Parallel Computing Toolbox

CHAPTER OBJECTIVES

This Chapter introduces the key features of MATLAB's Parallel Computing Toolbox. Task- and data-parallel applications can be parallelized using the features provided by Parallel Computing Toolbox. After reading this Chapter, you should be able to:

- understand the key features of Parallel Computing Toolbox.
- execute loop iterations in parallel using a parfor-loop.
- execute code in parallel on MATLAB workers using an *spmd* statement.
- use distributed and codistributed arrays across many MATLAB workers.
- work interactively with a communicating job using the *pmode* functionality.
- create and submit jobs on a computer cluster.

3.1 PRODUCT DESCRIPTION AND OBJECTIVES

MATLAB's Parallel Computing Toolbox allows users to solve computationally and data-intensive problems using multicore and multiprocessor computers, computer clusters, and GPUs. Users can use high-level MATLAB functions to parallelize applications without OpenMP, MPI, and CUDA programming. One of the important features of MATLAB's Parallel Computing Toolbox is that the same application can be executed on a simple core, a multicore processor, or a computer cluster without changing the code. Combining Parallel Computing Toolbox with Distributed Computing Server, users can run their MATLAB programs on computer clusters, grids, and clouds.

Other MATLAB products can take advantage of parallel computing resources and speedup users' codes. As of MATLAB R2015b, the toolboxes that have parallel capabilities are presented in Table 3.1 [12].

The toolboxes that provide GPU support will be presented in Chapter 5. Readers who are interested in utilizing Parallel Computing Toolbox in other toolboxes presented in Table 3.1 are directed to toolboxes user's guides provided by MathWorks. The rest of this chapter will present the most important programming features of the Parallel Computing Toolbox.

 Table 3.1 Parallel computing support in MATLAB and Simulink products

Table 5.1 Farallel Collips	uting support in MALLAB and Simulink products
Product name	Parallel capabilities
Simulink	Run multiple Simulink simulations using simcommand with parfor.
	Run multiple simulations in rapid accelerator mode using parfor with prebuilt Simulink models.
Embedded Coder	Generate and build code in parallel using model blocks.
Simulink Coder	Generate and build code in parallel using model blocks.
Bioinformatics Toolbox	Distribute pairwise alignments to a computer cluster using functions for progressive alignment of multiple sequences and pairwise distance between sequences.
Robust Control Toolbox	Tune fixed-structure control systems with the looptune, systune, and hinfstruct commands.
Simulink Control Design	Perform frequency response estimation of Simulink models.
Simulink Design Optimization	Estimate model parameters and optimize system response.
Image Processing	Improve performance in Batch Image Processor.
Toolbox	Improve performance of block processing tasks in blockproc function.
	Improve performance of many functions using a GPU.
Computer Vision System Toolbox	Improve performance of functions in bag-of-words workflow.
Global Optimization	Explore simultaneously local solution space in genetic
Toolbox Model-Based	algorithm, particle swarm, and pattern search solvers. Fit multiple models to experimental data.
Calibration Toolbox	Run of multiple optimizations in parallel.
Neural Network Toolbox	Improve performance of training and simulation.
	Improve performance of training and simulation using a GPU.
Optimization Toolbox	Accelerate gradient estimation in selected constrained nonlinear solvers.
	Launch parallel computations from optimtool GUI.
Statistics and Machine	Improve performance of many resampling functions.
Learning Toolbox	Fit multiclass support vector machines and other classifiers.
	Generate reproducible random streams in parallel.
	Improve performance of many D-optimal design generation functions.
	Improve performance of many functions with multiple
	starting points.
	Improve performance of many functions using a GPU.
Communications System Toolbox	Improve performance of many System objects using a GPU.
	MATLAB Compiler support for GPU System objects.
Phased Array System	Accelerate clutter model simulation using parfor.
Toolbox	Accelerate clutter model simulation using a GPU.
Signal Processing Toolbox	Improve performance of many functions using a GPU.
SystemTest	Run test iterations on multiple processors or machines.

The Parallel Computing Toolbox can be used to improve the performance of various types of applications:

- Applications that involve repetitive segments of code. Different iterations can be evaluated in parallel using parallel *for-loops* (Section 3.2). Applications can include many iterations that might not take long to execute or few iterations that take long to execute. The only restriction on parallel *for-loops* is that each iteration is not allowed to depend on any other iteration.
- Applications that run a series of independent tasks. Different tasks can be evaluated in parallel using parallel *for-loops* (Section 3.2).
- Applications evaluating the same code on multiple data sets. Multiple workers can be used to run the same code on multiple data sets (Section 3.3).
- Applications involving data that is too large for your computer's memory. Arrays
 can be distributed among multiple MATLAB workers, and each MATLAB worker
 can operate only on its part of the array (Sections 3.3 and 3.4).
- Applications that their execution in parallel can improve their performance (Sections 3.2 and 3.3).
- Applications that their execution on GPUs can improve their performance (Section 3.6).

3.2 PARALLEL FOR-LOOPS (parfor)

It is well-known that a *for-loops* is a series of statements that are executed over a range of values. Similarly, a *parfor-loop* in MATLAB is the same as the standard *for-loops* with the difference that the loop iterations can be executed in parallel. To understand better the concept of the *parfor-loop*, let's introduce some terminology. The MATLAB client is the MATLAB instance that initiated the execution of the *parfor-loop*, while the MATLAB worker is a MATLAB process that runs in parallel without a graphical user interface. Several MATLAB workers can be used to evaluate a *parfor-loop*. As of MATLAB R2015b, a maximum number of 512 MATLAB workers can be used in parallel. To set the number of the MATLAB workers on your machine, select the Parallel Preferences option from MATLAB's workspace (Fig. 3.1). Then, select the Cluster Profile Manager option (Fig. 3.2). Finally, select a cluster profile (the default is named 'local'), click on Edit, and enter a value for the number of workers that you want to use (Fig. 3.3). The default option is the number of cores. If multithreading is enabled, you can set the number of workers equal to the number of threads.

Before running a code that includes a *parfor-loop* or an *spmd* statement, you should open a parallel pool. To open a parallel pool using a specific cluster, e.g., 'local', and a specific size, e.g., 4, type the following command:

```
>> parpool('local', 4)
```

After running your code, you can shut down the parallel pool by typing the following commands (*gcp* returns the current parallel pool):

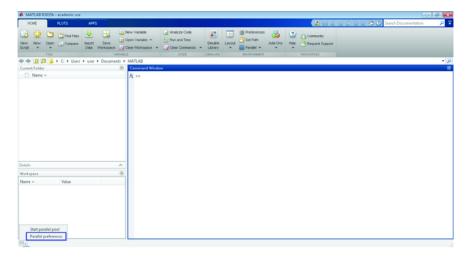


FIG. 3.1

Select the parallel preferences option.

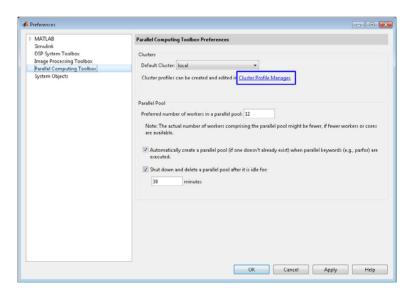


FIG. 3.2

Select the cluster profile manager option.

