Photometric Invariance

Light, Photometry, and Radiometric Image Formation

EECS 442 Computer Vision

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Readings: SZ 2.2.3, 4.1.2, 10.1

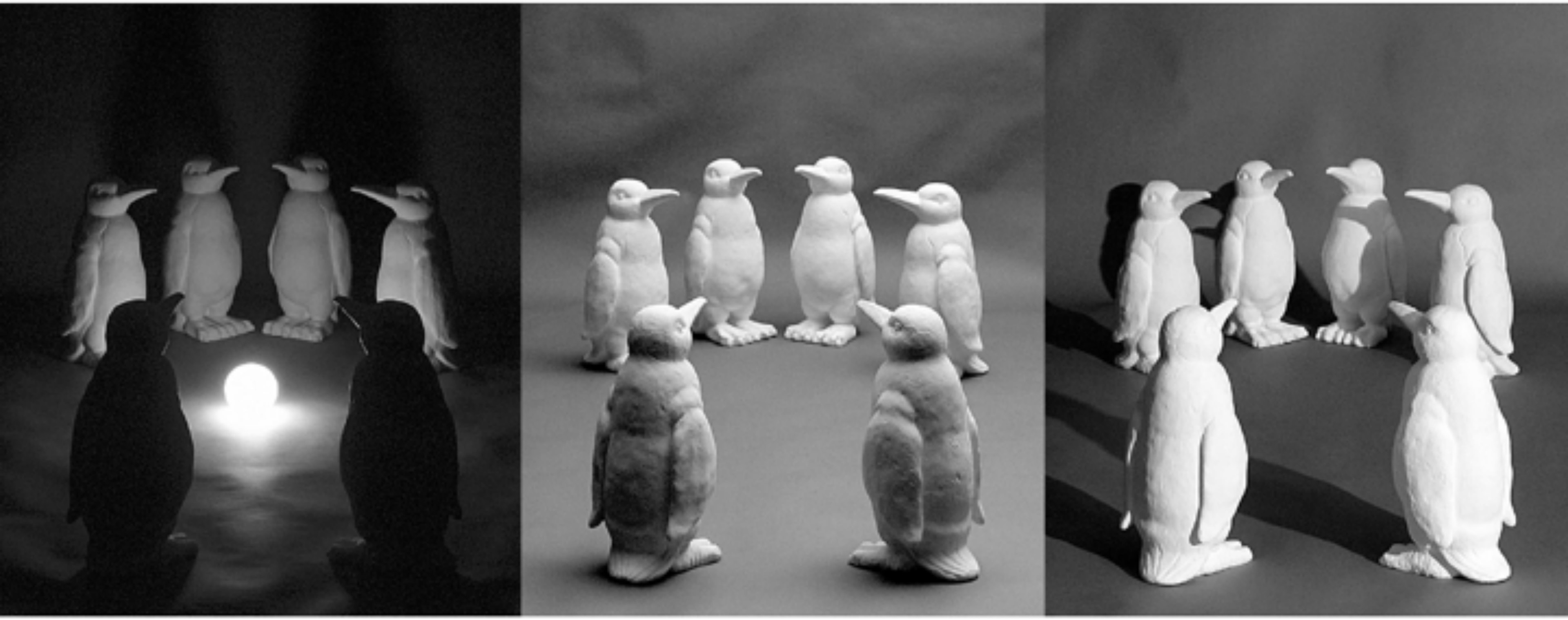
Challenges: scale variation

Source: Saverese, Fei-Fei, Fergus, and Torralba.

Challenges: viewpoint variation

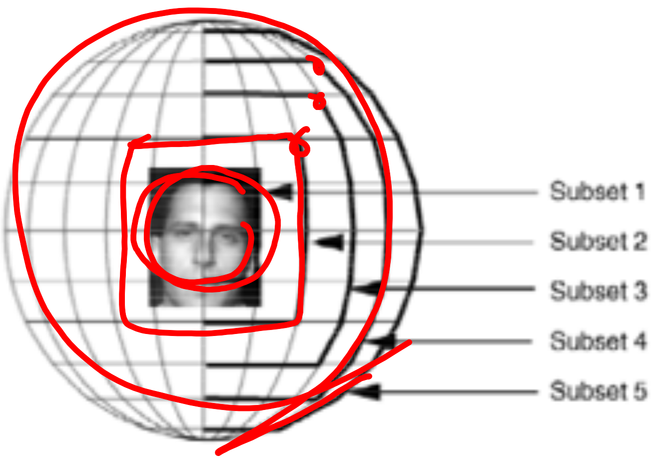
slide credit: Fei-Fei, Fergus & Torralba

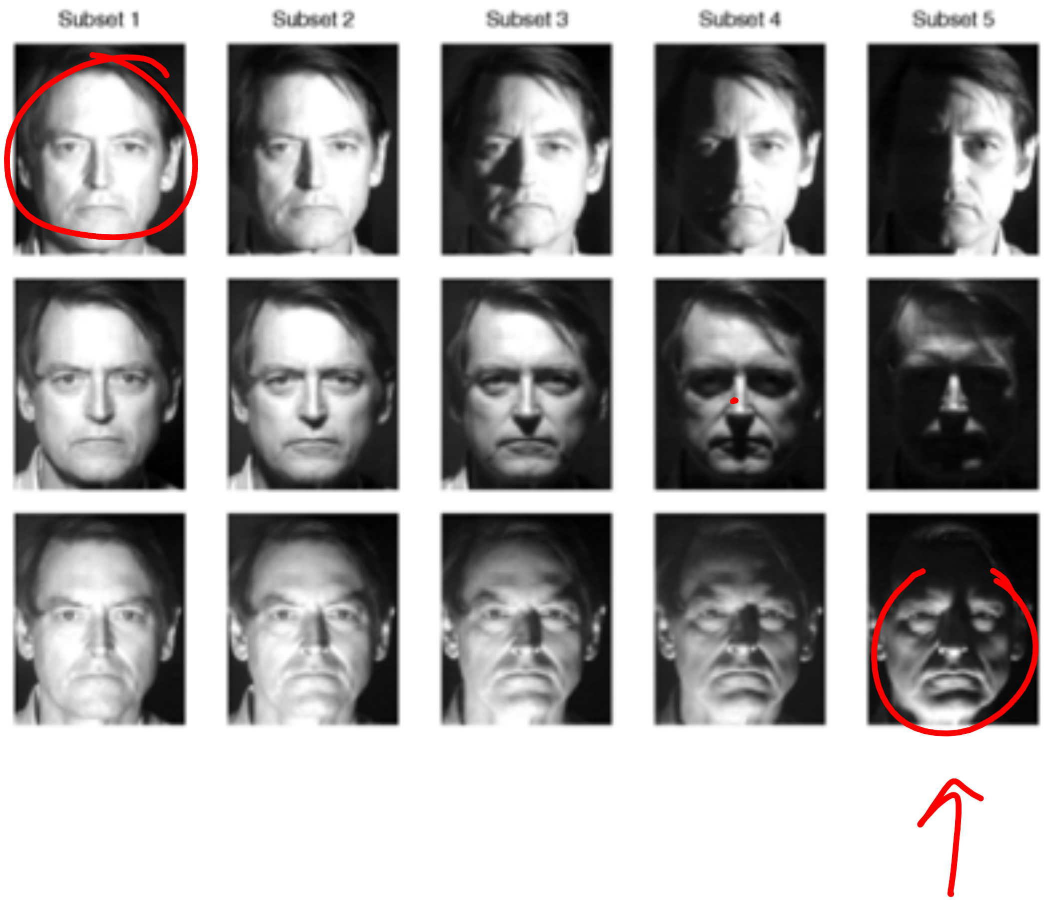
Challenges: illumination variation



Source: Saverese slides; Koenderink image.

Challenges: illumination variation: directionality





Source: Belhumeur et al. TPAMI 1997

Challenges: illumination variation: dynamic range

Source: Szeliski Book (Ch. 10)

Photometric and Radiometric Image Formation

Quantum “Catception”

Source: https://www.youtube.com/watch?v=2dRr-fnPCwM

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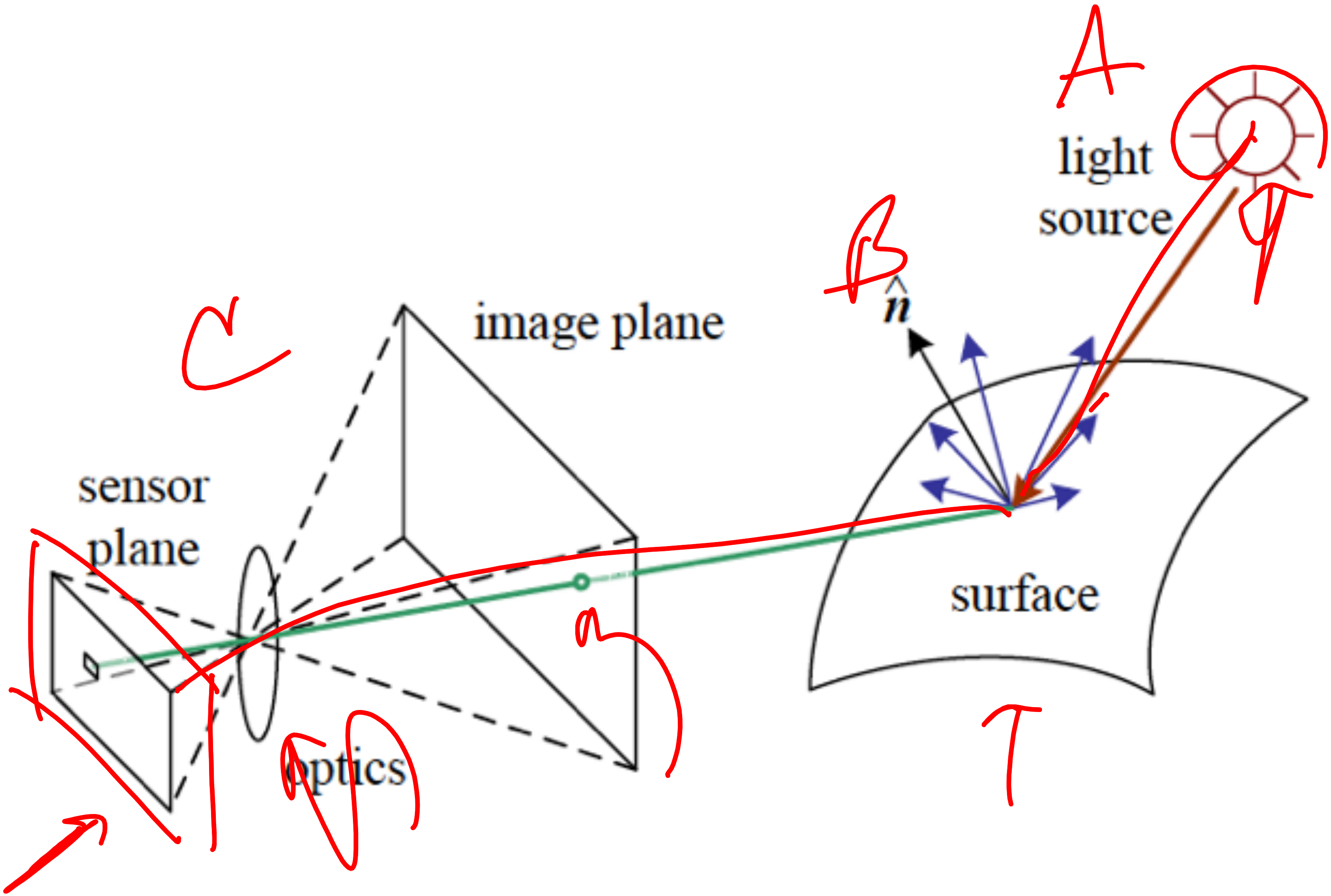
Photometric Image Formation



Source: Szeliski book (Fig. 2.14).

Photometric Image Formation

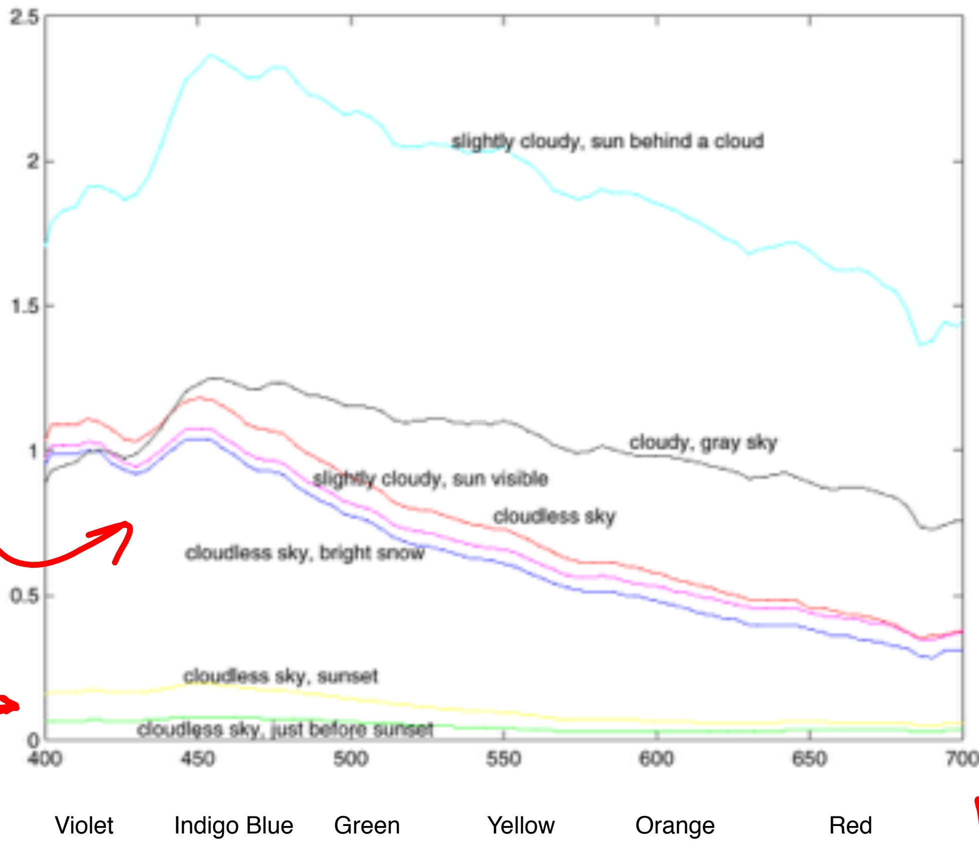
- Three components to pixel brightness
  - Illumination and light sources

  - Surfaces and reflection

  - Camera response

Lights in the real-world

- Real lights are complicated
  - Sun-light, incandescent bulbs, fluorescent bulbs

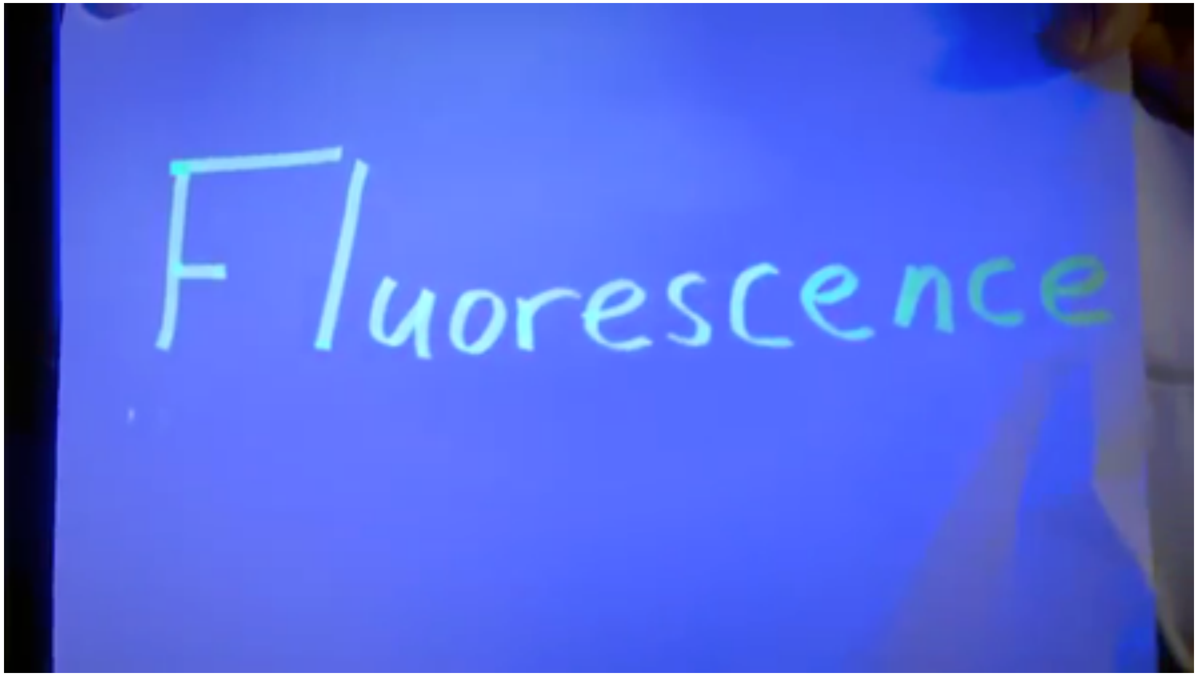


Measurements of relative spectral power of sunlight, made by J. Parkkinen and P. Silfsten. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm. The color names on the horizontal axis give the color names used for monochromatic light of the corresponding wavelength --- the “colors of the rainbow”. Mnemonic is “Richard of York got blisters in Venice”.

Source: Forsyth and Ponce slides

Lights in the real-world

- Real lights are complicated
  - Sun-light, incandescent bulbs, fluorescent bulbs
  - Different spectra
  - Different directions
  - Time-varying
- Fluorscence and biochemistry as well.



Source: https://www.youtube.com/watch?v=WnWIt0iz00A

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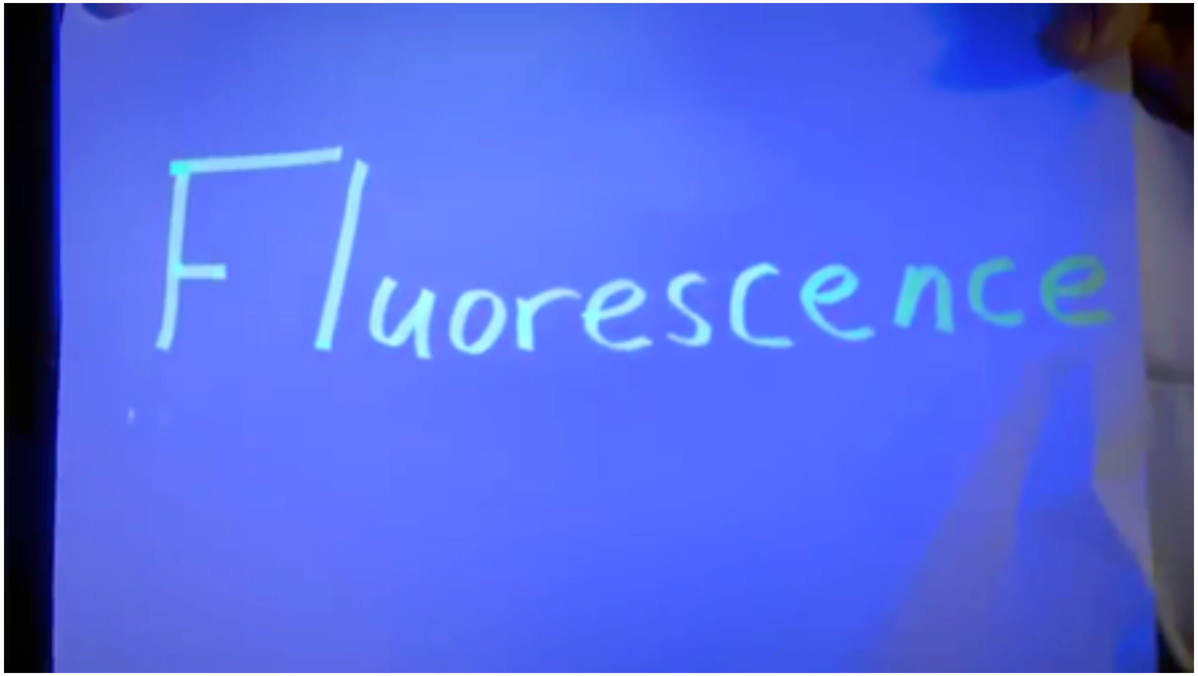

