Institute of Systems Science and Intelligent Control Technology 系统科学与智能控制技术研究所

鲁棒控制：建模、跟踪、抗扰、容错

周克敏
山东科技大学
电气与自动化工程学院
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（第四讲）

提纲

1. 古典控制基础
2. 鲁棒控制理论基础（继续）
3. 鲁棒控制在迟滞系统中应用
4. 高精度跟踪与抗扰控制
5. 故障诊断与容错控制
6. 教材2-16章

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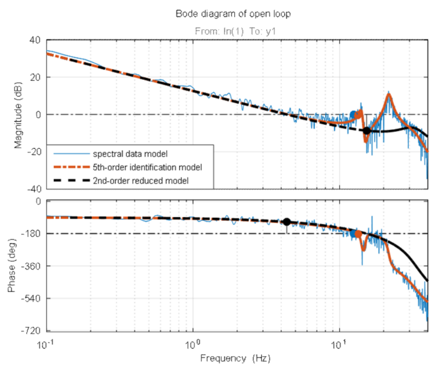
建模的重要性



此图来自华南理工闻成/苏为洲教授

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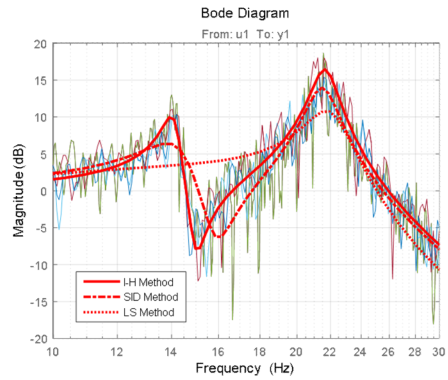
建模的重要性



此图来自华南理工闻成/苏为洲教授

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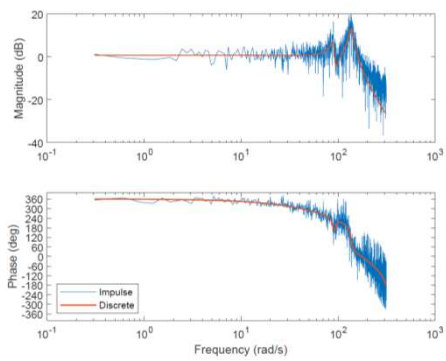
建模的重要性



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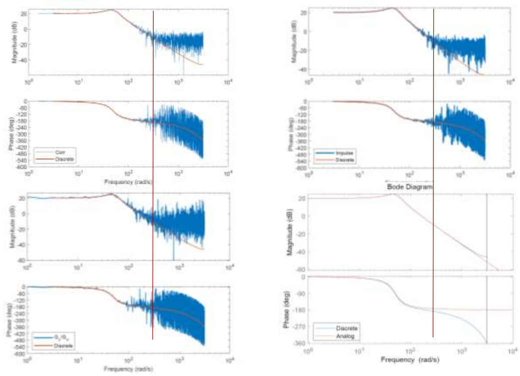
建模的重要性



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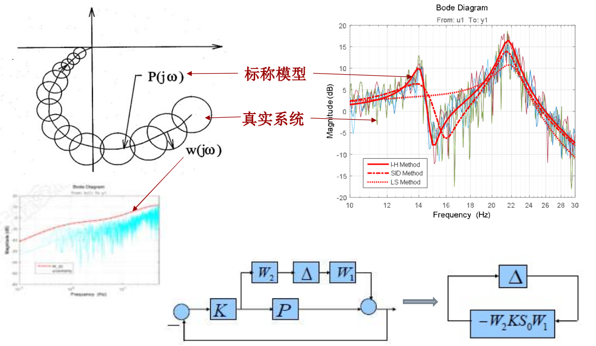
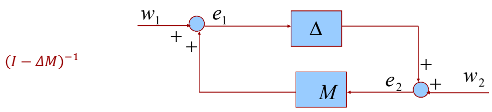
模型不确定性

$P_\Delta(s) = P(s) + w(s)\Delta(s), \quad \bar{\sigma}[\Delta(j\omega)] < 1, \forall\omega$



什么是鲁棒稳定性?

