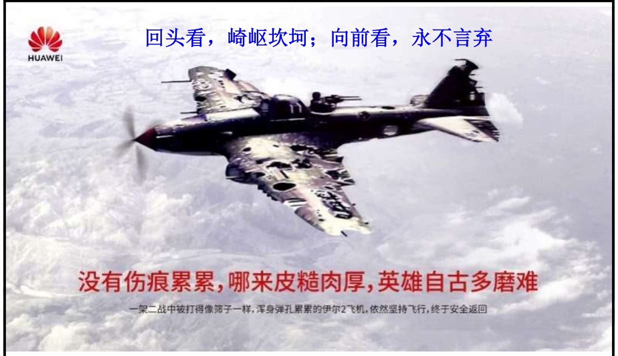

最可怕的敌人，就是没有坚强的信念

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Institute of Systems Science and Intelligent Control Technology 系统科学与智能控制技术研究所

鲁棒控制：建模、跟踪、抗扰、容错

周克敏
山东科技大学
电气与自动化工程学院
2020年5月19日
（第五讲）

提纲

1. 古典控制基础
2. 鲁棒控制理论基础（继续）
3. 鲁棒控制在迟滞系统中应用
4. 高精度跟踪与抗扰控制
5. 故障诊断与容错控制
6. 教材2-16章

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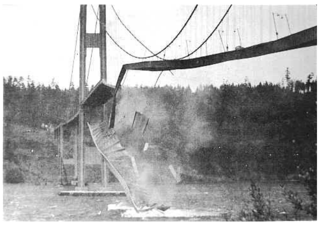
从频率响应看H_∞范数

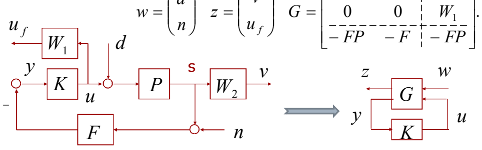


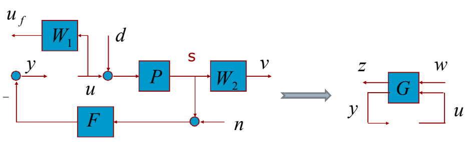
The Broughton Suspension Bridge spanned the River Irwell (UK) collapsed due to troops marching over the bridge in step.

抗扰

- 反馈系统:

$$w=\begin{pmatrix} d \\ n \end{pmatrix} \quad z=\begin{pmatrix} v \\ u_f \end{pmatrix} \quad G=\left[\begin{array}{cc:c} W_2P & 0 & W_2P \\ 0 & 0 & W_1 \\ \hdashline -FP & -F & -FP \end{array}\right].$$





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G(s) 的状态空间表示

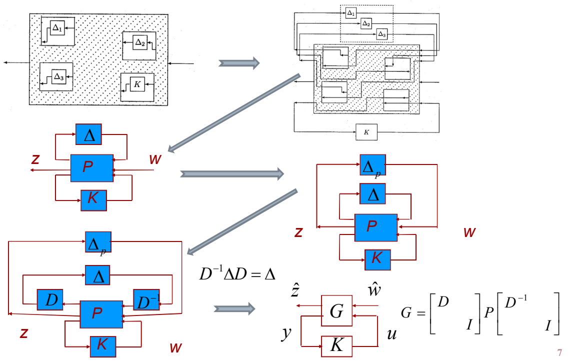
Never try to realize G(s) directly!

$$P:\begin{cases} \dot{x}_p = A_p x_p + B_p(d+u) \\ s = C_p x_p + D_p(d+u) \end{cases} \quad W_2:\begin{cases} \dot{x}_2 = A_2 x_2 + B_2 s = B_2C_px_p + A_2x_2 + B_2D_p(d+u) \\ v = C_2x_2 + D_2 s = D_2C_px_p + C_2x_2 + D_2D_p(d+u) \end{cases}$$

$$F:\begin{cases} \dot{x}_f = A_f x_f + B_f(n+s) = B_fC_px_p + A_fx_f + B_fD_pd + B_fn + B_fD_pu \\ -y = C_fx_f + D_f(n+s) = D_fC_px_p + C_fx_f + D_fD_pd + D_fn + D_fD_pu \end{cases} \quad W_1:\begin{cases} \dot{x}_1 = A_1x_1 + B_1u \\ u_f = C_1x_1 + D_1u \end{cases}$$

