

# CE100 Algorithms and Programming II

## Week-7 (Greedy Algorithms, Knapsack)

Spring Semester, 2021-2022

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# Greedy Algorithms, Knapsack

## Outline

- Greedy Algorithms and Dynamic Programming Differences
- Greedy Algorithms
  - Activity Selection Problem
  - Knapsack Problems
    - The 0-1 knapsack problem
    - The fractional knapsack problem

# Activity Selection Problem

# Activity Selection Problem

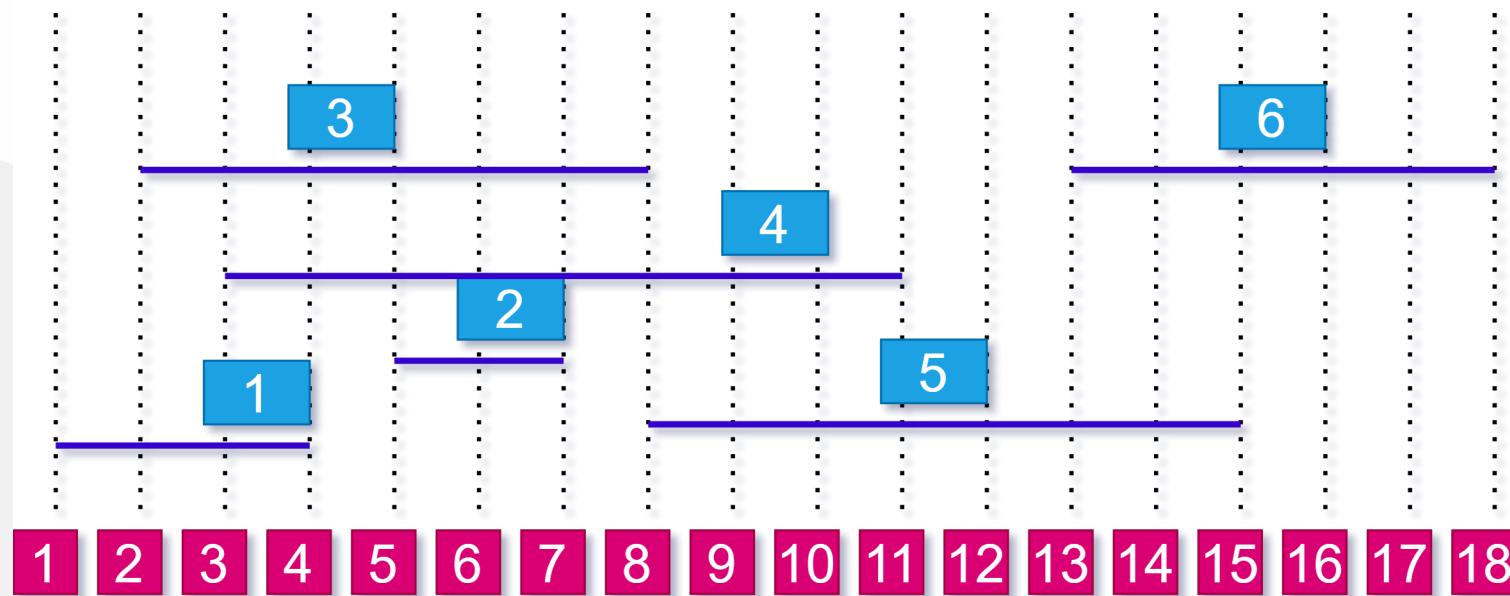
- We have:
  - A set of activities with fixed start and finish times
  - One shared resource (only one activity can use at any time)
- **Objective:** Choose the max number of compatible activities
- **Note:** Objective is to maximize the number of activities, not the total time of activities.
- **Example:**
  - *Activities:* Meetings with fixed start and finish times
  - *Shared resource:* A meeting room
    - *Objective:* Schedule the max number of meetings

# Activity Selection Problem

- **Input:** a set  $S = \{a_1, a_2, \dots, a_n\}$  of  $n$  activities
- $s_i$  : Start time of activity  $a_i$ ,
- $f_i$  : Finish time of activity  $a_i$   
Activity  $i$  takes place in  $[s_i, f_i)$
- **Aim:** Find max-size subset  $A$  of mutually *compatible* activities
  - Max number of activities, not max time spent in activities
  - Activities  $i$  and  $j$  are compatible if intervals  $[s_i, f_i)$  and  $[s_j, f_j)$  do not overlap, i.e., either  $s_i \geq f_j$  or  $s_j \geq f_i$

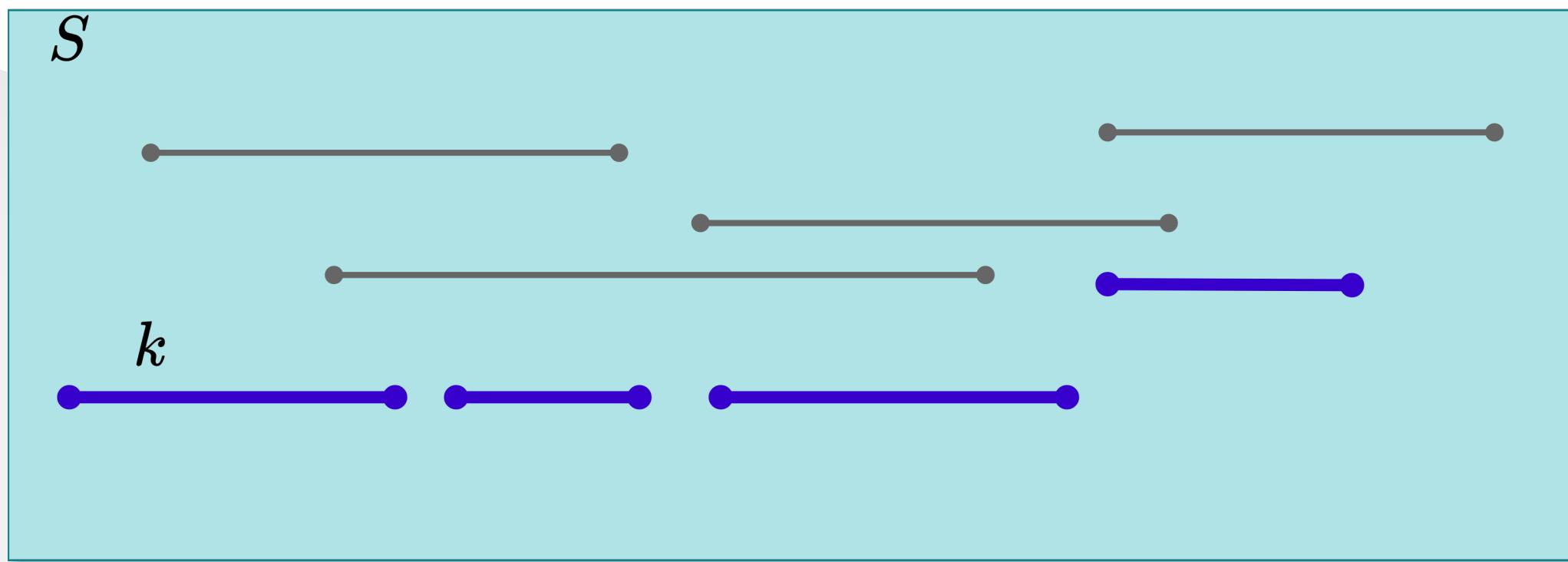
# Activity Selection Problem An Example

$$S = [1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)$$



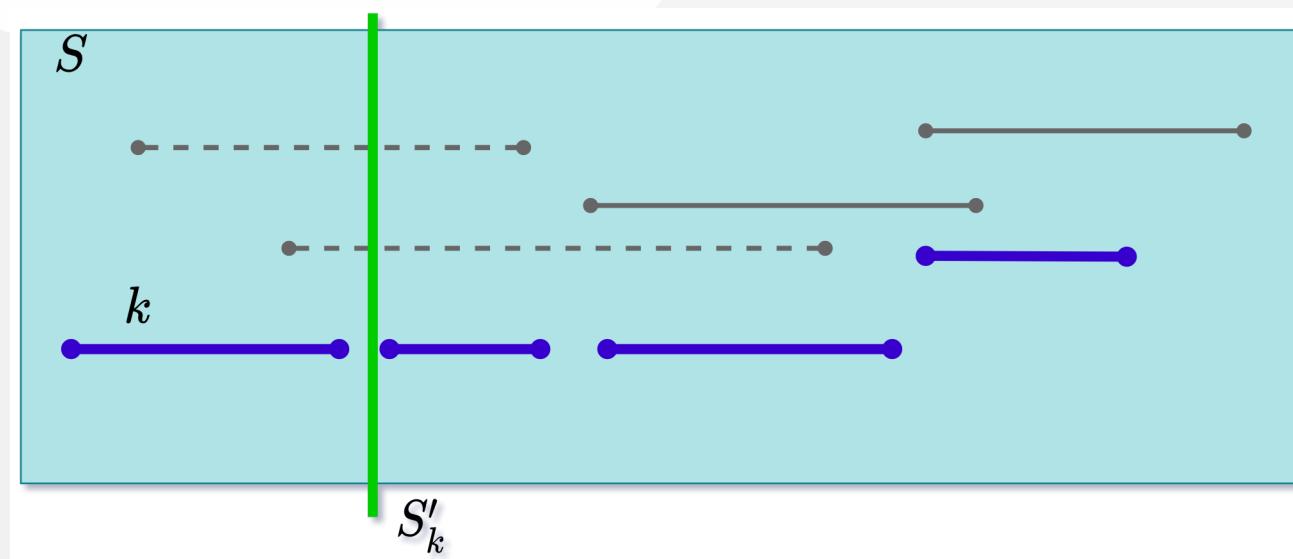
# Optimal Substructure Property

- Consider an optimal solution  $A$  for activity set  $S$ .
- Let  $k$  be the activity in  $A$  with the earliest finish time



