Exploring Proposer-Acceptor Disparity in Dynamic Matching

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Abstract

We investigate utility disparity between proposing and accepting sides in stable matching problems, with a focus on dynamic matching markets where agents arrive over time. Using discrete timesteps and the deferred acceptance algorithm, we simulate locally stable matches and observe that utility disparity worsens in dynamic ones. We also conduct experiments to confirm this trend and propose a cutoff utility–based heuristic to mitigate the disparity.

Introduction

This report discusses our experiments with modelling disparity in utilities of proposing and accepting sides in a stable matching problem. Larger disparity implies unfair treatment to one side compared to the other. Specifically, we are interested in *dynamic* matching markets — ones where different agents arrive at different times. If we formulate agent arrivals at discrete timesteps, we can apply the Deferred Acceptance algorithm (DAA) [Gale and Shapley, 1962] to create *locally stable matches*. DAA in a non-dynamic market leads to disparity in utilities of proposers and acceptors [Roth, 2007]. By setting up a small-scale experiment and following simplified assumptions, we conclude that this disparity increases in the dynamic market setting. We further describe a heuristic based on cutoff utility that aims to reduce this disparity.

Related Work

Although the specific problem we have explored has not been studied in depth, other notions of fairness and efficiency in the dynamic market setting have been introduced. For example, Doval [2022] introduced *dynamic stability* and showed that it always exists and is a necessary condition to ensure agents do not strategically delay the time at which they enter the market. Similarly, Ashlagi et al. [2022] studied the dynamics of a kidney exchange platform and showed that in sufficiently large markets, matching upon the first arrival (locally greedy matching) leads to shorter waiting times and more total matches than waiting longer to enhance the likelihood of a better match.

Problem Formulation

We formulate a dynamic market as a market where proposing or accepting agents may arrive at discrete timesteps. If the market has a finite horizon T, then the timesteps can be $t \in \{0, 1, 2, \ldots, T\}$. We use the Poisson distribution [Haight, 1967] to capture the market dynamics, which includes market *thickness*, or the distribution of available agents over time. Although the total number of agents on the proposing and accepting sides are equal (say, n), their arrival dynamics can wildly differ from each other. The rate parameter λ of the Poisson distribution helps us vary the arrival dynamics neatly and study a suite of cases where markets either start thin and gradually thicken or vice-versa. Appendix A describes how agent arrivals change as the rate parameter λ of the Poisson distribution is varied.

Further, unlike the classical DAA, where proposing and accepting agents have ranked preferences over each other, we define an agent i's utility over j as u_{ij}, which captures the measure of happiness agent i will get if they are matched to agent j. In this simple problem setup, we assume that $u_{ij} \in [0, 1]$.

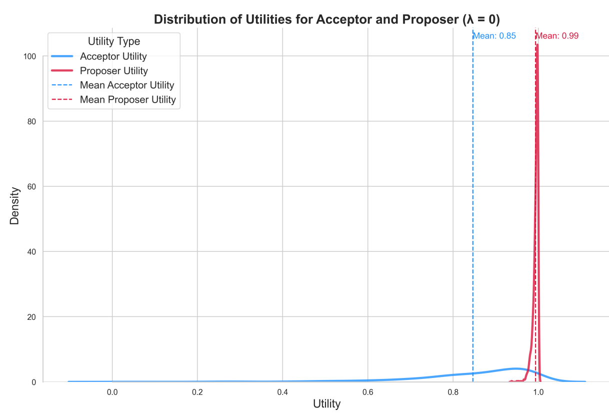
Finally, we further limit the scope of the problem to *locally stable matches*. In this setting,

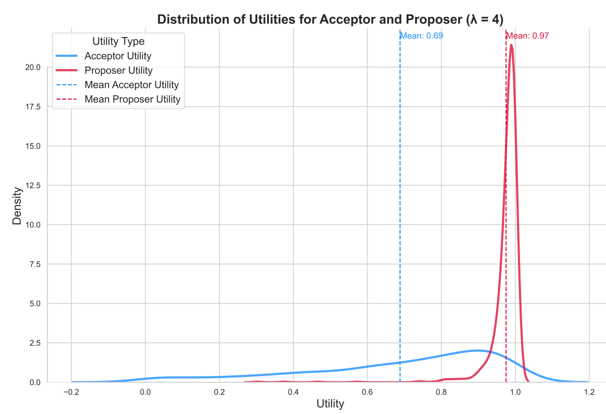
1. the DAA must run at every timestep t and *after* all the agents that were supposed to arrive at t have arrived. Therefore, at the end of every timestep, the DAA must create as many stable matches as possible.

