



ACC'16 Tutorial:

# Large-scale 3D Reconstruction from Images

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LONG QUAN, TIANWEI SHEN, JINGLU WANG

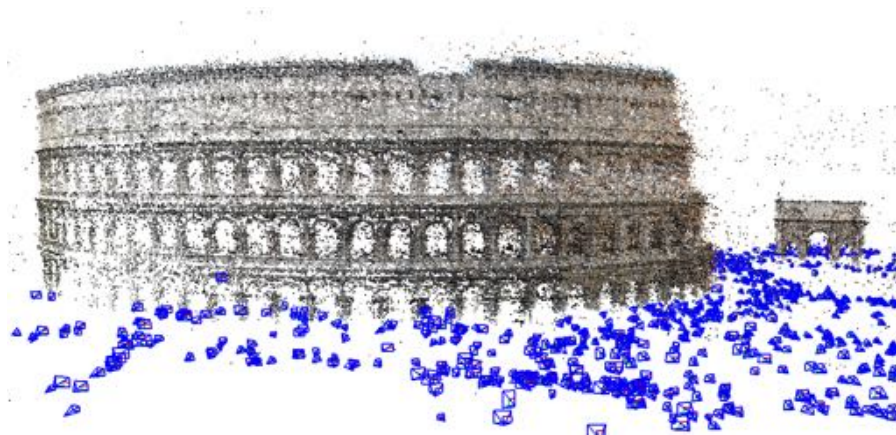


THE HONG KONG  
UNIVERSITY OF SCIENCE  
AND TECHNOLOGY

# Part I

## Tianwei Shen

# Large-scale Structure-from- Motion: A Modern Synthesis



# Outline

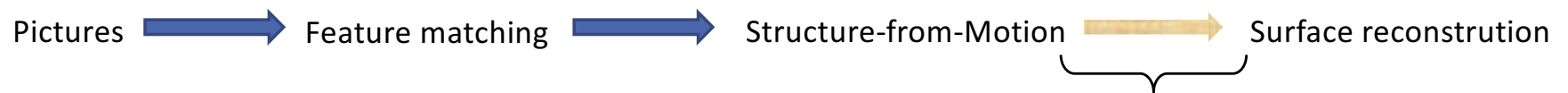
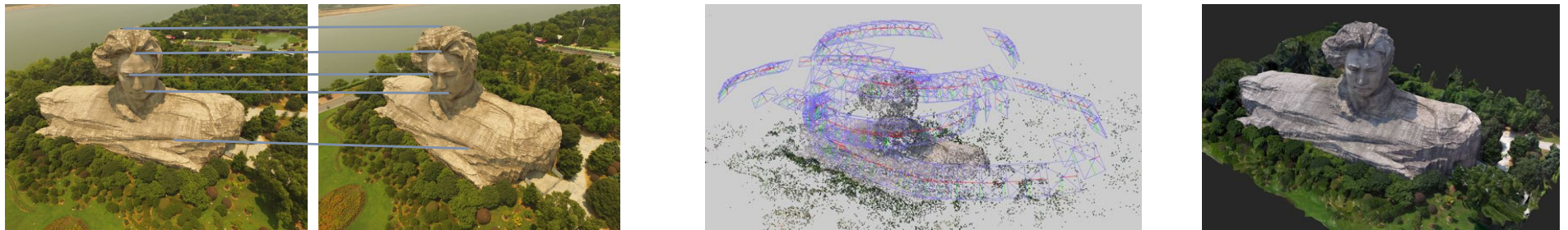
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- ❑ Introduction to Structure-from-Motion (SfM)
- ❑ Component I: Feature Detection and Matching
- ❑ Component II: From Feature matches to 3D
- ❑ Component III: Large-scale Bundle Adjustment
- ❑ Applications and Future Directions



# SfM - The entry point to 3D computer vision

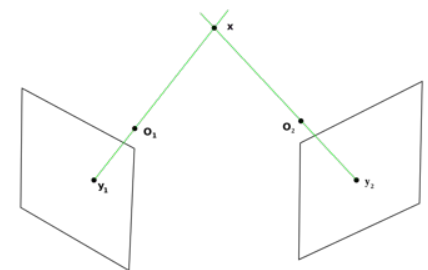
## □ From pictures to 3D scenes



Not covered in this talk

# Notations

- ❑ Views/Frames/Images:  $\{I_i\}$
- ❑ Features: 2D salient regions/blobs (edges, corners), e.g. SIFT
- ❑ Tracks: 3D point structures that correspond to 2D features in images
- ❑ Camera Intrinsic / Extrinsic:  $\{P_i\} \Rightarrow K [R T]$
- ❑ Residual error: distance between 2D features and 3D projection
- ❑ Triangulation: the process of determining a point in 3D space given its projections onto two, or more images



# A typical pipeline of SfM

- ❑ Feature extraction: images  $\{I_i\} \rightarrow$  local feature collections  $\{F_i\}$
- ❑ Feature matching:  $\{F_i\} \rightarrow$  match pairs  $\{M_{ij}\}$ , epipolar geometry  $\{f, h, R_{ij}, t_{ij}\}$
- ❑ Match graph construction:  $\{M_{ij}, R_{ij}, t_{ij}\} \rightarrow$  camera poses  $\{P_i\}$ , tracks  $\{p_k\}$ 
  - ❑ Graph initialization (select a robust initial match pair to build a metric reconstruction)
  - ❑ How we add edges to the match graph (global / incremental)
- ❑ Bundle adjustment:  $\{P_i\}, \{p_k\} \rightarrow$  optimized  $\{P_i\}, \{p_k\}$
- ❑ *Building Rome in a day* (2009) – the first practical large-scale SfM system



