EECS 442 F17 Images as Functions : Operations

3 Types

1. Range Operation
2. Range Maps
3. Domain Operations

$$I : \mathbb{R}^2 \to \mathbb{R}$$
$$\mathbb{Z}^2 \to \mathbb{Z}$$

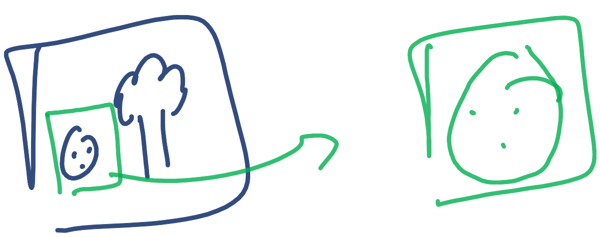
Range Operations

Λ is the lattice of pixel locations

A region $W \subseteq \Lambda \subseteq \mathbb{Z}^2$

↳ windows

$I[W] \to$ a new image where we save only the subset of Λ on W.



Binary W

I on Λ

$$B_W : \Lambda \to \{0,1\}$$

$$I \,.\!*\, B_W \doteq I[W]$$

Def: Range Operation

A function $f : W \times \underline{(\mathbb{Z}^2 \to \mathbb{Z})} \to \mathbb{R}$

in the discrete case.

or $f : W \times I \to \mathbb{R}$

Can include parameters to f as well

Def Linearity: A R.O. f is said to be linear if

$$f_W(\alpha_i I_i + \alpha_j I_j) = \alpha_i f_W(I_i) + \alpha_j f_W(I_j)$$

where $\alpha_i, \alpha_j \in \mathbb{R}$.

Example of Composition

Haar Operator

$$\text{haar}(I, \{W\}_i^N, \{\alpha\}_i^N) = \sum_{i=1}^{N} \alpha_i \, \text{sum}(I, W_i)$$

$\alpha_1, \alpha_2 \cdots \alpha_N \in \{-1, +1\}$

Range Map

Def. A range map operation

$$g : \underline{(W \times I \to \mathbb{R})} \times I \to J$$

for image J.

The range operation "kernel"

A kernel k is a matrix the same size as W whose values are real numbers : $k \in \mathbb{R}^{|W|}$.

Kernel operations are element-wise products over W followed by sums.

$$K \in \mathbb{R}^{n \times n} \qquad I[W] \in \mathbb{R}^{n \times n}$$

$$\mathrm{sum}\,(K \;.\!*\; I(W))$$

Vectorize k and $I[w]$:

$\vec{k} \in \mathbb{R}^{(n\times n)\times 1}$, $\overrightarrow{I[w]}$

$$\boxed{\vec{k}^T \overrightarrow{I[w]}}$$

Kernel operation as dot-product.

Def. Kernel range map is classically called convolution, $\otimes$

$$J(s,t) = k \otimes I[w] = \sum_{k=-m}^{m} \sum_{l=-n}^{n} k(k,l)\, I(s-k,t-l)$$

$$= \vec{k}^T \overrightarrow{I[w]}$$

Write $K \otimes I$ means we apply this as a map.

Ex. KRM : Discrete Image Derivatives

$$I(x,y)$$

def. $$\frac{\partial I(x,y)}{\partial x} = \lim_{h \to 0} \frac{I(x+h,y) - I(x,y)}{h}$$

discrete $$\frac{dI(x,y)}{dx} = \frac{I(x+h,y) - I(x,y)}{h}$$

when $h = 1$

$$\nabla_x I \qquad \frac{dI(x,y)}{dx} = I(x+1,y) - I(x,y)$$

1	2	20
3	3	3
0	0	0

I

1	18	
0	0	

J

∇_x

$$K = [-1,\ 1]$$