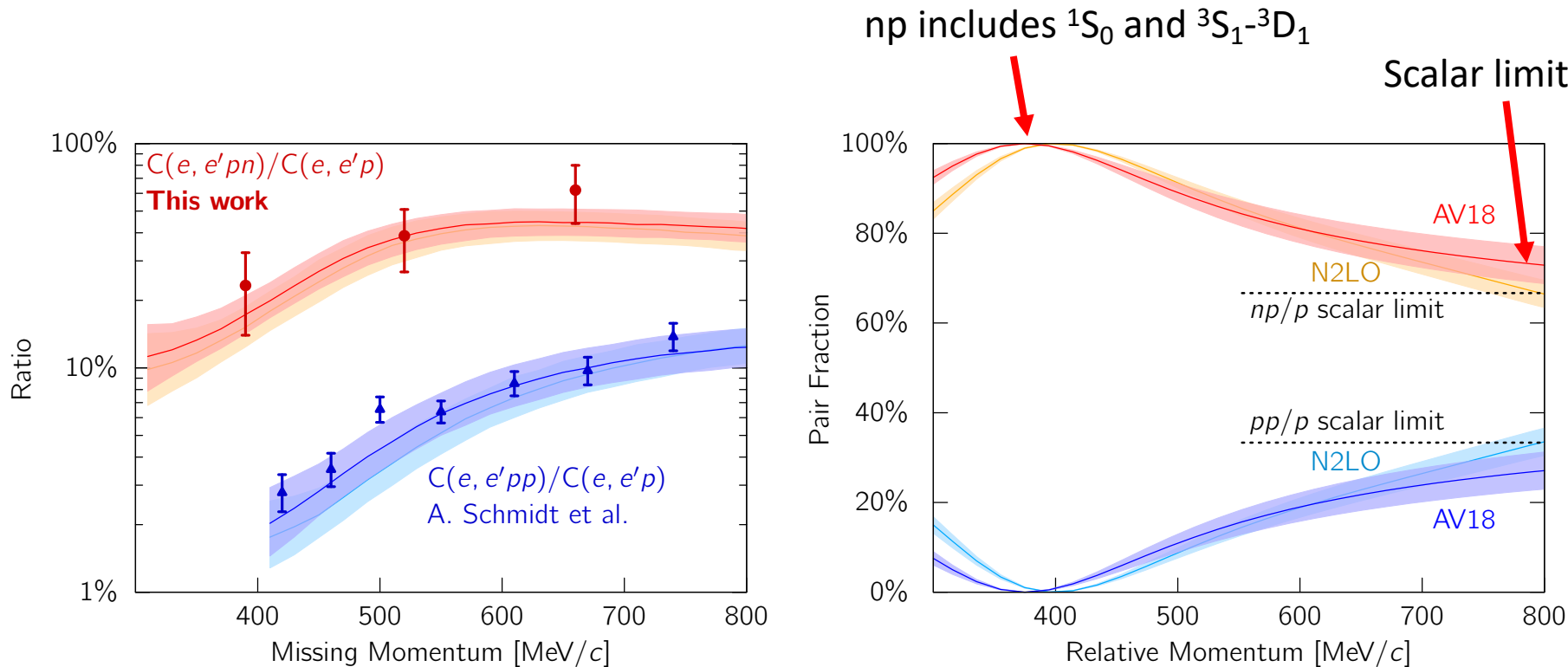
# Preliminary LDA results

- **Low RG resolution** calculations reproduce momentum distributions of AV18 data<sup>1</sup> (high RG resolution calculation)
  - **Low RG works well with simple approximations and is systematically improvable**
  - *Absolute normalization still a work in progress (scaled up by one overall factor)*



**Fig. 5:** Proton momentum distributions for <sup>12</sup>C, <sup>16</sup>O, and <sup>40</sup>Ca under LDA with AV18 and  $\lambda = 1.35$  fm<sup>-1</sup>.

