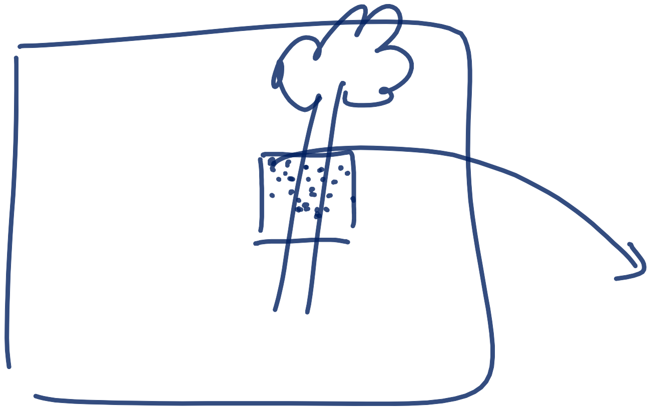
EECS 442 10/10/17

What is the SIFT Histogram?



Detector $\underline{DoG}$

Descriptor HOG

$\frac{dI}{dx}, \frac{dI}{dy} \rightarrow [1 \ -1]$ $[1 \ -1]^T$

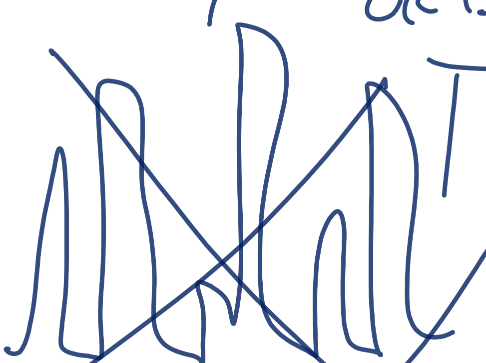
$I(u,v)$ $\left[\frac{dI(u,v)}{dx}, \frac{dI(u,v)}{dy}\right]$ | $\boxed{\theta}$

$\underline{dog}$

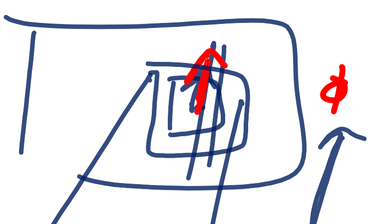
$\{\theta_{u,v} \ \ldots\} \rightarrow$

— BASIC HISTOGRAMMING —

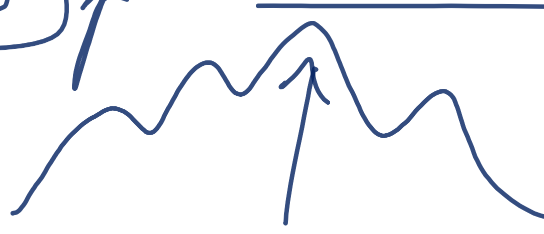
1 ORIENTATION ALIGNMENT



Kernel Density



ϕ ROTATE PATCH BY PHI



$\hat{\phi}$

2

4x4 → separate histogram

16 x 8

Why is the SIFT Histogram?

vectorization

$v_i \in \mathbb{R}^{m \times n}$ window size

Ω is a set of all ($m \times n$) patches of images

enforces types of invariance that make patches more necessarily matched in Ω

BEER FOAM ANALOGY

10×10

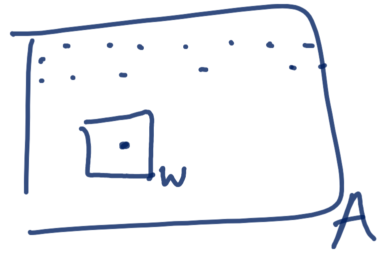
100

2^{100}

2) Within each subwindow of the patch, we have structural invariance given by the Histogram

Images as functions

$$I : \underline{\mathbb{Z}_A^2} \to \underline{\mathbb{R}}$$



$$I(x) \quad x \in \mathbb{Z}_A^2$$

valid pixels

new function on domain W(A)

$$\underline{P_{W(A)}(x)} \quad x \in W(A) \subseteq \mathbb{Z}_A^2$$

for a point x I compute a feature

$$\underline{v} \Leftarrow \underline{f(x)} \quad \text{e.g. HOG}$$

$$\longrightarrow \quad P_{W(A)}(x) \stackrel{?}{=} Q_{W(A)}(x)$$

HOW TO MATCH ON FUNCTIONS?

$$v_1 \stackrel{?}{=} v_2$$

matching on points.

$$\underline{\|v_1 - v_2\|_2^2}$$

MODULE 1 PART 2

Images as Points.