# SfM is just a large-scale optimization problem

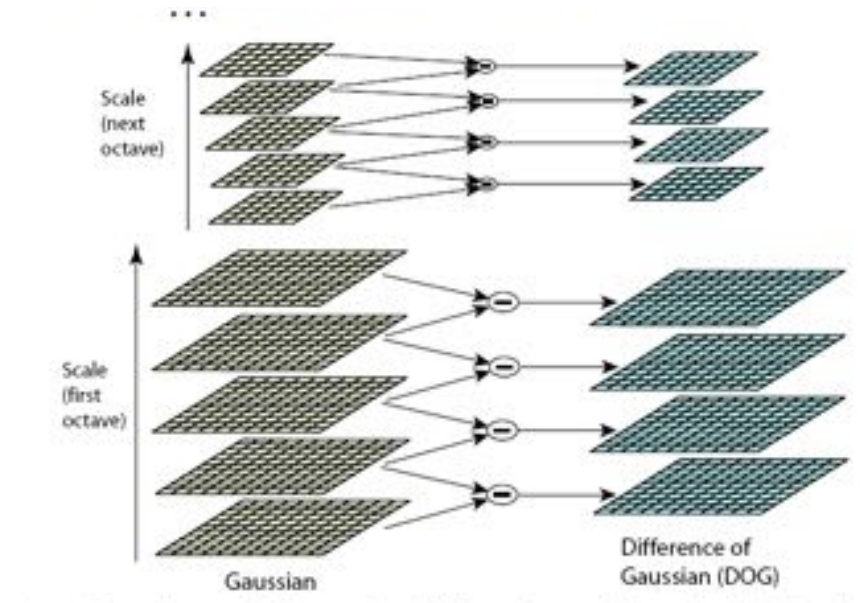
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- ☐ 2-view/3-view optimization (epipolar geometry)
- ☐ Match graph optimization
- ☐ Pose averaging
- ☐ Bundle adjustment (non-linear least squares)



# Topic I: Local Features and Matching

- Local feature - the basis for SfM
- Scale Invariant Feature Transform (SIFT)
  - Scale-space extrema detection
  - Keypoint localization
  - Orientation assignment
  - Keypoint description
- Invariant to translation, scaling and rotation





# Problems with feature matching

- ❑ Tradeoff: SIFT is not invariant under geometric transformations
- ❑ Problem1: Pairwise feature matching is costly.
- ❑ Problem2: Erroneous matches is evitable, thus robust estimation is used.
  - ❑ An extreme case:



Front-front match

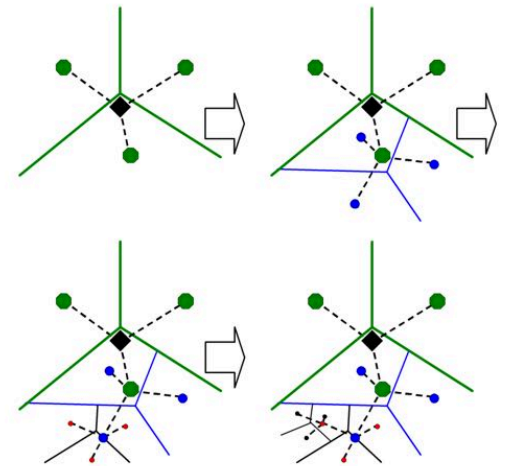


Erroneous front-back match



# To tackle problem 1: matching efficiency

- Use image retrieval to compute a candidate match set
- Vocabulary tree: train -> build -> match
- Reduce cost from  $O(n^2)$  to  $O(kn)$ ,  $k$  decided by users
- Main problem with this approach:
  - $k$  is not known beforehand
  - Too small  $k$  is not sufficient
  - Too large  $k$  slows down the process



[1] Nister, David, and Henrik Stewenius. "Scalable recognition with a vocabulary tree." *CVPR*. Vol. 2. IEEE, 2006.

# To tackle problem 1: matching efficiency

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## ☐ Other approaches:

- ☐ Relevance feedback and entropy minimization ([1] Lou et al. )
- ☐ Match features in larger pyramid scale ([2] Wu)
- ☐ Learning-based method to predict overlaps ([3] Schönberger et al. )
- ☐ A hashing-based cascading matching ([4] Cheng et al. )

[1] Lou, Y., Snavely, N., Gehrke, J.: Matchminer: Efficient spanning structure mining in large image collections. In: ECCV, pp. 45–58 (2012)

[2] Wu, C.: Towards linear-time incremental structure from motion. In: 3DV, pp. 127–134 (2013)

[3] Schönberger, J.L., Berg, A.C., Frahm, J.M.: Paige: Pairwise image geometry encoding for improved efficiency in structure-from-motion. In: CVPR, pp. 1009–1018 (2015)

[4] Cheng, Jian, et al. "Fast and accurate image matching with cascade hashing for 3d reconstruction." *CVPR*. 2014.



# To tackle problem 2: erroneous matches

- ❑ Identification and removal of erroneous epipolar geometry is a recent research focus for SfM.
- ❑ Can lead to catastrophic results for SfM.