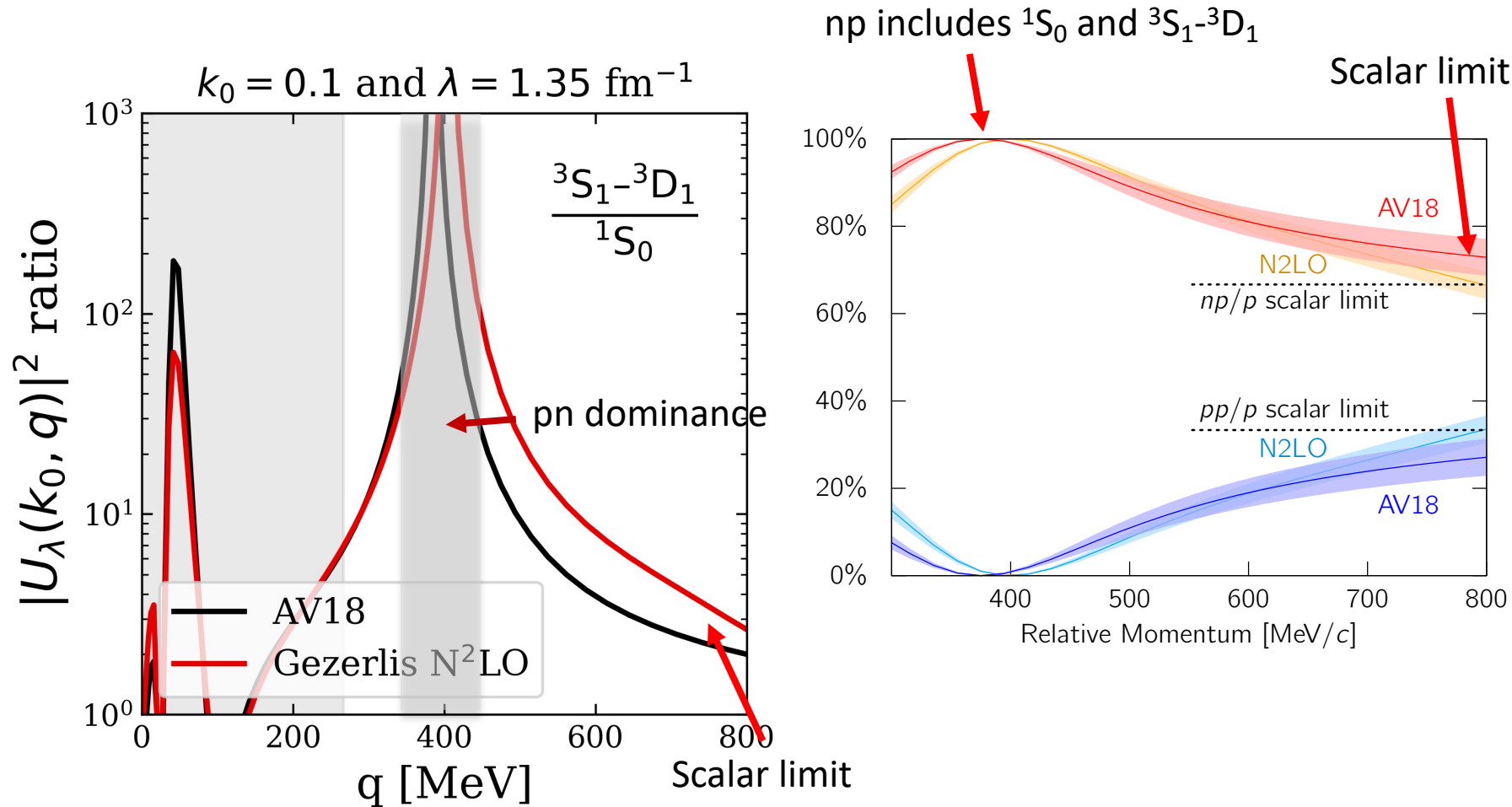
# Preliminary LDA results



**Fig. 6:** (a) Ratio of two-nucleon to single-nucleon electron-scattering cross sections for carbon as a function of missing momentum. (b) Fraction of np to p and pp to p pairs versus the relative momentum. Figure from CLAS collaboration publication<sup>1</sup>.

