

We want to minimize the function:

$$f(x_1, x_2) = \frac{1}{2} x_1^2 + x_2^2 + 2 x_1 + \frac{5}{2} x_2.$$

To do so, we set its first derivatives to zero:

$$\begin{aligned} \frac{\partial f}{\partial x_1} &= x_1 + 2 = 0, \\ \frac{\partial f}{\partial x_2} &= 2 x_2 + \frac{5}{2} = 0. \end{aligned}$$

These equations have the unique solution $x_1 = -2$ and $x_2 = -\frac{5}{4}$. The following contour plot illustrates the fact that this point is a minimizer:

