May 14, 2003

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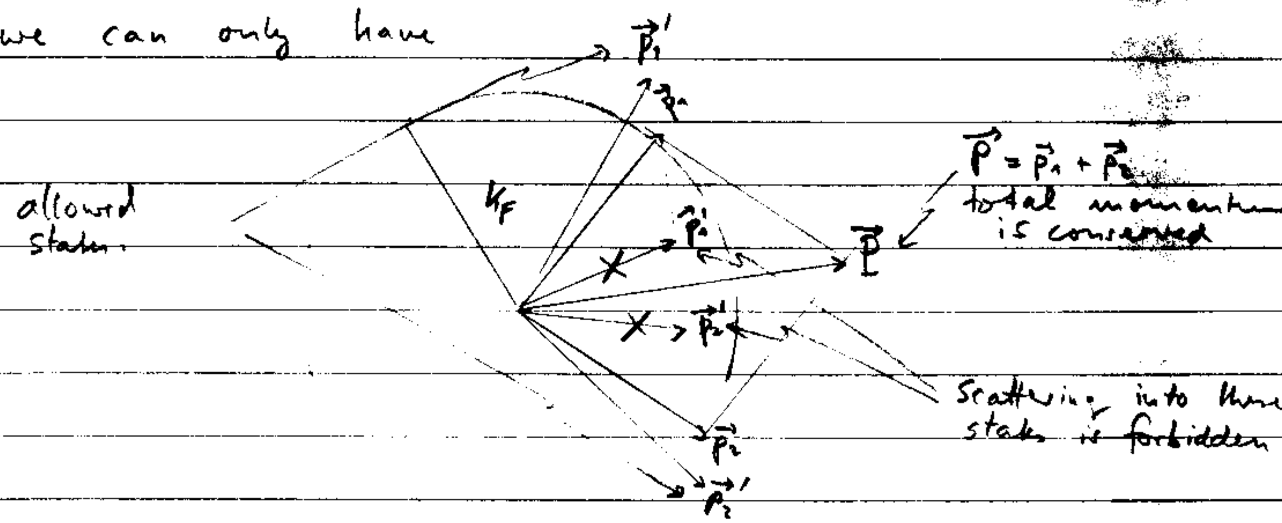
Scattering in the many-body system

We have, up to now, been discussing nucleon-nucleon scattering in free space. The situation is of course different for the scattering of nucleons in a nucleus or inside a neutron star, where there are many nucleons present, which will influence the scattering amplitude.

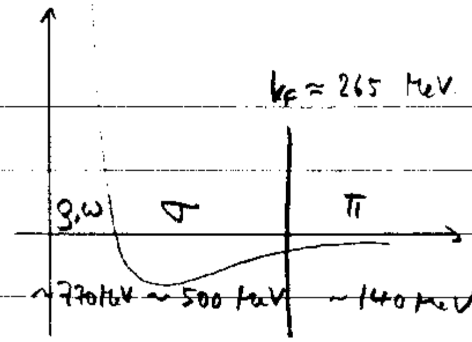
The difference to scattering in free space arises from the Pauli principle, i.e., two nucleons can only scatter into states which are not already occupied. At low temperatures, the nucleons occupy all states up to the Fermi energy, i.e., we can only have



We say the states in the Fermi sea are Pauli-blocked.

Before we continue and discuss how two-body scattering is modified in the medium, we first consider the scales in the problem.

In free space:



In the many-body system, there is one additional scale, the Fermi momentum. From elastic e^- - nucleus scattering, we know that the central densities in a nucleus are

$$\rho_0 \approx \frac{1}{6}\ \mathrm{fm}^{-3}$$ ($\frac{1}{6}$ of a nucleon per box of length 1 fm)

This corresponds to

$$\rho_0 = \frac{g}{6\pi^2} k_F^3$$ ($g = 4 \to 2$ spin, 2 isospin degrees of freedom)