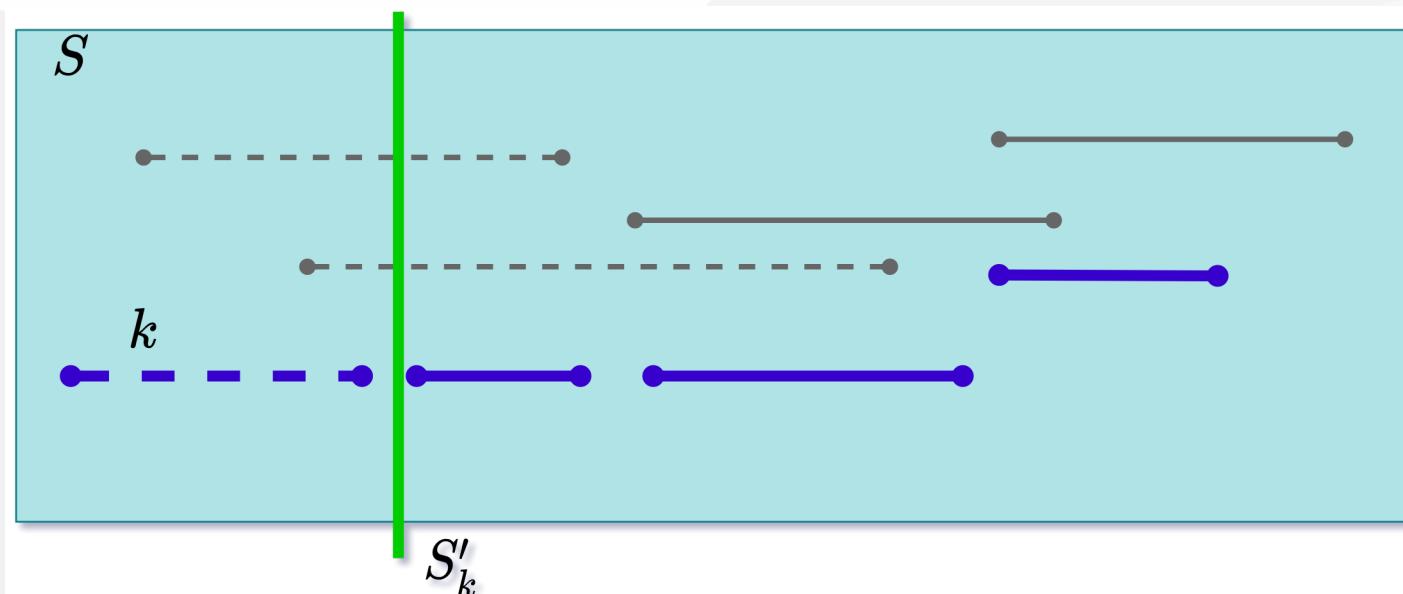
# Optimal Substructure Property

- Consider an optimal solution  $A$  for activity set  $S$ .
- Let  $k$  be the activity in  $A$  with the **earliest finish time**
- Now, consider the **subproblem**  $S'_k$  that has the activities that start after  $k$  finishes,  
i.e.  $S'_k = \{a_i \in S : s_i \geq f_k\}$
- What can we say about the optimal solution to  $S'_k$  ?



# Optimal Substructure Property

- Consider an optimal solution  $A$  for activity set  $S$ .
- Let  $k$  be the activity in  $A$  with the **earliest finish time**
- Now, consider the **subproblem**  $S'_k$  that has the activities that start after  $k$  finishes,  
i.e.  $S'_k = \{a_i \in S : s_i \geq f_k\}$
- $A - \{k\}$  is an optimal solution for  $S'_k$ . Why?



# Optimal Substructure

- **Theorem:** Let  $k$  be the activity with the earliest finish time in an optimal soln  $A \subseteq S$  then
  - $A - \{k\}$  is an optimal solution to subproblem
  - $S'_k = \{a_i \in S : s_i \geq f_k\}$
- **Proof (by contradiction):**
  - ▷ Let  $B'$  be an optimal solution to  $S'_k$  and
    - $|B'| > |A - \{k\}| = |A| - 1$
  - Then,  $B = B' \cup \{k\}$  is compatible and
    - $|B| = |B'| + 1 > |A|$
  - Contradiction to the optimality of  $A$

*Q.E.D.*

# Optimal Substructure

- **Recursive formulation:** Choose the first activity  $k$ , and then solve the remaining subproblem  $S'_k$
- How to choose the first activity  $k$ ?
  - DP, memoized recursion?
    - i.e. choose the  $k$  value that will have the max size for  $S'_k$
- DP would work,
  - but is it necessary to try all possible values for  $k$ ?

## Greedy Choice Property

- Assume (without loss of generality)  $f_1 \leq f_2 \leq \dots \leq f_n$ 
  - If not, sort activities according to their finish times in non-decreasing order
- **Greedy choice property:** a sequence of locally optimal (greedy) choices  $\Rightarrow$  an optimal solution
- How to choose the first activity **greedily** without losing optimality?

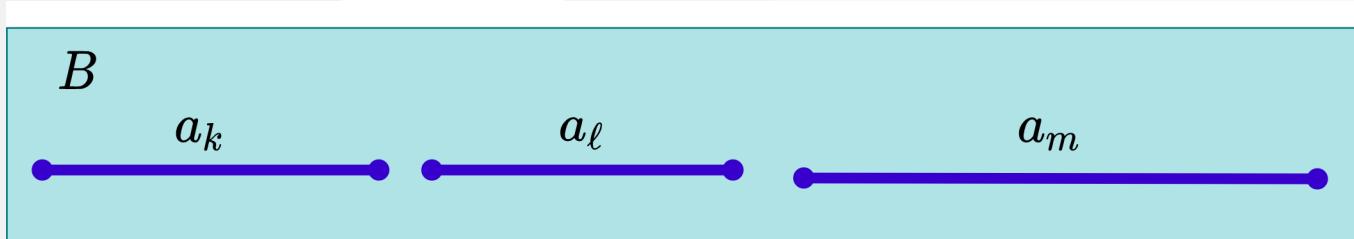
## Greedy Choice Property - Theorem

- Let activity set  $S = \{a_1, a_2, \dots, a_n\}$ , where  $f_1 \leq f_2 \leq \dots \leq f_n$
- **Theorem:** There exists an optimal solution  $A \subseteq S$  such that  $a_1 \in A$

In other words, the activity with the earliest finish time is guaranteed to be in an optimal solution.

# Greedy Choice Property - Proof

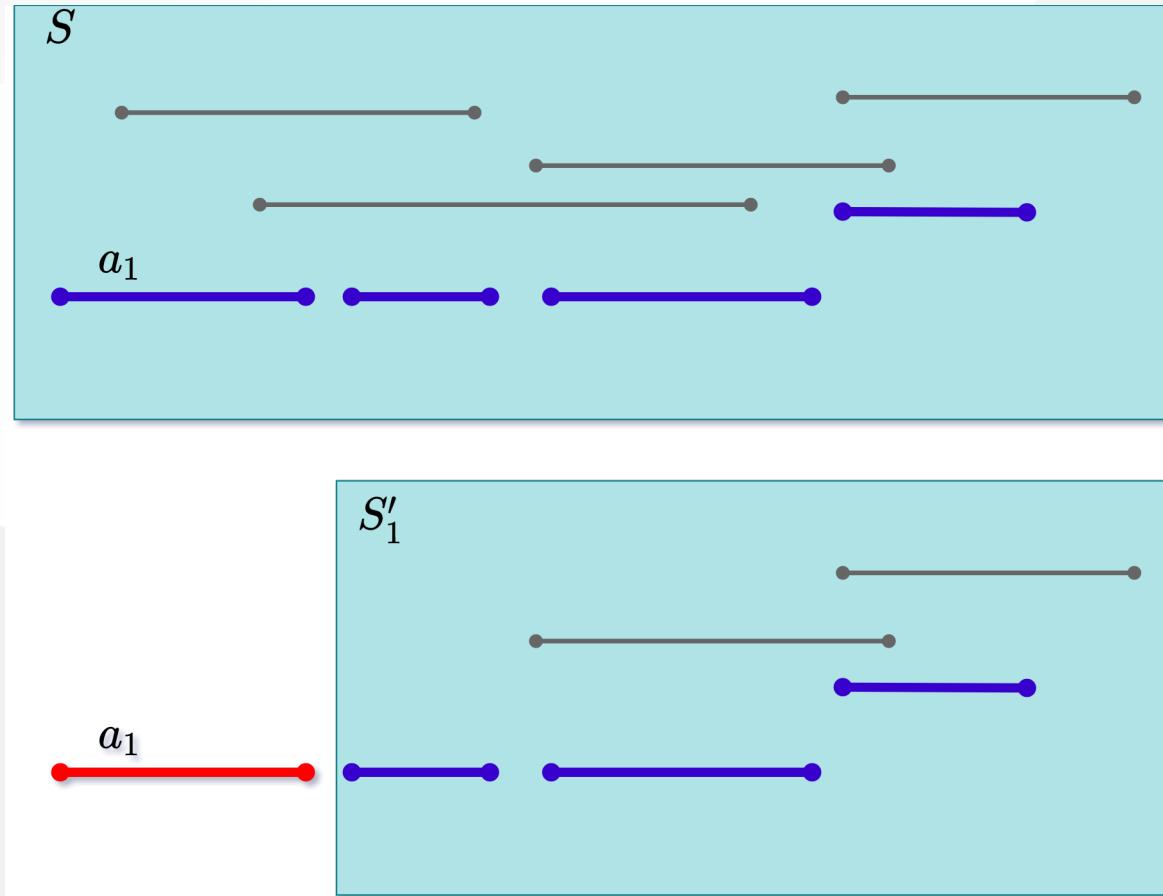
- **Theorem:** There exists an optimal solution  $A \subseteq S$  such that  $a_1 \in A$
- **Proof:** Consider an arbitrary optimal solution  $B = \{a_k, a_\ell, a_m, \dots\}$ , where  $f_k < f_\ell < f_m < \dots$ 
  - If  $k = 1$ , then  $B$  starts with  $a_1$ , and the proof is complete
  - If  $k > 1$ , then create another solution  $B'$  by replacing  $a_k$  with  $a_1$ . Since  $f_1 \leq f_k$ ,  $B'$  is guaranteed to be valid, and  $|B'| = |B|$ , hence also optimal



# Greedy Algorithm

- So far, we have:
  - **Optimal substructure property:** If  $A = \{a_k, \dots\}$  is an optimal solution, then  $A - \{a_k\}$  must be optimal for subproblem  $S'_{k'}$ , where  $S'_{k'} = \{a_i \in S : s_i \geq f_k\}$ 
    - Note:  $a_k$  is the activity with the earliest finish time in  $A$
  - **Greedy choice property:** There is an optimal solution  $A$  that contains  $a_1$ 
    - Note:  $a_1$  is the activity with the earliest finish time in  $S$

# Greedy Algorithm



*explained in the next slide..*

# Greedy Algorithm

- **Theorem:** There exists an optimal solution  $A \subseteq S$  such that  $a_1 \in A$
- Basic idea of the greedy algorithm:
  - Add  $a_1$  to  $A$
  - Solve the remaining subproblem  $S'_1$ , and then append the result to  $A$
- Remember arbitrary optimal solution explanation from previous sections (finish time order is important for  $a_1$  selection with start time and overlapping checking)
  - $B = \{a_k, a_\ell, a_m, \dots\}$ ,
  - where  $f_k < f_\ell < f_m < \dots$

# Greedy Algorithm for Activity Selection

## Definitions in Greedy Algorithm:

- $j$ : specifies the index of most recent activity added to  $A$
- $f_j = \text{Max}\{f_k : k \in A\}$ , max finish time of any activity in  $A$ ;
  - because activities are processed in non-decreasing order of finish times
- Thus,  $s_i \geq f_j$  checks the compatibility of  $i$  to current  $A$
- **Running time:**  $\Theta(n)$  assuming that the activities were already sorted.

