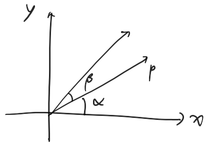
EECS 442 9/7 Lecture 2

Math Background

Matrices & Eigenvalues / vectors

Rotation matrix (2×2 in plane)

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



$$p = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cdot\cos\alpha \\ r\cdot\sin\alpha \end{bmatrix}$$

$$q = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r\cdot\cos(\alpha+\beta) \\ r\cdot\sin(\alpha+\beta) \end{bmatrix}$$

$$q = \begin{bmatrix} r(\cos\alpha\cos\beta - \sin\alpha\sin\beta) \\ r(\sin\alpha\cos\beta + \cos\alpha\sin\beta) \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\beta - y\sin\beta \\ x\sin\beta + y\cos\beta \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

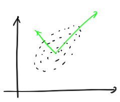
$$x^2 - 2\cos\beta\cdot x + 1 = 0$$

$$\frac{2\cos\beta \pm \sqrt{4\cos^2\beta - 4}}{2} = \cos\beta \pm \sqrt{\cos^2\beta - 1}$$

$$= \cos\beta \pm i|\sin\beta|$$

Eigenvectors

$$Av = \lambda v$$



$$A = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \\ x_n & y_n \end{bmatrix}$$

eigenvector of A^TA

Similarity transform

any non singular S

$$B = S^{-1}AS$$

Taylor's Expansion

let $f \in C^\infty(a-\varepsilon, a+\varepsilon)$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$= f(a) + f'(a)\cdot(x-a) + \frac{f''(a)}{2!}\cdot(x-a)^2 + \cdots$$

Multivariate

$$f(x,y) = f(a,b) + f_x(a,b)\cdot(x-a) + f_y(a,b)(x-b)$$

$$= f(a,b) + (\nabla f(a,b))^T\begin{bmatrix} x-a \\ x-b \end{bmatrix}$$

Consider a system of p linear equations in q unknowns.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1q}x_q = y_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2q}x_q = y_2$$
$$\vdots$$
$$a_{p1}x_1 + a_{p2}x_2 + \cdots + a_{pq}x_q = y_p.$$

$$Ax = y$$

If $p < q$ equations less than unknowns.

$p = q$

$p > q$ equations more than unknowns

Gaussian elimination (unique solution)

least squares no exact solutions

$$E(x) \doteq \sum_{i=1}^{p}(a_{i1}x_1 + \cdots + a_{iq}x_q - y_i)^2$$

$$= \|Ax - y\|_2^2 \quad \text{(Euclidean norm)}$$

$$x = (A^TA)^{-1}A^Ty$$

Lecture 3. Images as functions 9/12.

Three primary topics in the representation of images

1. image as functions
2. Image as coefficients/points
3. image as graphs.

1. Defining the classes of images

Def 1. A $\boxed{\text{perfect image}}$ is the continuous image generated by a physical process. We denote them with I.

$I: \mathbb{R}^2 \to \mathbb{R}^+$.

Remark: perfect images exist in abstraction only.

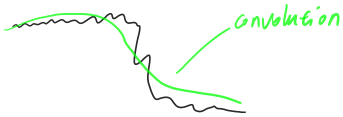
Eg. Lambertian Model is a classical model of surface reflectance.

→ The amount of light reflected off the surface

Why do I use Difference of Gaussian instead of Laplacian of Gaussian

$$\underbrace{G_1 \otimes I}_{J_1} - \underbrace{G_2 \otimes I}_{J_2} \Rightarrow \text{edges}$$

Better than Laplacian since it smooth



Range map

Geometric permitives

$$Ax+b \quad \Big| \quad \tilde{A} = \begin{bmatrix} A & b \\ 0 \; 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ 1 \end{pmatrix}$$

Projection plane.

intersection of two lines

$$\boxed{\begin{aligned} \tilde{x} &= \tilde{l}_1 \times \tilde{l}_2 \\ \tilde{l} &= \tilde{x}_1 \times \tilde{x}_2 \end{aligned}}$$

degree of freedom

rotation + similarity $\quad \bar{x}' = \begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix} \bar{x}$

DoF = 4

Affine $\quad \bar{x}' = A\bar{x} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \bar{x}$

DoF = 6

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} ax_1 + by_1 + cz_1 \\ dx_1 + ey_1 + fz_1 \\ z_1 \end{bmatrix} \begin{pmatrix} ax_2 + by_2 + cz_2 \\ dx_2 + ey_2 + fz_2 \\ z_2 \end{pmatrix}$$

$$px + qy + r \cdot z = c_1$$

$$px + qy + rz = c_2$$

$$\frac{x_1 - x_2}{p} = \frac{y_1 - y_2}{q} = \frac{z_1 - z_2}{r} \quad \text{line} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \quad ①$$

$$\frac{x_3 - x_4}{p} = \frac{y_3 - y_4}{q} = \frac{z_3 - z_4}{r} \quad \text{line} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} \begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix} \quad ②$$

① // ②

New lines: directions

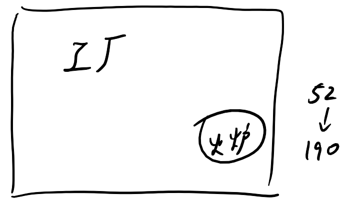
$$\begin{pmatrix} a(x_1 - x_2) + b(y_1 - y_2) + c(z_1 - z_2) \\ d(x_1 - x_2) + e(y_1 - y_2) + f(z_1 - z_2) \\ z_1 - z_2 \end{pmatrix}$$

$$\frac{x}{a(x_1 - x_2) + b(y_1 - y_2) + c(z_1 - z_2)} = \frac{y}{}$$

SRO

special range operation.