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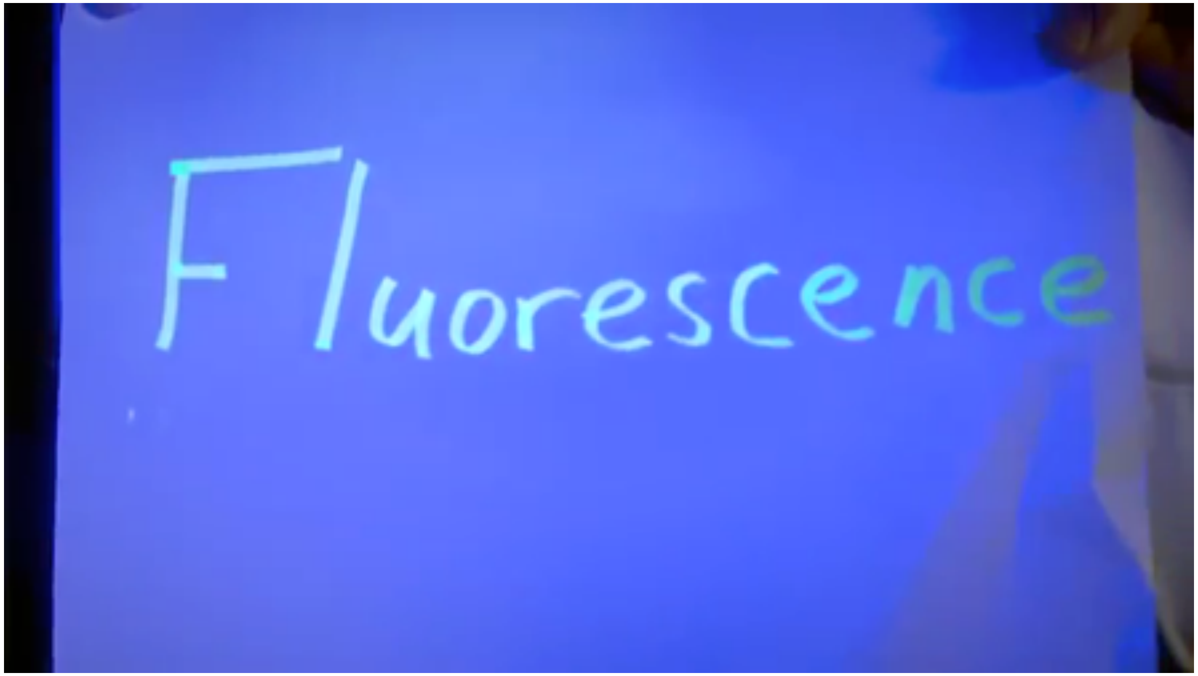


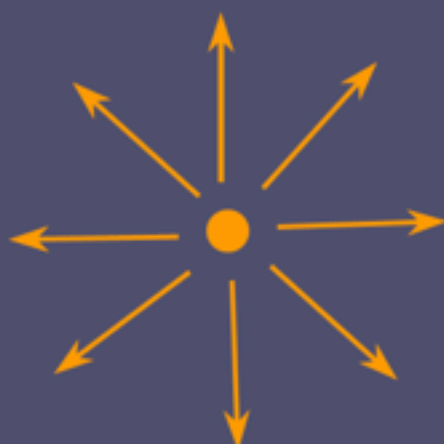
Source: https://www.youtube.com/watch?v=WnWIt0iz00A

Light Models

- Coarse approximations to real light
  - Point light
    - Directional
    - Spot
    - Has a location in space and a distribution over wavelengths $L(\lambda)$
  - Area lights
    - Light-fields
- Environment Map $L(\hat{\boldsymbol{v}}; \lambda)$
  - Maps incident light directions to color values

Source: Image from Pat Hanrahan slides on environment maps

Point Light



point light

Specified by:

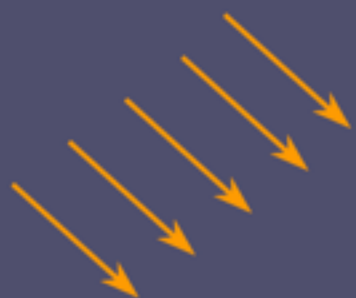
- **position (x,y,z)**
- **intensity (r,g,b)**

Radiates equal intensity in all directions

$$L = P_{light} - P_{surface}$$

Directional Light

point light
at infinity



Point light at infinity

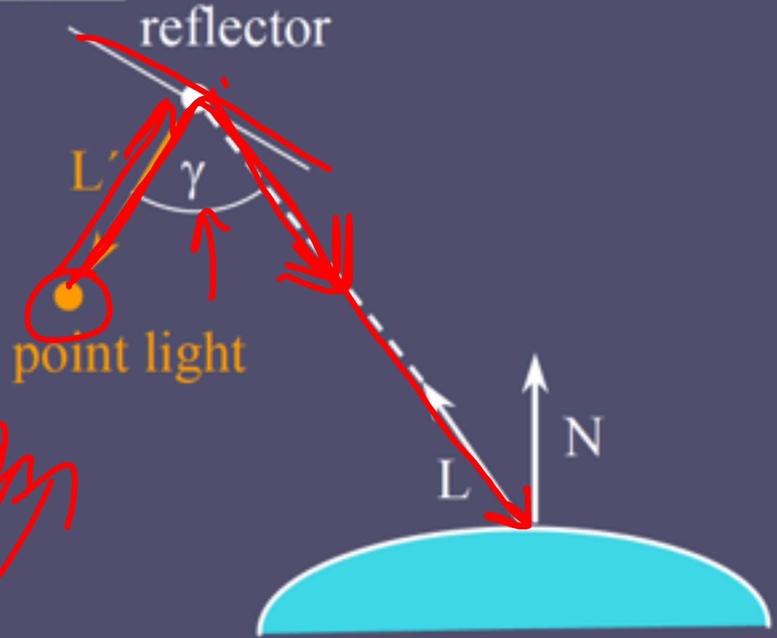
Specified by:

- **direction (x,y,z)**
- **intensity (r,g,b)**

All light rays are parallel

L = -direction

Spot (Warn) Light



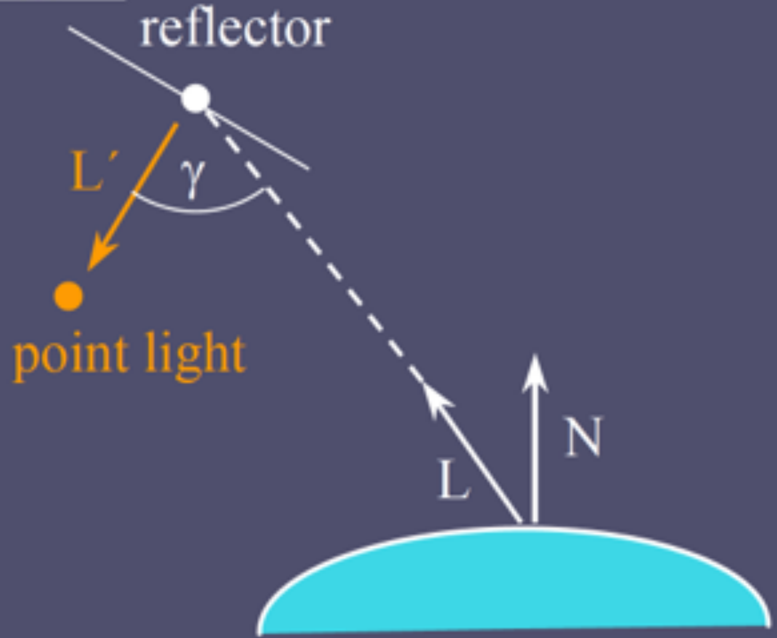
Specular reflection of point light source

Specified by:

- position of reflector
- position of point light (or direction to point light)
- intensity of point light
- falloff exponent

$$I_{warn} = I_{point}\cos^p\gamma = I_{point}(V'.R')^p = I_{point}(-L.L')^p$$

Spot (Warn) Light



Specular reflection of point light source

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Warn Light Profile and Examples

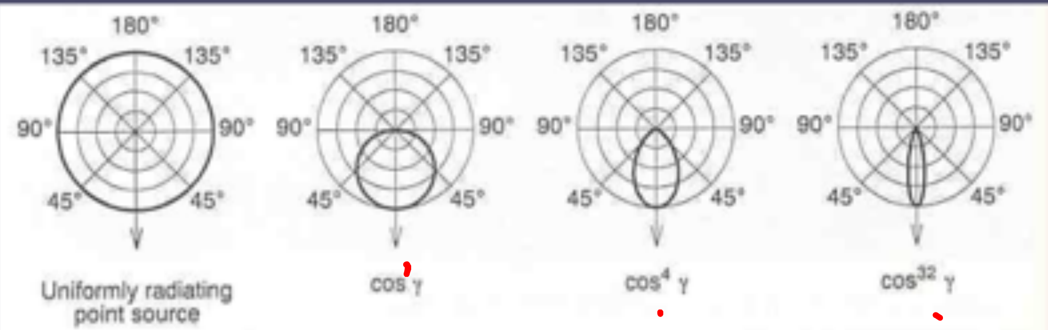


Fig. 16.14 Intensity distributions for uniformly radiating point source and Warn light source with different values of p.

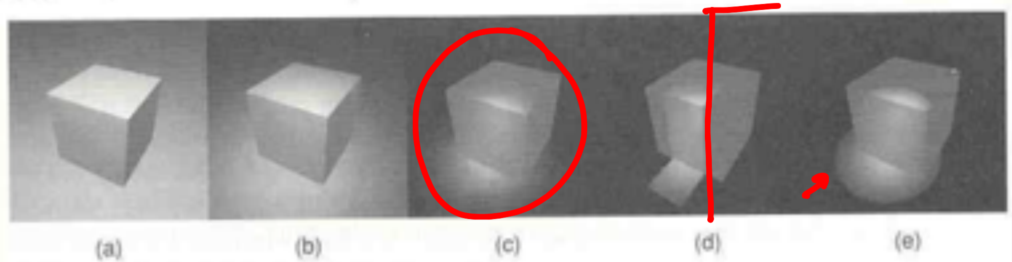
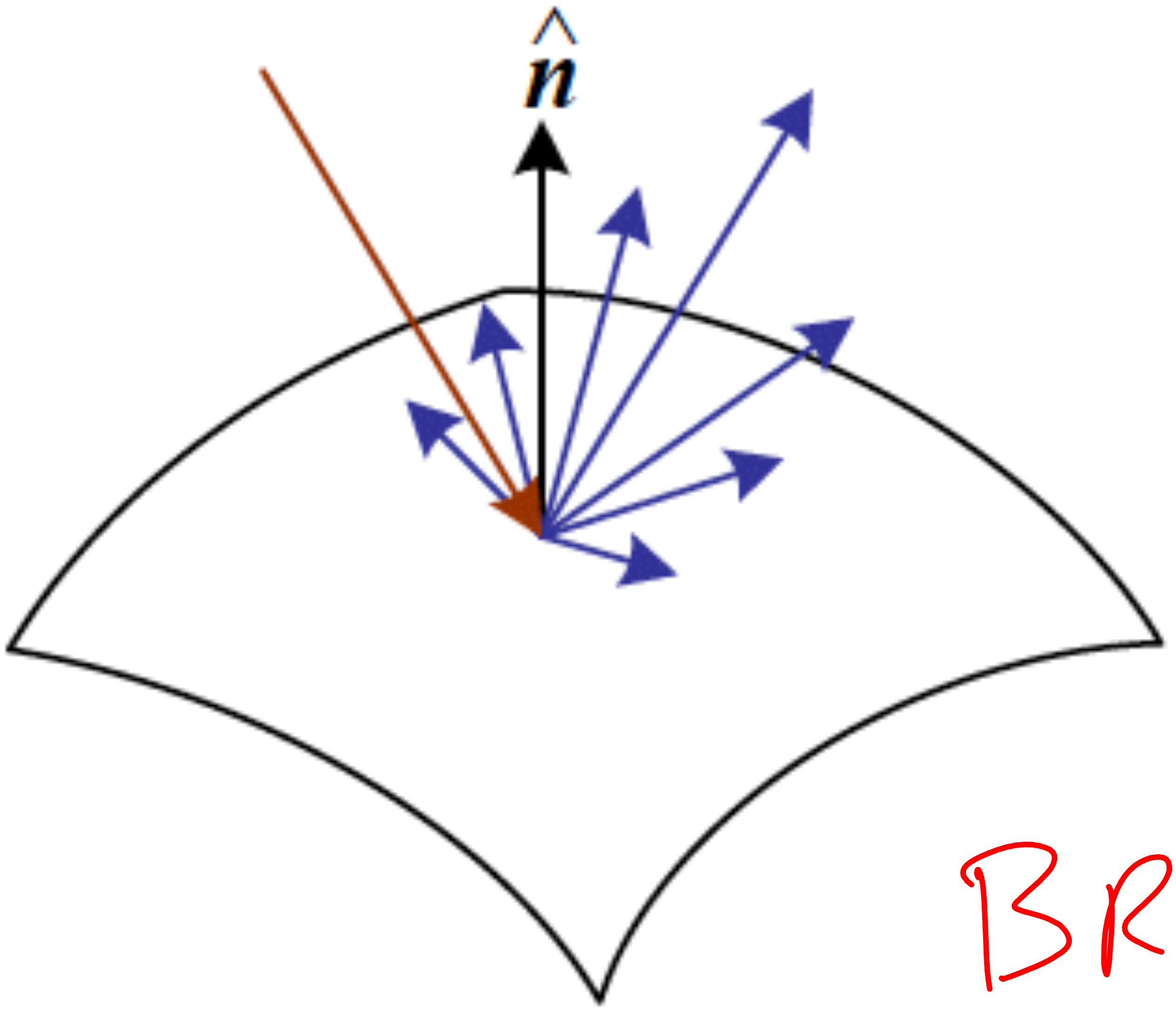


Fig. 16.15 Cube and plane illuminated using Warn lighting controls. (a) Uniformly radiating point source (or $p = 0$). (b) $p = 4$. (c) $p = 32$. (d) Flaps. (e) Cone with $\delta = 18°$. (By David Kurlander, Columbia University.)

From Foley, vanDam, Feiner, and Hughes, **Computer Graphics: Principles and Practice, 2nd edition, page 732, 733**

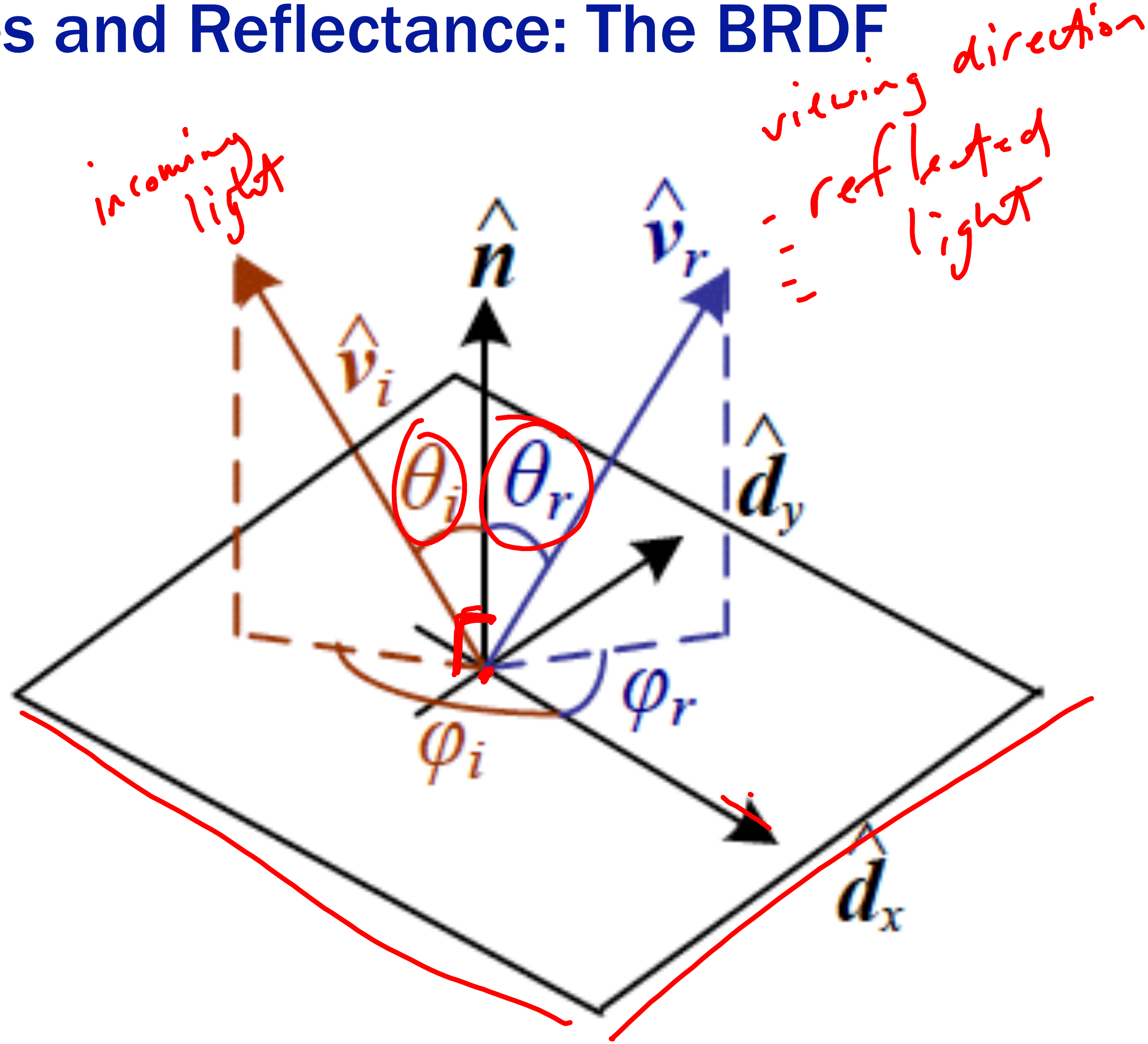
Surfaces and Reflectance



BRDF

Source: Szeliski book.

Surfaces and Reflectance: The BRDF

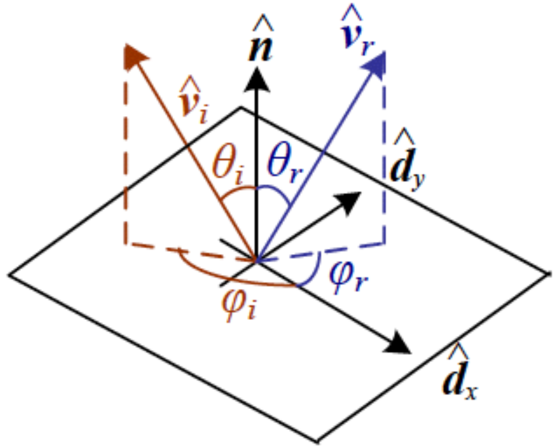


Source: Szeliski book.

The Bidirectional Reflectance Distribution Function

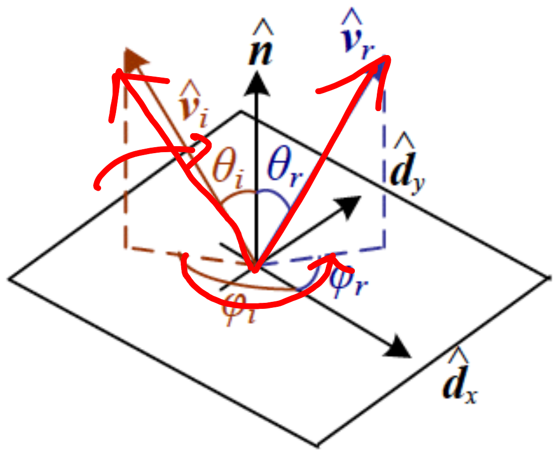
- A general model of light scattering

$$f_r(\theta_i, \varphi_i, \theta_r, \varphi_r; \lambda)$$



The Bidirectional Reflectance Distribution Function

- A general model of light scattering



$$f_r(\boxed{\theta_i, \varphi_i,} \theta_r, \varphi_r; \lambda)$$

Incident light parameters

The Bidirectional Reflectance Distribution Function

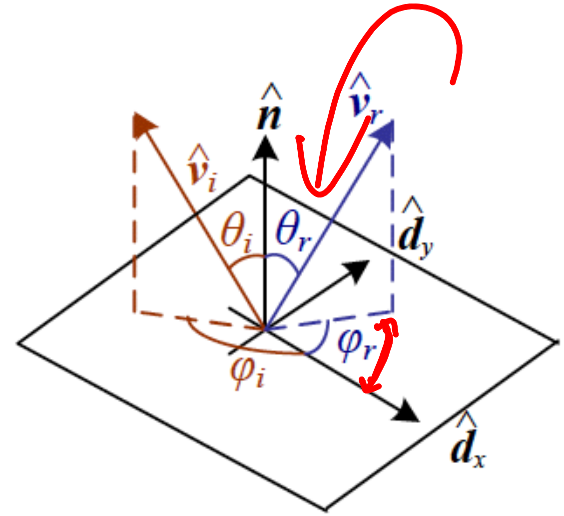
- A general model of light scattering

$$f_r(\boxed{\theta_i, \varphi_i;} \boxed{\theta_r, \varphi_r;} \lambda)$$

Incident light parameters

Reflected light parameters

viewing parameters



The Bidirectional Reflectance Distribution Function

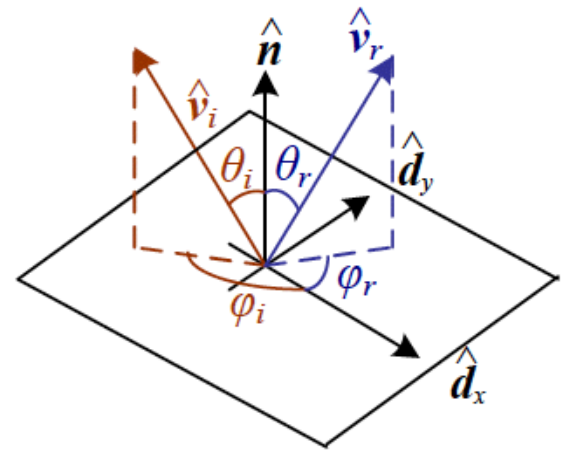
- A general model of light scattering

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r; \lambda)$$

Incident light parameters

Light parameter

Reflected light parameters



The Bidirectional Reflectance Distribution Function

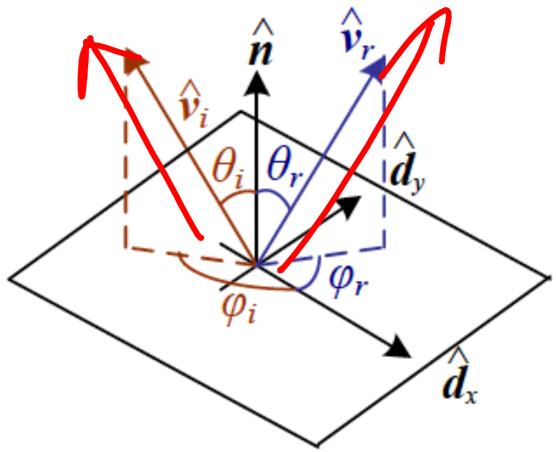
- A general model of light scattering

$$f_r(\boxed{\theta_i, \varphi_i}, \boxed{\theta_r, \varphi_r}; \boxed{\lambda})$$

Incident light parameters

Light parameter

Reflected light parameters



- Helmholtz reciprocity

The Bidirectional Reflectance Distribution Function

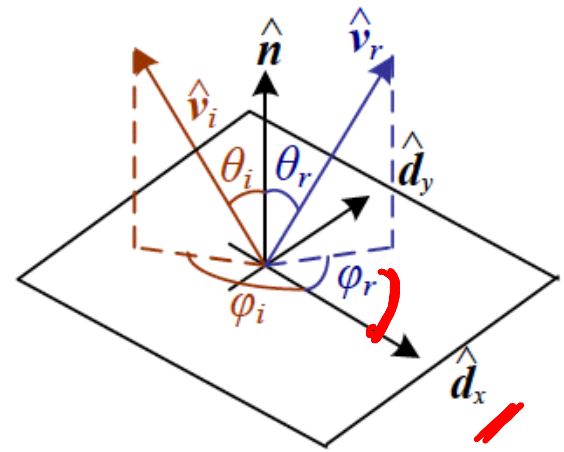
- A general model of light scattering

$$f_r(\boxed{\theta_i, \varphi_i}, \boxed{\theta_r, \varphi_r}; \boxed{\lambda})$$

Incident light parameters

Reflected light parameters

Light parameter

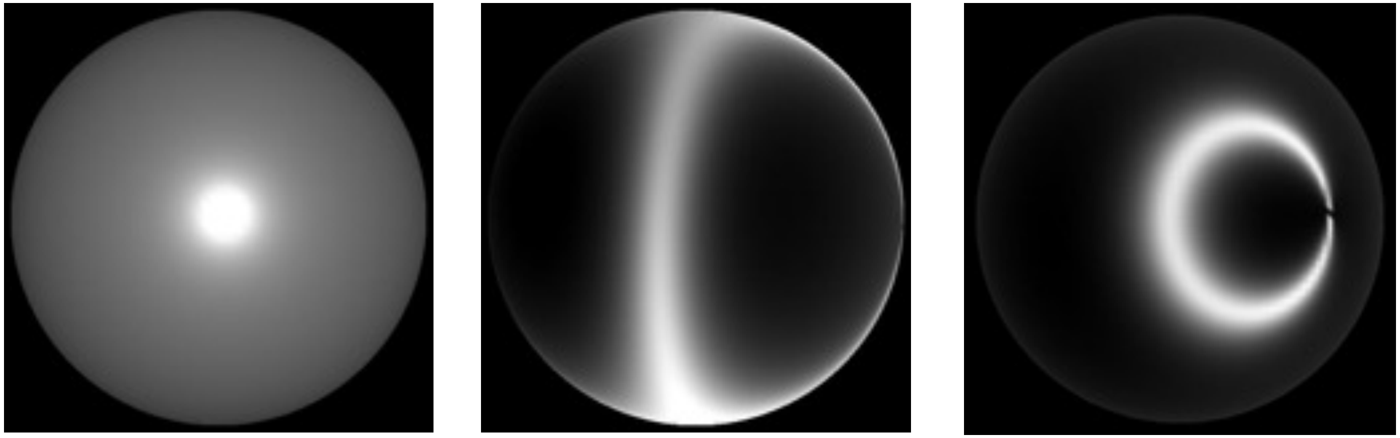


- Helmholtz reciprocity
- Simplified form for isotropic materials

$$f_r(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}}; \lambda)$$

BRDF

- Isotropic vs. Anisotropic

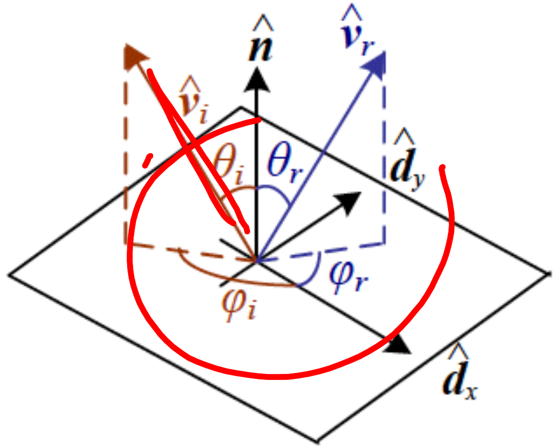


amount of light reflected depends on direction.

Source: Hanrahan slides.

BRDF

- Light exiting a surface point in a direction under a given lighting condition:



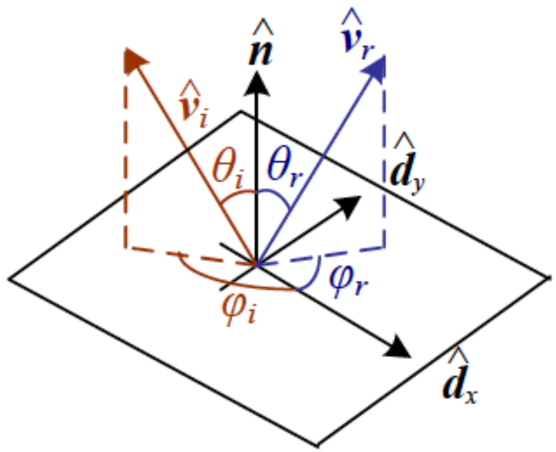
$$L(\hat{\boldsymbol{v}}_r;\lambda) = \int L_i(\hat{\boldsymbol{v}}_i;\lambda) f_r(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}};\lambda) \cos^+\theta_i d\hat{\boldsymbol{v}}_i$$

$$\cos^+\theta_i = \max(0, \cos\theta_i)$$

isotropic case

BRDF

- Light exiting a surface point in a direction under a given lighting condition:



$$L(\hat{\boldsymbol{v}}_r;\lambda) = \int L_i(\hat{\boldsymbol{v}}_i;\lambda) f_r(\hat{\boldsymbol{v}}_i,\hat{\boldsymbol{v}}_r,\hat{\boldsymbol{n}};\lambda) \cos^+\theta_i \, d\hat{\boldsymbol{v}}_i$$

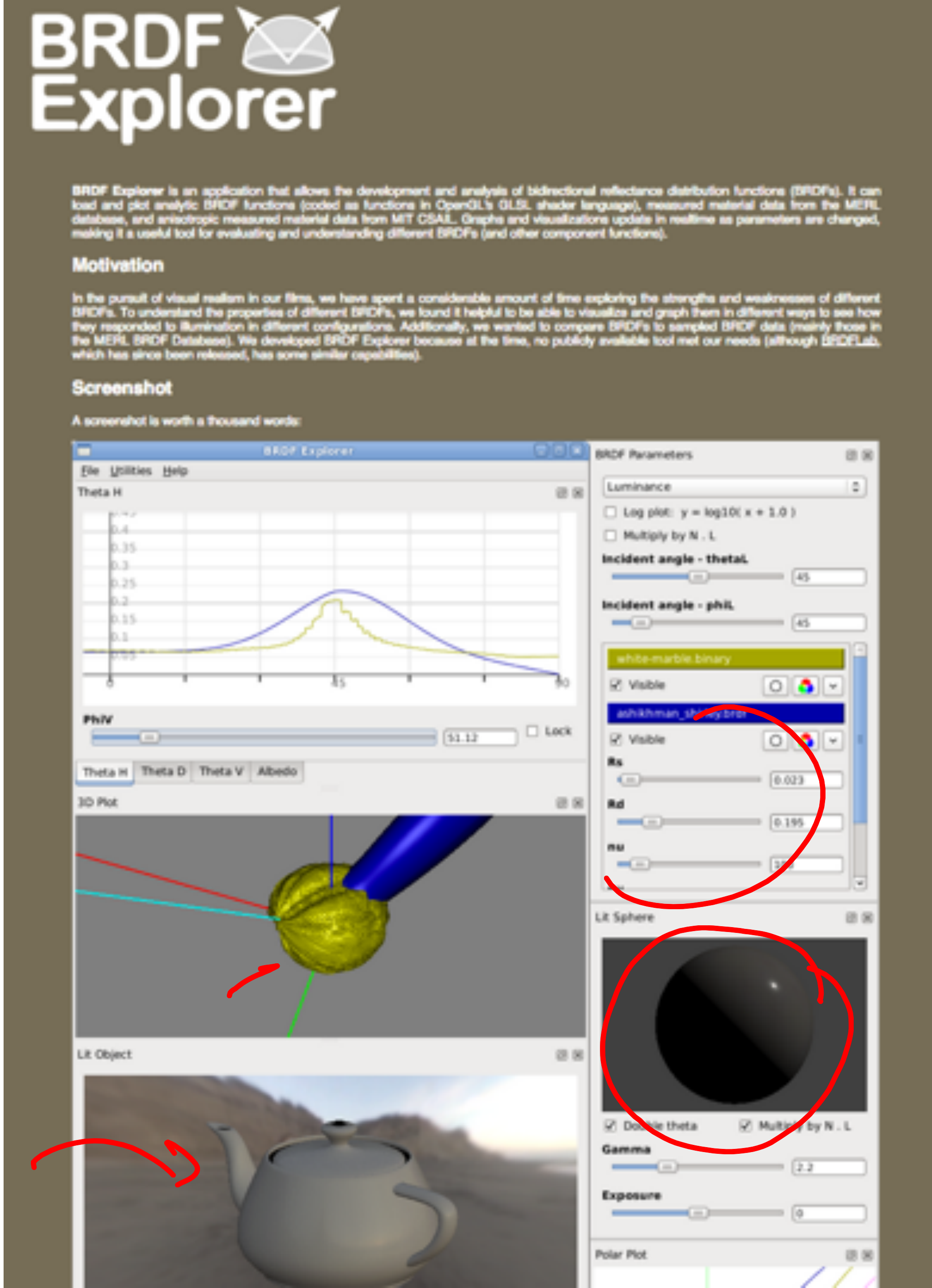
$$\cos^+\theta_i = \max(0,\cos\theta_i)$$

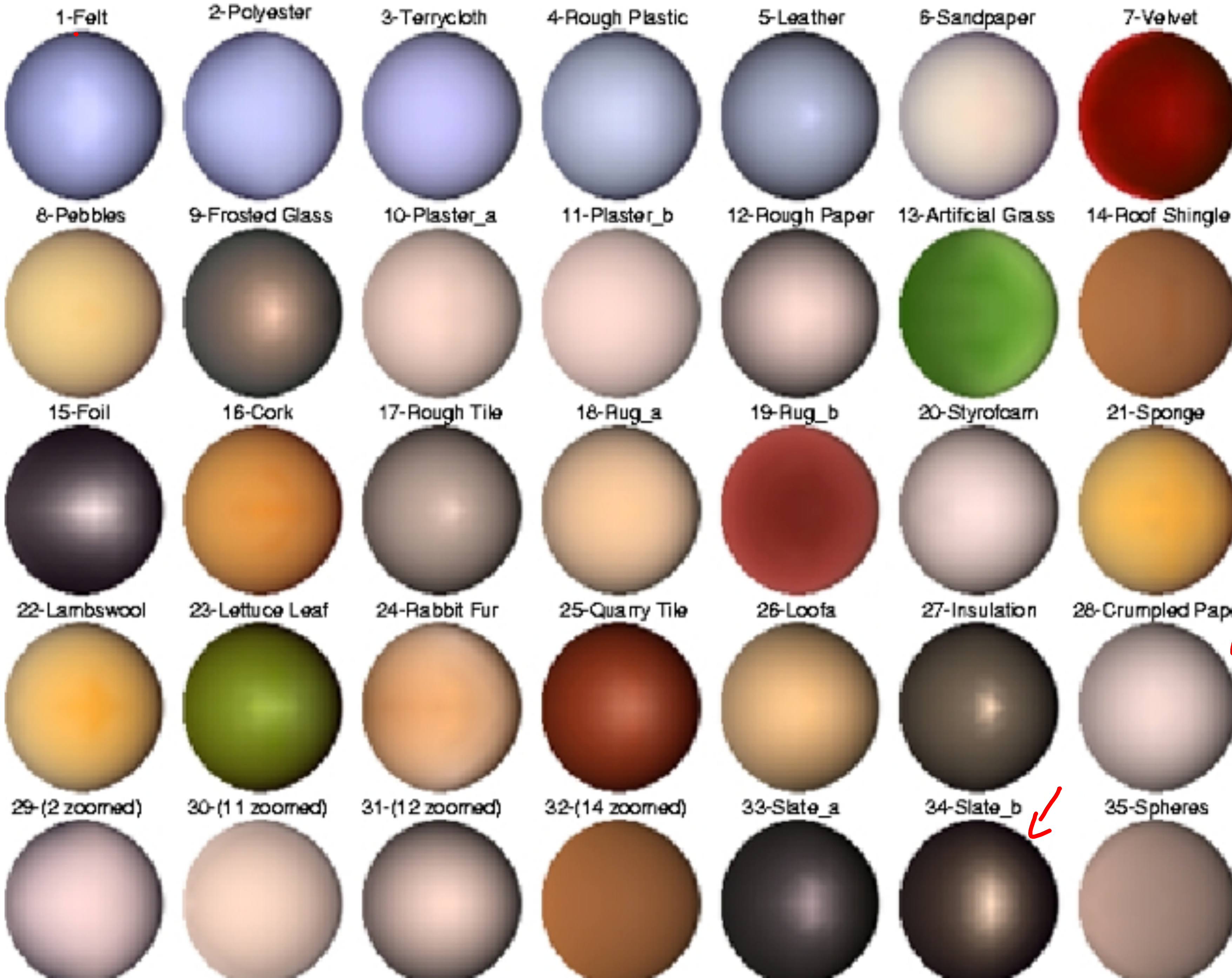
Foreshortening effect due to surface orientation

BRDF

- Understanding and modeling the BRDF is critical to realism in graphics as well as various computer vision applications.

- http://www.disneyanimation.com/technology/brdf.html





Source: BRDFs Rendered on Spheres. From http://www1.cs.columbia.edu/CAVE/software/curet/html/sphere_meas.html.

Diffuse or Lambertian Reflection

- Light is scattered uniformly in all directions.

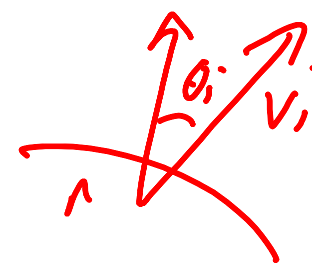
$\hat{v}_r$

$$f_d(\hat{\boldsymbol{v}}_i, \cancel{\hat{\boldsymbol{r}}_i}, \hat{\boldsymbol{n}}; \lambda) = f_d(\lambda)$$

- The amount of light depends on the angle between the incident light direction and the surface normal.
  – Lambert's cosine law

$$L_d(\hat{\boldsymbol{v}}_r; \lambda) = \sum_i L_i(\lambda) f_d(\lambda) \cos^+ \theta_i$$

$$= \sum_i \underline{L_i(\lambda)} f_d(\lambda) \underline{\left[\hat{\boldsymbol{v}}_i^\mathsf{T} \hat{\boldsymbol{n}}\right]^+}$$



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Diffuse Reflection Examples

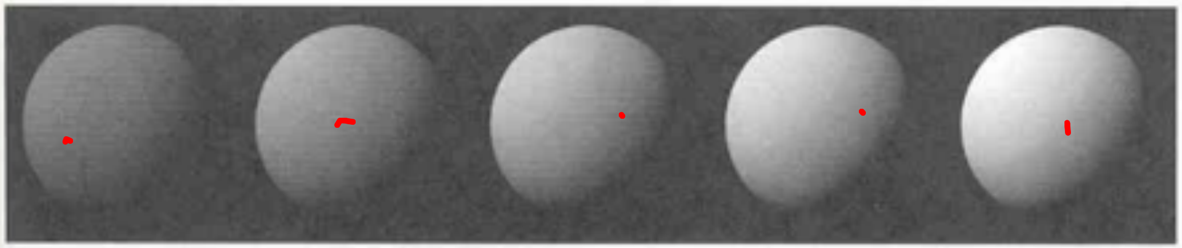


Fig. 16.3 Spheres shaded using a diffuse-reflection model (Eq. 16.4). For all spheres, $I_p = 1.0$. From left to right, $k_d = 0.4, 0.55, 0.7, 0.85, 1.0$. (By David Kurlander, Columbia University.)

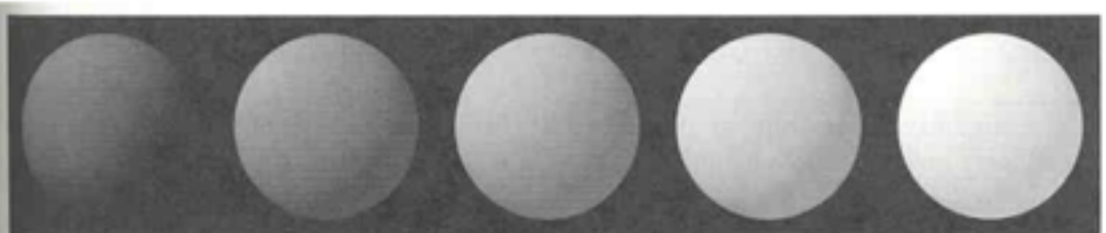
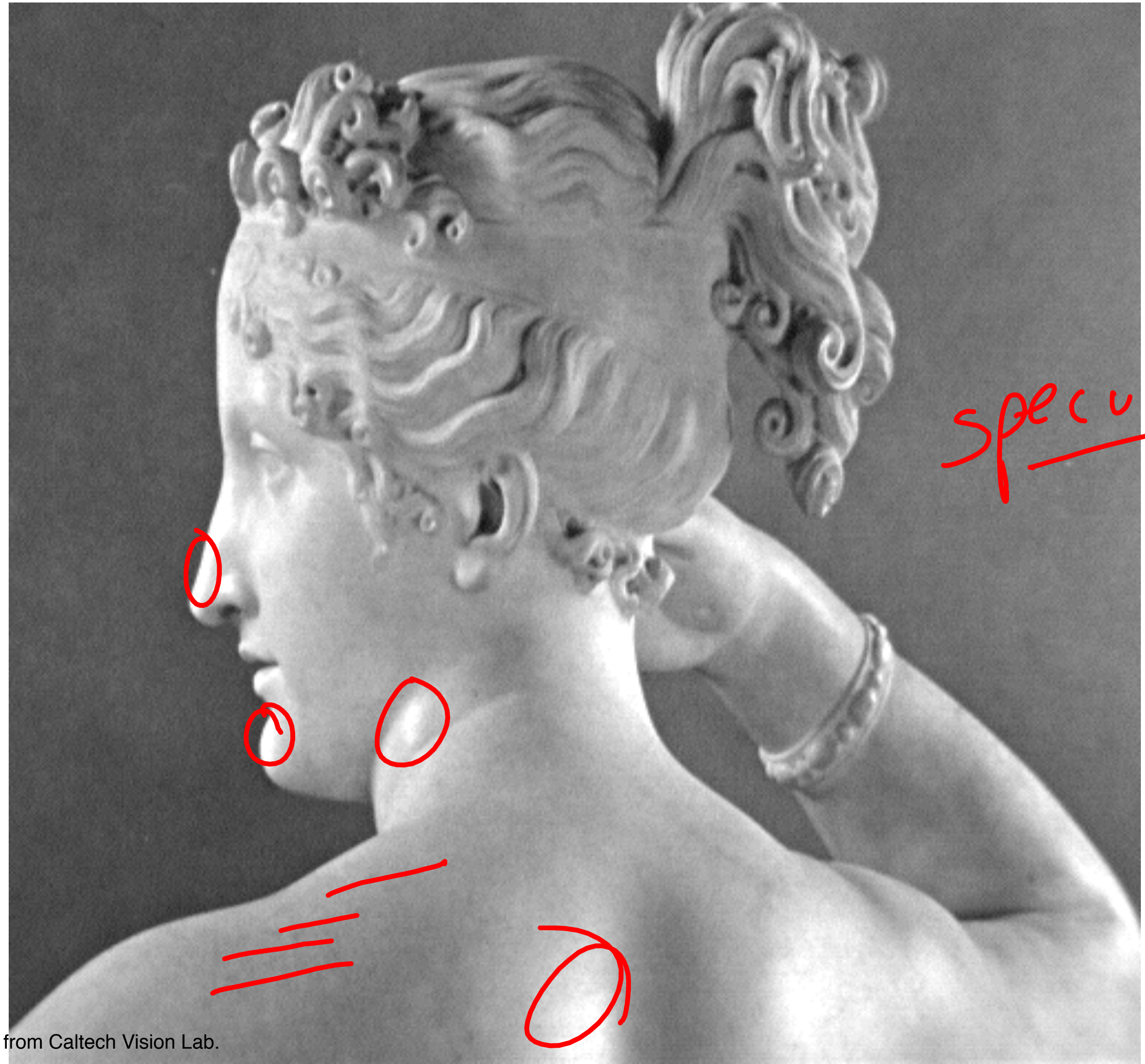


Fig. 16.4 Spheres shaded using ambient and diffuse reflection (Eq. 16.5). For all spheres, $I_a = I_p = 1.0$, $k_d = 0.4$. From left to right, $k_a = 0.0, 0.15, 0.30, 0.45, 0.60$. (By David Kurlander, Columbia University.)