2. at the end of each timestep, matched agents leave the market. In this sense, the deferred acceptance by the accepting agents is *local* — they cannot defer their acceptance over multiple timesteps.

To measure the disparity between the proposing and accepting agents, we calculate the difference in the mean utility of proposing and accepting agents as a function of arrival dynamics (the rate parameter λ of the Poisson distribution).

Experimental Setup and Results





We started by building our baseline, which replicated the problem formulation described above. For our experiments, we randomly drew utilities u_{ij} from $U[0, 1]$. We show our results for $n = 1000$ and $T = 20$, where n and T are the total number of proposing (or accepting) agents and the market horizon, respectively. $\lambda = 0$ is the classical DAA setting where all the agents are available at $t = 0$. Figures above show the distribution of utilities by the proposing and accepting agents for $\lambda = 0$ and $\lambda = 4$. It is no surprise that in the default DAA setting, the mean acceptor utility is lower than the proposer utility. As λ increases, the mean utilities of both the proposers and acceptors decrease, and their differences widen.

Acceptors are doubly disadvantaged — not only are they constrained to only accept/reject the proposal and strive to *bubble-up* their utility at a timestep t, but also, because of the local nature of deferred acceptance, they can easily get unlucky if they arrive in the market at a timestep t when they have low utilities over available proposers. From these plots (and additional plots in Appendix B), it is evident that the disparity measured as the difference between the mean utility of proposing and accepting sides increases as λ increases.

This led us to introduce a heuristic where the accepting agents have a cut-off utility. At each timestep t, agents with cut-off utility can call some matches unacceptable if their utility over their potential match is lower than their current cut-off utility. To ensure the heuristic finishes at time T with no unmatched agents, we drew the agent's cut-off utilities iid from $U[0, 1]$ and, with the passing of each timestep, reduced it linearly until all the agents had a cut-off of zero at T.

Introducing cut-off utility to acceptors only

We first introduced the cut-off utility to the accepting side as they were doubly disadvantaged in the baseline. It is worth noting that $\lambda = 0$ in this setting does not correspond to the classical DAA setting. Rather, a high value of λ (one that ensures all the agents arrive late at timestep T) corresponds to the classical stable matching problem. This one-sided cut-off put the accepting at an unfair advantage over reasonable values of λ, as their mean utilities remained higher than those of proposers until λ increased to a point where the market thickened only at the last few timesteps.

Cut-off utility to both acceptors and proposers

As a final experiment, we introduced cut-off utilities to both the proposers and acceptors. In this setting, the acceptors reject a proposal if their utility over the proposer is less than their cut-off utility. Similarly, the proposers do not propose to someone with a lower utility than their current cut-off utility. Just like before, both the acceptor and proposer cut-off utilities linearly decrease with time until they become zero at T. The plots below show that doing so diminishes the disparity in mean utilities. From these experiments, we conclude that the introduction of cut-off utilities that anneal with time leads to a fair group treatment in the dynamic matching problem while also ensuring that all agents eventually match in a given time horizon. We further conclude that a thick market increases the mean utility of both the proposing and accepting sides. This necessitates further exploration of mechanisms that ensure agent availability over time. To further illustrate our point, we include additional plots in Appendix C and Appendix D.

Future Work

In the future, we would like to relax the notion of locally stable matches to allow agents to remain in the market for some time, even after finding a match, and to re-enter the market. Our utilities do not account for waiting times by agents before they are matched. One way to improve upon this is to devise time-discounted agent utilities. In our experiments, we linearly annealed the cut-off utility to zero. A more realistic treatment of how agents adjust their cut-off utility would include factors like their utilities over their recent *interactions* with agents on the other side.