Avg = 162   Avg = 47



RMO

Laplacian of Gaussian.

$$\nabla^2 \big(f(x,y) \otimes G(x,y) \big) = \nabla^2 G(x,y) \otimes f(x,y)$$

$$\underbrace{G \otimes f}_{\text{blurred image}}$$

$$f_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2} \right) \qquad \sigma > 0 \quad x, y \in \mathbb{R}.$$

$$LoG(x,y) = -\frac{1}{\pi\sigma^4} \left[1 - \frac{x^2+y^2}{2\sigma^2} \right] \exp\left(-\frac{x^2+y^2}{2\sigma^2} \right)$$

$$\text{Laplacian} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\text{Gaussian}_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \frac{1}{9}$$

$$\text{Gaussian}_2 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \frac{1}{16}.$$

Photometric Invariance.

Light photometry radiometric image

Light models

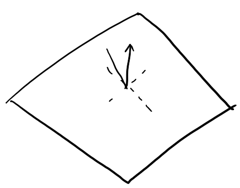
Coarse approximations to real light.

- Point light
  - · directional
  - · spot

Point light $\begin{cases} \text{position } (x, y, z) \\ \text{intensity } (r, g, b) \end{cases}$

Direction (point at infinity) $\begin{cases} \text{direction} \\ \text{intensity} \end{cases}$

BRDF



$$L(v_r; \lambda) = \int L_i(\hat{v}_i; \lambda) \cdot f_r(v_i, v_r, \hat{n}; \lambda) \cos^+\theta \, dv_i$$

$$f_d(v_i, v_r, \hat{n}; \lambda) \xlongequal{\text{simplify}} f_d(\lambda).$$

Specularity. (Mirror) reflection.

non-diffusion

Phong Shading model.

Automatic Gain Control: noise

shutter: rolling shutter is bad.

9/27 Discussion

Image Stiching

① Find feature points in both image

$\rightarrow$ structure tensor (reduction)

$I(x,y) \longrightarrow (x+u, y+v)$

$$I(x+u, y+v) = I(x,y) + \frac{\partial I(x,y)}{\partial x}\cdot u + \frac{\partial I(x,y)}{\partial y}\cdot v + \cdots \text{ higher order.}$$

only use linear terms

Calculate the difference.

$$E = \sum_{x,y\in W} \left(I(x+u, y+v) - I(x,y)\right)^2$$

$$\approx \sum_{x,y\in W} \left(\frac{\partial I}{\partial x}\cdot u + \frac{\partial I}{\partial y}\cdot v\right)^2$$

$$= [u, v] \underbrace{\begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}}_{H \,:\, \text{tensor structure.}} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\det(H - I\lambda) = \det\begin{pmatrix} h_{11}-\lambda & h_{12} \\ h_{21} & h_{22}-\lambda \end{pmatrix} = 0$$

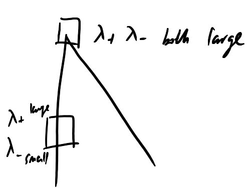
$$(h_{11}-\lambda)(h_{22}-\lambda) - h_{12}\cdot h_{21} = 0$$

$$\lambda = \frac{h_{11}+h_{22} \pm \sqrt{(h_{11}+h_{22})^2 - 4(h_{11}h_{22} - h_{21}h_{12})}}{2}$$

Window size: set manually.

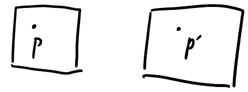
Harris detector property

λ_+ λ_- rotation invariant



$$\frac{\lambda_+\lambda_-}{\lambda_+ + \lambda_-} = \frac{\det H}{\mathrm{tr}\, H}$$

② match the feature



$$p = Hp'$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

projection

Pick many features (pairs of p, p')

want

$$\min_H \|p - Hp'\|_2^2$$

Recall the fitting

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix}$$

EECS 442

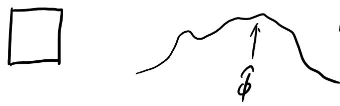
What is SIFT histogram

Detector DoG

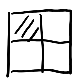
Descripter HoG

histogram oriented Gradients

1. Orientation alignment



rotate parameter



4x4 separate histogram.

within each subwindows of the patch

we have structured invariance given the histogram.

Module 1 Part 2.

Images as Points.

Ω is the set of all Images of a certain size.

Cartesian Basis.