$$G(s)=\left[\begin{array}{cccc|cc:c} A_p & 0 & 0 & 0 & B_p & 0 & B_p \\ B_2C_p & A_2 & 0 & 0 & B_2D_p & 0 & B_2D_p \\ B_fC_p & 0 & A_f & 0 & B_fD_p & B_f & B_fD_p \\ 0 & 0 & 0 & A_1 & 0 & 0 & B_1 \\ \hline D_2C_p & C_2 & 0 & 0 & D_2D_p & 0 & D_2D_p \\ 0 & 0 & 0 & C_1 & 0 & 0 & D_1 \\ \hdashline -D_fC_p & 0 & -C_f & 0 & -D_fD_p & -D_f & -D_fD_p \end{array}\right]$$

一般综合框架



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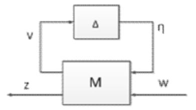
不确定系统和鲁棒稳定性



$z = F_u(M(s),\Delta)w$

$\Delta \in \mathbf{\Delta}$满足$\|\Delta\|_\infty \le 1$

$M_{22}(s)$为标称模型

$$F_u(M(s),\Delta) \triangleq M_{22}(s) + M_{21}(s)\Delta(I - M_{11}(s)\Delta)^{-1}M_{12}(s)$$

鲁棒稳定性： 当且仅当$(I - M_{11}(s)\Delta)^{-1}$
对所有$\Delta \in \mathbf{\Delta}$满足$\|\Delta\|_\infty \le 1$都稳定

哈里托诺夫定理

区间多项式集合 **(Kharitonov, 1978)**

$P = \{p(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n\text{:}\ a_i \in [\underline{a}_i, \overline{a}_i]\}$

稳定当且仅当以下四个多项式稳定

- $a_1(s) = \underline{a}_0 s^n + \underline{a}_1 s^{n-1} + \overline{a}_2 s^{n-2} + \overline{a}_3 s^{n-3} + \underline{a}_4 s^{n-4} + \cdots$
- $a_2(s) = \underline{a}_0 s^n + \overline{a}_1 s^{n-1} + \overline{a}_2 s^{n-2} + \underline{a}_3 s^{n-3} + \underline{a}_4 s^{n-4} + \cdots$
- $a_3(s) = \overline{a}_0 s^n + \underline{a}_1 s^{n-1} + \underline{a}_2 s^{n-2} + \overline{a}_3 s^{n-3} + \overline{a}_4 s^{n-4} + \cdots$
- $a_4(s) = \overline{a}_0 s^n + \overline{a}_1 s^{n-1} + \underline{a}_2 s^{n-2} + \underline{a}_3 s^{n-3} + \overline{a}_4 s^{n-4} + \cdots$

下下上上下下上上。。。。。。
下上上下下上上。。。。。。。
上下下上上下下。。。。。。。
上上下下上上下下。。。。。。

Robust Stability

Kharitonov Theorem

$$\mathcal{P} = \{a(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n : a_i \in [\underline{a}_i, \overline{a}_i]\}$$

is stable if and only if the following four are stable

$$a_1(s) = \underline{a}_0 s^n + \underline{a}_1 s^{n-1} + \overline{a}_2 s^{n-2} + \overline{a}_3 s^{n-3} + \underline{a}_4 s^{n-4} + \cdots$$
$$a_2(s) = \underline{a}_0 s^n + \overline{a}_1 s^{n-1} + \overline{a}_2 s^{n-2} + \underline{a}_3 s^{n-3} + \underline{a}_4 s^{n-4} + \cdots$$
$$a_3(s) = \overline{a}_0 s^n + \underline{a}_1 s^{n-1} + \underline{a}_2 s^{n-2} + \overline{a}_3 s^{n-3} + \overline{a}_4 s^{n-4} + \cdots$$
$$a_4(s) = \overline{a}_0 s^n + \overline{a}_1 s^{n-1} + \underline{a}_2 s^{n-2} + \underline{a}_3 s^{n-3} + \overline{a}_4 s^{n-4} + \cdots.$$

$$\mathcal{P} = \{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 : a_1 \in [3,4], a_2 \in [3,4], a_3 \in [2,3], a_4 \in [0.5,1]\}$$

$$a_1(s) = s^4 + 3s^3 + 4s^2 + 3s + 0.5$$
$$a_2(s) = s^4 + 4s^3 + 4s^2 + 2s + 0.5$$
$$a_3(s) = s^4 + 3s^3 + 3s^2 + 3s + 1$$
$$a_4(s) = s^4 + 4s^3 + 3s^2 + 2s + 1.$$

are all stable.