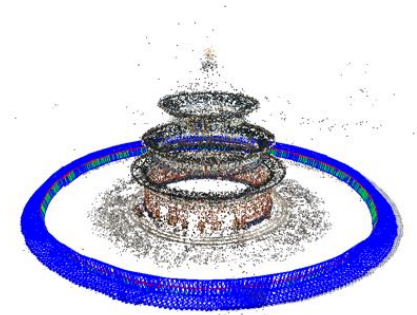
Front-front match



Erroneous front-back match



Wrong model



Correct model



# To tackle problem 2: erroneous matches

## □ Loop consistency [1]:

- Chained relative motion should be an identity map:  $R_{12}R_{23}R_{31} = I$
- Start from a full match graph
- Sample cycles from the full graph
- The problem is casted as a Bayesian inference task
- Strong assumption on variable independence

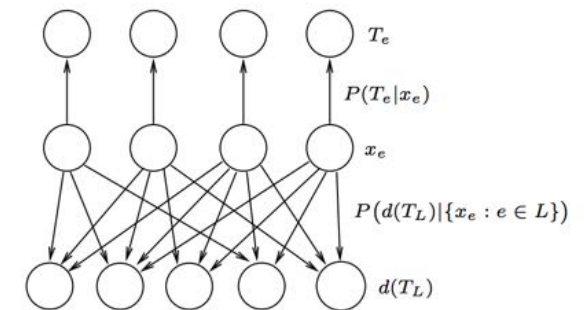
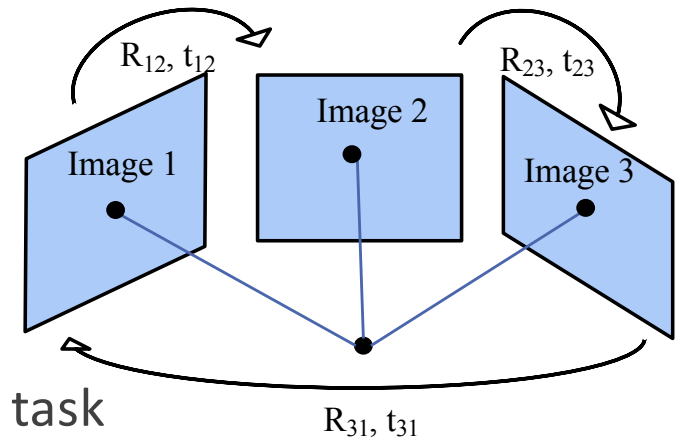


Figure 3. The Bayesian network for cycle inference.

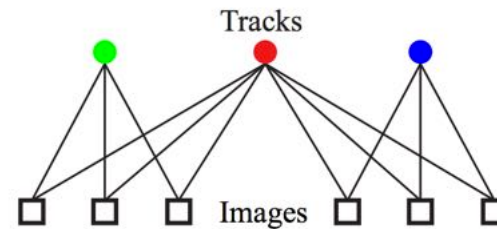
[1] Zach, Christopher, Manfred Klopschitz, and Manfred Pollefeys. "Disambiguating visual relations using loop constraints." *CVPR*. Vol. 2. 2010.

# To tackle problem 2: erroneous matches

## ❑ Other works:

- ❑ Sampling match graph based on missing correspondences and time stamp cue. [1]

- ❑ Analysis of visibility graph. [2]



- ❑ Splits the camera graph and then leverages conflicting observations. [3]

- [1] Roberts, Richard, et al. "Structure from motion for scenes with large duplicate structures." *CVPR*, IEEE, 2011.
- [2] Wilson, Kyle, and Noah Snavely. "Network principles for sfm: Disambiguating repeated structures with local context." *CVPR*, 2013.
- [3] J. Heinly, E. Dunn, and J.-M. Frahm, "Correcting for duplicate scene structure in sparse 3d reconstruction," in *ECCV*, pp. 780–795, 2014.

# Motivation: Solve two problems together

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- ☐ All disambiguation methods start from a relatively full match graph
- ☐ Construct an error-free match graph in a bottom-up fashion
- ☐ Select a sufficient match set that can guarantee a reconstruction
- ☐ Prevent additions of erroneous pairs



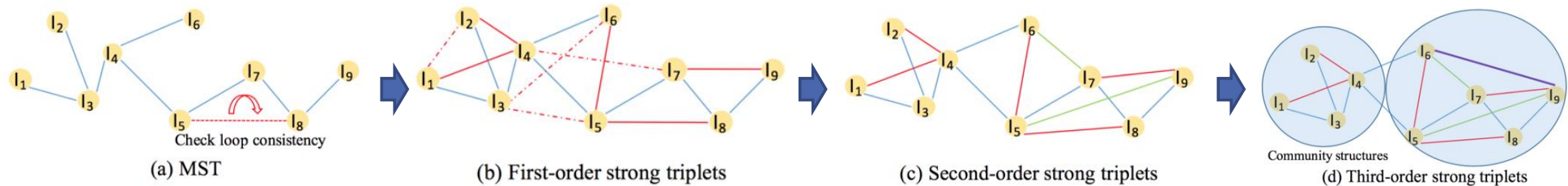
# Graph-based consistent matching for SfM

## □ Multi-stage matching process:

□ Stage 1: Starts from a minimal spanning tree based on vocabulary tree ranks

□ Stage 2: Expand the spanning tree with loop consistency guaranteed

□ Stage 3: Find loop closures by community detection



T. Shen, S. Zhu, T. Fang, R. Zhang, and L. Quan, “Graph-based consistent matching for structure-from-motion,” in *ECCV*, 2016.