Consider $n \times n$ Image

rotate the basis $\{V_i\}$

$$I_{2\times 2} = V_1\alpha_1 + V_2\alpha_2 + V_3\alpha_3 + V_4\alpha_4$$

$$= \begin{bmatrix}1&0\\0&0\end{bmatrix}\cdot 0 + \begin{bmatrix}0&1\\0&0\end{bmatrix}\cdot 128$$

$$+ \begin{bmatrix}0&0\\1&0\end{bmatrix}\cdot 255 + \begin{bmatrix}0&0\\0&1\end{bmatrix}\cdot 0$$

90° counterclock

$\begin{bmatrix}1&0\\0&0\end{bmatrix} \rightarrow$

Wavelets

Harr

Basis that has certain desirable properties

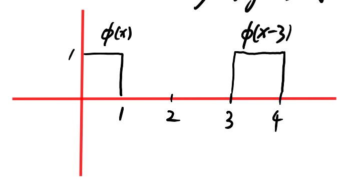
Compact support

Orthonormal basis

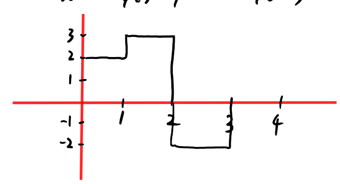
Def: Harr basis is an orthonormal basis with compact support

$$\phi(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{o/w} \end{cases}$$

We can construct many signals from $\phi(x)$



$f(x) = 2\phi(x) + 3\phi(x-1) - 2\phi(x-2)$



Can approximate continuous functions.

Def Harr wavelet in 1D

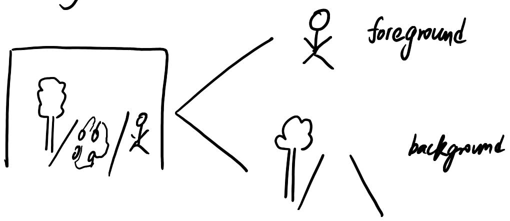
$$\psi(x) = \begin{cases} 1 & \text{if } 0 \le x \le \frac{1}{2} \\ -1 & \text{if } \frac{1}{2} < x \le 1 \\ 0 & \text{o/w} \end{cases}$$

Family of Harr wavelets

① translation $\psi(x-t)$

② stretching $2^{j/2}\psi(2^j x - t)$

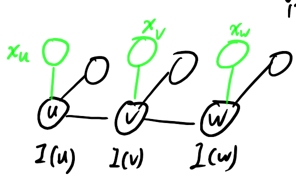
Case study



$$f(I(u);\theta) = \|I(u)-\theta\|_2$$

$\downarrow$ foreground potential

$\downarrow$ 178



$x_i \in \{0, 1\}$

background foreground

Set of $\{x_i\}_{i=1}^n$ is called a $\boxed{\text{configuration}}$

We have f: our foreground function

b: our background function

Define our binary variable $x_1, \dots, x_{mn}$

Define an energy function

$$E_1(x) = \sum_{u=1}^{mn} x_u f(I(u);\theta) + \sum_{u=1}^{mn} (1-x_u)\, b(I(u);\theta)$$

$\uparrow$ configuration

$$x^* = \arg\min_x E_1(x)$$

or

$$x_i = \begin{cases} 1 & \text{if } f(I(u);\theta) < b(I(u);\theta) \\ 0 & \text{o/w} \end{cases}$$

Potts model Eq

We need to regularize E_1 with a form that explores binary spatial relationship.

$$E_p(x, \partial) = E_1(x) + \sum_{u=1}^{mn} \sum_{v \in \partial(u)} \beta\, \mathbb{1}\{x_u == x_v\}$$

$\uparrow$ neighborhood

$\partial(u)$ yields set of neighbors

$$E_2(x;\partial) = E_1(x) + \sum_{u}^{mn} \sum_{v\in\partial(u)} |x_u - x_v| \; [1 - R(u,v,I)]$$

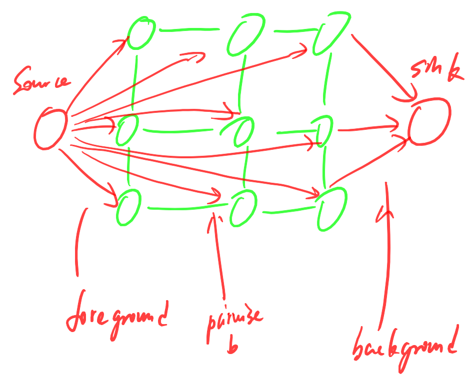
low for different weight

$$R(u,v;I) = \frac{1}{\alpha}\|I(u)-I(v)\|_2^2$$

$\uparrow$ normalize

Min cut - Max flow

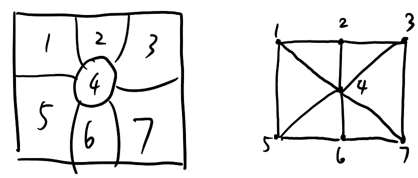
Augmented



① compute the reachable set S in the residual graph

② Assign any S to foreground

$V-S$ to background



	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

Similarity

$S = \sum \min(hist_1(i), hist_2(i))$

weight $= 1 - s$.