例： $p(s) = s^3 + a_1 s^2 + a_2 s + K$, $a_1 \in [2,3]$, $a_2 \in [3,4]$, $K > 0$

$p_1(s) = s^3 + 2s^2 + 4s + K$， 稳定当且仅当$K < 8$

$p_2(s) = s^3 + 3s^2 + 4s + K$， 稳定当且仅当$K < 12$

$p_3(s) = s^3 + 3s^2 + 3s + K$， 稳定当且仅当$K < 9$

$p_4(s) = s^3 + 2s^2 + 3s + K$， 稳定当且仅当$K < 6$

所以$K < 6$时，这个多项式稳定。

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Andrzej W. Olbrot

In Memoriam

Andrzej W. Olbrot

Andrzej W. Olbrot, Professor of Electrical and Computer Engineering at Wayne State University, was shot and killed on December 10, 1998, as he collected final exams. The accused murderer is a student who had failed the Ph.D. qualifying exam.

Dr. Olbrot, 52, obtained his engineering degree in 1970 from Warsaw Technical University. He completed his doctoral studies there in 1973 and joined the faculty of the Institute of Automatic Control. He defended his Habilitation in 1977 and became a Professor in the Institute in 1978. In 1988 he moved to Wayne State.

Dr. Olbrot was a dedicated, caring, and creative teacher. He was intrigued by the limitations of existing theories and techniques and was superb at finding simple examples to illustrate complex concepts.

In the '70s Dr. Olbrot's research concentrated on systems with time delays. Many of his papers contain seminal results; for example, in 1978 he originated the open-loop definitions of stabilizability and detectability, showed their connections to closed-loop concepts, and derived testable criteria for systems with general delay structure. Since 1980 his research focused on robust stability and stabilization for systems with parametric uncertainty. Dr. Olbrot is generally credited with originating a new direction in robust stability and control by showing in a workshop presentation in 1982 that interval uncertainty models can be treated with the help of Kharitonov's theorem. After Dr. Olbrot brought this result to the attention of the control community, hundreds of journal and conference papers have appeared giving extensions and refinements.

The world has lost a valuable and innovative control educator and researcher.

棱边定理

多项式集合

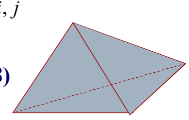
$$p(s,q)=a_0(q)s^n+\cdots+a_{n-1}(q)s+a_n(q),$$
$$q\in Q\subset R^m$$

$a_i(q)$ 是q的仿射函数，设$q^j\in Q$，$j=1,\ldots,2^m$为当q的分量分别取上界或下界时得到的多面体“顶点”。相应的特征多项式为$p(s,q^j)$，则特征多项式$p(s,q)$对所有的$q\in Q\subset R^m$都是稳定的当且仅当下列的棱边多项式

$$\lambda p(s,q^i)+(1-\lambda)p(s,q^j),\forall i,j$$

对 $\lambda\in[0,1]$都稳定。

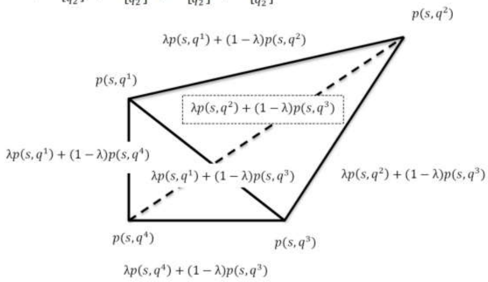
(Bartlett，Hollot, and Huang，1988)



m=2

例如，m=2, $q=\begin{bmatrix}q_1\\q_2\end{bmatrix}, q_1\in[q_1^-,q_1^+], q_2\in[q_2^-,q_2^+]$。那么我们有

$q^1=\begin{bmatrix}q_1^-\\q_2^-\end{bmatrix},\ q^2=\begin{bmatrix}q_1^-\\q_2^+\end{bmatrix},\ q^3=\begin{bmatrix}q_1^+\\q_2^-\end{bmatrix},\ q^4=\begin{bmatrix}q_1^+\\q_2^+\end{bmatrix}$

