NUMERICAL OPTIMIZATION: ASSIGNMENT 6

DEADLINE: the lab on 2024.05.13

In this assignment, we are going to look into Newton's method of finding roots of a function. The assignment should be fairly lightweight, given that there are holidays on May 1–3.

Additionally, you can hand in Assignment 5 for 100% of the points on May 6 and May 13.

1. 1.5 points Construct an interesting test set of 4 functions, consisting of:
   (a) a polynomial;
   (b) a trigonometric function or a combination of them;
   (c) a rational function (a ratio of two polynomials);
   (d) some combination of the three above.

   Plot these functions over a chosen (interesting) interval. Make sure that there is at least one root of the function in the interval. The more interesting functions, the better. Pick the interval for the rational function so that it is continuous within it.

2. 1 point Implement a function which, for set function f, its derivative f', and the starting point x_0, executes Newton's method until such an x_n is found that $|f(x_n)| < \epsilon$, $|x_n - x_{n-1}| < \theta$, or $n == N$ (where ϵ, θ, N are the implemented function's parameters. Test it on one of your cases from Task 1. Try different values of x_0.

3. 1 point In general, it's hard to always know a derivative of a function. Try exchanging the derivative with an approximation $f'(x) \approx (f(x+h) - f(x))/h$. Experiment for all four functions from Task 1. Try different values of x_0.

4. 1.5 points Try a different approximation of a derivative: $f'(x) \approx (f(x+h) - f(x-h))/(2h)$. Compare your results with the previous one. Try different values of x_0.