CS4999 – Lorem Ipsum

1 Algorithm Analysis and Data Structures

1.1 Complexity Theory

1.1.1 Big O Notation

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Sed do eiusmodut tempor incididunt labore et dolore magna aliqua. The **time complexity** can be expressed as:

$$O(n) = \lim_{n \to \infty} \frac{f(n)}{g(n)}$$

Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Using amortized analysis we can determine the *average cost of operations*.

Key Properties:
- Transitivity: If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$
- Sum rule: $O(f(n)) + O(g(n)) = O(\max(f(n), g(n)))$
- Product rule: $O(f(n)) \cdot O(g(n)) = O(f(n) \cdot g(n))$

> **ⓘ Complexity Classes**
>
> Pellentesque habitant morbi tristique senesque et netus et malesuada fames ac turpis egestas. Common complexity classes include:
> - P: Problems solvable in polynomial time
> - NP: Problems verifiable in polynomial time
> - NP-Complete: Hardest problems in NP

1.1.2 Binary Search Implementation

Duis aute irure dolor in **reprehenderit** in voluptate velit esse cillum dolore eu fugiat nulla pariatur. This algorithm uses the divide and conquer strategy:

```python
def binary_search(arr, target):
    left, right = 0, len(arr) - 1

    while left ≤ right:
        mid = (left + right) // 2

        if arr[mid] == target:
            return mid
        elif arr[mid] < target:
```

```
10          left = mid + 1
11        else:
12          right = mid - 1
13
14    return -1
```

> ⚠ **Time Complexity Analysis**
>
> Mauris blandit aliquet elit, at hendrerit urna semper vel. Binary search achieves $O(\log n)$ time complexity by eliminating half the search space in each iteration. Curabitur aliquet quam id dui posuere blandit.

1.2 Sorting Algorithms

1.2.1 QuickSort Analysis

Lorem ipsum dolor sit amet, the **average case time complexity** is $O(n \log n)$, sed consectetur adipiscing elit. The recurrence relation demonstrates the divide and conquer approach:

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

Vestibulum ante ipsum primis in faucibus orci luctus:

```
 1 void quickSort(int arr[], int low, int high) {
 2   if (low < high) {
 3     int pi = partition(arr, low, high);
 4
 5     quickSort(arr, low, pi - 1);
 6     quickSort(arr, pi + 1, high);
 7   }
 8 }
 9
10 int partition(int arr[], int low, int high) {
11   int pivot = arr[high];
12   int i = low - 1;
13
14   for (int j = low; j < high; j++) {
15     if (arr[j] < pivot) {
16       i++;
17       swap(arr[i], arr[j]);
18     }
19   }
20   swap(arr[i + 1], arr[high]);
21   return i + 1;
22 }
```

> 🗐 **Master Theorem Application**
>
> Vestibulum ante ipsum primis in faucibus orci *luctus et ultrices posuere cubilia curae*. For recurrences of the form $T(n) = aT\left(\frac{n}{b}\right) + f(n)$:
>
> 1. If $f(n) = O\left(n^{\log_b a - \varepsilon}\right)$ for some $\varepsilon > 0$, then $T(n) = \Theta\left(n^{\log_b a}\right)$
> 2. If $f(n) = \Theta\left(n^{\log_b a}\right)$, then $T(n) = \Theta\left(n^{\log_b a} \log n\right)$
> 3. If $f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right)$ for some $\varepsilon > 0$, then $T(n) = \Theta(f(n))$
>
> Proin eget tortor risus, donec sollicitudin **molestie malesuada**.

