1 Начальные данные:

1.1 Ковариантная метрика:

$$\begin{bmatrix} -1+\frac{r_0}{r} & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{r_0}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2(\theta) \end{bmatrix}$$

2 Базовая геометрия:

2.1 Контравариантная метрика:

$$\begin{bmatrix} -\frac{r}{r-r_0} & 0 & 0 & 0 \\ 0 & \frac{r-r_0}{r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\sin^2(\theta)} \end{bmatrix}$$

2.2 Символы Кристоффеля II рода:

$$\Gamma^t_{tt} = \Gamma^t_{tt} = 0$$

$$\Gamma^t_{rt} = \Gamma^t_{tr} = \frac{r_0}{2r(r-r_0)}$$

$$\Gamma^t_{rr} = \Gamma^t_{rr} = 0$$

$$\Gamma^t_{\theta t} = \Gamma^t_{t\theta} = 0$$

$$\Gamma^t_{\theta r} = \Gamma^t_{r\theta} = 0$$

$$\Gamma^t_{\theta\theta} = \Gamma^t_{\theta\theta} = 0$$

$$\Gamma^t_{\phi t} = \Gamma^t_{t\phi} = 0$$

$$\Gamma^t_{\phi r} = \Gamma^t_{r\phi} = 0$$

$$\Gamma^t_{\phi\theta} = \Gamma^t_{\theta\phi} = 0$$

$$\Gamma^t_{\phi\phi} = \Gamma^t_{\phi\phi} = 0$$

$$\Gamma^r_{tt} = \Gamma^r_{tt} = \frac{r_0(r-r_0)}{2r^3}$$

$$\Gamma^r_{rt} = \Gamma^r_{tr} = 0$$

$$\Gamma^r_{rr} = \Gamma^r_{rr} = -\frac{r_0}{2r(r-r_0)}$$

$$\Gamma^r_{\theta t} = \Gamma^r_{t\theta} = 0$$

$$\Gamma^r_{\theta r} = \Gamma^r_{r\theta} = 0$$

$$\Gamma^r_{\theta\theta} = \Gamma^r_{\theta\theta} = -r + r_0$$

$$\Gamma^r_{\phi t} = \Gamma^r_{t\phi} = 0$$

$$\Gamma^r_{\phi r} = \Gamma^r_{r\phi} = 0$$

$$\Gamma^r_{\phi\theta} = \Gamma^r_{\theta\phi} = 0$$

$$\Gamma^{r}_{\phi\phi} = \Gamma^{r}_{\phi\phi} = (-r + r_0)\sin^2(\theta)$$
$$\Gamma^{\theta}_{tt} = \Gamma^{\theta}_{tt} = 0$$
$$\Gamma^{\theta}_{rt} = \Gamma^{\theta}_{tr} = 0$$
$$\Gamma^{\theta}_{rr} = \Gamma^{\theta}_{rr} = 0$$
$$\Gamma^{\theta}_{\theta t} = \Gamma^{\theta}_{t\theta} = 0$$
$$\Gamma^{\theta}_{\theta r} = \Gamma^{\theta}_{r\theta} = \frac{1}{r}$$
$$\Gamma^{\theta}_{\theta\theta} = \Gamma^{\theta}_{\theta\theta} = 0$$
$$\Gamma^{\theta}_{\phi t} = \Gamma^{\theta}_{t\phi} = 0$$
$$\Gamma^{\theta}_{\phi r} = \Gamma^{\theta}_{r\phi} = 0$$
$$\Gamma^{\theta}_{\phi\theta} = \Gamma^{\theta}_{\theta\phi} = 0$$
$$\Gamma^{\theta}_{\phi\phi} = \Gamma^{\theta}_{\phi\phi} = -\frac{\sin(2\theta)}{2}$$
$$\Gamma^{\phi}_{tt} = \Gamma^{\phi}_{tt} = 0$$
$$\Gamma^{\phi}_{rt} = \Gamma^{\phi}_{tr} = 0$$
$$\Gamma^{\phi}_{rr} = \Gamma^{\phi}_{rr} = 0$$
$$\Gamma^{\phi}_{\theta t} = \Gamma^{\phi}_{t\theta} = 0$$
$$\Gamma^{\phi}_{\theta r} = \Gamma^{\phi}_{r\theta} = 0$$
$$\Gamma^{\phi}_{\theta\theta} = \Gamma^{\phi}_{\theta\theta} = 0$$
$$\Gamma^{\phi}_{\phi t} = \Gamma^{\phi}_{t\phi} = 0$$
$$\Gamma^{\phi}_{\phi r} = \Gamma^{\phi}_{r\phi} = \frac{1}{r}$$
$$\Gamma^{\phi}_{\phi\theta} = \Gamma^{\phi}_{\theta\phi} = \frac{1}{\tan(\theta)}$$
$$\Gamma^{\phi}_{\phi\phi} = \Gamma^{\phi}_{\phi\phi} = 0$$

