Exercise 1

Generate n observations from a $N(\theta, \sigma^2)$ distribution.

x = rnorm(n, mean = ..., sd = ...)

sd, not var!!!

$\hat{\theta}_1 =$ mean(x)

$\hat{\theta}_2 =$ median(x)

$\hat{\theta}_3 =$ sum(w * x)

$w = (w_1, \dots, w_n)$: $\sum_{i=1}^{n} w_i = 1$

$0 \le w_i \le 1$

$$\hat{\theta}_4 = \sum_{i=1}^{n} w_i X_{i:n}$$

$(X_{1:n}, \dots, X_{n:n})$ - order statistics

sort()

$$w_i = \varphi(\Phi^{-1}(\tfrac{i-1}{n})) - \varphi(\Phi^{-1}(\tfrac{i}{n}))$$

φ - pdf, Φ - cdf of $N(0,1)$

dnorm(qnorm())

$K = 10\,000$

$X_1^1 \dots X_{50}^1$	$\hat{\theta}^1$	$(\theta - \hat{\theta}^1)^2$	$\hat{\theta}^1 - \theta$
$\dots$	$\dots$	$\dots$	$\dots$
$X_1^K \dots X_{50}^K$	$\hat{\theta}^K$	$(\theta - \hat{\theta}^K)^2$	$\hat{\theta}^K - \theta$
	$\downarrow$	$\downarrow$	$\downarrow$
	var()	mean()	mean()
		$\frac{1}{K}\sum_{i=1}^{K}(\theta - \hat{\theta}^i)^2$	$\frac{1}{K}\sum_{i=1}^{K}\hat{\theta}^i - \theta$

MSE: $E(\theta - \hat{\theta})^2$ $\quad / E\xi \to \frac{1}{K}\sum_{i=1}^{K}\xi_i /$

Bias: $E\hat{\theta} - \theta$

Exercise 2

Discuss the command set. seed (1) . . .

a positive, integer number

rnorm( ), runif( ), sample( ), etc generate pseudo-random number sequences.

set. seed ( ) is used to get reproduce results. (the starting point, used in the generation of pseudo-random number sequence, is defined) seed number

Exercise 3+4

$X_1, \dots, X_n$ - random sample from logistic distribution with unknown shift parameter

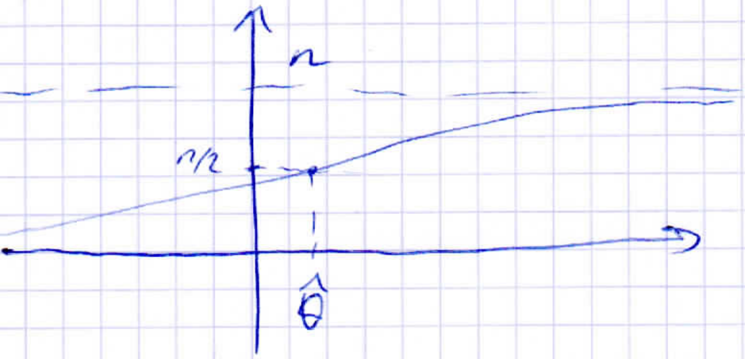
$$f(x, \theta) = \frac{e^{-(x-\theta)}}{(1+e^{-(x-\theta)})^2} \quad ; \quad x \in \mathbb{R}, \ \theta \in \mathbb{R}$$

$$L(\theta) = \prod_{i=1}^{n} f(x_i, \theta)$$

$$\ell(\theta) = \log L(\theta) = -\sum x_i + n\theta - $$
$$- 2\sum_{i=1}^{n} \log(1+e^{-(x_i-\theta)})$$

$$\frac{\partial \ell}{\partial \theta} = n - 2\sum_{i=1}^{n} \frac{e^{-(x_i-\theta)}}{1+e^{-(x_i-\theta)}} = 0.$$

$$\Rightarrow \underbrace{\sum_{i=1}^{n} \frac{e^{-(x_i-\theta)}}{1+e^{-(x_i-\theta)}}}_{g(\theta)} = \frac{n}{2}$$



$g(\theta) \to 0, \quad \theta \to -\infty$
$g(\theta) \to n, \quad \theta \to +\infty$

$g \uparrow\uparrow \Rightarrow$

$\Rightarrow \exists! \hat{\theta} : \quad g(\hat{\theta}) = \frac{n}{2}$

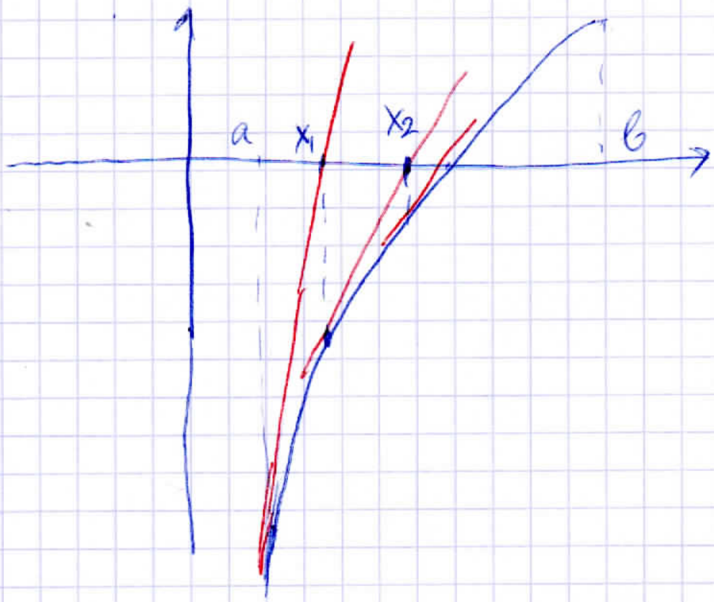
$\frac{\partial^2 \ell}{\partial \theta^2} < 0, \forall \theta \Rightarrow \hat{\theta}$ - MLE of θ

?? $\hat{\theta}$

Newton's method

$f : [a, b] \to \mathbb{R}, \qquad f \in C([a, b])$

1. $\exists!$ root on $[a, b]$
2. $f(a) f(b) < 0$, $\quad f', f''$ have constant sign.



I step

$x_0 \quad (= a, = b, \ldots)$

II step

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

. " "

kth step

$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

. " "

The stopping condition of succesive approximation method:

or

1. $f(x_k) \le \varepsilon$
2. $|x_{k+1} - x_k| < \varepsilon$

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logistic distribution $\approx$ normal distribution

$\theta_0 = \bar{x}$

$$\theta_1 = \theta_0 - \frac{g(\theta_0) - n/2}{g'(\theta_0)}$$

. . .

$$\theta_{k+1} = \theta_k - \frac{g(\theta_k) - n/2}{g'(\theta_k)},$$

. . .

$|g(\theta_k) - \frac{n}{2}| \le \varepsilon$ or $|\theta_{k+1} - \theta_k| < \varepsilon$