



Dep. of Mechanical Eng.

Robotics

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Homework 3

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Questions

The figure below shows a spherical robot using an RRP configuration to determine the position of the end effector. The required parameter values for the robot are specified in the figure:

Question 1

Build this robot model in SimMechanics. Apply the joint angle values according to the table below to your model and read the position of the end effector frame in the base frame from the software and enter it in the table below.

θ_1 (deg)	130	30	90
θ_2 (deg)	10	50	20
d (mm)	20	80	250
X (mm)			
Y (mm)			
Z (mm)			

(In this case the second revolute joint is at zero position, $L_1 = 31cm$, $L_2 = 30cm$.)

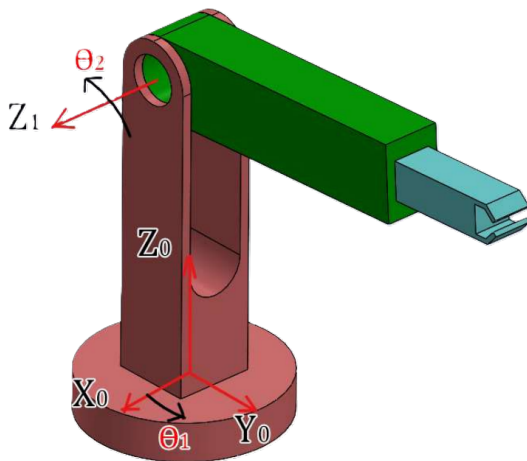


Figure 1: Isometric view of the robot.

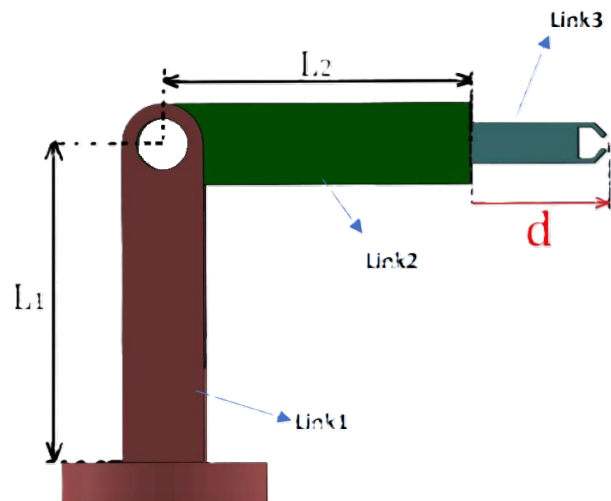


Figure 2: Front view of the robot.

Solution

I begin by placing the coordinates in the specified locations on the every link of the robot as the figure beside. By linking $Frame_0$ with World Frame via revolute joint, the rotation of the robot with respect to World Frame is available. Hence $Frame_1$ is defined in a manner that there is a revolute joint along the z -axis with $Frame_2$ and this joint performs the second rotation of robot's hand. Afterwards $Frame_3$ is defined such that there is a prismatic joint between $Link_2$ and $Link_3$ along the z -axis and this joint performs length of robot's end-effector hand which is identified as $Frame_4$.

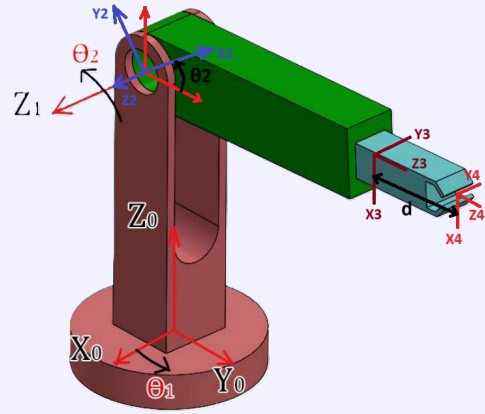


Figure 3: Frame Placement

Now the robot is modeled as below in Simulink:

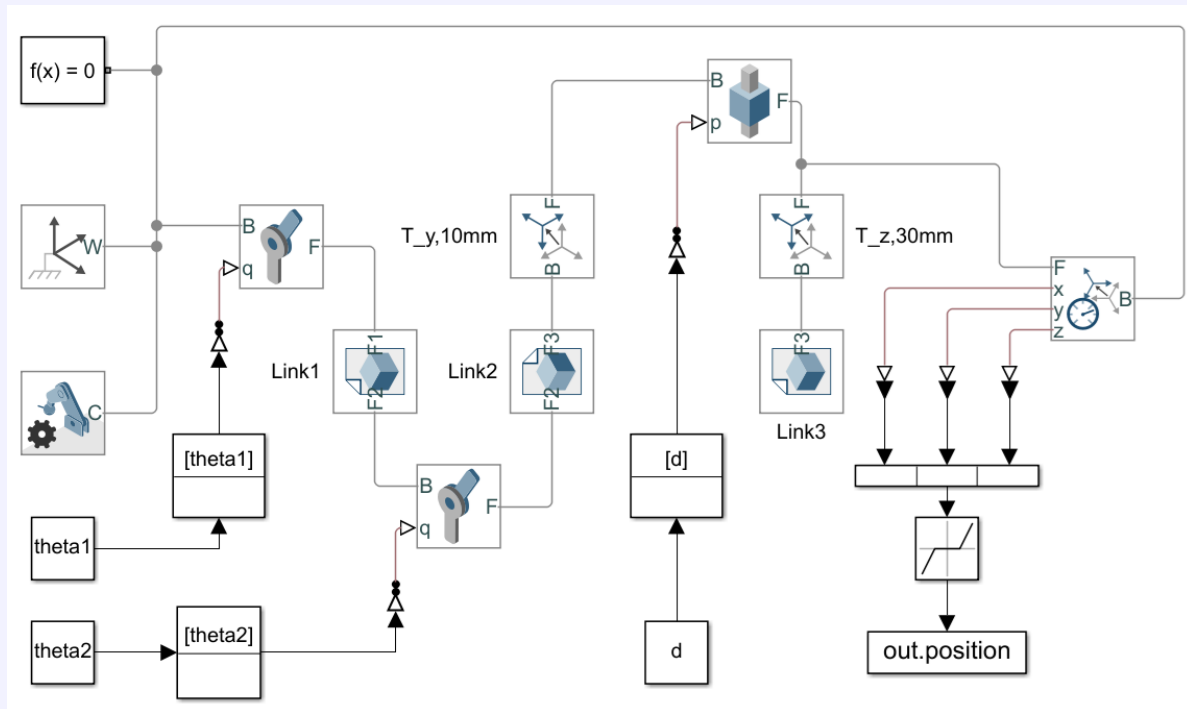


Figure 4: Simulink Model

At the start of every simulation, a pop-up window asks for the values of the robot's parameters, θ_1 , θ_2 , and d , to configure the robot according to the parameters. Using the transformation sensor, the end-effector's position is calculated and is concatenated in a vector called "pos" in the Base Workspace for every 6 simulation. The position of the robot's end-effector for every configuration is shown in the table below:

Config.	θ_1 (deg)	130	30	90
	θ_2 (deg)	10	50	20
	d (mm)	20	80	250
End-Eff.	X (mm)	-241.4101	-122.1296	-516.8309
	Y (mm)	-202.5671	211.5347	0
	Z (mm)	365.5674	601.0969	498.1111

Question 2

In MATLAB, create a function that performs inverse kinematics for this robot. The input to this function is the coordinates of the end point of the third link and its output is the three joint variables (as defined in part 1). Pass the obtained values for the end effector coordinates from section 1 to this function and report the joint variable values. Also test your function with the values in the table below:

θ_1 (deg)			
θ_2 (deg)			
d (mm)			
X (mm)	68.4	298.5	184.9
Y (mm)	12.0	-250.5	139.3
Z (mm)	703.9	535	86.3

Solution

Using the projection of $Link_2$ and $Link_3$ on the $x_0 - y_0$ plane and the plane passing through $Link_1$ and $Link_2$, we have:

$$\begin{aligned} z &= L_1 + (L_2 + d) \sin(\theta_2) \\ y &= (L_2 + d) \cos(\theta_2) \sin(\theta_1 + 90^\circ) \\ x &= (L_2 + d) \cos(\theta_2) \cos(\theta_1 + 90^\circ) \end{aligned}$$