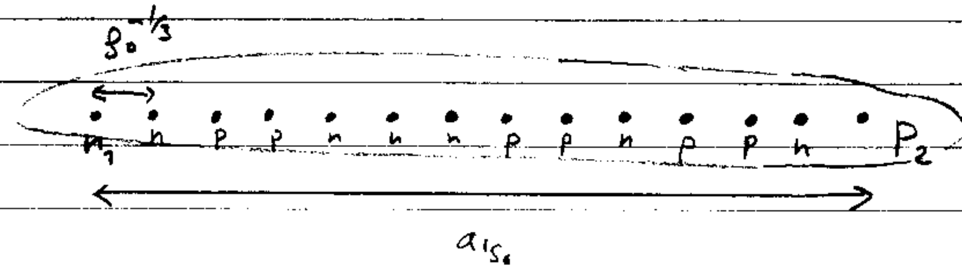
$$\Rightarrow k_F \approx 1.35\ \mathrm{fm}^{-1} \quad \text{or} \quad 265\ \mathrm{MeV}$$

This scale lies between the long-range π exchange and the short-range part of the interaction. Therefore, the details of the short-range part of the NN interaction cannot be resolved by nucleons of momentum k_F and $V_{\mathrm{low}\,k}$ is a good starting point for the many-body problem.

Recall also that the NN scattering length in the $^1S_0$ partial wave in free space is

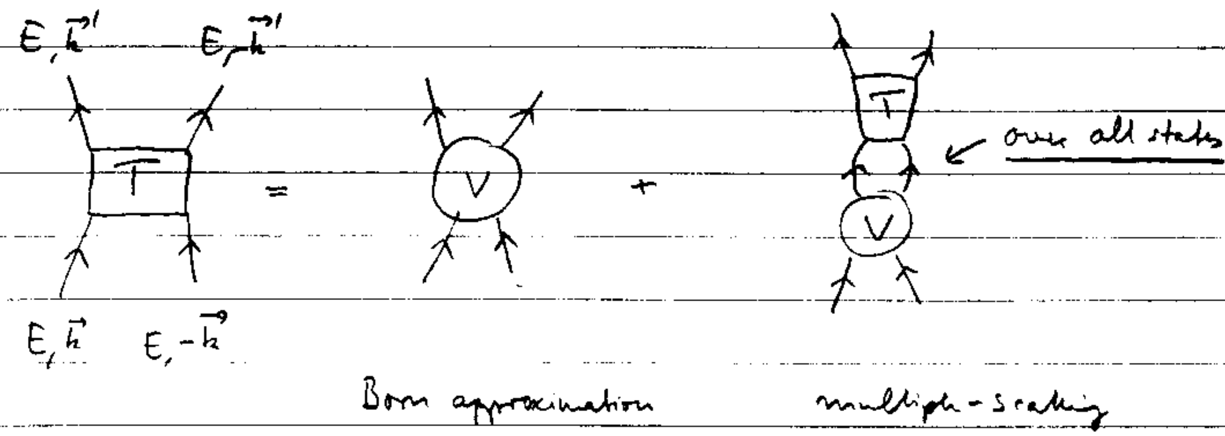
$$a_{^1S_0} \approx -23.73\ \mathrm{fm}$$

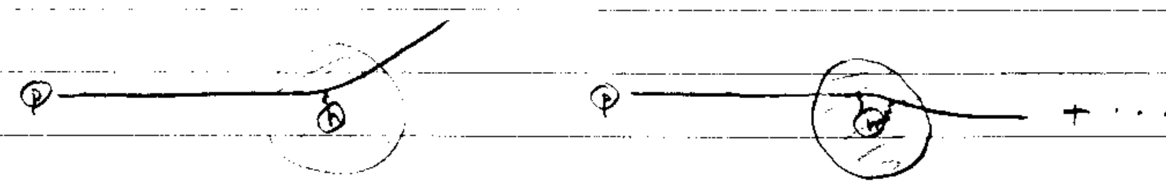
Thus, a natural question to ask is, what happens to this large structure in the presence of other particles. The interparticle spacing is $\sim \frac{1}{k_F} \approx 0.75$ fm or better $\rho_0^{-1/3} \approx 1.8$ fm, therefore there are many particles, which sit between two nucleons separated by $a_{^1S_0}$ (corresponds to distance over which two nucleons still see each other):