# Graph-based consistent matching for SfM

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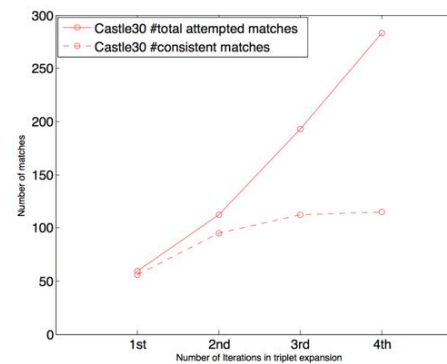
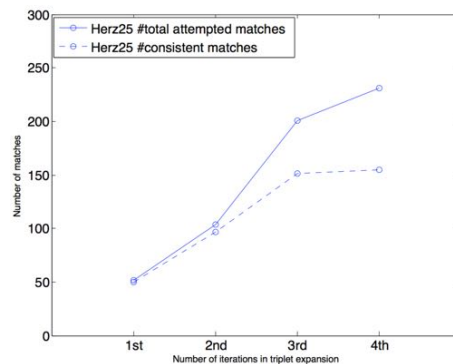
- Stage 1: start from a minimal spanning tree
  - The purpose is to quickly chain the views
  - A modified Kruskal's algorithm (online version): reject outliers
- Edge weight parameter  $w(e_{ij}) = \sqrt{\frac{Rank_i^2(j) + Rank_j^2(i)}{2}}$  given by vocabulary tree:

T. Shen, S. Zhu, T. Fang, R. Zhang, and L. Quan, "Graph-based consistent matching for structure-from-motion," in *ECCV*, 2016.



# Graph-based consistent matching for SfM

- Stage 2: Graph Expansion by Strong Triplets
  - Verifying all loops is hard to achieve, even verifying all triplets is  $O(n^3)$
  - Generate a consistent match graph in a bottom-up way
  - A empirical choice: traversing two steps starting from each node

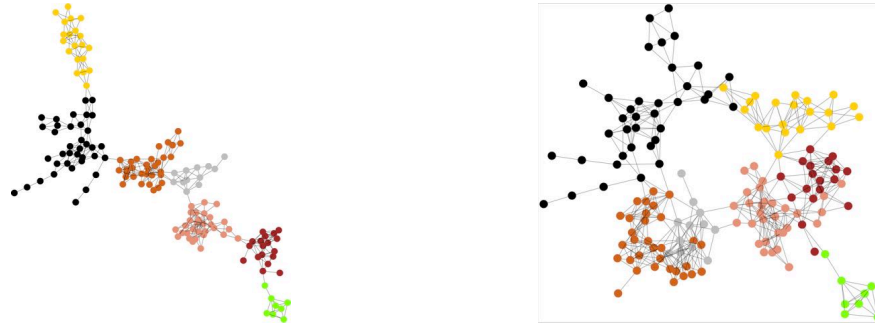


T. Shen, S. Zhu, T. Fang, R. Zhang, and L. Quan, “Graph-based consistent matching for structure-from-motion,” in *ECCV*, 2016.



# Graph-based consistent matching for SfM

- ❑ Stage 3: Community-Based Graph Reinforcement
  - ❑ Too sparse connection after triplet expansion
  - ❑ Longer loops are not verified
  - ❑ Community detection: divide a graph into groups with denser connections inside and sparser connections outside.

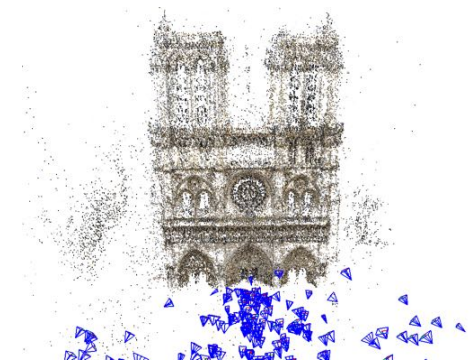
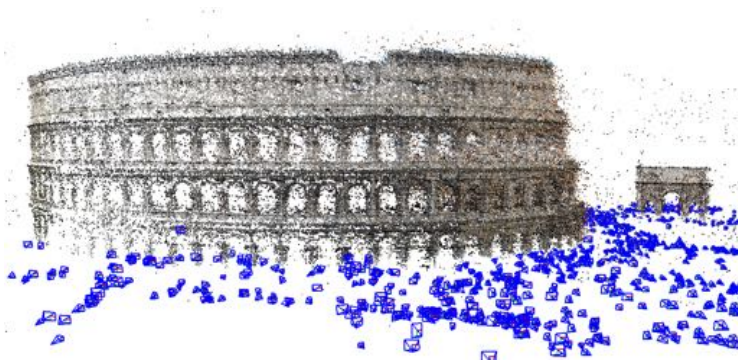


T. Shen, S. Zhu, T. Fang, R. Zhang, and L. Quan, “Graph-based consistent matching for structure-from-motion,” in *ECCV*, 2016.