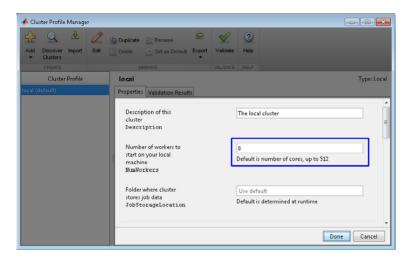


FIG. 3.3

Set the number of workers.

```
>> p = gcp;
>> delete(p)
```

A parfor-loop is initiated on the MATLAB client, and the MATLAB client sends the necessary data to the MATLAB workers. Each MATLAB worker performs the computations that were assigned to it and sends the results back to the MATLAB client. Finally, the MATLAB client pieces the results together. There is no guarantee that all MATLAB workers will be assigned the same number of iterations. If the number of MATLAB workers is equal to the number of the iterations, then each MATLAB worker performs one iteration. If there are more iterations, then some MATLAB workers may perform more iterations than others. Moreover, the iterations are evaluated in no particular order and independent of each other. For example, let's consider the following example:

```
parfor i = 1:8
     disp(i)
end
```

Using a parallel pool with 4 MATLAB workers, let's execute the preceding code twice (Fig. 3.4). The results show that the iterations are evaluated in no particular order.

Moreover, each iteration must be independent of all other iterations. For instance, the MATLAB worker evaluating the third iteration might not be the same MATLAB worker evaluating the fifth iteration. Thus, each iteration cannot use data used in other iterations. Let's take, for example, the following code in which the first *n* Fibonacci numbers are calculated.

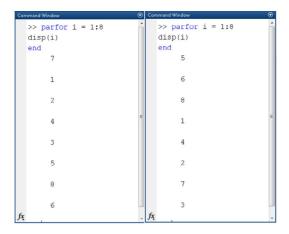


FIG. 3.4

Iterations evaluation in no particular order.

```
n = 10;
fib = zeros(1, n);
fib(1) = 0;
fib(2) = 1;
for i = 3:n
    fib(i) = fib(i - 1) + fib(i - 2);
end
```

As you may already know, the sequence F_n of Fibonacci numbers is defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$. The statement in each iteration depends on results that are calculated in previous iterations. For instance, to compute the fifth Fibonacci number (i = 5), the fourth Fibonacci number (i = 4) and the third Fibonacci number (i = 3) are needed. However, as mentioned earlier, each iteration must be independent of all other iterations, and the iterations are evaluated in no particular order. Thus, we cannot use *parfor* in this example. Even if we used the *parfor* to parallelize the *for-loop*, MATLAB outputs an error that cannot classify the variable *fib* in the *parfor-loop* (Fig. 3.5). Note that the calculation of the first n Fibonacci numbers can be calculated in parallel if we use Binet's formula,

$$f(n) = \frac{\left(1 + \sqrt{5}\right) - \left(1 - \sqrt{5}\right)}{2^n \sqrt{5}}, \text{ as already mentioned in Chapter 1.}$$

MATLAB classifies the variables found in a *parfor-loop* in one of the following categories:

(1) **Loop Variables**: Loop variables define the loop index value for each iteration. A loop variable is set at the beginning line of a *parfor-loop*:

```
Command Window

>> n = 10;
fib = zeros(1, n);
fib(1) = 0;
fib(2) = 1;
parfor i = 3:n
fib(i) = fib(i - 1) + fib(i - 2);
end

Error: The variable fib in a parfor cannot be classified.
See Parallel for Loops in MATLAB, "Overview".

fx
```

FIG. 3.5

Wrong use of parfor in non-independent iterations.

There are some restrictions while using a loop variable:

- A loop variable must evaluate to ascending consecutive integers. Hence, the parfor statements parfor i = 1:100 and parfor i = -10:10 are valid, while the parfor statements parfor i = -100:-1:1, parfor i = 1:2:100, and parfor i = 1:0:10 are not valid
- Assignments to the loop variable are not allowed. Hence, the following parfor-loop is not valid:

```
parfor i = 1:100
    i = i + 2; % not valid
    ...
end
```

• A loop variable cannot be indexed or subscripted. Hence, the following *parfor-loop* is not valid:

- **(2) Sliced Variables**: Sliced variables are arrays whose segments are manipulated by different workers on different loop iterations. Sliced variables are used to reduce the communication time between the MATLAB client and MATLAB workers because only those slices needed by a worker are sent to it when it starts working on a particular range of indices. Let *i* be a loop variable. A variable is sliced if it has all the following characteristics [5]:
 - The first level of indexing is enclosed in either parentheses '()' or braces '{}'. After the first level, any valid MATLAB indexing can be used. Hence, the variable A.p{i} is not sliced, while the variables A{i}, A(i), A(i, 5), and A(i).p are sliced.
 - Within the first-level parenthesis or braces, the list of indices is the same for all occurrences of a given variable. In the following code, variables *A* and *B* are not sliced because *A* is indexed by *i* and *i* + 1 in different places and *B* is indexed by (*i*, 1) and (*i*, 2) in different places.

- Within the list of indices for the variable, exactly one index involves the loop variable, and every other index is a scalar constant, a broadcast variable, a nested *for-loop* index, *colon*, or *end*. Hence, A(i, i + 1) and B(i, 20:30, end) are not sliced, while A(i, :, end) and B(i + 2, i, :, 3) are sliced.
- The sliced variable maintains a constant shape (no deletion or insertion operators are allowed). In the following code, variables *A* and *B* are not sliced because they change their shape by adding or deleting elements.

```
>> A = rand(100, 2);
>> B = rand(100);
>> parfor i = 1:100
        A(i, :) = []; % not sliced
        B(end + 1) = i; % not sliced
end
```

All sliced variables have the characteristics of being input or output. Some sliced variables have both characteristics. Sliced input variables are transmitted from the MATLAB client to the MATLAB workers, sliced output variables are transmitted from the MATLAB workers to the MATLAB client, and sliced input and output variables are transmitted in both directions. In the following code, variable A is a sliced input variable, variable B is a sliced output variable, and variable C is a sliced input and output variable.

```
>> A = rand(100);
>> C = rand(100);
>> parfor i = 1:100
    B(i) = A(i);
    if rand < 0.5
        C(i) = 0;
    end
end</pre>
```

(3) Broadcast Variables: Broadcast variables are variables other than loop and sliced variables that are defined before a *parfor-loop* and are used inside the loop but never modified. The MATLAB client sends the broadcast variables to all MATLAB workers at the start of a *parfor-loop*. In the following code, variables a and *B* are broadcast variables.

```
>> a = 0.4;
>> B = rand(101, 1);
>> C = zeros(100, 1);
>> parfor i = 1:100
        if B(i) + B(i + 1) < a
        C(i) = 0;</pre>
```

```
else
     C(i) = 1;
     end
end
```

Note that variable B is not a sliced variable because it is indexed by i and i+1 in different places. Large broadcast variables can cause a lot of communication between the MATLAB client and the MATLAB workers, and degrade performance. In some cases it is more efficient to use temporary instead of broadcast variables, if possible.