- 假设M（给定）和Δ都稳定. 什么是最小扰动Δ使系统不稳定?



所有传递函数:

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} (I - M\Delta)^{-1} & M(I - M\Delta)^{-1} \\ \Delta(I - M\Delta)^{-1} & (I - \Delta M)^{-1} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$(I - \Delta M)^{-1} = I + (I - \Delta M)^{-1}\Delta M = I + \Delta(I - M\Delta)^{-1}M$$

什么是鲁棒稳定性?

- 从标量系统说起

As δ goes from $0 \to \dfrac{1}{m}$

$$\begin{bmatrix} \dfrac{1}{1-m\delta} & \dfrac{m}{1-m\delta} \\ \dfrac{\delta}{1-m\delta} & \dfrac{1}{1-m\delta} \end{bmatrix} \to \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

- 因此重要的是保持

$$1 - m\delta \neq 0 \Leftrightarrow |\delta| < \frac{1}{|m|}$$

奇异值分解

- 对任何 $A \in F^{m\times n}$ 存在酉矩阵

$U=[u_1, u_2, \ldots, u_m] \in F^{m\times m}$, $V=[v_1, v_2, \ldots, v_n] \in F^{n\times n}$

使得 $A=U\Sigma V^*$ 其中 $\Sigma = \begin{bmatrix} \Sigma_1 & 0 \end{bmatrix}$ 或者 $\Sigma = \begin{bmatrix} \Sigma_1 \\ 0 \end{bmatrix}$

$\Sigma_1=\text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_r\}$ and

$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \geq 0$, $r=\min\{m, n\}$.

$\bar{\sigma}(A) = \sigma_{max}(A) = \sigma_1(A) =$ *A的最大奇异值*;

$\underline{\sigma}(A) = \sigma_{min}(A) = \sigma_r(A) =$ *A的最小奇异值*.

奇异值分解（续）

$$A = \sum_{i=1}^{r} \sigma_i u_i v_i^* = U_r \Sigma_r V_r^*$$

其中 $U_r=[u_1, u_2, \ldots, u_r]$, $V_r=[v_1, v_2, \ldots, v_r]$,

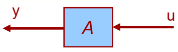
and $\Sigma_r=\text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_r\}$

Note that $u_j^* u_i = \delta_{ij}$ *and* $v_j^* v_i = \delta_{ij}$

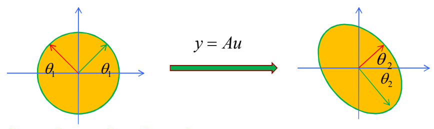
$Av_i = \sigma_i u_i$, $A^*u_i = \sigma_i v_i$ *(singular value equations)*

$A^*Av_i = \sigma_i^2 v_i$, $AA^*u_i = \sigma_i^2 u_i$ (eigenvalue equations)

奇异值分解与增益



$$A = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & \\ & \sigma_2 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix}.$$



$x_1 = v_1 = \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix}$, $\sigma_1 \begin{bmatrix} \cos\theta_2 \\ -\sin\theta_2 \end{bmatrix} = A \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix}$, $x_2 = v_2 = \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \end{bmatrix}$, $\sigma_2 \begin{bmatrix} \sin\theta_2 \\ \cos\theta_2 \end{bmatrix} = A \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \end{bmatrix}$

$$\|A\|_2 = \bar{\sigma}(A) = \max_{u \neq 0} \frac{\|Au\|}{\|u\|} \qquad \underline{\sigma}(A) = \min_{u \neq 0} \frac{\|Au\|}{\|u\|}.$$