# Graph-based consistent matching for SfM

## □ Results – Internet data

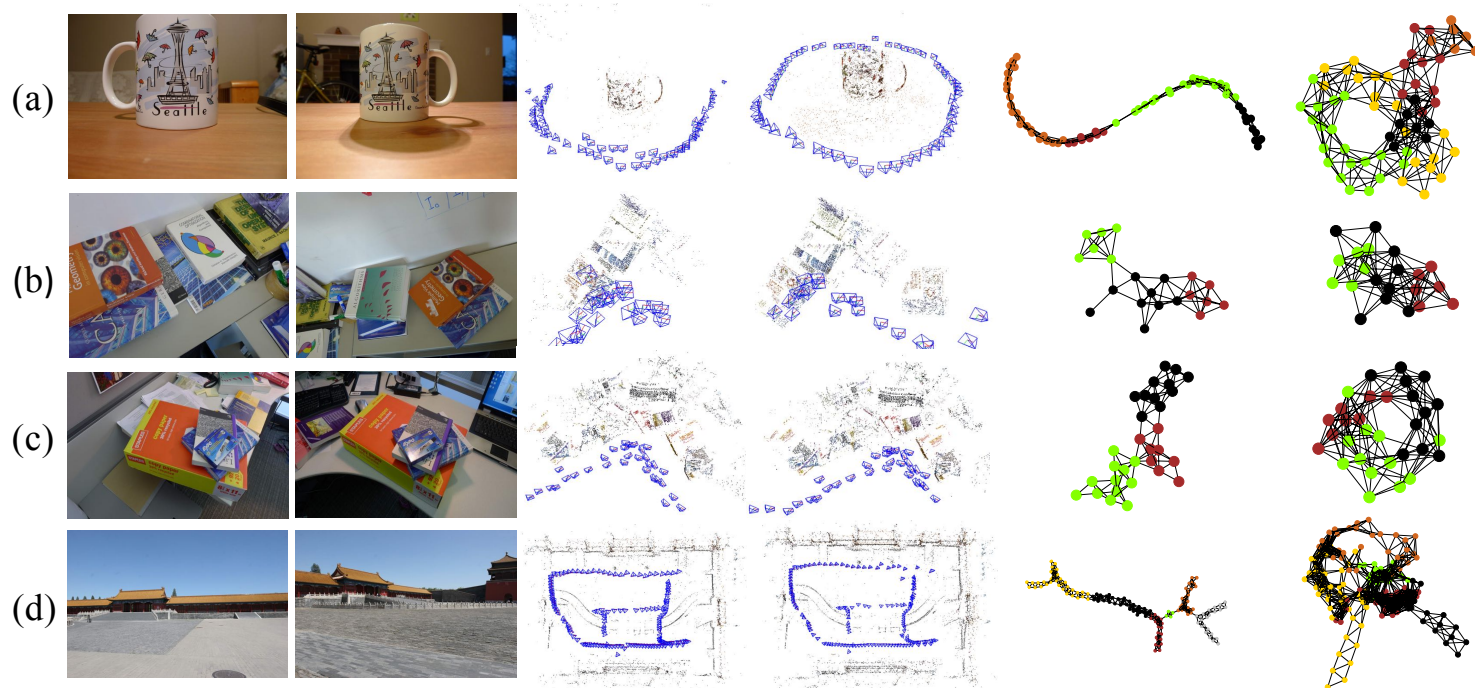


T. Shen, S. Zhu, T. Fang, R. Zhang, and L. Quan, “Graph-based consistent matching for structure-from-motion,” in *ECCV*, 2016.



# Graph-based consistent matching for SfM

## □ Results – ambiguity data



T. Shen, S. Zhu, T. Fang, R. Zhang, and L. Quan, “Graph-based consistent matching for structure-from-motion,” in *ECCV*, 2016.

# Future direction: learning local features

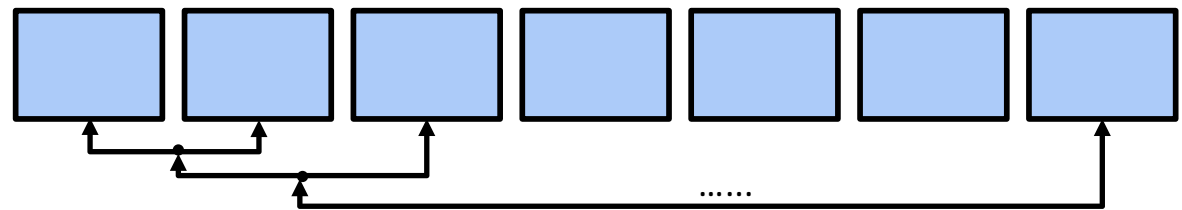
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- ☐ Feature is the most important factor in SfM accuracy
- ☐ Deep learning approaches: learning local feature descriptors
- ☐ Speed up matching and improve matching accuracy