2.3 Тензор кривизны Римана (4-ковариантный):

С учётом симметрий:

$$R_{abcd} = -R_{bacd} = -R_{abdc} = R_{cdab}$$

и первого тождества Бианки:

$$R_{abcd} + R_{acdb} + R_{adbc} = 0$$

его независимые компоненты имеют вид:

$$R_{trtr} = -\frac{r_0}{r^3}$$
$$R_{tr t\theta} = 0$$
$$R_{tr t\phi} = 0$$
$$R_{trr\theta} = 0$$
$$R_{trr\phi} = 0$$
$$R_{tr\theta\phi} = 0$$
$$R_{t\theta t\theta} = \frac{r_0\left(r - r_0\right)}{2r^2}$$
$$R_{t\theta t\phi} = 0$$
$$R_{t\theta r\theta} = 0$$
$$R_{t\theta r\phi} = 0$$
$$R_{t\theta\theta\phi} = 0$$
$$R_{t\phi t\phi} = \frac{r_0\left(r - r_0\right)\sin^2\left(\theta\right)}{2r^2}$$
$$R_{t\phi r\phi} = 0$$
$$R_{t\phi\theta\phi} = 0$$
$$R_{r\theta r\theta} = -\frac{r_0}{2r - 2r_0}$$
$$R_{r\theta r\phi} = 0$$
$$R_{r\theta\theta\phi} = 0$$
$$R_{r\phi r\phi} = -\frac{r_0\sin^2\left(\theta\right)}{2r - 2r_0}$$
$$R_{r\phi\theta\phi} = 0$$
$$R_{\theta\phi\theta\phi} = rr_0\sin^2\left(\theta\right)$$

2.4 Скалярный квадрат тензора Римана:

$$R_{abcd}R^{abcd} = \frac{12r_0^2}{r^6}$$

2.5 Тензор Риччи (ковариантный):

$$R_{tt} = R_{tt} = 0$$
$$R_{rt} = R_{tr} = 0$$
$$R_{rr} = R_{rr} = 0$$
$$R_{\theta t} = R_{t\theta} = 0$$
$$R_{\theta r} = R_{r\theta} = 0$$
$$R_{\theta\theta} = R_{\theta\theta} = 0$$
$$R_{\phi t} = R_{t\phi} = 0$$
$$R_{\phi r} = R_{r\phi} = 0$$
$$R_{\phi\theta} = R_{\theta\phi} = 0$$
$$R_{\phi\phi} = R_{\phi\phi} = 0$$

2.6 Скалярный квадрат тензора Риччи:

$$R_{ab}R^{ab} = 0$$

2.7 Скалярная кривизна:

$$R = 0$$

2.8 Тензор Эйнштейна:

$$G_{tt} = G_{tt} = 0$$
$$G_{rt} = G_{tr} = 0$$
$$G_{rr} = G_{rr} = 0$$
$$G_{\theta t} = G_{t\theta} = 0$$
$$G_{\theta r} = G_{r\theta} = 0$$
$$G_{\theta\theta} = G_{\theta\theta} = 0$$
$$G_{\phi t} = G_{t\phi} = 0$$
$$G_{\phi r} = G_{r\phi} = 0$$
$$G_{\phi\theta} = G_{\theta\phi} = 0$$
$$G_{\phi\phi} = G_{\phi\phi} = 0$$