(4) Reduction Variables: Reduction variables accumulate a value that depends on all the iterations together. This is the only exception to the rule that loop iterations must be independent. However, the value of the reduction variable is independent of the iteration order. Reductions variables appear on both sides of an assignment statement. Table 3.2 presents all the possible assignments in which a reduction variable can appear (*expr* is a MATLAB expression). In the following code, the usage of a reduction variable *X* is illustrated:

The value of variable X is not transmitted from the MATLAB client to MATLAB workers or from a MATLAB worker to another. Rather, each MATLAB worker performs a number of additions of A(i) and transmits back to the MATLAB client the result. The MATLAB client receives the results from each MATLAB worker and adds these into X. Some other rules for the reduction variables are the following:

Table 3.2 Possible assignments involving a reduction variable X

Assignment	Assignment
X= X + expr	X = expr + X
X = X - expr	
X = X .* expr	X = expr .* X
X = X * expr	X = expr * X
X = X & expr	X = expr & X
$X = X \mid expr$	X = expr X
X = [X, expr]	X = [expr, X]
X = [X; expr]	X = [expr; X]
X = min(X, expr)	X = min(expr, X)
X= max(X, expr)	X = max(expr, X)
X = union(X, expr)	X = union(expr, X)
X = intersect(X, expr)	X = intersect(expr, X)

• For any reduction variable, the same reduction function or operation must be used in all reduction assignments for that variable. For example, the following *parfor-loop* is not valid because the reduction assignment uses '+' in one instance and '*' in another.

```
>> X = 0;
>> parfor i = 1:100
    if rand < 0.5
        X = X + 5;
    else
        X = X * 2;
    end
end</pre>
```

On the other hand, the following *parfor-loop* is valid.

```
>> X = 0;
>> parfor i = 1:100
    if rand < 0.5
        X = X + 5;
    else
        X = X + i + 2;
    end
end</pre>
```

• If the reduction assignment uses '*' or '[,]' or '[,]', then in every reduction assignment for *X*, *X* must be consistently specified as the first argument or consistently specified as the second. For example, the following code is not valid because variable *X* does not occur in the same argument position in all reduction assignments.

```
>> X = 0;
>> parfor i = 1:100
    if rand < 0.5
        X = X * 5;
    else
        X = 2 * X;
    end
end</pre>
```

On the other hand, the following *parfor-loop* is valid.

```
>> X = 0;
>> parfor i = 1:100
    if rand < 0.5
        X = [X, 5];
    else
        X = [X, i + 2];
    end
end</pre>
```

• The logical operators && and || are not allowed in reduction assignments.

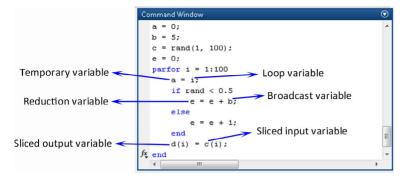


FIG. 3.6

Types of parfor variables.

(5) Temporary Variables: Temporary variables are variables that are the target of direct non-indexed assignments, but they are not reduction variables. Temporary variables exist only in a MATLAB worker's workspace and are not transferred back to the MATLAB client. In the following code, variables a and *b* are temporary variables.

```
>> a = 0;
>> parfor i = 1:100
    a = i;
    if rand < 0.5
       b = 0;
    else
       b = 1;
    end
end</pre>
```

To sum up, Fig. 3.6 presents an example where all different types of *parfor* variables are used.

If a *parfor-loop* contains a variable that cannot be uniquely classified into one of the preceding categories or a variable violates its category restrictions, MATLAB generates an error, as shown in Fig. 3.5. Understanding the different types of *parfor* variables will help you find performance degrades.

You should also consider the following limitations/concerns before parallelizing your application using a *parfor-loop*:

- A code including a *parfor-loop* may result in better execution times in a multicore machine but not in a computer cluster due to the communication cost.
- You cannot use a *parfor-loop* if an iteration depends on the results of other iterations.
- There might be no benefit to using a *parfor-loop* if there is a small number of simple calculations due to the communication cost.

- When running a *parfor-loop* in a computer cluster, the communication cost will be greater than running the same code on local MATLAB workers.
- Be careful when using *cd*, *addpath*, and *rmpath* on a *parfor-loop* as the MATLAB search path might not be the same on all MATLAB workers.
- If an error occurs in one iteration, then all MATLAB workers terminate.
- Be careful when using commands like *input* and *keyboard*, which are not strictly computational in nature, as they might not have a visible effect or might hang the corresponding MATLAB worker.
- Any graphical output, for example, a figure, will not be displayed at all if it is used inside a *parfor-loop*.
- If you want to use objects inside a loop, then explicitly assign them to a variable, as they are not automatically propagated to the client.
- If you want to call a function handle with the loop variable as an argument, use *feval*.
- The body of a *parfor-loop* cannot make reference to a nested function, but it can call a nested function by means of a function handle.
- The body of a *parfor-loop* cannot contain another *parfor-loop*. Hence, if you have a code with two nested *for-loops*, you can parallelize either of the nested loops. Usually, the best strategy is to convert the outer *for-loop* to a *parfor-loop*.
- The range of a *for-loop* nested in a *parfor-loop* must be defined by constant numbers or variables and not by using function calls, like *length(A)*.
- When using a nested *for-loop* variable for indexing a sliced variable, you must use the variable in plain form and not as part of an expression (e.g., you can use A(j) but not A(j + 1), where j is the nested *for-loop* variable and A a sliced variable).
- If you index into a sliced variable inside a nested *for-loop*, you cannot use that variable elsewhere in the *parfor-loop*.
- If you index into a sliced variable inside multiple *for-loops* (not nested) in a *parfor-loop*, they must loop over the same range of values.
- A sliced output variable can be used in only one nested for-loop.
- The body of a *parfor-loop* cannot contain *break* or *return* statements.
- You cannot clear variables from a MATLAB worker's workspace (e.g., clear x;), but instead you can set its value to empty (e.g., x = [];).
- You cannot use functions *eval*, *evalc*, *evalin* if the variables involved in these functions are not visible in the *parfor-loop* (e.g., they are declared prior to the *parfor-loop*).
- You cannot use *save* and *load* inside a *parfor-loop* unless the output of *load* is assigned to a variable.

There are mainly two types of applications that the use of a *parfor-loop* will improve their computation time:

- Applications that involve repetitive segments of code and include many iterations that do not take long to execute
- Applications that involve repetitive segments of code and include few iterations that take long to execute

Example: Monte Carlo simulation to approximate the area of a figure

A Monte Carlo simulation is a useful method to approximate the area of a figure. In some cases, it is difficult to find the exact area of a figure; so, this approximation method is very useful. A Monte Carlo method performs the following steps to find the area of a figure:

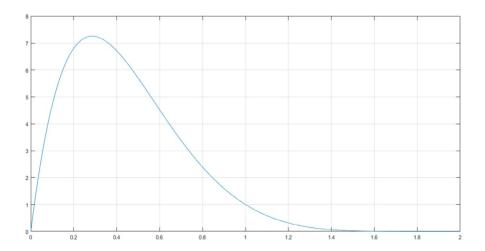
- Generates uniformly *n* random points on the square that encloses the figure and counts the number of points (*counter*) that lie inside the figure.
- Calculates the ratio *counter/n*, which approximates the ratio between the area of the figure and the square that encloses the figure.
- Computes the area of the given figure by multiplying the calculated ratio with the known area of the square that encloses the figure.

For instance, the previously mentioned Monte Carlo simulation will be applied to compute the area of the following definite integral:

$$\int_0^2 x(x-2)^6 dx$$

The area of this definite integral is the area under the curve $y = x(x-2)^6$ from x = 0 to x = 2 (Fig. 3.7).

A single-threaded MATLAB code to compute the area of the above definite integral is the following:



Flot of the curve $y = x(x-2)^6$ from x = 0 to x = 2.

```
1.
    function area = test(n)
2.
    % Filename: test.m
    % Description: This function computes the area of the
   % definite integral int(x * (x - 2) ^{\circ} 6, x = 0..2)
    % (single-threaded version)
    % Authors: Ploskas, N., & Samaras, N.
    % Syntax: area = test(n)
8.
    % Input:
9.
    % -- n: the number of random points to generate
10. % Output:
11. % -- area: the area of the integral
12.
13. counter = 0;
14. for i = 1:n
      x = 2 * rand;
15.
16.
      y = 8 * rand;
17.
      if(y \le x * (x - 2) ^ 6)
18.
            counter = counter + 1;
19.
     end
20. end
21. area = counter / n * 2 * 8;
22. end
```

The number of the uniformly n random points that will be generated is an important part of each Monte Carlo method. The higher the number, the more accurate will be the calculation of the area. So, let's use 1,000,000,000 random points to calculate the area. The *test* function runs in 188.89 seconds.