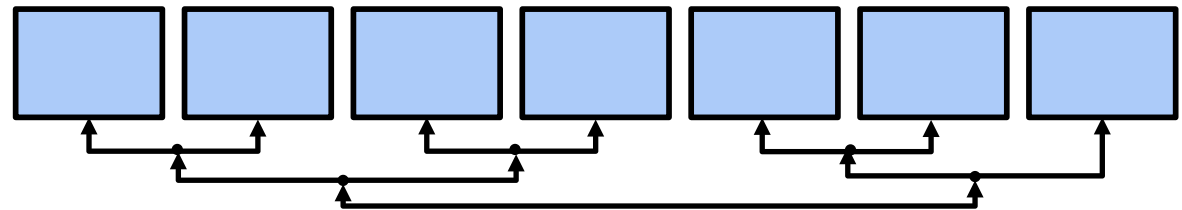


# Topic II: From Feature matches to 3D

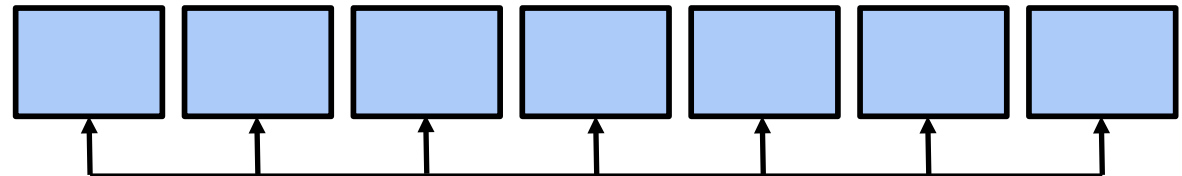
□ Incremental



□ Hierarchical



□ Global



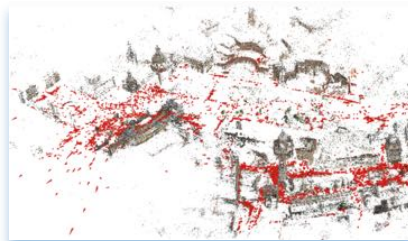


# Some Recent Representative Architectures

## Sequential/Incremental Approaches



Building Rome in a day



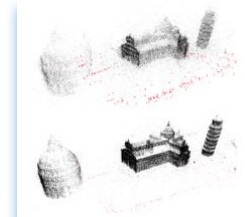
Colmap: SfM Revisited

## Hierarchical

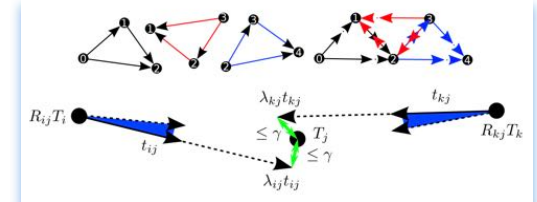


Randomized structure from motion based on atomic 3d models from camera triplets

## Global Approaches



Optimizing the Viewing Graph for Structure-from-Motion



Global Fusion of Relative Motions for Robust, Accurate and Scalable Structure from Motion.





# Three SfM Paradigms

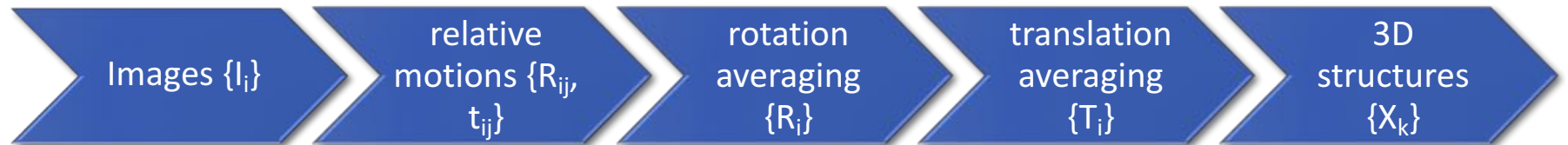
	Incremental	Hierarchical	Global
Feature extraction and matching	-	-	-
Match graph initialization	Initialized by carefully selected two-view	Atomic models	All views are treated equally
Image Registration	Perspective-n-Point (PnP), 2D-3D correspondences	3D-3D fusion	Rotation and translation averaging
Bundle adjustment	Iterative, many times	BA when merging	one time
Advantages	Robust	Fewer BA steps	Evenly-distributed errors
Disadvantages	Prone to drifting errors	Model merging, graph partition	Prone to noisy pairwise matches
Softwares	Bundler, openMVG, VisualSfM, MVE	Research papers	openMVG, Theia



# Key technique: motion averaging

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- ❑ Correct accumulating errors in chained pose estimation
- ❑ First rotation averaging, then translation averaging



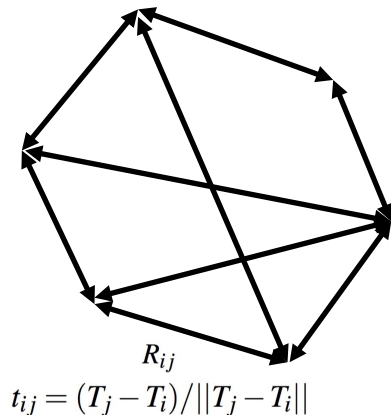
# Convex optimization in SfM

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- ☐ Convex optimization becomes popular because of its elegant mathematical forms and the existence of global minimum
- ☐ First investigated by Hartley et al. on triangulation
- ☐ Quasi-convex optimization by Ke et al. and Kahl, casted as an Second-Order Cone Programming (SOCP)
- ☐ Not practical due to its sensitivity to noises, but theoretically interesting



# Rotation averaging on a graph



Viewing Graph:  $G = (\mathcal{V}, \mathcal{E})$

Globally consistent rotation:  $R_{ij} = R_j R_i^{-1}, \quad \forall (i, j) \in \mathcal{E}$

Minimize Riemannian distance:  $d(\mathbf{X}, \mathbf{Y}) = ||\log(\mathbf{YX}^{-1})||$

Rotation average is non-convex

V. M. Govindu, "Lie-algebraic averaging for globally consistent motion estimation," in *CVPR*, vol. 1, pp. I-684, IEEE, 2004.



# Rotation averaging: other approaches

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- ❑ Quaternions parameterization (Martinec et al. [1])
- ❑ L1 norm based on Weiszfeld algorithm (Hartley et al. [2])

[1] D. Martinec and T. Pajdla, “Robust rotation and translation estimation in multiview reconstruction,” in *CVPR*, pp. 1–8, 2007.

[2] R. Hartley, K. Aftab, and J. Trumpf, “L1 rotation averaging using the weiszfeld algorithm,” in *CVPR*, pp. 3041–3048, IEEE, 2011.



# Translation averaging

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- Long been characterized as a convex optimization problem
- Min-max formulation, SOCP
- Same L-infinity drawbacks: prone to outliers



# Translation averaging

□ Considering observed points together (triplet bundle)

□ Re-projection error:  $\rho(t_i, X_j) = \left\| \left( \hat{x}_{ij}^{(1)} - \frac{R_i^{(1)T} X_j + t_i^{(1)}}{R_i^{(3)T} X_j + t_i^{(3)}}, \hat{x}_{ij}^{(2)} - \frac{R_i^{(2)T} X_j + t_i^{(2)}}{R_i^{(3)T} X_j + t_i^{(3)}} \right) \right\|_{\infty}$

□ Linear program minimal  
case with RANSAC

$$\begin{aligned} & \underset{\{t_i\}, \{X_j\}, \gamma}{\text{minimize}} && \gamma \\ & \text{subject to} && \rho(t_i, X_j) \leq \gamma, \\ & && R_i^{(3)T} X_j + t_i^{(3)} \geq 1, \\ & && t_i = (0, 0, 0) \quad \forall i, j. \end{aligned}$$