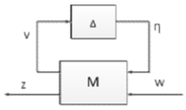


非结构实稳定半径

Δ为非结构实矩阵: $\mathbf{\Delta}=\mathbb{R}^{q_1\times p_1}$，闭环系统对所有Δ∈ **Δ**， $\bar\sigma(\Delta)\le 1$稳定, 当且仅当

$$\sup_{\omega}\inf_{r\in[0,1]}\sigma_2\left(\begin{bmatrix}\operatorname{Re}M_{11}(j\omega) & -r\operatorname{Im}M_{11}(j\omega)\\ \frac{1}{r}\operatorname{Im}M_{11}(j\omega) & \operatorname{Re}M_{11}(j\omega)\end{bmatrix}\right)<1$$

其中σ_2表示第二大奇异值。（**Qiu, Bernhardsson, Rantzer, Davison, Young, and Doyle， Automatica, 1995)**



二次稳定

$$\begin{aligned}\dot x&=Ax+B_1\eta+B_2w\\ v&=C_1x+D_{11}\eta+D_{12}w\\ z&=C_2x+D_{21}\eta+D_{22}w\\ \eta&=\Delta v\end{aligned}$$

二次稳定: 存在$P=P'>0$
使得$V(x)=x'Px$
满足 $\dot V(x)=2x'P\dot x<0$
对所有$x\neq 0$ 成立。

系统对所有非结构实矩阵$\mathbf{\Delta}=C^{q_1\times p_1}$, $\|\Delta\|_\infty\le 1$二次稳定的, 当且仅当

$$\|M_{11}(s)\|_\infty<1$$

(Popov, 1960; Khargonekar, Petersen, and Zhou, 1990)

此结论对**Δ** 时变实数, 复数, 时变（或时不变）动态系统亦成立（*与小增益定理等价*）

小μ定理

$$\Delta=\left\{\begin{matrix}diag[\phi_1 I_{n_1},\phi_2 I_{n_2},\cdots,\phi_{r_1} I_{n_{r1}},\delta_1 I_{k_1},\delta_2 I_{k_2},\cdots,\delta_{c_1} I_{k_{c1}},\Delta_1,\Delta_2,\cdots\Delta_F]:\\ \phi_i\in R,\delta_j\in C,\Delta_l\in C^{m_l x m_l}\end{matrix}\right\}$$

$$\mu_\Delta(N)=\frac{1}{\min\left\{\bar\sigma(\Delta):\Delta\in\mathbf{\Delta},\det(I-N\Delta)=0\right\}}$$

系统对结构不确定性Δ满足 $\Delta(j\omega)\in\mathbf{\Delta}$ $\|\Delta\|_\infty\le 1$稳定，当且仅当

$$\sup_{\omega\in R}\mu_\Delta(M_{11}(j\omega))<1$$

(Doyle, 1982)

μ的上下界

定义

$$\Phi=\left\{\Delta\in\mathbf{\Delta}:\phi_i\in[-1,1],|\delta_j|=1,\Delta_l^*\Delta_l=I_{m_l}\right\}$$

$$\Gamma=\left\{\begin{matrix}diag[\tilde D_1,\cdots,\tilde D_{r_1},D_1,\cdots,D_{c_1},d_1 I_{m_1},\cdots d_{F-1}I_{m_{F-1}},I_{m_F}]:\\ \tilde D_i\in C^{n_i x n_i},\tilde D_i=\tilde D_i^*>0,D_j\in C^{k_j x k_j},D_j=D_j^*>0,d_l\in R,d_l>0\end{matrix}\right\}$$

$$\Omega=\left\{diag[G_1,\cdots,G_{r_1},0,\cdots,0]:G_i=G_i^*\in C^{n_i x n_i}\right\}$$

$$\max_{Q\in\Phi}\rho_R(QN)=\mu_{\mathbf{\Delta}}(N)\le\inf_{D\in\Gamma,G\in\Omega}\min\left\{\beta:N^*DN+j(GN-N^*G)-\beta^2D\le 0\right\}$$
$$\le\inf_{D\in\Gamma}\bar\sigma(DND^{-1})$$