SVD for 图像压缩

```
% function IMAGESVD
clear
clc
FileName = input('Enter the name of your image(put it inside your current directory,sample.png for example):','s');
M = double(imread(FileName));
image_size = size(M);
%% RGB Decomposition
MR = M(:,:,1); % Red Matrix of Image
MG = M(:,:,2); % Green Matrix of Image
MB = M(:,:,3); % Blue Matrix of Image

display('Size of the corresponding Matrix is:'),image_size,
n = input('Enter Number of singular values that you want:'); %%% Number of Desired Singular Values

%% Singular Value Decomposition (M = USV')
[UR,SR,VR] = svd(MR); % SVD for MR
[UG,SG,VG] = svd(MG); % SVD for MG
[UB,SB,VB] = svd(MB); % SVD for MB
%% Image Reconstruction with n Largest Singular Values
MR1 = UR(:,1:n)*SR(1:n,1:n)*VR(:,1:n)';
MG1 = UG(:,1:n)*SG(1:n,1:n)*VG(:,1:n)';
MB1 = UB(:,1:n)*SB(1:n,1:n)*VB(:,1:n)';
%% Reconstruction of RGB Matrix
M1(:,:,1) = MR1;
M1(:,:,2) = MG1;
M1(:,:,3) = MB1;
%% Displaying Reconstructed Image
imshow(uint8(M1)),title('Reconstructed Image')
```

SVD for 图像压缩

image_size = 512 512 3

Red: MR=UR•SR•VR*
Green: MG=UG•SG•VG*
Blue: MB=UB•SB•VB*

512x512x3=786,432

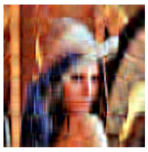
Each has 512 singular values. The first 50 singular values of MR are shown below:

SR=1.0e+04 *

7.6237	1.8165	1.3721	1.1314	0.9439
0.9053	0.6565	0.6132	0.4835	0.4605
0.4034	0.3777	0.3515	0.3328	0.3140
0.2964	0.2890	0.2803	0.2661	0.2522
0.2465	0.2236	0.2023	0.1985	0.1867
0.1801	0.1749	0.1677	0.1607	0.1517
0.1496	0.1458	0.1422	0.1361	0.1336
0.1312	0.1300	0.1243	0.1216	0.1178
0.1148	0.1132	0.1086	0.1076	0.1057
0.1034	0.1016	0.0999	0.0958	0.0926

50x(512+512+1)x3=153,750

Original n=512



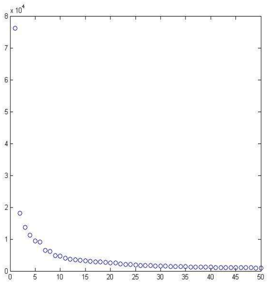
N=10

N=50

N=100

奇异值 (Red)

- 你可以用奇异值来决定压缩大小



鲁棒性概念

- Q: 对任何非奇异 $M\in F^{n\times n}$, 什么是最小扰动δ使得

  $I - M\delta$ 奇异?

  A: $\|\delta\|=1/|\lambda_1|$, $\delta=1/\lambda_1$

  $I - M\delta = T\, diag(0, \lambda_2/\lambda_1, \ldots, \lambda_n/\lambda_1)\, T^{-1}$

- Q: 对任何 $M\in F^{n\times m}$, 什么是最小扰动Δ 使得

  $I - M\Delta$ 奇异? $M=\sum_{i=1}^{r}\sigma_i u_i v_i^*$

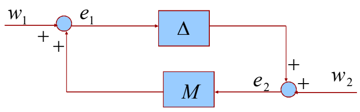
  A: $\|\Delta\|=1/\sigma_1$, $\Delta=(1/\sigma_1)v_1 u_1^*$

$$I - M\Delta = I - \left(\sum_{i=1}^{r}\sigma_i u_i v_i^*\right)\frac{1}{\sigma_1}v_1 u_1^* = I - u_1 u_1^*$$

$$=[u_1,\ldots,u_n]diag(0,1,\ldots,1)[u_1,\ldots,u_n]^*$$

反馈系统鲁棒性

- 如M 和 Δ 都稳定. 什么是最小扰动Δ 使系统不稳定?

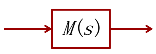


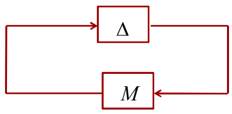
稳定性: *I-MΔ* 在闭右半平面非奇异.

