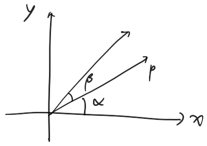
EECS 442 9/7 Lecture 2

Math Background.

Matrices & Eigenvalues/vectors

Rotation matrix (2×2 in plane)

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$




$$p = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \cdot \cos\alpha \\ r \cdot \sin\alpha \end{bmatrix}$$

$$q = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r \cdot \cos(\alpha+\beta) \\ r \cdot \sin(\alpha+\beta) \end{bmatrix}$$

$$q = \begin{bmatrix} r(\cos\alpha\cos\beta - \sin\alpha\sin\beta) \\ r(\sin\alpha\cos\beta + \cos\alpha\sin\beta) \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\beta - y\sin\beta \\ x\sin\beta + y\cos\beta \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

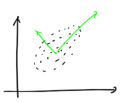
$$x^2 - 2\cos\beta \cdot x + 1 = 0$$

$$\frac{2\cos\beta \pm \sqrt{4\cos^2\beta - 4}}{2} = \cos\beta \pm \sqrt{\cos^2\beta - 1}$$

$$= \cos\beta \pm i|\sin\beta|$$

Eigenvectors

$$Av = \lambda v$$



$$A = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \\ x_n & y_n \end{bmatrix}$$

eigenvector of $A^TA$

Similarity transform

any non singular $S$

$$B = S^{-1}AS$$

Taylor's Expansion

let $f \in C^\infty(a-\varepsilon, a+\varepsilon)$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$= f(a) + f'(a)\cdot(x-a) + \frac{f''(a)}{2!}\cdot(x-a)^2 + \cdots$$

Multivariate

$$f(x,y) = f(a,b) + f_x(a,b)\cdot(x-a) + f_y(a,b)(x-b)$$

$$= f(a,b) + (\nabla f(a,b))^T \begin{bmatrix} x-a \\ x-b \end{bmatrix}$$

Consider a system of $p$ linear equations in $q$ unknowns.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1q}x_q &= y_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2q}x_q &= y_2 \\ &\vdots \\ a_{p1}x_1 + a_{p2}x_2 + \cdots + a_{pq}x_q &= y_p. \end{aligned}$$

$$Ax = y$$

If $p < q$ equations less than unknowns.

$p = q$ Gaussian elimination (unique solution)

$p > q$ equations more than unknowns — least squares, no exact solutions

$$E(x) \doteq \sum_{i=1}^{p} (a_{i1}x_1 + \cdots + a_{iq}x_q - y_i)^2$$

$$= \|Ax - y\|_2^2 \quad \text{(Euclidean norm)}$$

$$x = (A^TA)^{-1}A^Ty$$