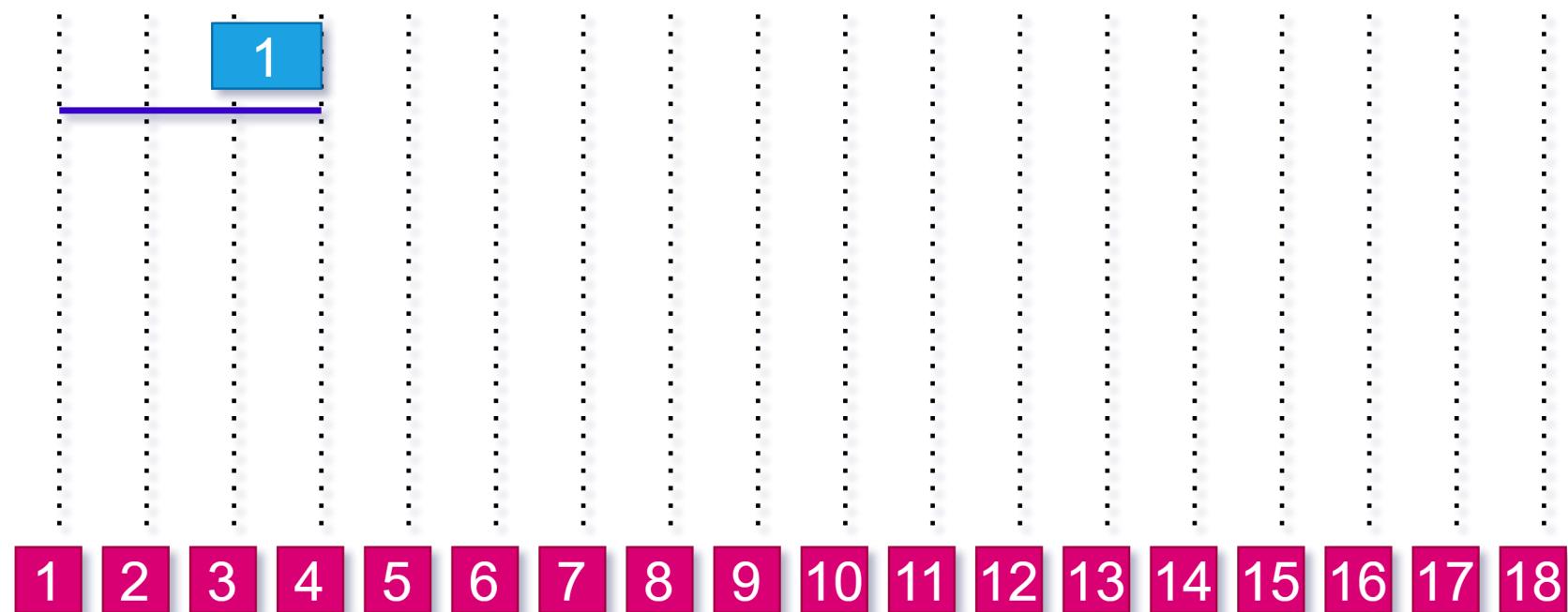
# Greedy Algorithm for Activity Selection

Pseudocode for Greedy Algorithm:

```
GAS( $s, f, n$ ) {  
     $A \leftarrow \{1\}$   
     $j \leftarrow 1$   
    for  $i \leftarrow 2$  to  $n$  do  
        if  $s_i \geq f_j$  then  
             $A \leftarrow A \cup \{i\}$   
             $j \leftarrow i$   
        endif  
    endfor  
}
```

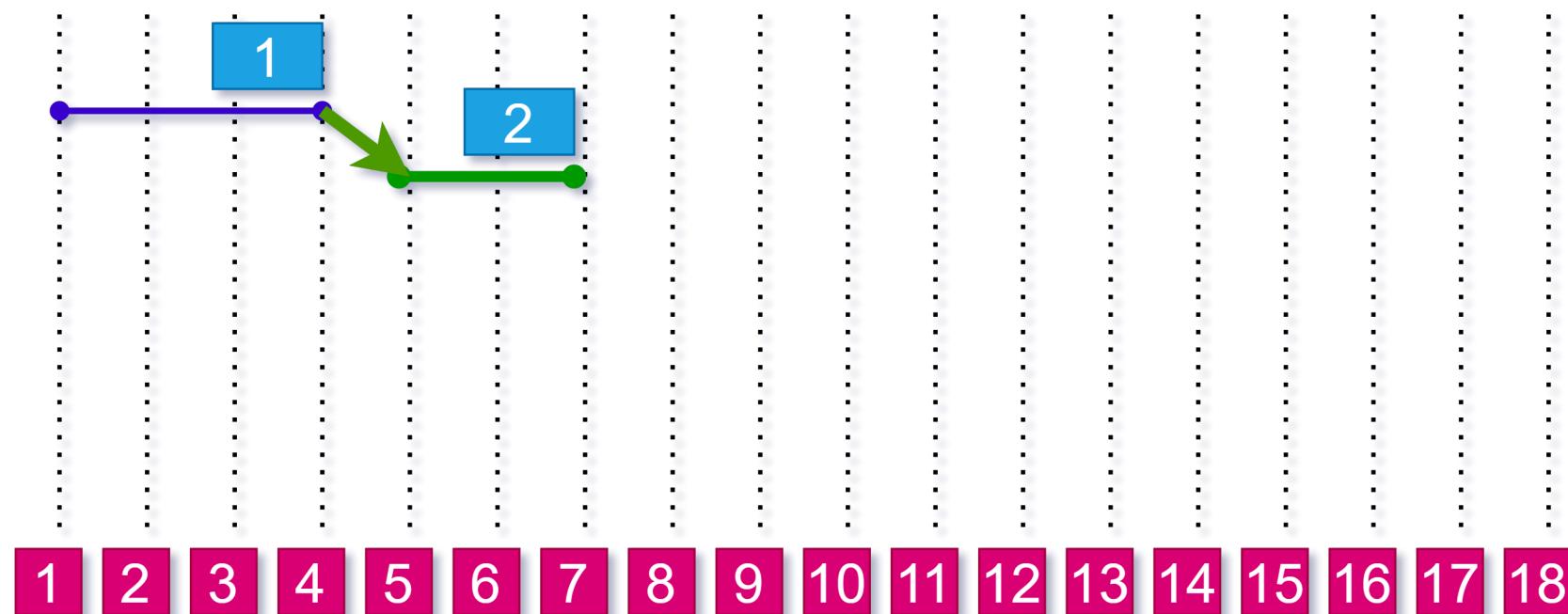
# Greedy Algorithm for Activity Selection, An Example (Step-1)

$$f_j = 0 \quad S = \{[1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)\}$$



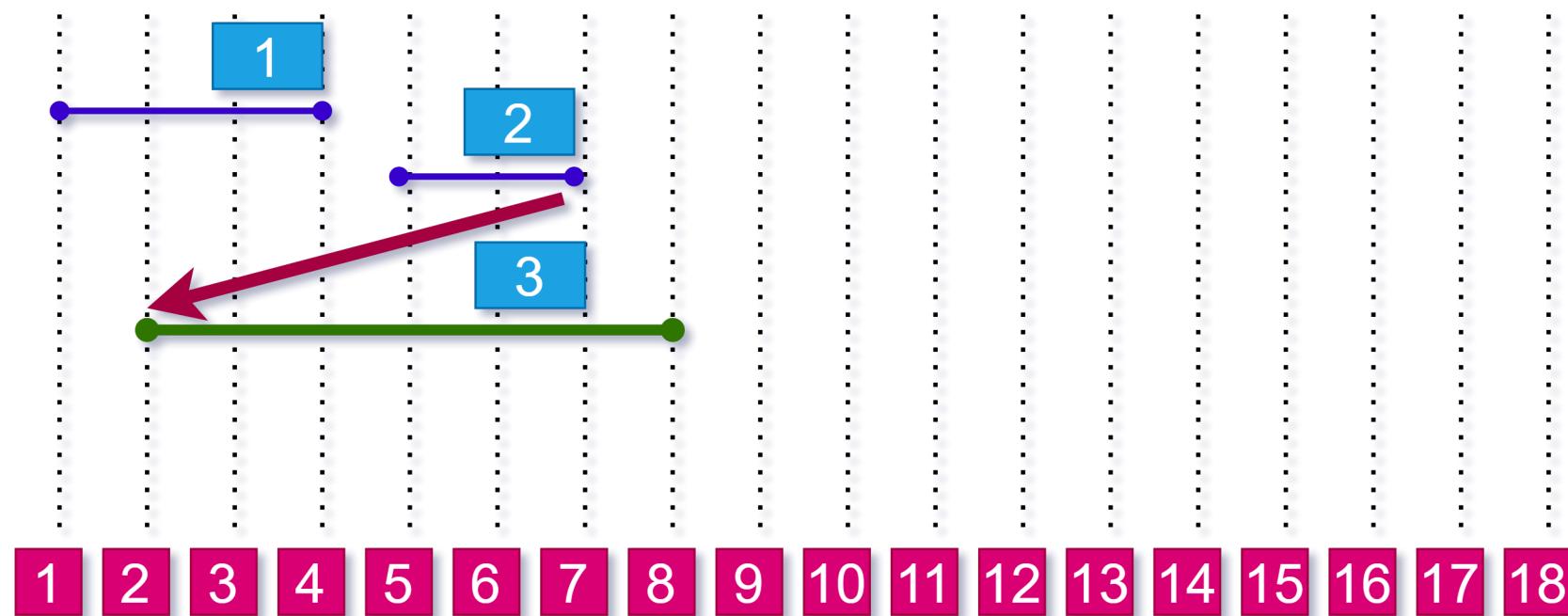
## Greedy Algorithm for Activity Selection, An Example (Step-2)

$$f_j = 4 \quad S = \{[1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)\}$$



