Isospin composition of the high-momentum fluctuations in nuclei from asymptotic momentum distributions

- Employ Low-order Correlation operator Approximation (LCA) to compute the SRC contribution to the single-nucleon momentum distribution and ratios of A-to-d momentum distributions.

- p becomes more correlated as N increases in asymmetric nuclei. Hen et. al. say the protons "speed up".

- $a_2^{exp}(A)$ can be evaluated with ratios of bound-nucleon probability distributions where $r_{12} \to 0$ ($p_{12} \to \infty$)

$$a_2(A) = \lim_{p_{12} \to \infty} \frac{n^A(p_{12}, \Lambda)}{n^d(p_{12}, \underset{\substack{\uparrow \\ \text{UV regulator scale}}}{\Lambda})}$$

- Factorization : $\underset{\substack{\uparrow \\ \text{contacts}}}{C^A_{NN'}(\Lambda)} \, |\psi_{NN'}(r_{12}, \Lambda)|^2$

- Single-nucleon momentum distribution :

$$n^A(\vec{p}) \sim \langle \Psi_A | a^\dagger_{\vec{p}} a_{\vec{p}} | \Psi_A \rangle \qquad (6)$$

$\uparrow$ G.S. of A

- LCA: $|\Psi_A\rangle \rightarrow |\Phi_A\rangle$ (simple wave function)

Complexity of $|\Psi_A\rangle$ into operator $\hat{G}$

$$|\Psi_A\rangle = \frac{1}{\sqrt{\underbrace{\langle \Phi_A | \hat{G}^\dagger \hat{G} | \Phi_A \rangle}_{\equiv \sqrt{N}}}} \hat{G} |\Phi_A\rangle \qquad (7)$$

$\uparrow$ Slater determinant

$\uparrow$ SRC operator

$$\hat{G} = \hat{S} \left(\prod_{i<j=1}^{A} [1 - g_c(r_{ij}) + f_{t\tau}(r_{ij}) \hat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j + \right.$$

$\uparrow$ Tensor operator

$$\left. f_{\sigma\tau}(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \right)$$

$$= \hat{S} \left(\prod_{i<j=1}^{A} [1 + \hat{G}_{ij}(r_{ij})] \right) \qquad (8)$$

$\uparrow$ Symmetrization operator

(go up to order $O(\hat{G}^2)$)

$$\Rightarrow \quad n^A(\vec{p}) \sim \langle \Psi_A | a^\dagger_{\vec{p}} a_{\vec{p}} | \Psi_A \rangle$$

$$= N \langle \Phi_A | \hat{G}^\dagger a^\dagger_{\vec{p}} a_{\vec{p}} \hat{G} | \Phi_A \rangle$$

$$= N \left\langle \Phi_A \right| G^\dagger(?) \, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}} \, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}} \, a_{\frac{\vec{K}}{2}-\vec{k}''} \, a_{\frac{\vec{K}}{2}+\vec{k}''} \times$$

$$a^\dagger_{\vec{p}} \, a_{\vec{p}} \, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}'''} \, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}'''} \, a_{\frac{\vec{K}}{2}-\vec{k}'} \, a_{\frac{\vec{K}}{2}+\vec{k}'} \, G(?) \left| \Phi_A \right\rangle$$

$$a_{\frac{\vec{K}}{2}-\vec{k}''} \, a_{\frac{\vec{K}}{2}+\vec{k}''} \, a^\dagger_{\vec{p}} \, a_{\vec{p}} \, a^\dagger_{\frac{\vec{K}}{2}+\vec{k}'''} \, a^\dagger_{\frac{\vec{K}}{2}-\vec{k}'''}$$

$$= \delta_{\frac{\vec{K}}{2}-\vec{k}'', \vec{p}} \; \delta_{\frac{\vec{K}}{2}+\vec{k}'', \frac{\vec{K}}{2}+\vec{k}'''} \; \delta_{\vec{p}, \frac{\vec{K}}{2}-\vec{k}'''}$$

Set $\vec{k}'' = \vec{k}'''$ from 2^{nd} δ

$$= \delta_{\frac{\vec{K}}{2}-\vec{k}'', \vec{p}} \; \delta_{\vec{p}, \frac{\vec{K}}{2}-\vec{k}''}$$

So $\frac{\vec{K}}{2} - \vec{k}'' = \vec{p}$

I would expect $\hat{G}^\dagger(\vec{k}, \vec{k}'') \to \hat{G}^\dagger(\vec{k}, \vec{p})$ but they write $\hat{G}^\dagger_{12}\left(\frac{\vec{K}}{2} + \vec{k} - \vec{p}\right)$. How do they get this?