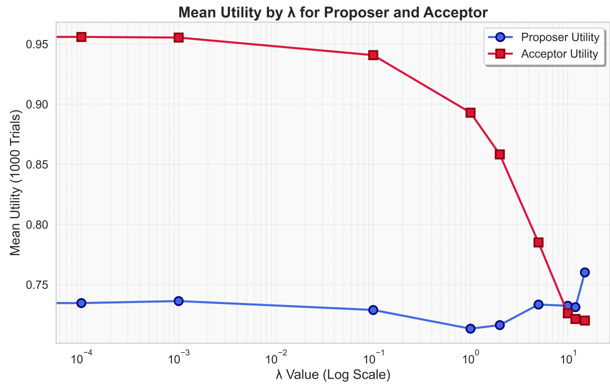


Figure 1: Acceptor-only cut-off

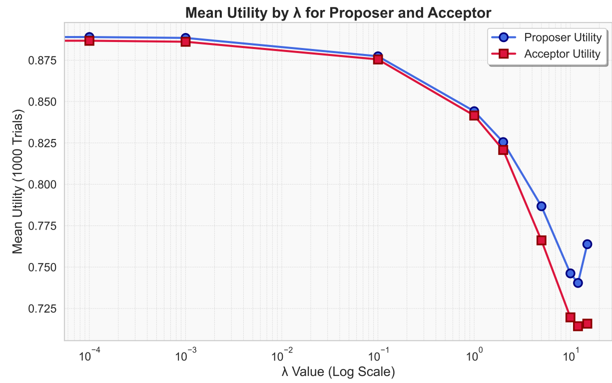


Figure 2: Acceptors and proposers cut-off

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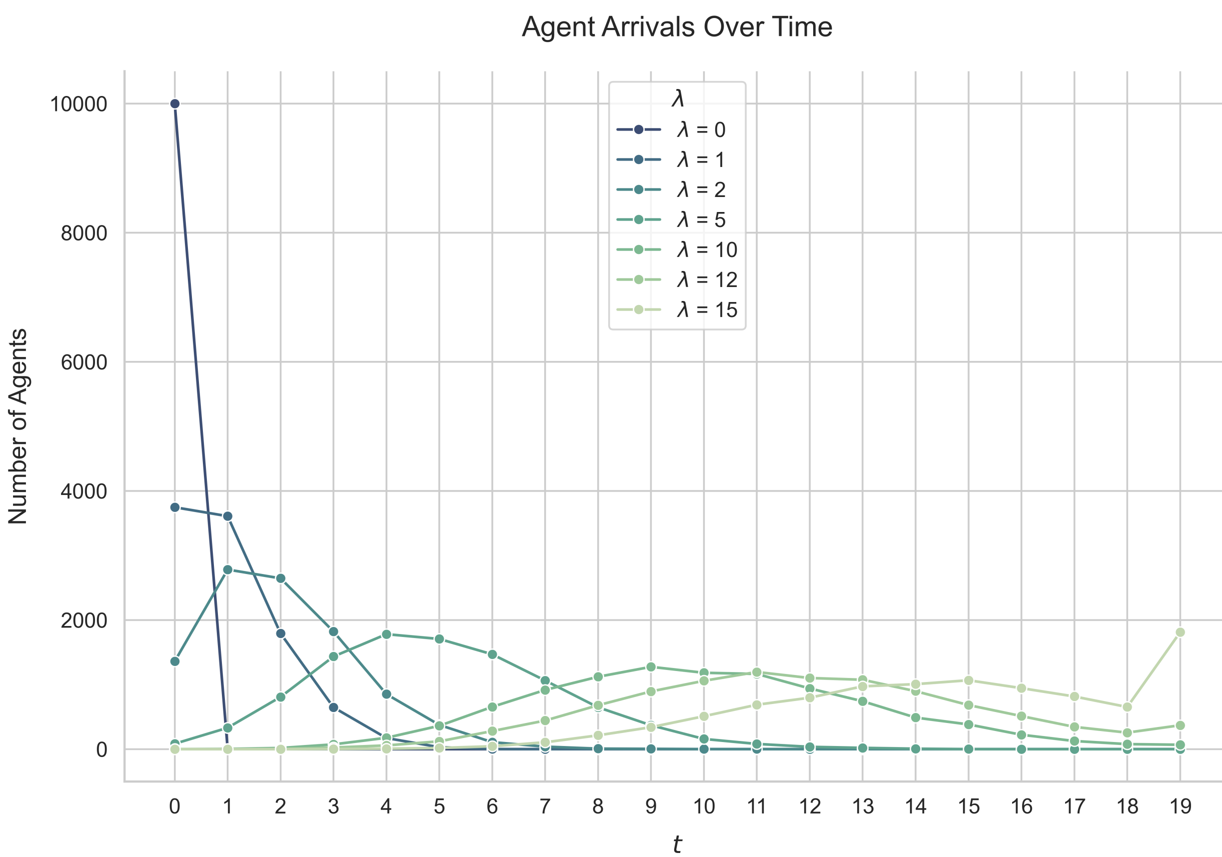
A Poisson distribution to capture market dynamics