当**N**是一个传递函数矩阵时，则有

$$\sup_{\omega\in R}\mu_\Delta(N(j\omega))\le\inf_{D(s),D^{-1}(s)\in H_\infty,D(j\omega)\in\Gamma}\|D(s)N(s)D^{-1}(s)\|_\infty$$

μ的上下界

$$\Delta = \begin{Bmatrix} diag[\phi_1 I_{n_1}, \phi_2 I_{n_2}, \cdots, \phi_{r_1} I_{n_{r1}}, \delta_1 I_{k_1}, \delta_2 I_{k_2}, \cdots, \delta_{c_1} I_{k_{c1}}, \Delta_1, \Delta_2, \cdots \Delta_F]: \\ \phi_i \in R, \delta_j \in C, \Delta_l \in C^{m_l x m_l} \end{Bmatrix}$$

$$\{\delta I:\ \delta \in C\} \subseteq \mathbf{\Delta} \subseteq \{\Delta:\ \Delta \in C^{pxm}\}$$

$$\rho(N) \le \mu_{\Delta}(N) \le \bar{\sigma}(N)$$

$$\min\left\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - N\Delta) = 0\right\}$$

$$= \min\left\{\alpha : \Delta \in \mathbf{\Delta}, \det(I - \alpha N\Delta) = 0, \bar{\sigma}(\Delta) \le 1\right\}$$

$$= \{\max_{\bar{\sigma}(\Delta)\le 1} \rho_R(N\Delta)\}^{-1} = \max_{Q\in\Phi} \rho_R(NQ)\}^{-1}$$

μ的上下界

$$\Delta = \begin{Bmatrix} diag[\phi_1 I_{n_1}, \phi_2 I_{n_2}, \cdots, \phi_{r_1} I_{n_{r1}}, \delta_1 I_{k_1}, \delta_2 I_{k_2}, \cdots, \delta_{c_1} I_{k_{c1}}, \Delta_1, \Delta_2, \cdots \Delta_F]: \\ \phi_i \in R, \delta_j \in C, \Delta_l \in C^{m_l x m_l} \end{Bmatrix}$$

$$\min\left\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - N\Delta) = 0\right\}$$

$$= \min\left\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - ND^{-1}\Delta D) = 0\right\}$$

$$= \min\left\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - DND^{-1}\Delta) = 0\right\}$$

$$\ge \{\bar{\sigma}(DND^{-1})\}^{-1}$$

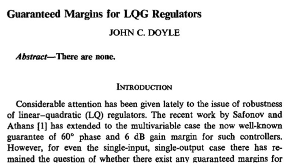
$$\mu_{\Delta}(N) \le \inf_{D\in\Gamma} \bar{\sigma}(DND^{-1})$$

μ和k_m的故事

- 70年代末至90年代中期鲁棒控制发展的黄金时代，Micheal G. Safonov 和John C. Doyle都是这个时代的代表人物。
- Safonov和他的导师Michael Athans是一群最早研究多变量系统稳定裕度并在1980引入了对角扰动稳定裕度k_m
- John C. Doyle于1977从MIT的电气工程拿到学士和硕士，于1984年从加州大学伯克利拿到数学博士，师从数学家Donald Erik Sarason。是80年代各个控制会议最受关注的学者，经常在国际学术大会上和Michael Athans, George Zames, Isaac Horwitz等发生争论。有他参与的会场基本上拥挤不堪，数百人是常事。他是个运动狂热者，保持有数项世界纪录。他在MIT做学生时代发表的关于LQG稳定裕度文章（1978年）被广泛地引用。

μ和k_m的故事

- Doyle在1982年文章中正式引入了一个多变量稳定裕度的测量，μ。它是矩阵奇异值的一个推广，叫做结构奇异值（structured singular value）。而这个μ和k_m是相关的。实际上，μ是k_m的倒数：$\mu = \frac{1}{k_m}$。
- 现在μ是鲁棒控制领域众所周知的，而k_m已经被大众所遗忘。



Guaranteed Margins for LQG Regulators

JOHN C. DOYLE

Abstract—There are none.

INTRODUCTION

Considerable attention has been given lately to the issue of robustness of linear–quadratic (LQ) regulators. The recent work by Safonov and Athans [1] has extended to the multivariable case the now well-known guarantee of 60° phase and 6 dB gain margin for such controllers. However, for even the single-input, single-output case there has remained the question of whether there exist any guaranteed margins for