From Foley, vanDam, Feiner, and Hughes, **Computer Graphics: Principles and Practice, 2nd edition, page 725**

Diffuse Vs. Specularity

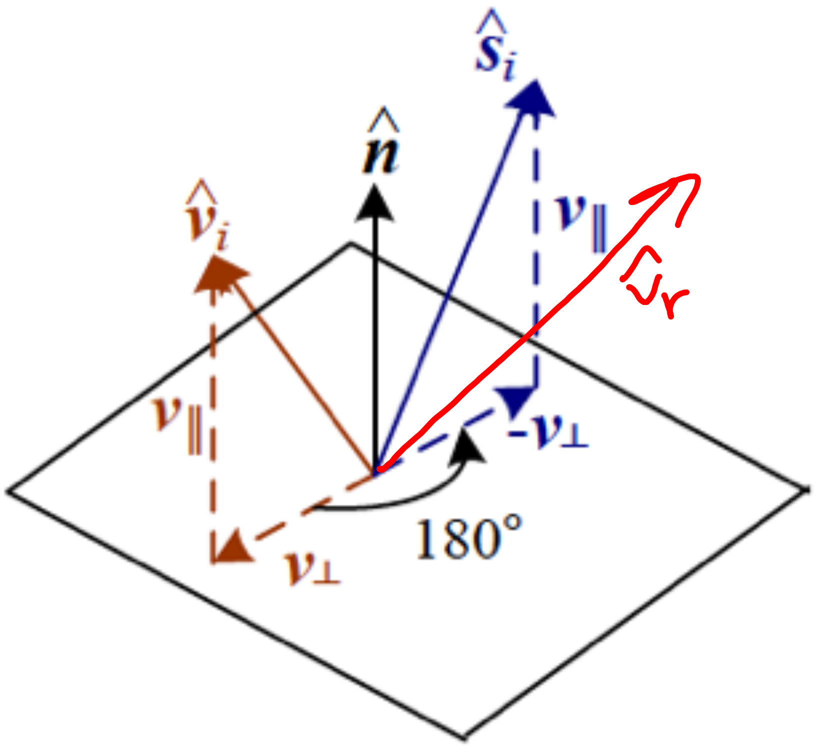


specularities

Source: Paolina Image from Caltech Vision Lab.

Specular (or Mirror) Reflections

- Specularity depends strongly on the direction of the outgoing light.
- Mirror-like reflection: incoming light is reflected off the surface in a single direction (which is the rotation of 180 degree around the surface normal).



Phong Shading Model

$$
\begin{aligned}
L_{\text{Phong}}(\hat{\boldsymbol{v}}_r; \lambda) = {} & k_a(\lambda) L_a(\lambda) + \\
& k_d(\lambda) \sum_i L_i(\lambda) f_d(\lambda) \left[\hat{\boldsymbol{v}}_i^{\mathsf{T}} \hat{\boldsymbol{n}}\right]^+ + \\
& k_s(\lambda) \sum_i L_i(\lambda) \left[\hat{\boldsymbol{v}}_r^{\mathsf{T}} \hat{\boldsymbol{s}}_i\right]^{k_e}
\end{aligned}
$$

Phong Shading Model

Ambient Light

$$L_{\text{Phong}}(\hat{\boldsymbol{v}}_r; \lambda) = \boxed{k_a(\lambda) L_a(\lambda)} +$$

$$k_d(\lambda) \sum_i L_i(\lambda) f_d(\lambda) \left[\hat{\boldsymbol{v}}_i^{\mathsf{T}} \hat{\boldsymbol{n}}\right]^+ +$$

$$k_s(\lambda) \sum_i L_i(\lambda) \left[\hat{\boldsymbol{v}}_r^{\mathsf{T}} \hat{\boldsymbol{s}}_i\right]^{k_e}$$

Phong Shading Model

Ambient Light

Specular Reflectance

$$L_{\text{Phong}}(\hat{\boldsymbol{v}}_r;\lambda) = \boxed{k_a(\lambda)L_a(\lambda)} +$$

Lambertian Model

$$k_d(\lambda)\sum_i L_i(\lambda) f_d(\lambda)\left[\hat{\boldsymbol{v}}_i^{\mathsf{T}}\hat{\boldsymbol{n}}\right]^+ +$$

$$\boxed{k_s(\lambda)\sum_i \underline{L_i(\lambda)}\left[\hat{\boldsymbol{v}}_{\underline{r}}^{\mathsf{T}}\hat{\boldsymbol{s}}_i\right]^{k_e}}$$

-> scalar constant

$$\hat{\boldsymbol{s}}_i = \boldsymbol{v}_{\|} - \boldsymbol{v}_{\perp} = (2\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^{\mathsf{T}} - \boldsymbol{I})\boldsymbol{v}_i$$

Phong Shading Model

Ambient Light

Diffuse Reflectance

Specular Reflectance

$$L_{\text{Phong}}(\hat{\boldsymbol{v}}_r;\lambda) = \boxed{k_a(\lambda)L_a(\lambda)} + \boxed{k_d(\lambda)\sum_i L_i(\lambda) f_d(\lambda)\left[\hat{\boldsymbol{v}}_i^{\mathsf{T}}\hat{\boldsymbol{n}}\right]^+ +} \boxed{k_s(\lambda)\sum_i L_i(\lambda)\left[\hat{\boldsymbol{v}}_r^{\mathsf{T}}\hat{\boldsymbol{s}}_i\right]^{k_e}}$$

$$\hat{\boldsymbol{s}}_{\boldsymbol{i}} = \boldsymbol{v}_{||} - \boldsymbol{v}_{\perp} = (2\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^{\mathsf{T}} - \boldsymbol{I})\boldsymbol{v}_{\boldsymbol{i}}$$

Phong Illumination Example

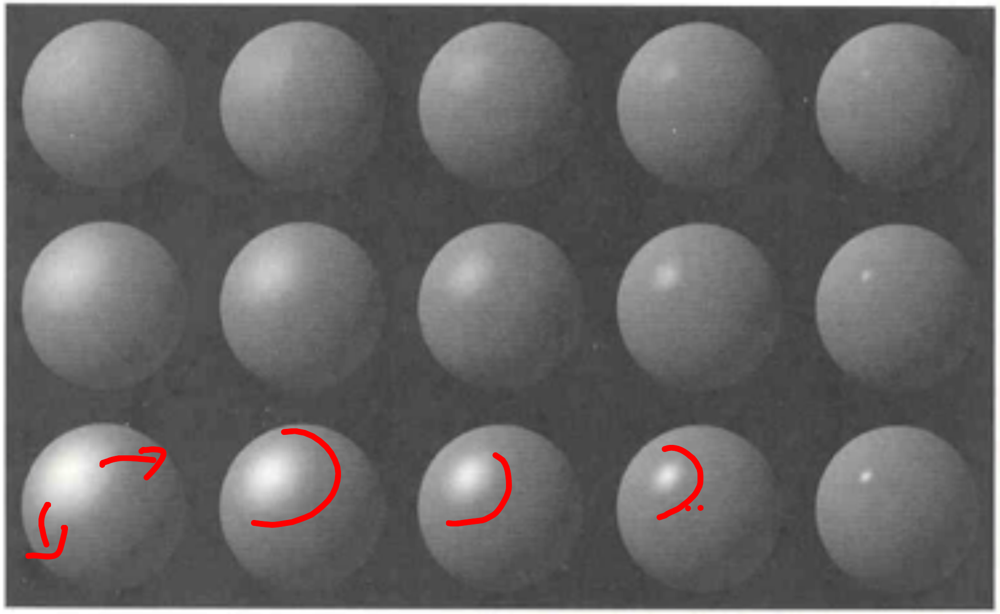


Fig. 16.10 Spheres shaded using Phong's illumination model (Eq. 16.14) and different values of k_s and n. For all spheres, $I_a = I_p = 1.0$, $k_a = 0.1$, $k_d = 0.45$. From left to right, n = 3.0, 5.0, 10.0, 27.0, 200.0. From top to bottom, k_s = 0.1, 0.25, 0.5. (By David Kurlander, Columbia University.)

larger ks the more diffuse the specularity.

***From Foley, vanDam, Feiner, and Hughes,* Computer Graphics: Principles and Practice, 2nd edition, page 730**

Torrance and Sparrow Shading

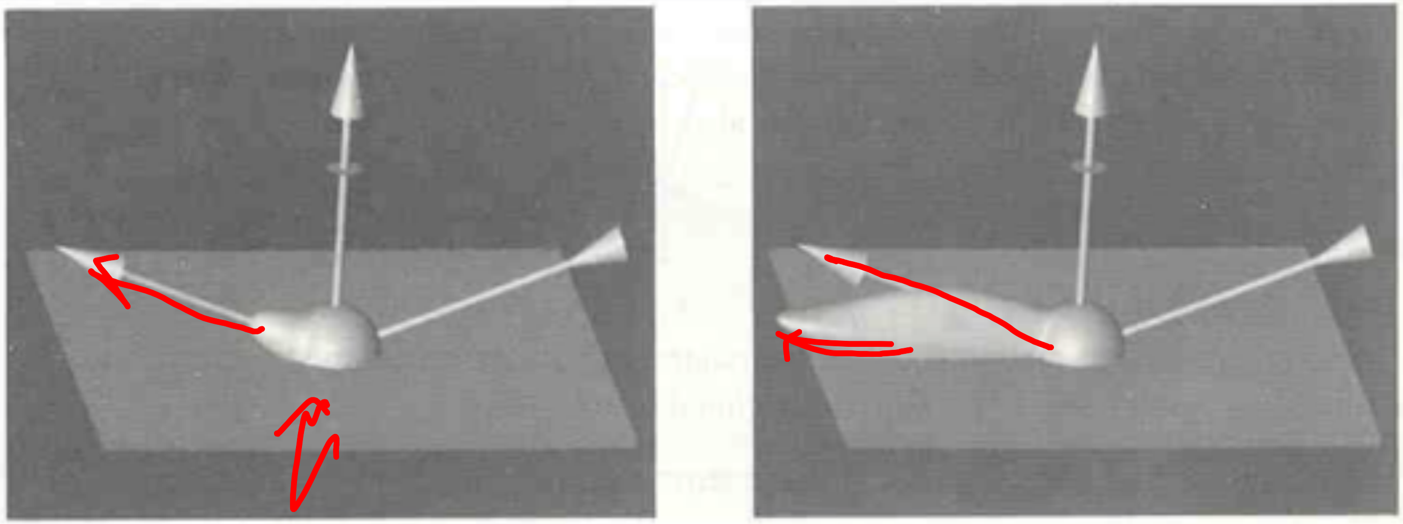
- Phong shading used a power of the cosine of the angle law

$$f_s(\theta_s;\lambda) = k_s(\lambda)\cos^{k_e}\theta_s$$

- The Torrance and Sparrow model uses a Gaussian

$$f_s(\theta_s;\lambda) = k_s(\lambda)\exp\left(-c_s^2\theta_s^2\right)$$

Phong vs. Cook/Torrance Example



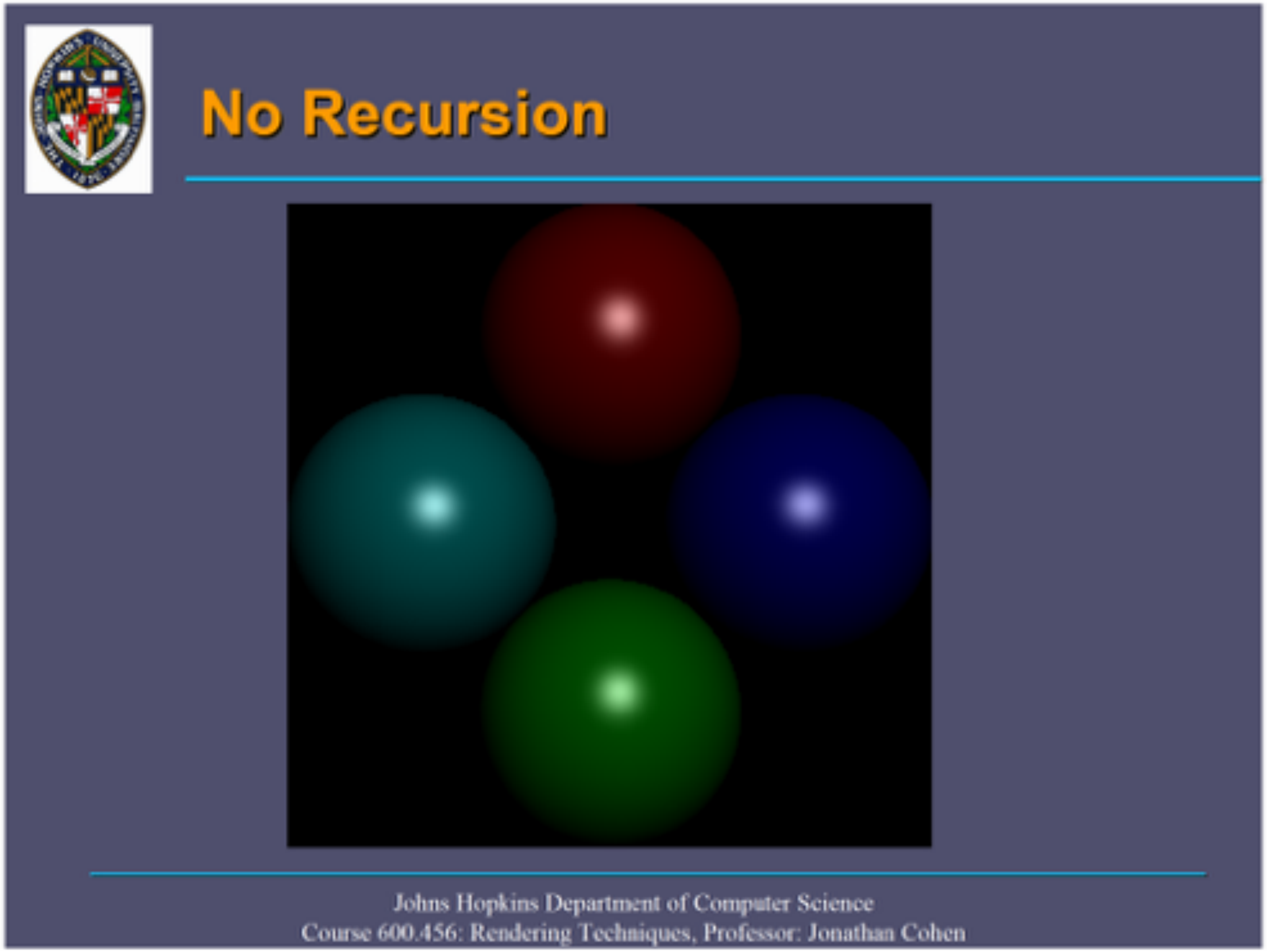
(a) Phong model

(b) Torrance-Sparrow model

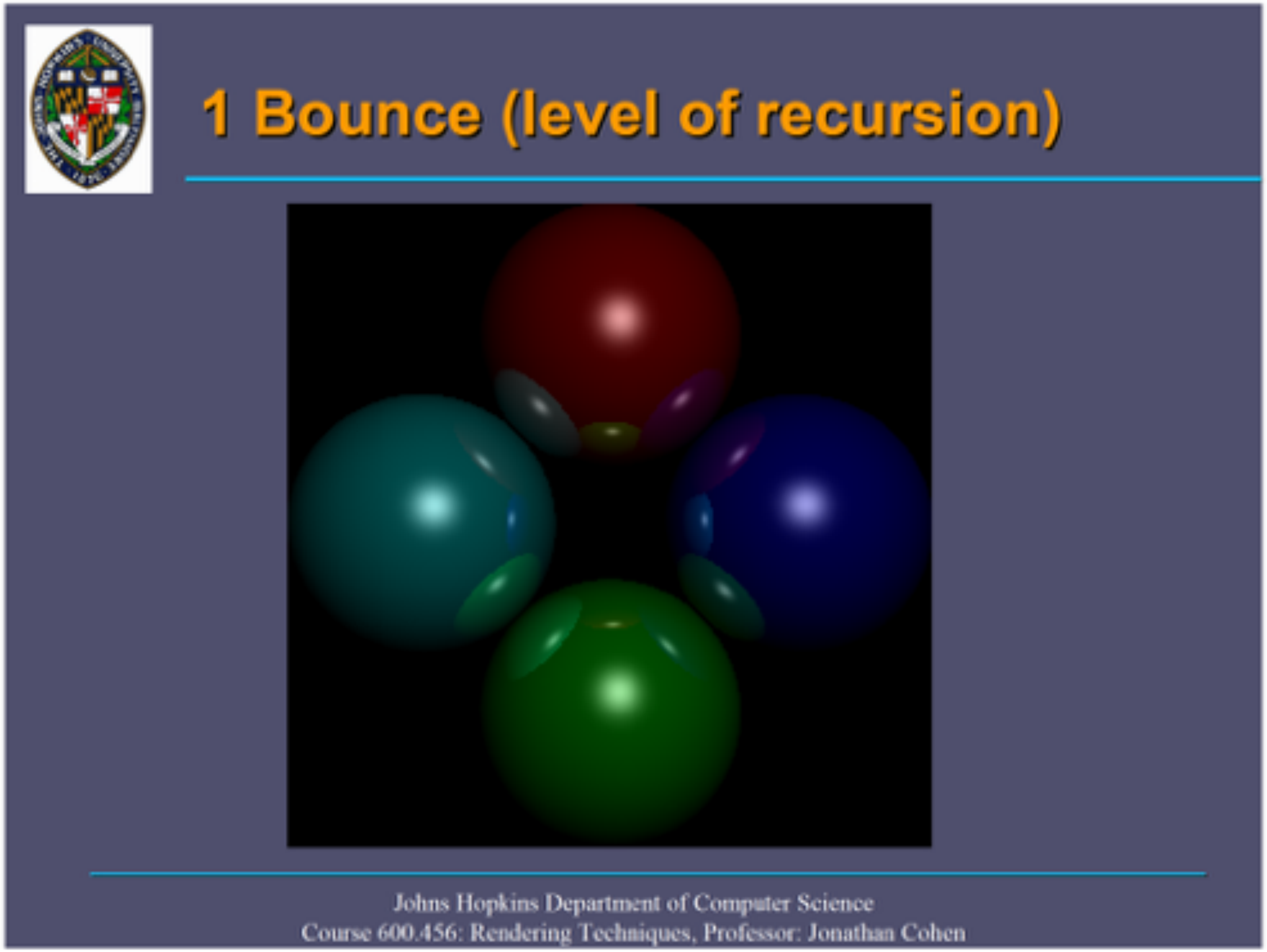
Fig. 16.44 Comparison of Phong and Torrance–Sparrow illumination models for light at a 70° angle of incidence. (By J. Blinn [BLIN77a], courtesy of the University of Utah.)