The smallest destabilizing $\Delta\in RH_\infty$

$$\|\Delta\|_\infty = \frac{1}{\sup_\omega \sigma_1(M(j\omega))} = \frac{1}{\|M\|_\infty}$$

小增益定理



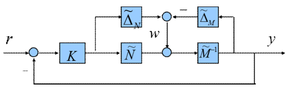


设 $M\in(RH_\infty)^{p\times q}$ （稳定传递函数）

$\|M\|_\infty<1$ ⟺ 闭环系统对所有 $\Delta\in RH_\infty$, $\|\Delta\|_\infty\le 1$ 稳定

关键是回路增益小于1: $\|M\Delta\|_\infty<1$

互质分式不确定性



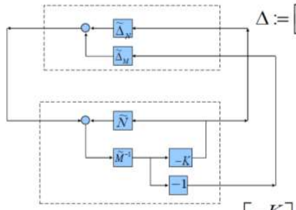
- Let $P=\tilde{M}^{-1}\tilde{N}$ be stable left coprime factorization and let K stabilize P. Suppose

$$\Pi=(\tilde{M}+\tilde{\Delta}_M)^{-1}(\tilde{N}+\tilde{\Delta}_N),\quad \Delta:=\begin{bmatrix}\tilde{\Delta}_N & \tilde{\Delta}_M\end{bmatrix}$$

with $\tilde{\Delta}_M,\tilde{\Delta}_N\in RH_\infty$. Then the closed-loop system is well-posed and internally stable for all $\|\Delta\|_\infty<1$ if and only if

$$\left\|\begin{bmatrix}K\\ I\end{bmatrix}(I+PK)^{-1}\tilde{M}^{-1}\right\|_\infty\le 1.$$

互质分式不确定性



$$\Delta:=\begin{bmatrix}\tilde{\Delta}_N & \tilde{\Delta}_M\end{bmatrix}$$

$$\begin{bmatrix}-K\\ I\end{bmatrix}(I+PK)^{-1}\tilde{M}^{-1}$$

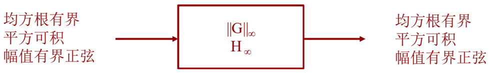
鲁棒控制的困惑

- 保守，难理解，很难设计

$$\|G\|_\infty:=\sup_{w\in L_2}\frac{\|Gw\|_2}{\|w\|_2}=\sup_{\omega\in\mathbb{R}}\bar{\sigma}[G(j\omega)]$$

$$\|w\|_2:=\sqrt{\int_0^\infty\|w(t)\|^2\,dt}$$

$$\|w\|_P:=\sqrt{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\|w(t)\|^2\,dt}$$



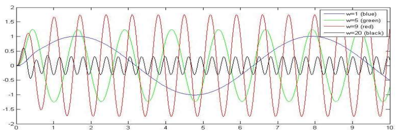
从频率响应看H_∞范数

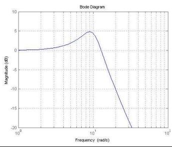
y(t)=|G(jω)|sin(ωt+∠G(jω)) *u(t)=sin ωt*

- 频率响应：正弦输入信号的稳态响应
- H_∞范数：所有正弦输入信号稳态响应的最大幅值

$$G(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\qquad \text{peak frequency } \omega_{peak}=\sqrt{1-2\xi^2}\,\omega_n=9.0554$$

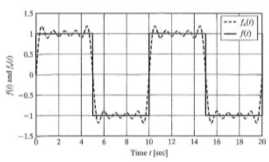
$$\omega_n=10,\xi=0.3\qquad \text{peak frequency response } M=\frac{1}{2\xi\sqrt{1-\xi^2}}=1.7471$$





为什么正弦？（阶跃，方波）

- 周期足够长的方波在一个有限时间区间可以认为是个阶跃。
- 任何一个方波可以用无穷多个离散频率的正弦函数来逼近



$$f(t)=\sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\sin\frac{(2n+1)\pi}{T}t$$

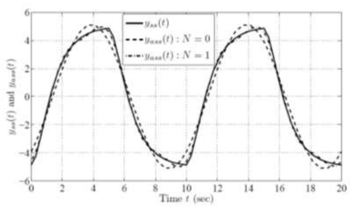
$$f_a(t)=\sum_{n=0}^{N}\frac{4}{(2n+1)\pi}\sin\frac{(2n+1)\pi}{T}t$$