P. Moulon, P. Monasse, and R. Marlet, “Global fusion of relative motions for robust, accurate and scalable structure from motion,” in *ICCV*, pp. 3248–3255, 2013



# Translation averaging

□ Then global translation averaging

□ Formulation under L-infinity: 
$$\begin{aligned} & \underset{\{T_i\}, \{\lambda_{ij}\}, \gamma}{\text{minimize}} && \gamma \\ & \text{subject to} && \|T_j - R_{ij}T_i - \lambda_{ij}t_{ij}\|_{\infty} \leq \gamma, \\ & && \lambda_{ij} \geq 1, \forall i, j \\ & && T_1 = (0, 0, 0). \end{aligned}$$

□ Minimizing two-side of  $\lambda_{ij}t_{ij} = T_j - R_{ij}T_i$

P. Moulon, P. Monasse, and R. Marlet, “Global fusion of relative motions for robust, accurate and scalable structure from motion,” in *ICCV*, pp. 3248–3255, 2013





# Translation averaging: robust formulation

- A small robust L1 formulation improvement: consider L1 norm of the re-projection error vector:

$$(\cdots, \rho(t_i, X_j), \cdots)$$

$$\begin{aligned} & \underset{\{t_i\}, \{X_j\}, \gamma}{\text{minimize}} \quad \gamma \\ & \text{subject to} \quad \rho(t_i, X_j) \leq \gamma, \\ & \quad R_i^{(3)} X_j + t_i^{(3)} \geq 1, \\ & \quad t_i = (0, 0, 0) \quad \forall i, j. \end{aligned}$$



$$\begin{aligned} & \underset{\{t_i\}, \{X_j\}, \{\gamma_i\}}{\text{minimize}} \quad \sum_i \gamma_i \\ & \text{subject to} \quad \rho(t_i, X_j) \leq \gamma_i, \\ & \quad R_i^{(3)} X_j + t_i^{(3)} \geq 1, \\ & \quad t_i = (0, 0, 0) \quad \forall i, j. \end{aligned}$$

$$\begin{aligned} & \underset{\{T_i\}, \{\lambda_{ij}\}, \gamma}{\text{minimize}} \quad \gamma \\ & \text{subject to} \quad \|T_j - R_{ij} T_i - \lambda_{ij} t_{ij}\|_\infty \leq \gamma, \\ & \quad \lambda_{ij} \geq 1, \quad \forall i, j \\ & \quad T_1 = (0, 0, 0). \end{aligned}$$



$$\begin{aligned} & \underset{\{T_i\}, \{\lambda_{ij}\}, \{\gamma_{(i,j)}\}}{\text{minimize}} \quad \sum_{(i,j)} \gamma_{(i,j)} \\ & \text{subject to} \quad \|T_j - R_{ij} T_i - \lambda_{ij} t_{ij}\|_\infty \leq \gamma_{(i,j)}, \\ & \quad \lambda_{ij} \geq 1, \quad \forall i, j \\ & \quad T_1 = (0, 0, 0). \end{aligned}$$

Tianwei Shen. Convex Modelling of Motion Estimation in Structure-from-Motion. (unpublished report)



# Translation averaging: comparison

## □ RobustL1 outperforms other methods



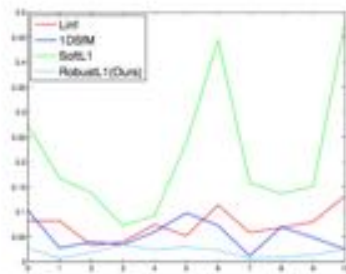
(a) fountain-P11 dataset



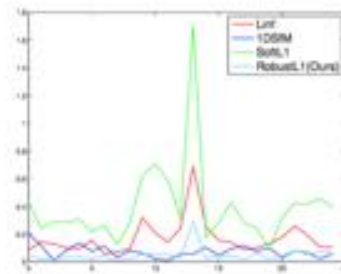
(b) Herz-Jesu-P25 dataset



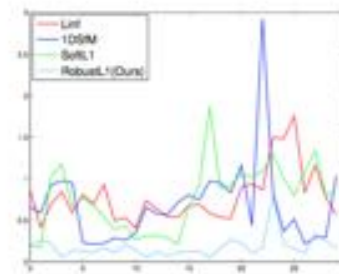
(c) castle-P30 dataset



(d) fountain-P11 per camera error



(e) Herz-Jesu-P25 per camera error



(f) castle-P30 per camera error



# Translation averaging: comparison

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- ❑ However, advantage is not evident after bundle adjustment (BA)
- ❑ Also, the problem scale is larger
- ❑ Future direction: no large-scale benchmark datasets for testing
- ❑ A potential useful settings is SLAM, where BA is costly



# Topic 3: Bundle adjustment

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- Joint optimization of camera poses and 3D tracks

$$\min_{P_i \in \mathcal{Q}} \sum_{i=1}^m \sum_{j=1}^n v_{ij} f(u_{ij} - \Pi(P_i, X_j))$$

- Error model:  $f(\Delta z_{ij}) = \frac{1}{2} \Delta z_{ij}^T W_{ij} \Delta z_{ij}$

$$\Delta z_{ij} = u_{ij} - \Pi(P_i, X_j)$$



# Bundle adjustment

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## □ Levenberg-Marquardt algorithm

□ Taylor expansion:  $f(x + \delta x) \approx f(x) + g^T \delta x + \frac{1}{2} \delta x^T H \delta x, g \equiv \frac{df}{dx}(x), H \equiv \frac{d^2f}{dx^2}(x)$

□ Newton step:  $\frac{d}{dx}f(x + \delta x) \approx H \delta x + g = 0 \implies \delta x = -H^{-1}g$

□ New value:  $f(x + \delta x) \approx f(x) - \frac{1}{2}g^T H^{-1}g$

□ Damped Newton's methods:  $(H + \lambda W)\delta x = -g$



# Bundle adjustment

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- ❑ Large-scale endeavors

  - ❑ Multi-core bundle adjustment [1]

  - ❑ Distributed settings [2]

- ❑ Essentially a non-linear least square problem, thus generally useful for other vision problems.