From Foley, vanDam, Feiner, and Hughes, **Computer Graphics: Principles and Practice, 2nd edition, page 768**

Wait a minute; light bounces…a lot



Wait a minute; light bounces…a lot



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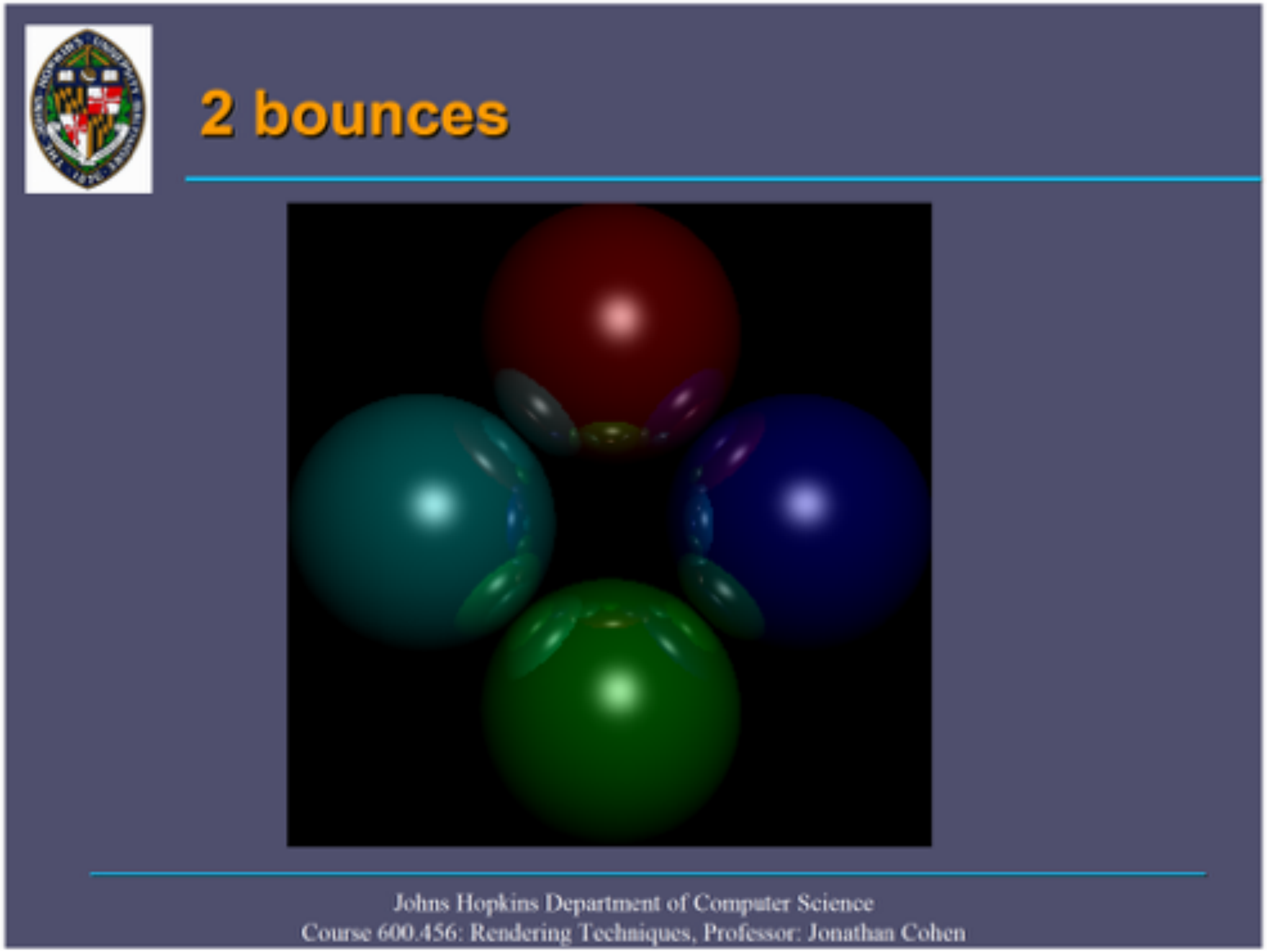

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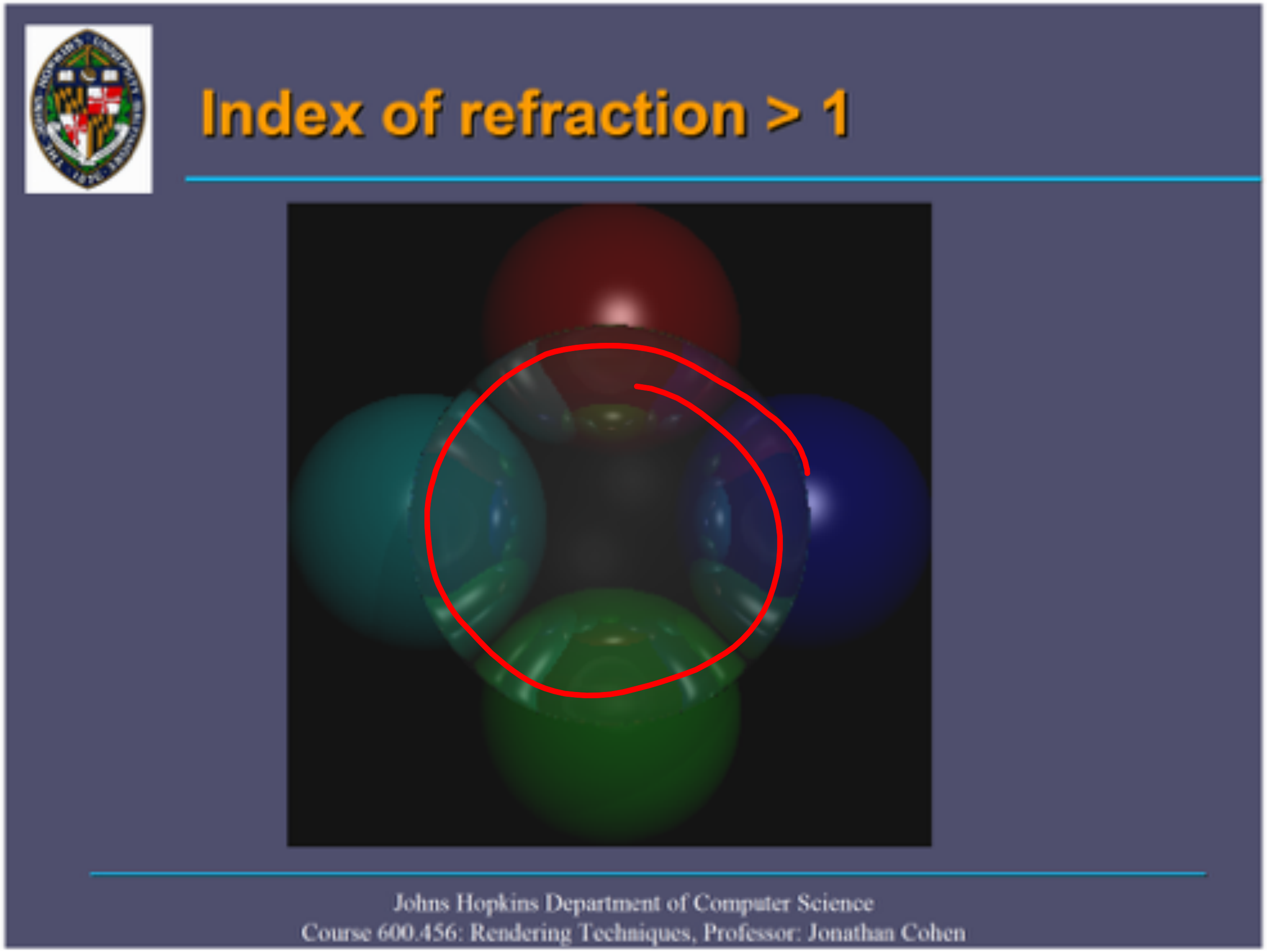


Camera Response and Color

A Word On Computer-Imaging

- Video imaging has gone from an exotic technology to everyday commodity.

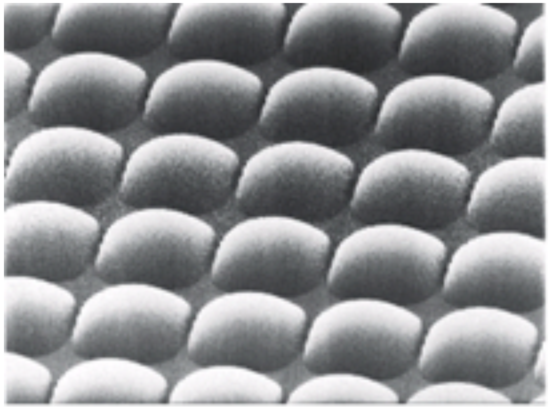
- Originally (since ~1930) NTSC standard
  - 480 x 640 YUV
  - Interlaced

- Now, a wide variety of resolutions and quality
  - VGA (= NTSC)
  - SVGA (= 600x800)
  - XVGA (= 768x1024)
  - SXGA (=1024x1280)
  - UGA (= 1200x1600)
  - HD (= 1080x1960)
  - SHD (=1080x1960x2)

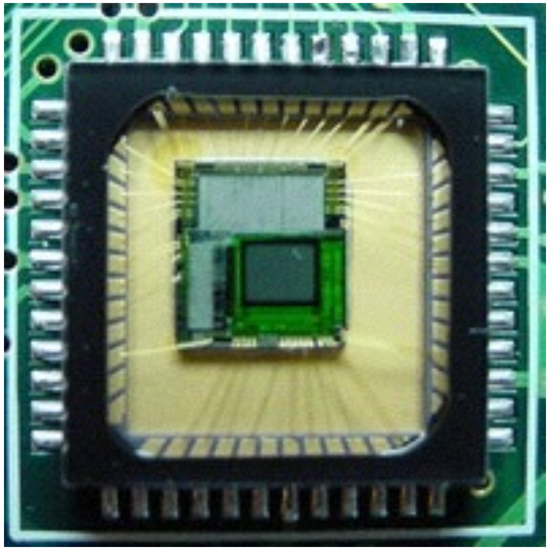
Source: G Hager Slides

How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness

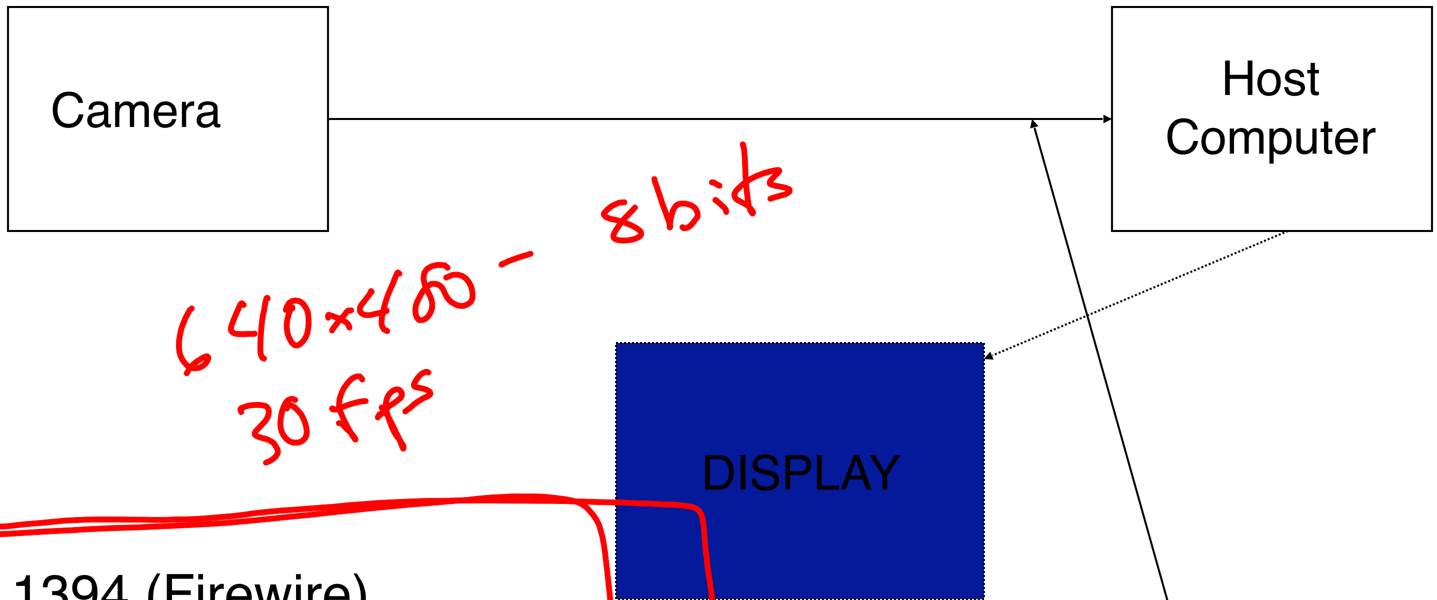


- Sensor types:
  - CCD (charge-coupled device)
    - most common
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed



Source: G Hager Slides

A Modern Digital Camera



640x480 - 8 bits
30 fps

IEEE 1394 (Firewire)
400 Mbit/sec sync/async transfer
Supports device control

USB 2.0
480 Mbit/sec (~280Mbit/sec in practice)
Less flexible, but simpler to implement

USB 3 960 Mbit/s.

Source: G Hager Slides

Other Issues

- **Automatic Gain Control (AGC)**: adjusting amplification and black level to get a "good fit" of the incident light power to the range of the image

noise

- **Shuttering**: Electronic "switch" that controls how long the CCD is "exposed."

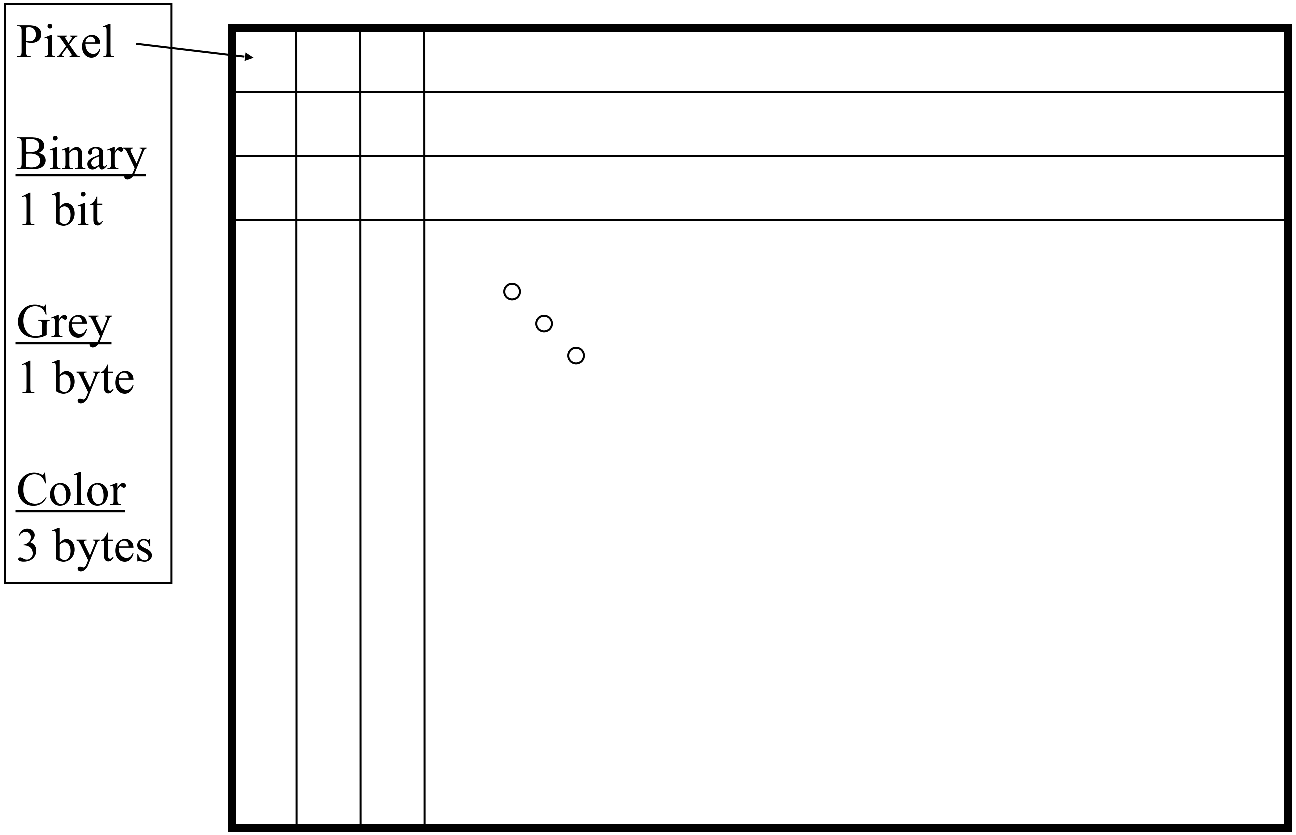
rolling shutter is bad...

- **White balance**: Adjustment of the mapping from measured spectral quantities to image RGB quantities.
- **Vignetting**: darkening of the image around the corners due to various reasons, including light blockage from lens barrel, filters and coatings on the lens, etc.

What's going on here?

Source: G Hager Slides

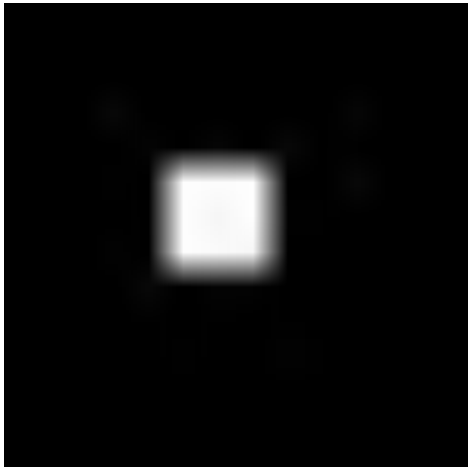
THE ORGANIZATION OF A 2D IMAGE

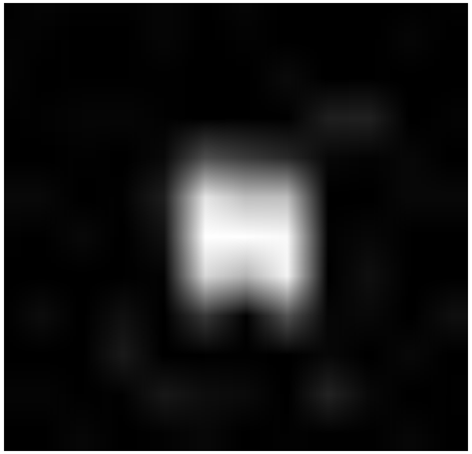


Source: G Hager Slides

Storing Images and Compression

- Non-lossy schemes
  - pbm/pgm/ppm/pnm
    - code for file type, size, number of bands, and maximum brightness
  - tif (lossless and lossy versions)
  - bmp
  - gif (grayscale) 256 bits
  - png
- Lossy schemes
  - gif (color)
  - jpg
    - uses Y Cb Cr color representation; subsamples the color
    - Uses DCT on result
    - Uses the fact the human system is less sensitive to color than spatial detail





Source: G Hager Slides

Storing Images and Compression

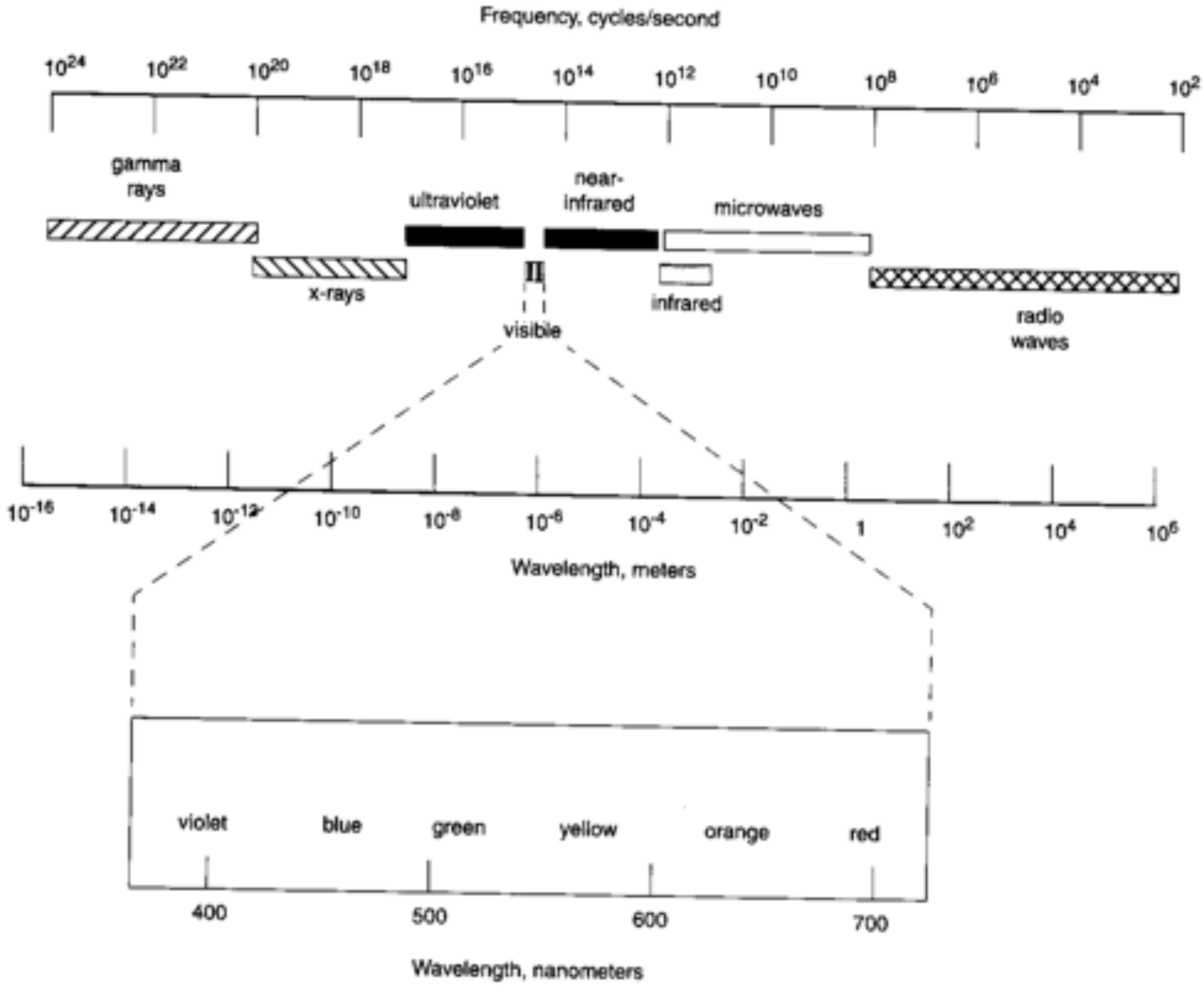
Storing Images and Compression

Storing Images and Compression

Color

Source: G Hager Slides

What is Color?