Someone could argue that the use of vectorized code will improve the execution time of the function *test*. The equivalent vectorized function is the following:

```
function area = testVectorized(n)
1.
    % Filename: testVectorized.m
2.
    % Description: This function computes the area of the
    % definite integral int(x * (x - 2) ^{\circ} 6, x = 0..2)
    % (vectorized version)
    % Authors: Ploskas, N., & Samaras, N.
7.
    % Syntax: area = testVectorized(n)
    % Input:
   % -- n: the number of random points to generate
10. % Output:
11. % -- area: the area of the integral
12.
13. x = 2 * rand(n, 1);
14. y = 8 * rand(n, 1);
15. counter = sum(y < (x .* (x - 2) .^ 6));
16. area = counter / n * 2 * 8;
17. end
```

The *testVectorized* function runs in 56.45 seconds, a \sim 3.35× speedup compared to the *test* function. This is a very good speedup and it was expected because the vectorized function is executed in multiple threads. However, the *testVectorized*

function requires 16 Gb of RAM to create and store variables *x* and *y*. This is a huge amount of memory that is not available in many computers.

Assuming that our computer has a smaller amount of memory, we can modify the *test* function to take advantage of the vectorized code. Instead of generating vectors x and y with 1,000,000,000 values, we will generate them with 10,000,000 values and perform the *for-loop* 100 times. So, the following code only uses 160 MB for variables x and y.

```
1.
    function area = test2(n)
2.
     % Filename: test2.m
3. % Description: This function computes the area of the
    % definite integral int(x * (x - 2) ^{\circ} 6, x = 0..2)
5. % (single-threaded semi-vectorized version)
6. % Authors: Ploskas, N., & Samaras, N.
7. % Syntax: area = test2(n)
8. % Input:
   % -- n: the number of random points to generate
9.
10. % Output:
11. % -- area: the area of the integral
12.
13. n = n / 100;
14. counter = 0;
15. for i = 1:100
16. x = 2 * rand(n, 1);
17. y = 8 * rand(n, 1);
17.
       y = 8 * rand(n, 1);
18. counter = counter + sum(y < (x .* (x - 2) .^ 6));
19. end
20. area = counter / (n * 100) * 2 * 8;
21. end
```

The *test2* function runs in 56.83 seconds, a \sim 3.32× speedup compared to the *test* function.

Initially, let's use a *parfor-loop* to parallelize the *test* function.

```
function area = testparfor(n)
   % Filename: testparfor.m
    % Description: This function computes the area of the
3.
4. % definite integral int(x * (x - 2) ^{\circ} 6, x = 0..2)
5. % (multi-threaded version using parfor)
6. % Authors: Ploskas, N., & Samaras, N.
7. % Syntax: area = testparfor(n)
   % Input:
8.
   % -- n: the number of random points to generate
9.
10. % Output:
11. % -- area: the area of the integral
12.
13. counter = 0;
14. parfor i = 1:n
```

The *testparfor* function runs in 36.12 seconds using 8 MATLAB workers, a \sim 5.23× speedup compared to the *test* function. We cannot achieve a linear speedup of 8× due to the communication cost.

Next, let's use a *parfor-loop* to parallelize the *test2* function.

```
function area = testparfor2(n)
1.
2.
     % Filename: testparfor2.m
    % Description: This function computes the area of the
     % definite integral int(x * (x - 2) ^{\circ} 6, x = 0..2)
    % (multi-threaded semi-vectorized version using parfor)
6.
    % Authors: Ploskas, N., & Samaras, N.
7.
    % Syntax: area = testparfor2(n)
8.
    % Input:
    % -- n: the number of random points to generate
9.
10. % Output:
11. % -- area: the area of the integral
12.
13. n = n / 100;
14. counter = 0;
15. parfor i = 1:100
       x = 2 * rand(n, 1);
16.
17.
       y = 8 * rand(n, 1);
18.
       counter = counter + sum(y < (x .* (x - 2) .^6));
20. area = counter / (n * 100) * 2 * 8:
21. end
```

The *testparfor2* function runs in 25.95 seconds using 8 MATLAB workers, a \sim 7.28× speedup compared to the *test* function and a \sim 2.19× speedup compared to the *test2* function.

3.3 SINGLE PROGRAM MULTIPLE DATA (spmd)

The single programming multiple data (*spmd*) statement allows seamless interleaving of serial and parallel programming. A block of code to run simultaneously on multiple MATLAB workers can be defined in an *spmd* statement. An *spmd* statement is used when we want to execute an identical code on multiple data. Each MATLAB worker will execute the same code on different data. Before and after an *spmd* statement, the code is executed on the MATLAB client, and the code inside an *spmd* statement is executed on the MATLAB workers. Hence, the general form of an *spmd* statement is the following:

```
... % statements executed on the MATLAB client
spmd
... % statements executed on multiple MATLAB workers
end
... % statements executed on the MATLAB client
```

You can also define the number of MATLAB workers to be used in an *spmd* statement:

In the first case (spmd(n)), the spmd statement requires that n MATLAB workers will run the spmd block of code. If the pool is large enough, but n MATLAB workers are not available, the statement waits until enough MATLAB workers are available. In the second case (spmd(m, n)), the spmd statement requires a minimum of m MATLAB workers, and it uses a maximum of n MATLAB workers, if available in the pool.

Each MATLAB worker used in an *spmd* statement has a unique value of *labindex* that can be used to run codes on specific MATLAB workers or access unique data. The total number of MATLAB workers used in an *spmd* statement can be obtained using the *numlabs* value. For example, we can create different sized arrays depending on *labindex*, as shown in the following code:

```
spmd
  if labindex == 1
    A = rand(3, 3);
  elseif labindex == numlabs
    A = rand(4, 4);
  else
    A = rand(2, 2);
  end
  maximum = max(max(A));
end
```

In the preceding example *A* and *maximum* are *composite* objects (Fig. 3.8). *Composite* objects contain references to the values stored on the MATLAB workers and can be retrieved on the MATLAB client using cell-array indexing (Fig. 3.9).

There are two ways to create composite objects:

• Define variables on MATLAB workers inside an *spmd* statement. These variables are accessible by the MATLAB client after the *spmd* statement, as shown in the previous example (Figs. 3.8 and 3.9).

```
Lab 1: class = double, size = [3
   Lab 2: class = double, size = [2
   Lab 3: class = double, size = [2
  Lab 4: class = double, size = [2
   Lab 5: class = double, size = [2
   Lab 6: class = double, size = [2
   Lab 7: class = double, size = [2
                                     21
   Lab 8: class = double, size = [4 4]
>> maximum
maximum =
   Lab 1: class = double, size = [1 1]
   Lab 2: class = double, size = [1
   Lab 3: class = double, size = [1
                                     11
   Lab 4: class = double, size = [1
   Lah 5: class = double, size = [1
```

FIG. 3.8

Composite objects.

```
Command Window

>> A{1}

ans =

0.4488    0.0538    0.6766
 0.6926    0.1998    0.7744
 0.8541    0.4874    0.3975

>> maximum{1}

ans =

0.8541

fx
```

FIG. 3.9

Retrieving *composite* objects on the MATLAB client.

Use the *Composite* function on the MATLAB client. The MATLAB client may
create a *composite* object prior to an *spmd* statement, and the MATLAB workers
can access the *composite* object inside an *spmd* statement.