# Greedy Algorithm for Activity Selection, An Example (Step-3)

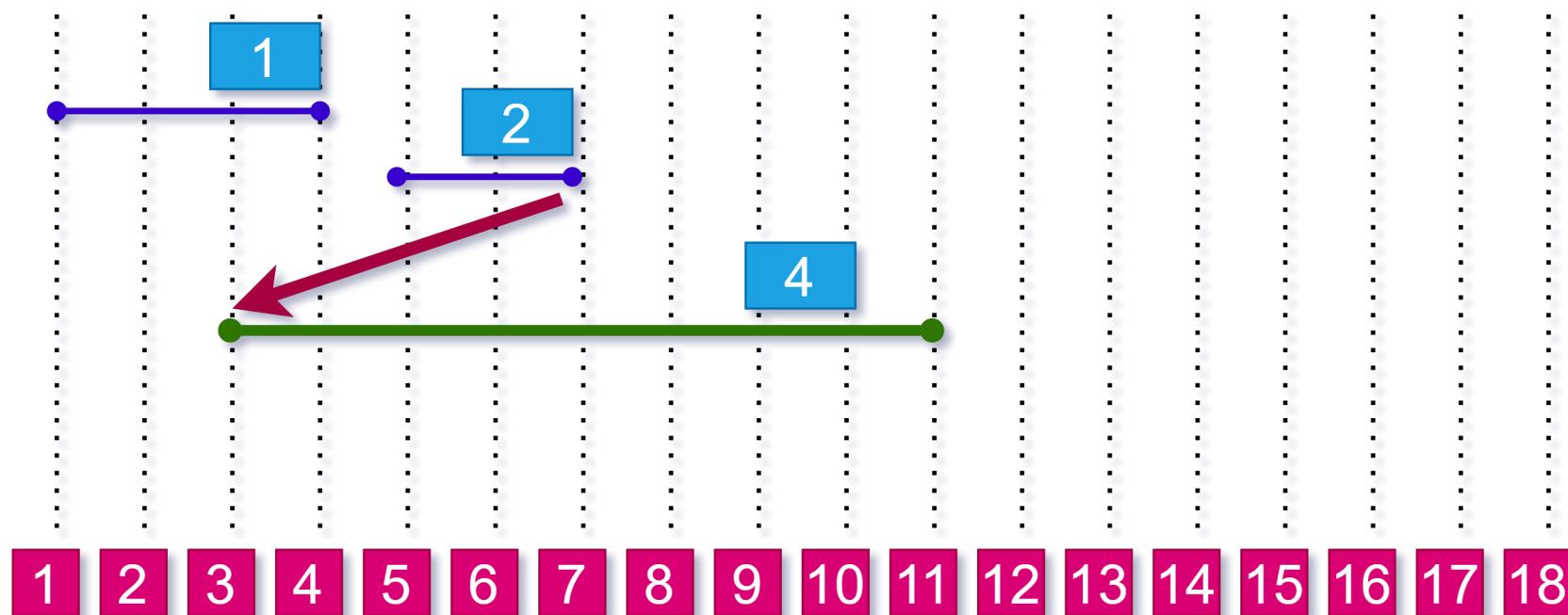
$$f_j = 7 \quad S = \{[1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)\}$$



# Greedy Algorithm for Activity Selection, An Example (Step-4)

$$f_j = 7$$

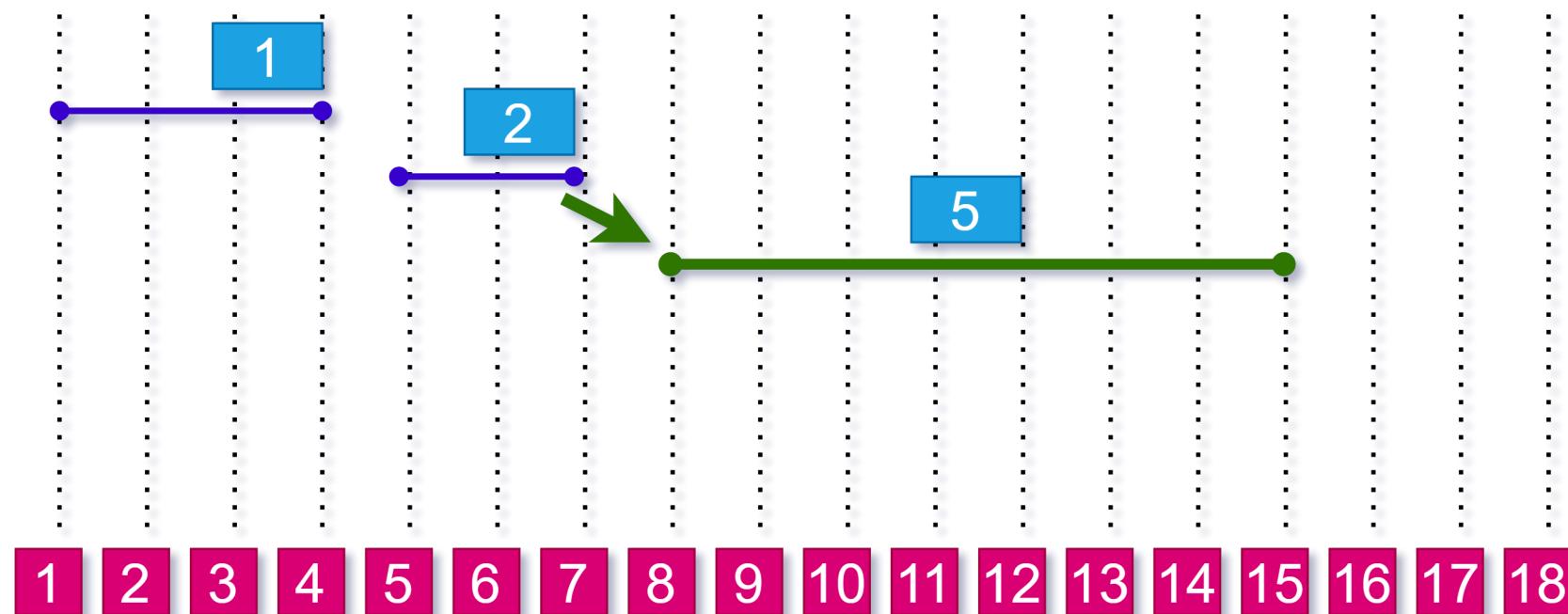
$$S = \{[1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)\}$$



# Greedy Algorithm for Activity Selection, An Example (Step-5)

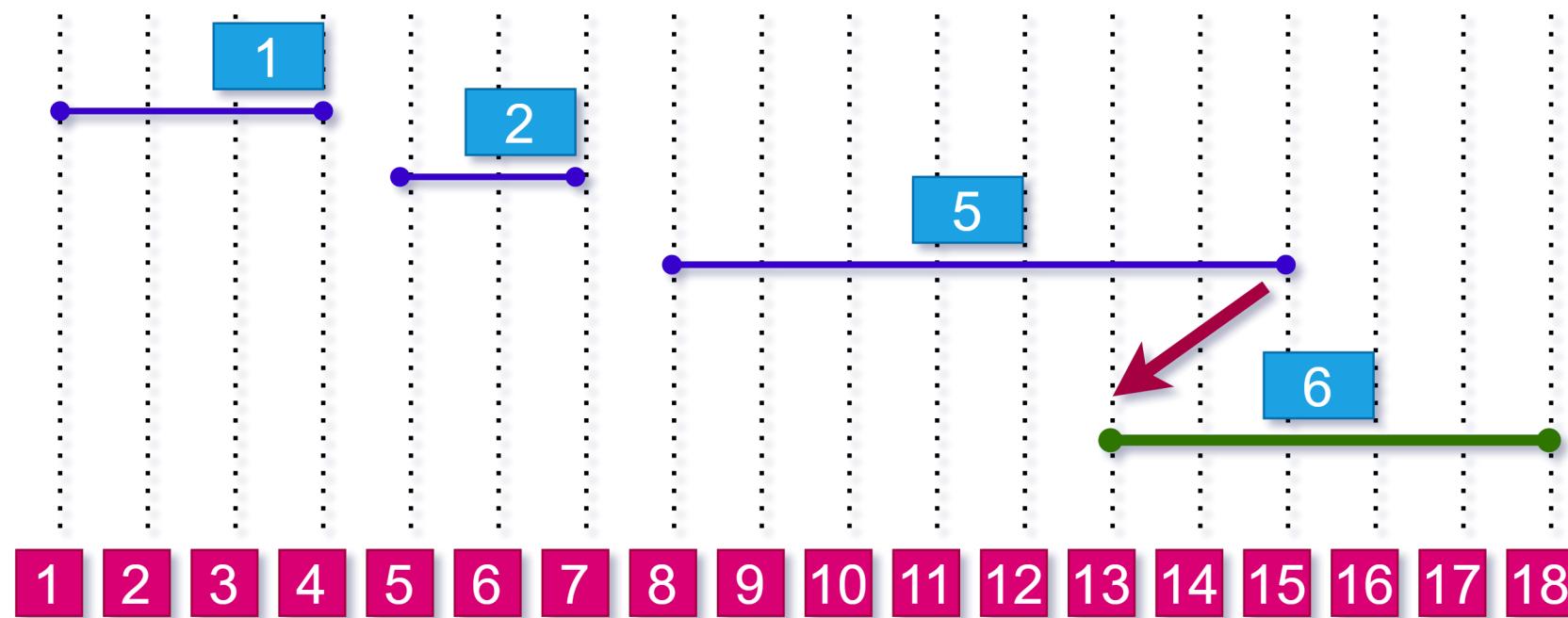
$$f_j = 7$$

$$S = \{[1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)\}$$



# Greedy Algorithm for Activity Selection, An Example (Step-6)

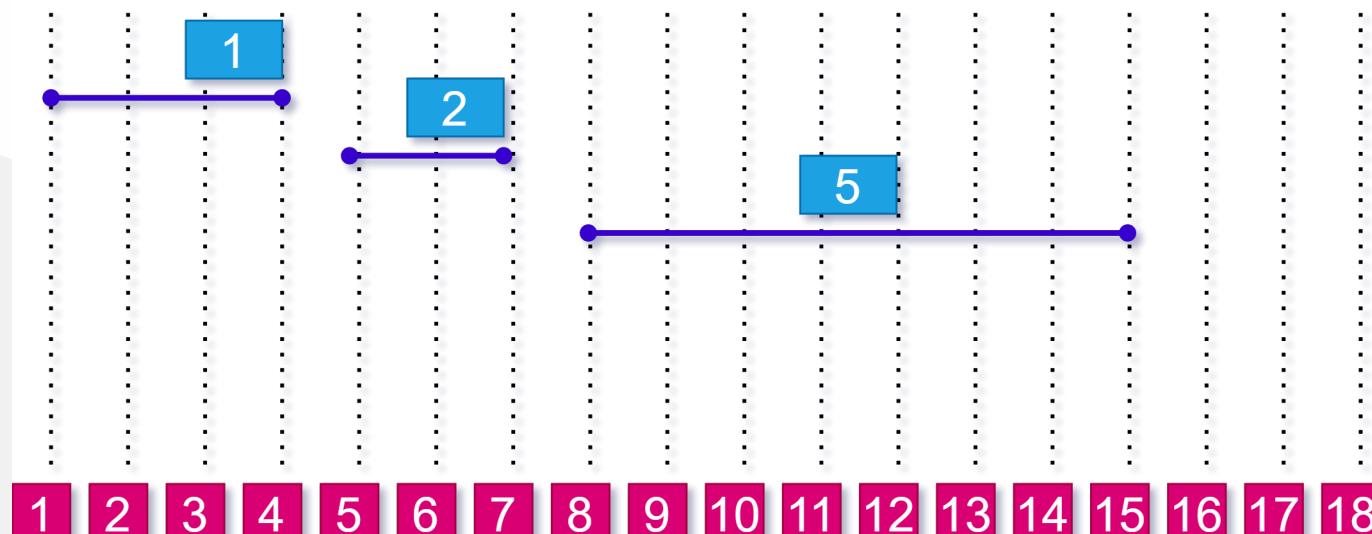
$$f_j = 15 \quad S = \{[1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)\}$$