Source: G Hager Slides

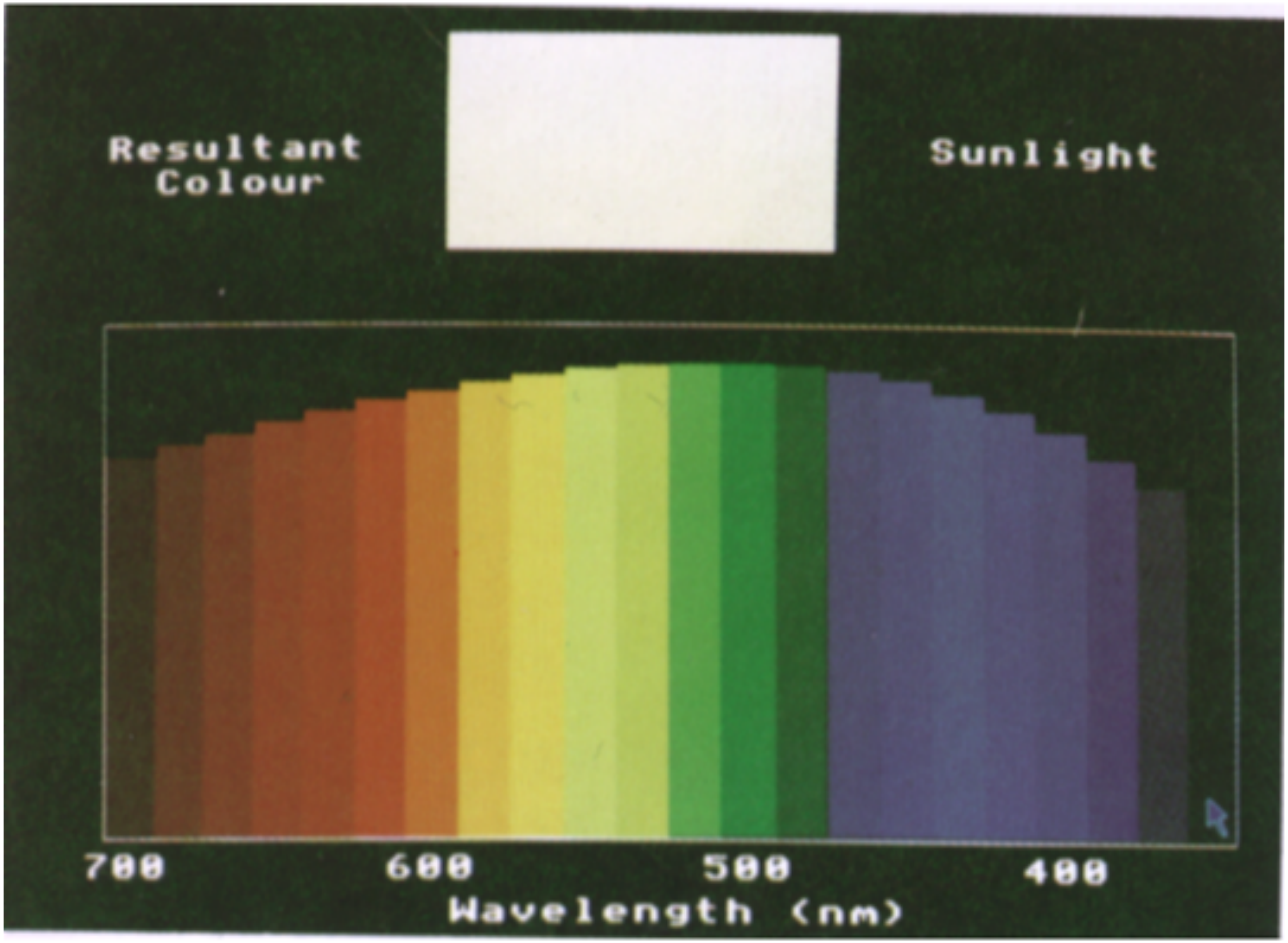
What is Color?

- We almost never see a “pure” wavelength of light; rather a mixture of wavelengths, each with a different “power”

- Only some colors occur as pure wavelengths; many are mixtures of pure colors (e.g. white)

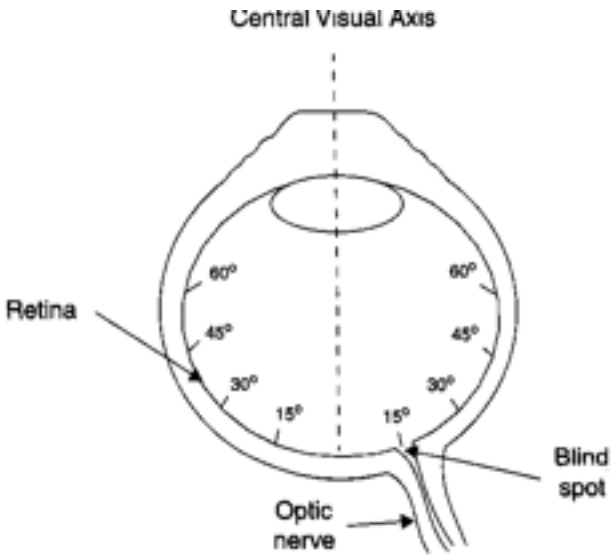
Source: G Hager Slides

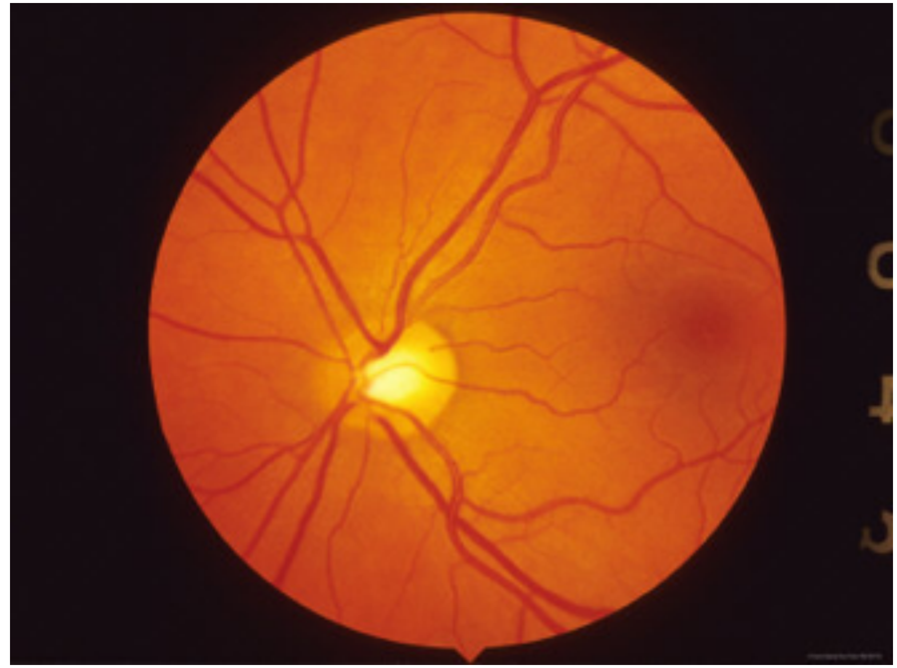
Sunlight

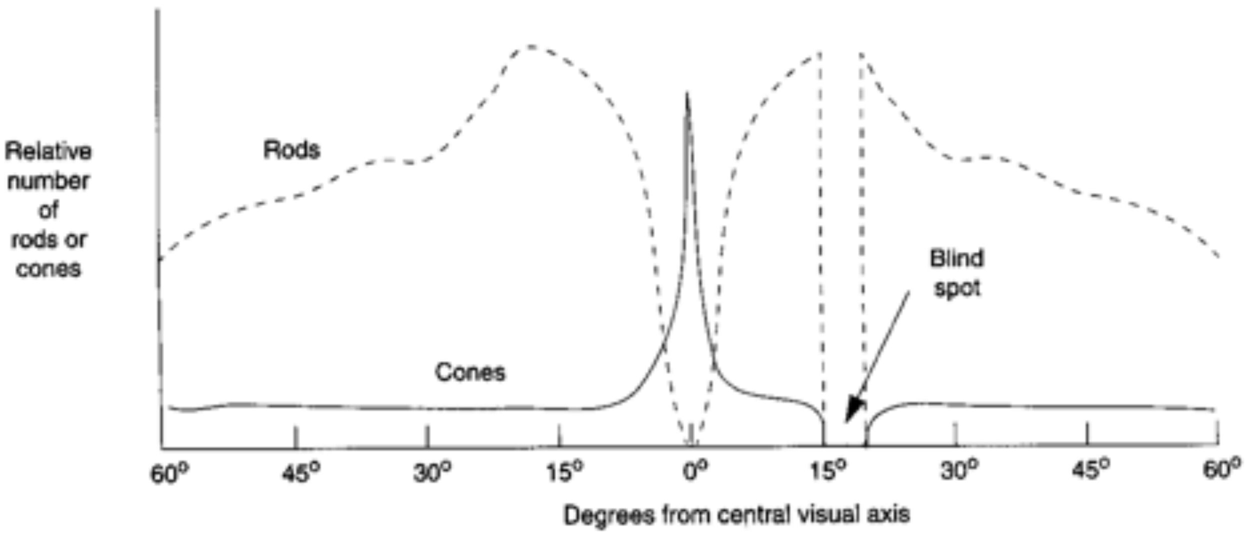


Source: G Hager Slides

Example: The Human Eye

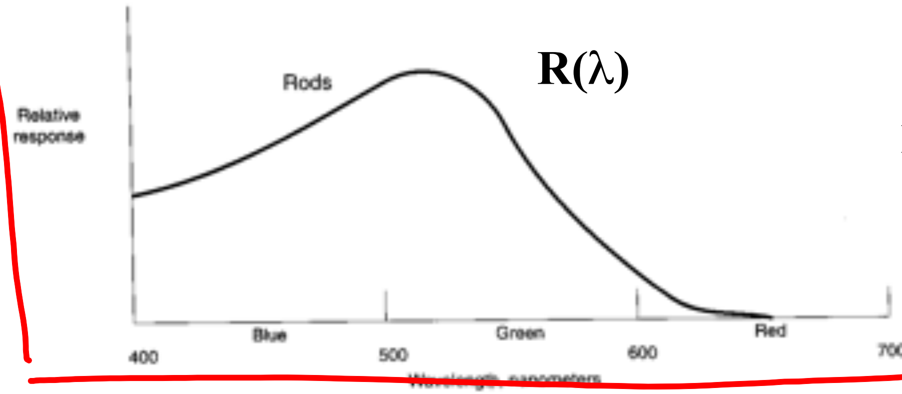




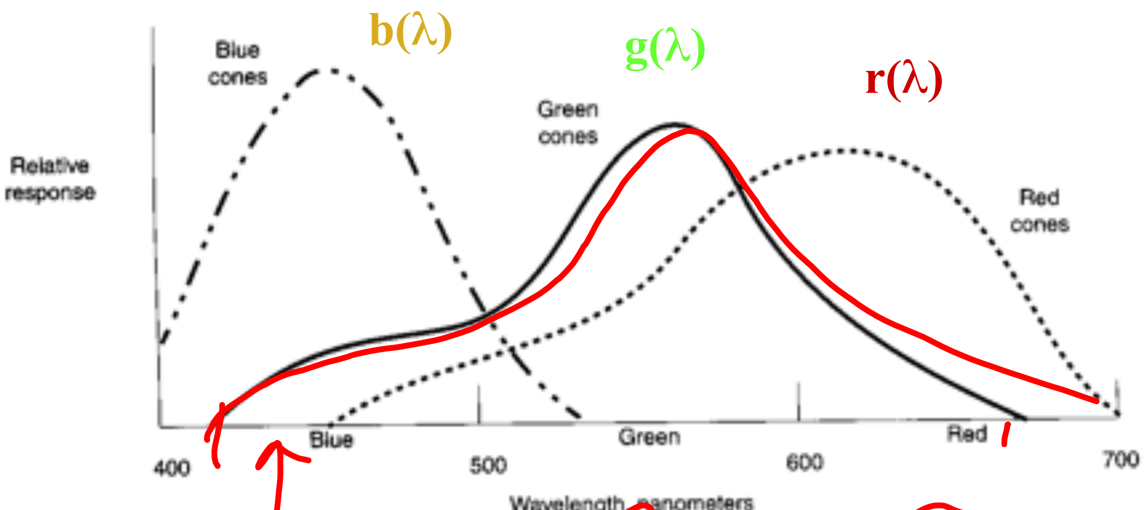


Source: G Hager Slides

The Human Eye Response



$$\text{BRIGHTNESS} = \int_{\lambda=400\,nm}^{\lambda=700\,nm} R(\lambda)\, I(\lambda)\, d\lambda$$



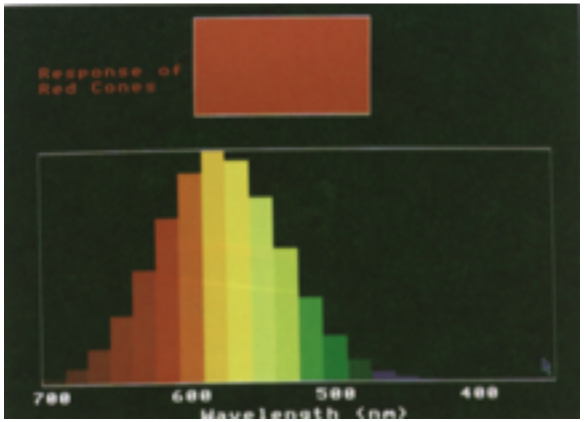
$$\text{RED} = \int_{\lambda=400\,nm}^{\lambda=700\,nm} r(\lambda)\, I(\lambda)\, d\lambda$$

$$\text{GREEN} = \int_{\lambda=400\,nm}^{\lambda=700\,nm} g(\lambda)\, I(\lambda)\, d\lambda$$

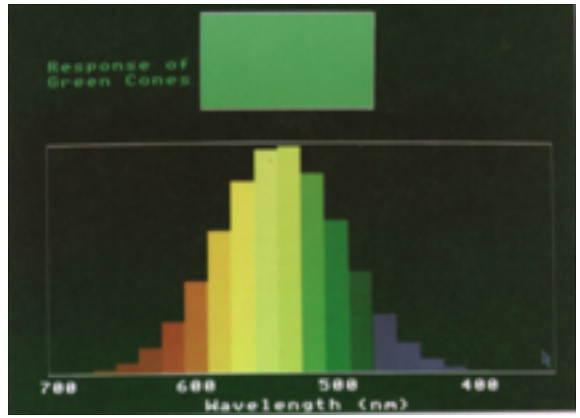
$$\text{BLUE} = \int_{\lambda=400\,nm}^{\lambda=700\,nm} b(\lambda)\, I(\lambda)\, d\lambda$$

Source: G Hager Slides

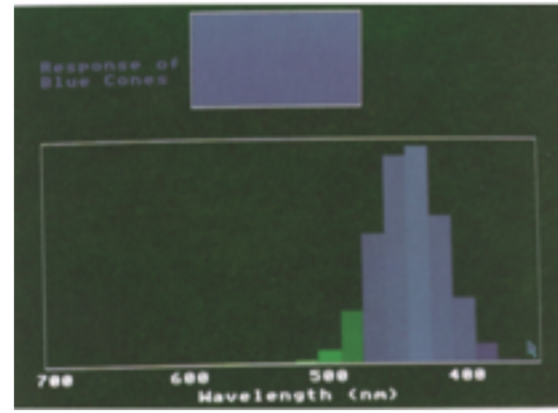
Color receptors



“Red” cone

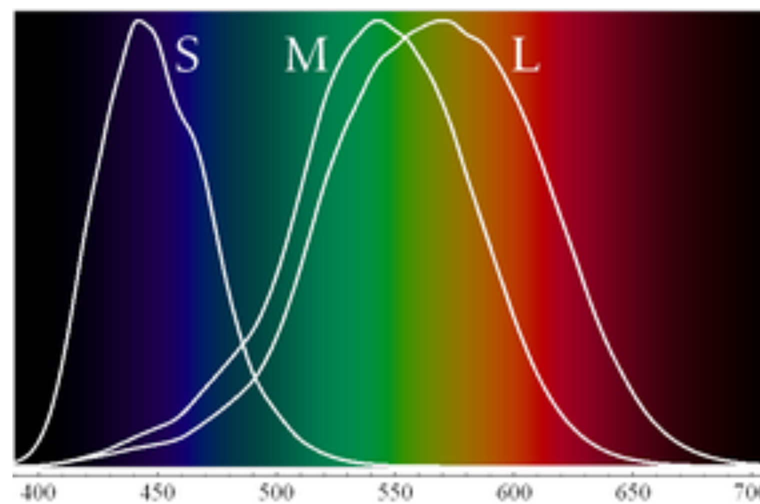


“Green” cone



“Blue” cone

Principle of univariance: cones give the same amount of response to different wavelengths -- a single cone cannot distinguish color. Output of cone is obtained by summing probability of absorption over wavelengths.