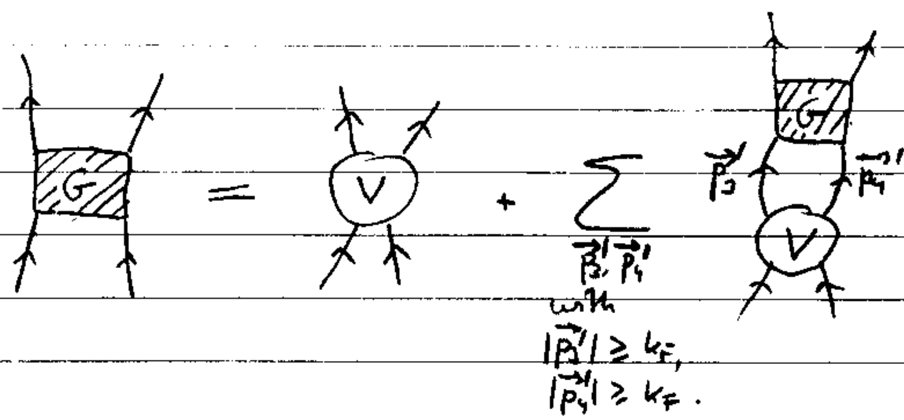
How does this effect the scattering of neutron n_1 and proton p_2?

In free-space, the Lippmann-Schwinger equation can be written diagrammatically as

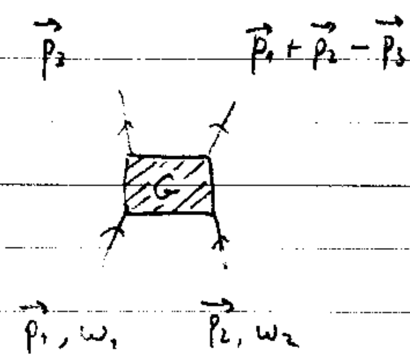




In the many-body system, the multiple scattering into particle-particle intermediate states has to be modified to account for the Pauli-blocking. This was addressed by Brueckner, Bethe, Goldstone and coworkers, who introduced the terminology "G-matrix" for the solution of the integral equation in the particle-particle channel with appropriate Pauli-blocking.



We know that the interaction is momentum-conserving, so we can label the lines by



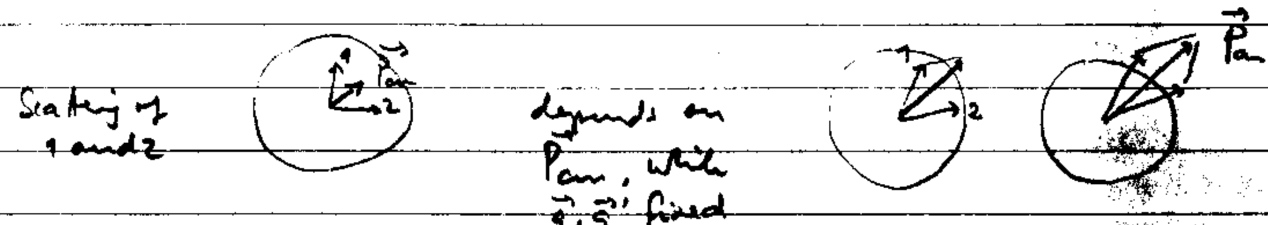
We again chose $\vec p_1 = \vec P/2 + \vec q/2 + \vec q\,'/2$, $\vec p_2 = \vec P/2 - \vec q/2 - \vec q\,'/2$, $\vec p_3 = \vec P/2 + \vec q\,'/2 - \vec q/2$

$$\Rightarrow \vec p_4 = \vec P/2 - \vec q\,'/2 + \vec q/2$$

The two-body scattering amplitude in the medium can depend on the center-of-mass momentum $\vec{P}$

$$G(\vec{q}, \vec{q}\,', \vec{P})$$

since the Fermi sea breaks Galilean invariance



Moreover, G depends on the energy of the incoming particle pair, $\omega_1 + \omega_2$, just as $T(E)$. For particles on the Fermi surface, $\omega_1 + \omega_2 = 2\mu$.

The phase space for the intermediate state can be depicted by writing $\vec{p}_3\,' = \vec{p}\,'' + \vec{P}/2$ and $\vec{p}_4\,' = \vec{P}/2 - \vec{p}\,''$ in the cm system.
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