# Greedy Algorithm for Activity Selection, An Example (Step-7)

## Final Solution

$$S = \{[1, 4), [5, 7), [2, 8), [3, 11), [8, 15), [13, 18)\}$$



$$A = \{1, 2, 5\}$$

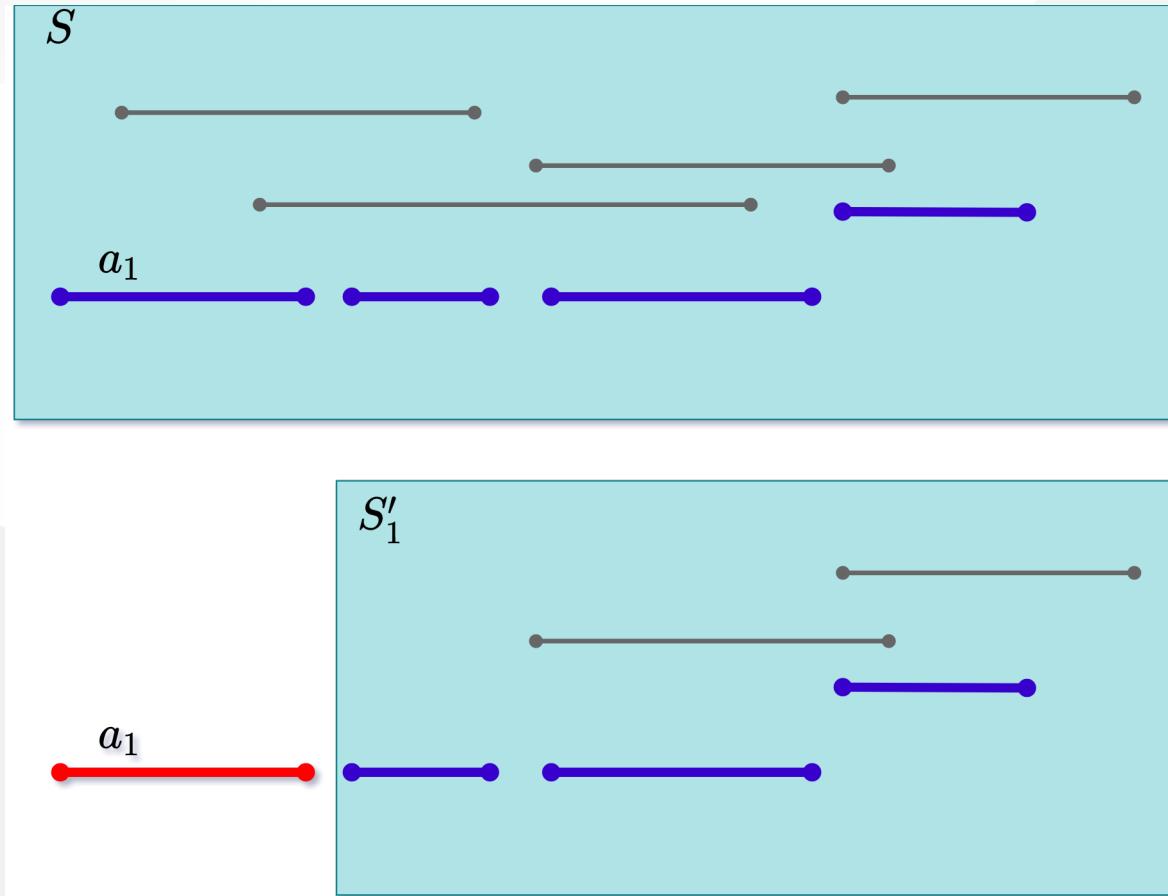
## Comparison of DP and Greedy Algorithms

## Reminder: DP-Based Matrix Chain Order

$$m_{ij} = \underset{i \leq k < j}{\text{MIN}} \{m_{ik} + m_{k+1,j} + p_{i-1}p_kp_j\}$$

- We don't know ahead of time which  $k$  value to choose.
- We first need to compute the results of subproblems  $m_{ik}$  and  $m_{k+1,j}$  before computing  $m_{ij}$
- The selection of  $k$  is done based on the **results of the subproblems**.

# Greedy Algorithm for Activity Selection



*explained in the next slide..*

## Greedy Algorithm for Activity Selection

- Make a greedy selection in the beginning:
  - Choose  $a_1$  (the activity with the earliest finish time)
- Solve the remaining subproblem  $S'_1$  (all activities that start after  $a_1$ )

## Greedy vs Dynamic Programming

- Optimal substructure property exploited by both **Greedy** and **DP** strategies
- **Greedy Choice Property:** A sequence of locally optimal choices  $\Rightarrow$  an optimal solution
  - We make the choice that seems best at the moment
  - Then solve the subproblem arising after the choice is made
- **DP:** We also make a choice/decision at each step, but the choice may depend on the optimal solutions to subproblems
- **Greedy:** The choice may depend on the choices made so far, but it cannot depend on any future choices or on the solutions to subproblems

## Greedy vs Dynamic Programming

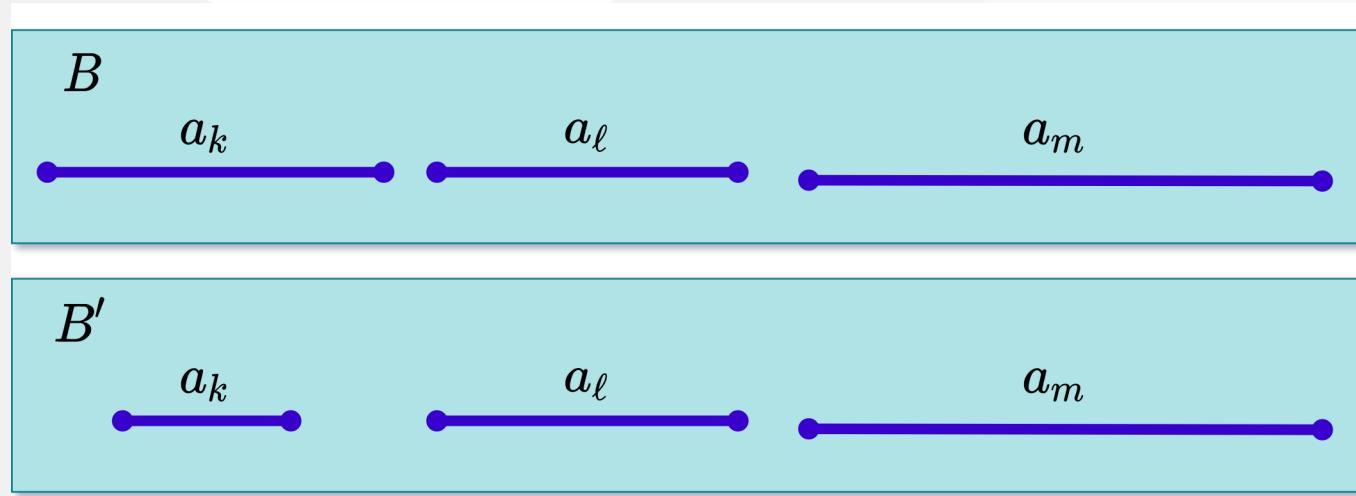
- DP is a bottom-up strategy (*use memory to store the results of subproblems*)
- Greedy is a top-down strategy (*make choices at each step*)
  - each greedy choice in the sequence iteratively reduces each problem to a similar but smaller problem

## Proof of Correctness of Greedy Algorithms

- Examine a globally optimal solution
- Show that this soln can be modified so that
  - (1) A greedy choice is made as the first step
  - (2) This choice reduces the problem to a similar but smaller problem
- Apply induction to show that a greedy choice can be used at every step
- Showing (2) reduces the proof of correctness to proving that the problem exhibits optimal substructure property

## Greedy Choice Property - Proof

- **Theorem:** There exists an optimal solution  $A \subseteq S$  such that  $a_1 \in A$
- **Proof:** Consider an arbitrary optimal solution  $B = \{a_k, a_\ell, a_m, \dots\}$ , where  $f_k < f_\ell < f_m < \dots$ 
  - If  $k = 1$ , then  $B$  starts with  $a_1$ , and the proof is complete
  - If  $k > 1$ , then create another solution  $B'$  by replacing  $a_k$  with  $a_1$ . Since  $f_1 \leq f_k$ ,  $B'$  is guaranteed to be valid, and  $|B'| = |B|$ , hence also optimal



## Elements of Greedy Strategy

- How can you judge whether
- A greedy algorithm will solve a particular optimization problem?
- **Two key ingredients**
  - Greedy choice property
  - Optimal substructure property

## Key Ingredients of Greedy Strategy

- **Greedy Choice Property:** A globally optimal solution can be arrived at by making locally optimal (greedy) choices
- In DP, we make a choice at each step but the choice may depend on the solutions to subproblems
- In **Greedy Algorithms**, we make the choice that seems best at that moment then solve the subproblems arising after the choice is made
  - The choice may depend on choices so far, but it cannot depend on any future choice or on the solutions to subproblems
- DP solves the problem bottom-up
- Greedy usually progresses in a top-down fashion by making one greedy choice after another reducing each given problem instance to a smaller one

## Key Ingredients: Greedy Choice Property

- We must prove that a greedy choice at each step yields a globally optimal solution
- The proof examines a globally optimal solution
- Shows that the soln can be modified so that a **greedy choice made as the first step** reduces the problem to a similar but smaller subproblem
- Then **induction** is applied to show that a greedy choice can be used at each step
- Hence, this induction proof reduces the proof of correctness to demonstrating that an optimal solution must exhibit **optimal substructure** property

## Key Ingredients: Greedy Choice Property

- How to prove the greedy choice property?
  - Consider the greedy choice  $c$
  - Assume that there is an optimal solution  $B$  that doesn't contain  $c$ .
  - Show that it is possible to convert  $B$  to another optimal solution  $B'$ , where  $B'$  contains  $c$ .
- Example: Activity selection algorithm
  - Greedy choice:  $a_1$  (the activity with the earliest finish time)
  - Consider an optimal solution  $B$  without  $a_1$
  - Replace the first activity in  $B$  with  $a_1$  to construct  $B'$
  - Prove that  $B'$  must be an optimal solution

## Key Ingredients: Optimal Substructure

- A problem exhibits optimal substructure if an optimal solution to the problem contains within it optimal solutions to subproblems
- Example: Activity selection problem  $S$ 
  - If an optimal solution  $A$  to  $S$  begins with activity  $a_1$  then the set of activities

$$A' = A - \{a_1\}$$

- is an optimal solution to the activity selection problem

$$S' = \{a_i \in S : s_i \geq f_1\}$$

- where  $s_i$  is the start time of activity  $a_i$  and  $f_i$  is the finish time of activity  $a_i$

