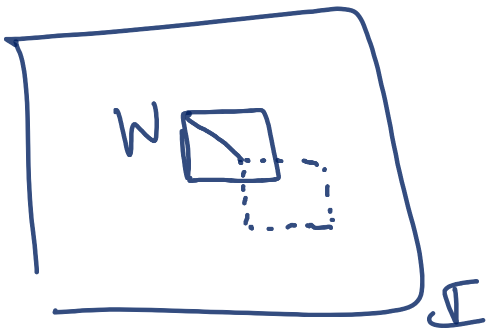
442 9/26 In-Class

How to measure local uniqueness.



Region, window W

Domain Operator T

$$\rightarrow E(T) = \sum_{(x,y)\in W} \Big(I(T(x,y)) - I(x,y) \Big)^2$$

Eg T_T : Translation $\Longrightarrow \begin{pmatrix} u \\ v \end{pmatrix}$

$$I(T_T(x,y)) = I(x+u, y+v)$$

$T_R : (\theta)$

$$I(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$$

Use Translation

$$I(T_T(x,y)) = I(x+u, y+v)$$

First order approximation to

@ $u=0$, $v=0$

$$\underline{I(x+u,y+v)} \overset{\text{Linear}}{\approx} I(x,y) + \frac{\partial I}{\partial x}(x,y)\cdot u + \frac{\partial I}{\partial y}(x,y)\cdot v$$

Sum of Squared Differences

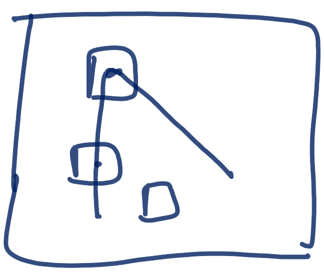
$$\underline{E}(I_T, I) = \sum_W \left[\underline{I}(x+u,y+v) - I(x,y)\right]^2$$

$\boxed{\text{Max over Ws}}$

$$\approx \sum_W \left[\cancel{I(x,y)} + \frac{\partial I}{\partial x}(x,y)\cdot u + \frac{\partial I}{\partial y}(x,y)\, v - \cancel{I(x,y)}\right]^2$$

$$\rightarrow \; = \sum_W \left[I_x \cdot u + I_y \cdot v\right]^2$$

where $I_j = \frac{\partial I}{\partial j}(x,y) \;;\; j = \{x,y\}$



expand square

$$= \sum_{(x,y)\in W} \left(\begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x(x,y)^2 & I_x(x,y) I_y(x,y) \\ I_x(x,y) I_y(x,y) & I_y(x,y)^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right)$$

$$= \begin{bmatrix} u, v \end{bmatrix} \begin{bmatrix} \underbrace{\sum_{x,y\in W} I_x(x,y)^2}_{h_{11}} & \underbrace{\sum I_x I_y}_{h_{12}} \\ \underbrace{\sum I_x I_y}_{h_{21}} & \underbrace{\sum I_y^2}_{h_{22}} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

STRUCTURE TENSOR

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

Look at the intrinsic structure of H
Through eigendecomposition

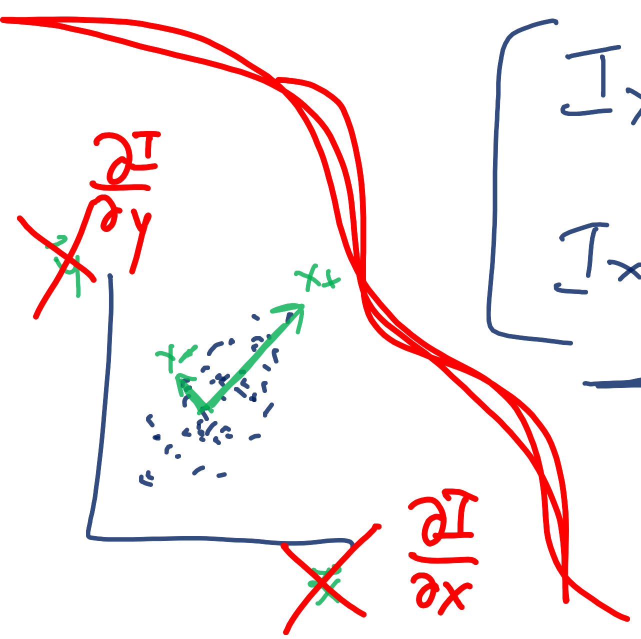
$$\det[H - \lambda I] = \det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$$

$$\downarrow$$

$\lambda_+ \quad \lambda_-$,

$\underline{x_+ \quad x_-}$ solving $(H - \lambda_+ I)\, x_+ = 0$

$$\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \qquad \begin{bmatrix} I_x \\ I_y \end{bmatrix}$$



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