Table 3.3 Functions used by MATLAB workers to communicate with each other

Function	Description
gcat	Global concatenation of an array performed across all MATLAB workers.
gop	Global reduction using binary associative operation performed across all MATLAB workers.
gplus	Global addition performed across all MATLAB workers.
labBarrier	Block execution until all MATLAB workers reach this call.
labBroadcast	Send data to all MATLAB workers or receive data sent to all MATLAB workers.
labProbe	Test to see if messages are ready to be received from other MATLAB worker.
labReceive	Receive data from another MATLAB worker.
labSend	Send data to another MATLAB worker.
labSendReceive	Simultaneously send data to and receive data from another MATLAB worker.

```
>> spmd
          maximum = max(max(A));
end
```

The *composite* objects are retained on the MATLAB workers until they are cleared on the MATLAB client or until the pool is closed. Moreover, multiple *spmd* statements can use *composite* objects defined in previous *spmd* statements.

The MATLAB workers can communicate with each other using a number of functions. First of all, *distributed* and *codistributed* arrays can be used to partition large data sets. *Distributed* and *codistributed* arrays will be thoroughly presented in Section 3.4. Moreover, a number of functions can be used from the MATLAB workers to communicate with each other (Table 3.3).

You should also consider the following limitations/concerns before parallelizing your application using an *spmd* statement:

- When running an *spmd* statement in a computer cluster, the communication cost will be greater than running the same code on local MATLAB workers.
- Any graphical output, for example, *plot*, will not be displayed at all because the MATLAB workers are sessions without graphical output.
- Be careful when using *cd*, *addpath*, and *rmpath* on an *spmd* statement as the MATLAB search path might not be the same on all MATLAB workers.
- If an error occurs on a worker, then all MATLAB workers terminate.
- The body of an *spmd* statement cannot make any direct reference to a nested function, but it can call a nested function by means of a variable defined as a function handle to the nested function.

- The body of an *spmd* statement cannot define an anonymous function, but it can reference an anonymous function by means of a function handle.
- The body of an *spmd* statement cannot directly contain another *spmd* statement, but it can call a function that contains another *spmd* statement. However, the inner *spmd* statement runs serially in a single thread on the worker running its containing function.
- The body of a *parfor-loop* cannot contain an *spmd* statement and an *spmd* statement cannot contain a *parfor-loop*.
- The body of an *spmd* statement cannot contain *break* and *return* statements.
- The body of an *spmd* statement cannot contain global or persistent variable declarations.

There are mainly two types of applications in which the use of an *spmd* statement will improve their computation time:

- Applications that take a long time to execute: several MATLAB workers compute solutions simultaneously.
- Applications that use large data sets: data is distributed to multiple MATLAB workers.

Example: Monte Carlo simulation to approximate the area of a figure

Let's consider again the example presented in Section 3.2. Initially, let's use an *spmd* statement to parallelize the *test* function.

```
function area = testspmd(n)
1.
2.
    % Filename: testspmd.m
    % Description: This function computes the area of the
4. % definite integral int(x * (x - 2) ^{\circ} 6, x = 0..2)
5. % (multi-threaded version using spmd)
6. % Authors: Ploskas, N., & Samaras, N.
7. % Syntax: area = testspmd(n)
8. % Input:
9. % -- n: the number of random points to generate
10. % Output:
11. % -- area: the area of the integral
12.
13. spmd(8)
14. iterations = floor(n / 8);
15. if 1
16.
17. end
        if labindex <= mod(n, 8)</pre>
            iterations = iterations + 1;
18. counter = 0;

19. for i = 1:ite

20. x = 2 *
      for i = 1:iterations
20.
            x = 2 * rand;
21.
            v = 8 * rand:
22.
            counter = counter + sum(y < (x * (x - 2) ^ 6));
23. end
24. end
25. counterStruct = qplus(counter);
```

The *testspmd* function runs in 35.20 seconds using 8 MATLAB workers, a \sim 5.37× speedup compared to the *test* function and almost the same execution time with the *testparfor* function.

Next, let's use an *spmd* statement to parallelize the *test2* function.

```
function area = testspmd2(n)
1.
     % Filename: testspmd2.m
     % Description: This function computes the area of the
3.
    % definite integral int(x * (x - 2) ^{\circ} 6, x = 0..2)
4.
     % (multi-threaded semi-vectorized version using spmd)
5.
6.
    % Authors: Ploskas, N., & Samaras, N.
7.
    % Syntax: area = testspmd2(n)
8.
    % Input:
9.
    % -- n: the number of random points to generate
10. % Output:
11. % -- area: the area of the integral
12.
13. n = n / 100;
14. spmd(8)
       iterations = floor(100 / 8);
15.
     iterations = floor(100 / 8
if labindex <= mod(100, 8)</pre>
16.
            iterations = iterations + 1;
17.
18.
       end
19.
     counter = 0,
for i = 1:iterations
    rand(n,
       counter = 0:
20.
21.
             x = 2 * rand(n, 1);
22.
             y = 8 * rand(n, 1);
23.
             counter = counter + sum(y < (x .* (x - 2) .^6));
24.
       end
25. end
26. counterStruct = gplus(counter);
27.
     counter = 0;
28. for i = 1:8
29.
        counter = counter + counterStruct{i};
30. end
31. area = counter / (n * 100) * 2 * 8;
32. end
```

The *testparfor2* function runs in 25.79 seconds using 8 MATLAB workers, a \sim 7.32× speedup compared to the *test* function, a \sim 2.20× speedup compared to the *test2* function and almost the same execution time with the *testparfor2* function.

3.4 DISTRIBUTED AND CODISTRIBUTED ARRAYS

The MATLAB client can create a distributed array and distribute it across all MATLAB workers. A codistributed array can be created inside an spmd statement and partitioned among the MATLAB workers. The difference between distributed and codistributed arrays is one of perspective. Codistributed arrays are partitioned among the MATLAB workers from which you execute code to create or manipulate them. Distributed arrays are partitioned among workers in the parallel pool. When a codistributed array is created in an spmd statement, it can be accessed as a distributed array on the MATLAB client. When a distributed array is created on the MATLAB client, it can be accessed as a codistributed array inside an spmd statement. The details of distribution cannot be controlled when creating a distributed array; a distributed array is distributed in one dimension, along the last non-singleton dimension, and as evenly as possible along that dimension among the MATLAB workers. On the other hand, the details of distribution can be controlled when creating a codistributed array.

A distributed array can be created using one of the following ways:

• Use the *distributed* function to distribute an existing array from the MATLAB client's workspace to the MATLAB workers of a parallel pool. In the following example, *A* and *B* are *distributed* arrays. Each MATLAB worker stores only a part of *A* and *B* (Fig. 3.10).

```
Command Window

Lab 1:

This worker stores A(:,1:25).

LocalPart: [100x25 double]
Codistributor: [1x1 codistributor1d]

This worker stores B(:,1:25).

LocalPart: [100x25 double]
Codistributor: [1x1 codistributor1d]

Lab 2:

This worker stores A(:,26:50).

LocalPart: [100x25 double]
Codistributor: [1x1 codistributor1d]

This worker stores B(:,26:50).
```

FIG. 3.10

• Use any of the overloaded *distributed* object methods to directly construct a *distributed* array on the MATLAB workers (Table 3.4). In the following example, A and B are *distributed* arrays. Each MATLAB worker stores only a part of A and B (Fig. 3.10).

• Create a *codistributed* array inside an *spmd* statement and access it as a *distributed* array outside the *spmd* statement (see examples for *codistributed* arrays later in this section).