This plot displays the dynamic pattern of agent arrivals over time, characterized by discrete time steps t on the horizontal axis and the number of agents on the vertical axis. Each line represents a different rate parameter λ of the Poisson distribution, which governs the stochastic process of agent arrivals.

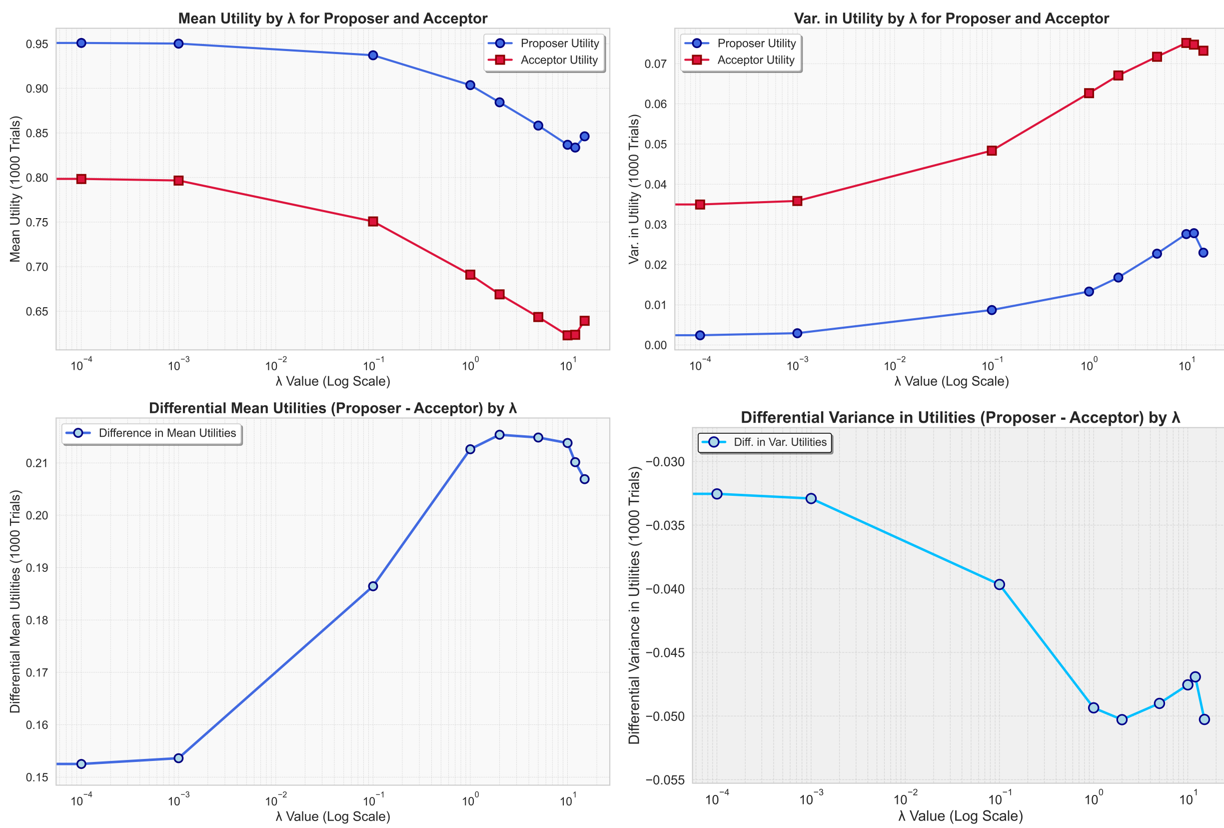
The line corresponding to $\lambda = 0$ shows a steep initial spike where all agents arrive at the very beginning of the time frame. As λ increases, the arrival curves flatten and spread out, resulting in a more distributed entry of agents across time steps. This is most apparent for $\lambda = 15$, where arrivals are more evenly spread, although still displaying some variability as indicated by the smaller peaks and troughs.