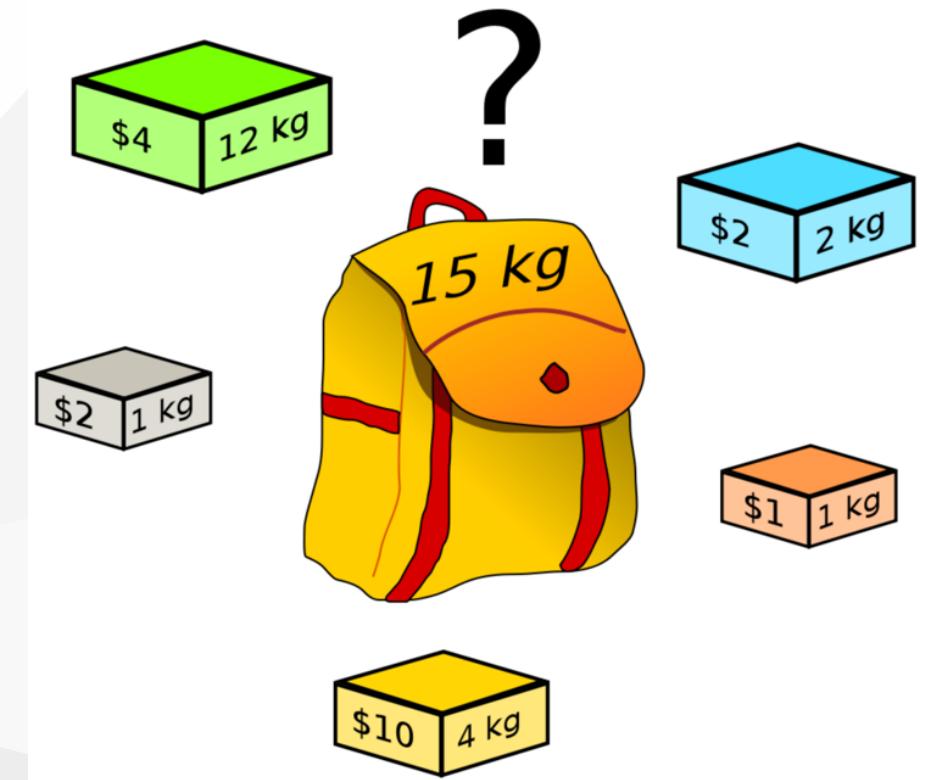
## Key Ingredients: Optimal Substructure

- Optimal substructure property is exploited by both Greedy and dynamic programming strategies
- Hence one may
  - Try to generate a dynamic programming soln to a problem when a greedy strategy suffices inefficient
  - Or, may mistakenly think that a greedy soln works when in fact a DP soln is required incorrect
- **Example:** Knapsack Problems( $S$ ,  $w$ )

# Knapsack Problems

## Knapsack Problem

- Each item  $i$  has:
  - weight  $w_i$
  - value  $v_i$
- A thief has a knapsack of weight capacity  $w$
- Which items to choose to maximize the value of the items in the knapsack?



## Knapsack Problem: Two Versions

- The 0-1 knapsack problem:
  - Each item is discrete.
  - Each item either chosen as a whole or not chosen.
  - Examples: *TV, laptop, gold bricks, etc.*
- The fractional knapsack problem:
  - Can choose fractional part of each item.
  - If item  $i$  has weight  $w_i$ , we can choose any amount  $\leq w_i$
  - Examples: *Gold dust, silver dust, rice, etc.*

# Knapsack Problems

- **The 0-1 Knapsack Problem( $S, W$ )**
  - A thief robbing a store finds  $n$  items  $S = \{I_1, I_2, \dots, I_n\}$ , the  $i$ th item is worth  $v_i$  dollars and weighs  $w_i$  pounds, where  $v_i$  and  $w_i$  are integers
  - He wants to take as valuable a load as possible, but he can carry at most  $W$  pounds in his knapsack, where  $W$  is an integer
  - The thief cannot take a fractional amount of an item
- **The Fractional Knapsack Problem ( $S, W$ )**
  - The scenario is the same
  - But, the thief can take fractions of items rather than having to make binary (0 – 1) choice for each item

## Optimal Substructure Property for the 0-1 Knapsack Problem ( $S, W$ )

- Consider an optimal load  $L$  for the problem  $(S, W)$ .
- Let  $I_j$  be an item chosen in  $L$  with weight  $w_j$
- Assume we remove  $I_j$  from  $L$ , and let:

$$L'_j = L - \{I_j\}$$

$$S'_j = S - \{I_j\}$$

$$W'_j = W - w_j$$

- Q: *What can we say about the optimal substructure property?*



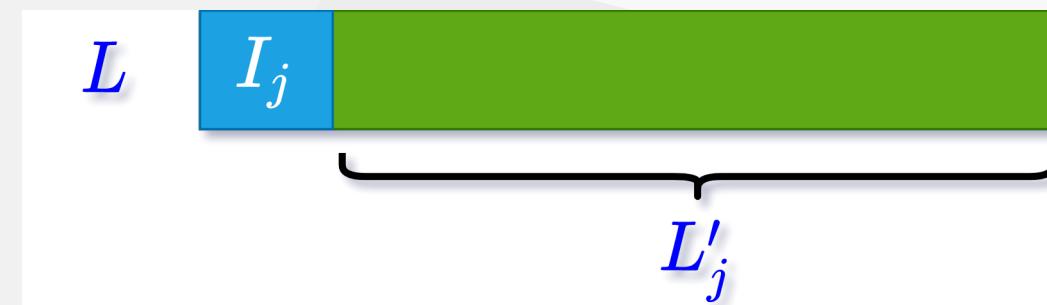
# Optimal Substructure Property for the 0-1 Knapsack Problem (S, W)

$$L'_j = L - \{I_j\}$$

$$S'_j = S - \{I_j\}$$

$$W'_j = W - w_j$$

- Optimal substructure property:
  - $L'_j$  must be an optimal solution for  $(S'_j, W'_j)$
- Why?
  - If we remove item  $j$  from  $L$ , we can construct a new optimal solution  $L'_j$  for  $(S'_j, W'_j)$
  - If  $L'_j$  is optimal, then  $L$  must be optimal



## Optimal Substructure Property for the 0-1 Knapsack Problem ( $S, W$ )

$$L'_j = L - \{I_j\}$$

$$S'_j = S - \{I_j\}$$

$$W'_j = W - w_j$$

- Optimal substructure:  $L'_j$  must be an optimal solution for  $(S'_j, W'_j)$
- **Proof:** By contradiction, assume there is a solution  $B'_j$  for  $(S'_j, W'_j)$ , which is better than  $L'_j$ .
  - We can construct a solution  $B$  for the original problem  $(S, W)$  as:  $B = B'_j \cup I_j$ .
  - The total value of  $B$  is now higher than  $L$ , which is a contradiction because  $L$  is optimal for  $(S, W)$ .
- *Q.E.D.*

## Optimal Substructure Property for the Fractional Knapsack Problem (S, W)

- Consider an optimal solution L for (S, W)
- If we remove a weight  $0 < w \leq w_j$  of item  $j$  from optimal load  $L$  and let:
  - The remaining load

$$L'_j = L - \{w \text{ pounds of } I_j\}$$

- must be a most valuable load weighing at most

$$W'_j = W - w$$

- pounds that the thief can take from

$$S'_j = S - \{I_j\} \cup \{w_j - w \text{ pounds of } I_j\}$$

- That is,  $L'_j$  should be an optimal soln to the

Fractional Knapsack Problem( $S'_j, W'_j$ )

## Knapsack Problems

- Two different problems:
  - 0-1 knapsack problem
  - Fractional knapsack problem
- The problems are similar.
  - Both problems have optimal substructure property.
- Which algorithm to solve each problem?

## Fractional Knapsack Problem

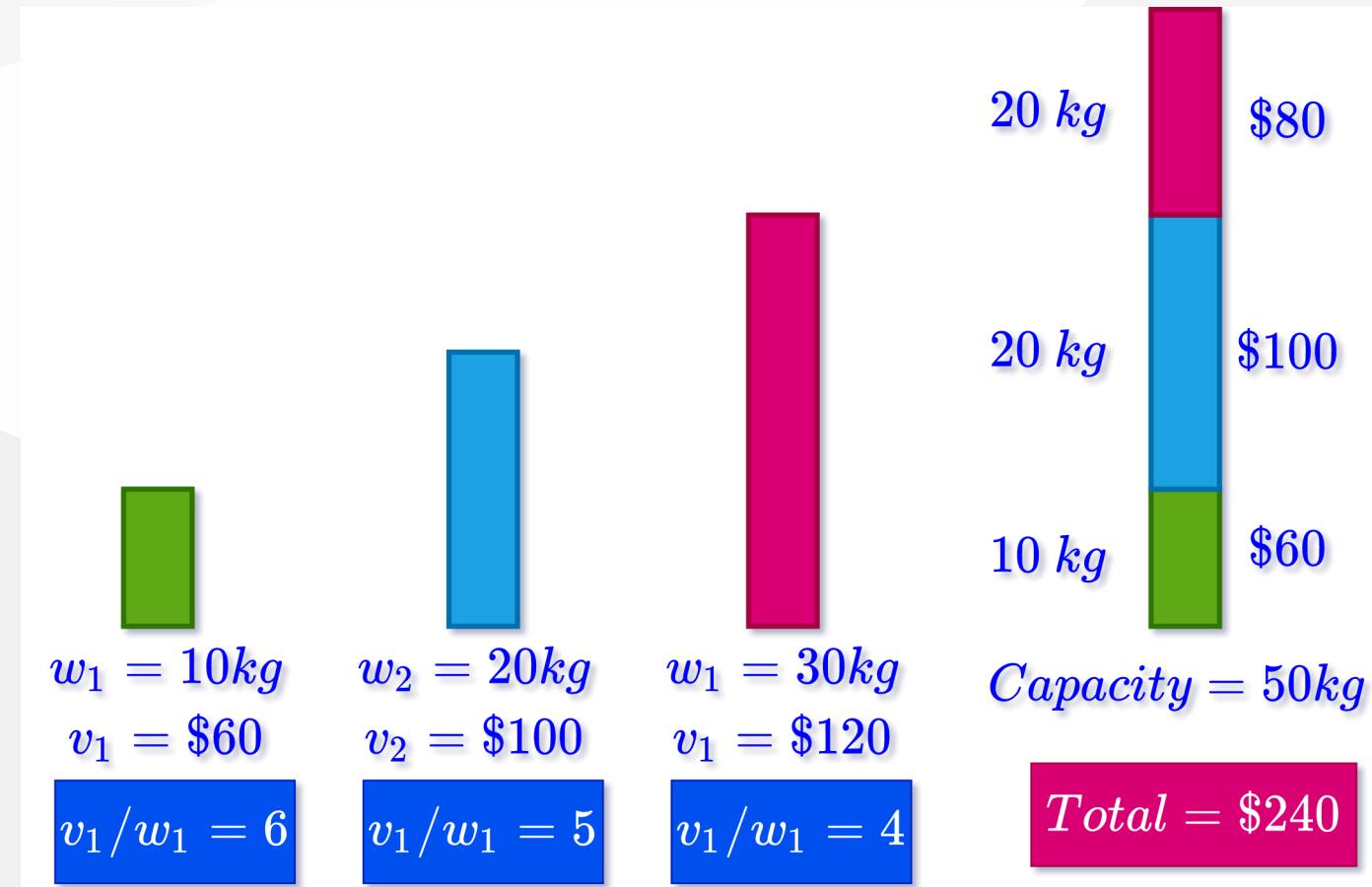
- Can we use a greedy algorithm?
- Greedy choice: Take as much as possible from the item with the largest value per pound  $v_i/w_i$
- Does the greedy choice property hold?
  - Let  $j$  be the item with the largest value per pound  $v_j/w_j$
  - Need to prove that there is an optimal load that has as much  $j$  as possible.
  - **Proof:** Consider an optimal solution  $L$  that does not have the maximum amount of item  $j$ . We could replace the items in  $L$  with item  $j$  until  $L$  has maximum amount of  $j$ .  $L$  would still be optimal, because item  $j$  has the highest value per pound.