A codistributed array can be created using one of the following ways:

• Use the *codistributed* function inside an *spmd* statement, a communicating job or *pmode* to codistribute data already existing on the MATLAB workers running that job. In the following example, *A* and *B* are *codistributed* arrays. Each MATLAB worker stores only a part of *A* and *B* (Fig. 3.10).

```
>> parpool('local', 4);
>> spmd
          A = rand(100, 100);
          A = codistributed(A);
          B = A * 2;
end
>> delete(gcp);
```

In the preceding code, codistributed(A), which is the same as codistributed(A), codistributor('1d', 2)), tells MATLAB to distribute array A along its second dimension, that is, columns. If you want to distribute array A along its first dimension, that is, rows, use codistributed(A, codistributor('1d', 1)) (Fig. 3.11).

As an alternative to distributing by a single dimension of rows or columns, you can distribute an array by blocks using '2dbc' or two-dimensional block-cyclic distribution. In the following example, A and B are distributed with a 50×50 block in 2-by-2 arrangement (Figs. 3.12 and 3.13).

```
>> parpool('local', 4);
>> spmd
         A = rand(100, 100);
```

 Table 3.4 Overloaded MATLAB functions for distributed arrays

	Create a distributed cell array.
distributed colon(a	
d, b)	Create a distributed array from the vector a:d:b.
distributed.eye (Create a distributed identity matrix.
distributed.false	Create a distributed array of logical zeros.
distributed.Inf	Create a distributed array with Inf values in all elements.
distributed.linspace (Create a distributed linearly spaced vector.
distributed.logspace (Create a distributed logarithmically spaced vector.
distributed.NaN	Create a distributed array with Nan values in all elements.
distributed.ones (Create a distributed array of ones.
	Create a distributed array of uniformly distributed pseudo-random numbers.
I	Create a distributed array of uniformly distributed pseudo-random integer numbers.
	Create a distributed array of normally distributed pseudo-random numbers.
distributed.spalloc	Allocate space for a sparse distributed array.
distributed.speye	Create a distributed sparse identity array.
	Create a distributed sparse array of uniformly distributed pseudorandom values.
	Create a distributed sparse array of normally distributed pseudorandom values.
distributed.true (Create a distributed array of logical ones.
distributed.zeros (Create a distributed array of zeros.
eye(, 'distributed')	Create a distributed identity matrix.
false(, 'distributed')	Create a distributed array of logical zeros.
Inf(, 'distributed')	Create a distributed array with Inf values in all elements.
NaN(, 'distributed')	Create a distributed array with Nan values in all elements.
ones(, 'distributed')	Create a distributed array of ones.
	Create a distributed array of uniformly distributed pseudo-random numbers.
` '	Create a distributed array of uniformly distributed pseudo-random integer numbers.
` '	Create a distributed array of normally distributed pseudo-random numbers.
true(, 'distributed')	Create a distributed array of logical ones.
zeros(, 'distributed')	Create a distributed array of zeros.

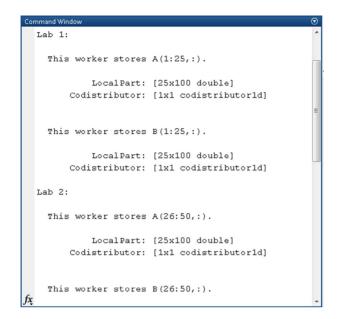


FIG. 3.11

Codistributed arrays along their first dimension.

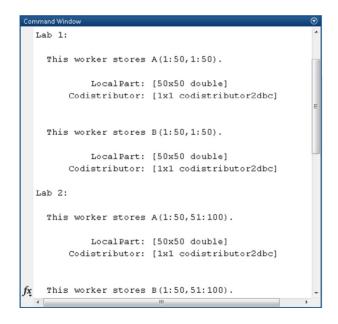


FIG. 3.12

Codistributed arrays using '2dbc' distribution.

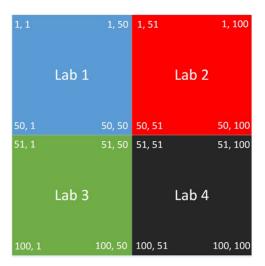


FIG. 3.13

Codistributed arrays using '2dbc' distribution.

```
A = codistributed(A, codistributor('2dbc', [2 2], 50));
B = A * 2;
>> end
>> delete(gcp);
```

• Use any of the overloaded *codistributed* object methods to directly construct a *codistributed* array on the MATLAB workers (Table 3.5).

```
>> parpool('local', 4);
>> spmd
          A = rand(100, 100, codistributorld());
          B = A * 2;
>> end
>> delete(gcp);
```

• Create a *distributed* array outside an *spmd* statement and access it as a *codistributed* array inside the *spmd* statement running on the same pool (see examples for *distributed* arrays earlier in this section).

Indexing into a non-distributed array is straightforward; each dimension is indexed within the range of 1 to the final subscript given by the *end* keyword. The length of any dimension can be determined using either *size* or *length* function. On the other hand, these values are not so easily obtained for *codistributed* arrays because the index range depends on the distribution scheme that was used to distribute the *codistributed* array. MATLAB provides the *globalIndices* function, which provides a correlation between the local and global indexing of the *codistributed* array.