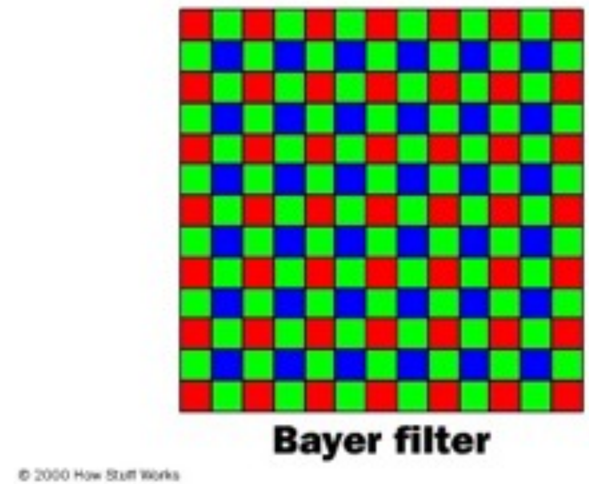


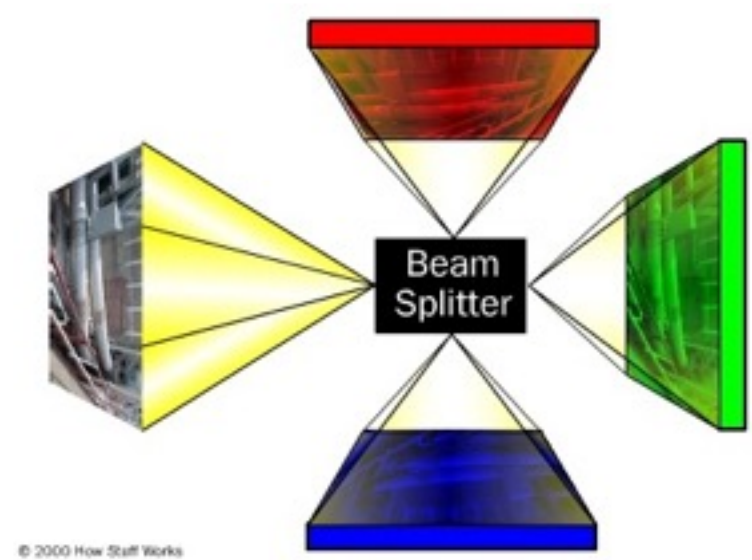
Source: G Hager Slides

How Color Cameras Work

- 1 CCD cameras
  - A **Bayer** pattern is placed in front of the CCD
  - A **Demosaicing** process reads the pixels in a region and computes color and intensity

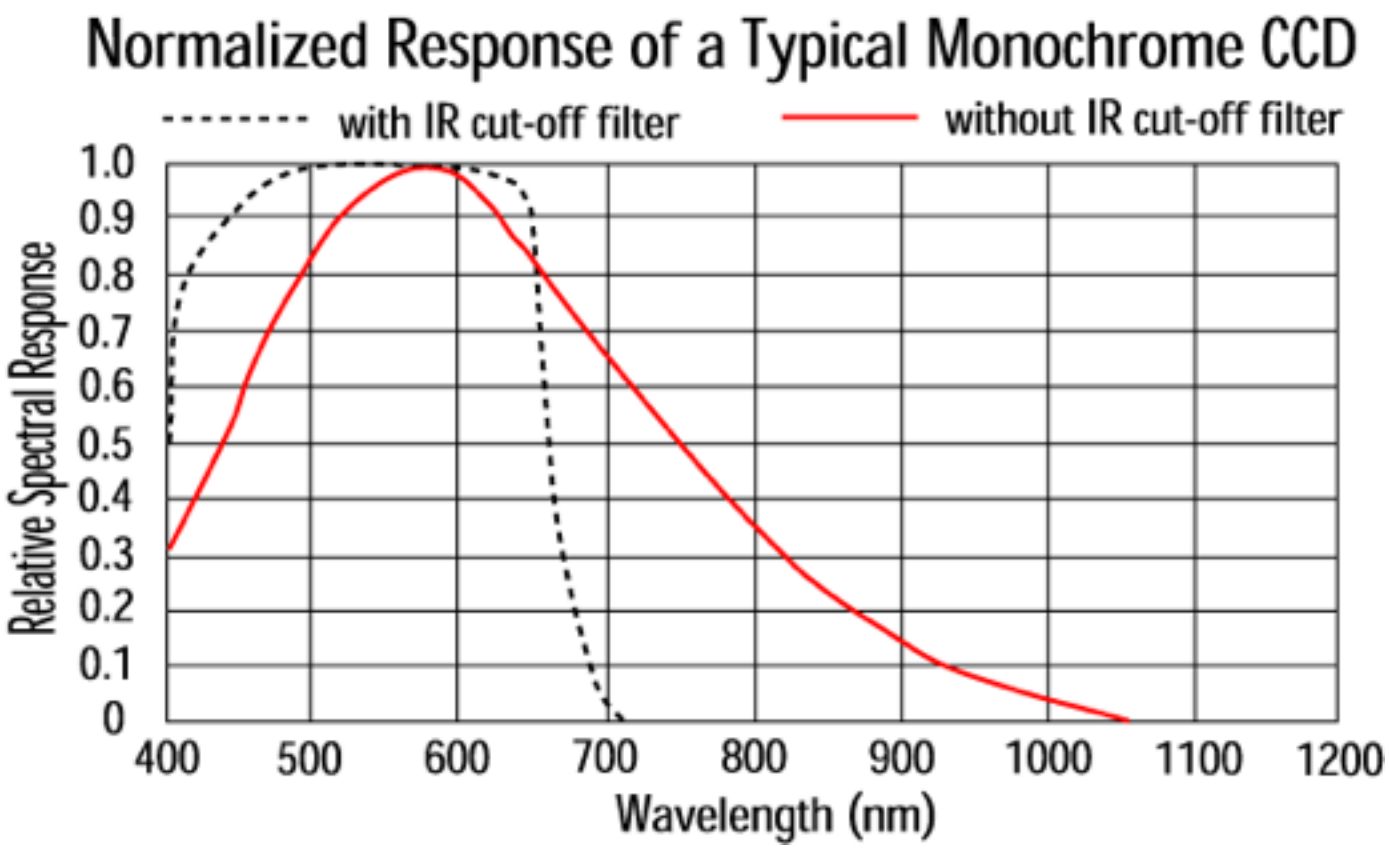
- 3 CCD camera use a beam splitter and 3 separate CCDs
  - higher color fidelity
  - needs lots of light
  - requires careful alignment of ccds





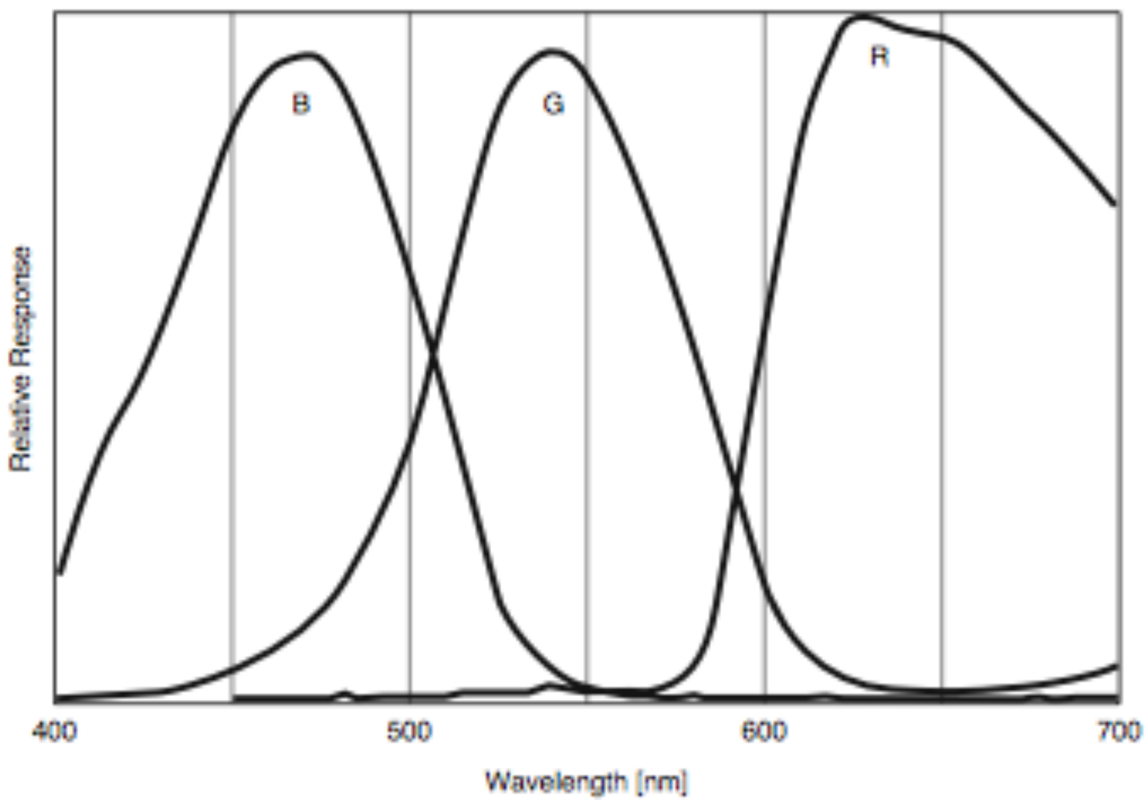
Source: G Hager Slides

Unfiltered CCD Response



Source: G Hager Slides

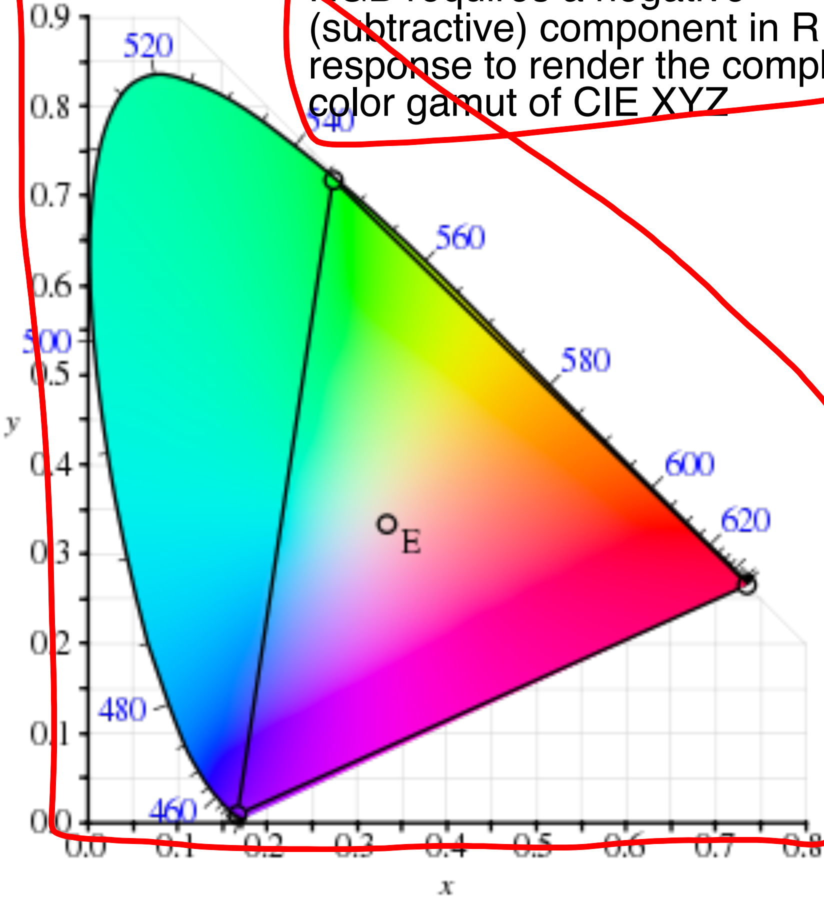
One Chip CCD Response
(Sony DFW V500)

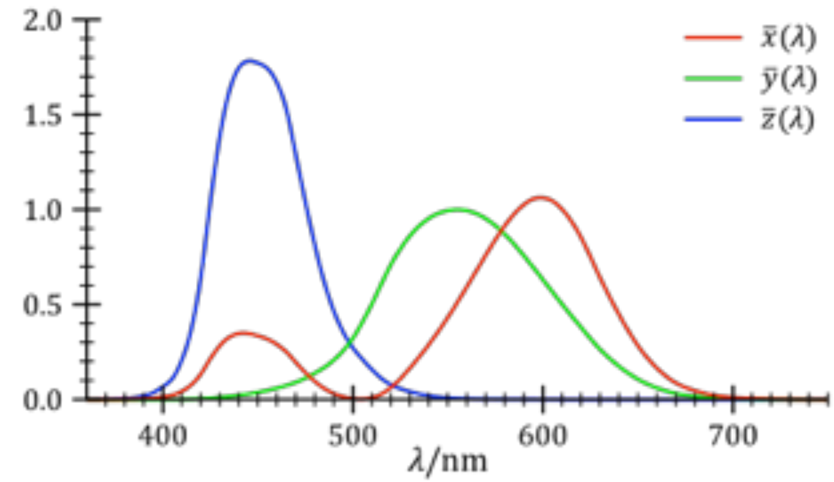


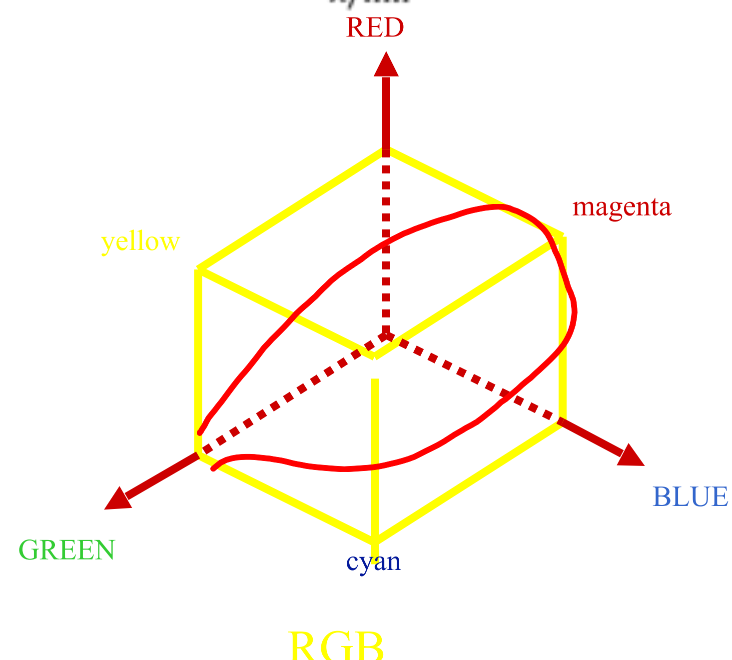
Source: G Hager Slides

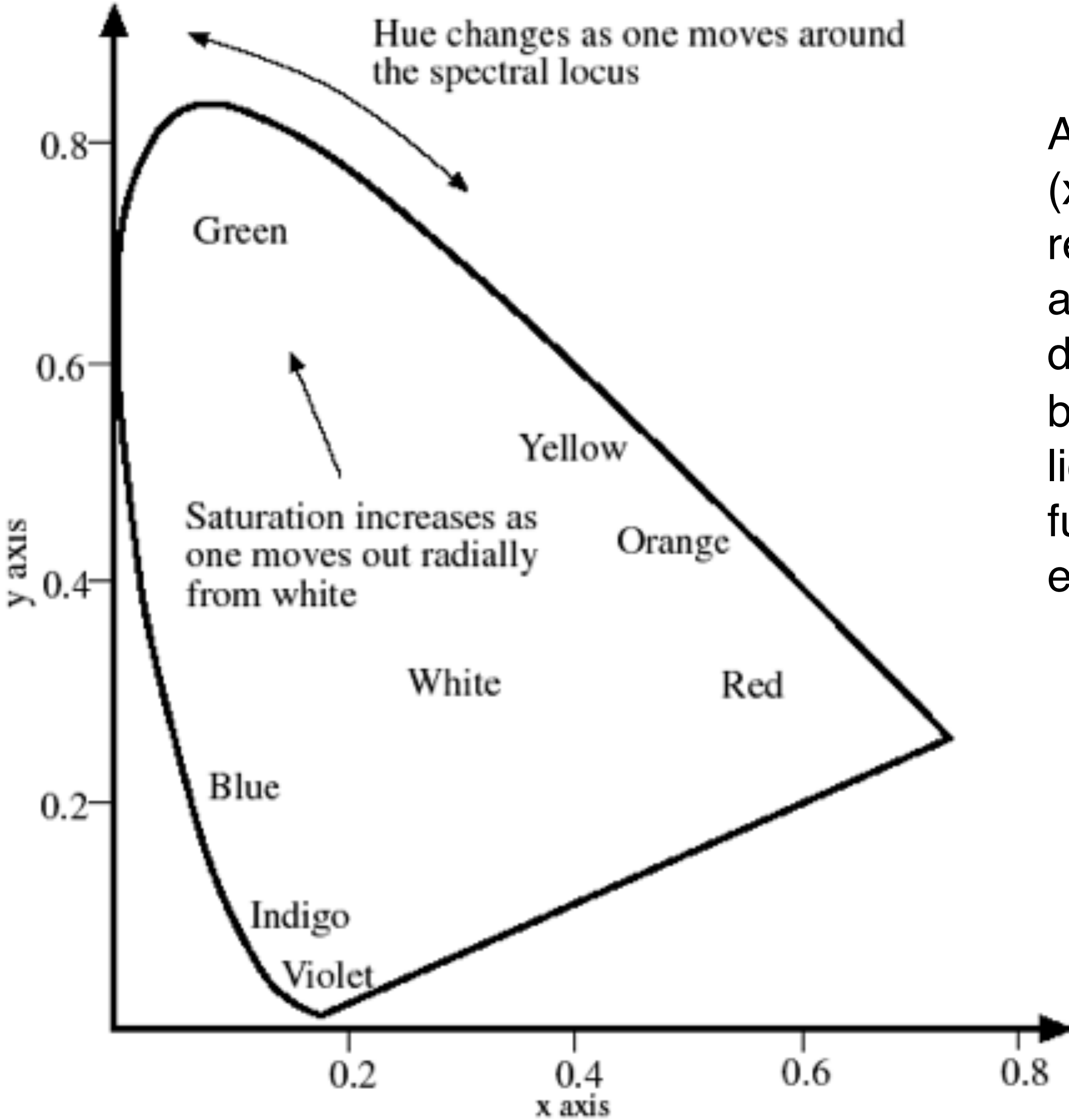
Standard Linear Color Systems

- Several standards are used to define "color" based on specific spectral response functions
  - CIE (Commission International d'Eclairage) establishes standards
  - CIE XYZ is a popular standard with everywhere positive response
  - RGB requires a negative (subtractive) component in R response to render the complete color gamut of CIE XYZ

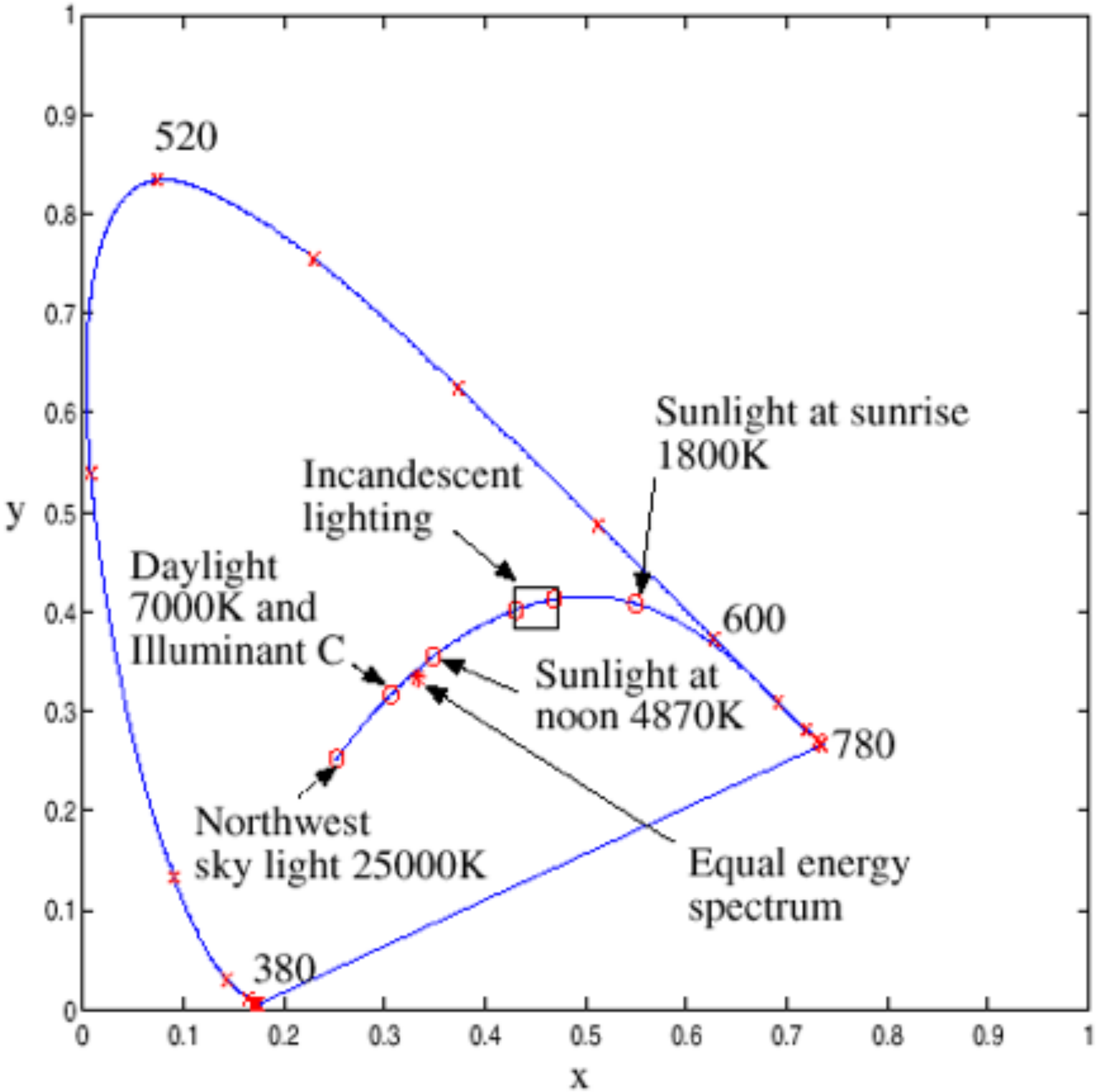








A qualitative rendering of the CIE (x,y) space. The blobby region represents visible colors. There are sets of (x, y) coordinates that don't represent real colors, because the primaries are not real lights (so that the color matching functions could be positive everywhere).