方波和正弦稳态响应

$$P(s)=\frac{10}{(s+1)(s+2)}$$

$$f(t)=\sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\sin\frac{(2n+1)\pi}{T}t$$

$$f_a(t)=\sum_{n=0}^{N}\frac{4}{(2n+1)\pi}\sin\frac{(2n+1)\pi}{T}t$$



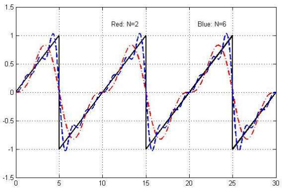
$$y(t)=\sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\left|P(j\frac{(2n+1)\pi}{T})\right|\sin(\frac{(2n+1)\pi}{T}t+\angle P(j\frac{(2n+1)\pi}{T}))$$

$$y_a(t)=\sum_{n=0}^{N}\frac{4}{(2n+1)\pi}\left|P(j\frac{(2n+1)\pi}{T})\right|\sin(\frac{(2n+1)\pi}{T}t+\angle P(j\frac{(2n+1)\pi}{T}))$$

只有前面几个频率是关键!

为什么正弦？（斜坡，齿波）

- 周期足够长的齿波在一个有限时间区间可认为是个斜坡信号。
- 任何一个齿波可以用无穷多个离散频率的正弦函数来逼近。



$$f(t)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n\pi}\sin\frac{2n\pi t}{T}$$

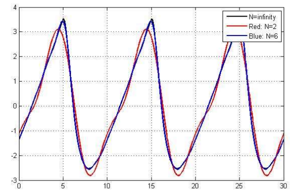
$$f_a(t)=\sum_{n=1}^{N}(-1)^{n+1}\frac{2}{n\pi}\sin\frac{2n\pi t}{T}$$

齿波和正弦稳态响应

$$P(s)=\frac{10}{(s+1)(s+2)}$$

$$f(t)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n\pi}\sin\frac{2n\pi t}{T}$$

$$f_a(t)=\sum_{n=1}^{N}(-1)^{n+1}\frac{2}{n\pi}\sin\frac{2n\pi t}{T}$$



$$y(t)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}2}{n\pi}\left|P(j\frac{2n\pi}{T})\right|\sin(\frac{2n\pi}{T}t+\angle P(j\frac{2n\pi}{T}))$$

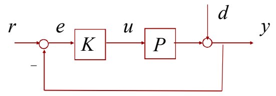
$$y_a(t)=\sum_{n=1}^{N}\frac{(-1)^{n+1}2}{n\pi}\left|P(j\frac{2n\pi}{T})\right|\sin(\frac{2n\pi}{T}t+\angle P(j\frac{2n\pi}{T}))$$

仅首几个频率重要！

任意信号？

- 任何一个周期信号可以用无穷多个离散频率的正弦函数来逼近。
- 任何一个连续信号可以用无穷多个连续频率的正弦函数来逼近。

跟踪与抗扰



- 跟踪正弦信号

  如果 $r(t)=\sin\omega_0 t$，稳态跟踪误差为

  $e_{ss}(t)=|T_{er}(j\omega_0)|\sin(\omega_0 t+\angle T_{er}(j\omega_0))$ $T_{er}(s)=(I+PK)^{-1}$
- 抗击正弦扰动

  如果 $d(t)=\sin\omega_0 t$，稳态输出响应为

  $y_{ss}(t)=|T_{yd}(j\omega_0)|\sin(\omega_0 t+\angle T_{yd}(j\omega_0))$ $T_{yd}(s)=(I+PK)^{-1}$
- 如果 ω_0 可以在一个范围W内变化，则最大可能误差为

  $$\max_{\omega\in W}|T_{er}(j\omega)| \text{ or } \max_{\omega\in W}|T_{yd}(j\omega)|$$

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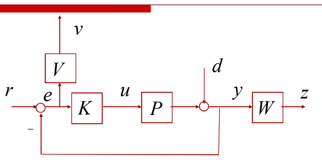
跟踪与抗扰(续)