 Table 3.5 Overloaded MATLAB functions for codistributed arrays

codistributed.colon(a, d, b) codistributed.colon(a, d, b) codistributed.colon(a, d, b) codistributed.eye codistributed.false codistributed.linspace codistributed.rand codistributed.rand codistributed.rand codistributed.randi codistributed.spalloc codistributed.spalloc codistributed.spalloc codistributed.spalloc codistributed.sprand codistributed.sprand codistributed.sprand codistributed.sprand codistributed.rand codistributed.rand codistributed.rand codistributed.sprand codistributed.sprand codistributed.rand codistributed.rand codistributed.rand codistributed.sprand codistributed.sprand codistributed.sprand codistributed.rand codistributed.rand codistributed.rand codistributed.rand codistributed.rand codistributed.rand codistributed.sprand codistributed sprane array of normally distributed pseudo-random values. Create a codistributed sprane array of logical ones. Create a codistributed array of logical ones. Create a codistributed array with lnf values in all elements. Create a codistributed array of logical paces. Create a codistributed array of ones. Create a codistribut	Function	Description
codistributed.colon(a, d, b) codistributed.lase codistributed.linspace codistributed.linspace codistributed.linspace codistributed.linspace codistributed.linf codistributed.NaN create a codistributed linearly spaced vector. Create a codistributed lingarithmically spaced vector. Create a codistributed array with lnf values in all elements. codistributed.rand create a codistributed array of ones. codistributed.rand create a codistributed array of ones. Create a codistributed array of uniformly distributed pseudo-random numbers. codistributed.randi codistributed.randi codistributed.spalloc codistributed.spalloc codistributed.speye codistributed.sprand codistributed.sprand codistributed.sprand codistributed.sprandn codistributed.sprandn codistributed.true codistributed.yrand codistributed.yrand codistributed.rand codistributed.rand codistributed.sprandn codistributed.sprandn codistributed.sprandn codistributed.sprandn codistributed.yrand create a codistributed array of normally distributed pseudo-random values. Create a codistributed array of logical ones. Create a codistributed array of logical pros. Create a codistributed array of logical pros. Create a codistributed array of ones. Create a codistributed array of logical pros. Create a codistributed array of ones. Create a codistributed array of normally distributed pseudo-random numbers. Create a codistributed array of normally distributed pseud	codistributed.build	Build a codistributed array from local parts.
codistributed.eye codistributed.false codistributed.linspace codistributed.linspace codistributed.linspace codistributed.logspace codistributed.logspace codistributed.NaN Create a codistributed array with Inf values in all elements. codistributed.rand codistributed.rand codistributed.rand codistributed.rand codistributed.spalloc codistributed.spalloc codistributed.spare codistributed.spare codistributed.spare codistributed.spare codistributed.spare codistributed.sprand codistributed array of normally distributed pseudo-random values. Create a codistributed array of logical ones. Create a codistributed array of logical zeros. Create a codistributed array with Nan values in all elements. Create a codistributed array with Nan values in all elements. Create a codistributed array of ones. Create a codistributed array of ones. Create a codistributed array of ones. Create a codistributed array of logical peros. Create a codistributed array of logical peros.	codistributed.cell	Create a codistributed cell array.
codistributed.false codistributed.linspace codistributed.logspace codistributed.lnf codistributed.lnf codistributed.NaN codistributed.ones codistributed.rand codistributed.randi codistributed.randi codistributed.spalloc codistributed.spalloc codistributed.sprand codistributed.true codistributed.array codistributed.array codistributed.array codistributed.array codistributed.array codistributed.array codistributed array codistributed ar	codistributed.colon(a, d, b)	Create a codistributed array from the vector a:d:b.
codistributed.linspace codistributed.logspace codistributed.lnf codistributed.NaN codistributed.NaN codistributed.ones codistributed.rand codistributed.randi codistributed.randi codistributed.randi codistributed.spalloc codistributed.spalloc codistributed.spalloc codistributed.sprand codistributed array codistri	codistributed.eye	Create a codistributed identity matrix.
codistributed.logspace codistributed.lnf codistributed.NaN Create a codistributed array with Inf values in all elements. Create a codistributed array with Nan values in all elements. Create a codistributed array of ones. Create a codistributed array of uniformly distributed pseudo-random numbers. Codistributed.randi Create a codistributed array of distributed pseudo-random integer numbers. Codistributed.spalloc codistributed.spalloc codistributed.sprand Create a codistributed array of normally distributed pseudo-random numbers. Allocate space for a sparse codistributed array. Create a codistributed sparse identity array. Create a codistributed sparse array of uniformly distributed pseudo-random numbers. Allocate space for a sparse codistributed array. Create a codistributed sparse array of uniformly distributed pseudo-random values. Create a codistributed sparse array of uniformly distributed pseudo-random values. Create a codistributed sparse array of normally distributed pseudo-random values. Create a codistributed array of logical ones. Create a codistributed array of logical ones. Create a codistributed array of logical zeros. Create a codistributed array with Inf values in all elements. Create a codistributed array with Nan values in all elements. Create a codistributed array of ones. Create a codistributed array of uniformly distributed pseudo-random numbers. Create a codistributed array of ones. Create a codistributed array of uniformly distributed pseudo-random numbers. Create a codistributed array of ones. Create a codistributed array of normally distributed pseudo-random numbers. Create a codistributed array of normally distributed pseudo-random numbers. Create a codistributed array of normally distributed pseudo-random numbers. Create a codistributed array of normally distributed pseudo-random numbers. Create a codistributed array of normally distributed pseudo-random numbers. Create a codistributed array of normally distributed pseudo-random numbers.	codistributed.false	Create a codistributed array of logical zeros.
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codistributed.randi Create a codistributed array of uniformly distributed pseudo-random numbers. Create a codistributed array of distributed pseudo-random integer numbers. Create a codistributed array of normally distributed pseudo-random numbers. Allocate space for a sparse codistributed array. Create a codistributed sparse identity array. Create a codistributed sparse array of uniformly distributed pseudo-random values. Create a codistributed sparse array of uniformly distributed pseudo-random values. Create a codistributed sparse array of normally distributed pseudo-random values. Create a codistributed array of logical ones. Create a codistributed array of logical ones. Create a codistributed array of logical zeros. Create a codistributed array of logical zeros. Create a codistributed array with Inf values in all elements. Create a codistributed array of ones.	codistributed.NaN	•
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, ,	sparse(, codist)	Create a sparse codistributed matrix.
zeros(, 'codistributed') Create a codistributed array of zeros.		Create a codistributed array of logical ones.
	zeros(, 'codistributed')	Create a codistributed array of zeros.

Moreover, MATLAB offers for drange loop to iterate through a codistributed array, as shown below:

Finally, many functions in MATLAB are overloaded so that they operate on *codistributed* arrays in the same way they operate on non-distributed arrays. For a full list, see [5].

3.5 INTERACTIVE PARALLEL DEVELOPMENT (pmode)

MATLAB provides the *pmode* functionality where you can work interactively with a communicating job running simultaneously on multiple MATLAB workers. *pmode* and *spmd* are very similar; the main difference to *spmd* is that *pmode* does not allow you to freely interleave serial and parallel work as *spmd* does. When a *pmode* session is terminated, its job is destroyed and all data stored on the MATLAB workers lost. Commands that are typed at the *pmode* prompt are executed on all MATLAB workers at the same time. Each worker has its own workspace. All communication functions supported in *spmd* statements can also be used in *pmode* and the variables can be transferred between the MATLAB client and the MATLAB workers. As with *spmd*, MATLAB workers are sessions without display. Moreover, the MATLAB workers running *pmode* can be on a computer cluster.

We can start *pmode* using the local profile with 4 local workers (Fig. 3.14):

```
>> pmode start local 4
```

Codistributed arrays can be distributed among the MATLAB workers (Fig. 3.15):

```
>> A = rand(100, 100, codistributor());
```

The *gather* function can be used to gather an entire array into the workspace of all MATLAB workers. Moreover, the *client2lab* function can be used to copy a variable from the MATLAB client to all/specified MATLAB workers, while *lab2client* function can be used to copy a variable from a MATLAB worker to the MATLAB client. Note that a *codistributed* array cannot be transferred from a MATLAB worker to the MATLAB client because the MATLAB worker stores only a local portion of the *codistributed* array. To overcome that limitation, the *gather* function must first be used to assemble the entire array into the workspace of all MATLAB workers and then use the *lab2client* function to transfer the *codistributed* array to the MATLAB client.

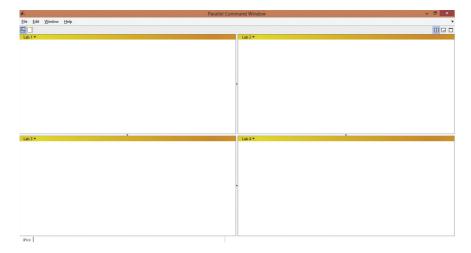


FIG. 3.14

pmode's parallel command window.



FIG. 3.15

Codistributed array in pmode's parallel command window.

3.6 GPU COMPUTING

For the sake of completeness, we have also included this section about GPU computing because the Parallel Computing Toolbox enables you to program MATLAB to use GPUs. However, we will not present MATLAB's GPU capabilities in this section because GPU computing will be extensively discussed in the next chapters of this book.

3.7 CLUSTERS AND JOB SCHEDULING

The Parallel Computing Toolbox and MATLAB Distributed Computing Server let you solve task- and data-parallel algorithms on many multicore and multiprocessor computers. A job is a large operation that you need to perform in MATLAB. A job can be divided into segments called tasks. These tasks can be identical or not. The MATLAB session in which the job is defined is called the MATLAB client. The Parallel Computing Toolbox should be installed on the MATLAB client. If the job is executed on a local cluster, no other MATLAB toolbox is needed. On the other hand, if the job is executed on a computer cluster, the MATLAB Distributed Computing Server should be installed on the machines of the cluster (Fig. 3.16). A job scheduler is needed to coordinate the execution of jobs and the evaluation of their tasks. MATLAB provides the MATLAB Job Scheduler (MJS) for this reason. However, the use of the MJS is optional; third-party schedulers, such as Microsoft Windows HPC Server and IBM Platform LSF, can also be used [13].