1.3 Graph Theory

1.3.1 Dijkstra's Algorithm

Sed porttitor lectus nibh, cras ultricies ligula **sed magna dictum porta**. We can represent the graph using an adjacency matrix:

```
 1  function dijkstra(graph, start) {
 2    const distances = {};
 3    const visited = new Set();
 4    const pq = new PriorityQueue();
 5
 6    for (let node in graph) {
 7      distances[node] = Infinity;
 8    }
 9    distances[start] = 0;
10    pq.enqueue(start, 0);
11
12    while (!pq.isEmpty()) {
13      const current = pq.dequeue();
14
15      if (visited.has(current)) continue;
16      visited.add(current);
17
18      for (let neighbor in graph[current]) {
19        const distance = distances[current] + graph[current][neighbor];
20        if (distance < distances[neighbor]) {
21          distances[neighbor] = distance;
22          pq.enqueue(neighbor, distance);
23        }
24      }
25    }
26
27    return distances;
28  }
```

> *Aside* 1.1: Historical Context
>
> Vivamus magna justo, lacinia eget consectetur sed, convallis at tellus. Edsger Dijkstra developed this algorithm in 1956. Quisque velit nisi, pretium ut lacinia in, elementum id enim.

1.3.2 Minimum Spanning Trees

Praesent sapien massa, convallis a pellentesque nec, egestas non nisi. **Kruskal's algorithm** complexity runs in polynomial time:

$$T(n) = O(E \log V)$$

Where E represents edges and V represents vertices. Nulla porttitor accumsan tincidunt. The greedy approach is employed here.

> ## Greedy Algorithms
>
> Donec rutrum congue leo eget malesuada. Both Kruskal's and Prim's algorithms use the greedy approach:
>
> - At each step, make the locally optimal choice
> - For MST: Always select the minimum weight edge that doesn't create a cycle
> - Greedy choice leads to globally optimal solution
>
> Curabitur non nulla sit amet nisl tempus convallis quis ac lectus.

1.4 Dynamic Programming

1.4.1 Longest Common Subsequence

Vivamus magna justo, lacinia eget consectetur sed, convallis at tellus. The DP table construction uses memoization:

```python
 1  def lcs(X, Y):
 2    m, n = len(X), len(Y)
 3    dp = [[0] * (n + 1) for _ in range(m + 1)]
 4
 5    for i in range(1, m + 1):
 6      for j in range(1, n + 1):
 7        if X[i-1] == Y[j-1]:
 8          dp[i][j] = dp[i-1][j-1] + 1
 9        else:
10          dp[i][j] = max(dp[i-1][j], dp[i][j-1])
11
12    return dp[m][n]
```

Nulla porttitor accumsan tincidunt. Space complexity can be *optimized* to $O(\min(m, n))$ using rolling arrays. This demonstrates optimal substructure.

> ### 🗎 Optimal Substructure
>
> Vestibulum ac diam sit amet quam vehicula elementum sed sit amet dui. A problem exhibits optimal substructure if an optimal solution can be constructed from optimal solutions of its subproblems. Pellentesque in ipsum id orci porta dapibus.

1.4.2 Knapsack Problem

Donec sollicitudin molestie malesuada. The recurrence relation shows optimal substructure:

$$K(i, w) = \max\big(K(i - 1, w), v_i + K(i - 1, w - w_i)\big)$$

Proin eget tortor risus, where v_i is value and w_i is weight of item i.

0/1 Knapsack Implementation

Lorem ipsum dolor sit amet, consectetur adipiscing elit:

```python
def knapsack(weights, values, capacity):
    n = len(weights)
    dp = [[0] * (capacity + 1) for _ in range(n + 1)]

    for i in range(1, n + 1):
        for w in range(capacity + 1):
            if weights[i-1] <= w:
                dp[i][w] = max(values[i-1] + dp[i-1][w-weights[i-1]],
                               dp[i-1][w])
            else:
                dp[i][w] = dp[i-1][w]

    return dp[n][capacity]
```

Sed porttitor lectus nibh, time complexity is $O(nW)$ where W is capacity.

1.5 Conclusion

Sed porttitor lectus nibh. **Cras ultricies ligula** sed magna dictum porta. Quisque velit nisi, pretium ut lacinia in, elementum id enim. Donec rutrum congue leo eget malesuada.

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