* For $\hat{G}$ at $G(G^0) = 1$, they get

$$n^A_{IPM}(\vec{p}) \approx \sum_{N \in \{p,n\}} \sum_{\alpha} \left\langle N\alpha \right| a^\dagger_{\vec{p}} a_{\vec{p}} \left| N\alpha \right\rangle \qquad (10)$$

$\alpha \in$ {single-particle states of the Slater determinant}

$$n^A_{SRC}(\vec{p}) \approx \sum_{NN' \in \{p,n\}} \sum_{\alpha\beta} \sum_{\vec{K}\vec{k}\vec{k}'} \hat{G}^\dagger_{12}\left(\frac{\vec{K}}{2} + \vec{k} - \vec{p}\right) \hat{G}_{12}\left(\frac{\vec{K}}{2} + \vec{k}' - \vec{p}\right) \times$$

$$\langle N_\alpha, N'_\beta | a^\dagger_{\frac{\vec{K}}{2}+\vec{k}} a^\dagger_{\frac{\vec{K}}{2}-\vec{k}} a_{\frac{\vec{K}}{2}+\vec{k}'} a_{\frac{\vec{K}}{2}-\vec{k}'} | N_\alpha, N'_\beta \rangle \qquad (11)$$

* In computing n^A, must integrate (11) over k, k', K
* Restricted to spherically symmetric nuclei: $n^A(\vec{p}) = n^A(p)$

$$n^A(p) \equiv \underbrace{n^A_{pp}(p) + n^A_{pn}(p)}_{\text{proton part}} + \underbrace{n^A_{nn}(p) + n^A_{np}(p)}_{\text{neutron part}} \qquad (12)$$

- Identify pair contributions to (10) by rewriting

$$n^A_{IPM}(\vec{p}) \sim \sum_{N \in \{p,n\}} \sum_{N' \in \{p,n\}} \sum_\alpha \sum_\beta \sum_{\vec{p}'} \langle N_\alpha | a^\dagger_{\vec{p}} a_{\vec{p}} | N_\alpha \rangle \times \langle N'_\beta | a^\dagger_{\vec{p}'} a_{\vec{p}'} | N'_\beta \rangle \qquad (13)$$

- Integrating over $\vec{p}'$ where $\int dp\, p^2 n^A(p) = A$

$$pp \to \frac{Z(Z-1)}{A-1}, \quad pn \to \frac{NZ}{A-1}, \quad nn \to \frac{N(N-1)}{A-1}, \quad np \to \frac{NZ}{A-1}$$

- Probability distribution

$$P^A(p) = \frac{p^2 n^A(p)}{A} \qquad \left(\int dp\, P^A(p) = 1 \right) \qquad (15)$$

$$P^A(p) = P^A_{pp}(p) + P^A_{pn}(p) + P^A_{np}(p) + P^A_{nn}(p) \qquad (16)$$

- Use HO single-particle states $|N\alpha\rangle$ "as they offer the possibility to separate the pair's relative and center-of-mass motions in the pair wave functions $|N\alpha, N'\beta\rangle$ with the aid of Moshinsky brackets." Not sure what this means.

- Figure 2: $\rho^A(p)$ vs p [fm^{-1}] for ^{40}Ca, ^{48}Ca, ... with total, pp, nn, pn+np

- Figure 3: $\frac{\text{pp SRC pairs}}{\text{pn SRC pairs}}$ vs Mass number A

- Figure 4: $a_2(A)$ vs A where $a_2(A) = \frac{\int_2^\infty dp\, \rho^A(p)}{\int_2^\infty dp\, \rho^d(p)}$ (20)

- Game plan: Calculate the IPM $\rho^A(p)$ for ^{40}Ca and ^{208}Pb using local density momentum distribution $\left(4\pi \int dr\, r^2\, \Theta(k_F(r) - p)\right.$ s.t. $\left.\int dp\, \rho^A(p) = 1\right)$ and compare to low-momentum parts of RCVC Fig. 2.

$$n^A_{IPM}(\vec{p}) \sim \sum_{NN'} \sum_{\alpha\beta} \sum_{\vec{p}'} \langle N\alpha | a^\dagger_{\vec{p}} a_{\vec{p}} | N\alpha\rangle \langle N'\beta | a^\dagger_{\vec{p}'} a_{\vec{p}'} | N'\beta\rangle$$

$$\rho^A_{IPM}(p) = \frac{p^2\, n^A_{IPM}(p)}{A} \qquad \left(\int_0^\infty dp\, \rho^A(p) = 1\right)$$

Use LDA:
$$\langle N\alpha | a^\dagger_{\vec{p}} a_{\vec{p}} | N\alpha\rangle = \Theta\left(k_F^N(r) - p\right)$$
$$\langle N'\beta | a^\dagger_{\vec{p}'} a_{\vec{p}'} | N'\beta\rangle = \Theta\left(k_F^{N'}(r) - p'\right)$$

$$\Lambda^{A,NN'}_{IPM}(p;\, k_F^N, k_F^{N'})$$

Gives $\displaystyle\sum_{p'} \to \frac{V}{(2\pi)^3}\int d^3p'\, \Theta(k_F^N(r)-p)\,\Theta(k_F^{N'}(r)-p')$

$$= \frac{4\pi}{3}(k_F^{N'})^3$$

$$\langle \Lambda^{A,NN'}_{IPM}(p)\rangle = 4\pi\int_0^\infty dr\, r^2\left[\frac{V}{(2\pi)^3}\,\frac{4\pi}{3}k_F^3(r)\times \Theta(k_F^N(r)-p)\right]$$

$$P^{A,NN'}_{IPM}(p) = \frac{p^2\langle \Lambda^{A,NN'}_{IPM}(p)\rangle}{A}$$

No! Use $\displaystyle\langle \Lambda^{A,N}_{IPM}(\vec p)\rangle = \sum_\alpha \langle\alpha|a^\dagger_{\vec p} a_{\vec p}|\alpha\rangle$

$$= \int d^3r\, \Theta(k_F^N(r)-p)$$

Then $\displaystyle P^A_{IPM}(p) = \frac{p^2\,\Lambda^A_{IPM}(p)}{A}$