Finding the configuration parameters:

$$\theta_1 = \text{atan2}(-x, y)$$

$$\theta_2 = \begin{cases} \text{atan2}((z - L_1) \sin(\theta_1 + 90^\circ), y) \\ \text{atan2}((z - L_1) \cos(\theta_1 + 90^\circ), x) \end{cases}$$

$$d = \begin{cases} \frac{z - L_1}{\sin(\theta_2)} - L_2 \\ \frac{y}{\cos(\theta_2) \sin(\theta_1 + 90^\circ)} - L_2 \\ \frac{x}{\cos(\theta_2) \cos(\theta_1 + 90^\circ)} - L_2 \end{cases}$$

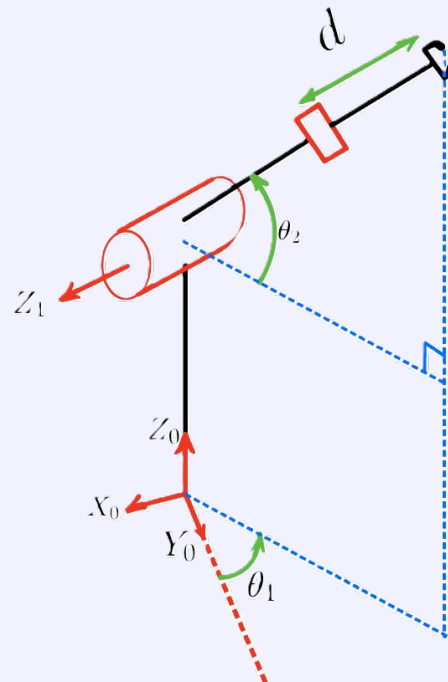


Figure 5: Frame Placement

Now we have multiple choices for finding the parameters of robot's configuration according to the equations. The choices are as below and the ideal ones are selected based on the singularity problem and negative length of d . With the procedure implemented in the MATLAB code below, the singularity points are handled efficiently:

```
function [ theta1 , theta2 , d] = RRP_InvPos( pos )
pos = round (pos , 3);
L1 =310;
L2 =300;
theta1=atan2d(-pos(1,:),pos(2,:));
theta2=atan2d((pos(3,:)-L1).*sind(theta1+90),pos(2,:));
id=abs(theta1)==0; % Singularity check
theta2(~id)=atan2d((pos(3,~id)-L1).*cosd(theta1(~id)+90),pos
    (1,~id));
d=(pos(3,:)-L1)./sind(theta2)-L2;
% Singularity and logicality check
singindx = isnan(d) | d<0;
d(singindx)=pos(2,singindx)./(cosd(theta2(singindx)).*sind(
    theta1(singindx)+90))-L2;
% Singularity and locicality check - again
remsingindx= (isnan(d) | d<0) & theta2~=90;
theta2(remsingindx)=atan2d((pos(3,remsingindx)-L1).*cosd(
    theta1(remsingindx)+90),pos(1,remsingindx))+180;
d(remsingindx)=pos(1,remsingindx)./(cosd(theta2(remsingindx))
    .*cosd(theta1(remsingindx)+90))-L2;
remsingindx= isnan(d) | d<0;
d(remsingindx)=pos(1,remsingindx)./(cosd(theta2(remsingindx))
    .*sind(theta1(remsingindx)+90))-L2;
theta1=round(theta1,3);
theta2=round(theta2,3);
d=round(d,3);
end
```

Robot's configuration according to its end-effector position is shown in the table below:

		Question 2			Question 1		
Config.	θ_1 (deg)	-80.049	-130.003	-53.006	130.000	30.000	90.000
	θ_2 (deg)	80.001	30.002	-44.018	10.000	50.000	20.000
	d (mm)	99.975	149.975	21.923	20.000	80.000	250.000
End-Eff.	X (mm)	68.4	298.5	184.9	-241.4101	-122.1296	-516.8309
	Y (mm)	12.0	-250.5	139.3	-202.5671	211.5347	0
	Z (mm)	703.9	535.0	86.3	365.5674	601.0969	498.1111

Conclusion

As shown in table above, Simulink model and MATLAB code are confirming each other.