[1] Wu, Changchang, et al. "Multicore bundle adjustment." *CVPR, 2011*.

[2] Eriksson, Anders, et al. "A Consensus-Based Framework for Distributed Bundle Adjustment." *CVPR, 2016*.



# What can we do with SfM?

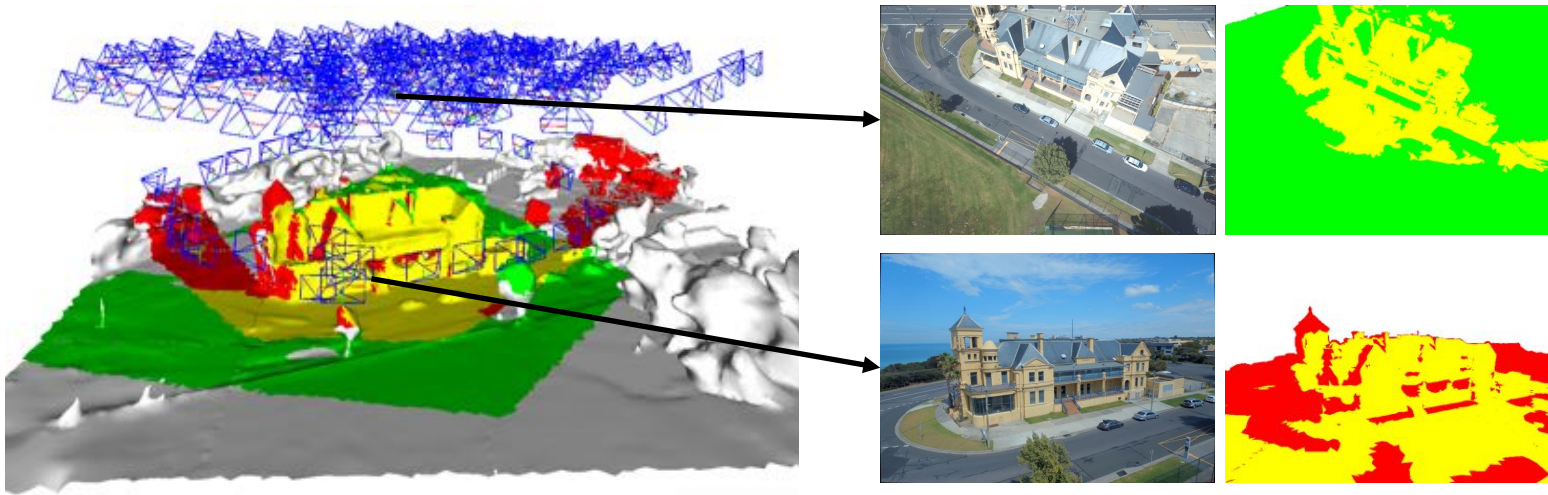
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- ☐ 3D reconstruction
- ☐ Simultaneous localization and mapping (SLAM)
- ☐ Test base for local features (distinctiveness, efficiency, matchability)
- ☐ Color correction for image collections
- ☐ Visual effects
- ☐ ...



# Application: Large-Scale Color Correction

- Motivation: Images captured for 3D reconstruction are color-inconsistent
- Optimize color of image collections, based on geometric information





# Application: Large-Scale Color Correction

□ Non-linear optimization on color histogram:

$$\begin{aligned} & \underset{\{s_i\}, \{o_i\}}{\text{minimize}} && \sum_{i,j,k} \rho\left(\frac{(s_i Q_{ij}^{(k)} + o_i) - (s_j Q_{ji}^{(k)} + o_j)}{s_i + s_j}\right)^2 \\ & \text{subject to} && 1 - \delta_s \leq s_i \leq 1 + \delta_s, -\delta_o \leq o_i \leq \delta_o, \forall i. \end{aligned}$$

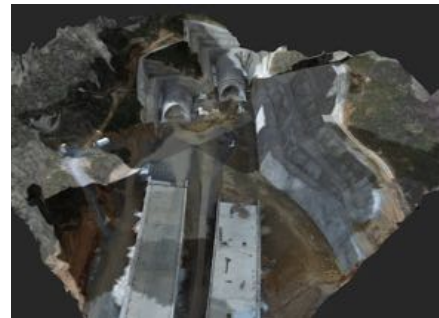
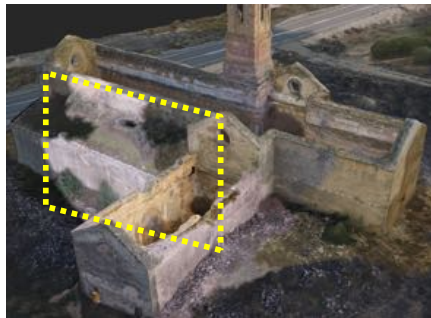
$$\rho(x) = \delta^2 (\sqrt{1 + (x/\delta)^2} - 1)$$



# Application: Large-Scale Color Correction

## □ Consistent texturing:

Before



After



# Final Remarks

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- ☐ Merge ground-level street-view images with aerial images
- ☐ Better local invariant features and efficient matching
- ☐ Distributed everything in SfM