For low λ values, such as 1 and 2, there are still relatively concentrated arrivals near the start, but as λ grows, the distribution becomes more uniform. At the highest λ values, we observe a gentle, undulating pattern without the sharp decline seen in the low-λ cases.

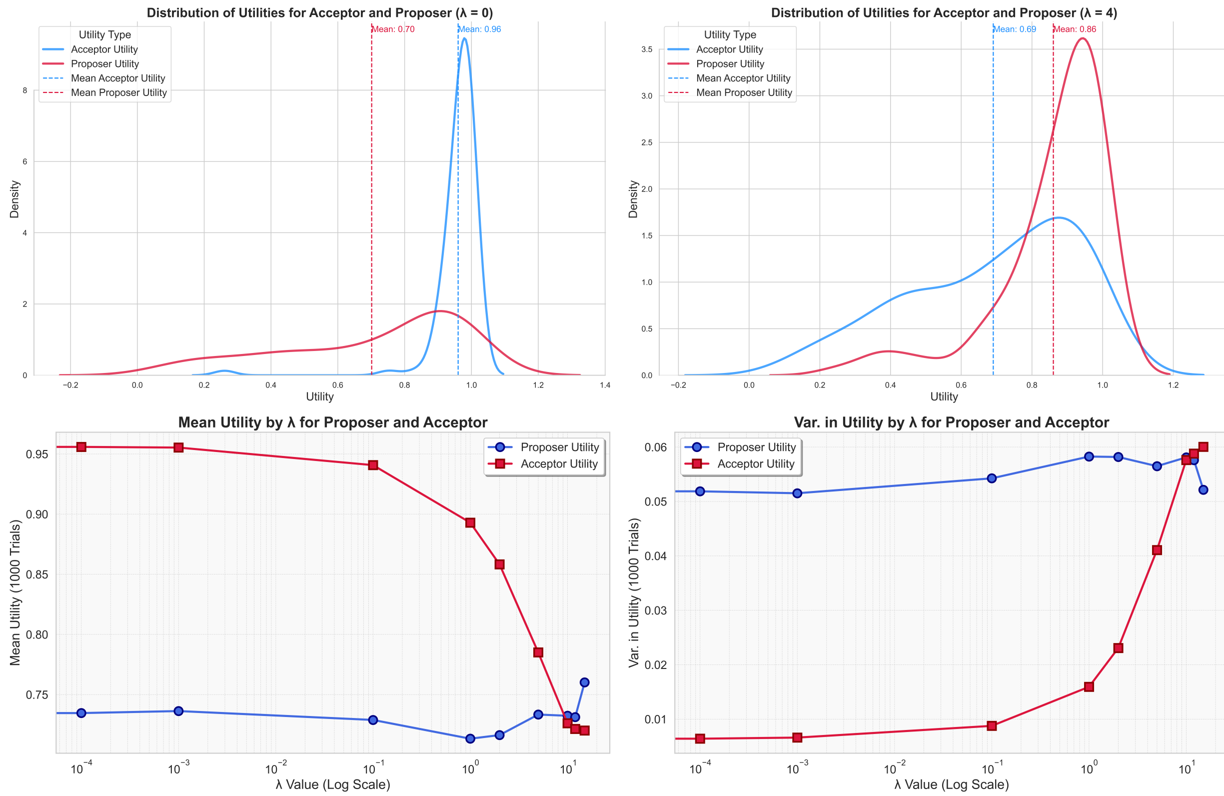
Thus, the rate parameter λ captures the temporal dynamics of agent arrival in a dynamic market.



B Results from experiments with the baseline



C Results from experiments with the cut-off heuristic (acceptor-only)



D Results from experiments with the cut-off heuristic (both acceptors and proposers)