The MJS can be run on any machine on the network. The MJS runs jobs in the order in which they are submitted, unless any job in its queue is promoted, demoted, canceled or deleted. The MJS distributes the tasks to MATLAB workers, and the MATLAB workers execute the tasks and return the results to the MJS. The MJS starts running a new job only when all tasks of a running job have been assigned to

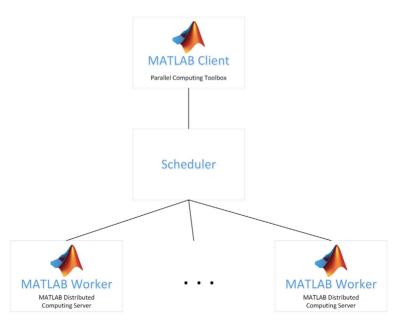


FIG. 3.16

Running a job on a computer cluster.

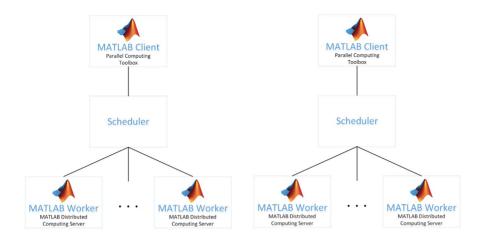


FIG. 3.17

Multiple MATLAB clients and MJSs.

MATLAB workers. In an independent job, the MATLAB workers evaluate tasks one at a time as available, perhaps simultaneously, perhaps not. In a communicating job, the MATLAB workers evaluate tasks simultaneously. Finally, the MJS returns all the results of all the tasks of a job to the MATLAB client.

In a large network, several MATLAB clients and MJSs may be used. Each client can create, run, and access jobs on any MJS, but a MATLAB worker is registered with and dedicated to only one MJS at a time (Fig. 3.17).

A cluster profile can be created either automatically (by discovering a MATLAB Distributed Computing Server cluster on the network) or manually. To create a cluster profile manually, on the *Home* tab in the *Environment* section, click $Parallel \rightarrow Manage\ Cluster\ Profiles$. The following information should be given to create a cluster profile:

- Description: the description of the cluster profile.
- Host: the hostname of the machine where the MJS is running (required).
- MJSName: the name of the MJS (required if multiple MJSs exist on the host machine).
- Username: the username for MJS access (required if the MJS uses security level 1–3).
- LicenseNumber: the license number.
- AutoAttachFiles: if code files will automatically be sent to the cluster.
- AttachedFiles: the files and folders to copy from the MATLAB client to cluster nodes.
- AdditionalPaths: the folders to add to the MATLAB workers' search path.
- NumWorkersRange: the range of number of MATLAB workers to run a job.

- CaptureDiary: if the Command Window output will be returned.
- RestartWorker: if the MATLAB workers will be restarted before a job will be executed.
- MaximumRetries: the maximum number of times that a job will try to run again if it fails.
- DefaultTimeout: the master timeout in seconds when a job or a task timeout is not defined.
- JobTimeout: the job timeout in seconds.
- TaskTimeout: the task timeout in seconds.
- JobFinishedFcn: the function that runs on the client when a job reaches the finished state.
- JobRunningFcn: the function that runs on the client when a job reaches the running state.
- JobQueuedFcn: the function that runs on the client when a job reaches the queued state.
- TaskFinishedFcn: the function that runs on the client when a task reaches the finished state.
- TaskRunningFcn: the function that runs on the client when a task reaches the running state.

When a job is created and executed, it progresses through the following stages:

- Pending: the job's first state is pending. This is when a job is defined by adding tasks to it.
- Queued: when a job is submitted, the MJS places the job in a queue.
- Running: when a job reaches the top of the queue, the MJS distributes the job's tasks to MATLAB workers for evaluation.
- Finished: when all of a job's tasks have been evaluated, the job is moved to the finished state.
- Failed: if a third-party scheduler is used instead of the MJS, a job might fail if the scheduler encounters an error while attempting to execute its commands or access necessary files.
- Deleted: when a job's data has been removed from its data location or from the MJS, the state of the job on the MATLAB client is deleted.

As already mentioned, two type of jobs can be created:

• **Independent job**: an independent job is one whose tasks do not directly communicate with each other, that is, its tasks are independent of each other. The tasks do not need to run simultaneously and a MATLAB worker might run several tasks of the same job in succession. The following example creates (*createJob* function) and executes (*submit* function) an independent job with three simple tasks (the calculation of a vector's maximum and minimum element and the calculation of the sum of a vector's elements). Finally, the results from all

tasks of the job are collected (fetchOutputs function) after the job is finished (wait function).

```
>> parallel.defaultClusterProfile('local');
>> c = parcluster();
>> j = createJob(c);
>> createTask(j, @max, 1, {rand(1, 10)});
>> createTask(j, @min, 1, {rand(1, 10)});
>> createTask(j, @sum, 1, {rand(1, 10)});
>> submit(j);
>> wait(j)
>> results = fetchOutputs(j)
```

• Communicating job: a communicating job is one in which the MATLAB workers can communicate with each other during the evaluation of their tasks. A communicating job consists of only a single task that runs simultaneously on several MATLAB workers, usually with different data. The task is duplicated on each MATLAB worker, so each MATLAB worker can perform the task on a different set of data or on a particular segment of a large data set. The following example creates (createCommunicatingJob function) and executes (submit function) a communicating job with a simple task (the calculation of the sum of a magic square). The task is defined in totalSum function which is attached to all MATLAB workers. The first MATLAB worker creates and sends to all other MATLAB workers a magic square (labBroadcast function), each of which calculates the sum of one column of the matrix. All of these column sums are combined to calculate the total sum of the elements of the original magic square (gplus function). Finally, the results from all tasks of the job are collected (fetchOutputs function) after the job is finished (wait function).

```
function total sum = totalSum
   if labindex == 1
        A = labBroadcast(1, magic(numlabs));
   else
       A = labBroadcast(1);
   colSum = sum(A(:, labindex));
   total sum = qplus(colSum);
end
>> parallel.defaultClusterProfile('local');
>> c = parcluster();
>> j = createCommunicatingJob(c, 'Type', 'SPMD');
>> j.AttachedFiles = {'totalSum.m'};
>> j.NumWorkersRange = 8;
>> createTask(j, @totalSum, 1, {})
>> submit(j);
>> wait(j)
>> results = fetchOutputs(j)
```

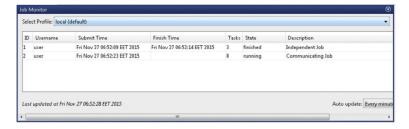


FIG. 3.18

Job monitor.

The jobs can be monitored using the Job Monitor by clicking $Parallel \rightarrow Monitor$ *Jobs* on the *Home* tab in the *Environment* section (Fig. 3.18).

3.8 CHAPTER REVIEW

This chapter presented the key features of MATLAB's Parallel Computing Toolbox. The Parallel Computing Toolbox allows users to solve computationally- and data-intensive problems using multicore and multiprocessor computers, computer clusters, and GPUs. The *parfor-loop* and the *spmd* statement, which enable users to parallelize their code, have been presented in detail. Moreover, the *pmode* functionality, where you can work interactively with a communicating job running simultaneously on multiple MATLAB workers, was also introduced. Finally, the combination of the Parallel Computing Toolbox with the Distributed Computing Server has been described to show how codes can be executed on computer clusters, grids, and clouds.

To sum up, the Parallel Computing Toolbox offers many features to execute task- and data-parallel applications on a single core, a multicore processor, or a computer cluster. In general, Parallel Computing Toolbox can be used to improve the performance of the following types of applications: (i) applications that involve repetitive segments of code, (ii) applications that run a series of independent tasks, (iii) applications evaluating the same code on multiple data sets, and (iv) applications involving data that is too large for your computer's memory.