## Greedy Solution to Fractional Knapsack

- (1) Compute the value per pound  $v_i/w_i$  for each item
- (2) The thief begins by taking, as much as possible, of the item with the greatest value per pound
- (3) If the supply of that item is exhausted before filling the knapsack, then he takes, as much as possible, of the item with the next greatest value per pound
- (4) Repeat (2-3) until his knapsack becomes full

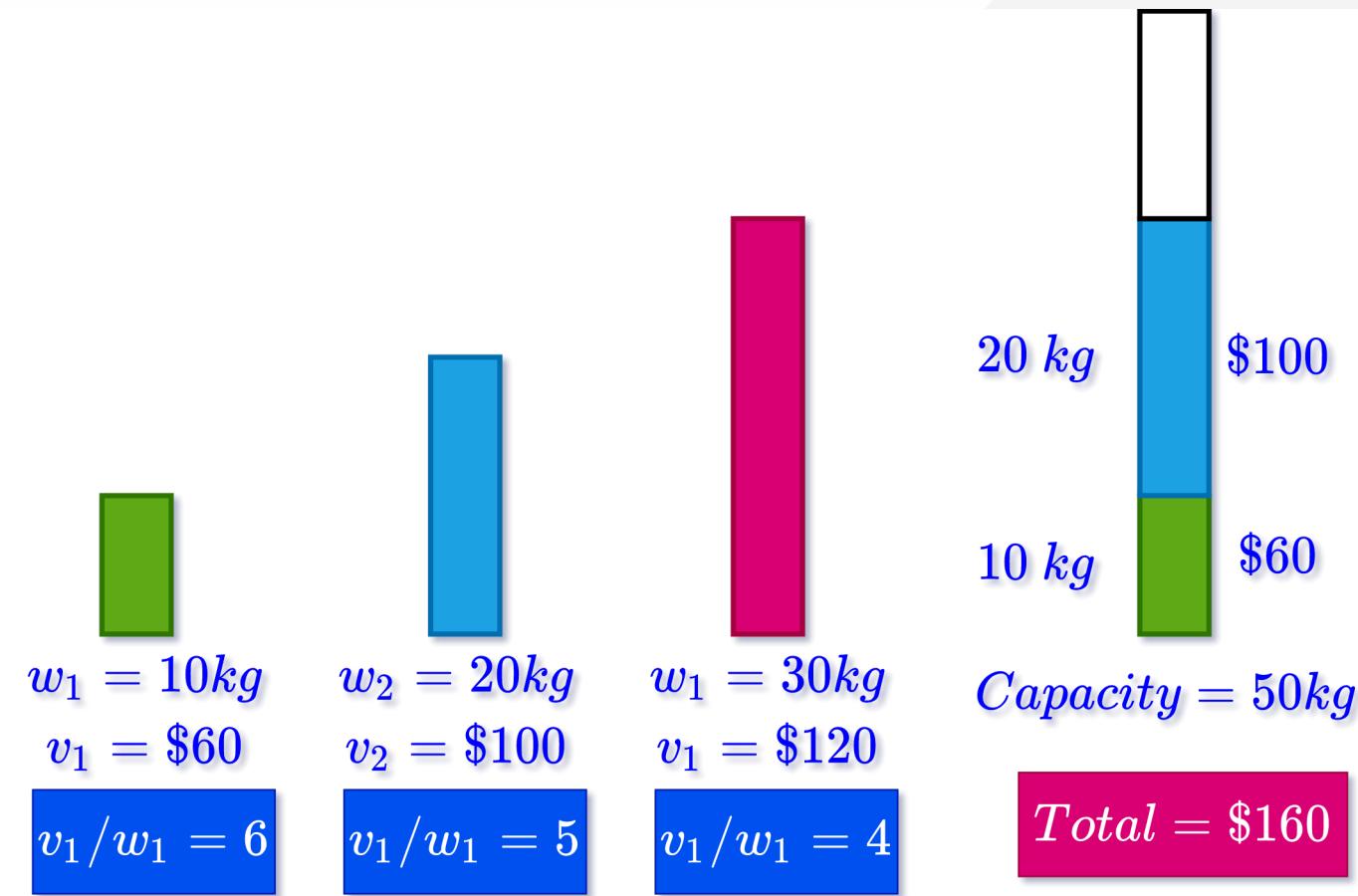
Thus, by sorting the items by value per pound the greedy algorithm runs in  $O(nlgn)$  time

## Fractional Knapsack Problem: Example



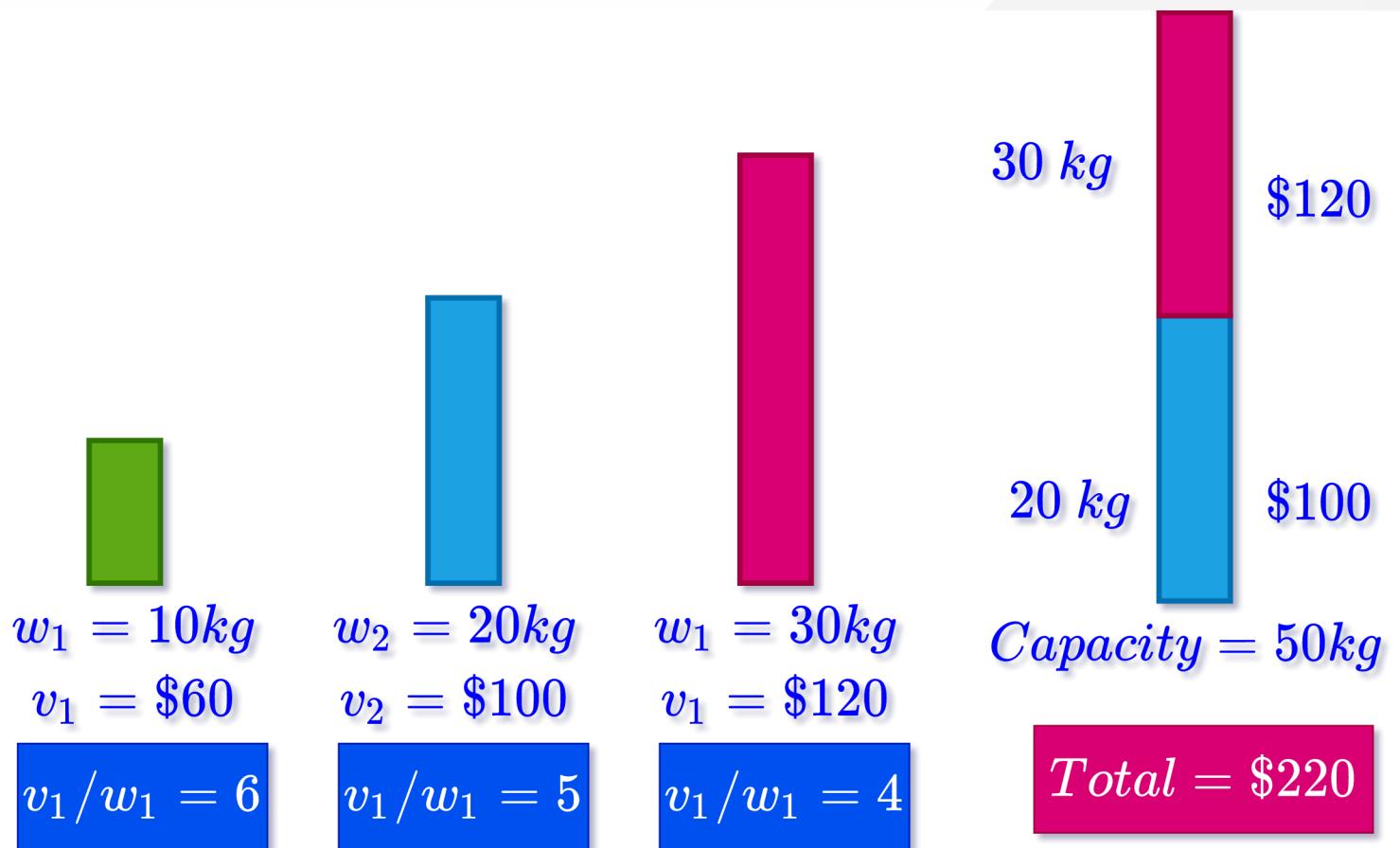
# 0-1 Knapsack Problem

- Can we use the same greedy algorithm?
  - Is there a better solution?



# 0-1 Knapsack Problem

- The optimal solution for this problem is:
  - This solution cannot be obtained using the greedy algorithm



## 0-1 Knapsack Problem

- When we consider an item  $I_j$  for inclusion we must compare the solutions to two subproblems
  - Subproblems in which  $I_j$  is included and excluded
- The problem formulated in this way gives rise to many
  - **overlapping subproblems** (a key ingredient of DP)
    - In fact, dynamic programming can be used to solve the **0-1 Knapsack problem**

## 0-1 Knapsack Problem

- A thief robbing a store containing  $n$  articles
  - $\{a_1, a_2, \dots, a_n\}$
- The value of  $i_{th}$  article is  $v_i$  dollars ( $v_i$  is integer)
- The weight of  $i_{th}$  article is  $w_i$  kg ( $w_i$  is integer)
- Thief can carry at most  $W$  kg in his knapsack
- Which articles should he take to maximize the value of his load?
- Let  $K_{n,W} = \{a_1, a_2, \dots, a_n : W\}$  denote 0-1 knapsack problem
- Consider the solution as a sequence of  $n$  decisions
  - i.e.,  $i_{th}$  decision: whether thief should pick  $a_i$  for optimal load.

