

Math Background.

Matrices & Eigenvalues/vectors

Rotation matrix (2x2 in plane)

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$



$$p = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \cdot \cos \alpha \\ r \cdot \sin \alpha \end{bmatrix}$$

$$q = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r \cdot \cos(\alpha + \beta) \\ r \cdot \sin(\alpha + \beta) \end{bmatrix}$$

$$q = \begin{bmatrix} r (\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\ r (\sin \alpha \cos \beta + \cos \alpha \sin \beta) \end{bmatrix}$$

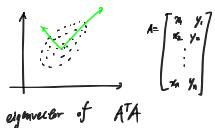
$$= \begin{bmatrix} x \cos \beta - y \sin \beta \\ x \sin \beta + y \cos \beta \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x^2 - 2 \cos \beta \cdot x + 1 = 0$$

$$\frac{2 \cos \beta \pm \sqrt{4 \cos^2 \beta - 4}}{2} = \cos \beta \pm \sqrt{\cos^2 \beta - 1} \\ = \cos \beta \pm i / |\sin \beta|$$

Eigen vectors

$$Av = \lambda v$$



Similarity transform

any non singular S

$$B = S^{-1}AS$$

Taylor's Expansion

let $f \in C^\infty(a-\varepsilon, a+\varepsilon)$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k \\ = f(a) + f'(a) \cdot (x-a) + \frac{f''(a)}{2!} \cdot (x-a)^2 + \dots$$

Multivariate

$$f(x,y) = f(a,b) + f_x(a,b) \cdot (x-a) + f_y(a,b) \cdot (y-b) \\ = f(a,b) + \begin{pmatrix} \nabla f(a,b) \end{pmatrix}^T \begin{bmatrix} x-a \\ y-b \end{bmatrix}$$

Consider a system of p linear equations in q unknowns.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1q}x_q = y_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2q}x_q = y_2$$

 \vdots

$$a_{p1}x_1 + a_{p2}x_2 + \dots + a_{pq}x_q = y_p$$

$$Ax = y$$

If $p \leq q$ equations less than unknowns.

$$\begin{cases} p = q \\ p > q \end{cases}$$

equations more than unknowns

Gaussian elimination (unique solution)

(least squares no exact solutions)

$$E(x) = \sum_{i=1}^p (a_{i1}x_1 + \dots + a_{iq}x_q - y_i)^2$$

$$= \|Ax-y\|_2^2 \quad (\text{Euclidean norm})$$

$$x = (A^T A)^{-1} A^T y$$

Three primary topics in the representation of images

1. image as functions
2. image as coefficients/pixels
3. image as graphs.

1. Defining the classes of images

Def 1. A **perfect image** is the continuous image generated by a physical process. We denote them with I .
 $I: \mathbb{R}^2 \rightarrow \mathbb{R}^+$.

Remark: perfect images exist in abstraction only.

Eg. Lambertian Model is a classical model of surface reflectance.

→ the amount of light reflected off the surface