A plot of the CIE (x,y) space. We show the spectral locus (the colors of monochromatic lights) and the black-body locus (the colors of heated black-bodies). I have also plotted the range of typical incandescent lighting.

Why specify color numerically?

- Accurate color reproduction is commercially valuable
  - Many products are identified by color ("golden" arches;
- Few color names are widely recognized by English speakers -
  - About 10; other languages have fewer/more, but not many more.
  - It's common to disagree on appropriate color names.
- Color reproduction problems increased by prevalence of digital imaging - eg. digital libraries of art.
  - How do we ensure that everyone sees the same color?

Another Linear Scheme for Representing Color

- Invented for color television (NTSC)
- Backward compatible with B/W TV
- Y given higher bandwidth than I/Q

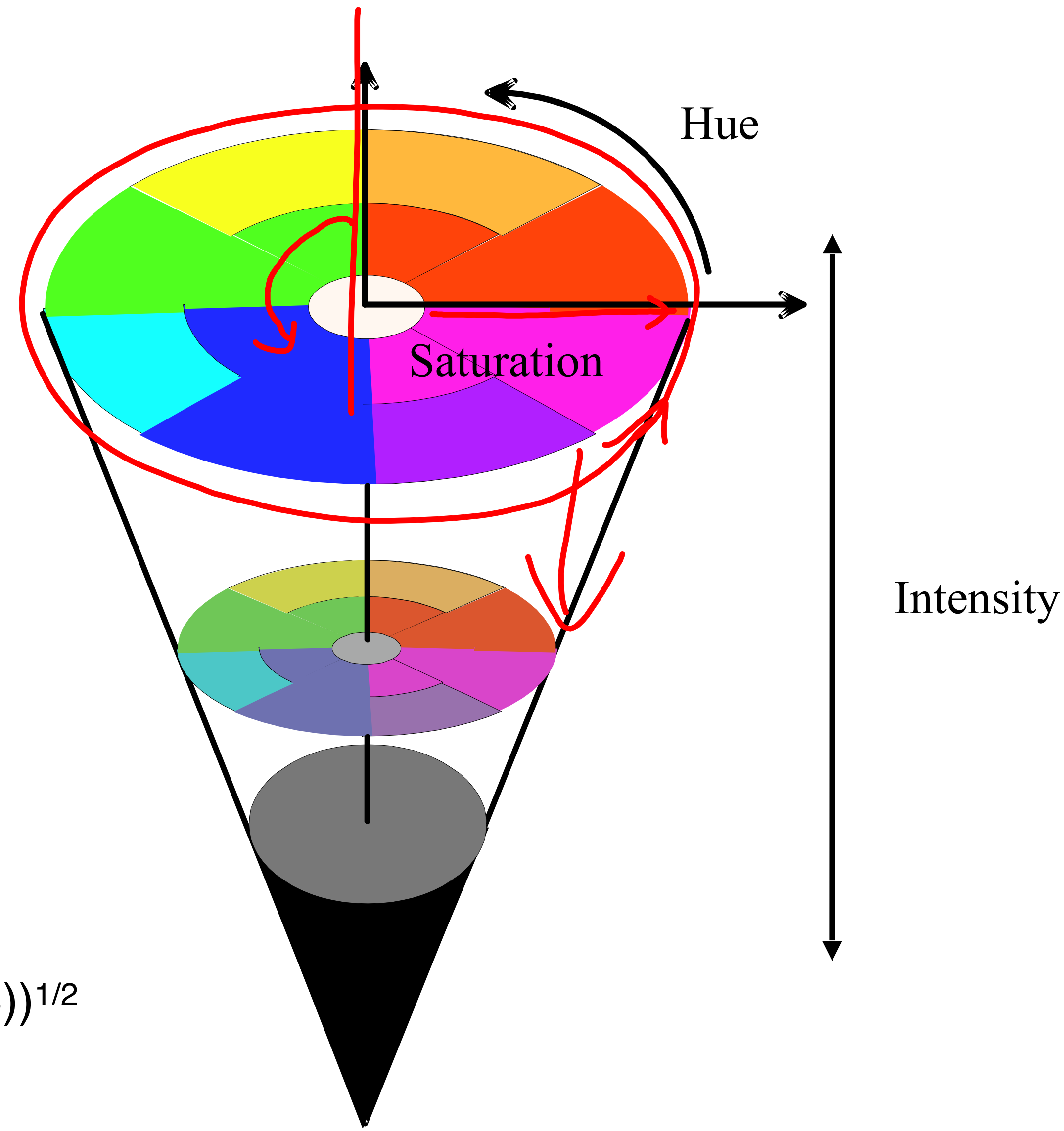
$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} .3 & .59 & .11 \\ .6 & -.28 & -.32 \\ .21 & -.52 & .31 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

YUV is similar to YIQ; PAL vs. NTSC
YCbCr is YUV but with a different reference level for Chrominance

Source: G Hager Slides

Dividing Up Color Space

HSI is a nonlinear representation of color space. Note the non-uniform treatment of color



$I = (R+G+B)/3$ or
$L = .3R + .6G + .1B$

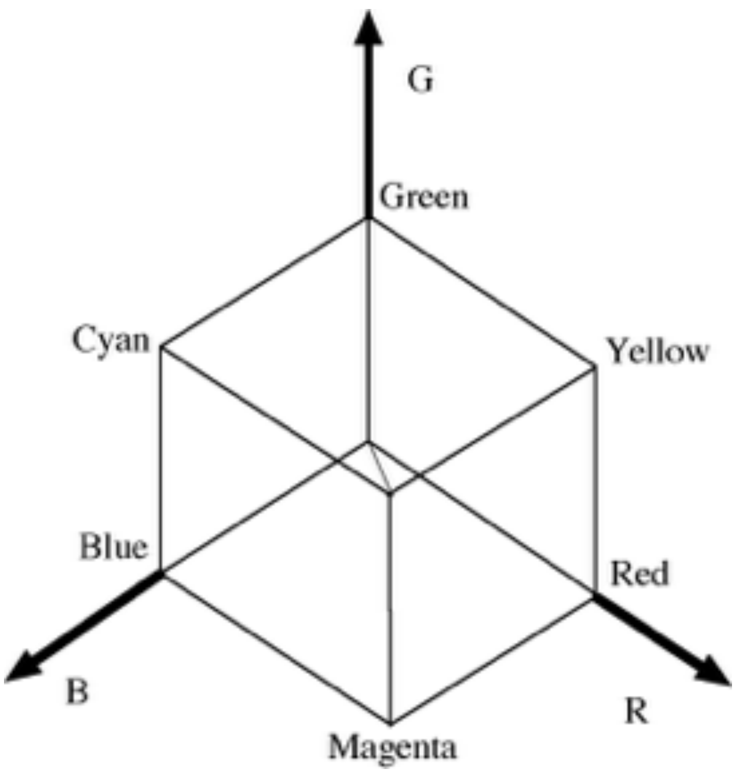
$S = 1 - 3\min(R,G,B)/I$

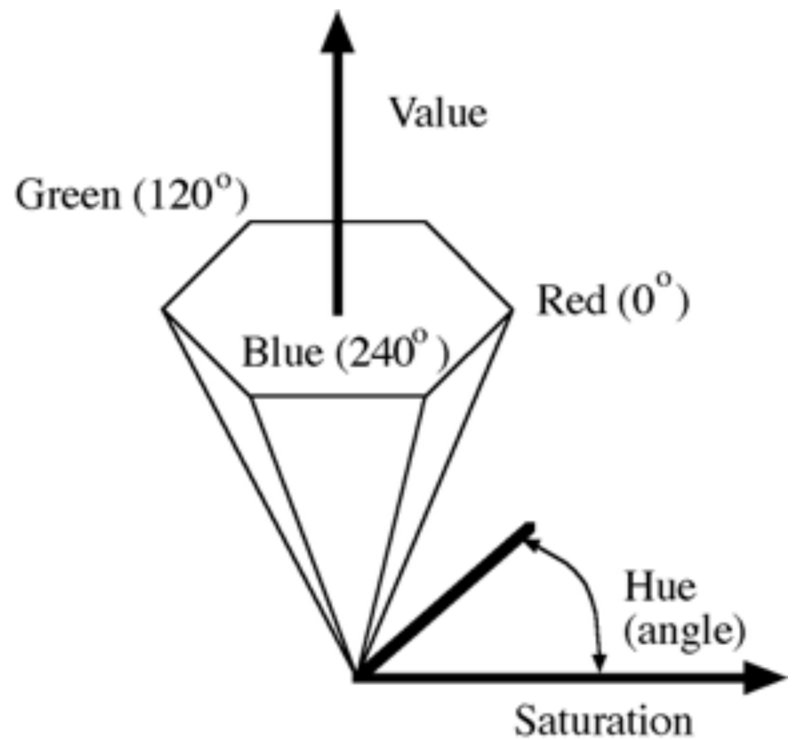
$$H = \begin{cases} \cos^{-1}(x) & \text{if } G > B \\ \pi - \cos^{-1}(x) & \text{if } G < B \end{cases}$$

$x = (R-G) + (R-B)/((R-G)^2 + (R-B)(G-B))^{1/2}$

Source: G Hager Slides

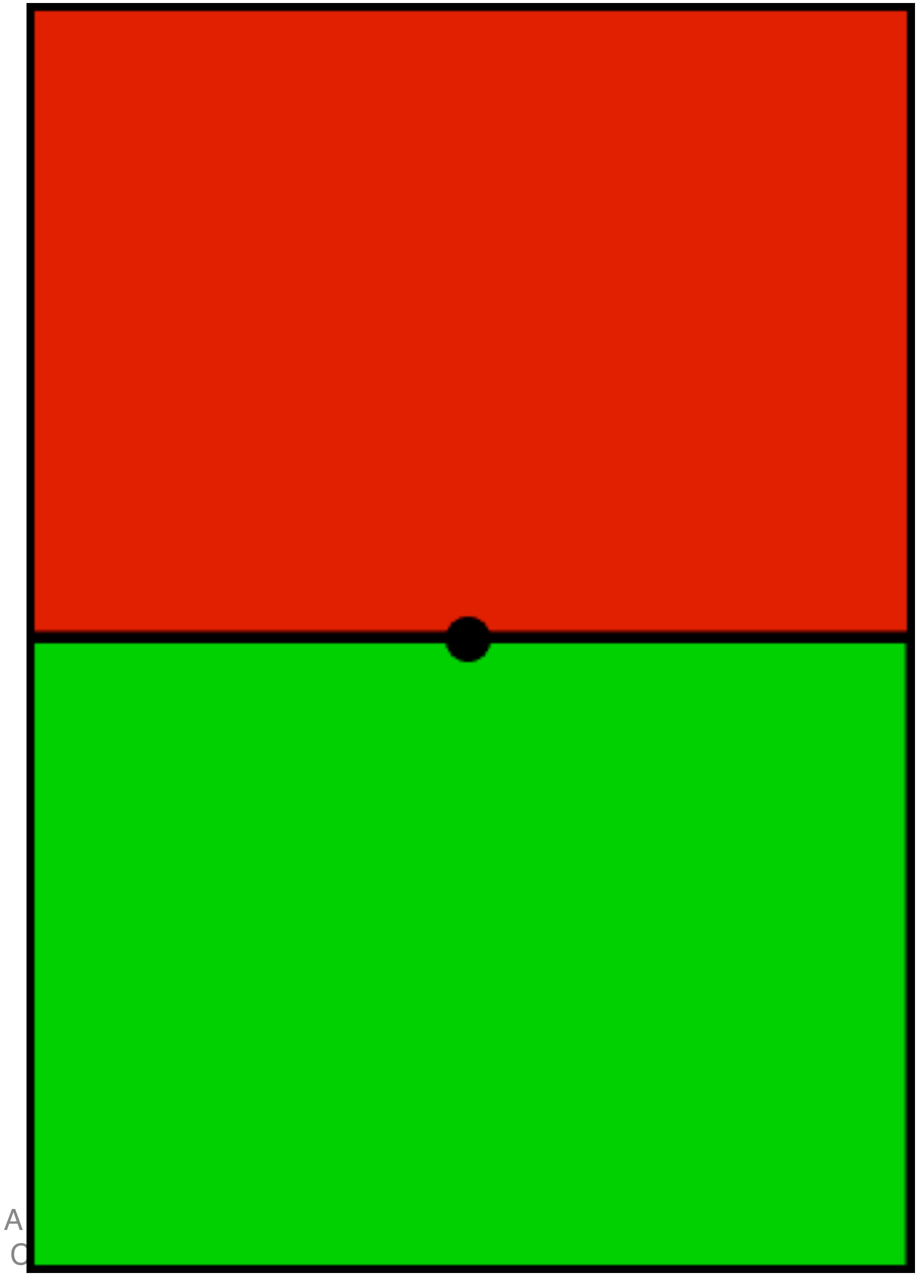
HSV hexcone

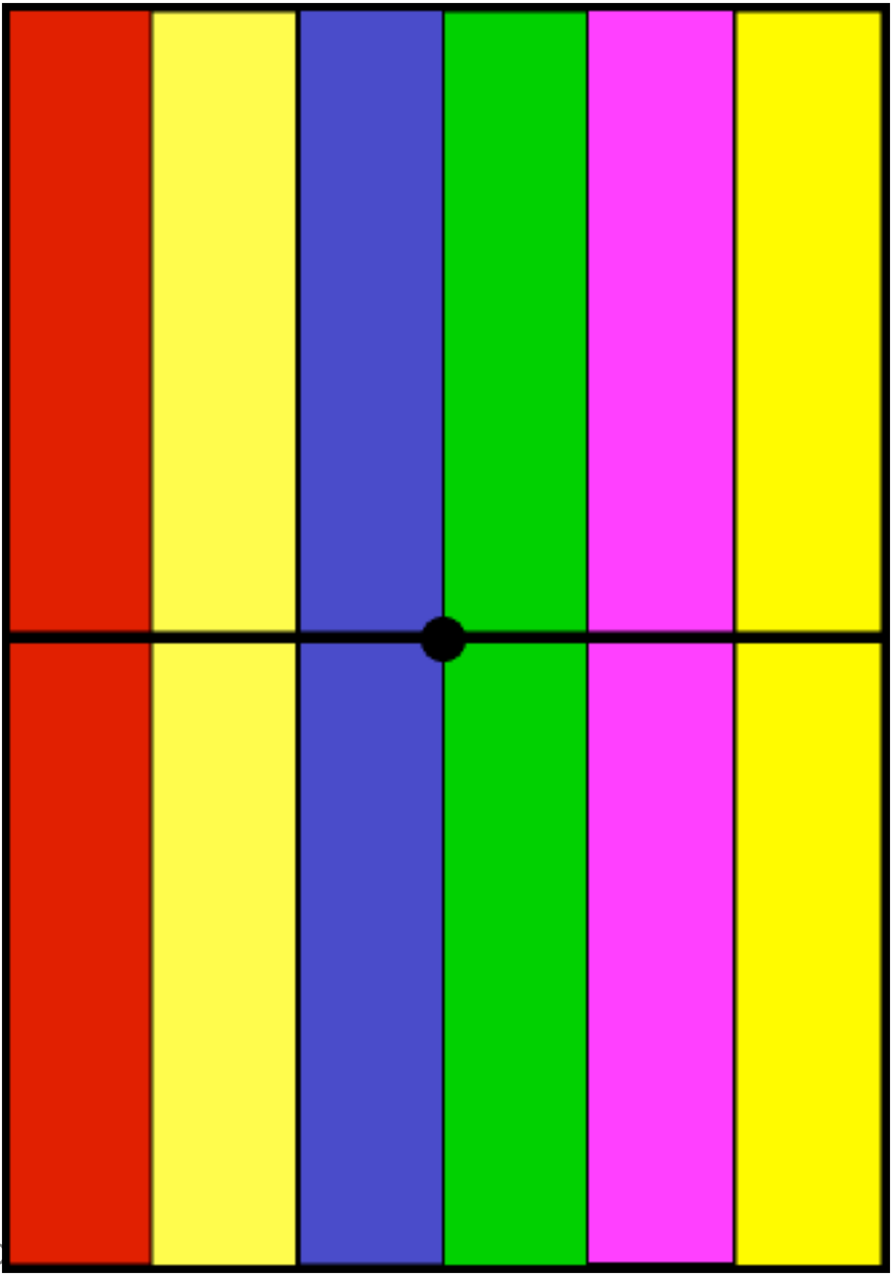

G
Green
Cyan
Yellow
Blue
Red
B
R
Magenta


Value
Green (120°)
Red (0°)
Blue (240°)
Hue
(angle)
Saturation

Color perception...

- It’s not all physics: as the following samples show.





XXXXXX	GREEN	GREEN
XXXXXX	BLUE	BLUE
XXXXXX	YELLOW	YELLOW
XXXXXX	PURPLE	PURPLE
XXXXXX	ORANGE	ORANGE
XXXXXX	RED	RED
XXXXXX	WHITE	WHITE
XXXXXX	PURPLE	PURPLE
XXXXXX	ORANGE	ORANGE
XXXXXX	BLUE	BLUE
XXXXXX	RED	RED
XXXXXX	GREEN	GREEN
XXXXXX	WHITE	WHITE
XXXXXX	YELLOW	YELLOW
XXXXXX	PURPLE	PURPLE
XXXXXX	RED	RED
XXXXXX	GREEN	GREEN
XXXXXX	BLUE	BLUE

Photometric Invariance in Practice

Save inferring the actual scene irradiance from highly radiometrically calibrated situations, what can we do to achieve photomeric invariance in practice?

An Affine Model of Photometric Variation

- Consider an image under an affine intensity transformation
  - An intensity of a spatial range operation at a single pixel
- For grayscale

$$I'(\cdot) = aI(\cdot) + b$$

- For color

$$I'(\cdot) = \mathbf{A}I(\cdot) + \mathbf{b} \quad \mathbf{A} \in \mathbb{R}^{3\times 3} \text{ and } \mathbf{b} \in \mathbb{R}^3$$

- What types of variations in the image do you expect to be accommodated by this model?

Photometric Invariance of DoG

- Is the DoG response (edges and blobls) invariant to affine photometric variation?

  Concretely, for image $\mathbf{I}$ and Gaussian kernels $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ with respective parameters σ_1 and σ_2, consider the two convolved images $\mathbf{G}_1 = \boldsymbol{\kappa}_1 \otimes \mathbf{I}$ and $\mathbf{G}_2 = \boldsymbol{\kappa}_2 \otimes \mathbf{I}$. We know that

$$\mathbf{J} = \mathbf{G}_2 - \mathbf{G}_1 = \boldsymbol{\kappa}_2 \otimes \mathbf{I} - \boldsymbol{\kappa}_1 \otimes \mathbf{I} = (\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes \mathbf{I}$$

  Now, consider an affine photometric operation on $\mathbf{I}$: $\hat{\mathbf{I}} = a\mathbf{I} + b$ for $a, b \in \mathbb{R}$.

For BG. (from 504)

Need to use linearity of convolution to show it is not invariant. Impacted by constant multiplier $a(\boldsymbol{\kappa}_2 \otimes \mathbf{I} - \boldsymbol{\kappa}_1 \otimes \mathbf{I})$. The constant $(\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1)b$ that arrives during derivation goes to 0 since each $\boldsymbol{\kappa}_2$ and $\boldsymbol{\kappa}_1$ integrate to 1 and the convolution is over the constant b. The question asks

$$(\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes \mathbf{I} \stackrel{?}{=} (\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes (a\mathbf{I} + b) \ . \tag{1}$$

Due to linearity of convolution we then rewrite this as

$$(\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes \mathbf{I} \stackrel{?}{=} a(\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes \mathbf{I} + (\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes b \tag{2}$$

where the second term $(\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes b$ is 0 because the convolution kernels each integrate to 1, leaving.

$$(\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes \mathbf{I} \stackrel{?}{=} a(\boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_1) \otimes \mathbf{I} \tag{3}$$

So, the answer is not invariant; scaled.

- Interpret:
  - Edges?
  - Blobs?

Harris features

Harris Features and Photometric Invariance

- Are Harris corners invariant to affine photometric variation?

$$I'(\cdot) = I(\cdot) + b$$

$$I'(\cdot) = aI(\cdot)$$

$$I'(\cdot) = aI(\cdot) + b$$

Harris Features and Photometric Invariance

- Harris corners are partially invariant to affine photometric variation.

Deriving this is part of your homework!

- Yes $$I'(\cdot) = I(\cdot) + b$$

- No $$I'(\cdot) = aI(\cdot)$$

Harris Features and Photometric Invariance

- Harris corners are partially invariant to affine photometric variation.

Deriving this is part of your homework!

- Yes $$I'(\cdot) = I(\cdot) + b$$

- No $$I'(\cdot) = aI(\cdot)$$