## Optimal Substructure Property

- Notation:  $K_{n,W}$ :
  - The items to choose from:  $\{a_1, \dots, a_n\}$
  - The knapsack capacity:  $W$
- Consider an optimal load  $L$  for problem  $K_{n,W}$
- Let's consider two cases:
  - $a_n$  is in  $L$
  - $a_n$  is not in  $L$

# Optimal Substructure Property

- Case 1: If  $a_n \in L$ 
  - What can we say about the optimal substructure?
    - $L - \{a_n\}$  must be optimal for  $K_{n-1, W-w_n}$
    - $K_{n-1, W-w_n}$ :
      - The items to choose from  $\{a_1, \dots, a_{n-1}\}$
      - The knapsack capacity:  $W-w_n$
  - Case 2: If  $a_n \notin L$ 
    - What can we say about the optimal substructure?
      - $L$  must be optimal for  $K_{n-1, W}$
      - $K_{n-1, W}$ :
        - The items to choose from  $\{a_1, \dots, a_{n-1}\}$
        - The knapsack capacity:  $W$

# Optimal Substructure Property

- In other words, optimal solution to  $K_{n,W}$  contains an optimal solution to:
  - either:  $K_{n-1,W-w_n}$  (if  $a_n$  is selected)
  - or:  $K_{n-1,W}$  (if  $a_n$  is not selected)

## Recursive Formulation

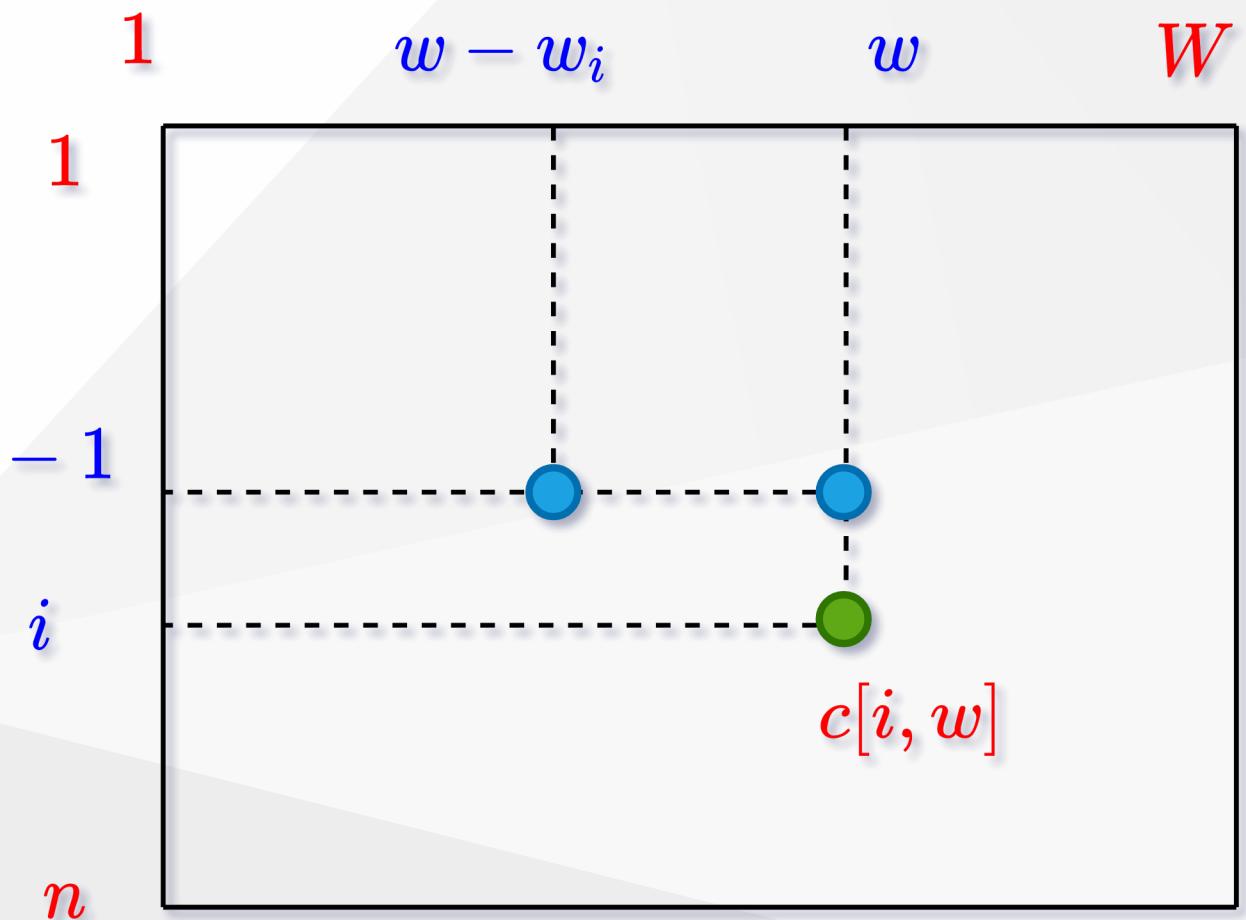
$$c[i, w] = \begin{cases} 0 & \text{if } i = 0, \text{ or } w = 0 \\ c[i - 1, w], & \text{if } w_i > w \\ \max\{v_i + c[i - 1, w - w_i], c[i - 1, w]\} & \text{otherwise} \end{cases}$$

# 0-1 Knapsack Problem

- Recursive definition for value of optimal soln:
  - This recurrence says that an optimal solution  $S_{i,w}$  for  $K_{i,w}$ 
    - either contains  $a_i \Rightarrow c(S_i, w) = v_i + c(S_{i-1, w-w_i})$
    - or does not contain  $a_i \Rightarrow c(S_i, w) = c(S_{i-1}, w)$
  - If thief decides to pick  $a_i$ 
    - He takes  $v_i$  value and he can choose from  $\{a_1, a_2, \dots, a_{i-1}\}$  up to the weight limit  $w - w_i$  to get  $c[i - 1, w - w_i]$
  - If he decides not to pick  $a_i$ 
    - He can choose from  $\{a_1, a_2, \dots, a_{i-1}\}$  up to the weight limit  $w$  to get  $c[i - 1, w]$
  - The better of these two choices should be made

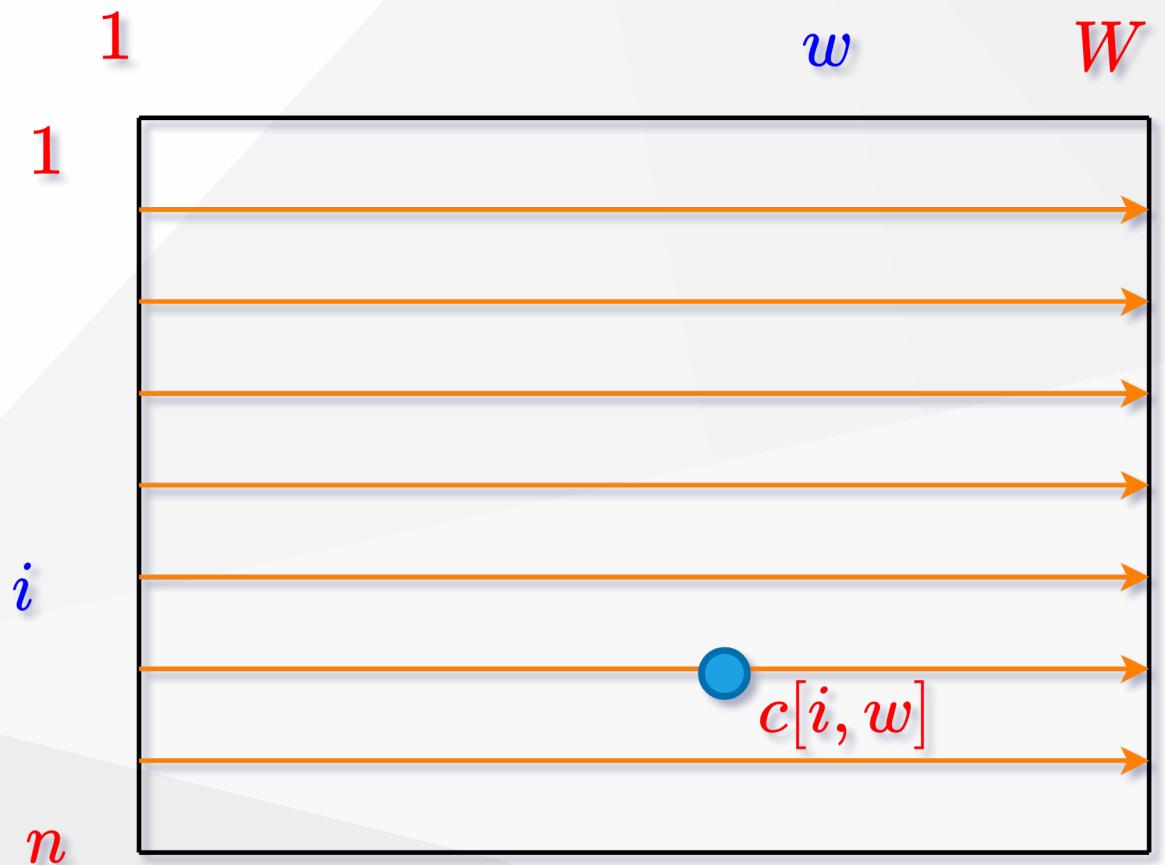
## Bottom-up Computation

- Need to process:
  - $c[i, w]$
- after computing:
  - $c[i - 1, w]$ ,
  - $c[i - 1, w - w_i]$ 
    - for all  $w_i < w$



## Bottom-up Computation

```
for  $i \leftarrow 1$  to  $n$  do  
    for  $w \leftarrow 1$  to  $W$  do  
        ...  
         $c[i, w] \leftarrow \dots$   
        ...
```



## DP Solution to 0-1 Knapsack

- $c$  is an  $(n + 1) \times (W + 1)$  array;  $c[0 \dots n : 0 \dots W]$
- **Note** : table is computed in row-major order
- **Run time:**  $T(n) = \Theta(nW)$

## DP Solution to 0-1 Knapsack

$\text{KNAP0-1}(v, w, n, W)$

for  $\omega \leftarrow 0$  to  $W$  do

$c[0, \omega] \leftarrow 0$

    for  $i \leftarrow 0$  to  $m$  do

$c[i, 0] \leftarrow 0$

    for  $i \leftarrow 0$  to  $m$  do

        for  $\omega \leftarrow 1$  to  $W$  do

            if  $w_i \leq \omega$  then

$c[i, \omega] \leftarrow \max\{v_i + c[i - 1, \omega - w_i], c[i - 1, \omega]\}$

            else

$c[i, \omega] \leftarrow c[i - 1, \omega]$

return  $c[m, W]$

## Constructing an Optimal Solution

- No extra data structure is maintained to keep track of the choices made to compute  $c[i, w]$ 
  - i.e. The choice of whether choosing item  $i$  or not
- Possible to understand the choice done by comparing  $c[i, w]$  with  $c[i - 1, w]$ 
  - If  $c[i, w] = c[i - 1, w]$  then it means item  $i$  was assumed to be not chosen for the best  $c[i, w]$

# Finding the Set $S$ of Articles in an Optimal Load

$\text{SOLKNAP0-1}(a, v, w, n, W, c)$

$i \leftarrow n; \omega \leftarrow W$

$S \leftarrow \emptyset$

*while*  $i \leftarrow 0$  *do*

*if*  $c[i, \omega] = c[i - 1, \omega]$  *then*

$i \leftarrow i - 1$

*else*

$S \leftarrow S \cup \{a_i\}$

$\omega \leftarrow \omega - w_i$

$i \leftarrow i - 1$

*return*  $S$

## References

- [Introduction to Algorithms, Third Edition | The MIT Press](#)
- [Bilkent CS473 Course Notes \(new\)](#)
- [Bilkent CS473 Course Notes \(old\)](#)

*–End – Of – Week – 7 – Course – Module –*