$\underline{\Omega}$ is the <u>set</u> of all images of a certain size

Cartesian Basis

Consider an $m \times n$ image

↳ nothing more than a matrix of values

$$\begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \vdots & \ddots & & \\ & & & \alpha_{mn} \end{bmatrix}$$

$\mathbb{R}^{(m\times n)\times 1}$

α is an $m \times n$ length vector of coefficients

Call each α_i a coefficient.

Consider a set of images $\{V_1, V_2 \ldots, V_{mn}\}$

Call these "Cartesian Images"

→ they have zeros everywhere except for 1 location

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

V_i has a 1 at the ith pixel location

Given these definitions

$$\{\alpha_1 \ldots \alpha_{mn}\} \doteq \vec{\alpha}$$

$$\{V_1 \ldots V_{mn}\}$$

we can rewrite the image I as

$$I = \sum_{i=1}^{mn} V_i \, \alpha_i \qquad (*)$$

Example: Rotate the bases $\{V_i\}$

$$I_{2\times 2} \doteq V_1\alpha_1 + V_2\alpha_2 + V_3\alpha_3 + V_4\alpha_4 \qquad (*)$$

$$\begin{bmatrix} 0 & 128 \\ 255 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} 0 + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} 128 + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} 255 + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} 0$$

Operator Q_R

<u>90°CCW</u> Rotates the domain of images

$$Q_R \circ I = Q_R \circ V_1\alpha_1 + Q_R \circ V_2\alpha_2 + Q_R \circ V_3\alpha_3 + Q_R \circ V_4\alpha_4$$

$$\begin{bmatrix} 128 & 0 \\ 0 & 255 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} 0 + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} 128 + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} 255 + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} 0$$

NOTE: the bases are rotated.
the data, the coefficients are the <u>same</u>

→ For a linear operator Q, we can change the basis $\{V\}$ and maintain the coefficients

$$I = \sum_{i=1}^{mn} V_i \alpha_i$$

$$Q \circ I = Q \circ \sum_{i=1}^{mn} V_i \alpha_i$$

$$Q \circ I = \sum_{i=1}^{mn} Q \circ V_i \alpha_i$$

linearity of a

Same.

Definition 2

The set of images, e.g. $\vec{\alpha} = \{\alpha_1, \ldots, \alpha_{mn}\} \in \Omega$, vector addition and scalar multiplication, and the field of reals $\mathcal{F} = \underline{\mathbb{R}}$, form a **vector space** $V_{\vec{\alpha}}$

1 Closed under vector addition and scalar multiplication $\vec{\alpha}, \vec{\beta} \in \Omega$, $a \in \mathbb{R}$

$\mathbb{Z}^2 \to \underline{\mathbb{R}}$

$\longrightarrow a\vec{\alpha} \in \Omega$

$\longrightarrow \vec{\alpha} + \vec{\beta} \in \Omega$

2 Addition commutes

$$\vec{\alpha} + \vec{\beta} = \vec{\beta} + \vec{\alpha}$$

3 Scalar addition ~~and~~ multiplication are associative $\vec{\alpha}, \vec{\beta}, \vec{\gamma} \in \Omega$

$$(\vec{\alpha}+\vec{\beta})+\vec{\gamma} = \vec{\alpha} + (\vec{\beta}+\vec{\gamma})$$

$$a(b\vec{\alpha}) = (ab)\vec{\alpha} \qquad a, b \in \mathbb{R}$$

4 Identities exist

$$z \in \Omega \quad \text{s.t.} \quad \vec{\alpha} + z = \vec{\alpha}$$

$$0 \in \mathbb{R} \quad \text{s.t.} \quad \vec{\alpha} \cdot 0 = \vec{\alpha}$$

5 Inverse exists for addition

$$\forall \alpha \in \Omega \quad \exists \beta \quad \text{s.t.} \quad \vec{\alpha} + \vec{\beta} = \vec{z}$$

image of zeros

6 scalar multiplication is distributive

$$a(\vec{\alpha}+\vec{\beta}) = a\vec{\alpha} + a\vec{\beta}$$

$$(a+b)\vec{\alpha} = a\vec{\alpha} + b\vec{\alpha}$$

define Euclidean Distance between images.

$$I, J \rightarrow \vec{\alpha}, \vec{\beta} \qquad \vec{\alpha} \in \mathbb{R}^{mn}$$

$$\|\vec{\alpha} - \vec{\beta}\|_2^2$$