- At **high RG resolution**, the tensor force and the repulsive core of the NN interaction kicks nucleon pairs into SRCs
- np dominates because the tensor force requires spin triplet pairs (pp are spin singlets)
- **Do we describe this physics at low RG resolution?**

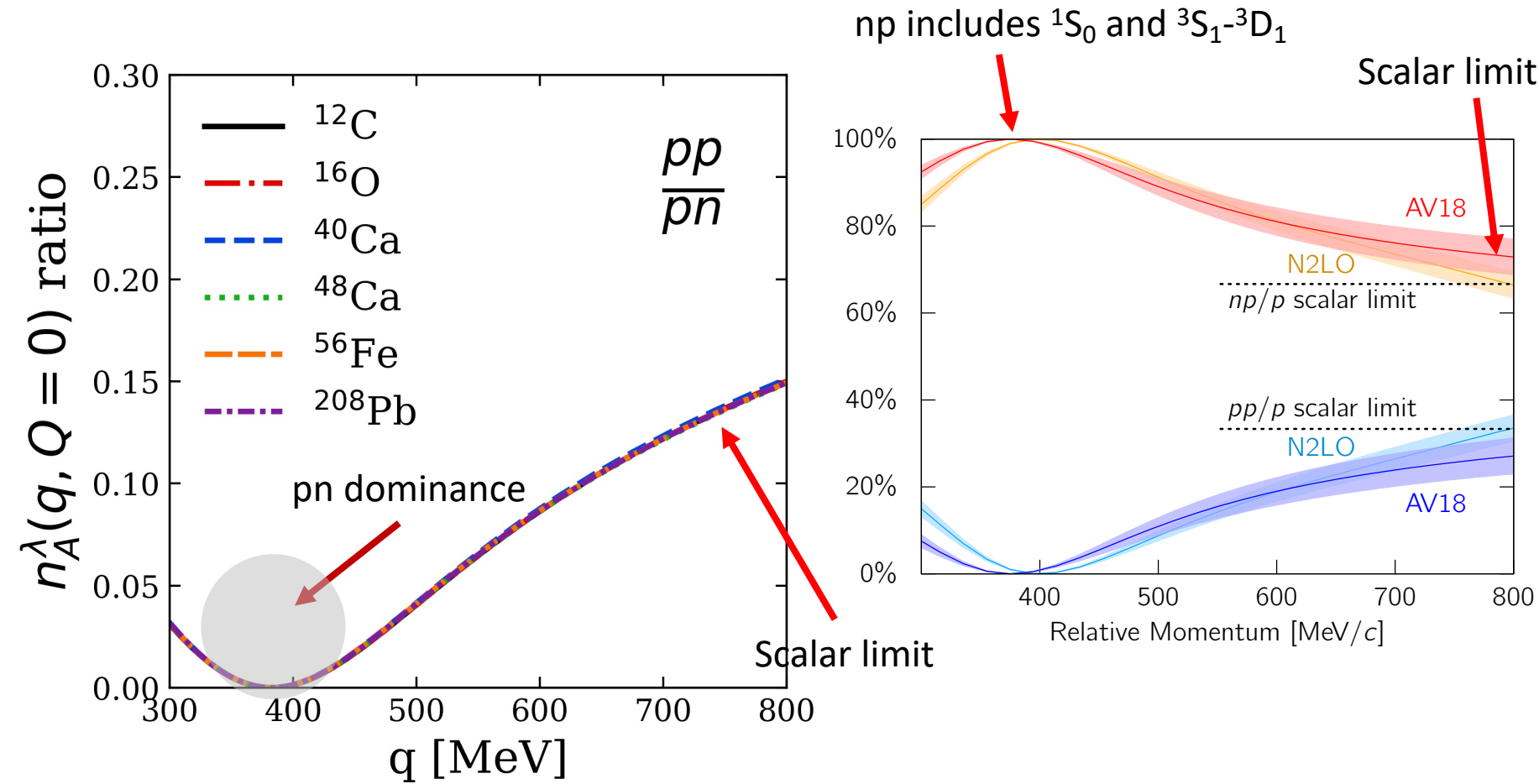
# Preliminary LDA results



- At **low RG resolution**, SRCs are suppressed in the wave function
- Consider the ratio of  ${}^3S_1$ - ${}^3D_1$  to  ${}^1S_0$  evolved momentum projection operator  $a_q^\dagger a_q$
- **This physics is established in the 2-body system!**
- **Can apply to any nucleus!**

**Fig. 7:**  ${}^3S_1$ - ${}^3D_1$  to  ${}^1S_0$  ratio of SRG-evolved momentum projection operators  $a_q^\dagger a_q$ .