- 跟踪正弦信号

  $|T_{er}(j\omega)|< 1/|V(j\omega)|$ $\Longleftrightarrow$ $\max_{\omega}|V(j\omega)T_{er}(j\omega)|=:\|V(s)T_{er}(s)\|_{\infty}$
- 抗击正弦扰动

  $|T_{yd}(j\omega)|< 1/|W(j\omega)|$ $\Longleftrightarrow$ $\max_{\omega}|W(j\omega)T_{yd}(j\omega)|=:\|W(s)T_{yd}(s)\|_{\infty}$

- 多变量系统怎么办？

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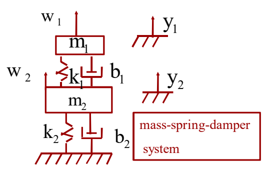
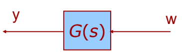
什么是H_∞范数？

- H_∞范数

$$\|G\|_{\infty}:=\sup_{\omega\in R}\bar{\sigma}[G(j\omega)]$$

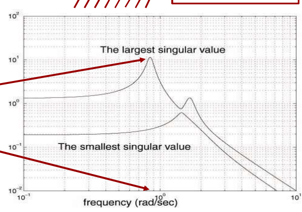
$$=\sup_{w\in L_2}\frac{\|Gw\|_2}{\|w\|_2}$$





$||G||_{\infty}=11.47$

$\omega_{max}=0.8483$



H_∞范数

- 输入输出信号的能量放大倍数：
- $\|G(s)\|_\infty = \sup_\omega \bar{\sigma}\left(G(j\omega)\right) = \sup_{w\in L_2} \frac{\|Gw\|_2}{\|w\|_2}$

  $\|G(s)\|_\infty = 11.47$并且在$\omega_{max} = 0.8483$达到峰值。奇异值分解

  $$G(j\omega_{max}) = \begin{bmatrix} 0.2524 - 10.6046i & 0.3623 - 2.9920i \\ 0.3623 - 2.9920i & 0.4640 - 0.8562i \end{bmatrix} = USV^*$$
- 其中

$$U = \begin{bmatrix} 0.0212 - 0.9611i & -0.2622 + 0.0839i \\ 0.0390 - 0.2725i & 0.9441 - 0.1813i \end{bmatrix} = [u_1 \quad u_2],$$

$$S = \begin{bmatrix} 11.47 & 0 \\ 0 & 0.2591 \end{bmatrix},$$

$$V = \begin{bmatrix} 0.9614 & -0.2753 \\ 0.2733 - 0.0329i & 0.9545 - 0.1151i \end{bmatrix} = [v_1 \quad v_2]$$

H_∞范数

那么

$$u_1 = \begin{bmatrix} 0.0212 - 0.9611i \\ 0.0390 - 0.2725i \end{bmatrix} = \begin{bmatrix} 0.9614e^{-j1.5487} \\ 0.2753e^{-j1.4288} \end{bmatrix},$$

$$v_1 = \begin{bmatrix} 0.9614 \\ 0.2733 - 0.0329i \end{bmatrix} = \begin{bmatrix} 0.9614 \\ 0.2753e^{-j\,0.12} \end{bmatrix}$$

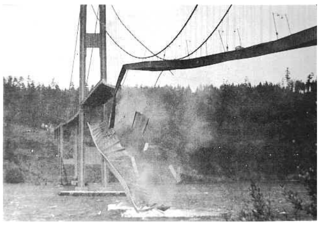
因此如果我们的输入信号为

$$\mathbf{w(t)} = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} = \begin{bmatrix} 0.9614\sin(0.8483t) \\ 0.2753\sin(0.8483t - 0.12) \end{bmatrix}$$

那么系统的稳态输出为

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = 11.47 \times \begin{bmatrix} 0.9614\sin(0.8483t - 1.5487) \\ 0.2753\sin(0.8483t - 1.4288) \end{bmatrix}$$

从频率响应看H_∞范数



The Broughton Suspension Bridge spanned the River Irwell (UK) collapsed due to troops marching over the bridge in step.