# Preliminary LDA results

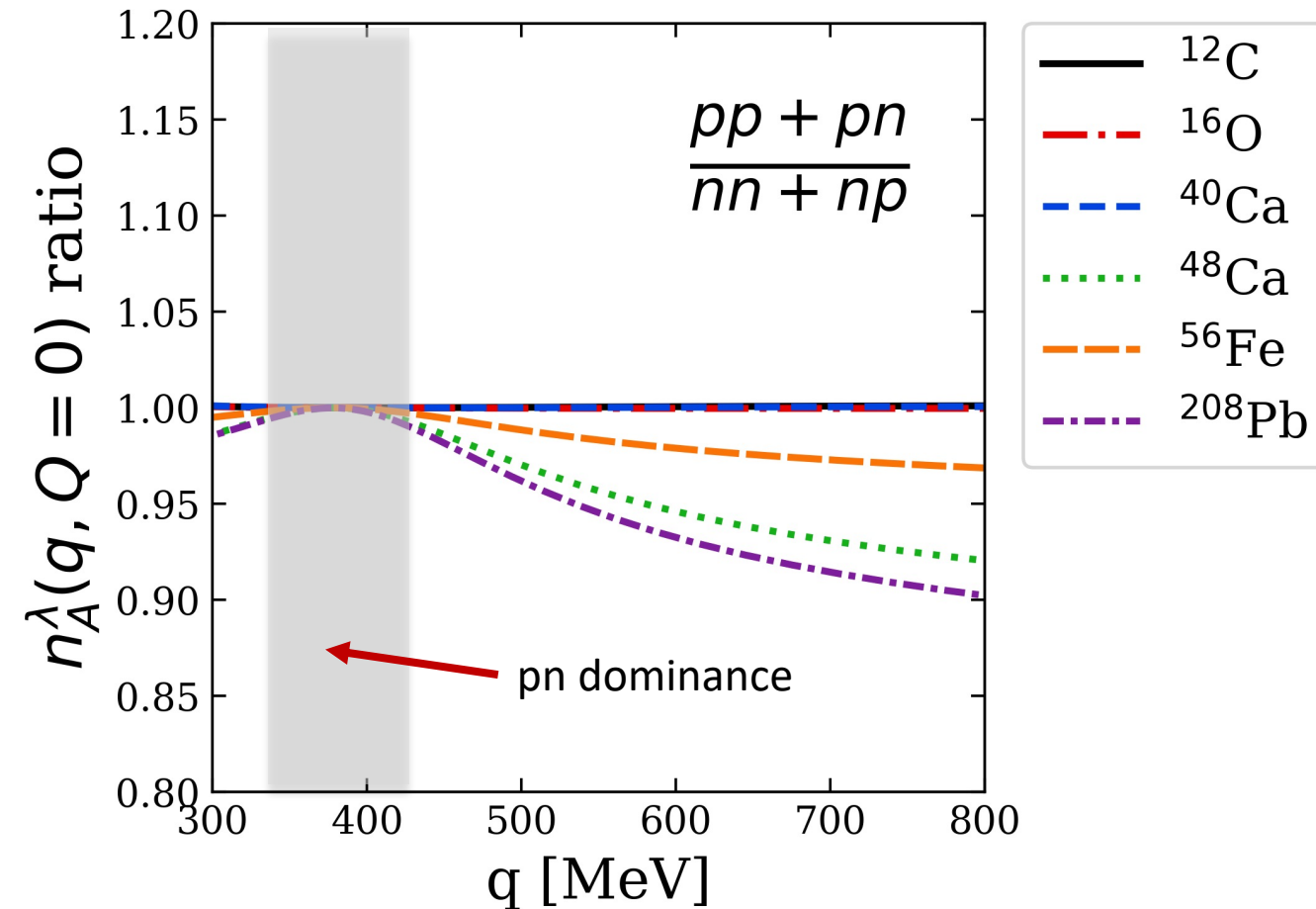


- Reproduces the characteristics of cross section ratios using **low RG resolution** operator with simple approximations

**Fig. 8:** pp/pn ratio of pair momentum distributions under LDA with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ .



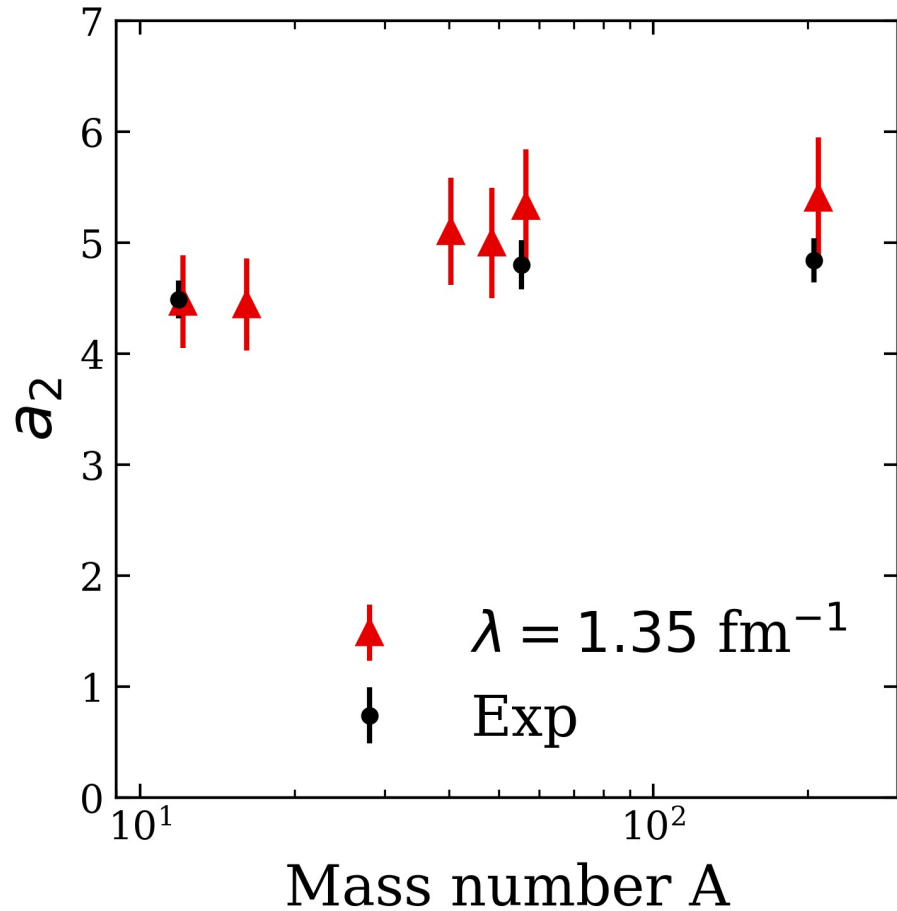
# Preliminary LDA results



- Ratio  $\sim 1$  independent of  $N/Z$  in pn dominant region
- Ratio  $< 1$  for nuclei where  $N > Z$  and outside pn dominant region

**Fig. 9:**  $(pp+pn)/(nn+np)$  ratio of pair momentum distributions under LDA with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$ . Anthony Tropiano, APS April 2021 Meeting

# Preliminary LDA results



**Fig. 10:**  $a_2$  scale factors using single-nucleon momentum distributions under LDA with AV18 and  $\lambda = 1.35 \text{ fm}^{-1}$  compared to experimental values<sup>1</sup>.

- SRC scale factors

$$a_2 = \lim_{q \rightarrow \infty} \frac{P^A(q)}{P^d(q)} \approx \frac{\int_{\Delta p^{high}} dq P^A(q)}{\int_{\Delta p^{high}} dq P^d(q)}$$

where  $P^A(q)$  is the single-nucleon probability distribution in nucleus  $A$  with error bars from varying  $\Delta p^{high}$

- Decent agreement with experiment<sup>1</sup> and LCA calculations<sup>2</sup> but need to further test systematics

<sup>1</sup>B. Schmookler et al. (CLAS), Nature **566**, 354 (2019)  
<sup>2</sup>J. Ryckebusch et al., Phys. Rev. C **100**, 054620 (2019)

# Summary and outlook

- Simple approximations work and are systematically improvable at low RG resolution
- Results suggest that we can analyze high-energy nuclear reactions using low RG resolution structure (e.g., shell model) and consistently evolved operators
  - Matching resolution scale between structure and reactions is crucial!
- Ongoing work:
  - Extend to cross sections and test scale/scheme dependence of extracted properties
  - Further investigate how final state interactions and physical interpretations depend on the RG scale
  - Apply to more complicated knock-out reactions (SRG with optical potentials) – see Mostofa Hisham's talk (X13.00004)